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Localisation in the EnKF

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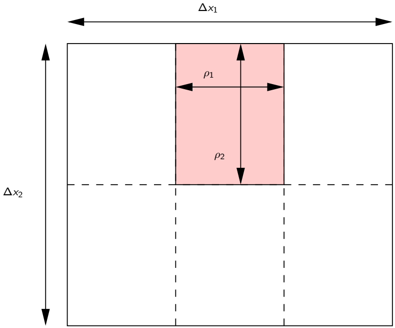
Outline

Reasoning

Two common reasons:

- "spurious covariances"
  $\sigma \sim \frac{1}{m^{1/2}}$
- ensemble rank versus model subspace dimension

$$m_{eff} \approx m \frac{\Delta \mathbf{x}_1}{\rho_1} \frac{\Delta \mathbf{x}_2}{\rho_2} \dots \frac{\Delta \mathbf{x}_N}{\rho_N}$$



Problems

- Q: what is the "price" paid for localisation?
  - dynamical ballance
  - optimality
- Note two different problems:
  1. Mean update ↔ local curve fitting
  2. Covariance update

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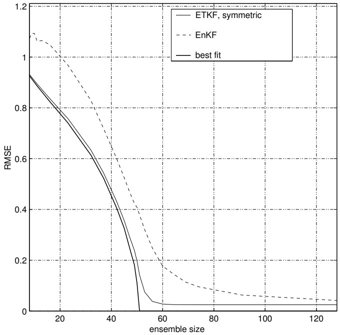

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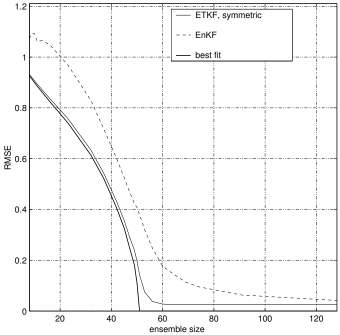

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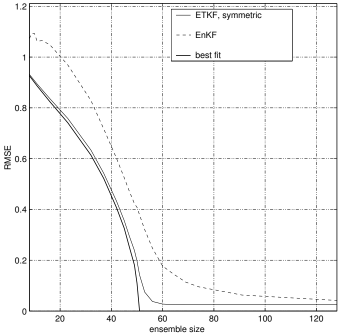

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Historical overview

- Covariance localisation: Hamill and Whitaker (2001); Houtekamer and Mitchell (2001)
- Local analysis: Evensen (2003); Anderson (2003); Ott et al. (2004)
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Some conventions

Indices

$(\mathbf{X})_i$ – ith column of $\mathbf{X}$
$(\mathbf{X})_{i,:}$ – ith row of $\mathbf{X}$
$(\mathbf{X})_{ij}$ – the element in ith row and jth column of $\mathbf{X}$
$(\mathbf{x})_i$ – ith element of $\mathbf{x}$

Matrix operations

$\mathbf{A} \circ \mathbf{B}$ – Schur (Hadamard, element-wise) product of $\mathbf{A}$ and $\mathbf{B}$

Localisation specific

$\overset{i}{\mathbf{A}}$ – ensemble tapered around ith element
$\overset{i}{\mathbf{f}}$ – taper function for the ith element
$\overset{i}{\mathbf{F}}$ – taper matrix, $\overset{i}{\mathbf{F}} = [\overset{i}{\mathbf{f}}, \overset{i}{\mathbf{f}} \dots \overset{i}{\mathbf{f}}]$, $\quad \overset{i}{\mathbf{A}} = \mathbf{A} \circ \overset{i}{\mathbf{F}}$
$\overset{\{o\}}{\mathbf{H}}$ – $\mathbf{H}$ with all rows but oth zeroed

Convenience

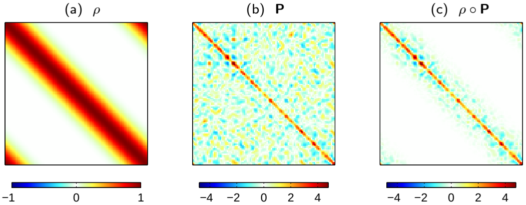
$\mathbf{A}$ – scaled ensemble anomalies: $\mathbf{A} \leftarrow \frac{1}{(m-1)^{1/2}} \mathbf{A}$

