Instability in Magnetic Materials with a Dynamical Axion Field

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Weather in Tokyo (past 2 weeks)

22 22/19	23 29/21	24 32/24	25 33/25	26 35/27	27 34/27	28 33/28
29 33/27	30 33/28	31 34/27	1	2	3	4

8月

日	月	火	水	木	金	土
29	30	31	1 33/27	2 35/27	3 34/27	4 32/26
5 34/27	6	7	8	9	10	11

U(1) gauge theory

Quantum Electrodynamics =
U(1) gauge theory in 3+1 dimensions

$$\mathcal{L}_{\mathrm{EM}} = \frac{1}{8\pi}(\vec{E}^2 - \vec{B}^2) \propto F_{\mu\nu}F^{\mu\nu}$$

Gauge invariance also allows

$$\mathcal{L}_{\theta} = \frac{\alpha}{4\pi^2}\theta\vec{E}\cdot\vec{B} \propto \epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$$

which breaks *T*-reversal and inversion symmetry

Topological Term

In a closed space-time with periodic boundary conditions

$$S_\theta = \int d^4x \, \mathcal{L}_\theta = \theta \times \text{integer}$$

θ-term is a topological term;
$\theta \sim \theta + 2\pi$ (in the bulk)

T-reversal invariance ⇒ $\theta = 0, \pi \pmod{2\pi}$

Inducing a Magnetic Monopole with Topological Surface States

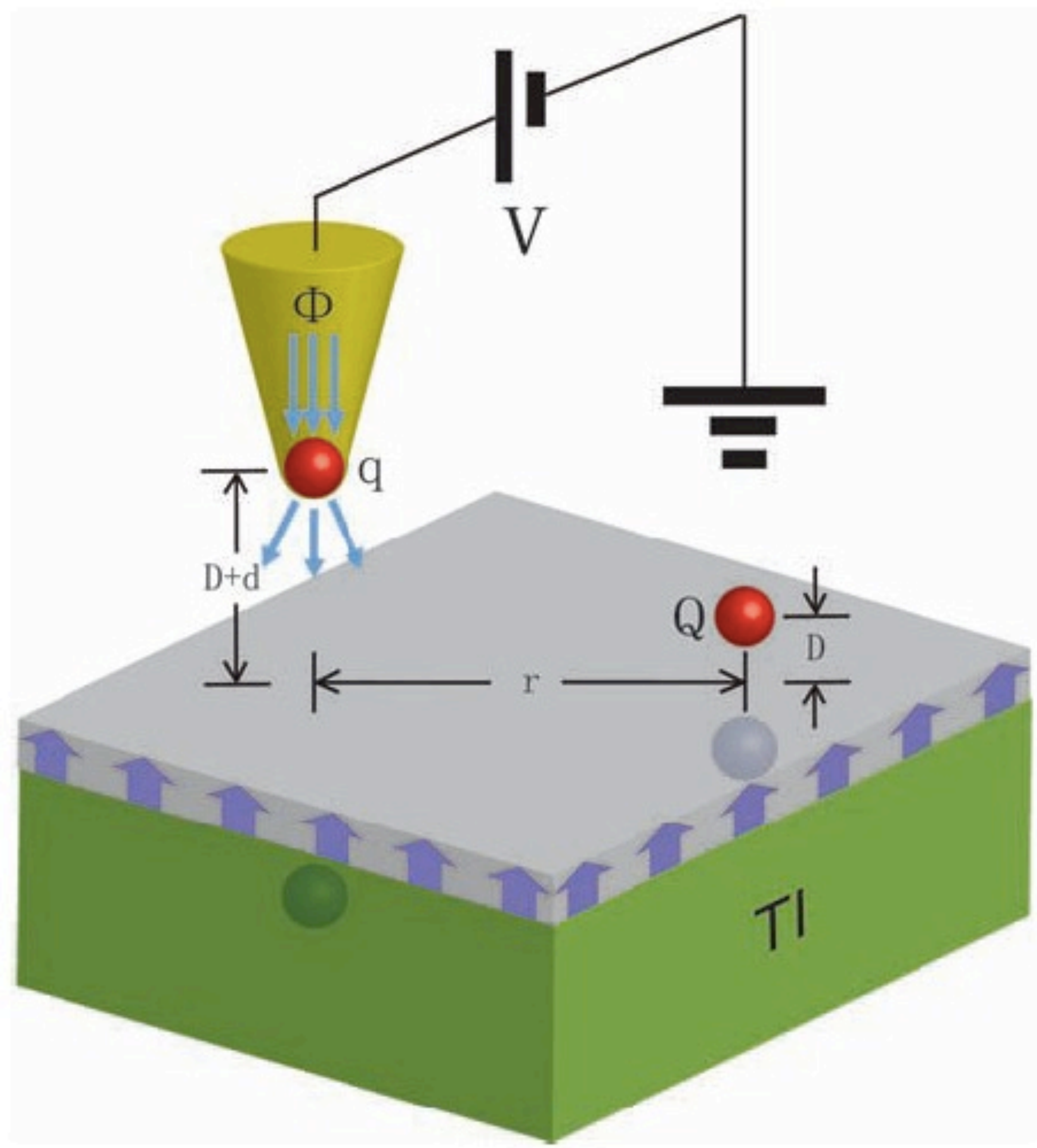
Xiao-Liang Qi,[1] Rundong Li,[1] Jiadong Zang,[2] Shou-Cheng Zhang[1]*

(PRB 2008/ Science 2009)

Z_2 topological insulator

low-energy effective theory

Electrodynamics with $\theta=\pi$



θ-term in Particle Physics

Similar term in QCD

Generic value of θ : breaks T-reversal

(and thus CP symmetry)

$\Rightarrow$ neutron will have electric dipole moment

(which is not observed)

Experimental bound: $\theta < 5 \times 10^{-10}$

Why this is so small? - "strong CP problem"

cf.) CP violation in CKM matrix (weak interaction)

Axion

Proposal (Peccei-Quinn 1977)

Introduce pseudoscalar, dynamical field which couples to $\epsilon_{\mu\nu\rho\sigma}F^{\mu\nu}F^{\rho\sigma}$

$\Rightarrow$ θ effectively becomes a dynamical field

Dynamical θ-field relaxes into the lowest-energy state, which is θ=0

restoration of CP symmetry

Quantum of dynamical θ-field: **new particle "axion"**

not (yet) found in experiments, but a possible component of "dark matter"

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nature physics

Dynamical axion field in topological magnetic insulators

Rundong Li[1], Jing Wang[1,2], Xiao-Liang Qi[1] and Shou-Cheng Zhang[1]*

Z_2 topological insulator
(such as Bi_2Se_3)
doped with magnetic
impurities (such as Fe)

