Quantum dot in a two-dimensional topological superconductor: The two channel Kondo fixed point

Ting Pong Choy
Instituut-Lorentz, Universiteit Leiden

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Outline:

- Brief review of the effective low-energy Hamiltonian for a two dimensional topoloigical superconductor
- 1-channel Kondo problem of helical Majorana edge mode
- Tunneling between two topological superconductors via a quantum dot
- 2-channel Kondo problem of helical Majorana edge modes through quantum dot with strong Coulomb repulsion between two topological superconductors

Two dimensional topological superconductor

- Two dimensional spin-triplet superconductor
  - Superfluid phase of 3He
  - Strontium rutanate Sr2RuO4
  - Non-centrosymmetric superconductor
- Existence of Majorana modes on the boundary
- Wihtout time-reversal symmetry – chiral Majorana modes (n=0,1,2,....)
- With time-reversal symmetry – helical Majorana mode (n=0,1)

Edge state of 2D topological superconductor (chiral)

Bogoliubov-de Gennes (BdG) equation for a two-dimensional spinless px ± i py superconductor

$$\mathcal{H}_{\pm}^{\vartheta} = \begin{pmatrix} -\frac{\hbar^2}{2m}\partial^2 - \mu & \frac{e^{i\vartheta}}{2ik_F}\{\Delta(\boldsymbol{r}), \partial_{\pm}\} \\ \frac{e^{-i\vartheta}}{2ik_F}\{\Delta(\boldsymbol{r}), \partial_{\mp}\} & \frac{\hbar^2}{2m}\partial^2 + \mu \end{pmatrix}.$$

Wave function with open boundary condition

$$\begin{pmatrix} u_k(r) \\ v_k(r) \end{pmatrix} = e^{\pm iky} w_k(x) \begin{pmatrix} e^{i(2\theta+\pi)/4} \\ e^{-i(2\theta+\pi)/4} \end{pmatrix}$$

Energy dispersion $E_k = vk$

$w_k(x) \sim e^{-x/\xi_{SC}}$ - Normalized wave function localized on the edge

Majorana nature of the localized edge mode

Particle-hole symmetry -> another eigenstate with opposite eigenenergy

$$(v_k^*, u_k^*)^T \quad \text{with } E_k = -vk$$

Mode expansion of the field operator for |E|<Δ

$$\begin{pmatrix} \psi(\boldsymbol{r}) \\ \psi^\dagger(\boldsymbol{r}) \end{pmatrix} = \int_0^{k_F} dk \left[\hat{\gamma}_k \begin{pmatrix} u_k(\boldsymbol{r}) \\ v_k(\boldsymbol{r}) \end{pmatrix} + \hat{\gamma}_k^\dagger \begin{pmatrix} v_k^*(\boldsymbol{r}) \\ u_k^*(\boldsymbol{r}) \end{pmatrix} \right],$$

$$\psi(r) = ie^{i\theta}\psi^\dagger(r)$$

Rewritten in term of the conventional Majorana field operator

$$\psi(r) = e^{i\theta}e^{i\pi/4}\gamma(x)$$

$$\gamma(x) = \gamma^\dagger(x)$$

Majorana fermion!

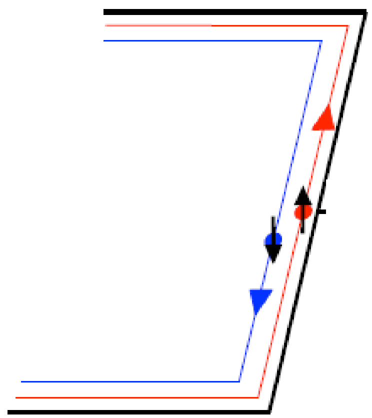
Two-dimensional topological superconductor with time-reversal symmetry (helical)

$$H = \begin{pmatrix} H_{+}^{\theta_\uparrow} & 0 \\ 0 & H_{-}^{\theta_\downarrow} \end{pmatrix}$$

Effective Hamiltonian for the low-lying edge modes (pseudospin)

$$H_0 = iv\int_{-\infty}^{\infty} dy\left(\gamma_\uparrow(y)\partial_y\gamma_\uparrow(y) - \gamma_\downarrow(y)\partial_y\gamma_\downarrow(y)\right)$$

Helical edge mode on the boundary



No local interaction term can be written within the low-lying Majorana modes

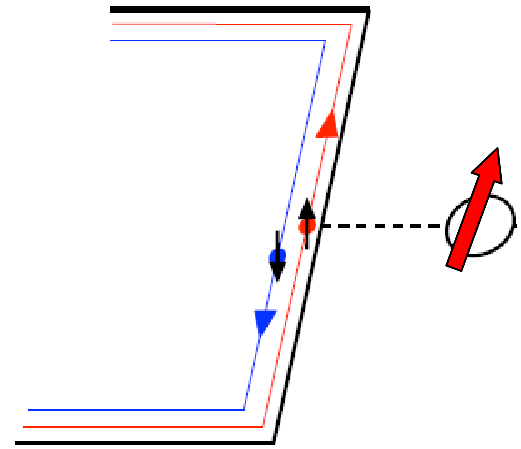
$$\rho_\uparrow\rho_\uparrow = \rho_\downarrow\rho_\downarrow = \rho_\uparrow\rho_\downarrow = 0$$

Kondo problem of helical Majorana mode

Shindou et. al.2010

Coupled to S=1/2 magnetic impurity (Ising-like)

$$H_{ex} = \frac{J}{2}\vec{S}\bullet\psi^{\dagger}_{\alpha}\vec{\sigma}_{\alpha\beta}\psi_{\beta} = iJS_z\gamma_{\uparrow}\gamma_{\downarrow}$$



Bosonization

$$\Psi(y) = (\gamma_{\uparrow}(y) + i\gamma_{\downarrow}(-y)) = \frac{1}{2\pi\alpha}Ue^{i\Phi(y)}$$

Ohmic dissipative two-state Hamiltonian

$$H = H_0 + H_{ex} = \frac{v}{2\pi}\int(\partial_y\Phi)^2 dy + \frac{J}{2\pi}S_z(\partial_y\Phi)\Big|_{y=0} + hS_x$$

Transverse fiield

Emery-Kivelson transformation:

$$W = e^{i\sqrt{\varepsilon}\Phi(0)S_z}$$

$$H_o^{'} = WH_0W^{\dagger} = \frac{v}{2\pi}\int(\partial_y\Phi)^2 dy + h\left(S^+e^{i\sqrt{\varepsilon}\Phi(0)} + S^-e^{-i\sqrt{\varepsilon}\Phi(0)}\right)$$

