# *Quantum Hall states and fractional topological insulator states in strained graphene*

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Fractional topological phases and broken time reversal symmetry in strained graphene, P. Ghaemi, J.C., D.N. Sheng, and A. Vishwanath, Phys. Rev. Lett. **108**, 266801 (2012)

# Quick history of Topological phases of electrons in crystals

Integer and Fractional Quantum Hall effect **(1981,1983)**

**2D electrons in strong magnetic field: Si, GaAs, graphene,...**

$\downarrow$

Topological insulators **with strong spin-orbit coupling**

**2D:** graphene, HgTe quantum wells **(2005,2007)**

**3D:** BiSb, Bi2Se3, Bi2Te3, HgTe under strain **(2006,2008)**

Zoology of Chern insulators with **no overall/net magnetic field**

**2D:** Haldane 1988

Various models of **Fractional Chern insulators** **(2011)**

# Rough/partial classification of topological phases in the absence of a global uniform field

| Time-reversal Symmetry | Free electrons | **Interacting electrons** |
| --- | --- | --- |
| **NO** | Chern insulator<br>No Landau levels<br>Dispersive Bloch bands<br>with nonzero Chern number | **Fractional Chern insulators**<br>Flattened Bloch band |
| **YES** | Topological insulator<br><br>Experimental evidences:<br>HgTe, Bi2Se3, Bi2Te3 | **Fractional topological insulators** |

**Our motivation** was to propose **graphene under strain** **as:**

**1) A possible experimental platform** to realize **topological phases** in the **absence of external magnetic field**

**2) A system with a competition between Time-reversal symmetric phases and Time-reversal breaking phases**

**3) A possible** (valley) Fractional Topological Insulator **under some fine-tuning of the interactions**

# Outline

## Pseudomagnetic fields in strained/deformed graphene

**Large valley-dependent** fields

Time-reversal (TR) **invariance** (*real magnetic field = 0*)

## Experimental signatures (short review)

Observation of large pseudo-fields: **60T, 100T, 300T,... !**

**Pseudo Landau level structure (PLL)**

## Interaction driven phases (our theoretical work)

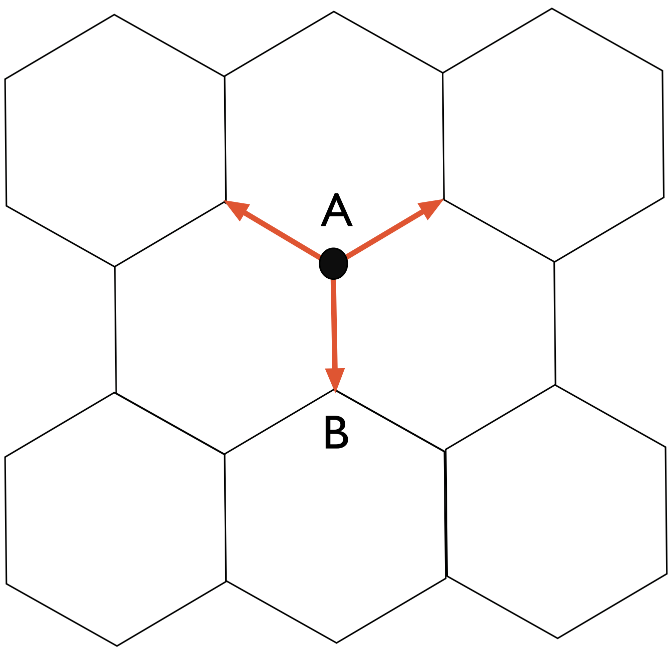
Effet of the **Coulomb interaction** in a **partially filled PLL**

Fractional quantum Hall states (**breaks** TR)

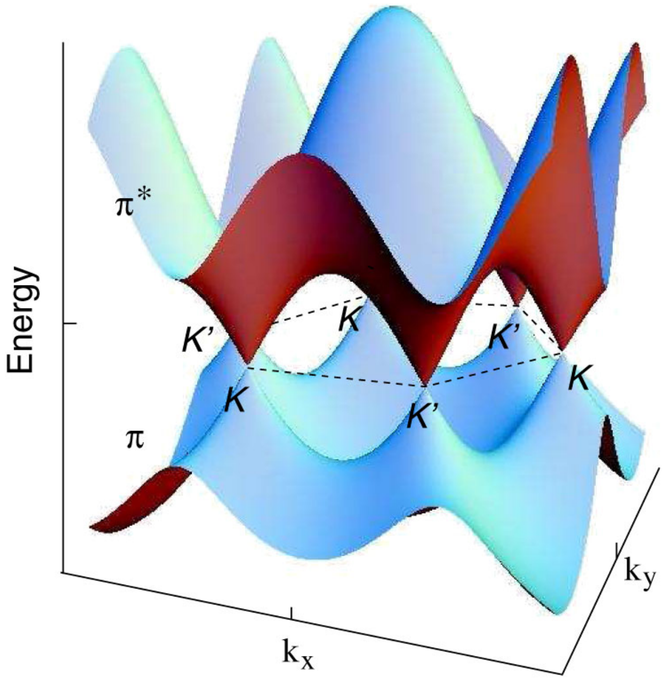
Fractional topological insulators (TR **invariant** state)

# Graphene (unstrained and B=0)

Honeycomb lattice




Band structure (pi orbitals)




Two sublattices A and B

2 Dirac points at K and K'

# Microscopic Tight binding model:

$$H_0 = \sum_{\mathbf{r}_{mn}} \sum_{a=1,2,3} t\, a^\dagger(\mathbf{r}_{mn}) b(\mathbf{r}_{mn} + \boldsymbol{\delta}_a) + h.c.$$

# Low energy theory: Dirac Hamiltonian

$$\boxed{H_\xi = v_F(\xi p_x \sigma_x + p_y \sigma_y)}$$

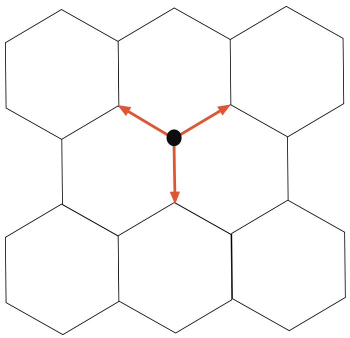
*valley*

*momentum from +/-K*

*Pauli matrices act on sublattice index A,B*

## Smooth deformation of the bonds

$$H_0 = \sum_{\mathbf{r}_{mn}} \sum_{i=1,2,3} (t + \delta t_i(\mathbf{r}_{mn}))(a^\dagger(\mathbf{r}_{mn}) b(\mathbf{r}_{mn} + \boldsymbol{\delta}_i) + h.c.)$$



no intervalley coupling

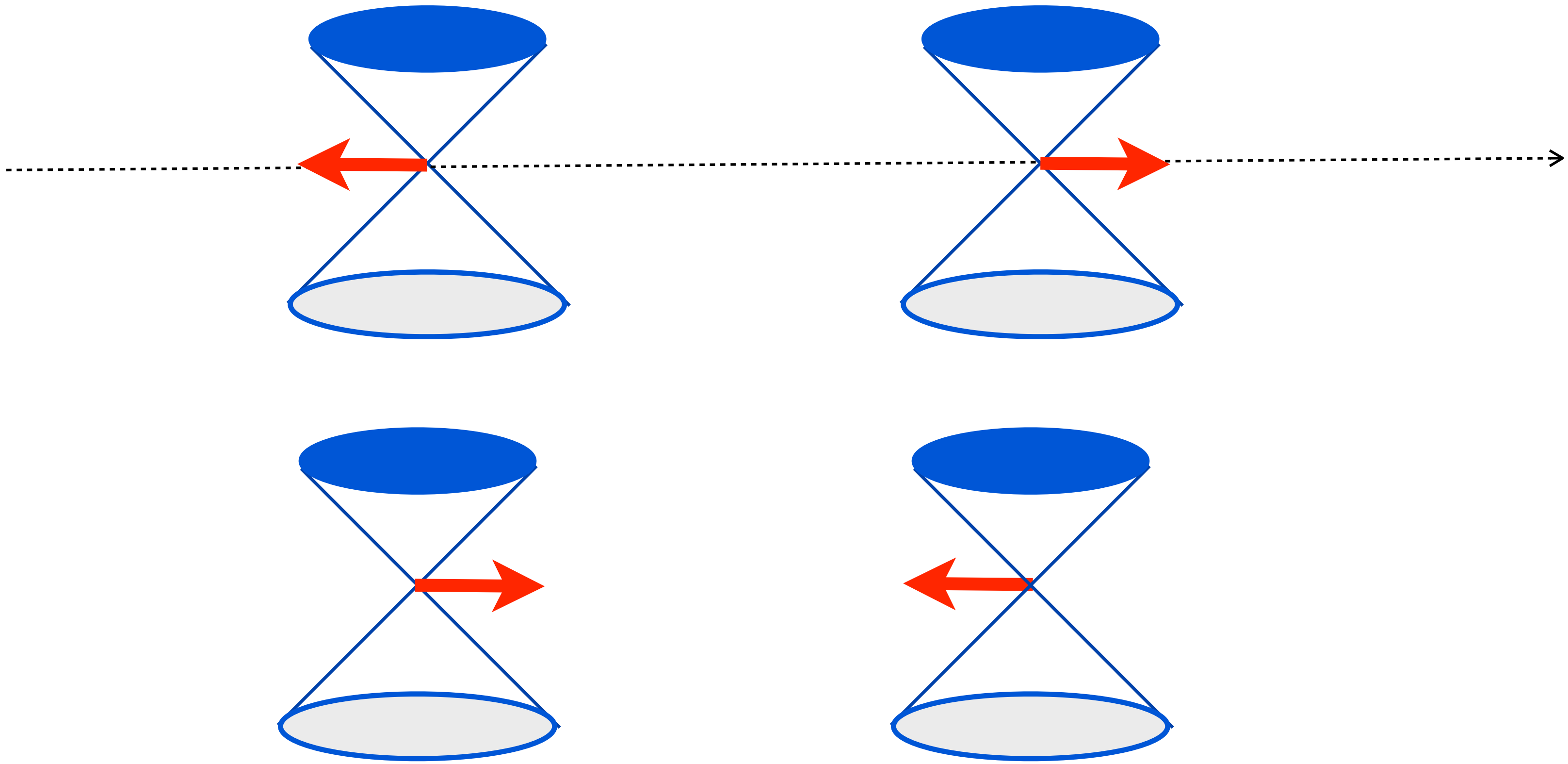
## Low energy theory: Dirac Hamiltonian

$$\boxed{H_\xi = v_F(\xi \Pi^\xi_x \sigma_x + \Pi^\xi_y \sigma_y)}$$

$$\boldsymbol{\Pi}^\xi = \mathbf{p} + \xi \delta \mathbf{p}$$

# Dirac point motion

$$\mathbf{\Pi}^{\xi} = \mathbf{p} + \xi \delta \mathbf{p}$$

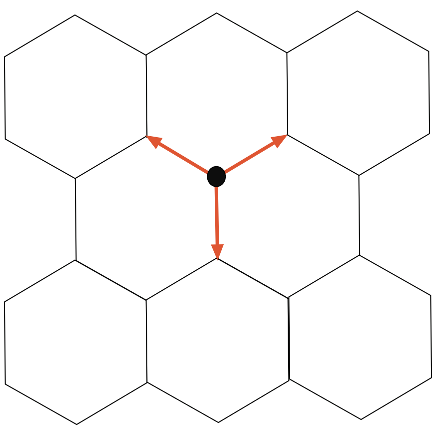


*Dirac points stay time-reversed partners*

# Momentum shift = Potential vector

$$\boldsymbol{\Pi}^{\xi} = \mathbf{p} + \xi\delta\mathbf{p} \rightarrow \xi e\mathbf{A}$$

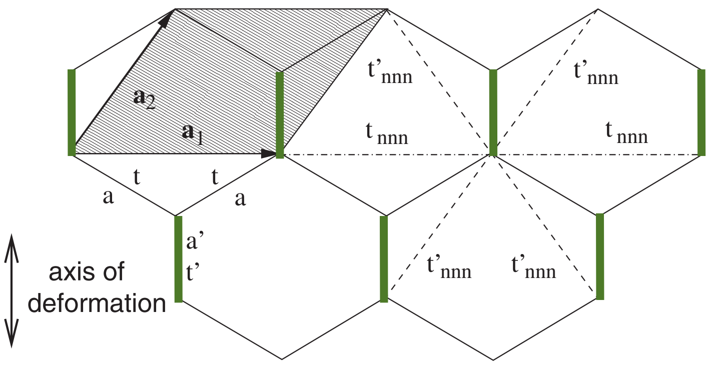
## Induced vector potentials are opposite in the valleys

