Conformal Field Theory of Composite Fermions in the QHE

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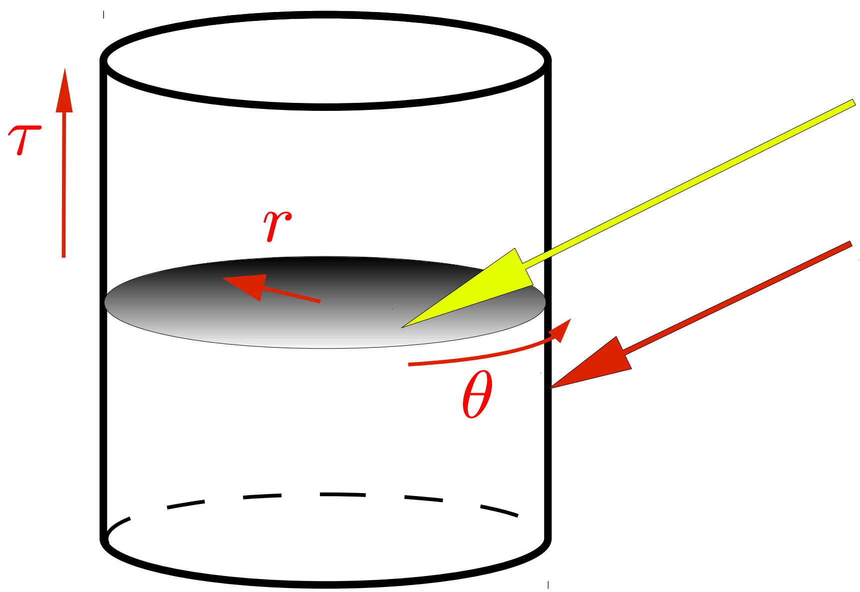
Outline

- Introduction: wave functions, edge excitations and CFT
- CFT for Jain wfs: Hansson et al. results
- CFT for Jain wfs: W-infinity minimal models
- independent derivation of Jain wfs from symmetry arguments
- CFT suggests non-Abelian statistics of quasi-holes

CFT descriptions of QHE



$z = re^{i\theta}$ plane (bulk excit)

$\zeta = e^{\tau} e^{i\theta}$ cylinder (edge excit)

$\langle \phi_1 \;\; \phi_2 \rangle_{CFT}$

$(z_1 - z_2)^{\alpha^2}$ anyon wavefunction

$(\zeta_1 - \zeta_2)^{\alpha^2}$ edge-excit. correlator

- equivalence of descriptions: analytic continuation from the circle,

  use map CFT $\longleftrightarrow$ Chern-Simons theory in 2+1 dim

- general CFT is U(1) x neutral:
  - wavefunctions: spectrum of anyons and braiding matrices
  - edge correlators: physics of conduction experiments

Non-Abelian fractional statistics

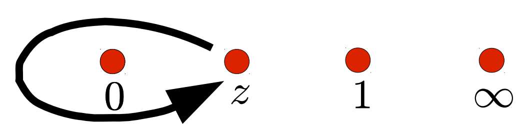
- $\nu = \frac{5}{2}$ described by Moore-Read "Pfaffian state" ~ Ising CFT x U(1)
- Ising fields: I identity, ψ Majorana = electron, σ spin field = q-hole
- fusion rules:
  - $\psi \cdot \psi = I$ electrons fuse into Bosonic bound state
  - $\boxed{\sigma \cdot \sigma = I + \psi}$ 2 channels of fusion = 2 conformal blocks

$\langle \sigma(0)\sigma(z)\sigma(1)\sigma(\infty)\rangle = a_1 F_1(z) + a_2 F_2(z)$ hypergeometric

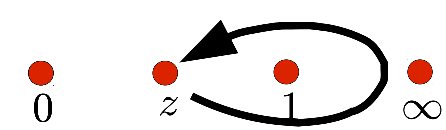
$\Rightarrow$ state of 4 anyons is two-fold degenerate (Moore, Read '91)

