Topological Quantum Phenomena in
Condensed Matter with Broken Symmetries

Surface Majorana Fermions in Topological Superconductors

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TOYOTA


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- Yoichi Ando (Osaka University)

Outline

1. Majorana fermions in various systems

2. Surface Majorana fermions in superconducting topological insulator $Cu_xBi_2Se_3$

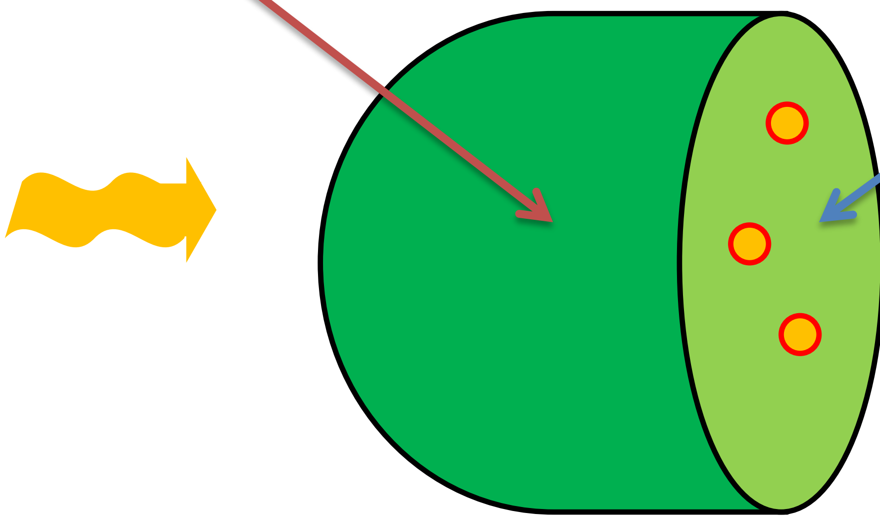
3. Summary

What is topological superconductor ?

Topological superconductors

Bulk:
gapped state with
non-zero **topological #**

Boundary:
gapless state with
Majorana condition



The gapless boundary state = Majorana fermion

[Read-Green(00), Roy-Stone, Qi et al (09)]

Majorana Fermion

Dirac fermion with Majorana condition

1. Dirac Hamiltonian

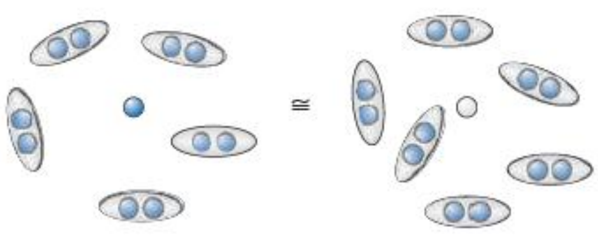
$$\mathcal{H}(\boldsymbol{k}) = \boldsymbol{\sigma} \cdot \boldsymbol{k}, \text{ or } \quad \mathcal{H}(k_x) = c k_x$$

2. Majorana condition

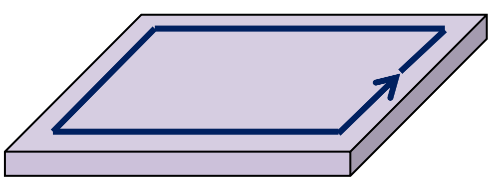
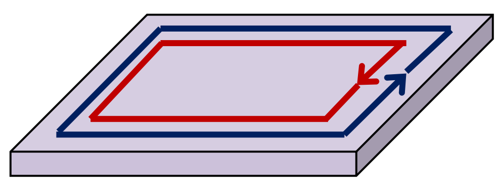
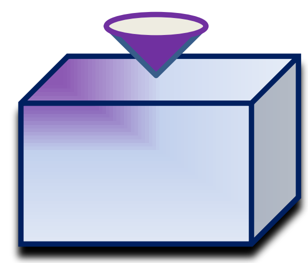
$$\Psi = C\Psi^*$$