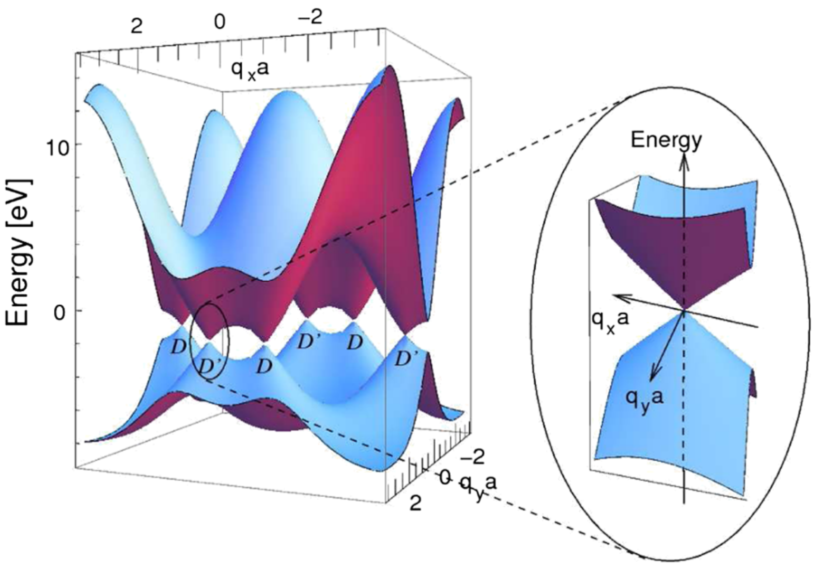


$$ev_F(A_x(\mathbf{x}) + iA_y(\mathbf{x})) = \sum_{a=1,2,3} \delta t_a(\mathbf{x}) e^{i\mathbf{K}.\boldsymbol{\delta}_a}$$

$$\begin{aligned} ev_F A_x &= -\frac{1}{2}(\delta t_1 + \delta t_2) + \delta t_3, \\ ev_F A_y &= \frac{\sqrt{3}}{2}(\delta t_1 - \delta t_2), \end{aligned}$$

# Uniform deformation




Dirac points **shifted** but **NO pseudomagnetic field** (curl A=0)


M. O. Goerbig, J.-N. Fuchs, G. Montambaux, and F. Piéchon, PRB **78,** 045415 (2008)
G. Montambaux, F. Piéchon, J.-N. Fuchs, and M. O. Goerbig, PRB **80,** 153412 (2009)
Pics from M.O. Goerbig, RMP **23**, 1193 (2011)

# Non uniform deformation yields finite fields

Arbitrary deformation leads in general to:

**Non uniform** field and complicated band structure...

Class of deformations yielding to

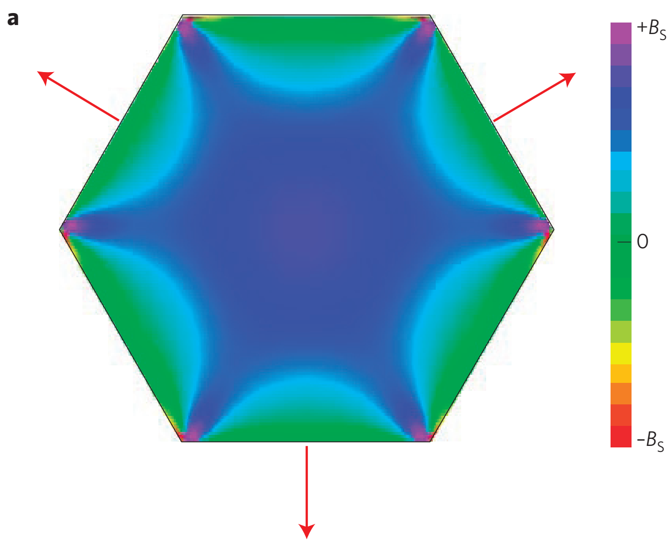
**Uniform** magnetic field in a given valley

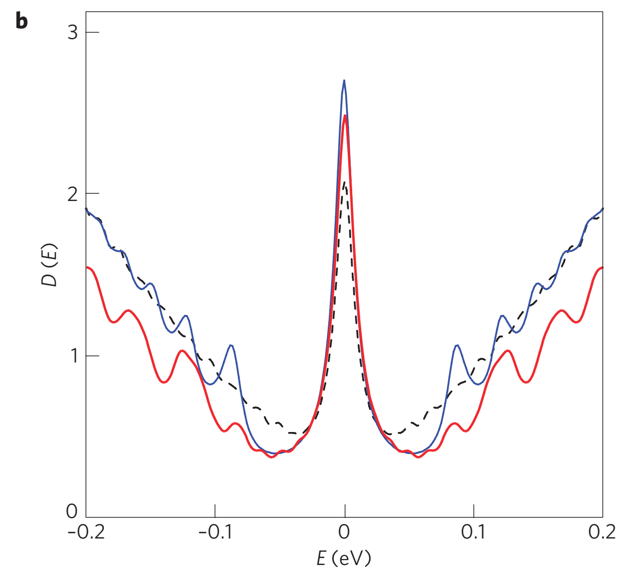
Dirac cones are expected to split into **flat Landau levels**

F. Guinea, M.I. Katsnelson, and A.K. Geim, Nature Physics, 2009

# (Pseudo) Landau levels in zero field

$$\boxed{E_n = \xi\sqrt{2e\hbar v_F^2 B|n|}}$$



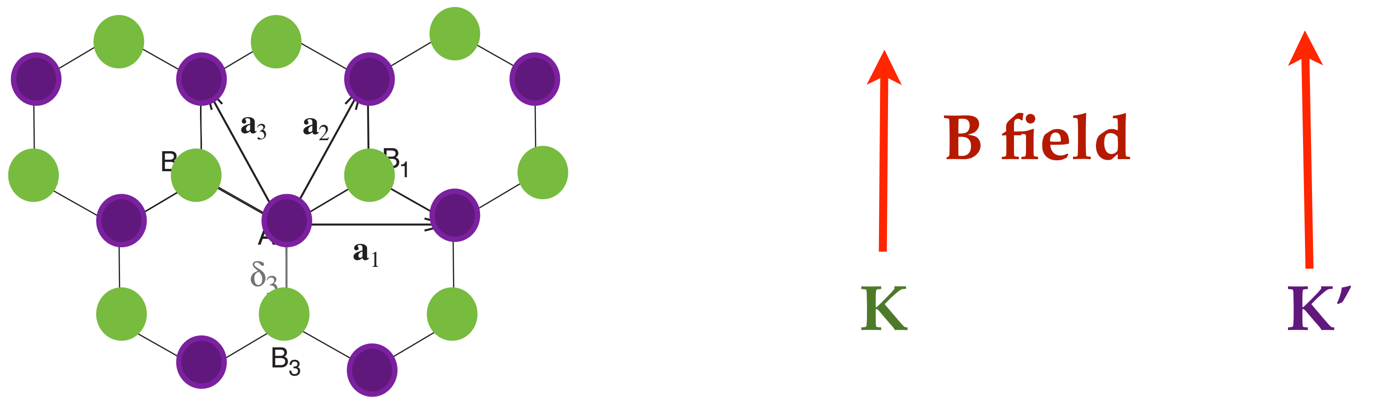

Triaxial strain / deformation

Density of states

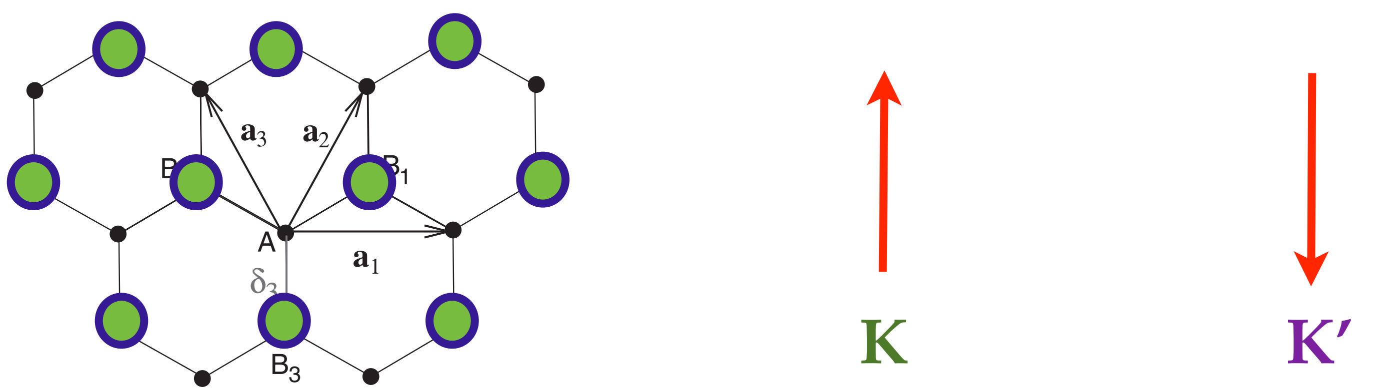
F. Guinea, M.I. Katsnelson, and A.K. Geim, Nature Physics, 2009

# Relation valley-sublattice (n=0 Landau level)

Real magnetic field: **K** on B sublattice and **K′** on A sublattice.




Pseudo field: Both K and K′ on **the same sublattice (B).**



## Two natural questions:

1- Experimental evidences of those large fields and associated Landau levels ?

2- Can we have Fractional Quantum Hall states in the partially filled Landau levels induced by pure strain ?

# PART II:
# Experiments on deformed graphene systems

Crommie (Berkeley): **real graphene** (Science 2011)

Lin He (Beijing University): **real bilayer graphene** (ArXiv 2012)

Manoharan (Stanford): **molecular graphene** (Nature 2012)

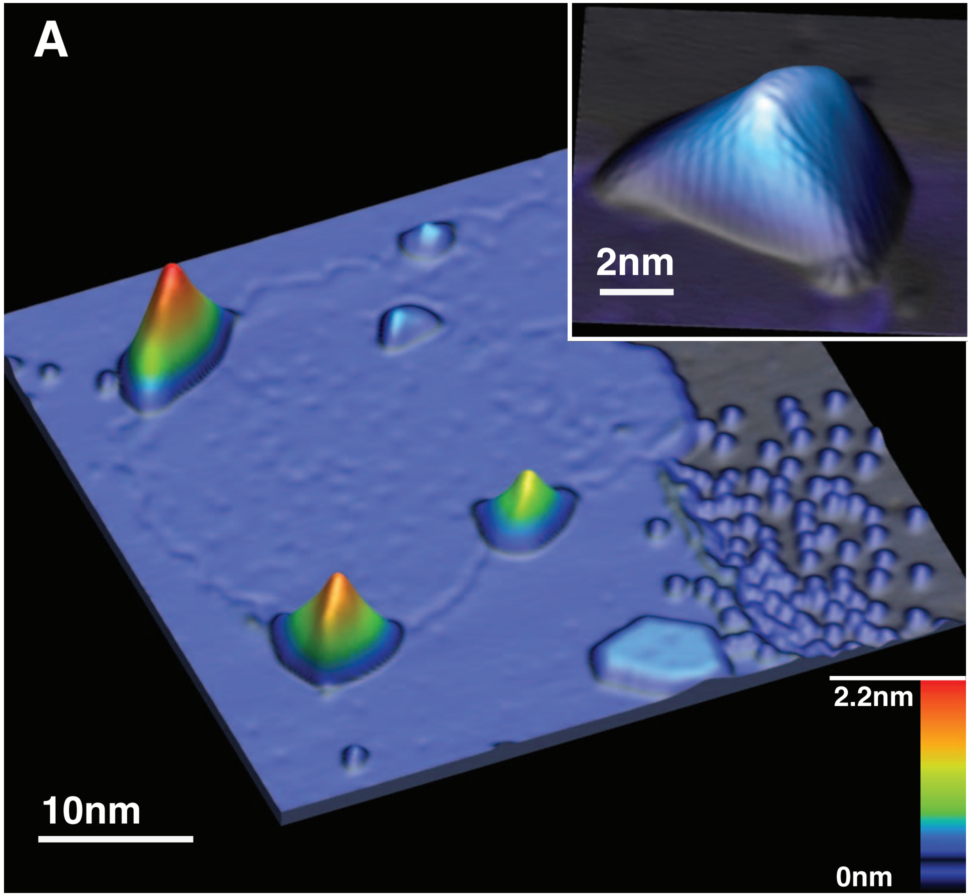
Esslinger (ETH Zurich): **fermionic cold atoms** (Nature 2012)

