Topological Josephson junctions and non-abelian solitons

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Ben-Gurion University of the Negev

Nordita, July 30, 2012

Outline

Josephson junctions

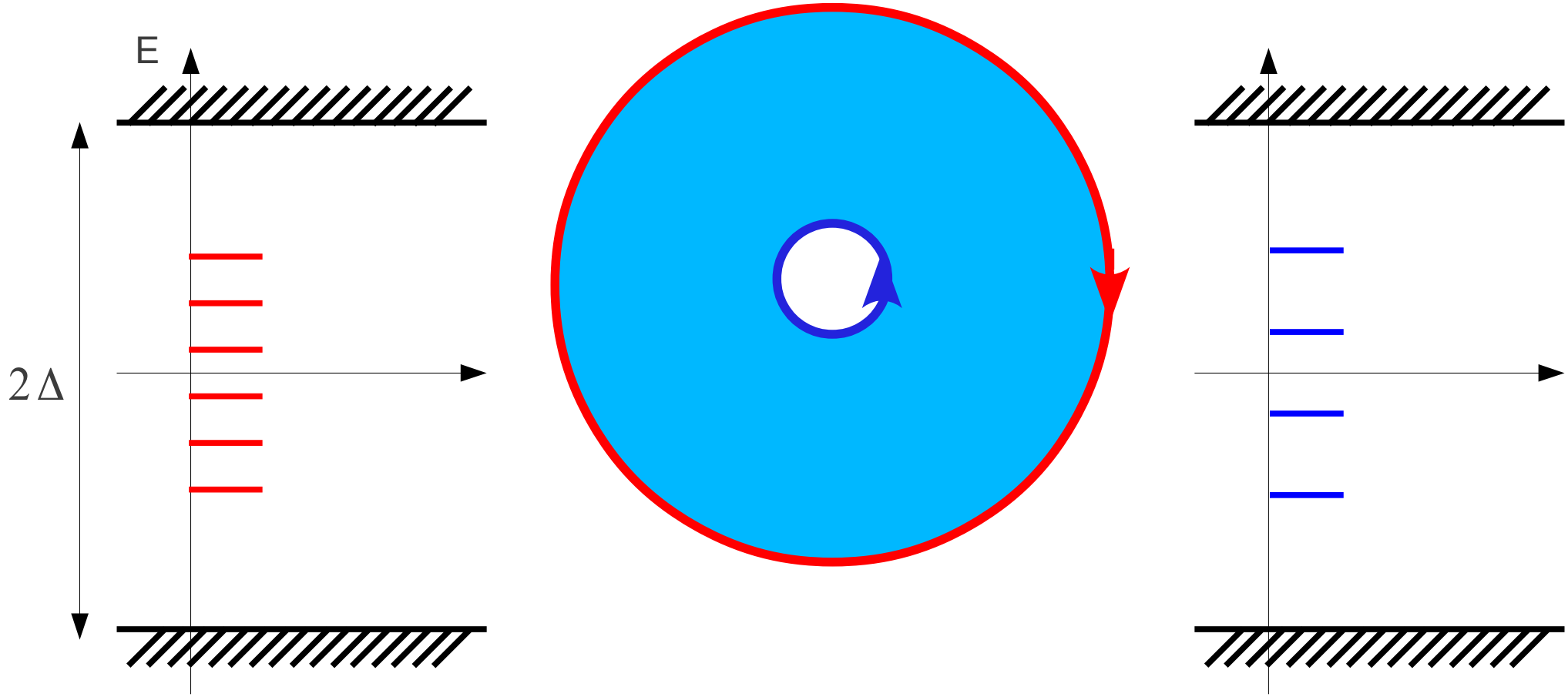
- **The known knowns**
  Abrikosov vortices in topological superconductors carry Majorana modes and satisfy non-abelian statistics
- **What we want to achieve**
  a measurement of non-abelian statistics of vortices
- **Strategy**
  use interference experiments / thermodynamics
- **Problems to be addressed**
  Abrikosov vortices often behave classically

Parallel effort – the quantum Hall effect

- **Detection of neutral edge states**

Topological superconductors

topological superconductor = a superconductor with bulk gap + chiral Majorana edge state



Read and Green

Topological superconductors

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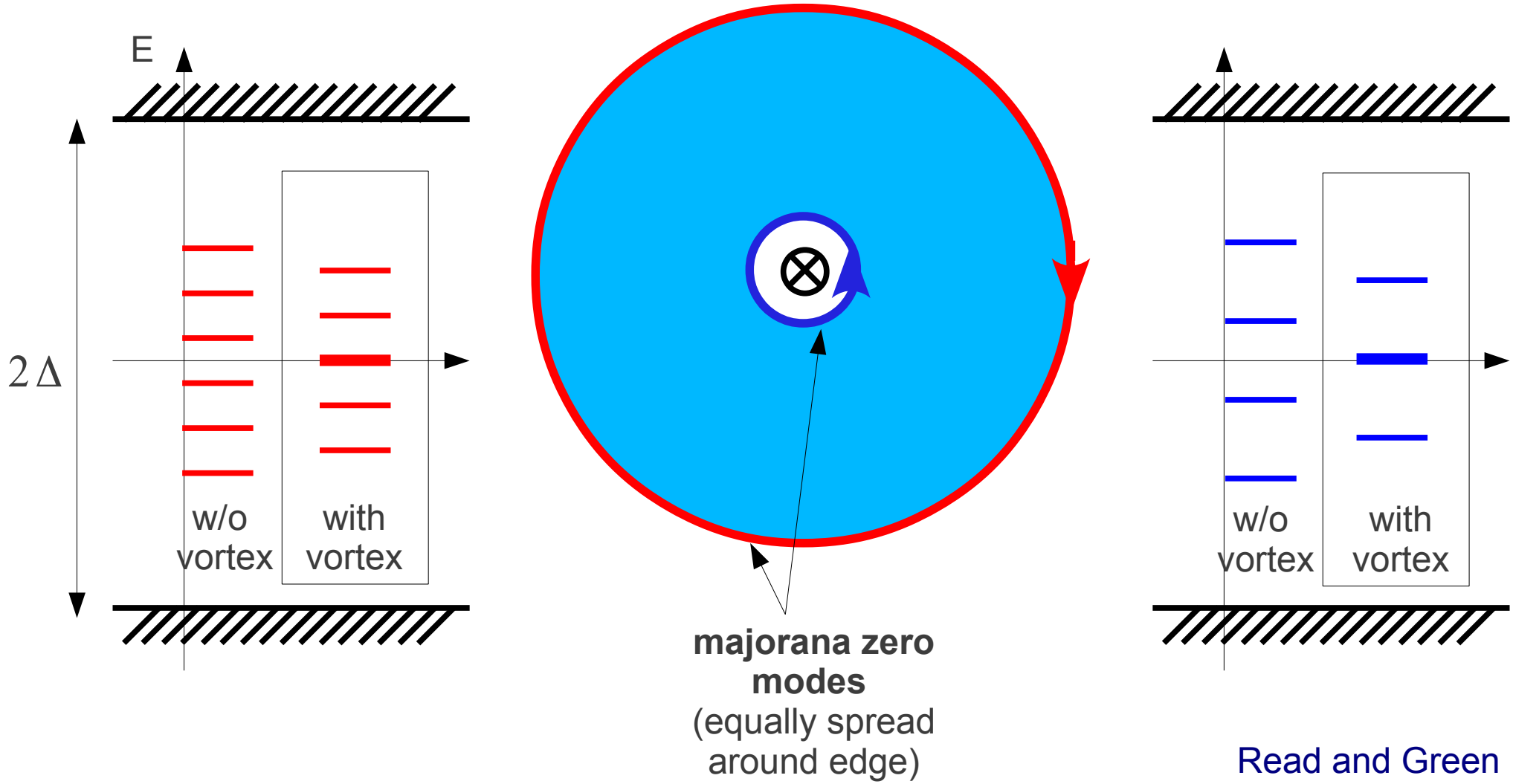


Read and Green

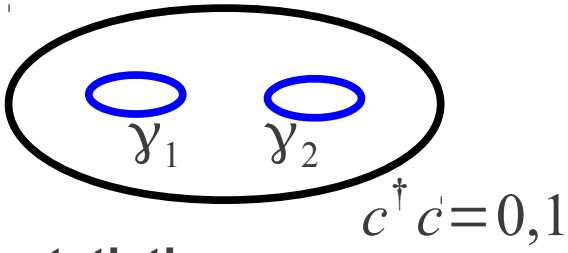
Non-Abelian statistics

Solve BdG for **bound quasi-particle states** on the background of a vortex

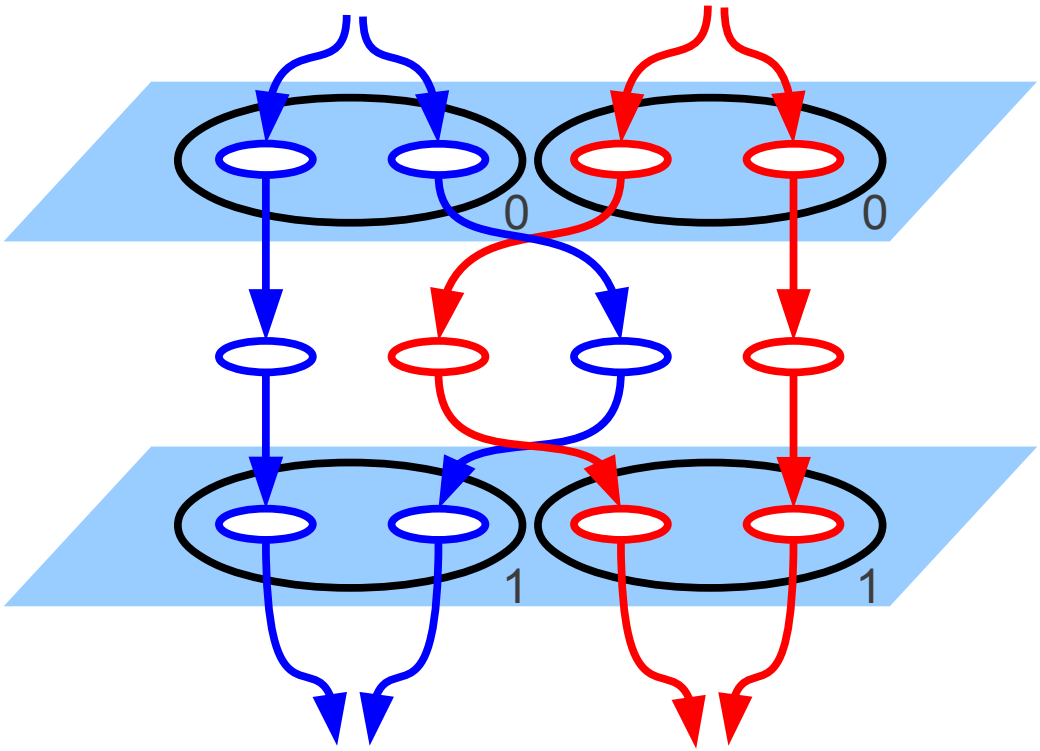
For topological superconductors: a solution at zero energy, **a Majorana zero mode**

$$\gamma_1^\dagger = \gamma_1 = c + c^\dagger$$

$$\gamma_2^\dagger = \gamma_2 = -i(c - c^\dagger)$$



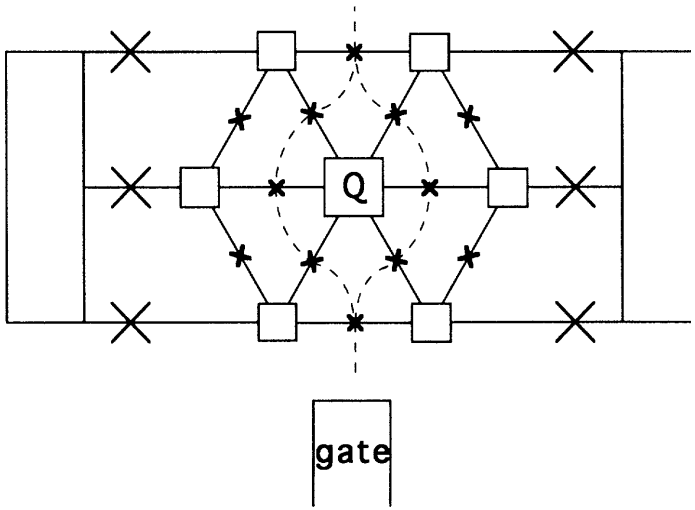
Vortices carrying Majorana core states satisfy non-Abelian statistics