particle = antiparticle

[Wilczek(09)]



different bulk topological
= different Majorana fermions

2+1D time-reversal breaking SC	2+1D time-reversal invariant SC	3+1D time-reversal invariant SC
1st Chern # (TKNN(82), Kohmoto(85), Volovik(87))	Z_2 number (Kane-Mele(06), Qi et al(09))	3D winding # (Volovik, Schnyder et al (08))
1+1D **chiral** edge Majorana fermion 	1+1D **helical edge Majorana fermion** 	2+1D **helical** surface Majorana fermion 
Sr_2RuO_4	Noncentosymmetric SC (MS-Fujimto(09), Tanaka et al (09))	^{3}He B (K.Nagai et al)

Which system become a topological SC ?

- Spin-triplet (odd-parity) superconductors

  Volovik (86), Read-Green(00)

- Superconducting states with SO interaction

  MS, Physics Letters B535,126 (03)

  MS, Takahashi, Fujimoto PRL(09) PRB(10)

Spin-triplet (odd parity) SCs

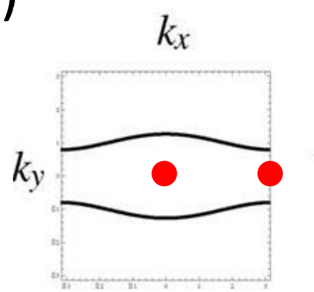
A simple criterion for topological SC

If the number of time-reversal invariant momenta enclosed by the Fermi surface is odd, the spin-triplet SC is (strongly) topological.

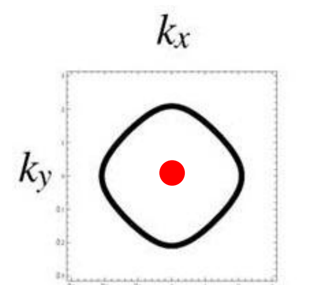
[Sato (09), Sato (10), Fu-Berg (10)]

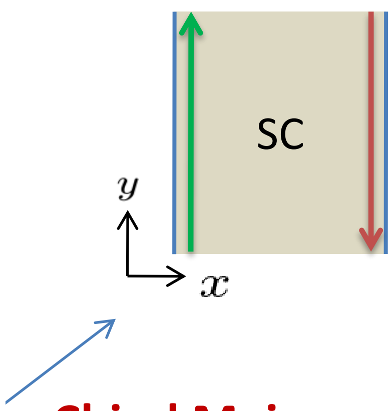
2D spinless SC)

Even



Odd



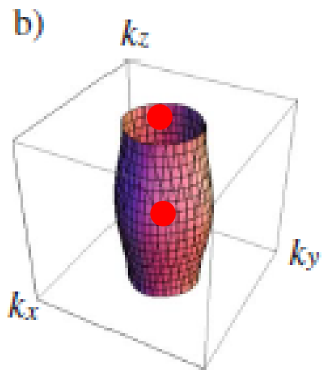


Chiral Majorana mode

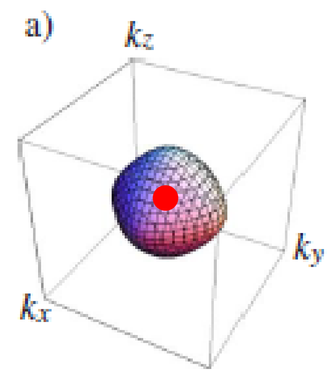
$$\Delta(\boldsymbol{k}) = k_x + ik_y$$

3D time-reversal invariant spin-triplet SC)

Even

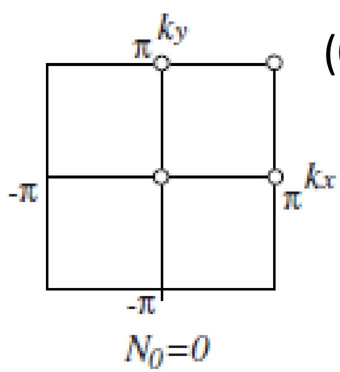


Odd



BW gap fn.