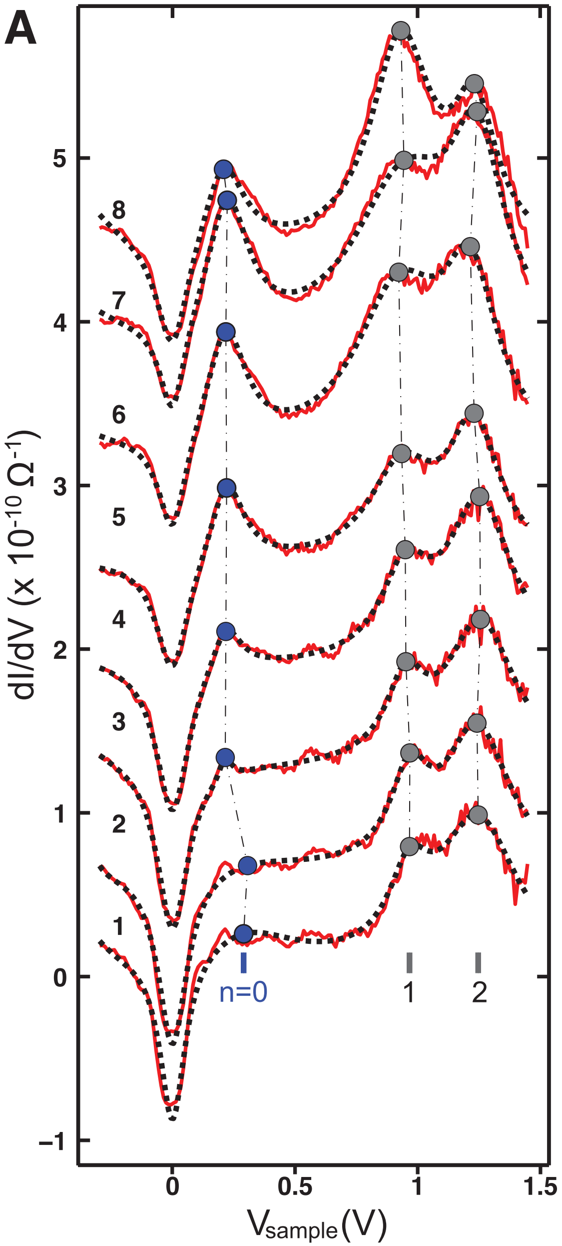
# Strained graphene (Crommie, Berkeley)

Real graphene on top of a Pt substrate

↓

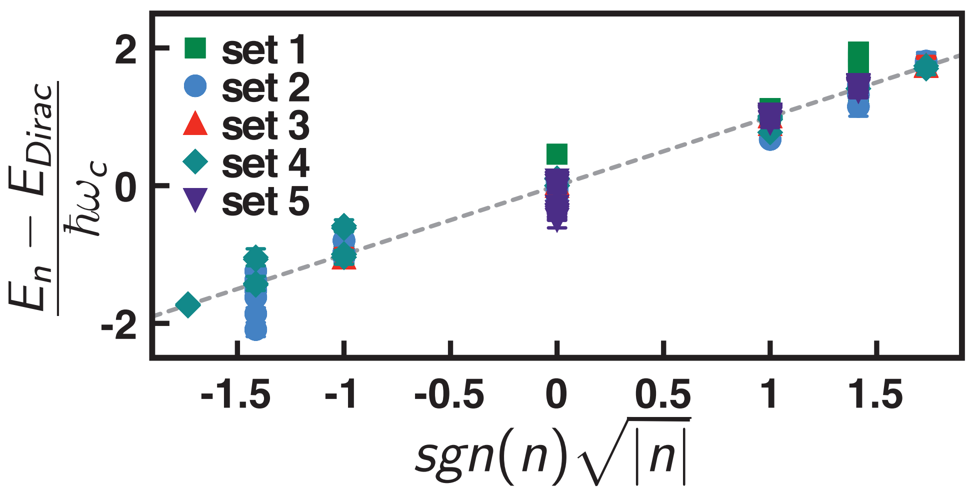
Nanoscale bubbles scanned by STM





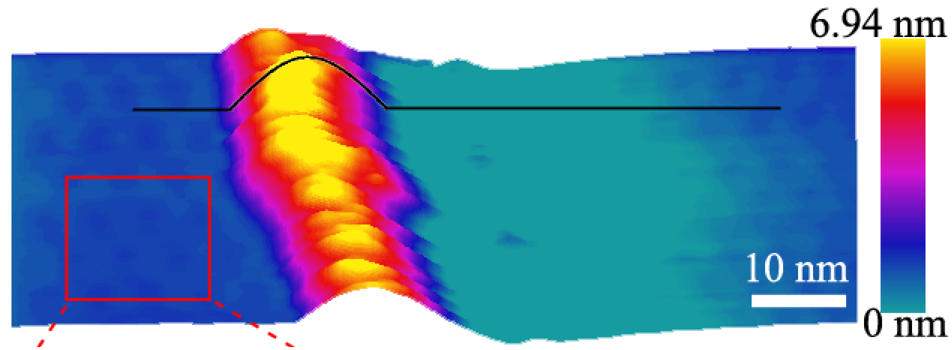

# Density of states by STM: Pseudo Landau Levels

Typical fields: 300 Tesla !



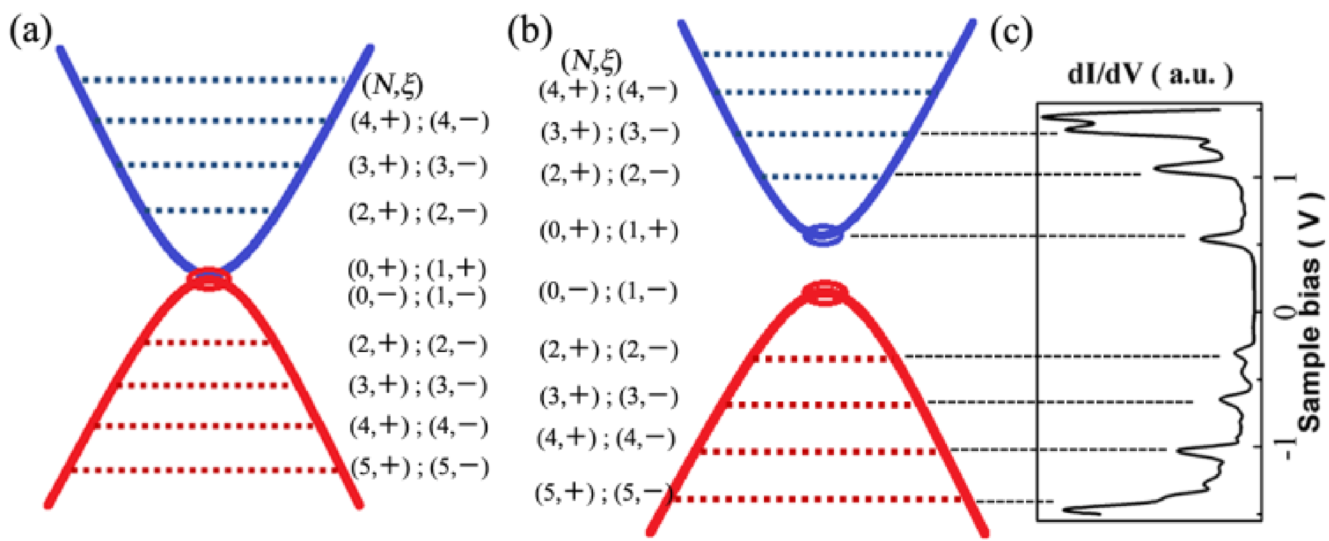
# Twisted graphene bilayer (Lin He, Beijing)

Graphene bilayer on top of a Rh substrate




Twist creates a

Nanoscale ridge




Non relativistic Landau levels

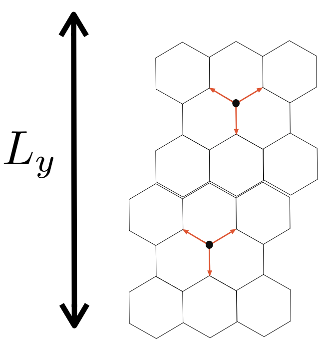
## Why strained induced fields are so large in graphene ?

$$B_{eff} \simeq \frac{\Phi_0}{a^2} \cdot \frac{\delta a}{\delta y}$$

$10^5$ Tesla

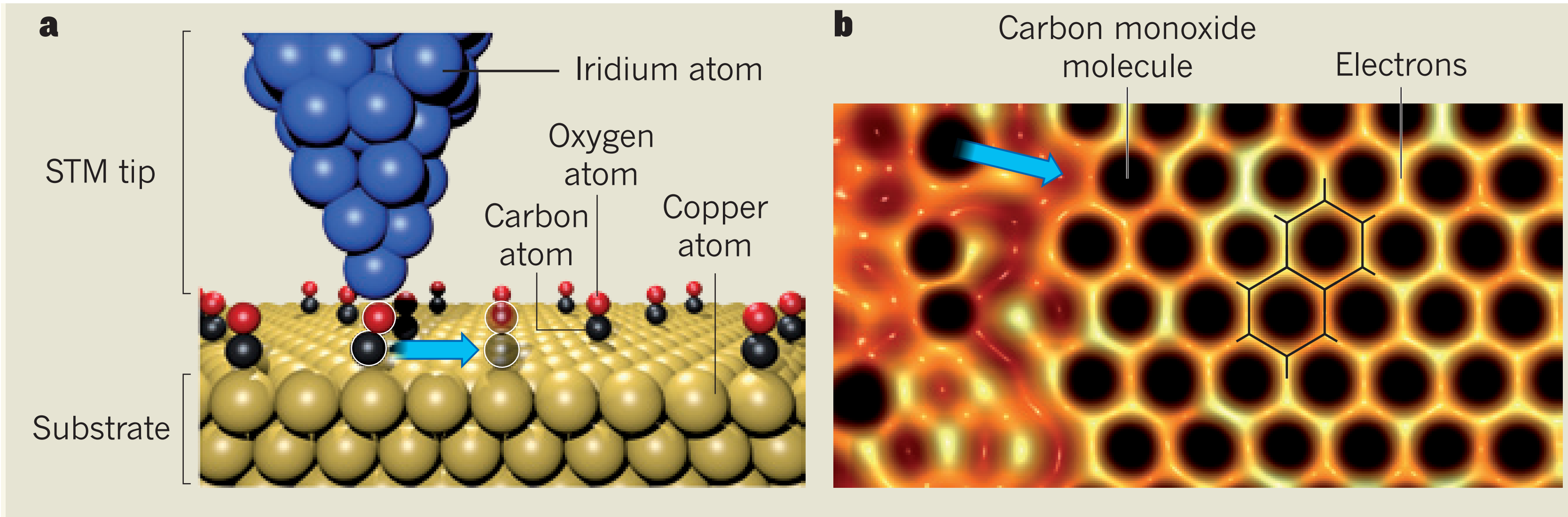
$10^{-3}$

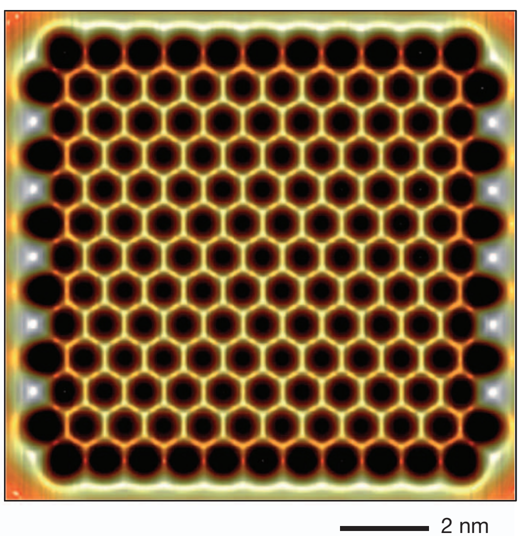
But with a limitation on the strained region **size**