Non-adaptive localisation: the two methods

1. Covariance localisation (covariance filtering)

Houtekamer and Mitchell (2001); Whitaker and Hamill (2002)

$$\mathbf{P} \rightarrow \boldsymbol{\rho} \circ \mathbf{P}$$



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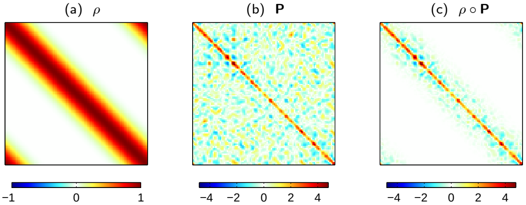
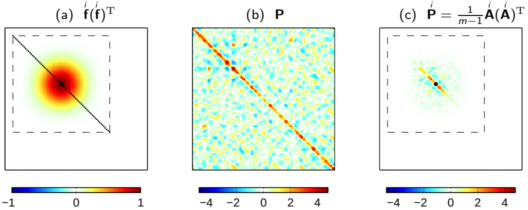


2. Local analysis

(Evensen, 2003; Anderson, 2003; Ott et al., 2004; Hunt et al., 2007)

$$i : \mathbf{A} \rightarrow \overset{i}{\mathbf{A}}$$



Covariance localisation: the technique

$$\boxed{\mathbf{P} \rightarrow \boldsymbol{\rho} \circ \mathbf{P}}$$

$$\mathbf{K} = (\rho \circ \mathbf{P}^f)(\mathbf{H})^{\mathrm{T}} \left[\mathbf{H}(\rho \circ \mathbf{P}^f)(\mathbf{H})^{\mathrm{T}} + \mathbf{R}\right]^{-1}$$

$(\rho \circ \mathbf{P}^f)(\mathbf{H})^{\mathrm{T}}$:

$$(\boldsymbol{\rho} \circ \mathbf{P}^f)\,(\mathbf{H})^{\mathrm{T}} \approx \sum_{o=1}^{p} \overset{i_o}{\mathbf{A}}{}^f\,(\overset{\{o\}}{\mathbf{HA}}{}^f)^{\mathrm{T}}$$

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EnKF:	$\mathbf{A}^a = \mathbf{A}^f + \mathbf{K}(\mathbf{D} - \mathbf{HA}^f)$	- OK
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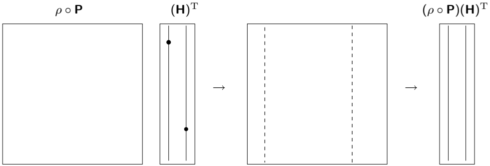

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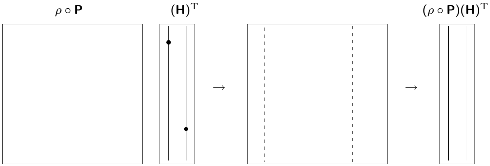

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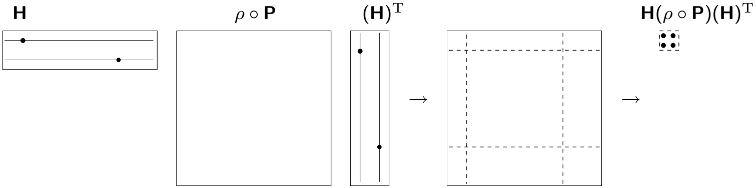

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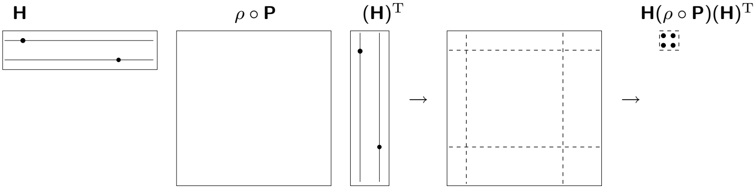