- statistics of anyons ~ analytic continuation $\Rightarrow$ 2x2 unitary matrix

$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z e^{i2\pi}) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z)$$



$$\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(1 + (z-1)e^{i2\pi}) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} F_1 \\ F_2 \end{pmatrix}(z)$$



- Topological Quantum Computation (Nayak et al. '07)

Jain composite fermion

$$\Psi_{\nu=\frac{1}{p+1}} = \prod_{i<j}(z_i - z_j)^p \prod_{i<j}(z_i - z_j) = \prod_{i<j}(z_i - z_j)^p \; \Psi_{\nu=1}, \qquad p \text{ even}$$

- Correspondence FQHE $\longleftrightarrow$ IQHE

$$\frac{N_\Phi}{N_e} = \frac{1}{\nu} = p + 1 \qquad\qquad \frac{1}{\nu^*} = 1$$

$$B \qquad\qquad B^* = B - p\,\rho\,\Phi_o$$

- generalize to n filled Landau levels

$$\frac{1}{\nu} = p + \frac{1}{n} \qquad\qquad \frac{1}{\nu^*} = \frac{1}{n}$$

$$\boxed{\Psi_{\nu=\frac{n}{np+1}} = \mathcal{P}_{LLL} \prod_{i<j}(z_i - z_j)^p \; \Psi_{\nu^*=n}}$$

- composite fermion: quasiparticle feeling the reduced B^*
- many experimental confirmations — no definite theory
- $\Psi_{\nu=\frac{n}{np+1}}$ written directly in LLL using projection $\bar{z}_i \to \partial_{z_i}$ in $\Psi_{\nu^*=n}$

(Jain, Kamilla '97)

CFT for Jain: Hansson et al. ('07-'10)

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{p+1}\ \partial_{z_1}\cdots\partial_{z_{N/2}} \prod^{N/2} z_{ij}^{p+1} \prod^{N/2} (z_i - w_j)^p\right]$$

$$\frac{1}{\nu} = p + \frac{1}{2}$$

$$w_{ij} = w_i - w_j$$

$$z_{ij} = z_i - z_j$$

- result based on non-trivial algebraic identites
- recover Abelian two-component edge theory

$$K = \begin{pmatrix} p+1 & p \\ p & p+1 \end{pmatrix}$$

(Wen, Zee; Read,....)

$$\boxed{\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\left\langle(\partial_{z_1}V_+)\cdots(\partial_{z_{N/2}}V_+)\,V_-\cdots V_-\right\rangle\right]}$$

$$V_\pm = e^{i\sqrt{p+\frac{1}{2}}\,\varphi}\, e^{\pm i\frac{1}{\sqrt{2}}\phi}$$

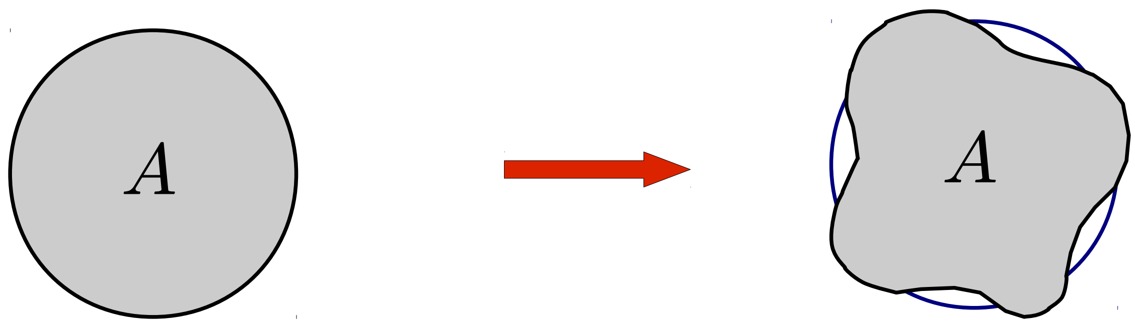
$U(1)$ neutral

- but there is more:
  - $\mathcal{A}$: two fermions V_+, V_- → one fermion
  - descendant fields needed for non-vanishing result, yield correct "shift"
- Next: find improved CFT that complete the derivation