$$L_y \ll \frac{\Phi_0}{|B|}\frac{1}{a} = \frac{l_B^2}{a} \simeq 500nm$$

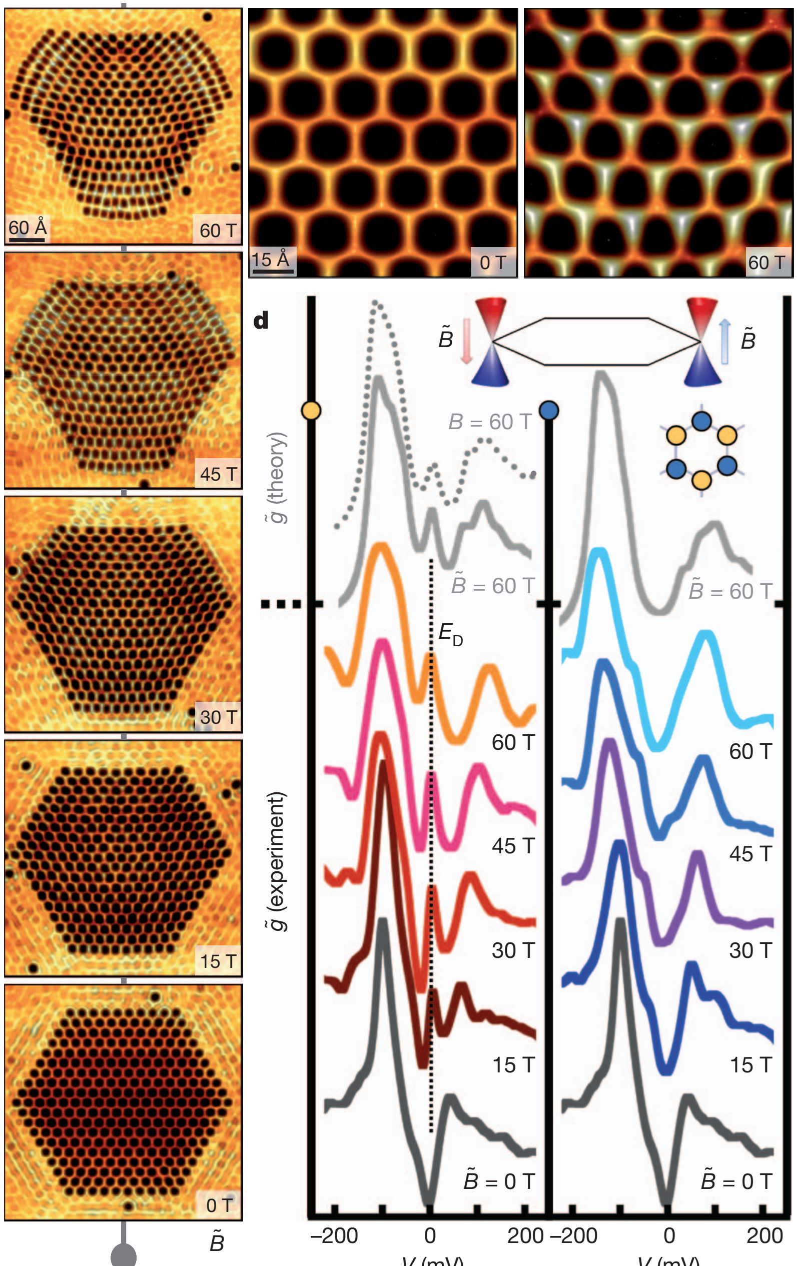
# Molecular graphene (Manoharan, Stanford)




100-1000 molecules deposited one by one on Cu surface

Lattice spacing a=2 nm
10 times larger than atomic graphene
10 times smaller than patterned 2DEG




# DOS by STM

Triaxial deformation

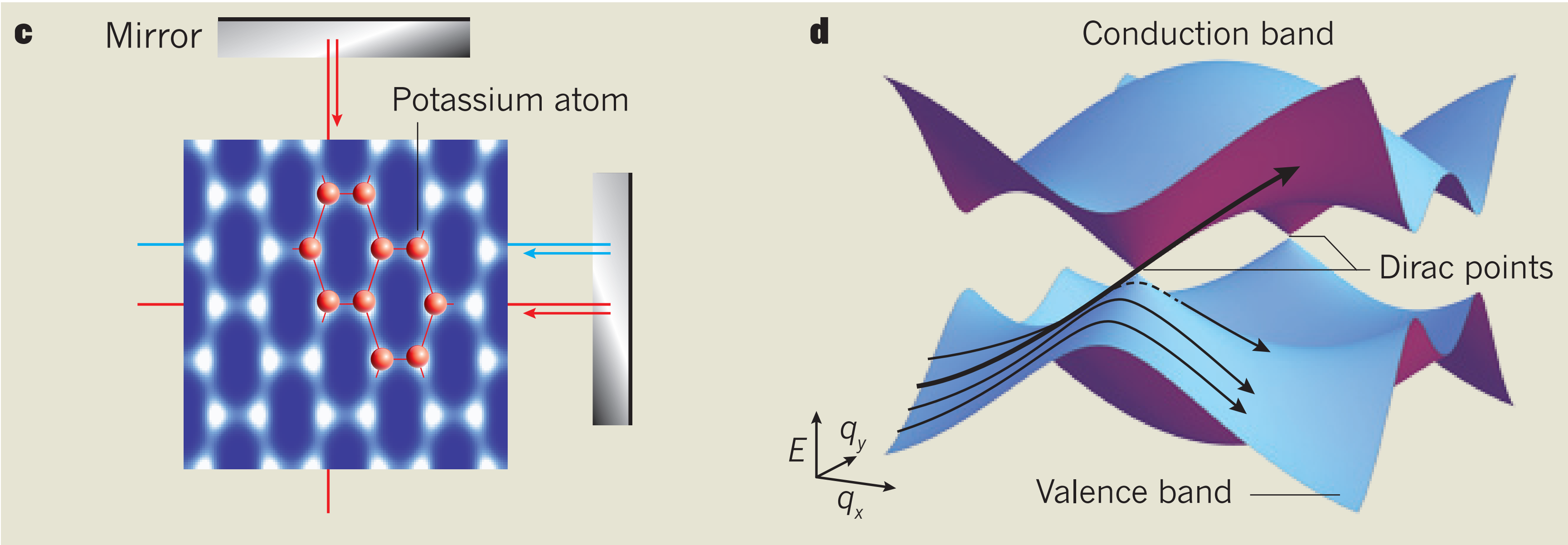
Typical pseudofields: 60 Tesla ! in «pseudographene»

Density of free electrons fixed

Lattice constant of the CO molecules grid can be varied

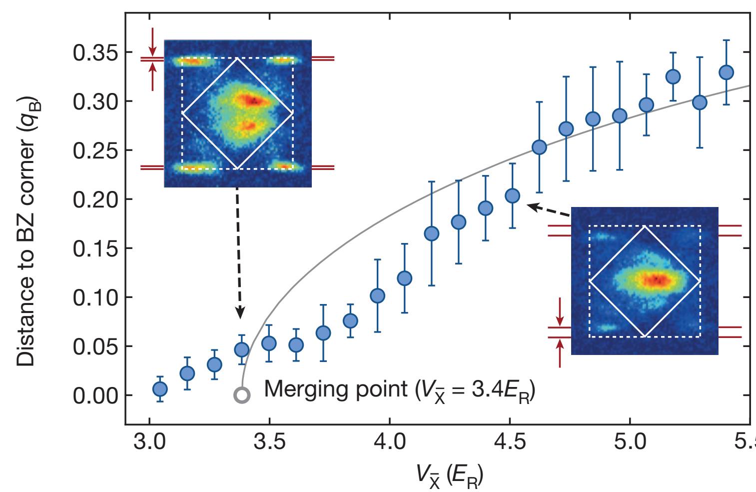
**Highly controlled system**

# Fermionic cold atoms (Esslinger, ETH Zurich)




**Uniform deformation**

**Dirac point motion**



# PART III:
# Interactions in a partially filled pseudo Landau level

**Interaction Hamiltonian**

**Integer** filling of spin-valley subbands

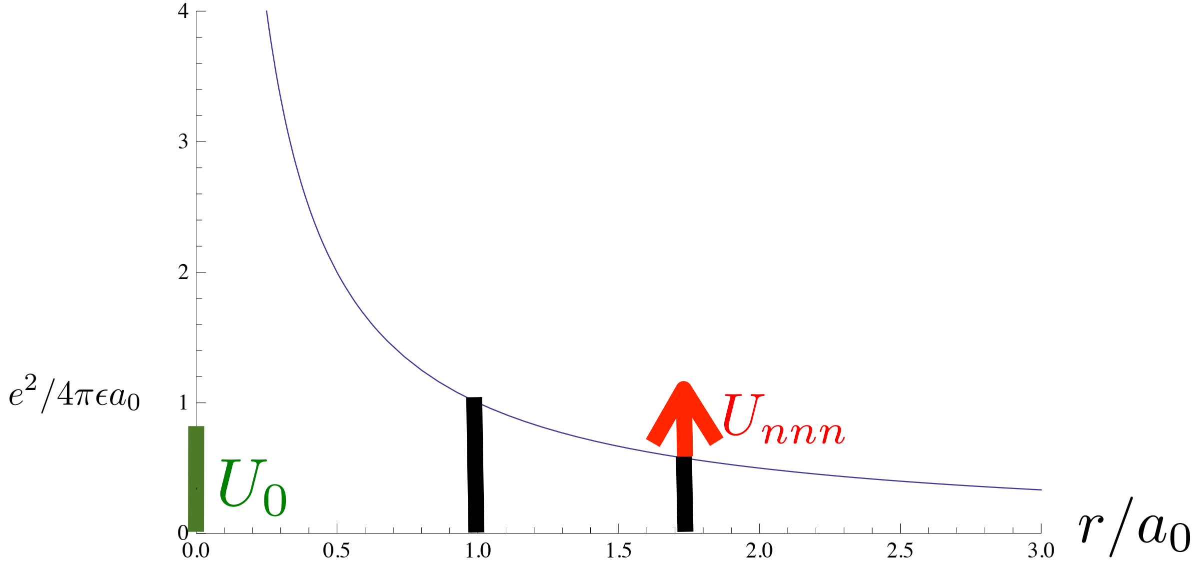
**Fractional** filling of the spin-valley degenerated PLL

# Interaction Hamiltonian

Long range Coulomb interaction: $V(\mathbf{r}_i - \mathbf{r}_j) = e^2/4\pi\epsilon|\mathbf{r_i} - \mathbf{r_j}|$

$$H_{int} = \sum_{\mathbf{r}_i \neq \mathbf{r}_j} V(\mathbf{r}_i - \mathbf{r}_j) n(\mathbf{r}_i) n(\mathbf{r}_j) + U_0 \sum_{\mathbf{r}_i} n(\mathbf{r}_i) n(\mathbf{r}_i)$$

$$+ U_{nnn} \sum_{\langle \mathbf{r}_i, \mathbf{r}_j \rangle} n(\mathbf{r}_i) n(\mathbf{r}_j)$$



# Half-filled n=0 Pseudo Landau Level

Valley ferromagnet: $$\Psi_V = \prod_k c^\dagger_{R,k,\uparrow} c^\dagger_{R,k,\downarrow} |0\rangle$$

Spin ferromagnet: $$\Psi_S = \prod_k c^\dagger_{R,k,\uparrow} c^\dagger_{L,k,\uparrow} |0\rangle$$

Reminiscent of the same problem for a real field:

J. Alicea and M.P.A. Fisher, Phys. Rev. B **74**, 075422, 2006

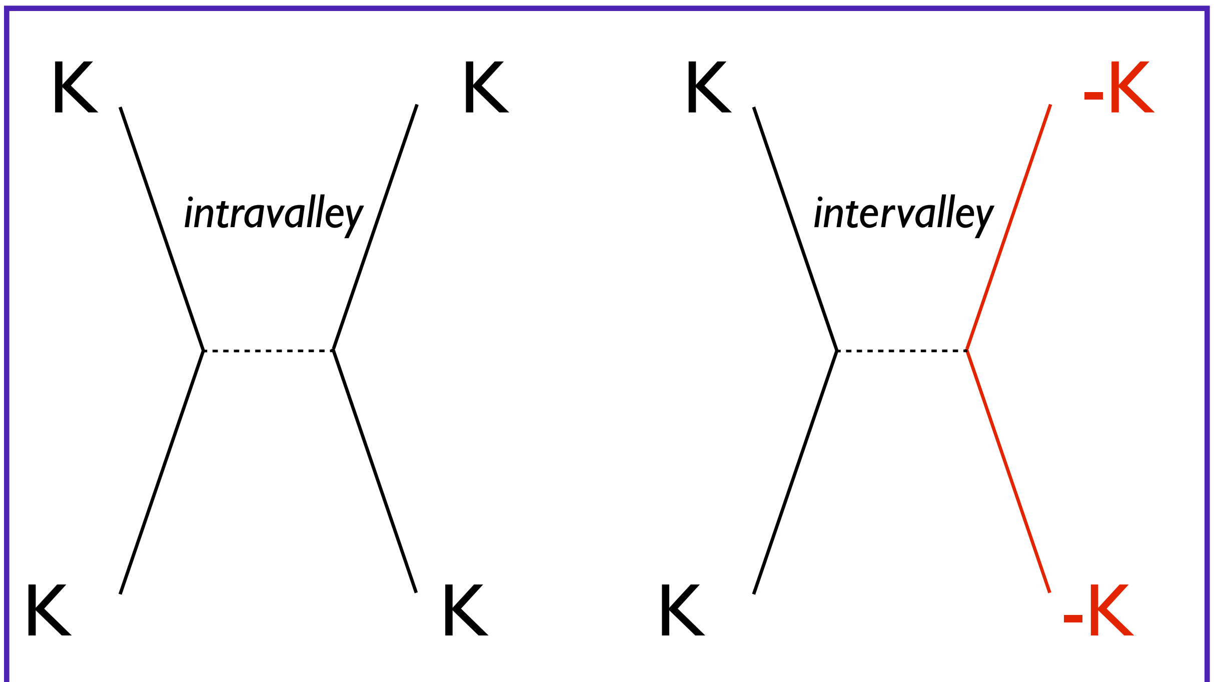
M. O. Goerbig, R. Moessner, and B. Douçot, Phys. Rev. B **74**, 161407, 2006

but here SU(4) symmetry is reduced to SU(2)xZ2

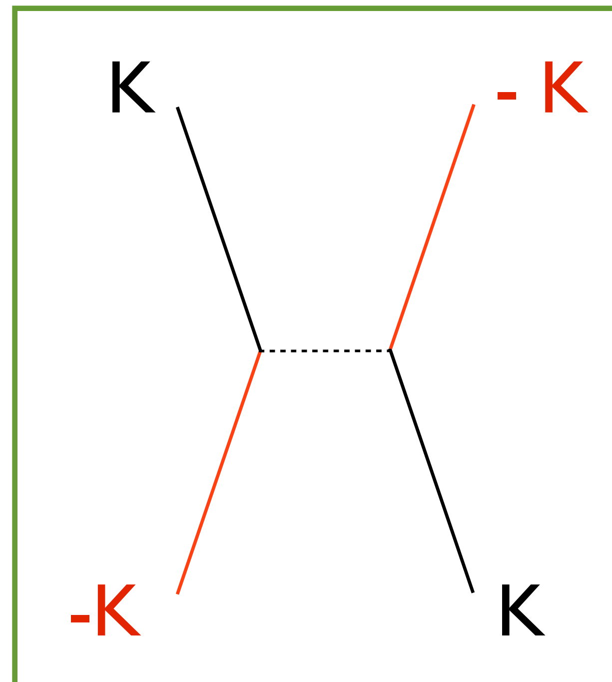
# Coulomb only

Dominant density-density terms: Spin ferromagnet and Valley Ising ferromagnet have the same Hartree-Fock energy

Backscattering terms favor the **Ising valley ferromagnet**

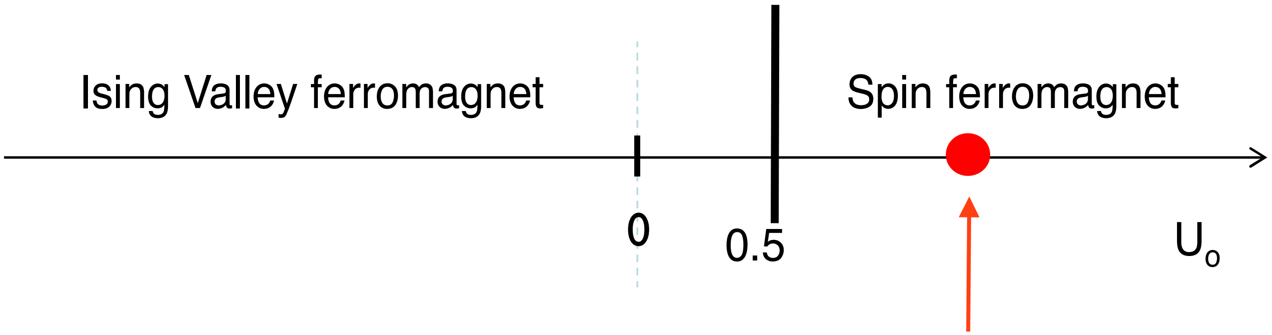



Some density-density terms




Backscattering term

# Coulomb + on-site Hubbard corrections U0

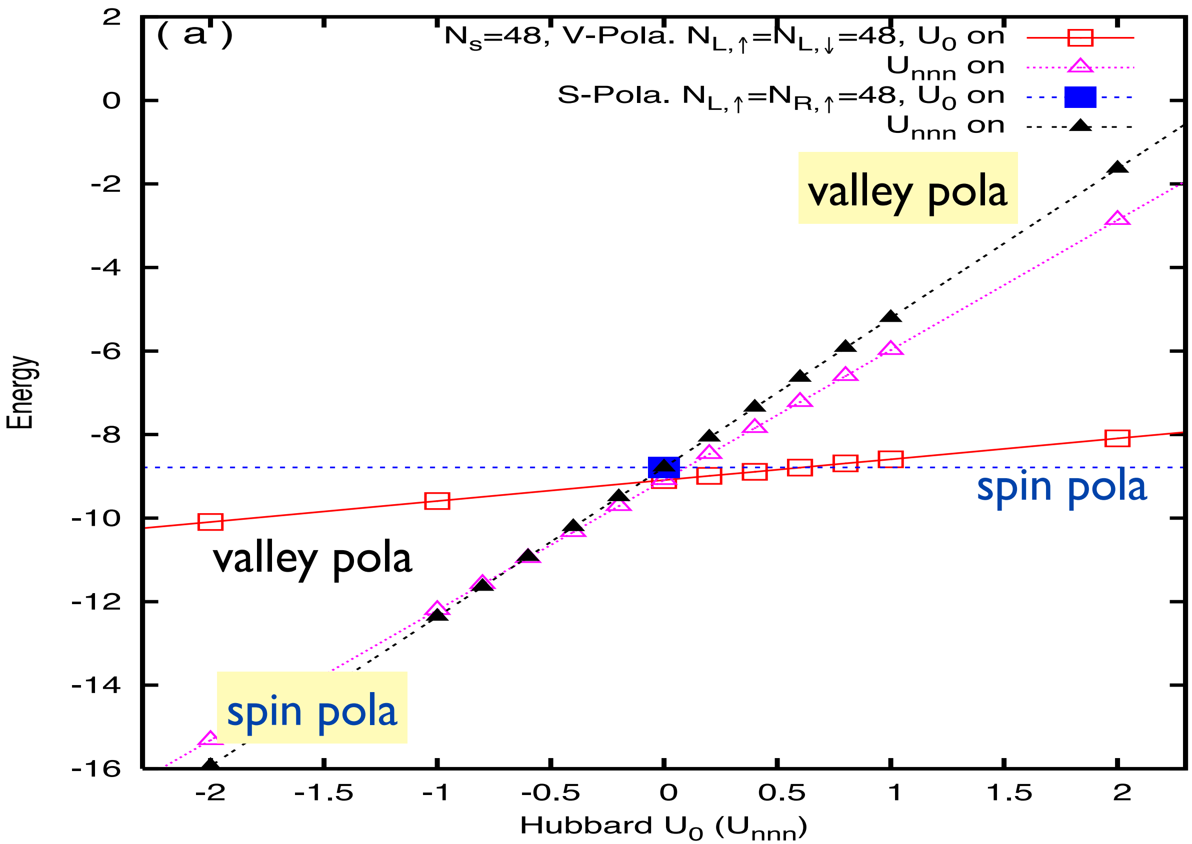



T.O. Wehling et al. PRL 106, 236805 (2011)

On-site repulsion favors the spin ferromagnet

No Zeeman effect at all (in contrast to real field)

# Numerical Hartree-Fock calculation of the groundstate energies (Donna Sheng)




Model with NNN correlations

**Model with on-site correlations (same model as previous the slide)**

Shows the sensitivity to local part of the interaction

# Fractional filling 2/3 graphene

**Two scenarios:**

**1)** Electrons **valley polarize** and realize a **FQH state in the large effective field of this valley.** **Time-reversal** symmetry is **spontaneously broken**.

**2)** Electrons **populate the two valleys**. **Time-reversal invariant state**.

# Graphene at fractional filling 2/3 (of the n=0 PLL)

**Exact diagonalization (Donna Sheng)**

$$N_e = 8$$
$$N_\phi = 12$$

$\Phi_0/48$ per hexagon on a 24*24 lattice

**1)** Groundstate and excited states **energies**

**2) Wavefunctions**

**3) Chern number**

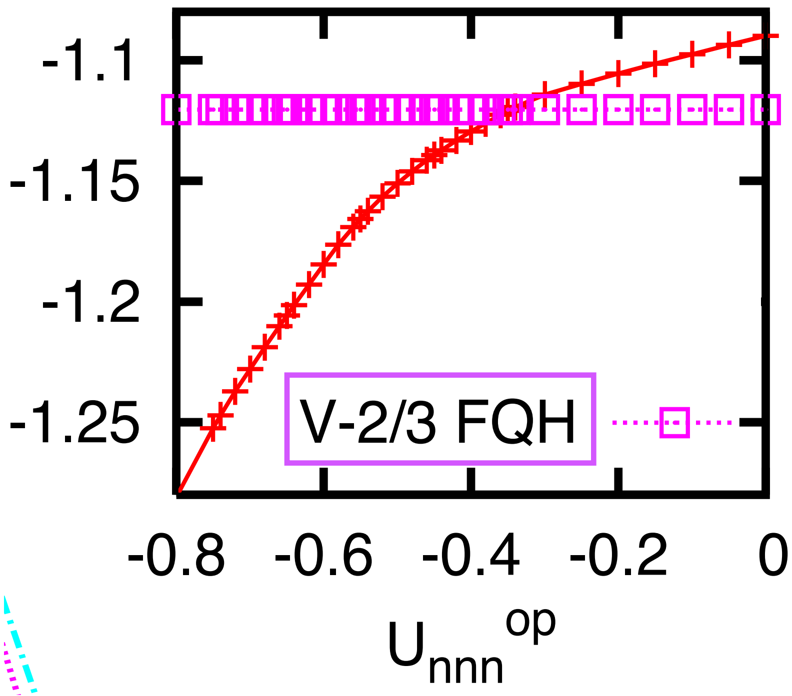
# Grounstate energy for Spinless electrons