"Topological Magnetic Insulator"

fluctuations of magnetic order play the role of dynamical axion field

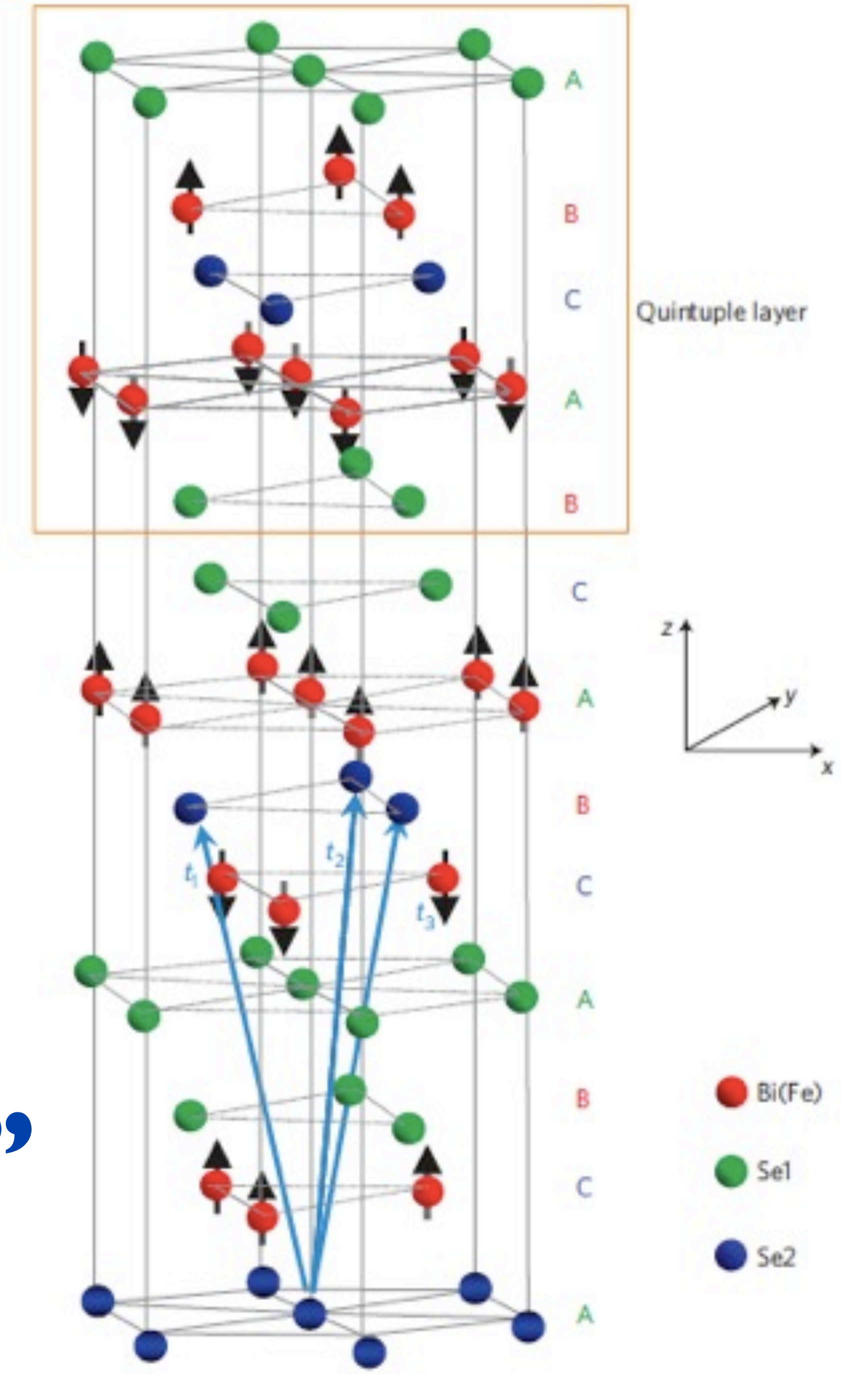


Figure 1 | Crystal structure of Bi(Fe)$_2$Se$_3$. Crystal structure of Bi(Fe)$_2$Se$_3$ with three primitive lattice vectors denoted as $\mathbf{t}_{1,2,3}$. A quintuple layer with Se1-Bi(Fe)1-Se2-Bi(Fe) 1′-Se1′ is indicated in the orange rectangle. The spin-ordering configuration giving rise to the Γ_5 mass is indicated by the black arrow, which is antiferromagnetic along the z direction and ferromagnetic within the xy plane.

The insulator does not need to be "topological" to have an "axion field"

Physics Letters A 372 (2008) 1141–1146

Relativistic analysis of magnetoelectric crystals: Extracting a new 4-dimensional P odd and T odd pseudoscalar from Cr_2O_3 data

Friedrich W. Hehl [a,*,1], Yuri N. Obukhov [a,2], Jean-Pierre Rivera [b], Hans Schmid [b]

magnetoelectric effect ↔ "axionic" θ term

fluctuations of magnetic moments may give rise to dynamical axion field [X-L. Qi, private commun.]

Effective theory of TMI

$$\mathcal{L} = \frac{1}{8\pi}\left(\epsilon\vec{E}^2 - \mu^{-1}\vec{B}^2\right) + \frac{\alpha}{4\pi^2}(\theta+\phi)\vec{E}\cdot\vec{B} + \\ + g^2 J\left((\partial_t\phi)^2 - \nu_i^2(\partial_i\phi)^2 - m^2\phi^2\right)$$

m: axion mass [$\sim$ 2 meV in Bi_2Se_3-Fe (LWQZ)]

ν: velocity of axion mode = spin wave velocity

Instability in gauge theory

AdS/CFT correspondence

Nakamura-Ooguri-Park (2010):

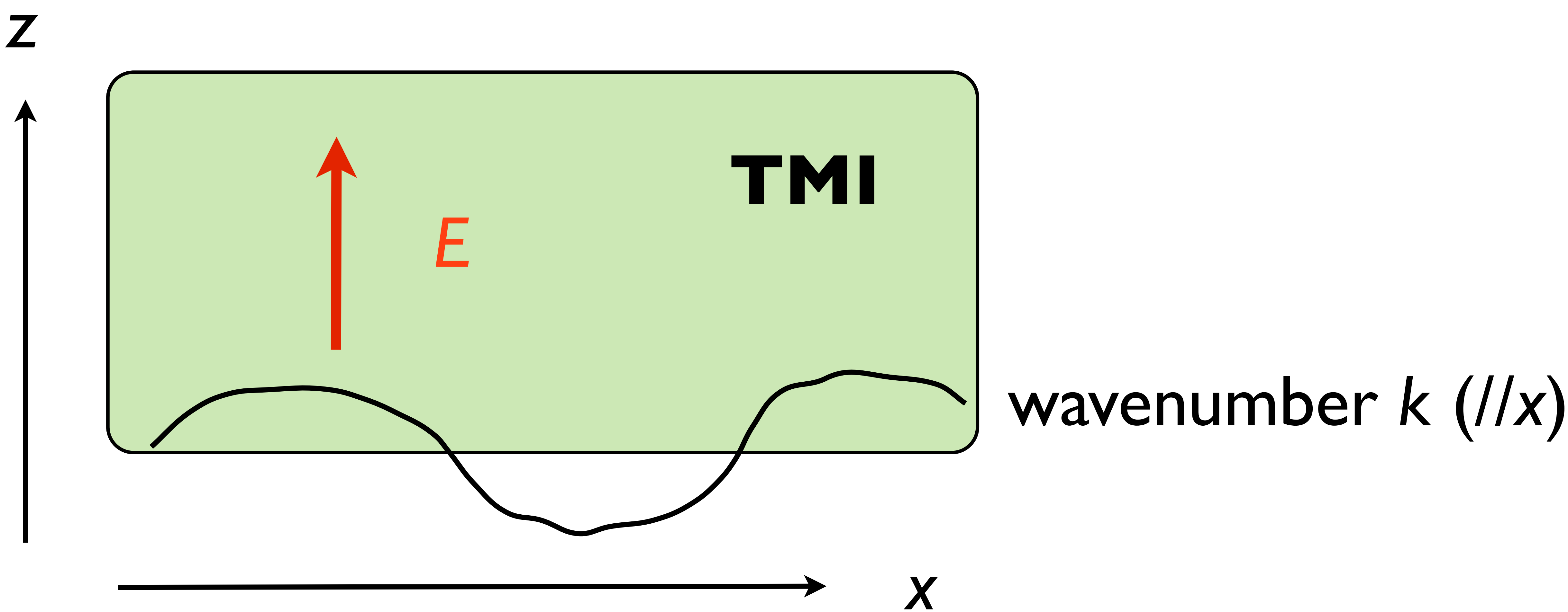
instability in Maxwell theory + Chern-Simons term in (4+1) dimensions