Renormalization group equation:

$$\frac{d\epsilon}{d\ln\tau} = -4\epsilon\eta^2, \quad \frac{d\eta}{d\ln\tau} = \frac{1}{2}(-\epsilon+2)\eta.$$

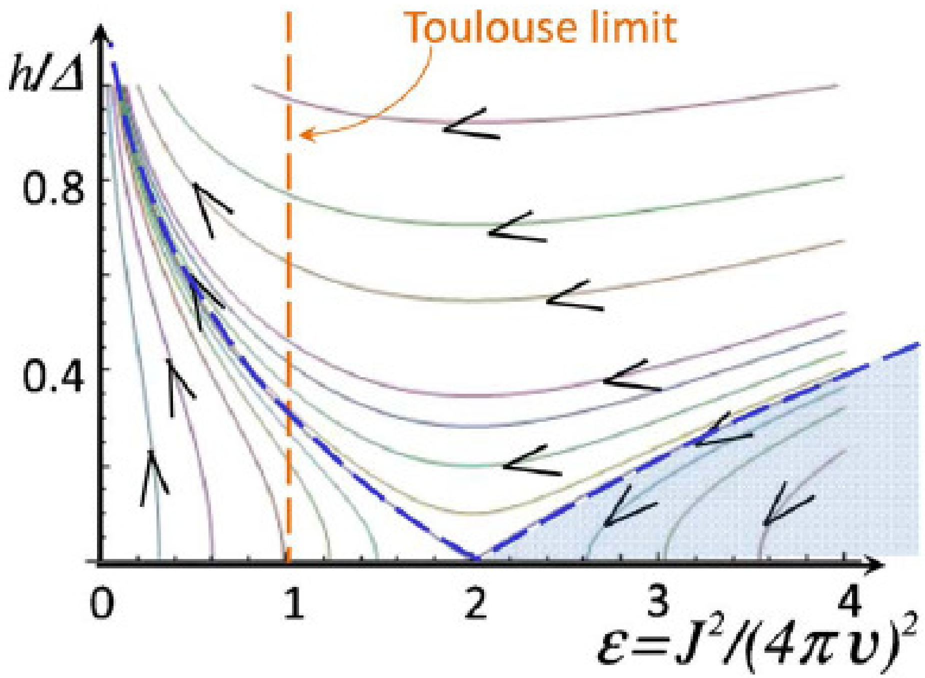
Dimensionless coupling constant

$$\varepsilon = (J/4\pi v)^2$$

$$\eta = h\alpha / v$$

Renormalization group flow

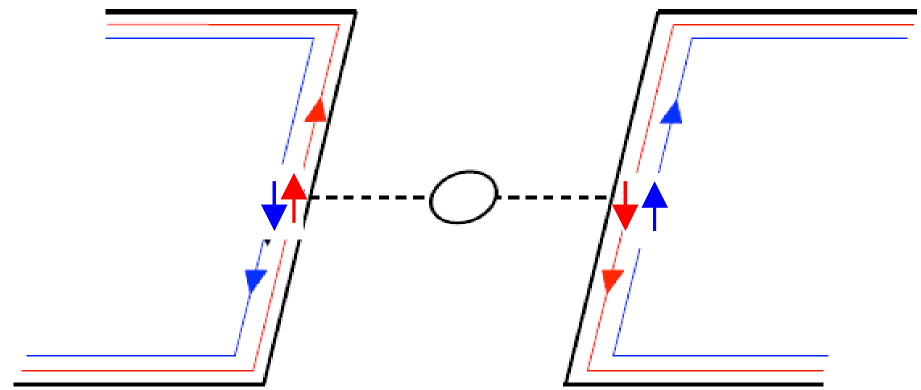
AFM Kondo singlet $\left\langle S^z \right\rangle_{h_z \to 0^+} = 0$



FM Kondo $\left\langle S^z \right\rangle_{h_z \to 0^+} \neq 0$

Tunneling between two helical SC through point contact

Asana et. al. (2010)



•Neutral Majorana fermions can interact with electric field because the superconducting phase enters into the relation between electron and Majorana fermion operators

$$\psi_{n\sigma}(r) = e^{\pm i\pi/4} e^{i\theta_n} \gamma_{n\sigma}(x)$$

•Josephson effect through the neutral Majorana quasiparticles

Kinetic energy for opposite helicity

$$H_0 = -iv_0 \sum_{n=1,2} \int dx \left(\gamma_{n\uparrow}(x)\partial_x \gamma_{n\uparrow}(x) - \gamma_{n\downarrow}(x)\partial_x \gamma_{n\downarrow}(x) \right)$$

Bosonization for two islands helical SC (opposite helicity)

Recombination of Majorana fermions

$$\Psi_R(x) = \gamma_{1\uparrow}(x) + i\gamma_{2\uparrow}(x)$$
$$\Psi_L(x) = \gamma_{1\downarrow}(x) + i\gamma_{2\downarrow}(x)$$

Bosonization $\Psi_{L,R}(x) = \frac{U_{L,R}}{\sqrt{2\pi\alpha}} e^{\pm i\phi_{L,R}(x)},$

Bosonized **'spinless'** fermion

$$H_0 = \frac{v_0}{2\pi}\int dx\left((\partial_x\phi(x))^2 + (\partial_x\theta(x))^2\right)$$
$$2\phi(x) = \phi_L(x) + \phi_R(x)$$
$$2\theta(x) = \phi_L(x) - \phi_R(x)$$

Possible interactions within the low-lying Majorana edge modes

$$H_1 = U_1 \int dx [\Psi^\dagger_{1\uparrow} \Psi_{1\downarrow} \Psi^\dagger_{2\uparrow} \Psi_{2\downarrow} + h.c.],$$

$$H_2 = U_2 \int dx [\Psi^\dagger_{1\uparrow} \Psi_{1\downarrow} \Psi_{2\uparrow} \Psi^\dagger_{2\downarrow} + h.c.].$$

$$V = g \int dx \gamma_{1\uparrow}(x) \gamma_{2\uparrow}(x) \gamma_{1\downarrow}(x) \gamma_{2\downarrow}(x),$$

$$g = 2(U_2 - U_1)$$

Bosonized Hamiltonian

$$H_0 + H_{\text{int}} = \frac{\tilde{v}}{8\pi} \int dx \frac{\{\partial_x \phi(x)\}^2}{K} + K\{\partial_x \theta(x)\}^2,$$