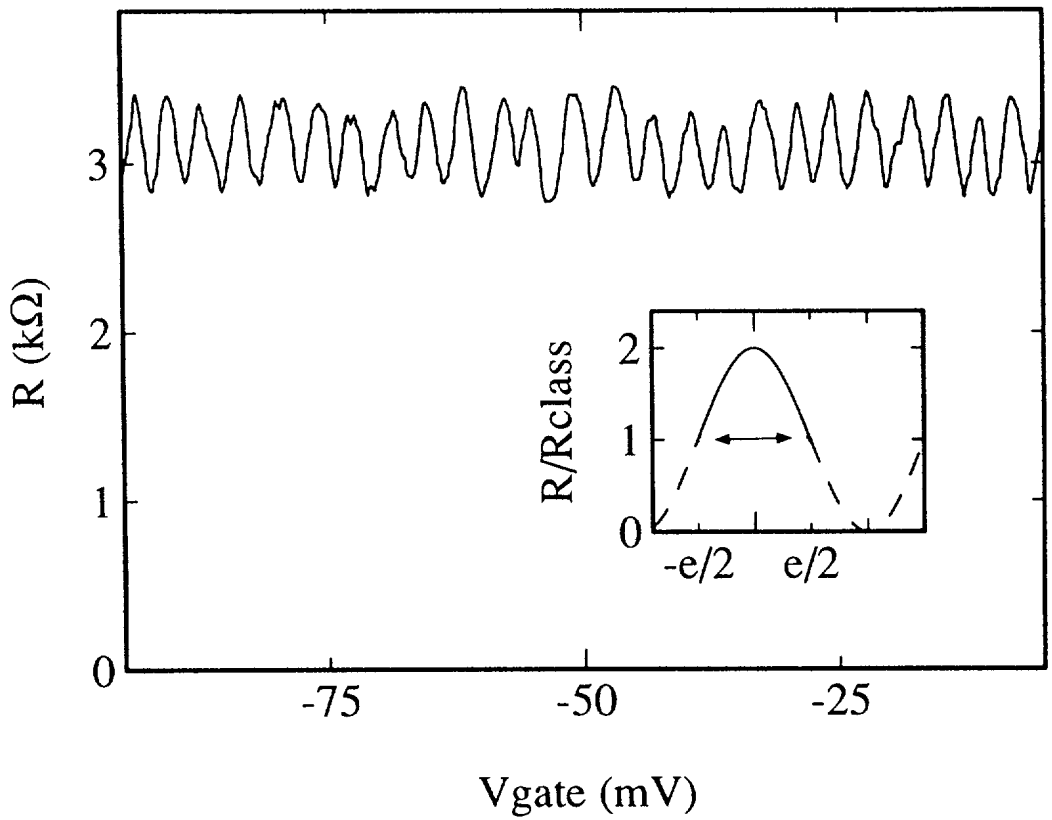


Moore & Read
Ivanov
Fradkin et al
Stern and Halperin
Bonderson et al
Alicea et al (1d)

Observation of the Aharonov-Casher Effect for Vortices in Josephson-Junction Arrays

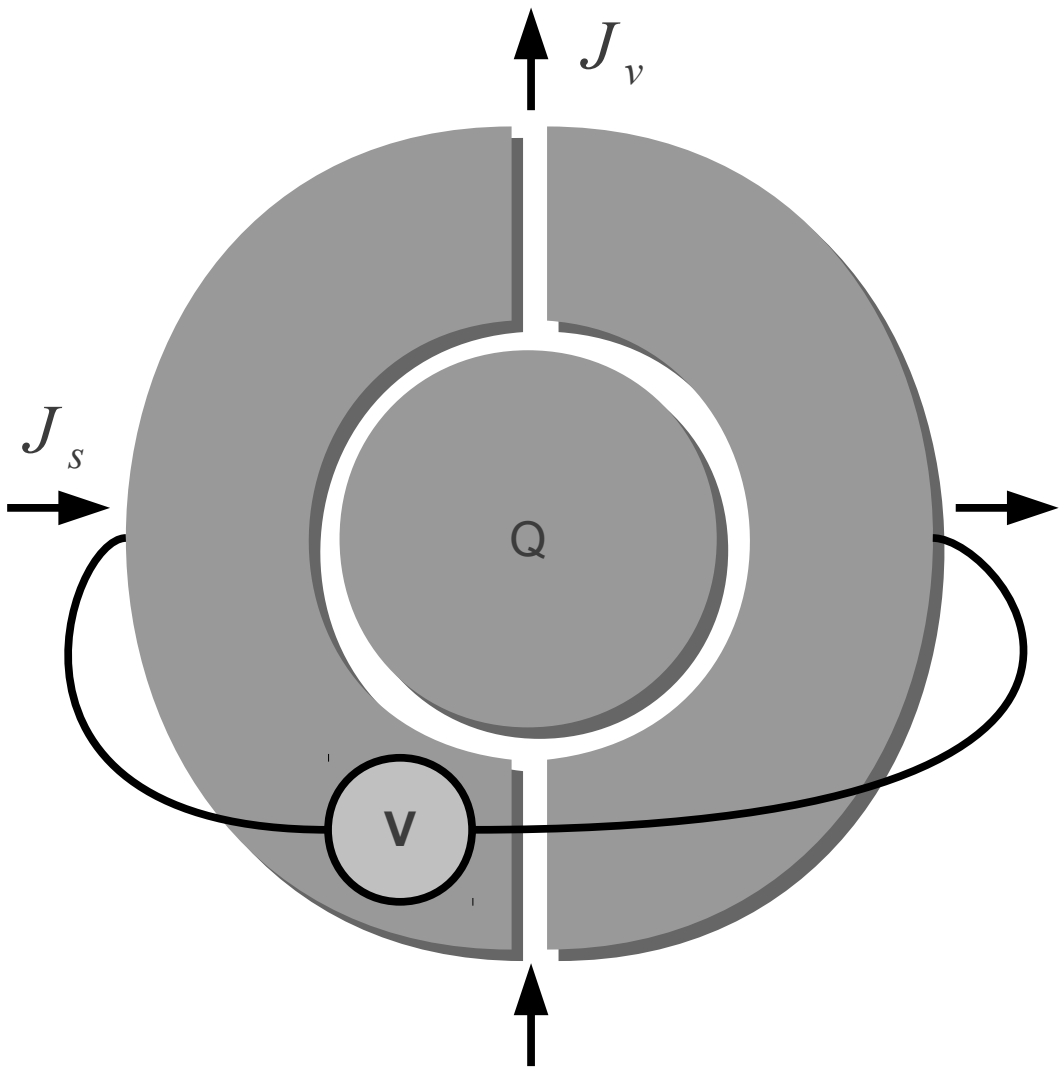
W. J. Elion, J. J. Wachters, L. L. Sohn, and J. E. Mooij

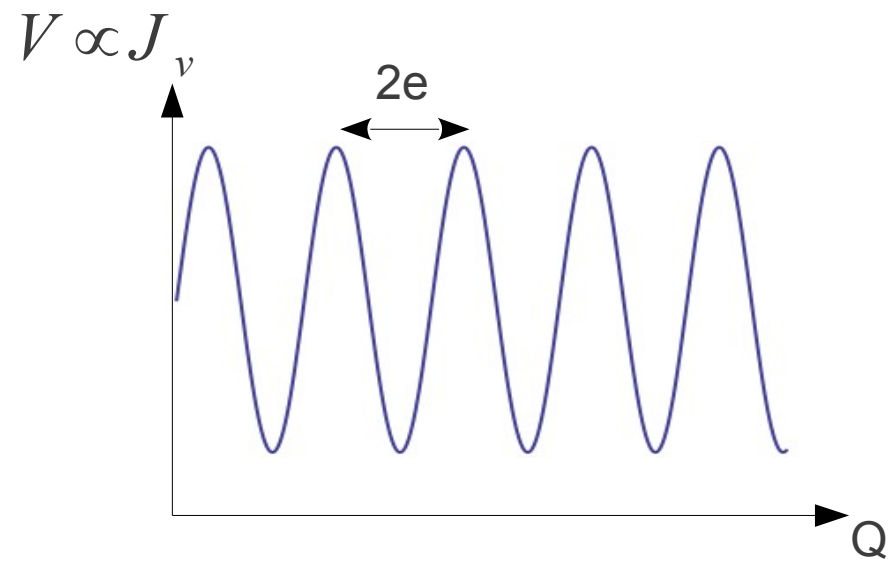




Experimental setup

- vortices created at bottom, pushed upwards by a supercurrent
- vortices generate measurable voltage from left to right
- vortex two-path interference term measures a geometric phase, $\varphi = Q\Phi$

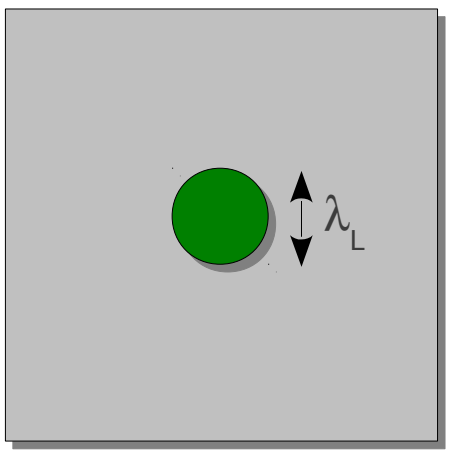


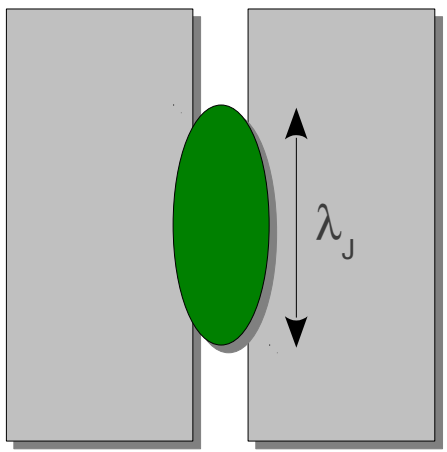


Hermon, Shnirman, Ben Jacob

Abrikosov vs Josephson vortices

	Abrikosov vortices	Josephson vortices
Core	metallic, many electrons, many CdGM states	insulating, no electrons, no CdGM states
Inertial mass	estimated to be high	calculated to be very small
Spatial extent	λ_L	λ_J

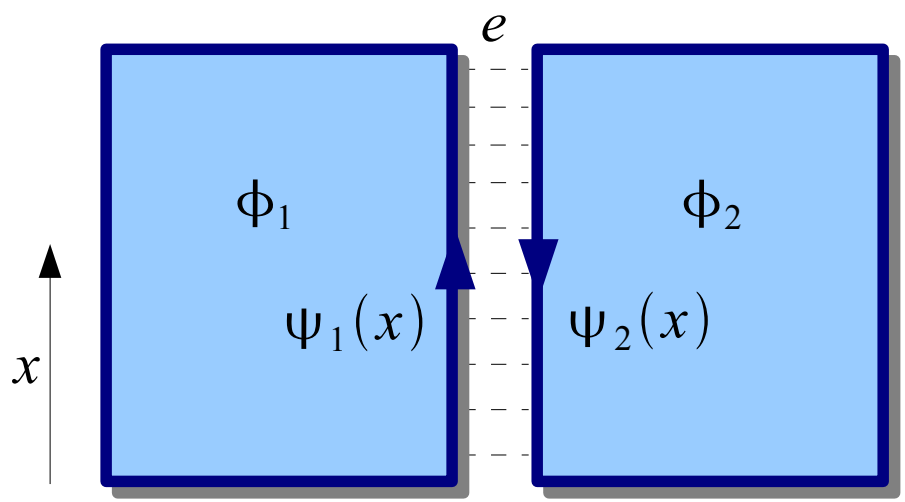




Josephson vortices have better prospects for interference experiments
BUT do they carry Majorana modes (zero energy CdGM state)?