dimensional reduction

Donos-Gauntlett / Bergman-Jokela-Lifschytz (2011):

instability in axionic electrodynamics in (3+1) dimensions

Axionic Polariton



"Axionic polariton" (= coupled axion+EM field)
in background *E*-field

cf.) Li-Wang-Qi-Zhang considered axionic polariton
in background *B*-field

Instability of Axionic Polariton

Dispersion of the axionic polariton in *E*-field:

$$\omega^2 = \tfrac{1}{2}[(c'^2 + \nu^2)k^2 + m^2] \pm \tfrac{1}{2}\sqrt{[(c'^2 - \nu^2)k^2 - m^2]^2 + 4m^2c'^2k^2E^2/E_{\rm crit}^2},$$

c': speed of light in the TMI

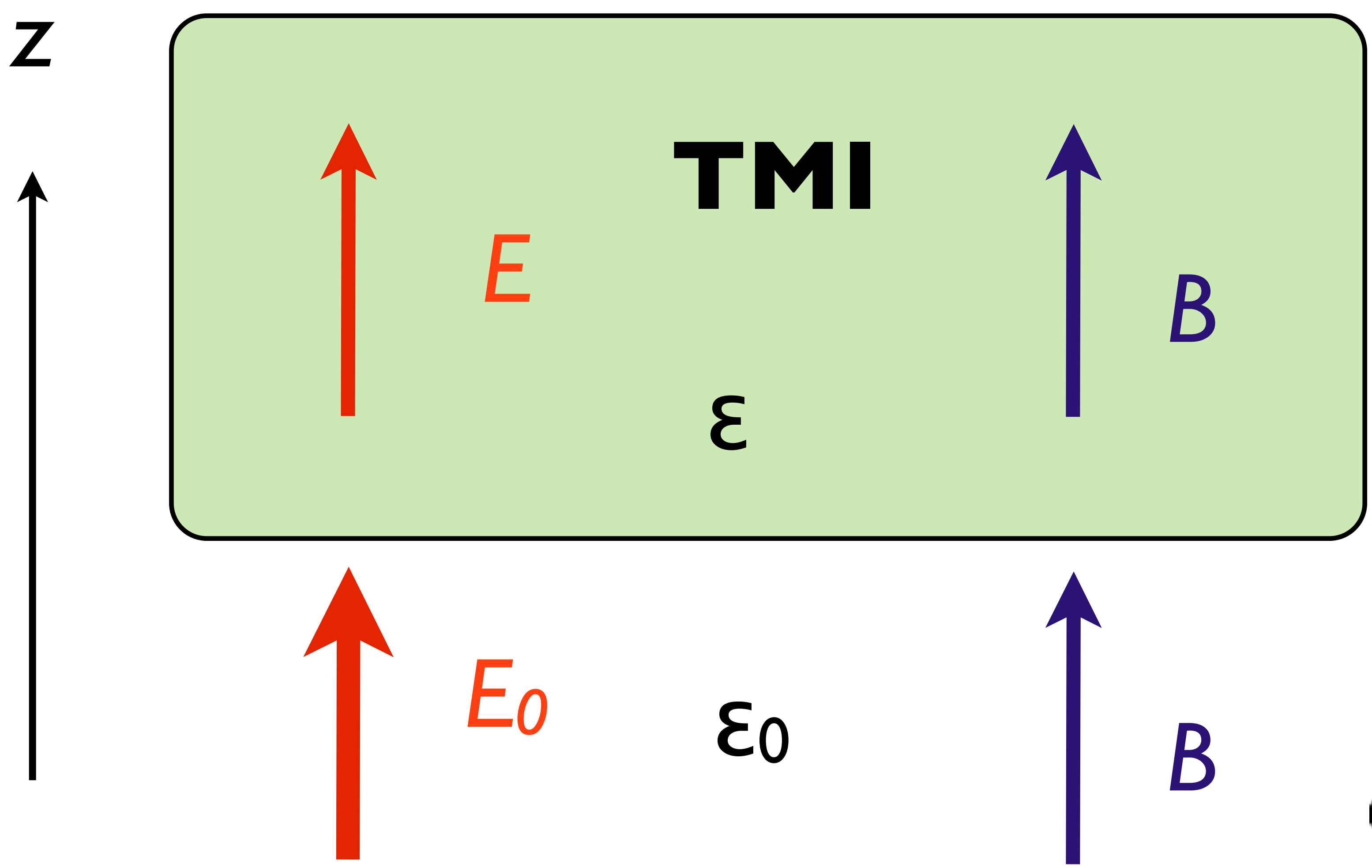
c' >> ν (spin wave velocity of the axion field)

$$E_{\rm crit} = \frac{m}{\alpha}\sqrt{\frac{(2\pi)^3 g^2 J}{\mu}}$$

ω acquires imaginary part if *E*>*E*crit, for $0 < k < \frac{m}{\nu}\sqrt{\left(\frac{E}{E_{\rm crit}}\right)^2 - 1}.$ (Instability!)

Where does it go?

Eventual fate of the system with the axionic instability ?



Neumann b.c.

$$\partial_z \phi = 0$$

(for simplicity)