Normalized velocity $v = \sqrt{1 - g'^2} v_0,$

Luttinger parameter $K = \sqrt{\frac{1-g'}{1+g'}},$ $g' = \frac{g}{8\pi v_0}.$

Tunneling between two helicla SC

$$H_T = -ta \sum_{\sigma,\sigma'} [\Psi^\dagger_{1,\sigma}(0)\{\sigma_0 + i\boldsymbol{\lambda}\cdot\boldsymbol{\sigma}\}_{\sigma,\sigma'}\Psi_{2,\sigma'}(0) + \Psi^\dagger_{2,\sigma}(0)\{\sigma_0 - i\boldsymbol{\lambda}\cdot\boldsymbol{\sigma}\}_{\sigma,\sigma'}\Psi_{1,\sigma'}(0)],$$

•Spin-orbital coupling at the point contact – **λ**
•Rashba-type SOI can be induced by appling an electric field on the point contact

Bosonized tunneling Hamiltonian **No Spin-flip (forward tunneling)**

$$H_T = \frac{ta}{\pi}\left[\sin(\frac{\varphi}{2})\partial\theta(x) - \lambda_3\cos(\frac{\varphi}{2})\partial\phi(x)\right]_{x=0} + \frac{i\eta_L\eta_R ta}{\pi\alpha_0}\left[\frac{\lambda_+}{2}\cos(\frac{\varphi}{2})\sin\theta(x) - \frac{\lambda_-}{2}\sin(\frac{\varphi}{2})\sin\phi(x)\right]_{x=0}$$

Spin-flip (backward tunneling)

Josephson current between two helical SC

$$J = \frac{2e}{\hbar}\frac{\partial H}{\partial \varphi}$$

$$\langle J \rangle = e\Delta\left[\frac{at}{\pi v}\right]^2 \sin\varphi\left[\frac{1}{K} - \lambda_3^2 K - \lambda_+^2 + \lambda_-^2\right],$$

Josephson current can be suppressed by spin-orbital coupling

Non-interacting s-wave Josephson junction $J \propto (1 + \boldsymbol{\lambda}^2)$

DC conductivity between two helical SC via the Josephson junction (Kubo formuia)

$$\sigma = -\lim_{\omega\to 0^+}[Q^R(\omega) - Q^R(0)]/(i\omega),$$

$$Q(\omega_n) = -\int_0^{1/T} d\tau e^{i\omega_n\tau}\langle J(\tau)J(0)\rangle,$$

DC conductivity via the point contact

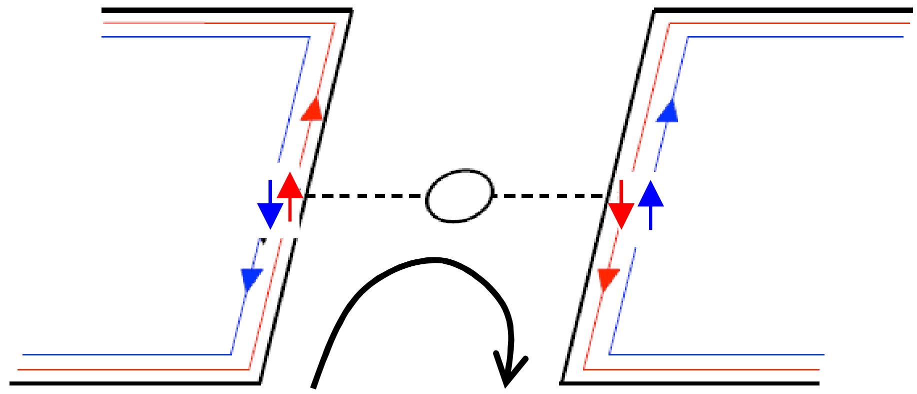
$$\frac{\sigma}{G_0} = \pi\frac{\sin^2(\varphi/2)}{K} + \lambda_+^2\cos^2\left(\frac{\varphi}{2}\right)D_\theta\left(\frac{T}{T_0}\right)^{2/K-2} + \pi\lambda_3 K\cos^2(\varphi/2) + \lambda_-^2\sin^2\left(\frac{\varphi}{2}\right)D_\phi\left(\frac{T}{T_0}\right)^{2K-2}.$$

		$\lambda = 0$	$\lambda \neq 0$
Equal helicity			
$\varphi = 0$	$K = 1$	0	const
	$K < 1$	0	T^{2K-2}
	$K > 1$	0	const
$\varphi \neq 0$	$K = 1$	const	const
	$K < 1$	$T^{2/K-2} \to 0$	T^{2K-2}
	$K > 1$	$T^{2/K-2}$	$T^{2/K-2}$
Opposite helicity			
$\varphi = 0$	$K = 1$	0	const
	$K < 1$	0	const
	$K > 1$	0	$T^{2/K-2}$
$\varphi \neq 0$	$K = 1$	const	const
	$K < 1$	const	T^{2K-2}
	$K > 1$	const	$T^{2/K-2}$

Asana et. al. (2010)

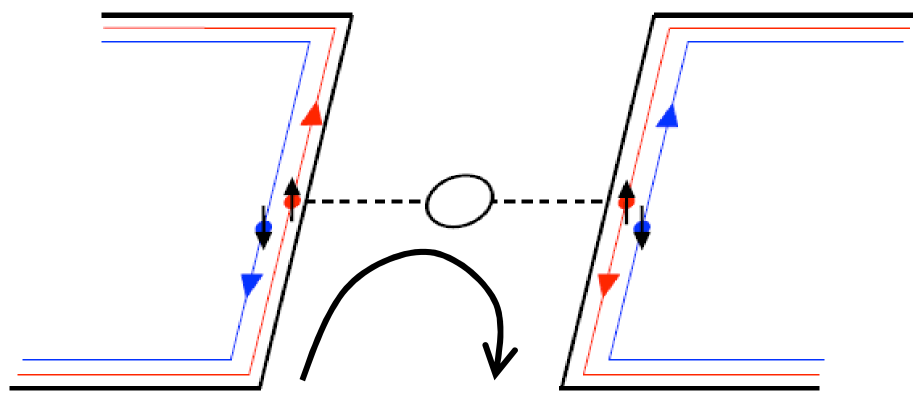
- Conductance depends on the relative helicity between two superconductors
- In the presence of SOI, electron interactions can drive the dc condcutance to infinite at zero temperature.