Long Topological Josephson Junctions

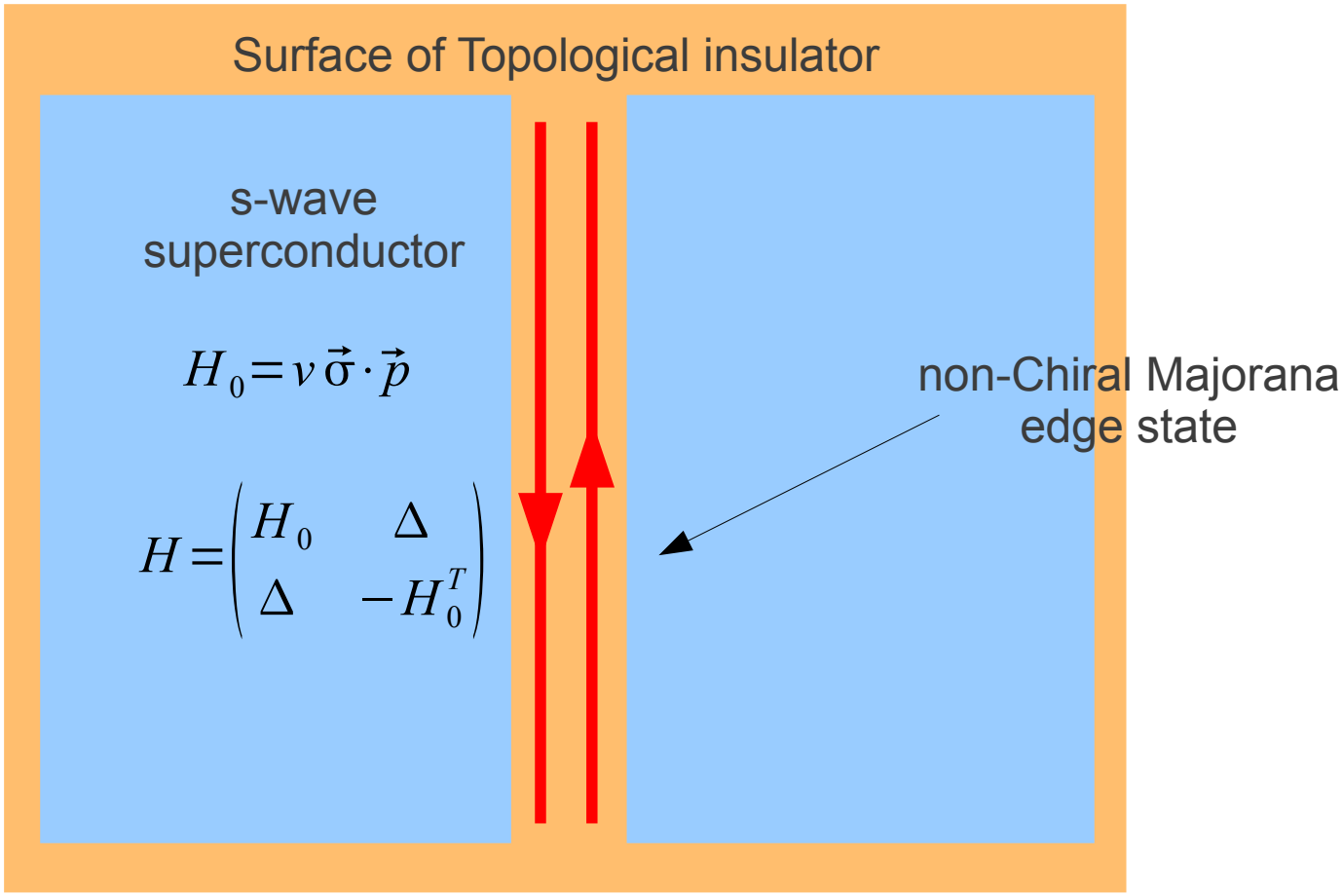
Projected Bogoliubov de Gennes equations for low lying quasi-particles



$$H = H^{\varphi} + H^{\psi} \quad \begin{cases} H^{\varphi} = \dfrac{\hbar}{g^2}\left[\dfrac{1}{2\bar{c}}(\partial_t \varphi)^2 + \dfrac{\bar{c}}{2}(\partial_x \varphi)^2 + \dfrac{\bar{c}}{\lambda_J^{\,2}}[1-\cos(\varphi)]\right] \\ H^{\psi} = i\,v \sum_j (-1)^j \int dx\, \psi_j(x)\,\partial_x \psi_j(x) + i\,m \int dx\, \psi_1(x)\,\psi_2(x) \cos\left[\dfrac{\varphi(x)}{2}\right] \end{cases}$$

E. Grosfeld and A. Stern, PNAS 108, 11810 (2011)

Topological insulator: surface states

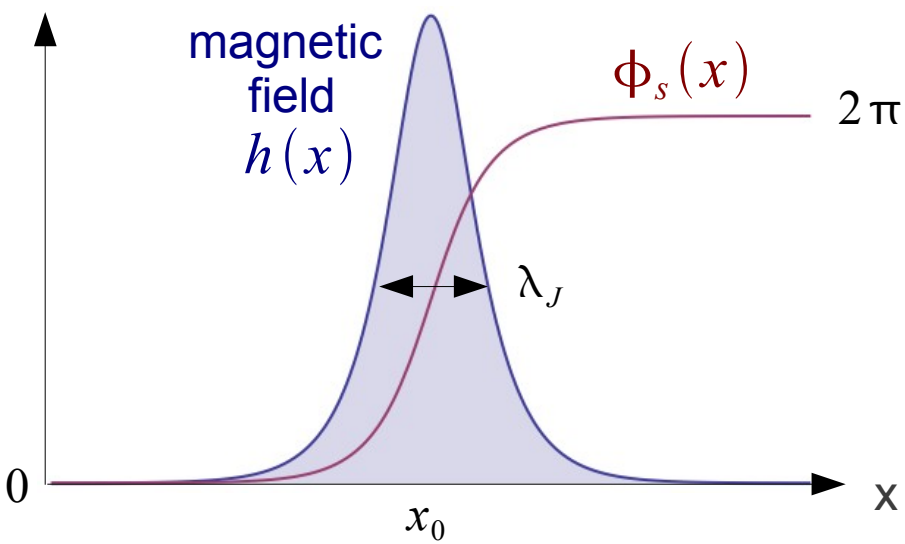


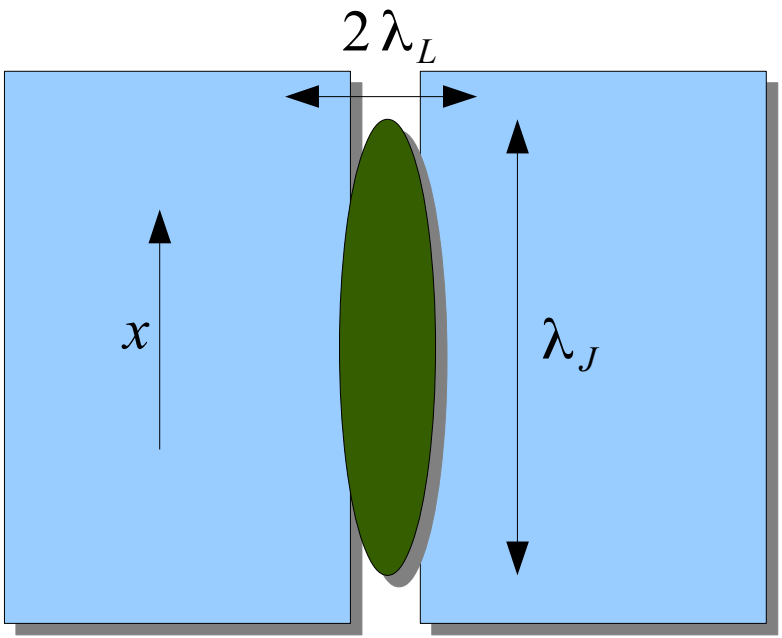
Fu & Kane

Solitons in Josephson Junctions

The solitonic solution is the Josephson vortex

- **Solitons** $$\phi_s(x) = 4\arctan\left[\exp\left(\frac{x - x_0}{\lambda_J}\right)\right]$$ (non-relativistic limit)





Excitations on the background of a soliton

$$H = \int dx\, \Psi^T \begin{pmatrix} i\,v\,\partial_x & m\tanh\left(\dfrac{x-x_0}{\lambda_J}\right) \\ m\tanh\left(\dfrac{x-x_0}{\lambda_J}\right) & -i\,v\,\partial_x \end{pmatrix} \Psi$$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}$$

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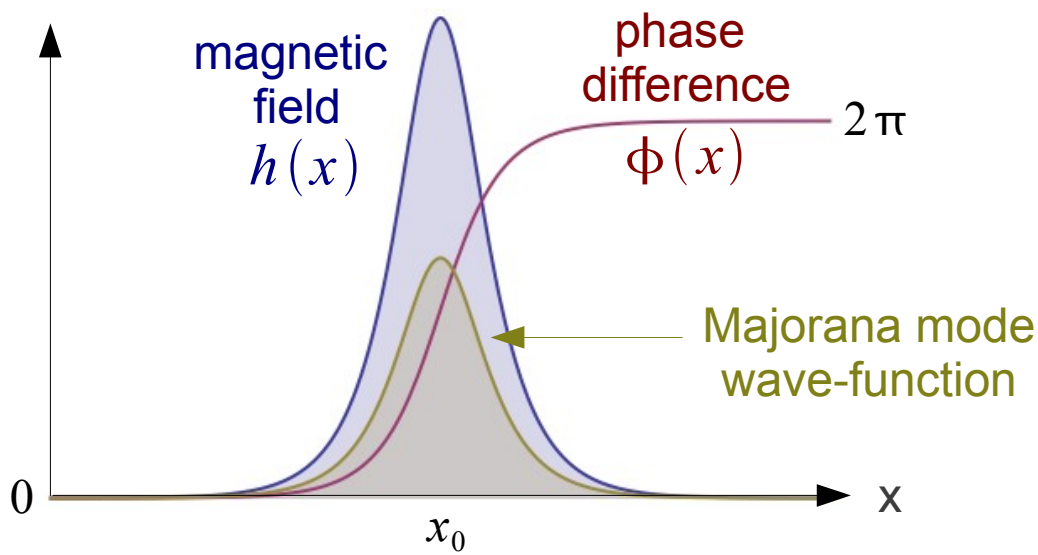
→ similar to **Jackiw-Rebbi Hamiltonian**, binding a localized zero mode

$$\gamma_J = \int dx\, f_{x_0}(x) \left[\psi_1(x) + sgn(m)\, \psi_2(x) \right] \qquad f_{x_0}(x) = \exp\left[-\frac{|m|}{v} \int_0^x dx' \tanh\left(\frac{x' - x_0}{\lambda_J} \right) \right]$$



Soliton – quantum mechanics

Quantize center of mass coordinate of (non-relativistic) soliton

- The soliton is a quantum particle with an effective Hamiltonian

$$H = E_0 + \frac{p^2}{2\mathrm{M}}$$

E. Grosfeld, preliminary

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- <u>The Aharonov-Casher effect</u>: a magnetic moment moving in an electric field

spatial charge distribution → a vector potential for soliton

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E. Grosfeld, preliminary

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  spatial charge distribution → a vector potential for soliton

$$H = E_0 + \frac{(p - A)^2}{2\mathrm{M}}$$

- <u>For topological superconductors</u>: the soliton's momentum is shifted in various ways

$$H = E_0 + \frac{\left[p - \left(\frac{Q}{2e} + i\gamma_1\gamma_2 \pm \frac{1}{16}\right)\frac{2\pi}{L}\right]^2}{2\mathrm{M}} + \ldots$$