$$\epsilon E + \frac{\alpha}{\pi}(\theta + \phi)B = \epsilon_0 E_0$$

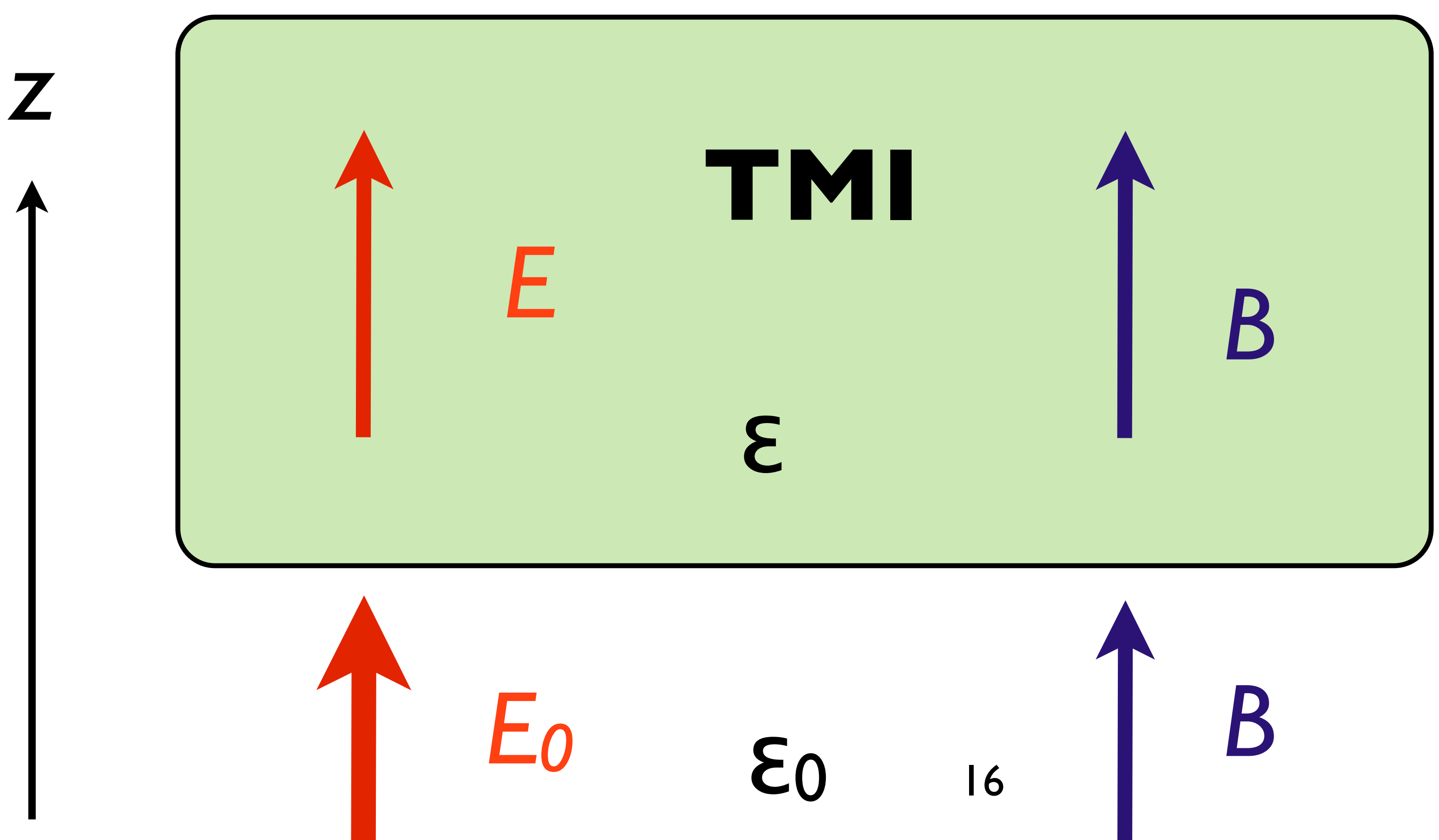
Uniform solution within TMI is allowed

Energy density in TMI

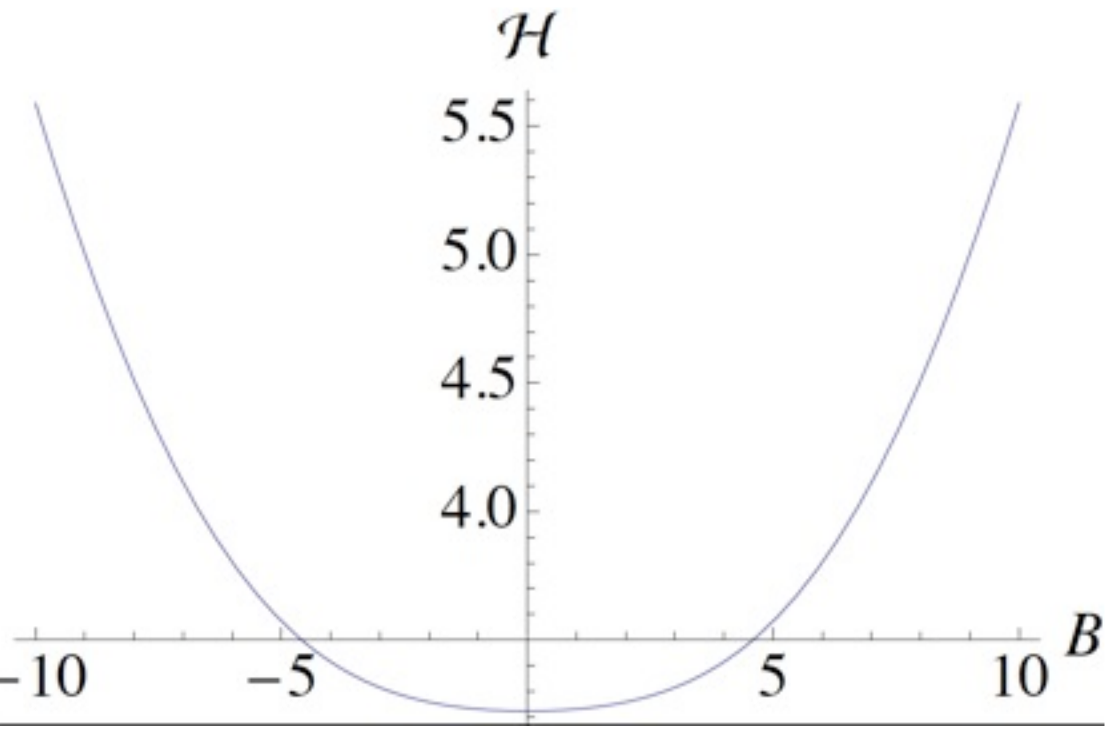
$$\mathcal{H} = \frac{1}{8\pi\epsilon}\frac{(\epsilon_0 E_0 - \alpha\theta B/\pi)^2}{1 + c'^2 B^2/E_{\rm crit}^2} + \frac{1}{8\pi\mu}B^2$$

For given external field E_0, find the magnetic field B which gives the groundstate!