$$d_i(\boldsymbol{k}) \propto k_i$$



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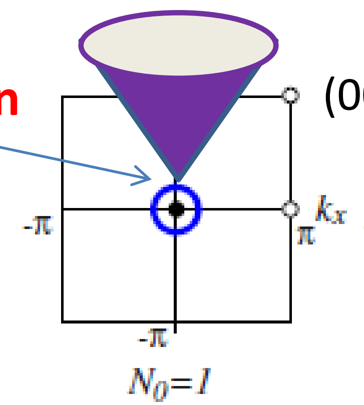
Majorana fermion



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With proper topology of the Fermi surface, spin-triplet SCs (or odd-parity SCs) naturally become topological.

Superconducting states with SO interaction

1. Dirac system with s-wave condensate MS, Physics Letters B535,126 (03)

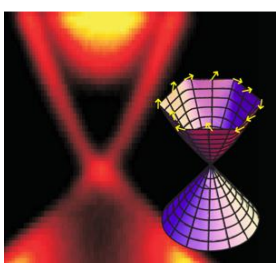
$$\mathcal{H} = \begin{pmatrix} -i\sigma_i\partial_i & \Phi^* \\ \Phi & -i\sigma_i\partial_i \end{pmatrix} \qquad \Phi = \Phi_0 f(r) e^{i\theta} \quad \text{vortex}$$

Zero mode in a vortex [Jackiw-Rossi (81), Callan-Harvey(85)]

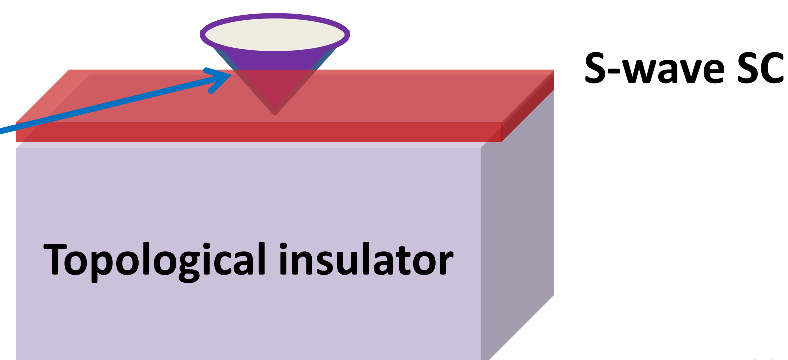
With Majorana condition → **Majorana zero mode**
non-Abelian anyon [MS(03)]

On the surface of topological insulator Fu-Kane PRL (08)

Bi_2Se_3 Hsieh et al., Nature (2009)



Dirac system with s-wave condensate



2. S-wave superconductor with Rashba SO interaction

[MS, Takahashi, Fujimoto PRL(09) PRB(10)]

Rashba SO

$$\mathcal{H}(\boldsymbol{k}) = \begin{pmatrix} \frac{(\hbar \boldsymbol{k})^2}{2m} - E_{\rm F} + \boxed{\boldsymbol{g_k} \cdot \boldsymbol{\sigma}} - \mu_{\rm B} H_z \sigma_z & i\Delta_0 \sigma_y \\ -i\Delta_0 \sigma_y & -\frac{(\hbar \boldsymbol{k})^2}{2m} + E_{\rm F} + \boxed{\boldsymbol{g_k} \cdot \boldsymbol{\sigma}^*} + \mu_{\rm B} H_z \sigma_z \end{pmatrix}$$

$$\mathcal{H}^{\rm D}(\boldsymbol{k}) = D\mathcal{H}(\boldsymbol{k})D^\dagger, \quad D = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i\sigma_y \\ i\sigma_y & 1 \end{pmatrix}$$