E. Grosfeld, preliminary

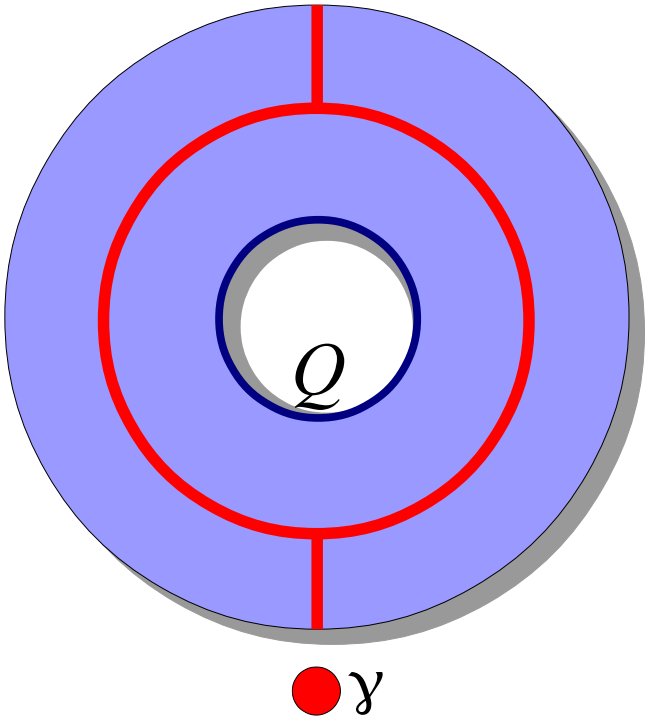
Proposed experiments

How to detect the Majorana mode carried by the soliton:

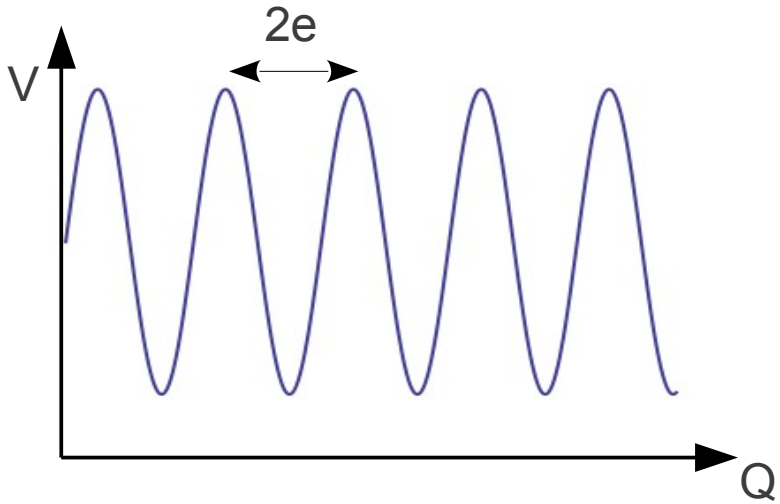
1. **Interference of a soliton beam**

2. **Through its effect on the capacitance of a junction**

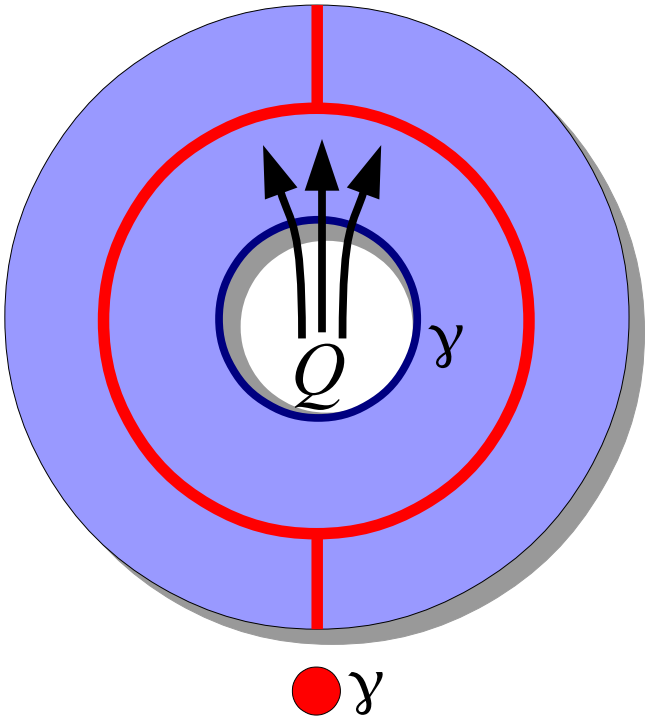
Vortex interferometry – topological superconductor



two knobs: **charge** AND **flux**



Vortex interferometry – topological superconductor



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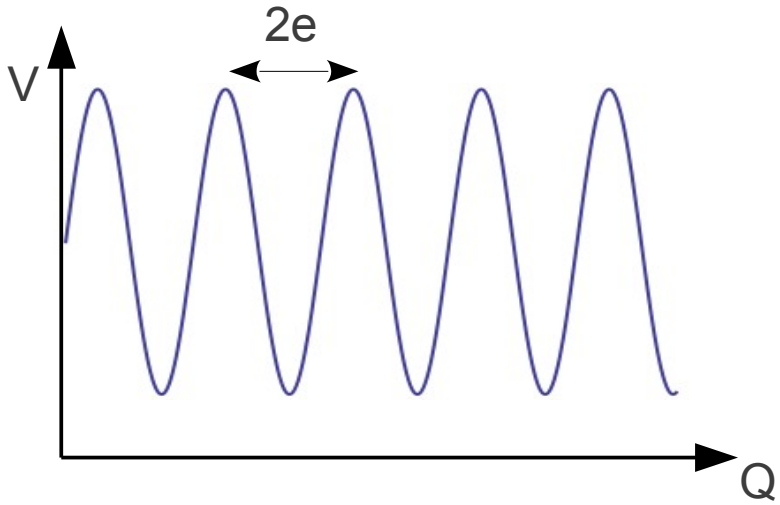
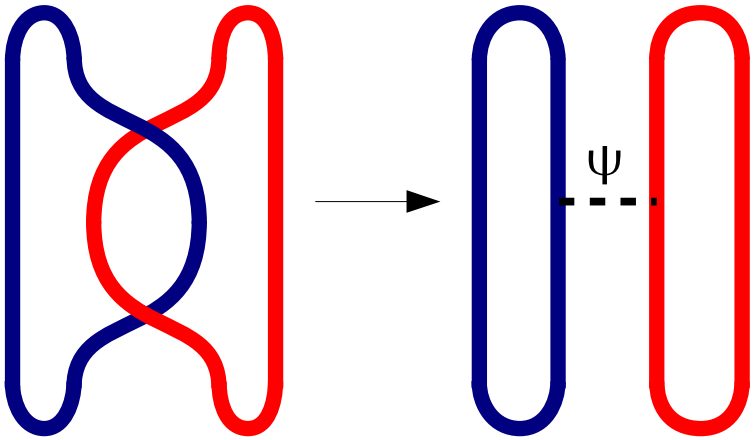
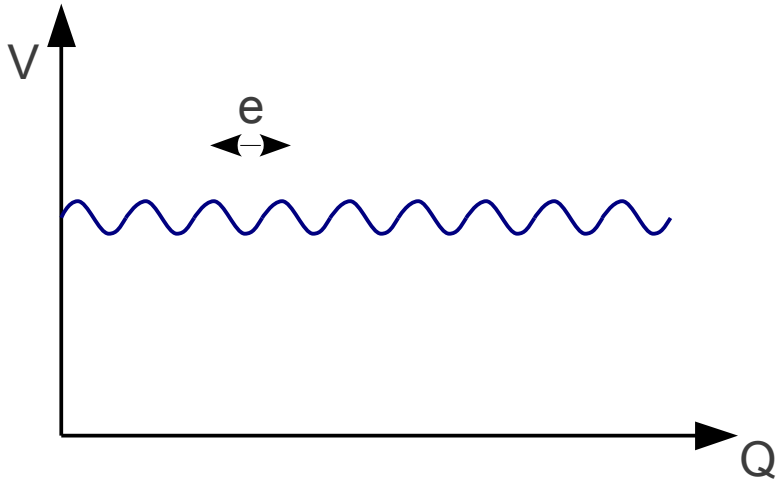




with a vortex,



Persistent motion of a soliton under a charge bias

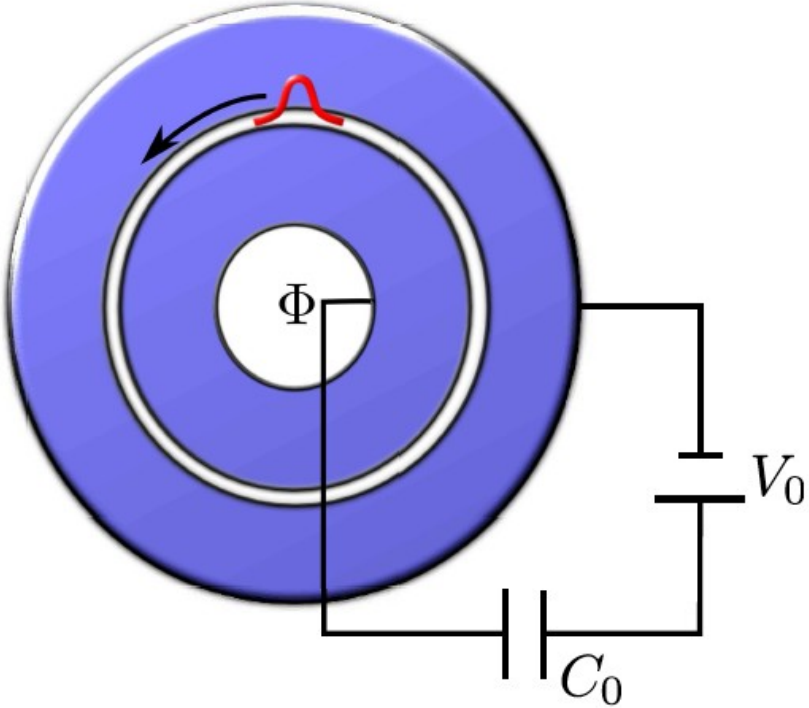
$$H = E_0 + \frac{(p-A)^2}{2\mathrm{M}}$$

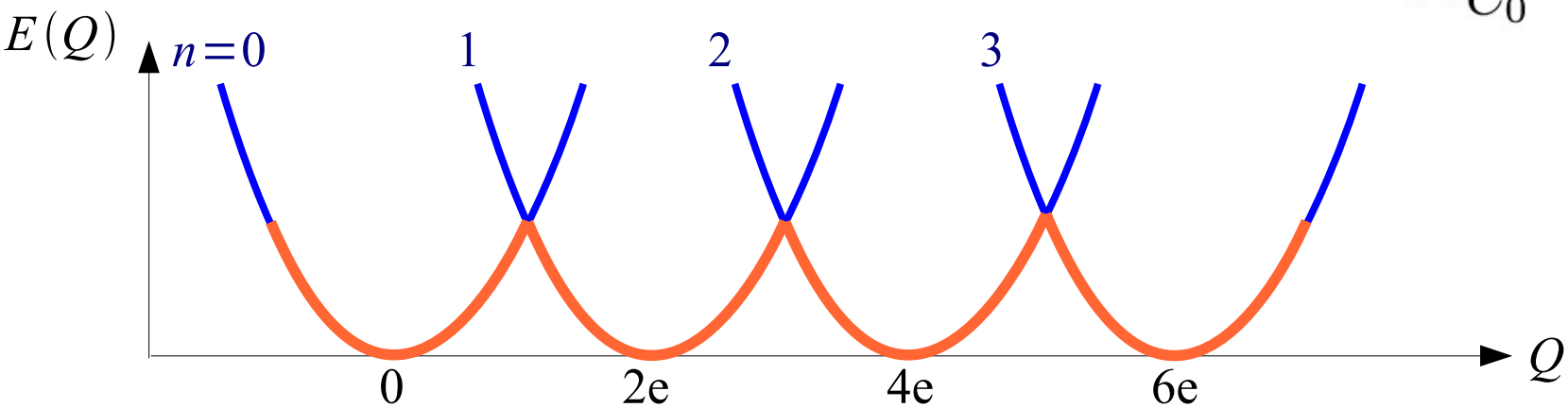
$$E_n(Q) = \frac{(2\pi\hbar)^2}{2ML^2}\left(n - \frac{Q}{2e}\right)^2$$