Opposite helicity



Backscattering with spin-flip

Equal helicity



Backscattering without spin-flip

Experiments on electron transport through quantum dots coupled to superconducting leads

[1] E. Scheer *et al.*, Phys. Rev. Lett. **78**, 3535 (1997).
[2] M. R. Buitelaar, T. Nussbaumer, and C. Schönenberger, Phys. Rev. Lett. **89**, 256801 (2002).
[3] M. R. Buitelaar *et al.*, Phys. Rev. Lett. **91**, 057005 (2003).
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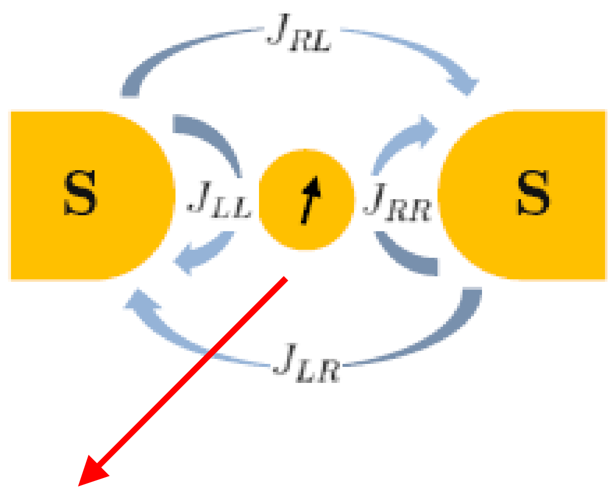
[6] H. B. Heersche *et al.*, Nature (London) **446**, 56 (2007).
[7] J. Xiang *et al.*, Nature Nanotech. **1**, 208 (2006).

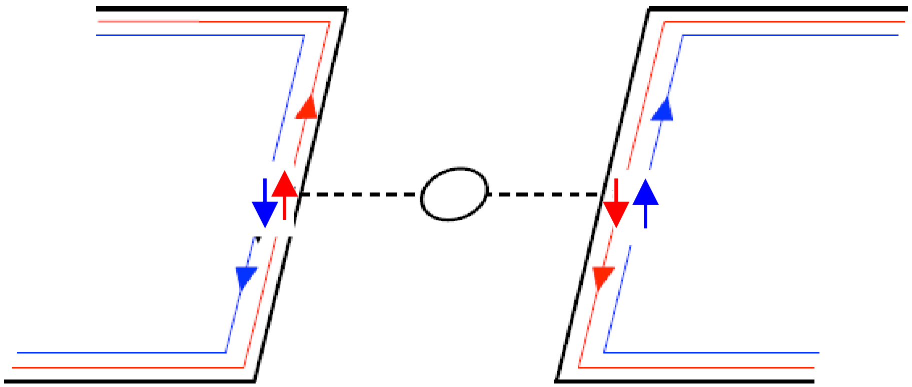


(BM Anderson 2011)

Carbon nanotube, molecules of Gadolinium (Gd) metallofullerenes

Tunneling between two helical SC through quantum dot with strong Coulomb repulsion (opposite helicity)

$d_\uparrow, d_\downarrow$ Localized state on the quantum dot



Anderson Hamiltonian

$$H_t = t\sum_{i=1,2}\sum_{\sigma}[d^\dagger_\sigma \Psi_{i\sigma}(0) + h.c.].$$

$$H_d = \sum_\sigma \epsilon d^\dagger_\sigma d_\sigma + U d^\dagger_\uparrow d_\uparrow d^\dagger_\downarrow d_\downarrow,$$

Two channel Kondo Hamiltonian

$$H_K = \sum_i J_1 \vec{S}\cdot\left(\Psi^\dagger_{i\alpha}\frac{\vec{\sigma}_{\alpha\beta}}{2}\Psi_{i\beta}\right) + \sum_{i\neq j} J_2 \vec{S}\cdot\left(\Psi^\dagger_{i\alpha}\frac{\vec{\sigma}_{\alpha\beta}}{2}\Psi_{j\beta}\right),$$

Bosonized Hamiltonian

Backward tunneling

$$H_K^o = \frac{J_1}{2\pi\alpha} S^x (iU_L U_R) \cos(\frac{2}{\sqrt{K}}\theta(0))$$
$$+ \frac{J_2^\theta}{2\pi\alpha} \cos\frac{\varphi}{2} S^y (iU_L U_R) \sin(\frac{2}{\sqrt{K}}\theta(0))$$
$$- \frac{J_2^\phi}{2\pi\alpha} \sin\frac{\varphi}{2} S^x (iU_L U_R) \sin(2\sqrt{K}\phi(0))$$
$$- \frac{J_2^z}{2\pi\sqrt{K}} \cos\frac{\varphi}{2} S^z \partial\theta(0).$$

Inter-channel process

$$\Psi_R(x) = \gamma_{1\uparrow}(x) + i\gamma_{2\uparrow}(x)$$
$$\Psi_L(x) = \gamma_{1\downarrow}(x) + i\gamma_{2\downarrow}(x)$$

$$\Psi_{L,R}(x) = \frac{U_{L,R}}{\sqrt{2\pi\alpha}} e^{\pm i\phi_{L,R}(x)},$$

Perturbative RG at weak coupling (J's small)

$$S_0 = \int d^2x \left(-\frac{i}{\pi}\partial_0\phi\partial_1\theta + \frac{1}{2\pi}[(\partial_1\theta)^2 + (\partial_1\phi)^2] \right),$$

$$\begin{aligned}
S_K^o = \int dx_0 \Big(& 2g_1 S^x (iU_L U_R) \cos(\frac{2}{\sqrt{K}}\theta) \\
& + 2g_2 S^y (iU_L U_R) \sin(\frac{2}{\sqrt{K}}\theta) \\
& - 2g_3 S^x (iU_L U_R) \sin(2\sqrt{K}\phi) - 2g_4 S^z \partial_1\theta \\
& - 2g_5 S^y (iU_L U_R) \cos(2\sqrt{K}\phi) - 2g_6 S^z \partial_1\phi \Big) .
\end{aligned}$$

Cumulant expansion

$$\frac{Z}{Z_0} = \langle e^{-S_K} \rangle_{0f}, = \int D\phi D\theta \exp\left(-S_0[\phi,\theta]\right) \times \exp\left(-\langle S_K \rangle_{0f} + \frac{1}{2}(\langle S_K^2 \rangle)_{0f} - \langle S_K \rangle_{0f}^2) + \ldots \right),$$