W-infinity symmetry

Area-preserving diffeomorphisms of incompressible fluid

$$\int d^2x \; \rho(x) = N = \rho_o A \quad \longrightarrow \quad A = \text{constant}$$



- W-infinity symmetry can be implemented in the edge CFT
- CFT with higher currents characteristic of 1d fermions (+ bosonization)

$$W^k =: \bar{F} \, (\partial_z)^k \, F :, \qquad W^0 = J, \qquad W^1 =: J^2 :\sim H, \qquad W^2 =: J^3 :, \quad \cdots$$

- representations completely known $\longrightarrow$ classification (V.Kac, A. Radul '92)
- $c = n$ generically reproduce Abelian theories $\widehat{U(1)}^n$ with K matrices
- but special representations for enhanced symmetry $\widehat{U(1)} \times \widehat{SU(n)}_1$

W-infinity minimal models

- repres. with extended symmetry are degenerate and allow for a projection: W_∞ minimal models (A.C., Trugenberger, Zemba '93-'99)

$$\widehat{U(1)}^n \longrightarrow \widehat{U(1)} \times \widehat{SU(n)}_1 \longrightarrow \widehat{U(1)} \times \frac{\widehat{SU(n)}_1}{SU(n)} = \widehat{U(1)} \times \mathcal{W}_n$$

- these edge theories reproduce Jain fillings, $\nu = \dfrac{n}{p\, n \pm 1}$
  with usual K matrices for charge and statistics
- extra projection of $SU(n)$ amounts to keeping edge excitations completely symmetric w.r.t. layer exchanges:
  - single electron excitation
  - reduced multiplicities of edge states
  - non-Abelian statistics of quasi-particles & electron (????)

FATAL ?.........NO !!

Ex: c=2 minimal model

$$\widehat{U(1)} \times \widehat{SU(2)}_1 \longrightarrow \widehat{U(1)} \times \frac{\widehat{SU(2)}_1}{SU(2)} = \widehat{U(1)} \times \mathrm{Vir}$$

$$c = 2, \quad \frac{1}{\nu} = p + \frac{1}{2}$$

Vir = SU(2) Casimir subalgebra

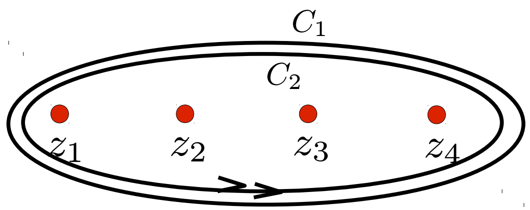
- take edge excitations symmetric w.r.t. two layers only
- neutral part is described by the Virasoro minimal model for $c \to 1$
- fields characterized by dimension $h = \frac{k^2}{4}$ i.e. total spin $s = \frac{k}{2}$; NO s_z
- electron has $s = \frac{1}{2}$
- identify two vertex operators by Dotsenko-Fateev screening operators

$$V_{\pm} = e^{\pm \frac{i}{\sqrt{2}}\phi}, \quad s_z = \pm\frac{1}{2}, \quad V_- \sim V_+ = Q_+ V_-, \quad Q_+ = J_0^+ = \oint du J^+(u)$$

Derivation of Jain wf in Hansson et al. form

- extend W-infinity minimal models from edge to bulk
- 4-el. wf has two channels, $\{\frac{1}{2}\}\times\{\frac{1}{2}\}=\{0\}+\{1\}$, given by choices of C_i

$$\Psi=\left\langle \oint_{C_1} J^+ \oint_{C_2} J^+ \, V_-(z_1)V_-(z_2)V_-(z_3)V_-(z_4)\right\rangle$$



- impose antisymm of electrons ➡ $\Psi=0$
- consider descendant with same charge: $J_0^+ \;\to\; J_{-1}^+,$ $\boxed{J_{-1}^+ V_- \sim \partial_z V_+}$

$$\boxed{\Psi'=\left\langle J_{-1}^+V_-(z_1)\,J_{-1}^+V_-(z_2)\,V_-(z_3)V_-(z_4)\right\rangle+\mathrm{perm}=\Psi_{\mathrm{Jain}}}$$

➡ Underlying theory of Jain wf is W-infinity minimal model + Fermi statistics for electrons

➡ Indipendent, exact derivation of Jain state from symmetry principles ➡ universality, robustness, etc.