$\Rightarrow$ Φ is automatically determined



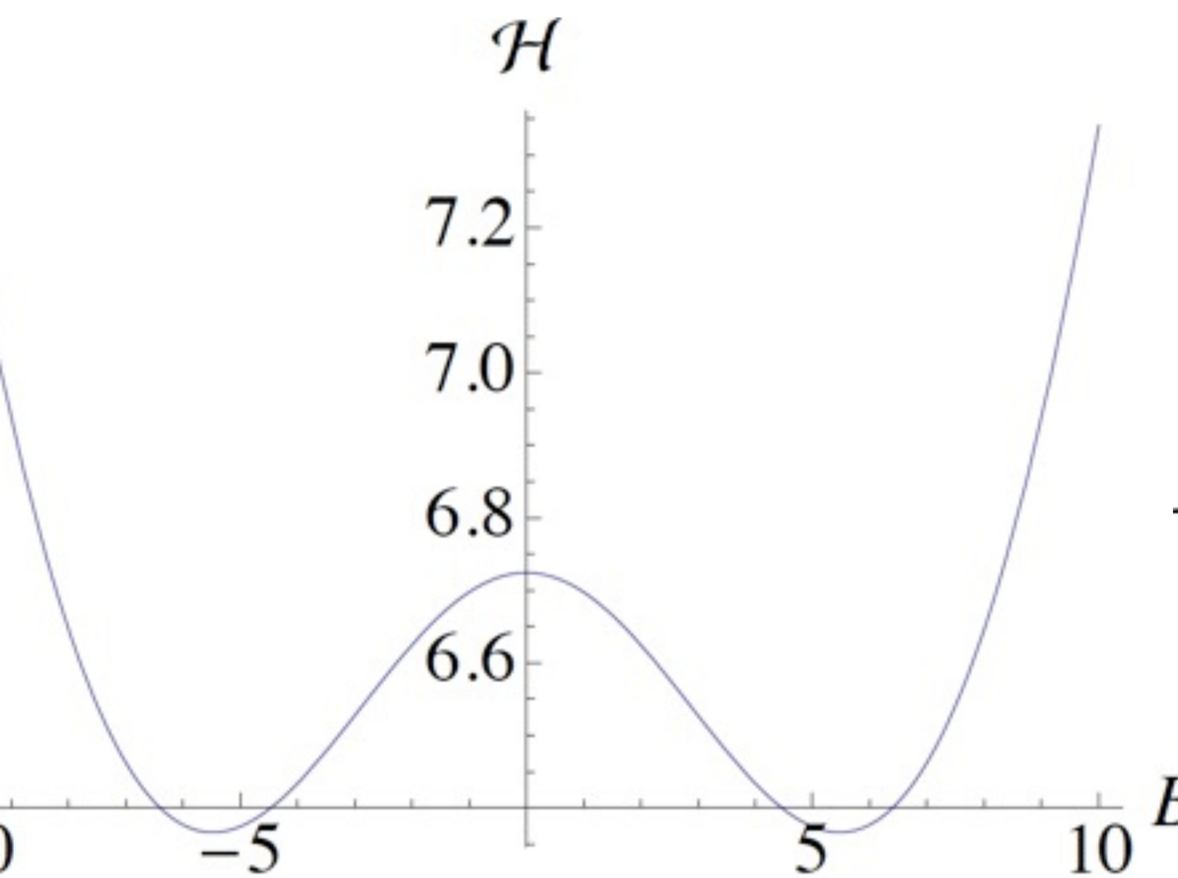
θ=0



$$E_0 = 0.9\frac{\epsilon}{\epsilon_0}E_{\text{crit}}$$

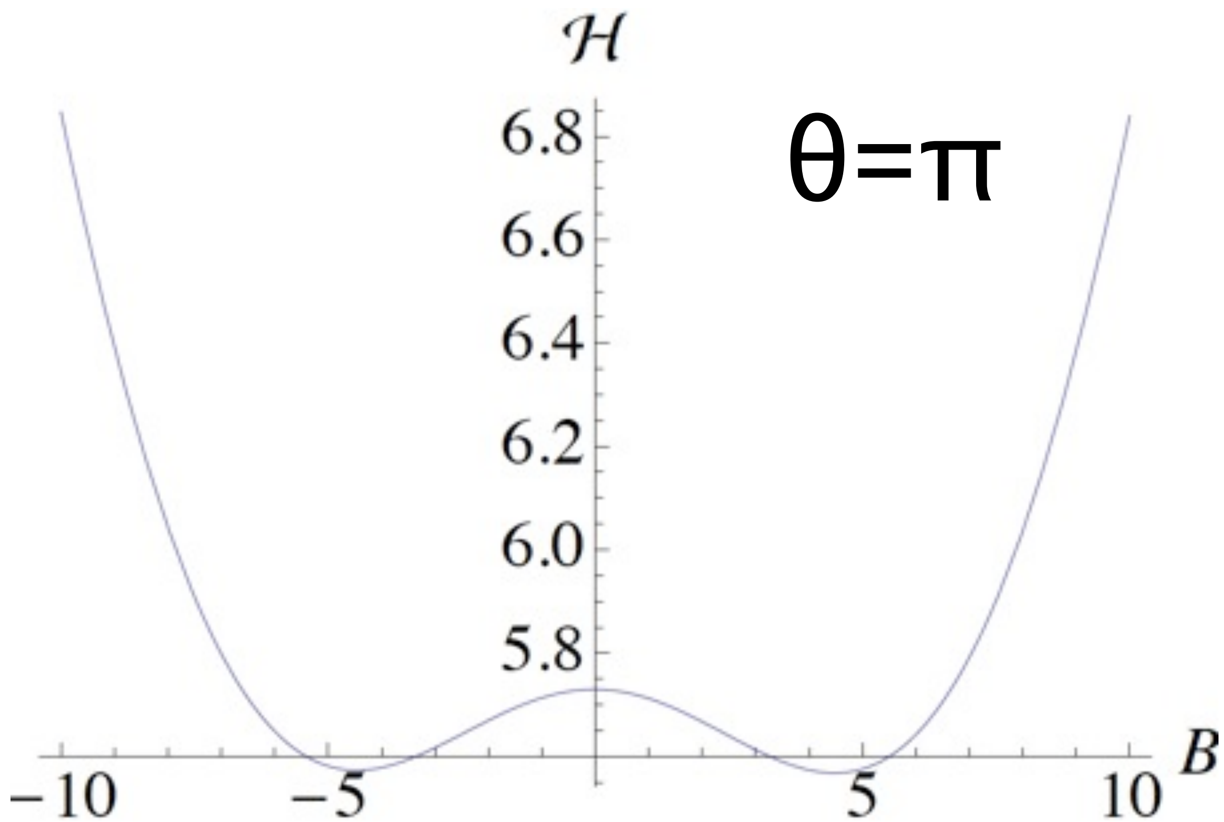
Second-order phase transition at

$$E_0 = \frac{\epsilon}{\epsilon_0}E_{\text{crit}}$$



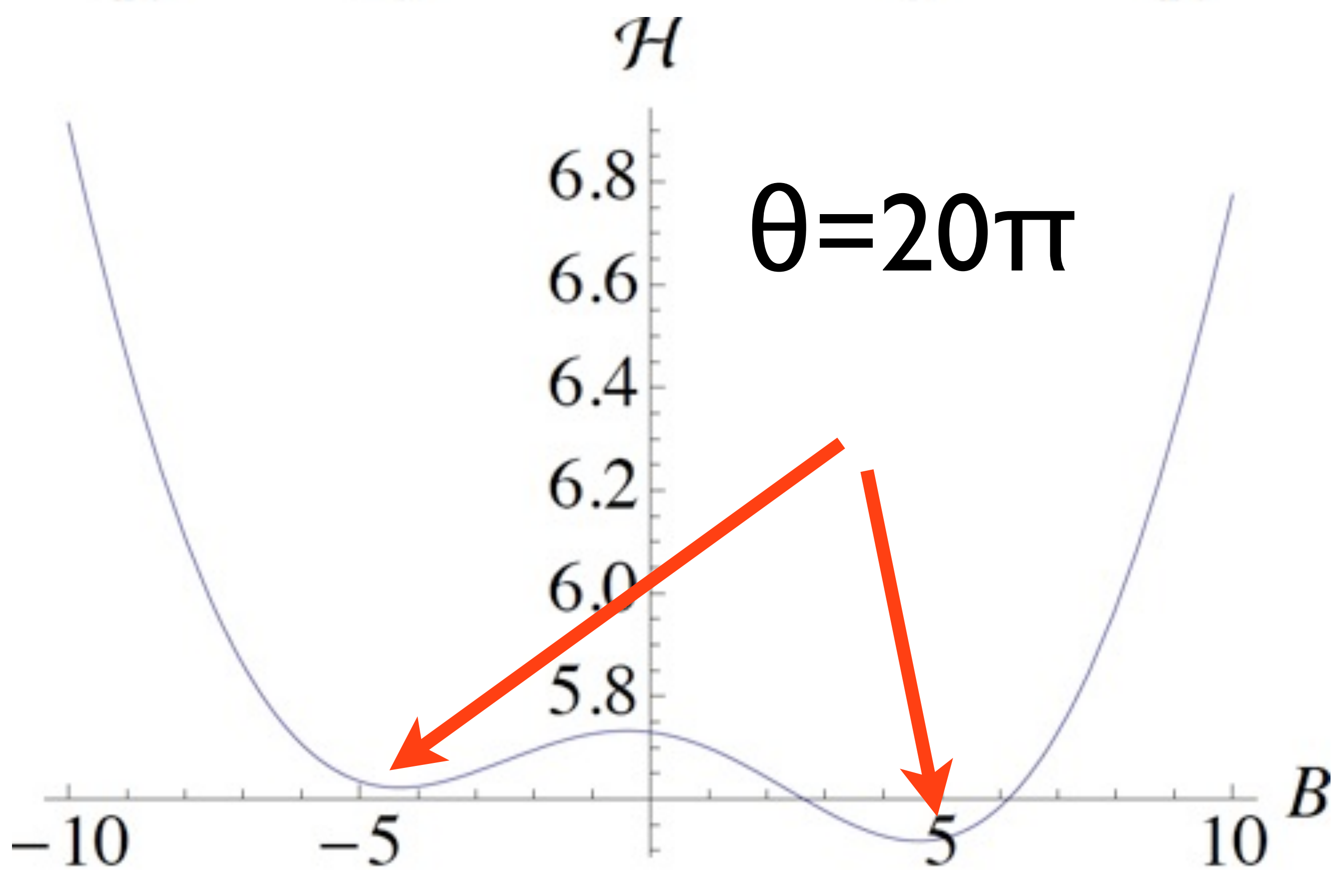
$$E_0 = 1.3\frac{\epsilon}{\epsilon_0}E_{\text{crit}}$$