1-loop RG equation

Bare value:

$$g_1 = J_1 / 4\pi v$$

$$g_2 = J_2^{\theta} \cos(\varphi / 2) / 4\pi v$$

$$g_3 = J_2^{\phi} \sin(\varphi / 2) / 4\pi v$$

$$g_4 = J_2^{z} \cos(\varphi / 2) / 4\pi v \sqrt{K}$$

$$g_5 = g_6 = 0$$

$$\frac{dg_1}{dl} = (1 - \frac{1}{K}) g_1 + C_1 \frac{g_2 g_6}{\sqrt{K}},$$

$$\frac{dg_2}{dl} = (1 - \frac{1}{K}) g_2 + C_2 \frac{g_1 g_6}{\sqrt{K}},$$

$$\frac{dg_3}{dl} = (1 - K) g_3 - C_3 \sqrt{K} g_5 g_4,$$

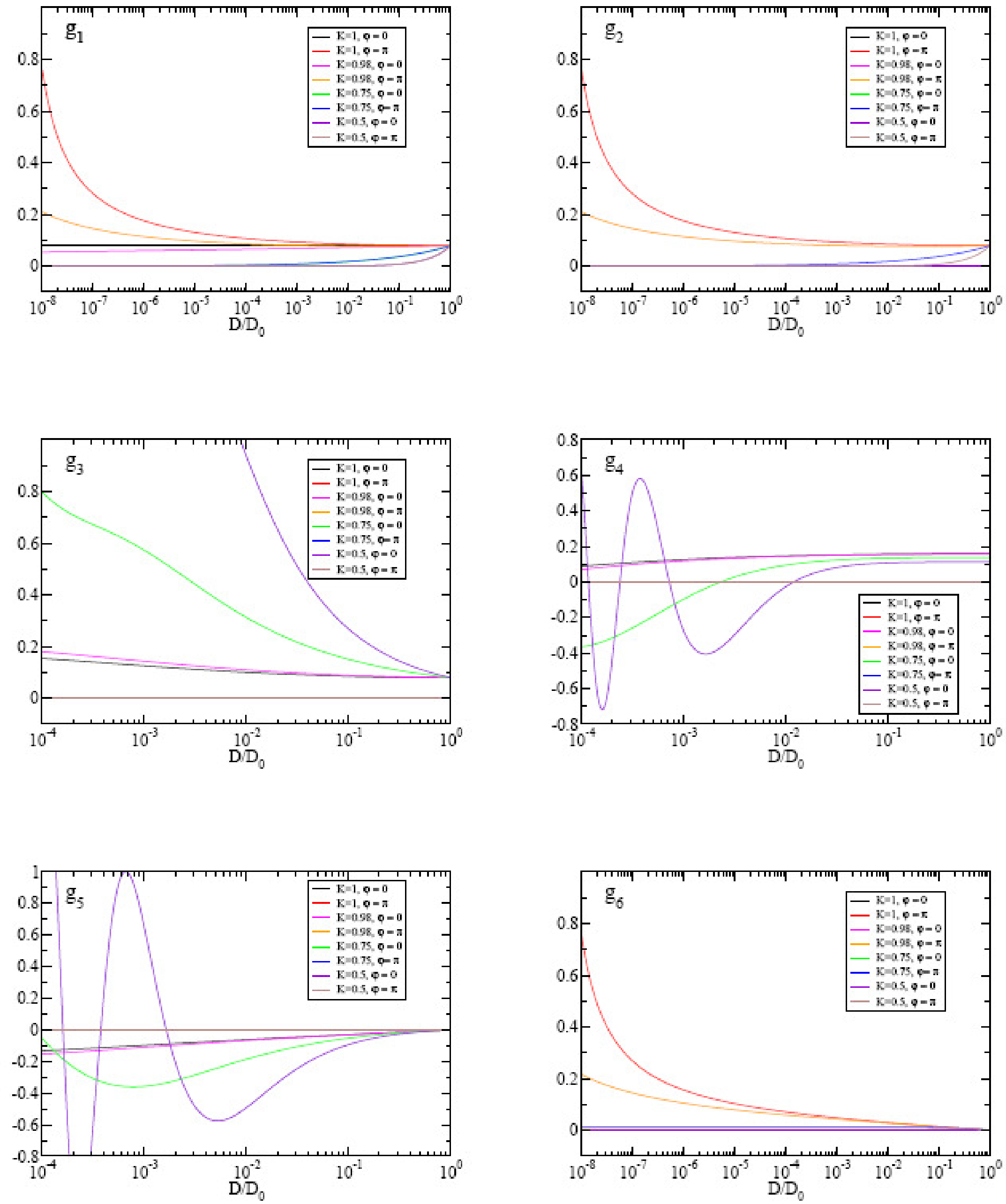
$$\frac{dg_4}{dl} = C_4 K^{3/2} g_3 g_5,$$

