Localisation in the EnKF

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Outline

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Non-adaptive localisation
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Conclusions
Reasoning

Two common reasons:

- “spurious covariances”
  \[ \sigma \sim \frac{1}{m^{1/2}} \]
- ensemble rank versus model subspace dimension

\[
m_{\text{eff}} \approx m \frac{\Delta x_1}{\rho_1} \frac{\Delta x_2}{\rho_2} \ldots \frac{\Delta x_N}{\rho_N}
\]

Problems

- Q: what is the “price” paid for localisation?
  - dynamical ballance
  - optimality

- Note two different problems:
  1. Mean update ↔ local curve fitting
  2. Covariance update
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- Covariance localisation: Hamill and Whitaker (2001); Houtekamer and Mitchell (2001)
  - Local analysis: Evensen (2003); Anderson (2003); Ott et al. (2004)
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Some conventions

Indices

\((X)_i\) — \(i\)th column of \(X\)
\((X)_{i,:}\) — \(i\)th row of \(X\)
\((X)_{ij}\) — the element in \(i\)th row and \(j\)th column of \(X\)
\((x)_i\) — \(i\)th element of \(x\)

Matrix operations

\(A \circ B\) — Schur (Hadamard, element-wise) product of \(A\) and \(B\)

Localisation specific

\(\hat{A}\) — ensemble tapered around \(i\)th element
\(\hat{f}\) — taper function for the \(i\)th element
\(\hat{F}\) — taper matrix, \(\hat{F} = [\hat{f}, \hat{f} \ldots \hat{f}]\), \(\hat{A} = A \circ \hat{F}\)
\(\{\circ\}\) — \(H\) with all rows but \(o\)th zeroed

Convenience

\(\hat{A}\) — scaled ensemble anomalies: \(\hat{A} = \frac{1}{(m-1)^{1/2}} A\)
Non-adaptive localisation: the two methods

1. Covariance localisation (covariance filtering)
   Houtekamer and Mitchell (2001); Whitaker and Hamill (2002)

\[
P \rightarrow \rho \circ P
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Non-adaptive localisation: the two methods

1. Covariance localisation (covariance filtering)
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\[ \mathbf{P} \rightarrow \rho \circ \mathbf{P} \]

2. Local analysis
   (Evensen, 2003; Anderson, 2003; Ott et al., 2004; Hunt et al., 2007)

\[ i : \mathbf{A} \rightarrow \hat{\mathbf{A}} \]
Covariance localisation: the technique

\[ \mathbf{P} \rightarrow \rho \circ \mathbf{P} \]

\[ \mathbf{K} = (\rho \circ \mathbf{P}^f)(\mathbf{H})^T \left[ \mathbf{H}(\rho \circ \mathbf{P}^f)(\mathbf{H})^T + \mathbf{R} \right]^{-1} \]

\[ (\rho \circ \mathbf{P}^f)(\mathbf{H})^T : \]

\[ (\rho \circ \mathbf{P}^f)(\mathbf{H})^T \approx \sum_{o=1}^{p} \mathbf{A}^f \mathbf{H}^T \{o\} \mathbf{A}^f \]

\[ \mathbf{H}(\rho \circ \mathbf{P}^f)(\mathbf{H})^T : \]

\[ \mathbf{H}(\rho \circ \mathbf{P}^f)(\mathbf{H})^T \approx (\mathbf{H} \rho \mathbf{H}^T) \circ \left[ (\mathbf{H} \mathbf{A}^f)(\mathbf{H} \mathbf{A}^f)^T \right] \]

CF and particular schemes:

- **EnKF**: \( \mathbf{A}^a = \mathbf{A}^f + \mathbf{K}(\mathbf{D} - \mathbf{H} \mathbf{A}^f) \) - OK
- **ESRF**: \( \mathbf{A}^a = (\mathbf{I} - \mathbf{K} \mathbf{H})^{1/2} \mathbf{A}^f \) - OK
- **ESRF**: \( \mathbf{A}^a = (\mathbf{I} + \mathbf{P}^T \mathbf{R}^{-1} \mathbf{H})^{-1/2} \mathbf{A}^f \) - OK
- **DEnKF**: \( \mathbf{A}^a = (\mathbf{I} - \frac{1}{2} \mathbf{K} \mathbf{H}) \mathbf{A}^f \) - OK
- **ETKF**: \( \mathbf{A}^a = \mathbf{A}^f \left[ \mathbf{I} + (\mathbf{HA}^f)^T \mathbf{R}^{-1} (\mathbf{HA}) \right]^{-1/2} \) - NOT POSSIBLE
Covariance localisation: the technique

\[ K = (\rho \circ P^f)(H)^T \left[ H(\rho \circ P^f)(H)^T + R \right]^{-1} \]

\[(\rho \circ P^f)(H)^T : \]

\[ (\rho \circ P^f)(H)^T \approx \sum_{i=1}^{p} \tilde{A}^f \{o\} \tilde{A}^f \]

\[ H(\rho \circ P^f)(H)^T : \]

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CF and particular schemes:

\begin{align*}
\text{EnKF:} & \quad A^a = A^f + K(D - HA^f) \quad \text{OK} \\
\text{ESRF:} & \quad A^a = (I - KH)^{1/2} A^f \quad \text{OK} \\
\text{ESRF:} & \quad A^a = (I + PH^T P^{-1} H)^{-1/2} A^f \quad \text{OK}
\end{align*}
Covariance localisation: the technique

\[ P \rightarrow \rho \circ P \]