M - the mass of the soliton

Q - the induced charge

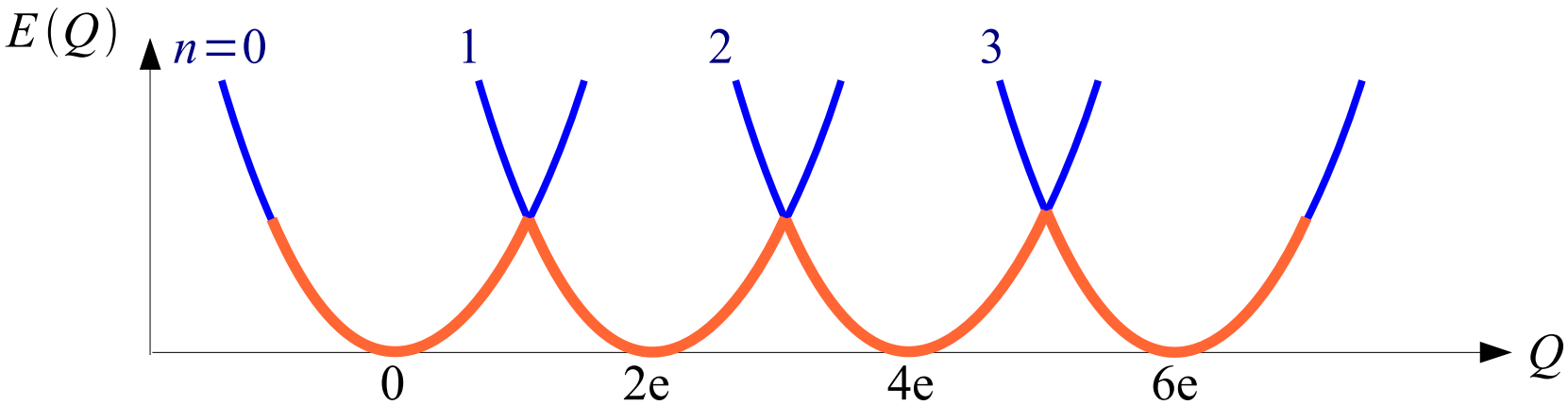
L - the perimeter of the junction

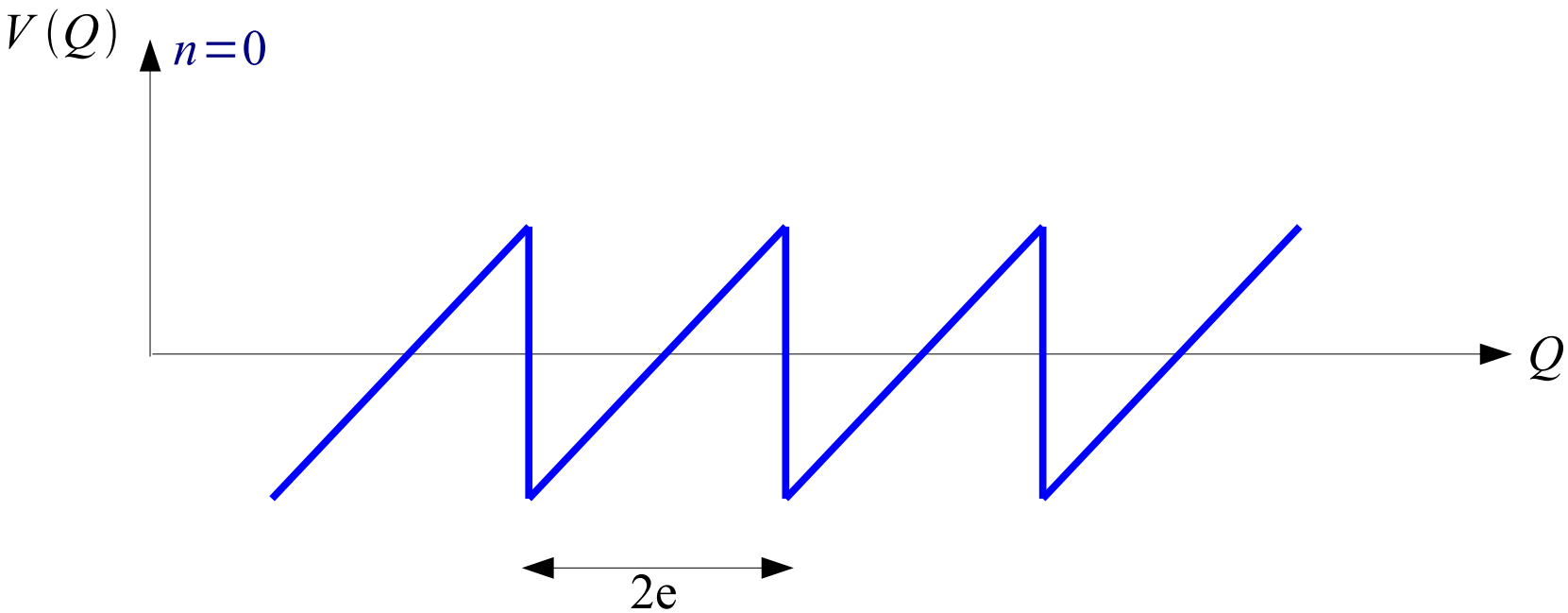




see, e.g., Hermon, Stern and Ben-Jacob 1994

A measurement of the non-linear capacitance of the junction



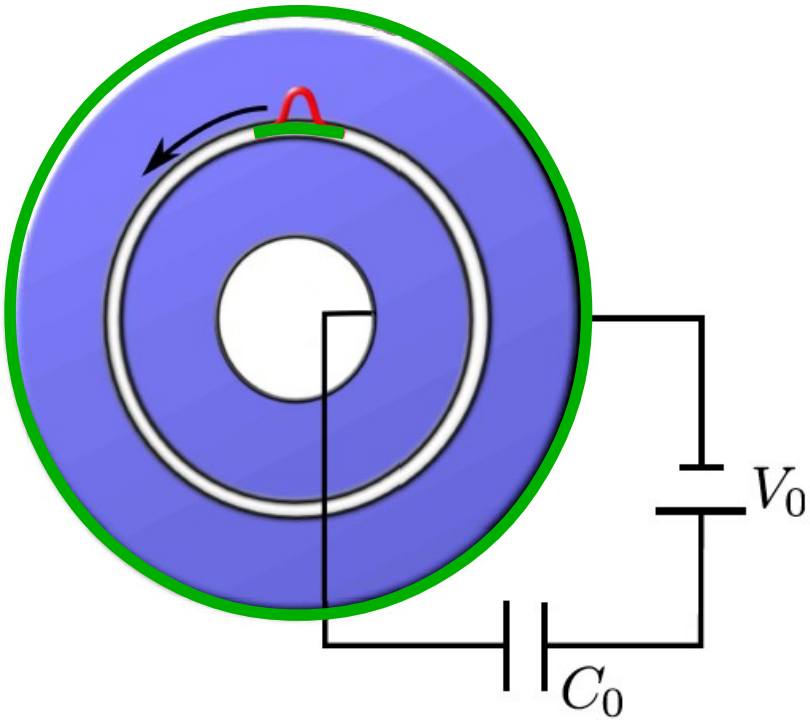


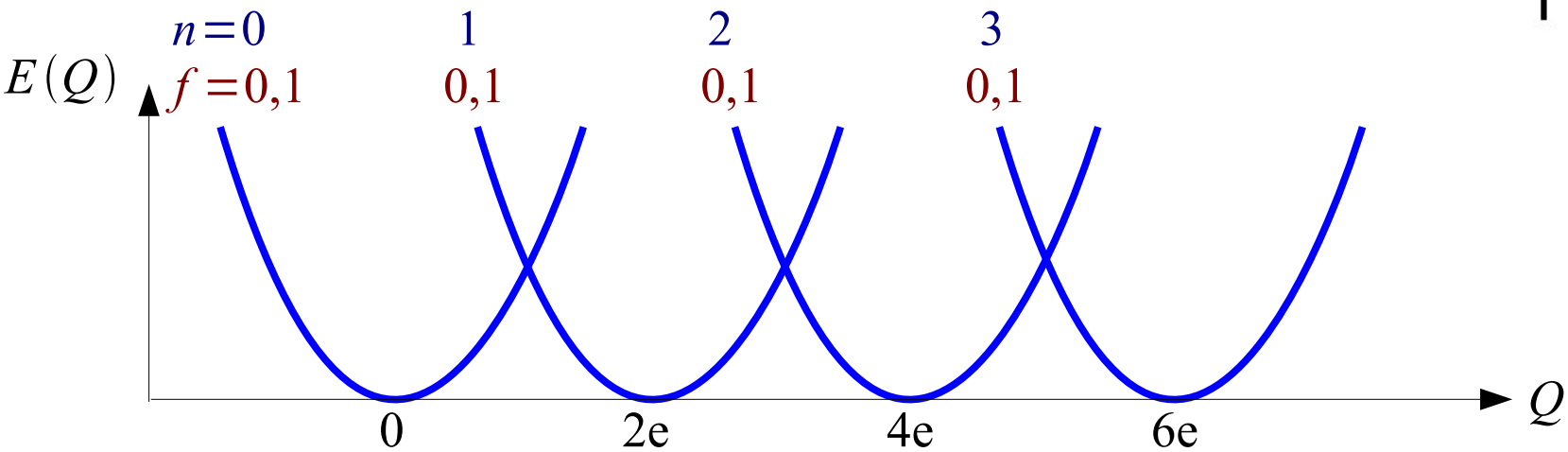
Persistent motion of a fluxon in a topological superconductor

n : difference of number of Cooper pairs between two SCs.