Jain wf vs. Pfaffian wf

$$\Psi_{\nu=\frac{2}{2p+1}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{p+1}\ \partial_{z_1}\cdots\partial_{z_{N/2}}\prod^{N/2} z_{ij}^{p+1}\prod^{N/2}(z_i-w_j)^p\right], \qquad \frac{1}{\nu}=p+\frac{1}{2}$$

- reminds of Paffian state in Abelian version (A.C, Georgiev, Todorov '01)

$$\Psi_{\mathrm{Pfaff}} = \mathcal{A}\left[\prod^{N/2} w_{ij}^{M+2}\ \prod^{N/2} z_{ij}^{M+2}\prod^{N/2}(z_i-w_j)^M\right],\quad M\ \mathrm{odd},\quad K=\begin{pmatrix} M+2 & M \\ M & M+2\end{pmatrix}$$

- same vanishing behaviour:

$$\Psi \sim z_{12}^{p-1}\left(z_{13}^2\ z_{14}^2\cdots\right), \qquad \frac{1}{\nu}=p+\frac{1}{2}$$

$$\Psi \sim (z_{12}\,z_{13}\,z_{23})^{p-1}\left(z_{14}^2\ z_{15}^2\cdots\right), \qquad \frac{1}{\nu}=p+\frac{1}{3}$$

→ $p=1$ Jain is excited state of the $M=0$ Pfaffian

- same pairing?
- fractional statistics?

cf. Simon, Rezayi, Cooper '07; Regnault, Bernevig, Haldane '09

Non-Abelian statistics of quasiholes

- smallest q-hole, e.g. $Q=\frac{1}{5}$ at $\nu=\frac{2}{5}$, has neutral part $s=\frac{1}{2}$:

→ two components $s_z=\pm\frac{1}{2}$ identified by the projection $H_+ \sim J_0^+ \, H_-$

q-holes in two layers are identified in W minimal models

- fusion $\begin{cases} H_\pm H_\pm \sim H_{s=1} \\ H_+ H_- \sim H_{s=0} = I \end{cases}$ two channels → non-Abelian statistics

- first non-trivial case is 4 q-holes: three independent states

$$\Psi_{(12,34)} = \mathcal{A}_{z_i} \left\langle H_+(\eta_1) H_+(\eta_2) H_-(\eta_3) H_-(\eta_4) \prod V_e(z_i) \right\rangle \; + (+ \leftrightarrow -)$$

$$\Psi_{(13,24)} = \mathcal{A}_{z_i} \left\langle (+-+-) \right\rangle , \qquad \Psi_{(14,23)} = \mathcal{A}_{z_i} \left\langle (+--+) \right\rangle$$

- they trasform among themselves under monodromy

→ multidimensional representation

Remarks

- $2k$ quasiholes have quantum dimension $d_k \sim \frac{2^{2k}}{\sqrt{k}}$ (but not Rational CFT)
- other Jain q-holes, $\langle(+ + ++)\rangle$, odd no., antisymm combinations, are projected out in the W minimal theory
- need suitable energetics to achieve this (as in Pfaffian)
- entaglement spectrum does not seem to show projection
- but edge is consistently non-Abelian (long-distance physics)
- Jain quasi-particles (after Hansson et al.) do fit in the W minimal theory (zero Km. product)

Conclusions

- CFT + W-infinity symmetry rederive Jain states

  → independently of composite fermion picture ←

- Jain states are consistent & universal
- same CFT hints at non-Abelian q-hole excitations
- open problems:
  - which model Hamiltonian?
  - relation to Gaffnian and mode counting
- experimental tests:
  - thermopower, if measure can be extended to higher B
  - puzzles in known experiments?