$$\frac{dg_5}{dl} = (1 - K) g_5 - C_5 \sqrt{K} g_3 g_4,$$

$$\frac{dg_6}{dl} = C_6 \frac{g_1 g_2}{K^{3/2}}.$$

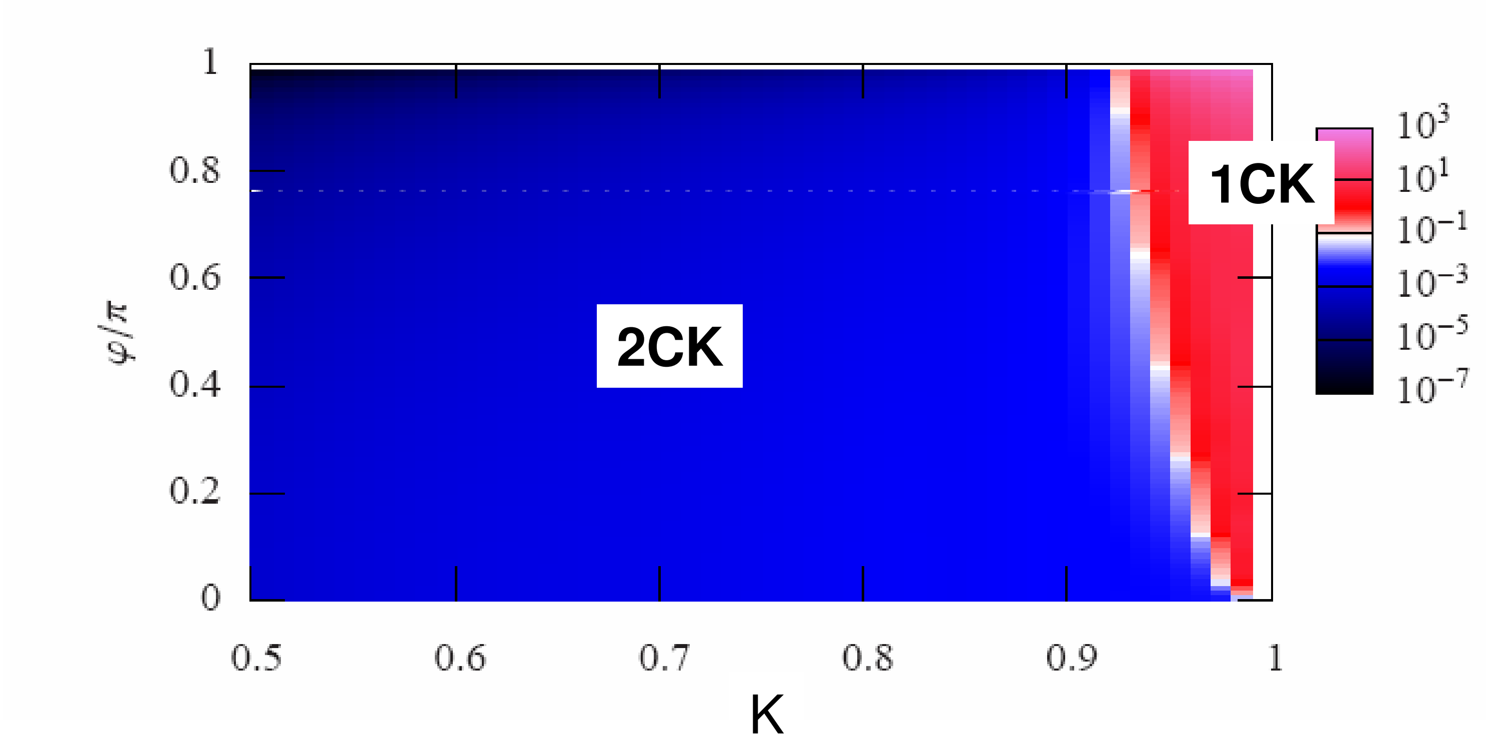
RG flow for different coupling constants

- For K<<1, the linear term dominates -> g3, g5 grow **(two channel Kondo fixed point)**
- For K->1, the quadratic term can compensate the linear term -> all coupling constants g's can grow **(one channel Kondo fixed point)**



Two different fixed points

$g_1(l^*)$ when $g_3(l^*) \sim 1$



•Quantum phase transition between 1CK and 2CK fixed points tuned by the superconducting phase difference

We have identified two possible Kondo fixed points from weak coupling perturbative renormalization group..

Problems

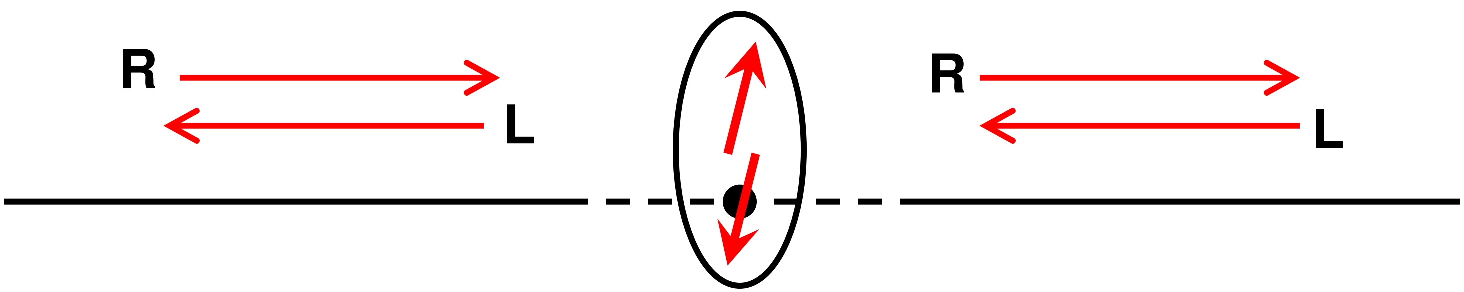
- Perturbative RG only valid at small coupling
- Stablility check of different fixed points are required

Stability of one-channel Kondo fixed point

$$J_1, J_2^{\theta}, J_2^{\phi}, J_2^{z} \to \infty$$

- Kondo singlet at x=0
- Two semi-infinite '**spinless**'Luttinger liquid

$$H_{1CK} = J\vec{S} \bullet \sum_{\alpha,\beta=L,R} \left(\Psi_{\alpha}^{\dagger} \frac{\vec{\sigma}_{\alpha\beta}}{2} \Psi_{\beta} \right)$$



$$\Psi_R(x) = \gamma_{1\uparrow}(x) + i\gamma_{2\uparrow}(x)$$

$$\Psi_L(x) = \gamma_{1\downarrow}(x) + i\gamma_{2\downarrow}(x)$$

Perturbation at the one-channel Kondo fixed point

Density-density interaction

$$H_\rho = u\left(\Psi_R^\dagger(0)\Psi_R(0) + \Psi_L^\dagger(0)\Psi_L(0)\right)^2$$

Scaling dimension: $[u] = 2$

Umklapp interaction

$$H_{um} = g_{um}\left(\Psi_R^\dagger(0)\Psi_R^\dagger(0)\Psi_L(0)\Psi_L(0) + h.c.\right)$$

Scaling dimension: $[g_{um}] = 4K$

Relevant when K<1/4

Stability of 2-channels Kondo fixed point $g_3 \to \infty$

Unperturbed action

$$S_{eff} = S_0 - \int dx_0 \left(2 g_3 S^x (i U_L U_R) \sin(2\sqrt{K}\phi(0)) \right)$$

Infinite g3 -> Φ(0) is pinned such that $\sin(2\sqrt{K}\phi(0)) = 1$

Chiral field representation

$$\phi(x) - \frac{\pi}{4\sqrt{K}} = \phi_L(x) + \phi_R(x),$$
$$\theta(x) = \phi_L(x) - \phi_R(x),$$

$$\phi_R(x) = \tilde{\phi}_R(x), \phi_L(x) = -\tilde{\phi}_R(-x) \text{ for } x > 0,$$
$$\phi_R(x) = -\tilde{\phi}_L(-x), \phi_L(x) = \tilde{\phi}_L(x) \text{ for } x < 0.$$