θ≠0



$$E_0 = 1.2\frac{\epsilon}{\epsilon_0}E_{\rm crit}$$

looks similar to θ=0 ?



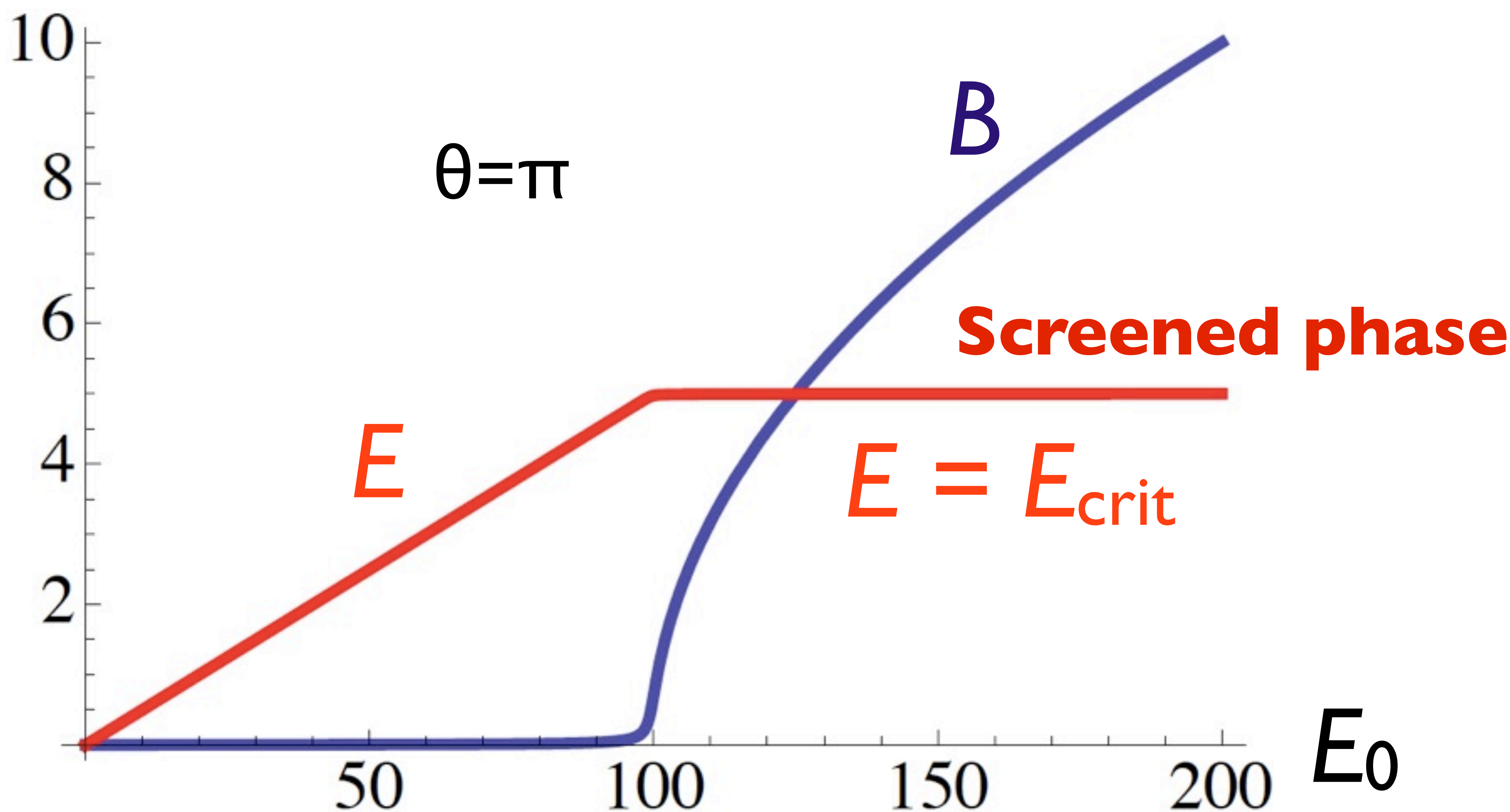
$$E_0 = 1.2\frac{\epsilon}{\epsilon_0}E_{\rm crit}$$

θ≠0 breaks the
B ↔ -*B* symmetry!

No SSB

(Smeared) transition for $\theta \neq 0$

cf.) Ooguri-Park 2010: $m = E_{crit} = 0$



Realization in TMI?

$$E_{\text{crit}} = \frac{m}{\alpha}\sqrt{\frac{(2\pi)^3 g^2 J}{\mu}}$$

Using LWQZ-estimate for Bi_2Se_3-Fe ($m \sim 2$ meV etc.)

$E_{\text{crit}} \sim 10^8$ V/m (perhaps too large)

Axion mass may be made smaller by tuning the system near the critical point of the magnetic order

g may also be made smaller?