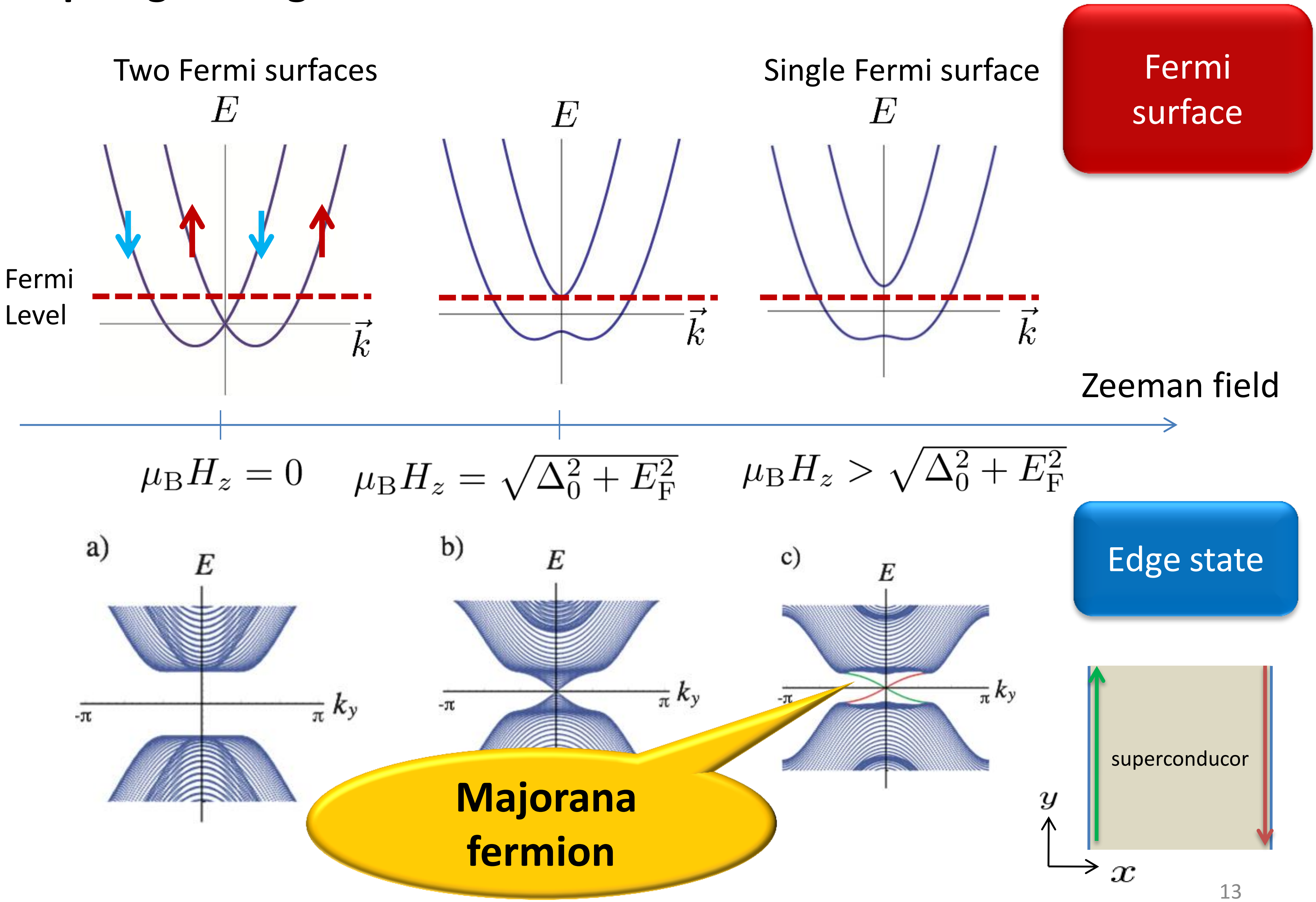
$$\mathcal{H}^{\rm D}(\boldsymbol{k}) = \begin{pmatrix} \Delta_0 - \mu_{\rm B} H_z \sigma_z & -i\left[\frac{(\hbar \boldsymbol{k})^2}{2m} - E_{\rm F}\right]\sigma_y - \boxed{i\boldsymbol{g_k} \cdot \boldsymbol{\sigma}\sigma_y} \\ i\left[\frac{(\hbar \boldsymbol{k})^2}{2m} - E_{\rm F}\right]\sigma_y + \boxed{i\boldsymbol{g_k}\sigma_y \cdot \boldsymbol{\sigma}} & -\Delta_0 + \mu_{\rm B} H_z \sigma_z \end{pmatrix}$$