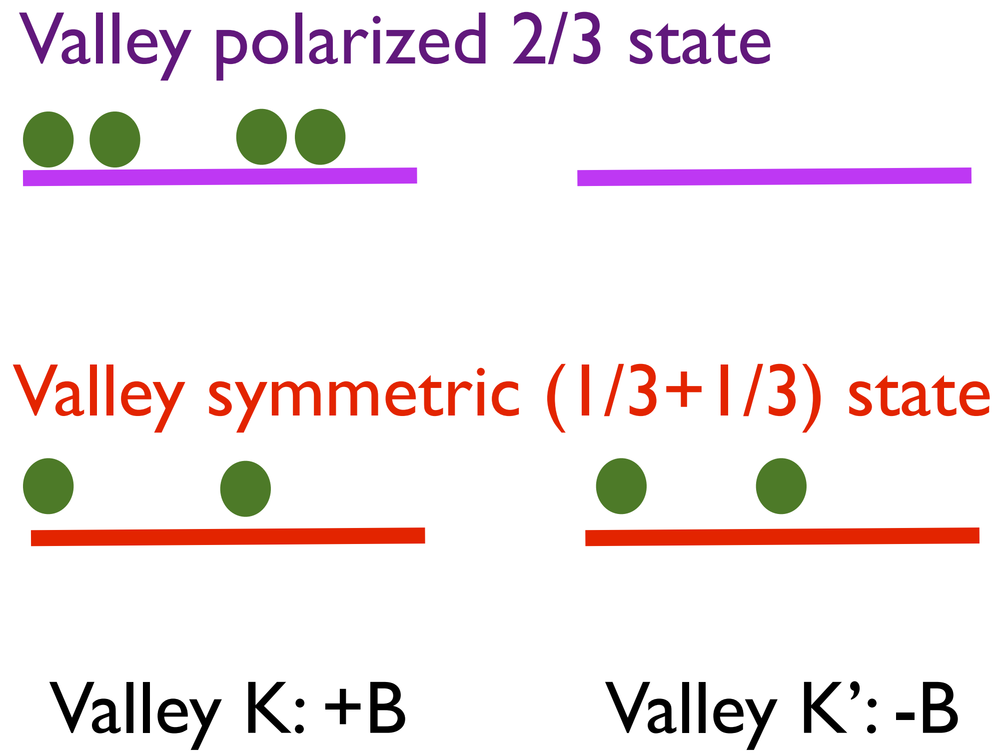
Valley polarized 2/3 state

Valley symmetric (1/3+1/3) state

Valley K: +B     Valley K': -B

# Grounstate energy for Spinless electrons




**Valley polarized state** is the most stable for pure Coulomb (U=0 points)

## Properties of the valley polarized state

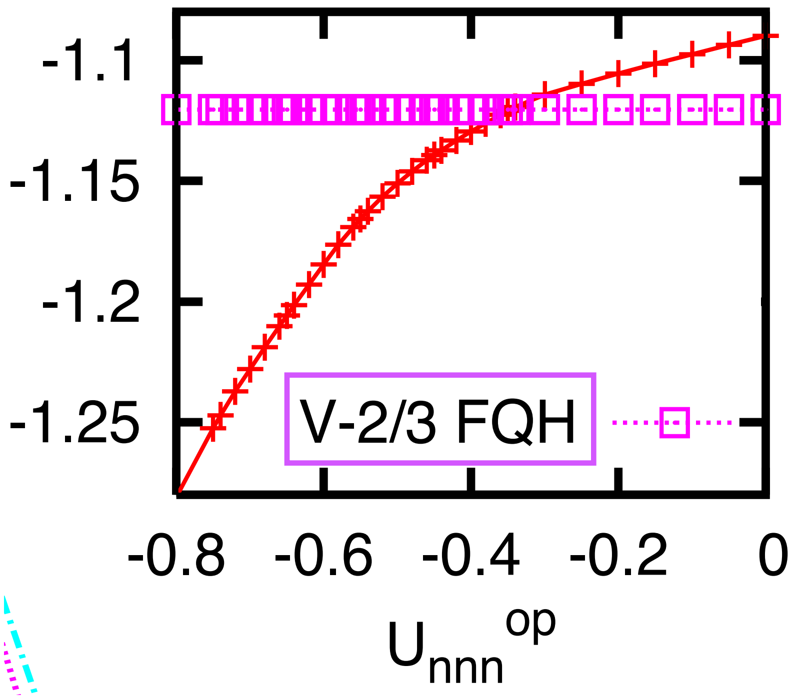
3-fold degenerate groundstate on the torus

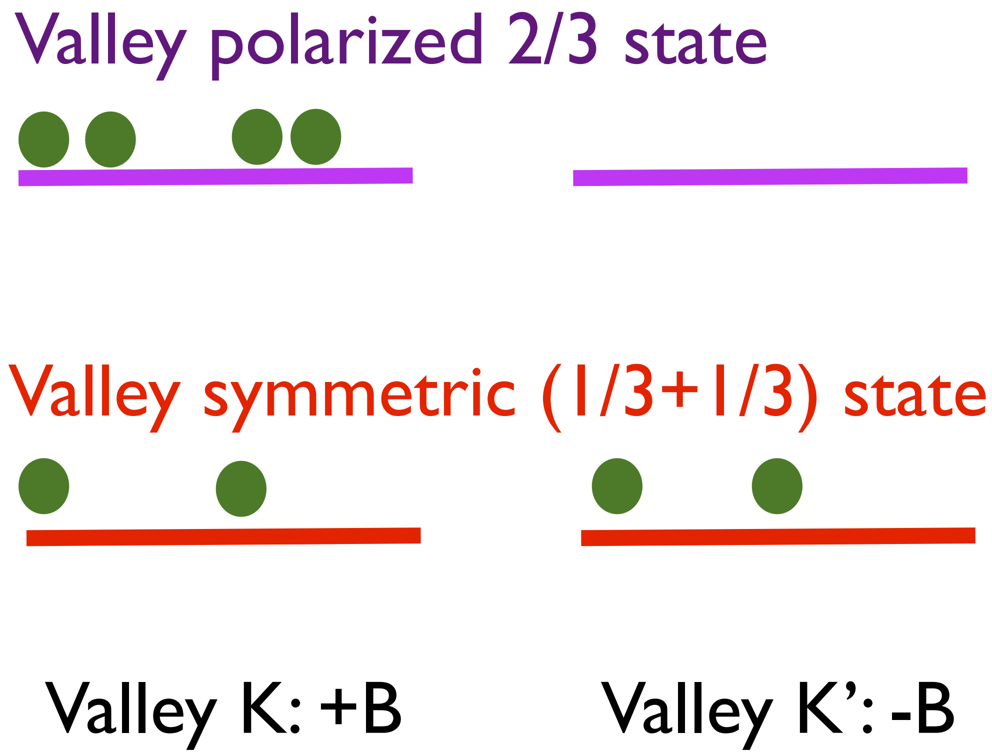
Total Chern number 2 for the 3-degenerated states
(calculated by a mesh method in phase twist space)

**This state is qualitatively similar to a FQH state at 2/3**
It is also a spin singlet state (for spinful electrons).
but realized in very large effective fields (up to hundreds of Tesla)

*How to destabilize this state ?*

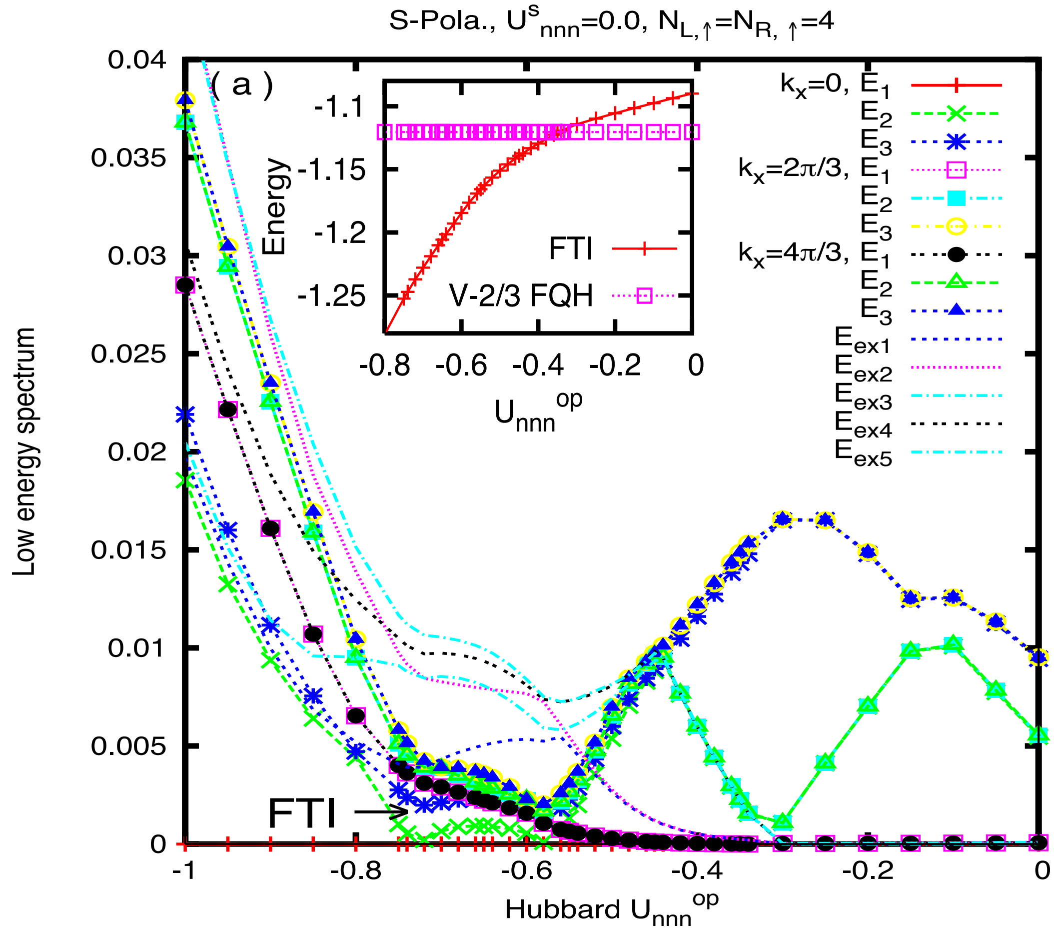
# Grounstate energy for Spinless electrons




**Valley symmetric state can be obtained by decreasing the next-nearest-neighbor interaction Unnn^op (between opposite valleys)**