For $E_{\text{crit}} \sim 10^5$ V/m and sample thickness $\sim$ 10 nm

$$\Delta V \sim 1 \text{ mV}$$

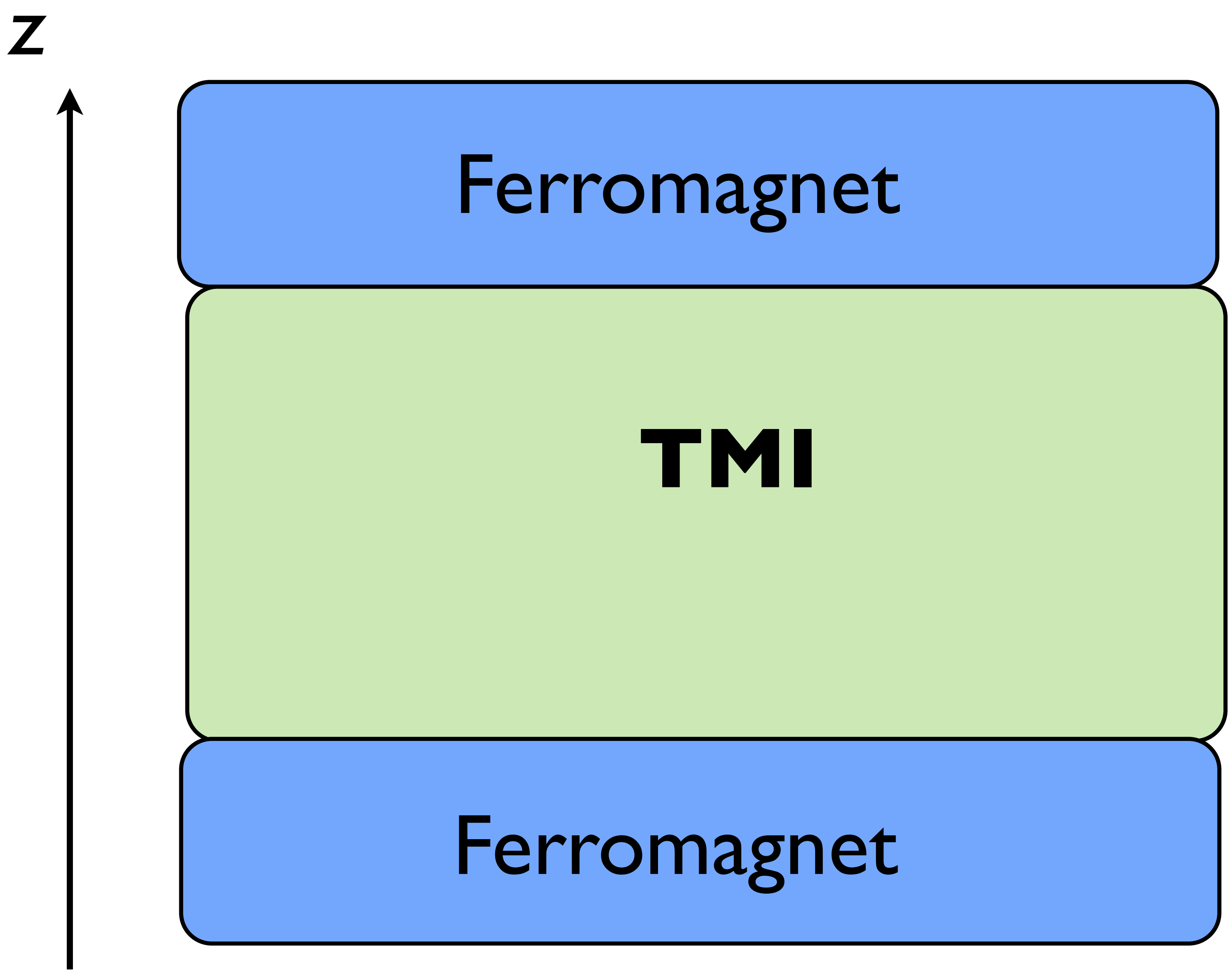
Surface Dirac mode?

If ΔV exceeds the surface mass gap m_5, additional screening due to the surface Dirac mode occurs
The dynamical axion gives an additional screening effect.

($m_5 \sim 1$ meV for Bi_2Se_3-Fe [LQWZ])

Can we separate the effect of dynamical axion field?

Magnetic coating



gives mass to surface Dirac mode, independently of the bulk axion mass

↓

dynamical axion effect may be observed while fine-tuning $m \to 0$

Magnetic coating induces magnetic order at the boundary, imposing Dirichlet b.c. $\phi = \phi_0$

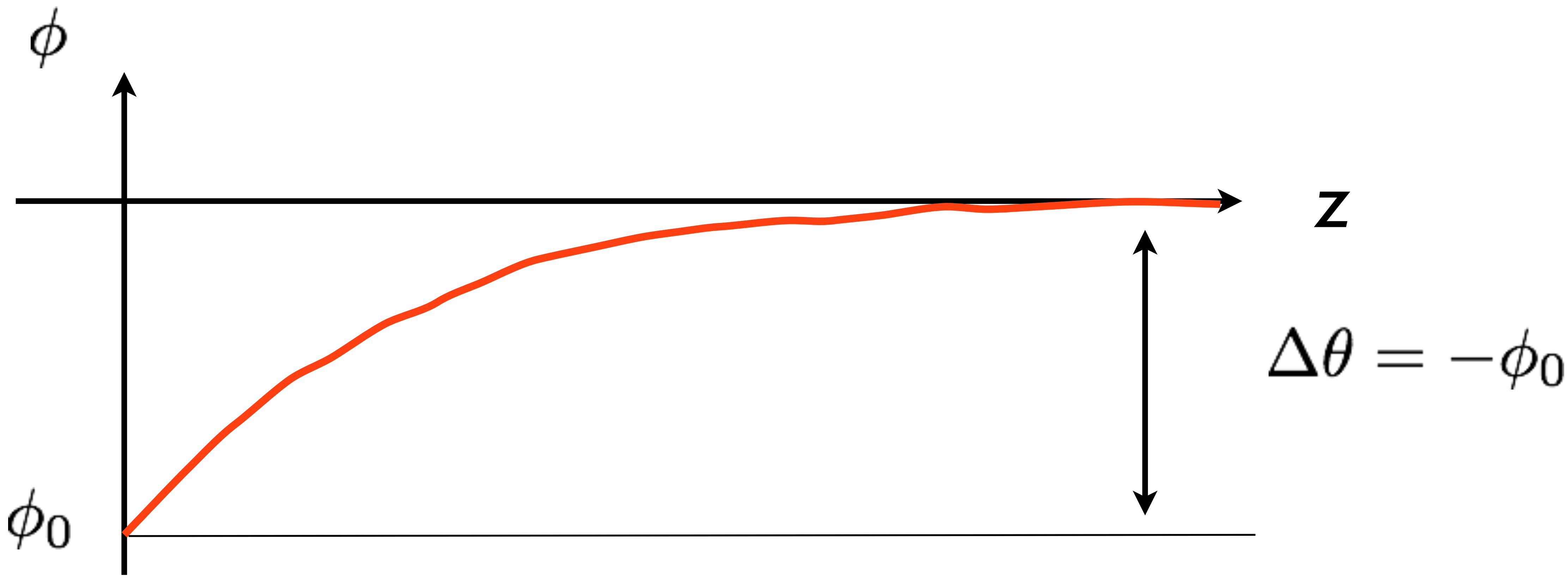
Using “trivial” insulator

Alternatively, we can use a topologically “trivial” insulator with a dynamical axion field (such as Cr_2O_3)

However, Cr_2O_3 has Coulomb repulsion $U \sim 5$eV, which may give axion mass of the same order (too big)