$\sin(2\sqrt{K}\phi(0)) = 1$ is automatically satisfied

Transformed Hamiltonian without any constraint at x=0

$$S_K^o = \int dx_0 \left(2g_1 S^x (iU_L U_R) \cos(\frac{2}{\sqrt{K}}(\tilde{\theta} - \tilde{\phi})) \right.$$
$$\left. +2g_2 S^y (iU_L U_R) \sin(\frac{2}{\sqrt{K}}(\tilde{\theta} - \tilde{\phi})) + 2g_6 S^z (\partial_1 \tilde{\theta} - \partial_1 \tilde{\phi}) \right),$$

$\tilde{\phi} = \tilde{\phi}_L + \tilde{\phi}_R$ and $\tilde{\theta} = \tilde{\phi}_L - \tilde{\phi}_R$

1-loop RG equation around 2CK fixed point

$$\frac{dg_1}{dl} = (1 - \frac{2}{K})g_1 + C_7 \frac{g_2 g_6}{\sqrt{K}},$$
$$\frac{dg_2}{dl} = (1 - \frac{5}{2K})g_2 + C_8 \frac{g_1 g_6}{\sqrt{K}},$$
$$\frac{dg_6}{dl} = -\frac{1}{2K} g_6 + C_9 \frac{g_1 g_2}{K^{3/2}},$$

Instanton effect at the two-channel Kondo fixed point

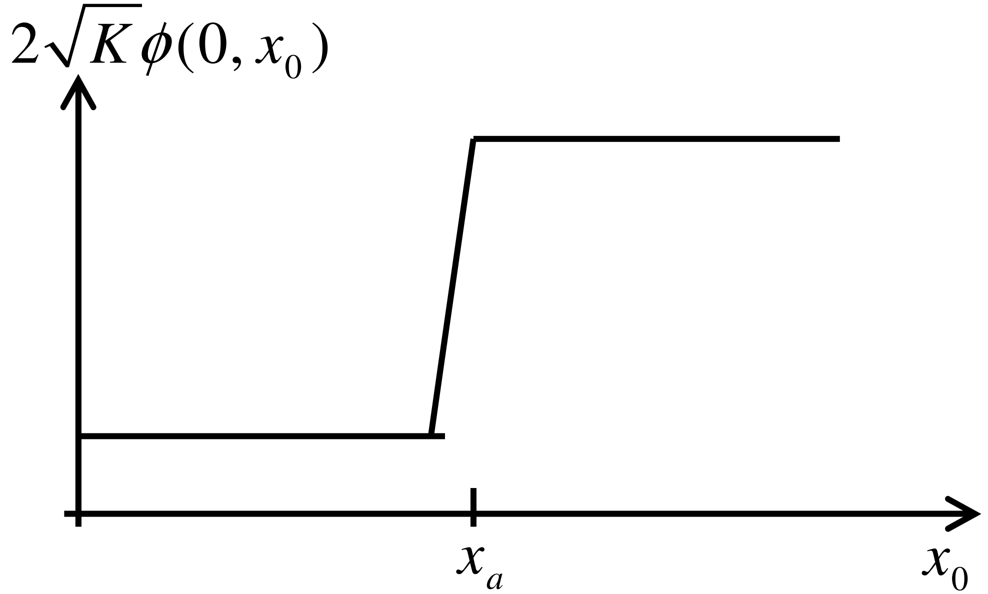
$$S_{eff} = S_0 - \int dx_0 \left(2 g_3 S^x (i U_L U_R) \sin(2\sqrt{K}\phi(0)) \right)$$

Tunneling event:

$$2\sqrt{K}\phi(0, x_0) = \pi / 2 + 2\pi\Theta(x_0 - x_a)$$

RG equation for the fugacity t of the instanton

$$\frac{dt}{dl} = (1 - \frac{1}{K})t$$



Effective action at the 2CK fixed point

$$S_{eff} = S_0 - \int dx_0 \left(2 g_3 S^x (i U_L U_R) \sin(2\sqrt{K}\phi(0)) \right)$$

$$H_{2CK} = \frac{J_2}{2} S^x \left(\psi_{1\uparrow}^\dagger \psi_{2\downarrow} + \psi_{1\downarrow}^\dagger \psi_{2\uparrow} + h.c. \right)$$ (Ising-like)

Residual entropy

$$S = \ln \sqrt{4K}.$$

$$S_{2CK} = \ln\sqrt{2K}$$ 2CK fixed point of topological insulator (*Law 2010*)

Josephson current at 2-channel Kondo fixed point

$$J^o = \frac{e}{h}\left(-\frac{J_2^\theta}{\alpha}\sin\frac{\varphi}{2}S^y(iU_LU_R)\sin\frac{2}{\sqrt{K}}\theta(0) - \frac{J_2^\phi}{\alpha}\cos\frac{\varphi}{2}S^x(iU_LU_R)\sin 2\sqrt{K}\phi(0) + \frac{J_2^z}{\sqrt{K}}\sin\frac{\varphi}{2}S^z\partial\theta(0)\right),$$

Supercurrent

$$\langle \mathrm{J}^0 \rangle = e\Delta\sin(\varphi)\left\{\frac{1}{2/K-1}\left(\frac{J_2^\theta}{v}\right)^2 + \frac{1}{2K-1}\left(\frac{J_2^\phi}{v}\right)^2 + \frac{1}{K}\left(\frac{J_2^z}{v}\right)^2\right\}$$

$$= e\Delta\sin(\varphi)\frac{(J_2^z)^2 - (J_2^\theta)^2 - (J_2^\phi)^2}{v^2} \quad (\mathrm{K=1})$$