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\[(\rho \circ P^f)(H)^T : \]

\[ \approx \sum_{o=1}^{p} A^f_i (H A^f)^T \]

\[ H(\rho \circ P^f)(H)^T : \]

\[ \approx (H \rho H^T) \circ \left[ (H A^f)(H A^f)^T \right] \]

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- ETKF: \( \mathbf{A}_a = \mathbf{A}_f \hat{\mathbf{I}} + (\mathbf{H} \mathbf{A}_f)^T \mathbf{R} - \mathbf{1}/2 - \mathbf{NOT POSSIBLE} \)
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  \[ A^a = A^f \left[ I + (HA^f)^T R^{-1} (HA) \right]^{-1/2} \]
  - NOT POSSIBLE
Possible inconsistency

1D case, $n = 100$, $r_{loc} = 20$, Gaspari and Cohn (1999) taper function

Step taper function
Possible inconsistency

1D case, \( n = 100, \ r_{loc} = 20 \), Gaspari and Cohn (1999) taper function

\[ \rho \text{ (non-periodic)} \quad \text{eig}(\rho) \quad \rho \text{ (periodic)} \quad \text{eig}(\rho) \]

2D case, \( n = 20 \times 20, \ r_{loc} = 5 \), Gaspari & Cohn taper function

\[ \rho \text{ (non-periodic)} \quad \text{eig}(\rho) \quad \rho \text{ (periodic)} \quad \text{eig}(\rho) \]
Local analysis

\[ i : \quad A \rightarrow \hat{A} \equiv A \circ [f \ldots f] \equiv A \circ \hat{f} \]

\[ x^a = x^f + K(d - Hx^f) \quad \rightarrow \quad x^a_i = x^f_i + (K)_{i,:}(d - Hx^f)_i \]
\[ A^a = A^f T \quad \rightarrow \quad (A^a)_{i,:) = (A^f)_{i,:} T \]

Equivalence of ensemble tapering and observation scaling:

\[ \hat{K} = A(H\hat{A})^T [(H\hat{A})(H\hat{A})^T + R]^{-1} \]
\[ = A(R^{-1/2}H\hat{A})^T [I + (R^{-1/2}H\hat{A})(R^{-1/2}H\hat{A})^T]^{-1} R^{-1/2} \]
\[ \hat{T} = [I + (R^{-1/2}H\hat{A})^T(R^{-1/2}H\hat{A})]^{-1/2} \]
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\[ x^a = x^f + K(d - Hx^f) \rightarrow x_i^a = x_i^f + (K)_i:(d - Hx_i^f) \]
\[ A^a = A^f T \rightarrow (A^a)_i = (A^f)_i T \]

\[ K = A\hat{H}A^T \left[ (\hat{H}A)(\hat{H}A)^T + R \right]^{-1} \]
\[ = A(R^{-1/2}HA)^T \left[ I + (R^{-1/2}HA)(R^{-1/2}HA)^T \right]^{-1} R^{-1/2} \]
\[ T = \left[ I + (R^{-1/2}HA)^T(R^{-1/2}HA) \right]^{-1/2} \]

- Equivalence of ensemble tapering and observation scaling:
Non-adaptive methods: conclusions

**Covariance localisation:**
- can only be applied to schemes formulated in state space
- can be inconsistent, depending on taper function
- can be formulated via either covariance filtering or tapering of ensemble anomalies
- computationally can be the most effective choice for intermediate systems experiment

**Local analysis:**
- can be used with any scheme
- is locally consistent
- can be formulated via either tapering of ensemble anomalies or scaling of observation error variance
- computationally can be relatively expensive, but is scalable as $O(n)$
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Adaptive methods

**Anderson (2007):**
- split ensemble into $N$ subensembles
- calculate gain matrices for each sub-ensemble
- calculate “regression confidence factor” $\alpha$
- use $K_s \rightarrow \alpha \circ K_s$, $s = 1, \ldots, N$

**Conclusions:**
- does not require distance measure between observation and state vector element
- Consistent formulation
- Optimises for sub-ensemble

**Bishop and Hodyss (2007, 2009):**

**Main idea:**
$$P \rightarrow \rho \circ P, \quad \rho = \rho(C)$$

**Conclusions:**
- does not require distance measure between observation and state vector element
- a lot of tuning to play with; rather expensive
- physically intuitive but *ad hoc*
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Anderson (2007):

- split ensemble into $N$ subensembles
- calculate gain matrices for each sub-ensemble
- calculate “regression confidence factor” $\alpha$

For each state vector element and each observation find the optimal regression coefficient (Kalman gain element) assuming that it is a random draw from the same distribution as $K_s$ for each sub-ensemble $s = 1, \ldots, N$, and find $\alpha$ to minimise

$$J = \sum_{i=1}^{N} \sum_{j \neq i} (\alpha K_i - K_j)^2$$

Solution:

$$\alpha(i, o) = \max \left\{ \frac{(\sum_s K_s)^2}{\sum_s K_s^2} - 1 \middle| \frac{\sum_s K_s^2}{N - 1}, 0 \right\}$$

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Diffusivity

Example: LA model (Oke et al., 2007)

- $n = 1000$, $m = 20$, $r_{loc} = 100$, 51 modes, $L = 100$
Conclusions

- Localisation in the EnKF is an ad-hoc modification of the analysis scheme.
- One must use localisation if the ensemble size is smaller than the model subspace dimension.
- Localisation makes the analysis schemes suboptimal and therefore inconsistent (sometimes - strongly) in regard to estimation of posterior covariance.
- Localisation makes it possible to recover the modal structure from observations even with a rank-deficient ensemble experiment.
- There are two main non-adaptive localisation schemes.
- These schemes are mainly equivalent and should be chosen by algorithmic convenience.
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