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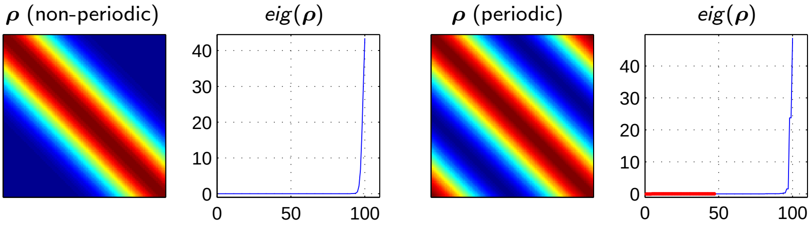
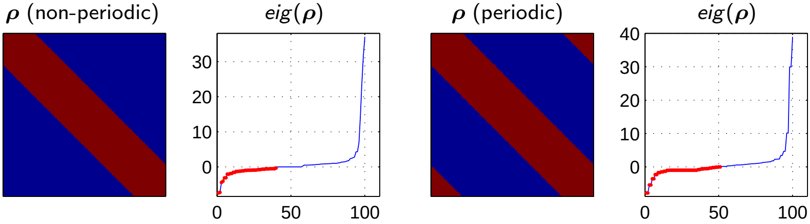
Possible inconsistency

1D case, $n = 100$, $r_{loc} = 20$, Gaspari and Cohn (1999) taper function



Step taper function



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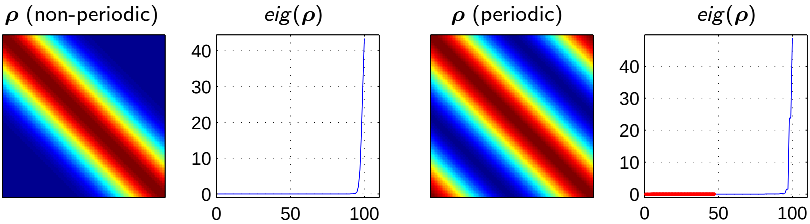
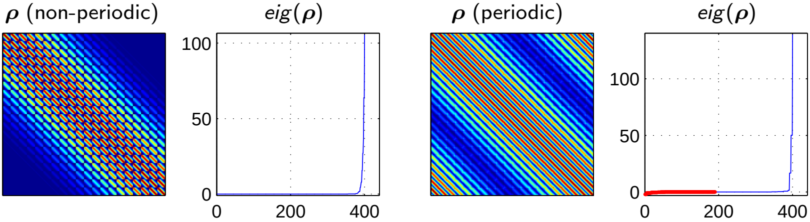


2D case, $n = 20 \times 20$, $r_{loc} = 5$, Gaspari & Cohn taper function



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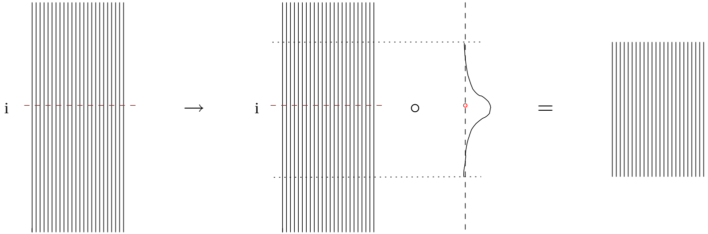

$$\begin{aligned} \mathbf{x}^a &= \mathbf{x}^f + \mathbf{K}(\mathbf{d} - \mathbf{H}\mathbf{x}^f) &\to\quad& \mathbf{x}^a_i = \mathbf{x}^f_i + (\overset{i}{\mathbf{K}})_{i,:}(\overset{i}{\mathbf{d}} - \mathbf{H}\overset{i}{\mathbf{x}^f}) \\ \mathbf{A}^a &= \mathbf{A}^f\mathbf{T} &\to\quad& (\mathbf{A}^a)_{i,:} = (\mathbf{A}^f)_{i,:}\overset{i}{\mathbf{T}} \end{aligned}$$

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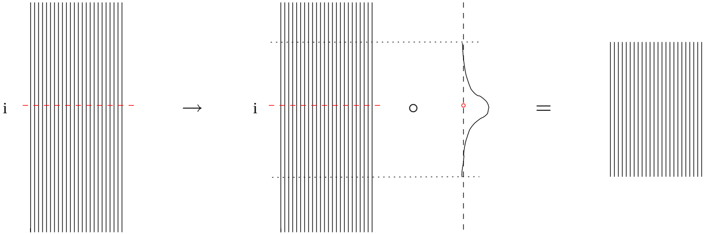