p-wave gap is induced by Rashba SO int.

Topological superconductivity can be obtained if we choose a suitable Fermi surface

Topological Edge state

Sato-Takahashi-Fujimoto (09, 10)

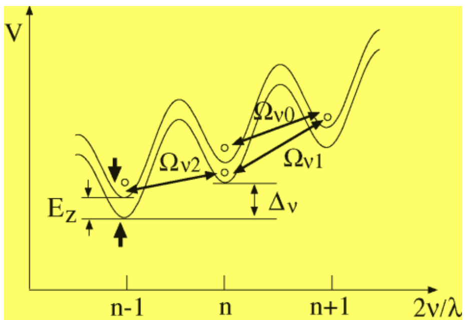


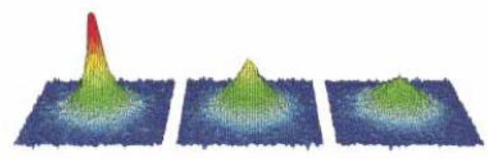
$$\boxed{\mu_{\rm B} H_z > \sqrt{\Delta_0^2 + E_{\rm F}^2}}$$ strong magnetic field is needed

One needs to avoid orbital depairing effect to realize Majorana fermion

a) s-wave superfluid of cold atoms with laser generated Rashba SO interaction

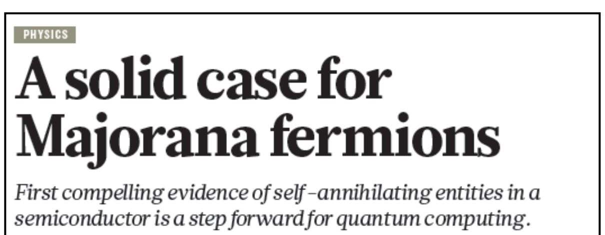
[Sato-Takahashi-Fujimoto PRL(09)]





b) 1D nanowire system with SO int.

R. M. Lutchyn et al. PRL(10), Y. Oreg et al, PRL(10)

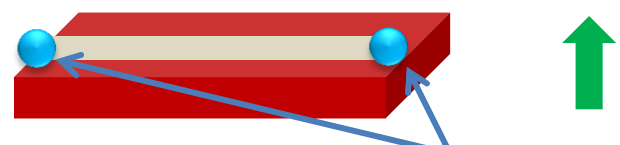

PHYSICS

A solid case for Majorana fermions

First compelling evidence of self-annihilating entities in a semiconductor is a step forward for quantum computing.

Kouwenhoven, Nature, News (12)

Magnetic field



Majorana fermion

- Majorana fermions are not merely a theoretical possibility now, but they are what we can realize somehow in experiments.

- Now I would like to discuss another Majorana fermions which can be realized experimentally.

Surface Majorana fermions in superconducting topological insulator $Cu_xBi_2Se_3$