f=0,1: occupation of the zero energy state

$$E_{n,f}(Q)=\frac{(2\pi\hbar)^2}{2ML^2}\left(n-\frac{Q}{2e}\right)^2$$





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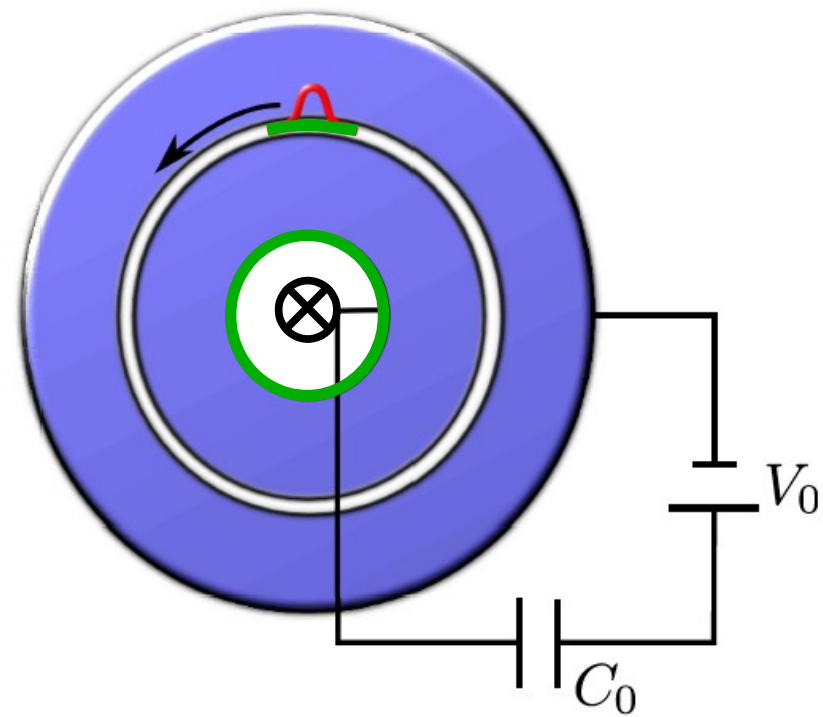
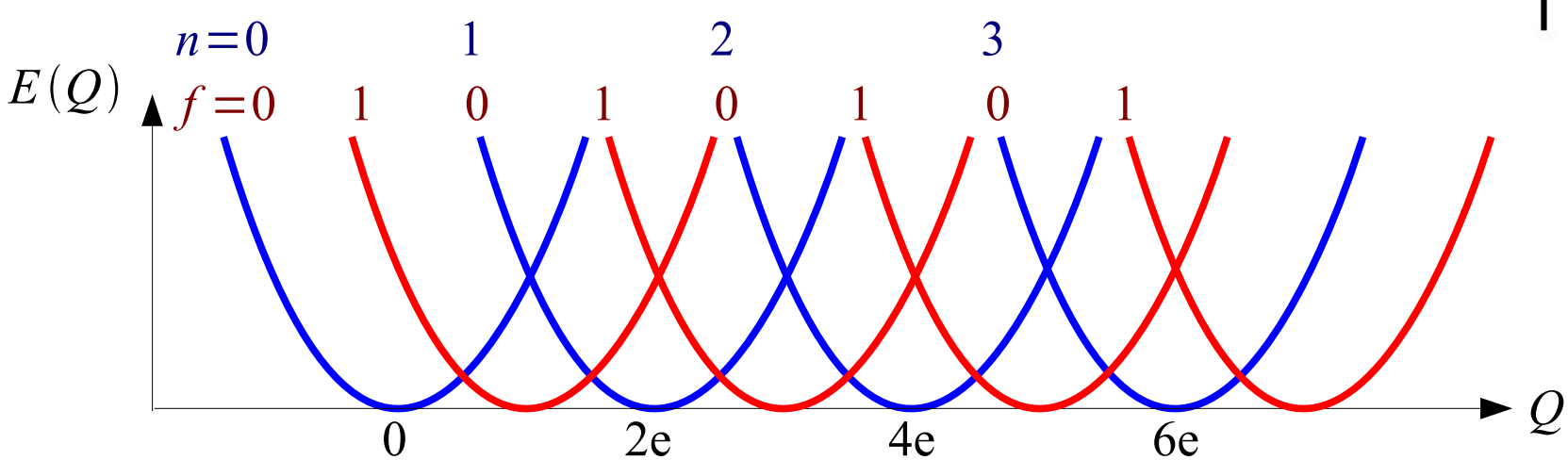
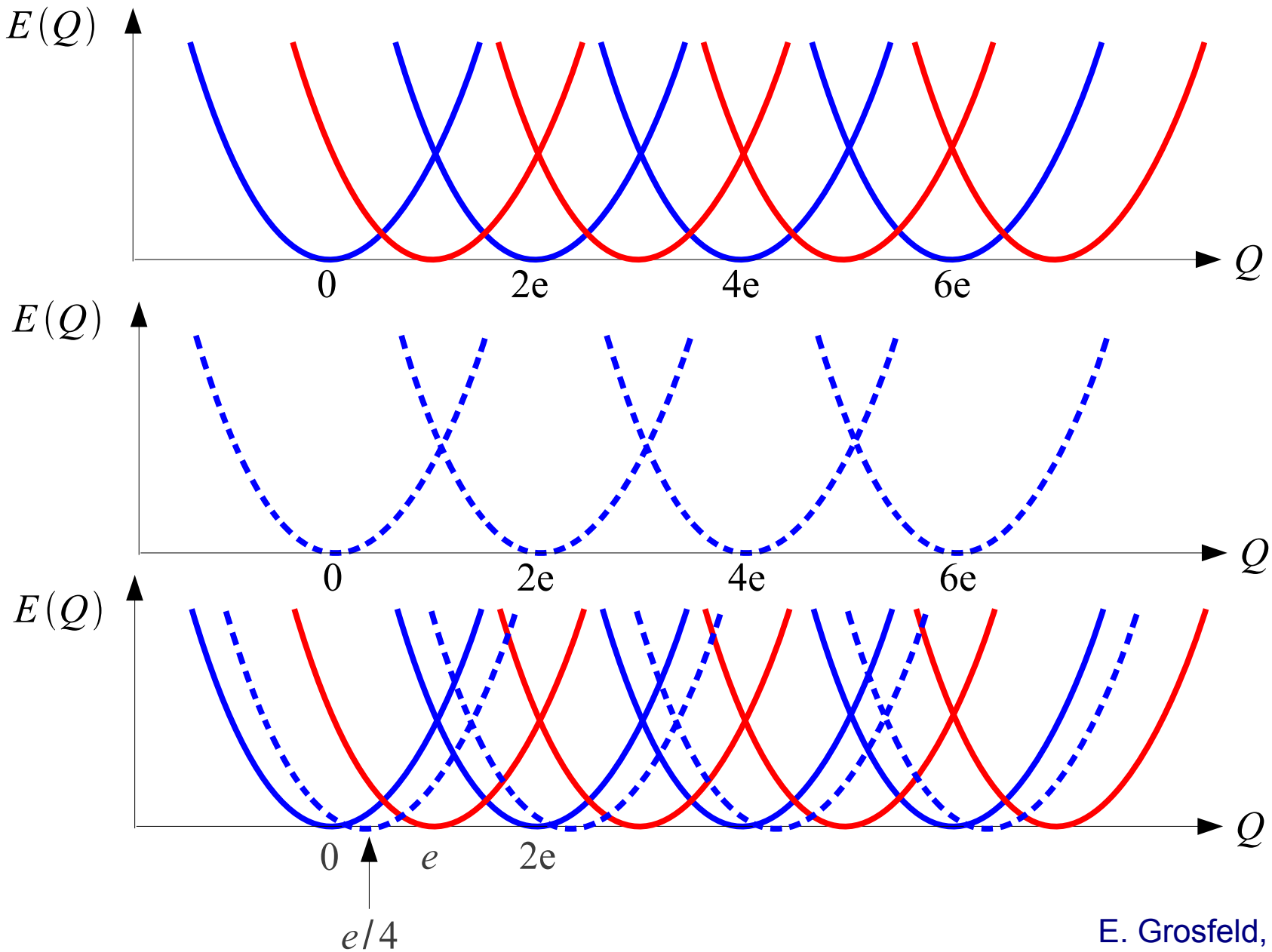
$$E_{n,f}(Q)=\frac{(2\pi\hbar)^2}{2ML^2}\left(n-\frac{f}{2}-\frac{Q}{2e}\right)^2$$