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Non-adaptive methods: conclusions

Covariance localisation:

- can only be applied to schemes formulated in state space
- can be inconsistent, depending on taper function
- can be formulated via either covariance filtering or tapering of ensemble anomalies
- computationally can be the most effective choice for intermediate systems

experiment

Local analysis:

- can be used with any scheme
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- computationally can be relatively expensive, but is scalable as $O(n)$

Non-adaptive methods: conclusions

Covariance localisation:

- can only be applied to schemes formulated in state space
- can be inconsistent, depending on taper function
- can be formulated via either covariance filtering or tapering of ensemble anomalies
- computationally can be the most effective choice for intermediate systems

experiment

Local analysis:

- can be used with any scheme
- is locally consistent
- can be formulated via either tapering of ensemble anomalies or scaling of observation error variance
- computationally can be relatively expensive, but is scalable as $O(n)$

Adaptive methods

Anderson (2007):

- split ensemble into N subensembles
- calculate gain matrices for each sub-ensemble
- calculate “regression confidence factor” α
- use $\mathbf{K}_s \rightarrow \alpha \circ \mathbf{K}_s, \quad s = 1, \ldots, N$

Conclusions:

- does not require distance measure between observation and state vector element
- Consistent formulation
- Optimises for sub-ensemble

Bishop and Hodyss (2007, 2009):

Main idea:

$$\mathbf{P} \rightarrow \rho \circ \mathbf{P}, \qquad \rho = \rho(\mathbf{C})$$

Conclusions:

- does not require distance measure between observation and state vector element
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For *each state vector element* and *each observation* find the optimal regression coefficient (Kalman gain element) assuming that it is a random draw from the same distribution as $\mathbf{K}_s$ for each sub-ensemble $s = 1, \ldots, N$, and find α to minimise

$$J = \sum_{i=1}^{N} \sum_{j \neq i}^{N} (\alpha \mathbf{K}_i - \mathbf{K}_j)^2$$

Solution:

$$\alpha(i, o) = \max \left\{ \frac{\frac{(\sum_s \mathbf{K}_s)^2}{\sum_s \mathbf{K}_s^2} - 1}{N - 1}, 0 \right\}$$

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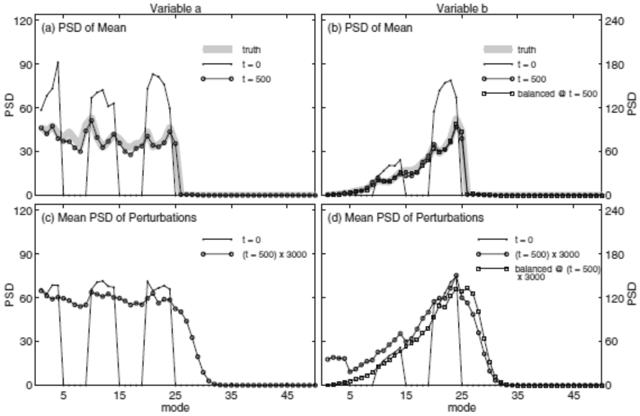
Diffusivity

Example: LA model (Oke et al., 2007)

- Linear advection (LA) model: Evensen (2004), http://enkf.nersc.no/Code/EnKF-Matlab
- $n = 1000,\ m = 20,\ r_{loc} = 100,\ 51\ \text{modes},\ L = 100$



Conclusions

- Localisation in the EnKF is an ad-hoc modification of the analysis scheme
- One must use localisation if the the ensemble size is smaller than the model subspace dimension
- Localisation makes the analysis schemes suboptimal and therefore inconsistent (sometimes - strongly) in regard to estimation of posterior covariance
- Localisation makes it possible to recover the modal structure from observations even with a rank-deficient ensemble experiment
- There are two main non-adaptive localisation schemes
- These schemes are mainly equivalent and should be chosen by algorithmic convenience

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