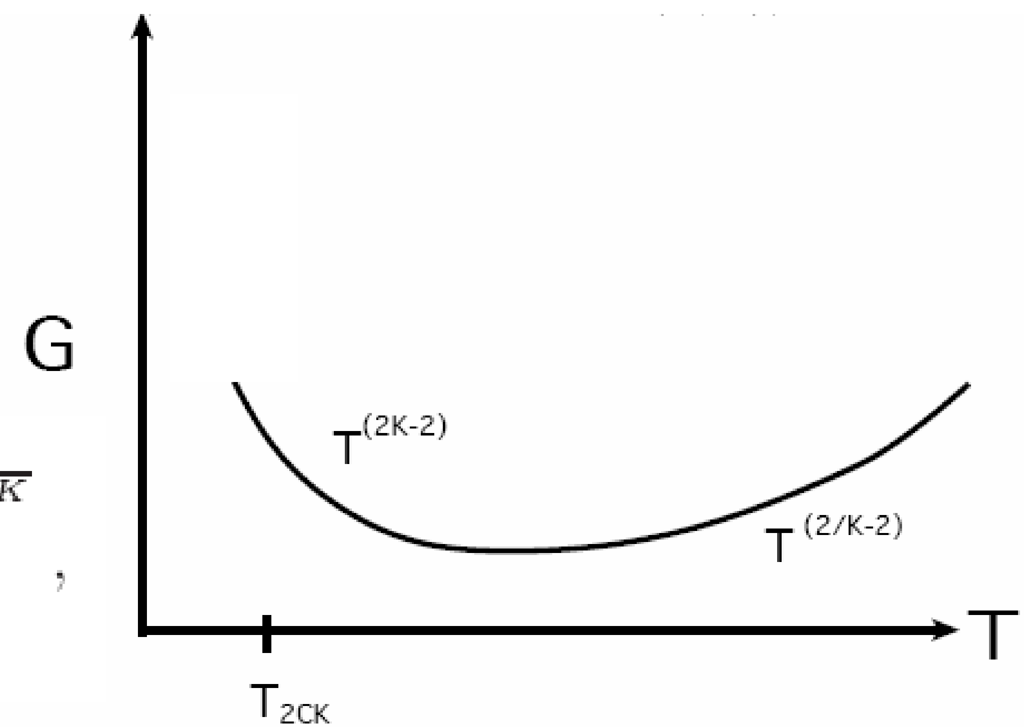
DC conductance across two helical supedrconducting islands

$$\frac{G}{G_0} = c_\theta (\frac{J_2^\theta}{\alpha T_c})^2 \sin^2\left(\frac{\varphi}{2}\right)\left(\frac{T}{T_c}\right)^{\frac{2}{K}-2} + c_\phi (\frac{J_2^\phi}{\alpha T_c})^2 \cos^2\left(\frac{\varphi}{2}\right)\left(\frac{T}{T_c}\right)^{2K-2} + c_z (\frac{J_2^z}{\alpha T_c})^2 \sin^2\left(\frac{\varphi}{2}\right),$$

$$G_0 = 2e^2/h$$

Kondo temperature

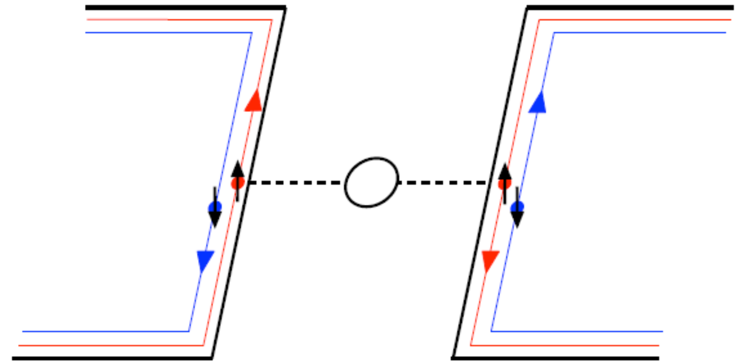
$$T_{2CK} \sim \Delta \left(\frac{J_2^\phi(0)\sin\frac{\varphi}{2}}{4\pi v}\right)^{\frac{1}{1-K}},$$



Two channel Kondo problem with equal helicity

Recombination of Majorana fermions

$$\Psi_R(x) = \gamma_{1\uparrow}(x) + i\gamma_{2\downarrow}(x)$$
$$\Psi_L(x) = \gamma_{1\downarrow}(x) + i\gamma_{2\uparrow}(x)$$



Bosonized Hamiltonian

$$\begin{aligned} H_K^e = &\frac{J_1}{2\pi\alpha}S^x(iU_LU_R)\cos(2\sqrt{K}\phi(0)) \\ &-\frac{1}{2\pi}\left(\frac{J_2^\theta}{\sqrt{K}}\sin\frac{\varphi}{2}S^x\partial\theta(0)+J_2^\phi\sqrt{K}\cos\frac{\varphi}{2}S^y\partial\phi(0)\right) \\ &+\frac{J_2^z}{2\pi\alpha}S^z\cos\frac{\varphi}{2}(iU_LU_R)\sin(2\sqrt{K}\phi(0)). \end{aligned} \qquad (3.6)$$

Three stable Kondo fixed points

$$J_i\text{'s} \to \infty \qquad \text{(1CK fixed point)}$$

$$J_1 \to \infty \qquad \text{(2CK fixed point)}$$

$$\sigma(T \to 0) \to 0$$

$$H_{2CK} = \frac{J_1}{2} S^x \left(\psi^\dagger_{1\uparrow}\psi_{1\downarrow} + \psi^\dagger_{2\downarrow}\psi_{2\uparrow} + h.c. \right)$$

$$J_2^z \to \infty \qquad \text{(2CK fixed point)}$$

$$\sigma(\mathrm{T} \to 0) \to \infty$$

$$H_{2CK} = \frac{J_2^z}{2} S^z \left(\psi^\dagger_{1\uparrow}\psi_{2\uparrow} - \psi^\dagger_{1\downarrow}\psi_{2\downarrow} + h.c. \right)$$

Comparsion with spinful Luttinger liquild and helical Luttinger liquid on two-channel Kondo problem

	spinful Luttinger liquild	Topological Insulator (helical)	Topological Superconductor (helical)
Number of species of fermions	8	4	2
QPT	$\mathrm{K} \to 1/2$	$\mathrm{K} \to 1^-$	$\mathrm{K} \to 1^-$
Residual entropy	$\ln\sqrt{2}$ (K=1)	$\ln\sqrt{2K}$	$\ln\sqrt{4K}$
	Emery 1992	*Law 2010*	

Conclusion:

•The 2D topological superconductor with time-reversal symmetry supports helical Majorana edge modes on the boundary
•If two helical superconductors are coupled via a quantum dot, the system exhibit both one-channel and two-channel Kondo fixed points which are tunalbe by the Luttinger parameter K and superconducting phase difference.
•Strong coupling fixed points are sensitive to the relative helicity between two helical superconductors