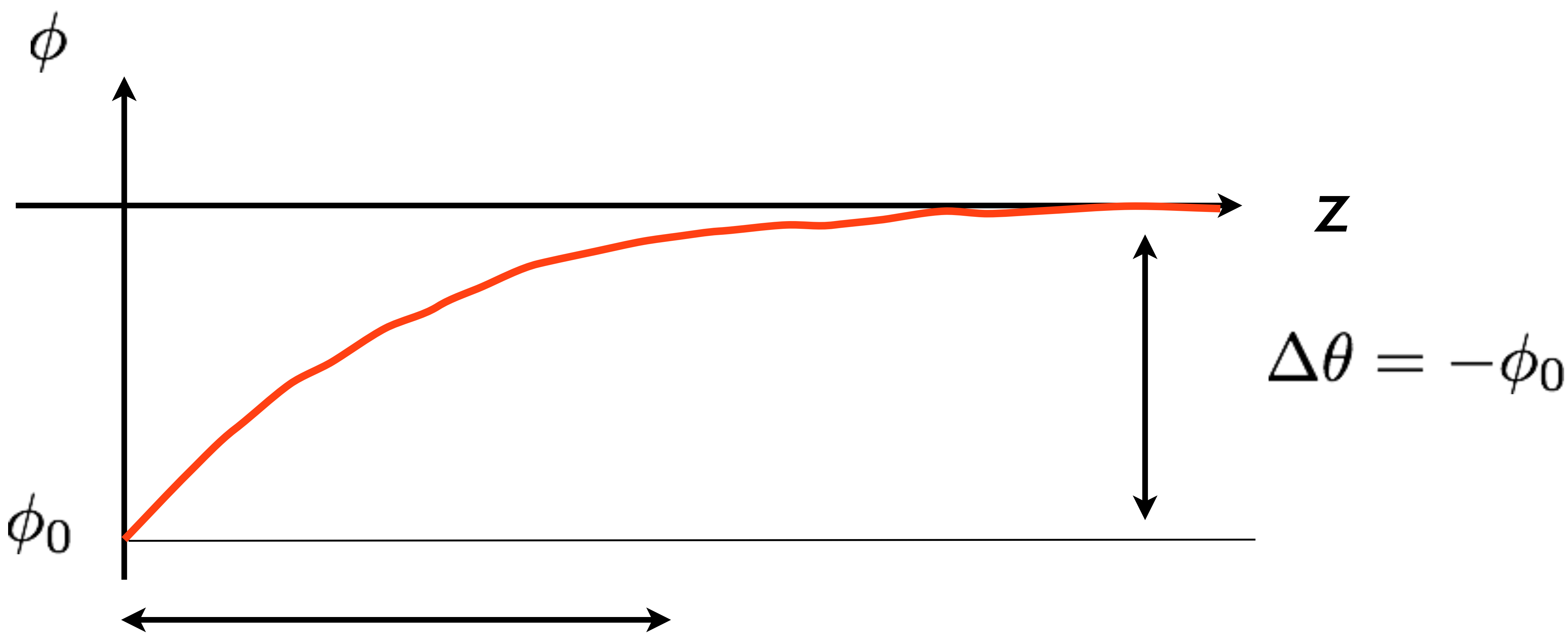
→ also need to fine-tune the axion mass towards zero (close to quantum criticality between magnetic and non-magnetic phases?)

Dirichlet b.c.



The solution is now z-dependent, but it asymptotically approaches the stationary solution for Neumann b.c., with the replacement $\theta = -\phi_0$

Screening Length



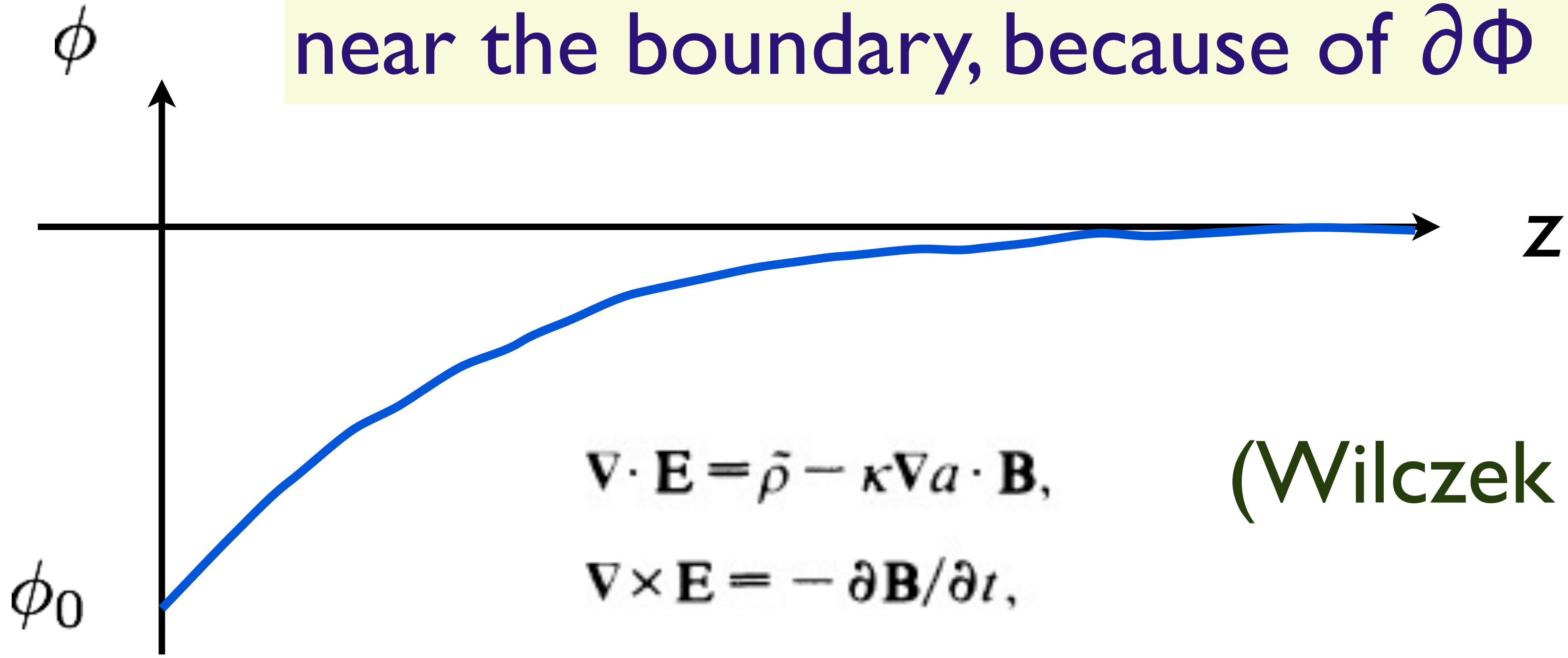
$$\xi = \frac{\nu}{m_{\mathrm{eff}}} = \frac{\nu}{\sqrt{m^2 + \frac{\alpha^2 B^2}{8\pi^3 g^2 J \epsilon}}}$$

< 10 QL
if $m \sim 0.01$ meV

and exchange $\sim$ 1 K

Why screening?

By generating *B*, screening charge is induced near the boundary, because of $\partial\Phi$



(Wilczek 1987)

$$\nabla\cdot\mathbf{E}=\tilde{\rho}-\kappa\nabla a\cdot\mathbf{B}, \tag{2}$$

$$\nabla\times\mathbf{E}=-\partial\mathbf{B}/\partial t, \tag{3}$$

$$\nabla\cdot\mathbf{B}=0, \tag{4}$$

$$\nabla\times\mathbf{B}=\partial\mathbf{E}/\partial t+\tilde{\mathbf{j}}+\kappa(\dot{a}\mathbf{B}+\nabla a\times\mathbf{E}), \tag{5}$$

where $\tilde{\rho},\tilde{j}$ are the ordinary (nonaxion) charge and current. We see that there is an extra charge density proportional to $-\nabla a\cdot\mathbf{B}$, and current density proportion-

Summary

Maxwell theory + dynamical axion has instability under strong E-field

The resulting stable state corresponds to complete screening of E above critical value

The screening accompanies (quasi-)SSB and generation of B-field

Realization in “axionic” insulators:
challenging but possible in principle