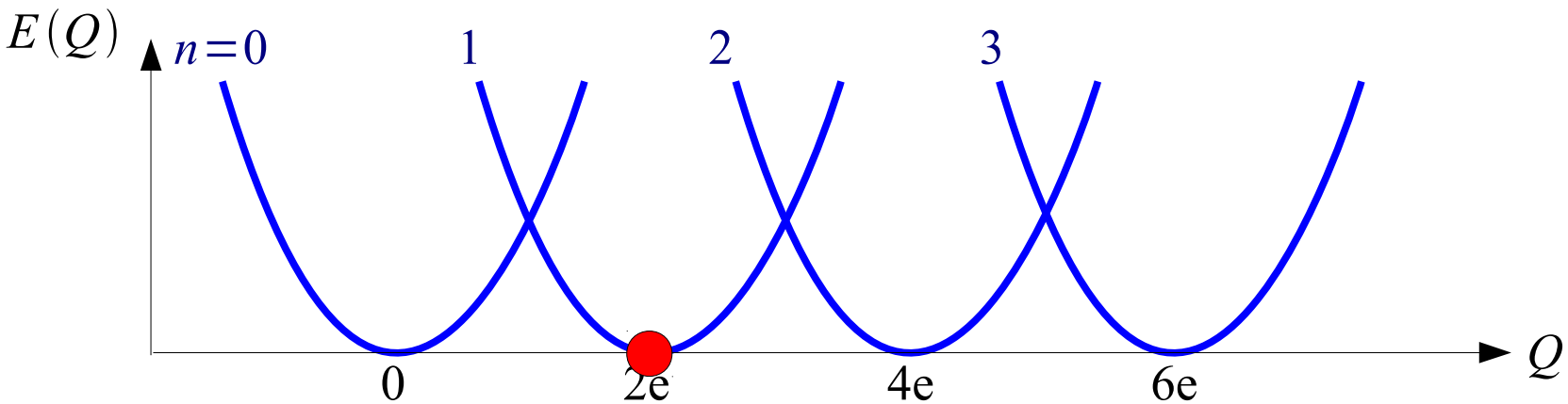


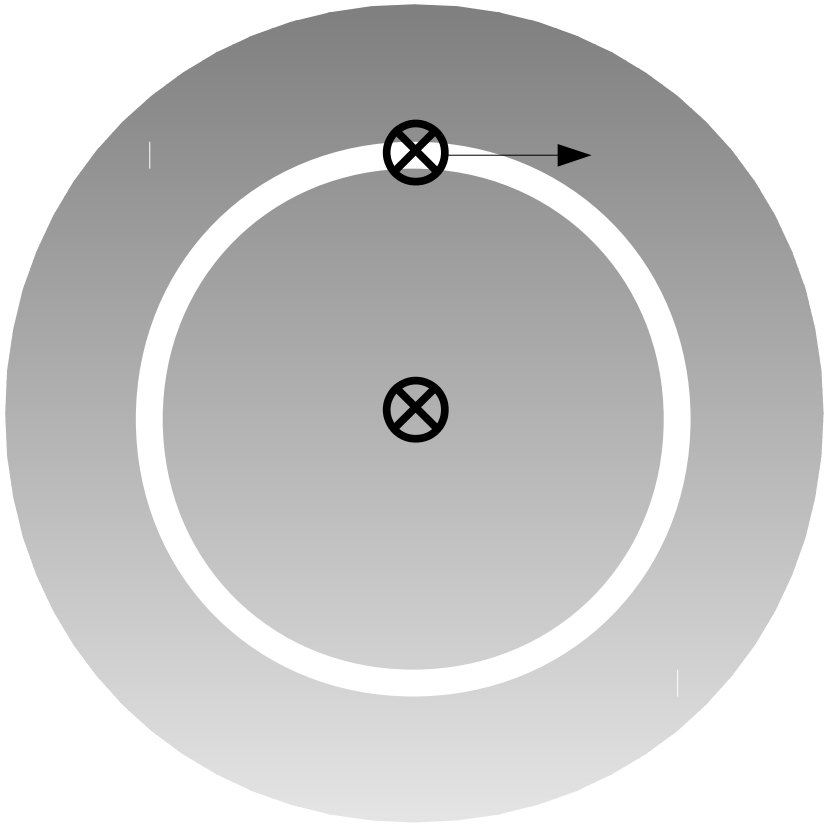


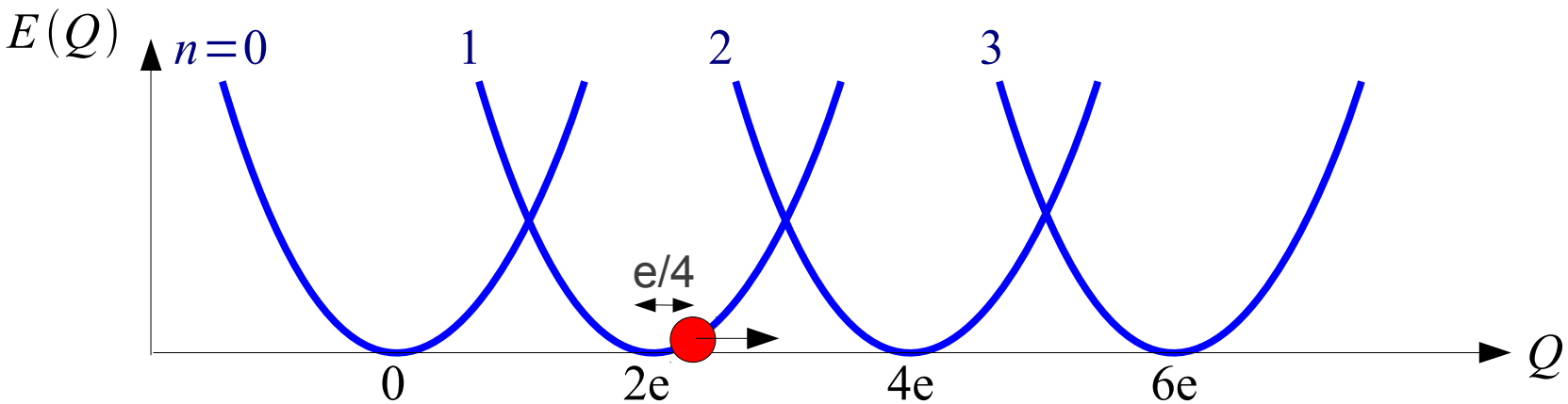
A universal momentum shift



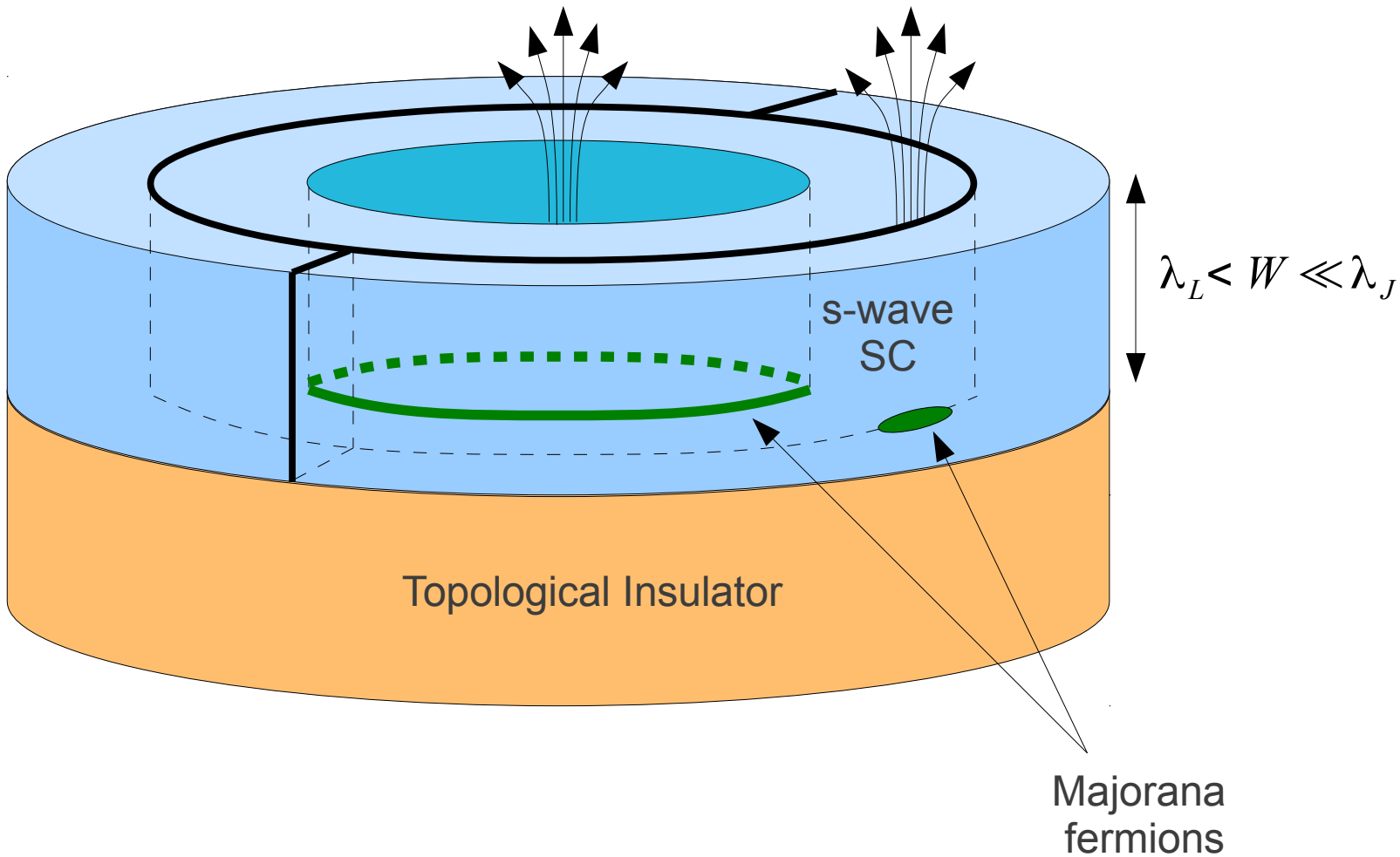
E. Grosfeld, preliminary


E(Q)
n=0
1
2
3
Q
0
2e
4e
6e




E(Q)
n=0
1
2
3
e/4
0
2e
4e
6e
Q

Topological insulator: interferometer



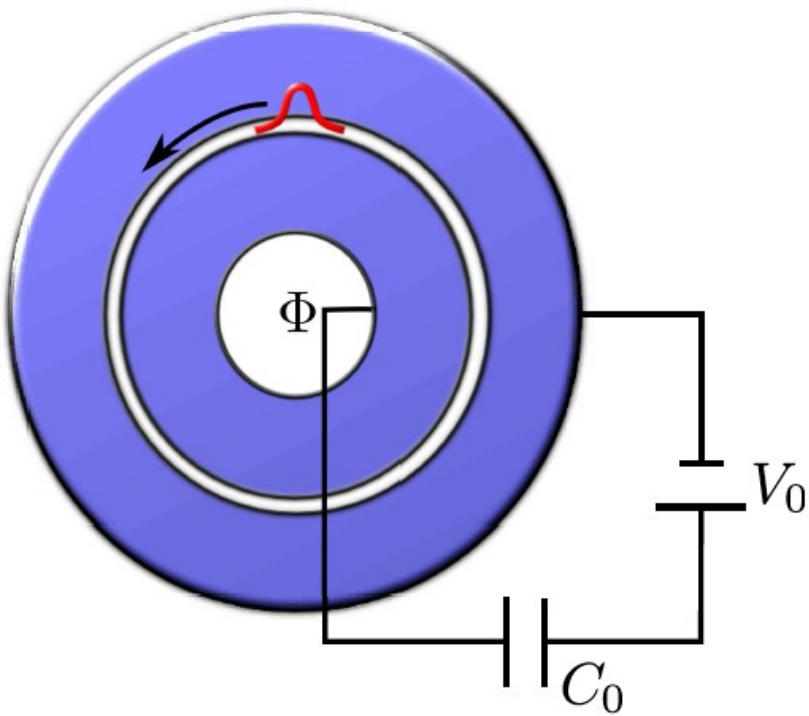
any other topological superconductor (with a single Majorana edge state) can work

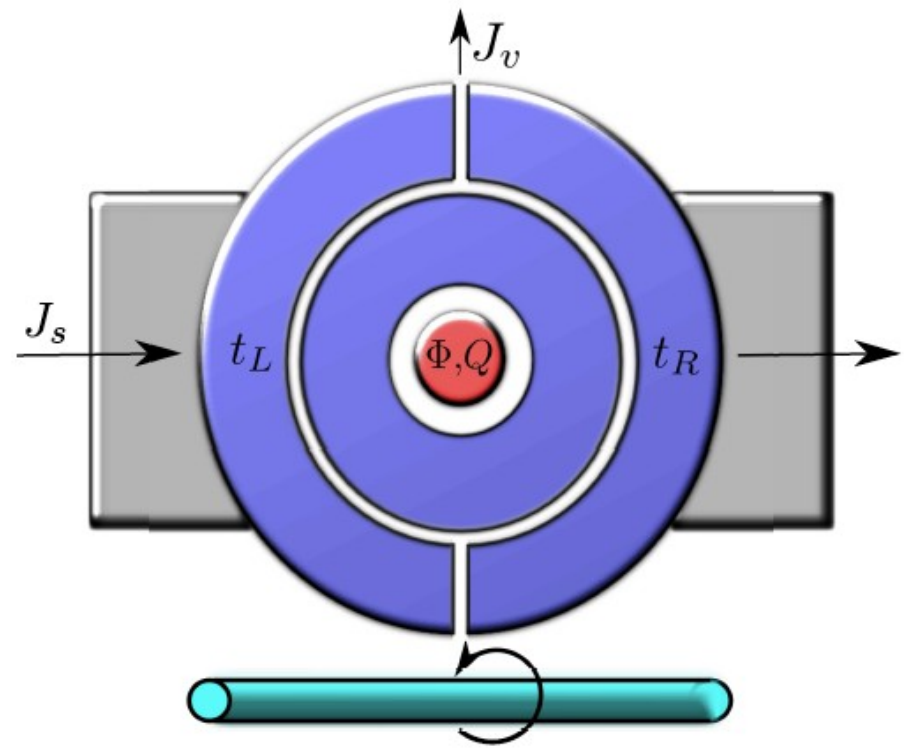
Summary so far

To fully characterize non-abelian statistics of solitons... either:

Detect e vs 2e periodicity of V(Q)
+ relative shift

Detect 2e-periodic oscillations of interference (no vortex)
VS (greatly suppressed) e-periodic oscillations
(with 1 vortex)
VS revival of 2e-periodic oscillations (with 2 vortices)



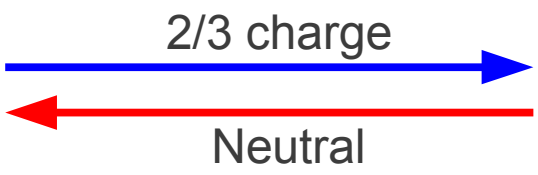


- E. Grosfeld, preliminary results
- E. Grosfeld and A. Stern, PNAS **108**, 11810 (2011)
- E. Grosfeld, B. Seradjeh and S. Vishveshwara, Phys. Rev. B **83**, 104513 (2011)

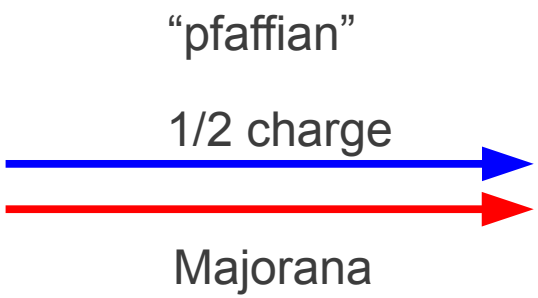
Parallel effort – quantum Hall effect

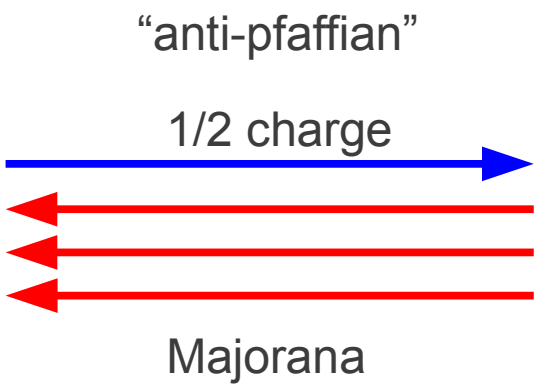
- How to measure the presence (and type) of neutral edge states in the quantum Hall effect