# Low energy spectrum for spinless electrons

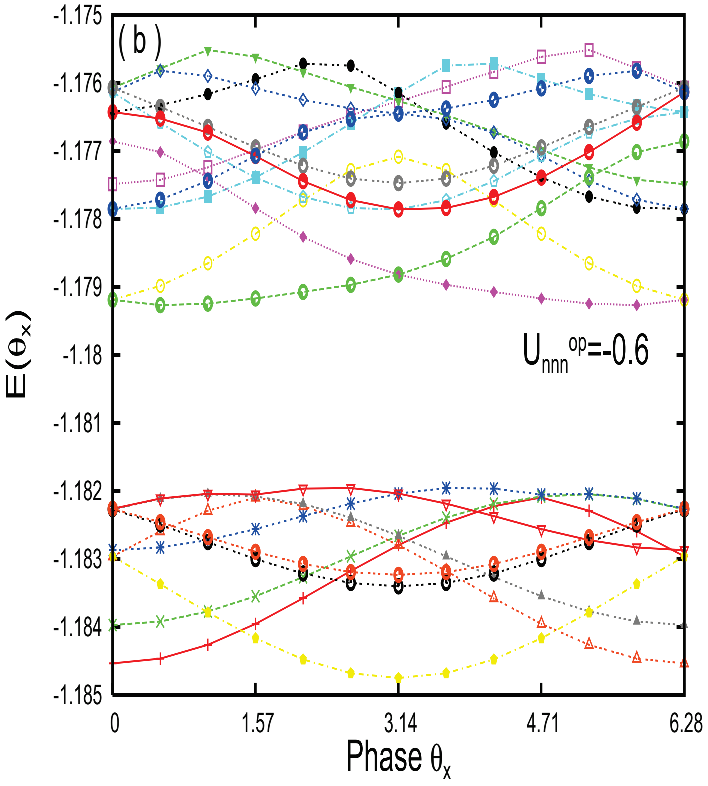



**9 fold** quasi-degenerate groundstate

**Valley fractional topological insulator (FTI)**

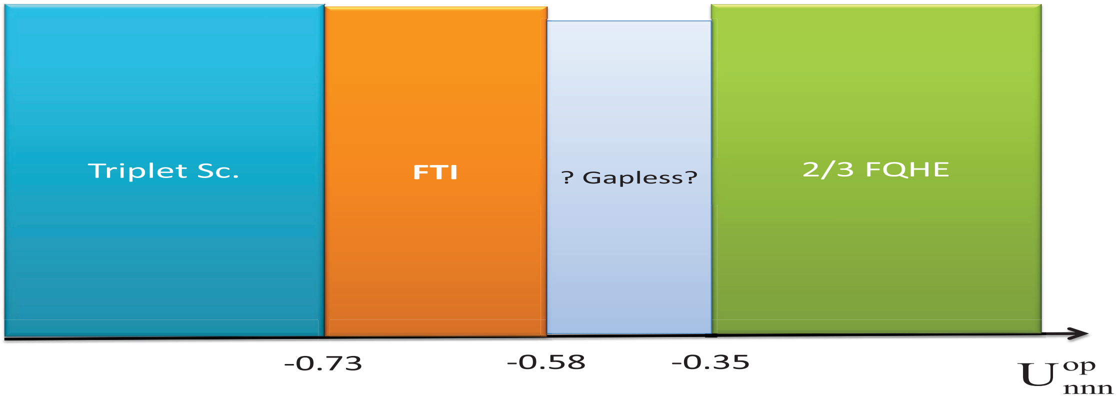
# Properties of this state

Boundary-phase twist



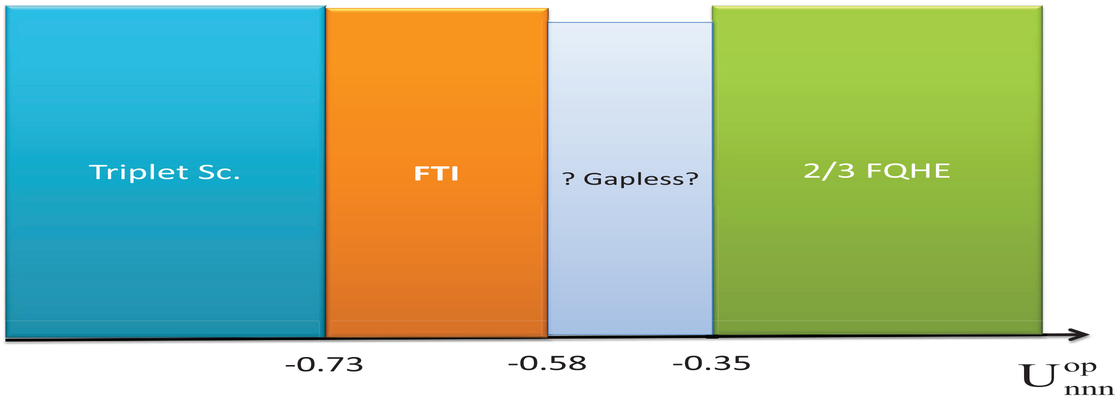

**9-fold degenerated valley Fractional Topological Insulator**

# Phase diagram: spinless case



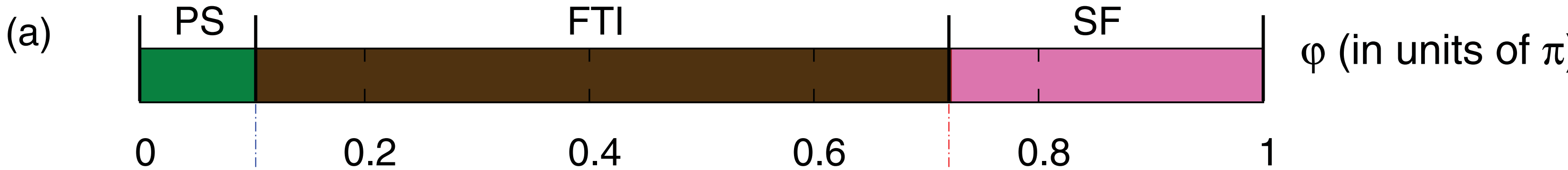

*opposite valley*

# Phase diagram: spinless case




*opposite valley*

## Spin FTI from Hua Chen and Kun Yang, PRB 85, 195113 (2012)

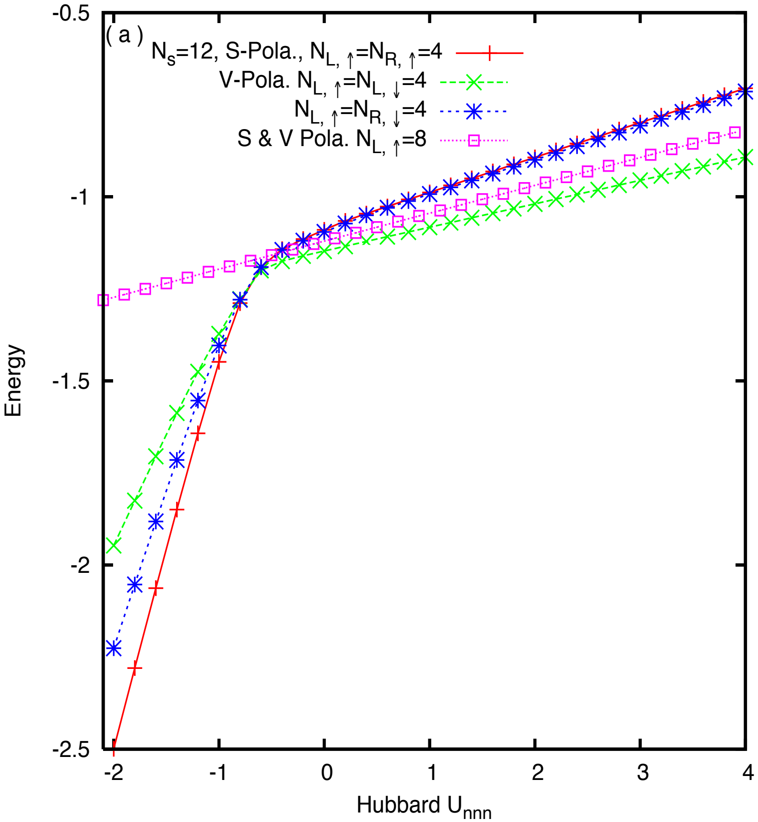



$$\boxed{\begin{aligned}\hat{\mathcal{H}}_{\text{int}} = \frac{1}{2}\sin(\varphi)\sum_{\{j_i\}\sigma} V^{\sigma\sigma}_{j_1 j_2 j_3 j_4} c^\dagger_{j_1\sigma} c^\dagger_{j_2\sigma} c_{j_3\sigma} c_{j_4\sigma} \\ + \frac{1}{2}\cos(\varphi)\sum_{\{j_i\}\sigma} V^{\sigma\bar{\sigma}}_{j_1 j_2 j_3 j_4} c^\dagger_{j_1\sigma} c^\dagger_{j_2\bar{\sigma}} c_{j_3\bar{\sigma}} c_{j_4\sigma}\end{aligned}}$$

*same spin*

*opposite spin*

# Grounstate energy for Spinful electrons




Zeeman energy is strictly zero

Valley K: +B

Valley K': -B

Four different states

**Valley polarized + spin singlet** for Unnn=0 (Pure Coulomb)

**Valley unpolarized + spin singlet** stabilized by Unnn<0 but no fractional topological insulator :-(

# Experiments vs theory

**Status of experiments:**

**- Spectroscopy of pseudo Landau levels**

**- Few flux quanta in samples**

**- No magnetotransport experiments so far**

**Possible improvements:**

**- Samples on insulating substrate**

**- Bigger samples (larger orbital degeneracy of the PLL)**

**Interactions in current experiments:**

- Strained graphene and molecular graphene: interactions are screened by metallic substrate

# Conclusions and perspectives

Real graphene: **valley polarized state** at partial filling. Spin (or valley) Hall ferromagnet at neutrality.

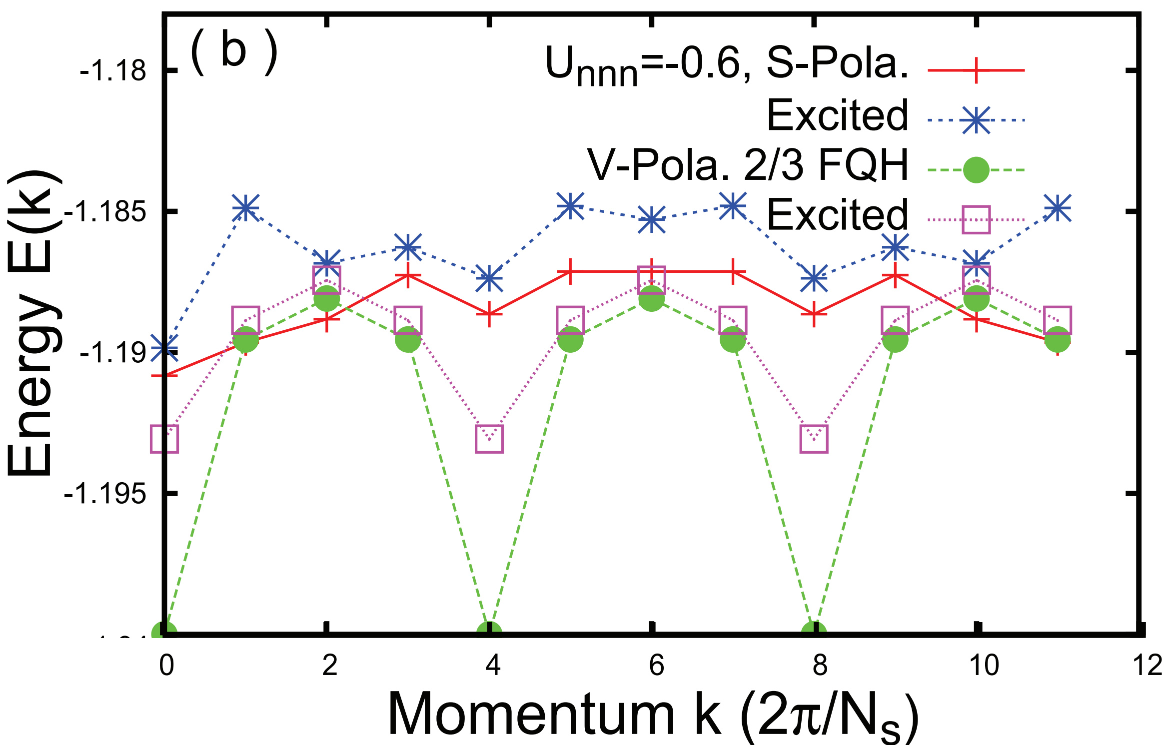
Tuned interactions: **valley Fractional Topological Insulator**

**New platforms** to generate **high fields** and **correlated phases.**

**Potentially** the richness of Quantum Hall physics but with additional competition between time-reversal breaking (FQH like) and time reversal invariant states.

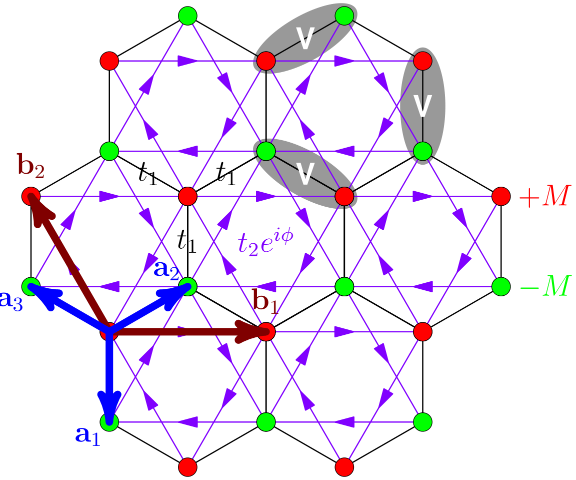
**Still a lot to study:** activation gaps, excitations, role of additional real magnetic field, superconductivity ...