- **Filling factor 2/3**: a charge mode + a counter-propagating neutral mode

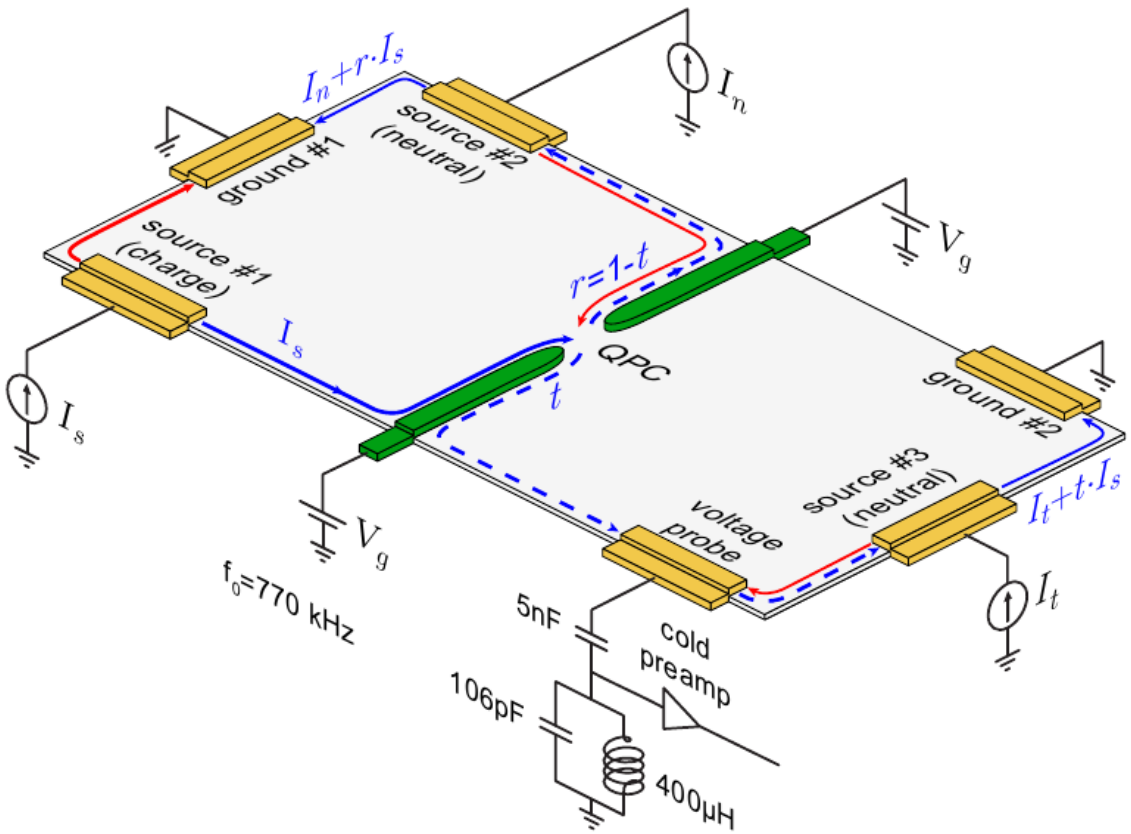


- **Filling factor 5/2**: a charge mode + Majorana modes





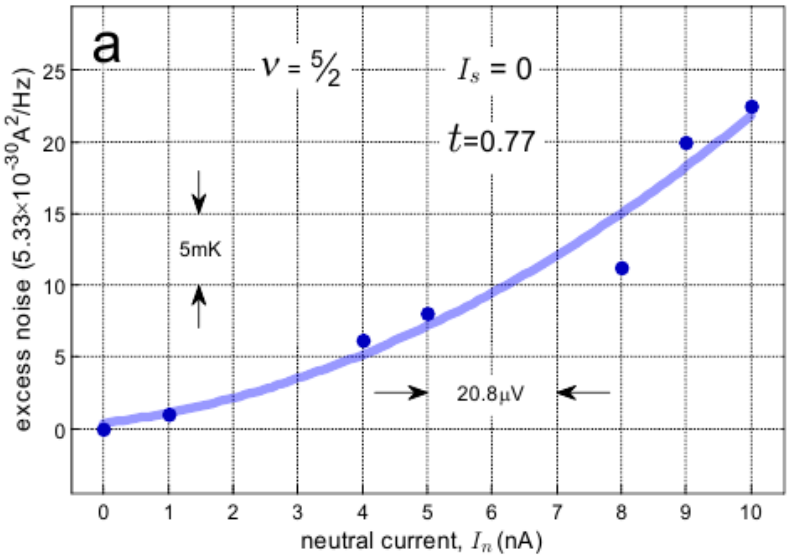
Noise measurements



Theory:
EG and S. Das, 2008
Feldman and Li, 2008

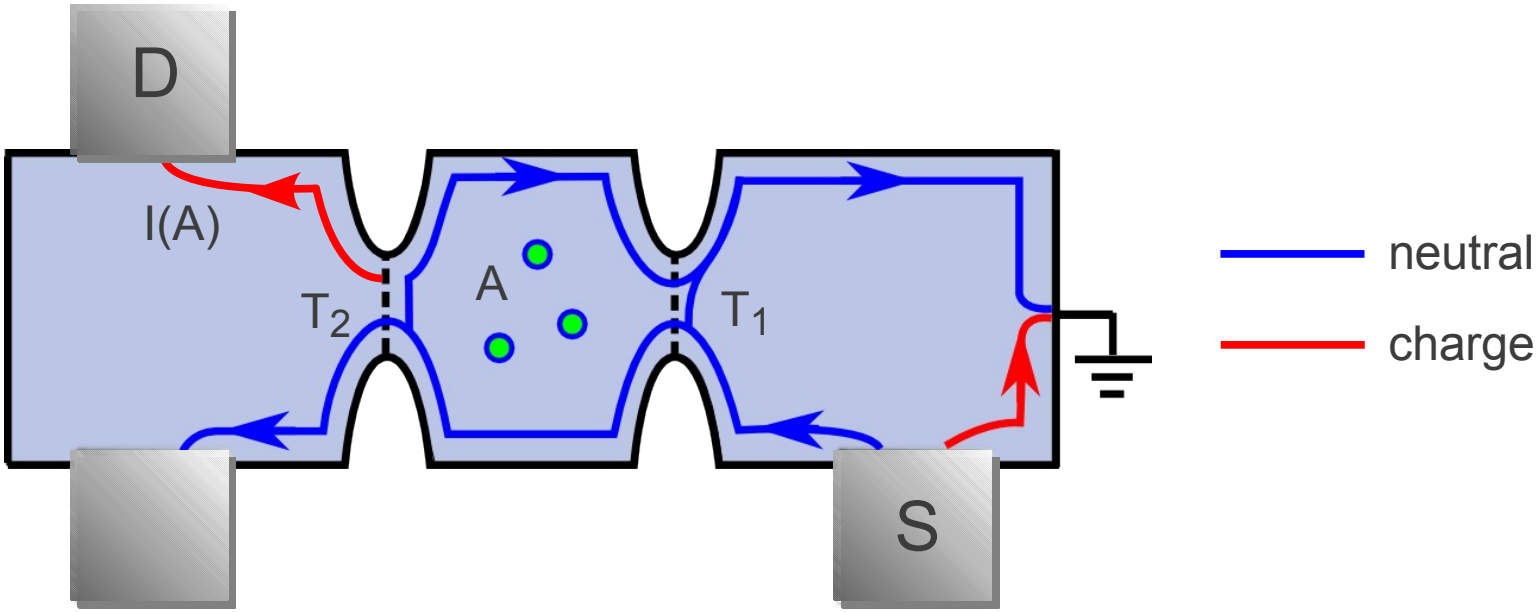
Experiment:
Bid et al, 2010

More theory:
Takei and Rosenow, 2010

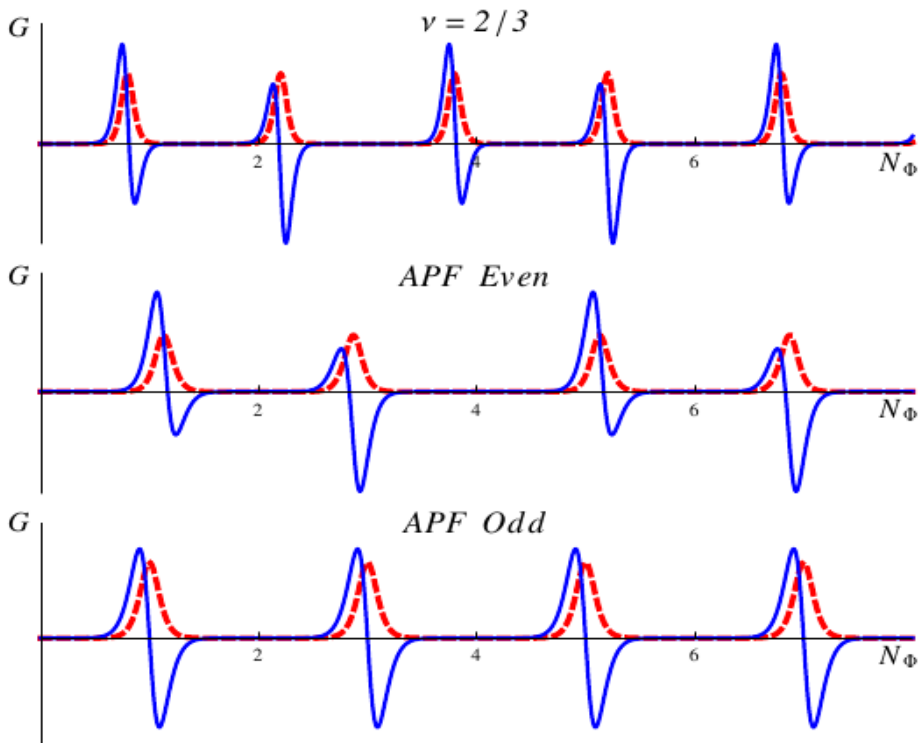


Improved version

- Inject charge current at S that is fully absorbed by the ground, measure at D
- Pumping of heat via the neutral edge state gives rise to a temperature drop across the dot
- Breaking particle-hole symmetry in the dot (via flux or a side gate) gives rise to a thermoelectric effect



- Series of thermoelectric peaks – different for the abelian 2/3 state and non-abelian 5/2 state – hence may help identify the state

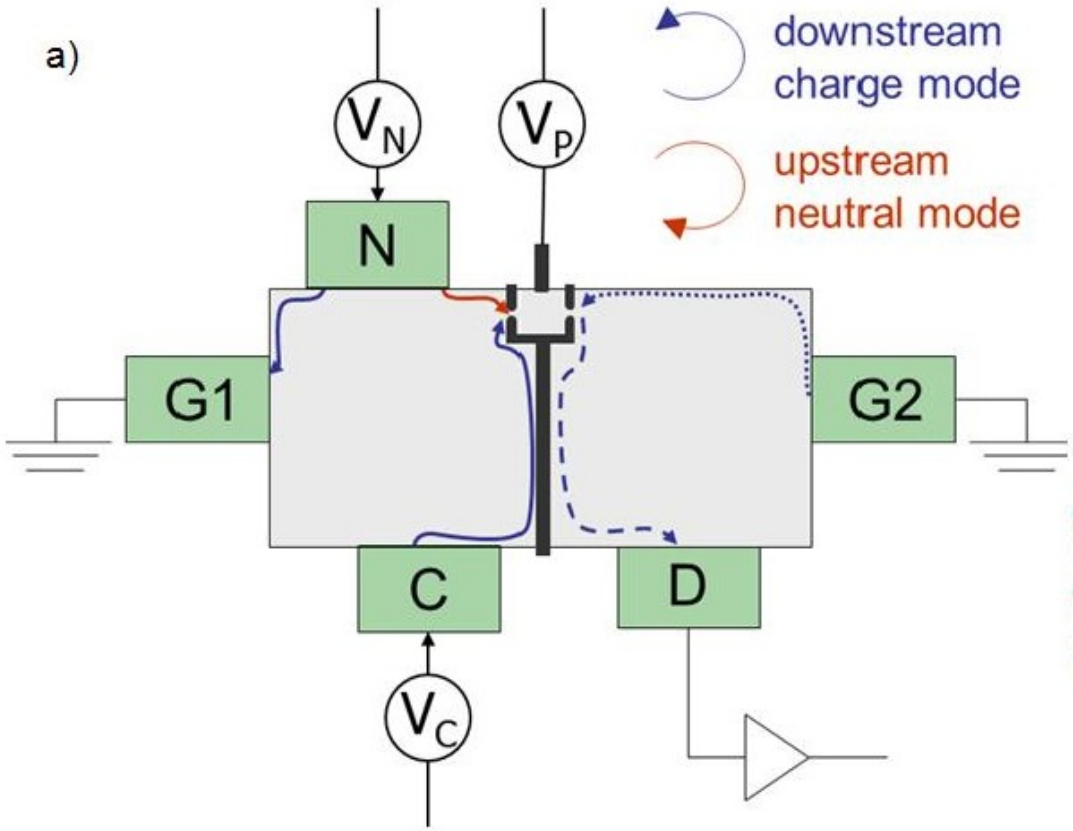


simpler detection of neutral edge states than previous methods
+ possibility to identify Luttinger (2/3) vs Majorana (5/2)

G. Viola, S. Das, EG, A. Stern, arXiv:1203.3813

Extracting net current from an upstream neutral mode in the fractional quantum Hall regime

I. Gurman[§], R. Sabo[§], M .Heiblum*, V. Umansky, D. Mahalu



Observe a series of thermoelectric peaks using the proposed setup!
Peaks appear at 2/3, disappear at 1 and 1/3

- Thank you for your attention!