**Thanks for your attention !**



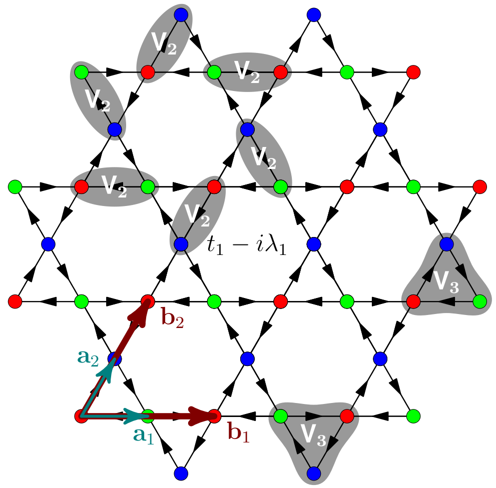
( b )
U_nnn=-0.6, S-Pola.
Excited
V-Pola. 2/3 FQH
Excited
Energy E(k)
-1.18
-1.185
-1.19
-1.195
0
2
4
6
8
10
12
Momentum k (2π/N_s)

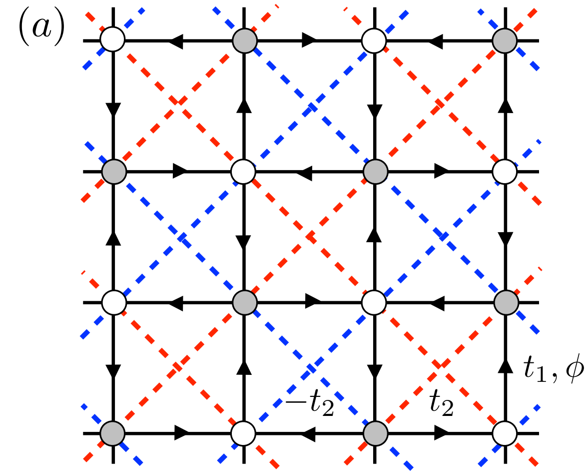
# Chern insulators (also Quantum Anomalous Hall phases)




Haldane model (1988)

Kagome lattice



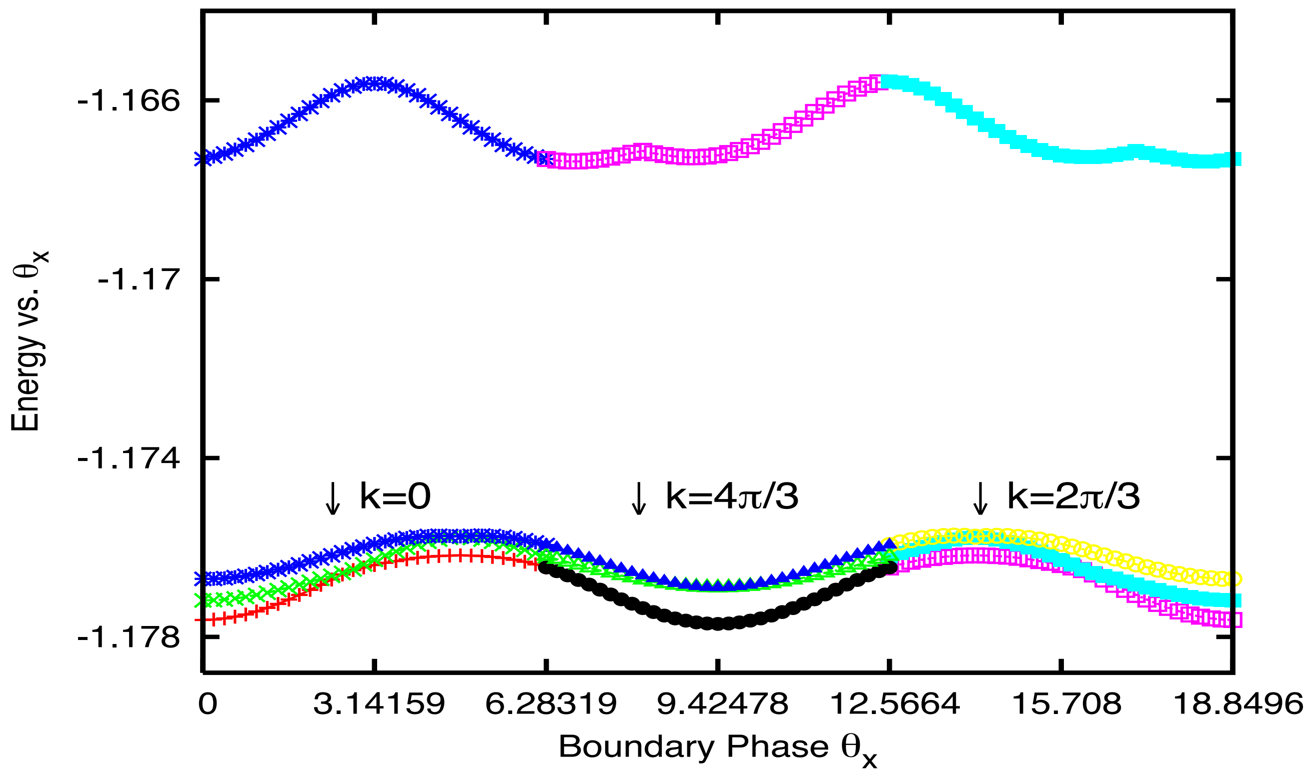

Chekerboard lattice

**Lattice model + local fluxes (complex hopping matrix elements)**

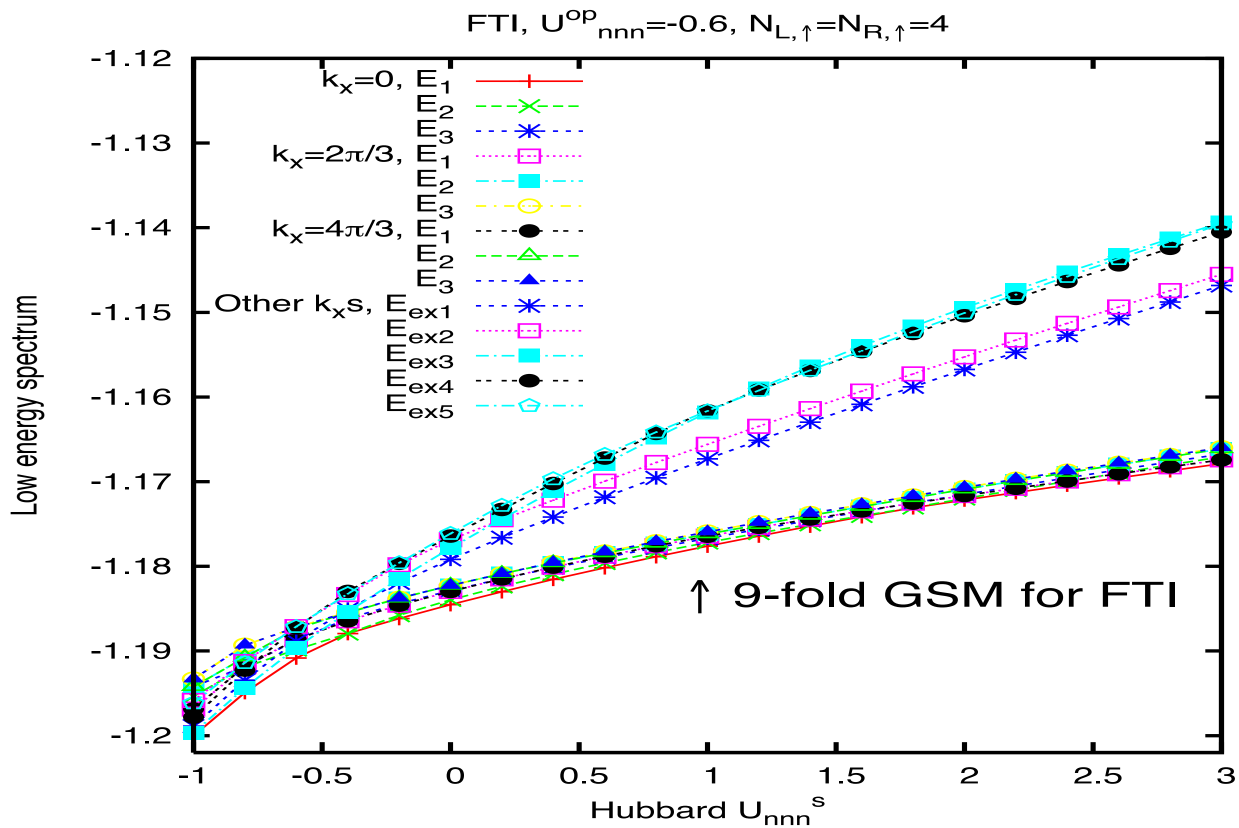
**Local fluxes break Time-Reversal symmetry and allow for Quantum Hall Effect for Bloch states (no Landau levels)**

Illustrations from Wu, Regnault, Bernevig, PRB 2012 and Lauchli et al. ArXiv 2012

# Valley FTI: decoupled 1/3 + 1/3*



# Properties of this valley FTI




Adiabatic continuity with the decoupled valley state

# Motivations for FTIs (no overall magnetic field)

Lattice system with Quantum Hall effect but no Landau levels

Are the states in FCIs similar to FQH states in some limit ?

Same serie of fractions or not ? Dependence on the underlying lattice model ?

FTI in a band with high Chern number N>1

etc...

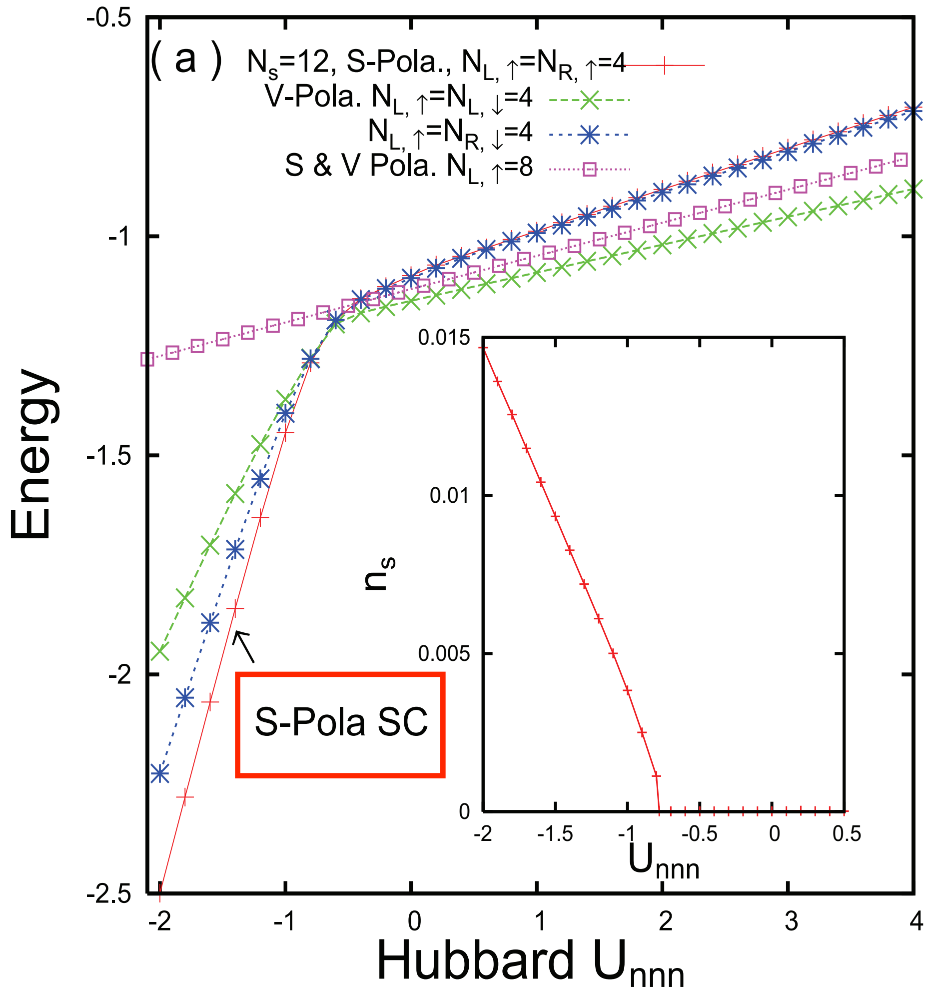
$$C = \frac{i}{4\pi} \int \int d\theta_x d\theta_y \left[ \langle \frac{\partial \Psi}{\partial \theta_x} | \frac{\partial \Psi}{\partial \theta_y} \rangle - \langle \frac{\partial \Psi}{\partial \theta_y} | \frac{\partial \Psi}{\partial \theta_x} \rangle \right]$$

$$\boxed{C^{\alpha,\beta} = \frac{i}{4\pi} \iint d\theta_x^\alpha d\theta_y^\beta \left[ \left\langle \frac{\partial \Psi}{\partial \theta_x^\alpha} \middle| \frac{\partial \Psi}{\partial \theta_y^\beta} \right\rangle - \left\langle \frac{\partial \Psi}{\partial \theta_y^\beta} \middle| \frac{\partial \Psi}{\partial \theta_x^\alpha} \right\rangle \right]}$$

# Tuned graphene: spinful case

Superfluid density

$$n_s = \frac{1}{2}\frac{\partial^2 E_g}{\partial \theta^2}$$




→ **Spin polarized superconductor**