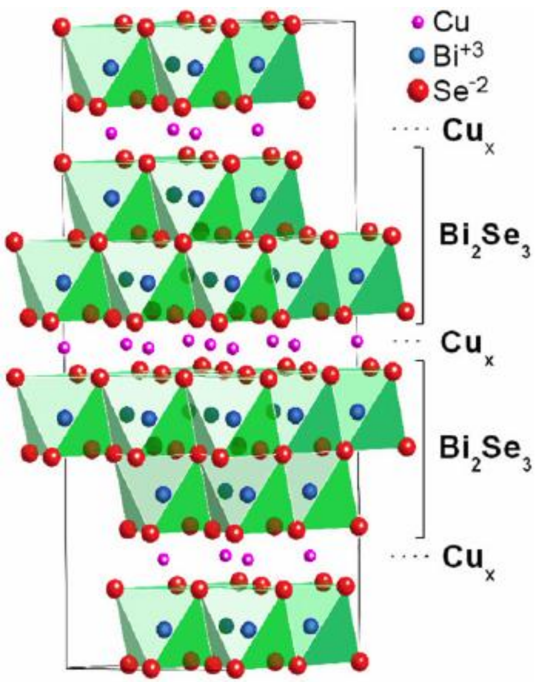
- The first observed Majorana fermions in solid state physics -

Sasaki, Kriener, Segawa, Yada, Tanaka, MS, Ando, Phys. Rev. Lett. 107, 217001 (11)
Yamakage, Yada, MS, Tanaka, Phys. Rev. B85, 180509(R) (12)

3D time-reversal invariant topological SC

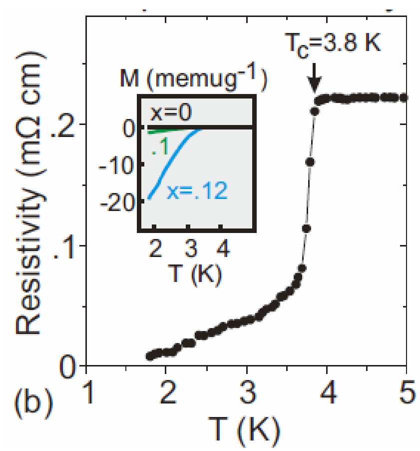
[Sasaki, Kriener, Segawa, Yada, Tanaka, MS, Ando PRL (11)]

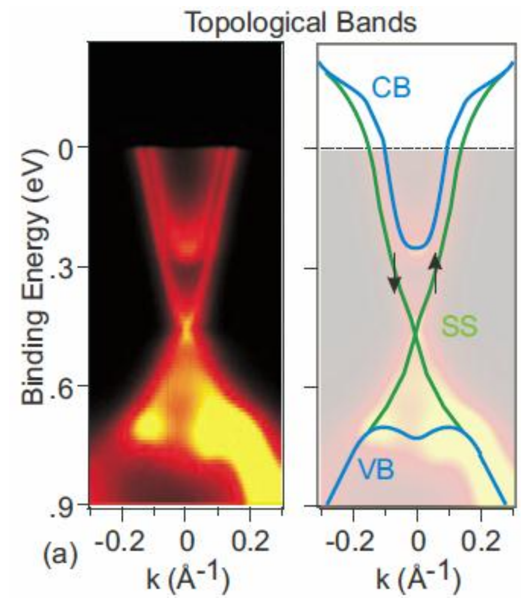
$Cu_xBi_2Se_3$



Hor et al., PRL (2010)

Superconducting only for $0.10 \leq x \leq 0.30$



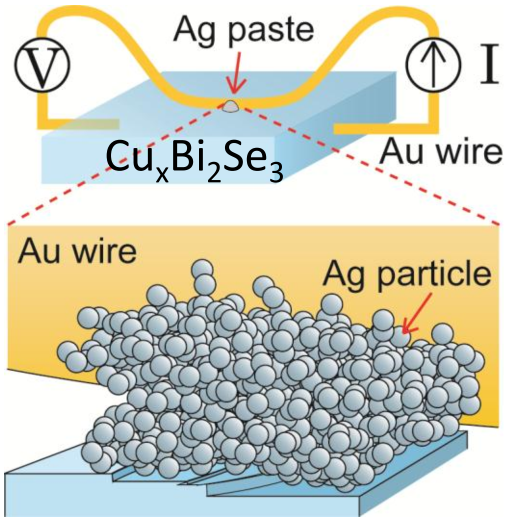


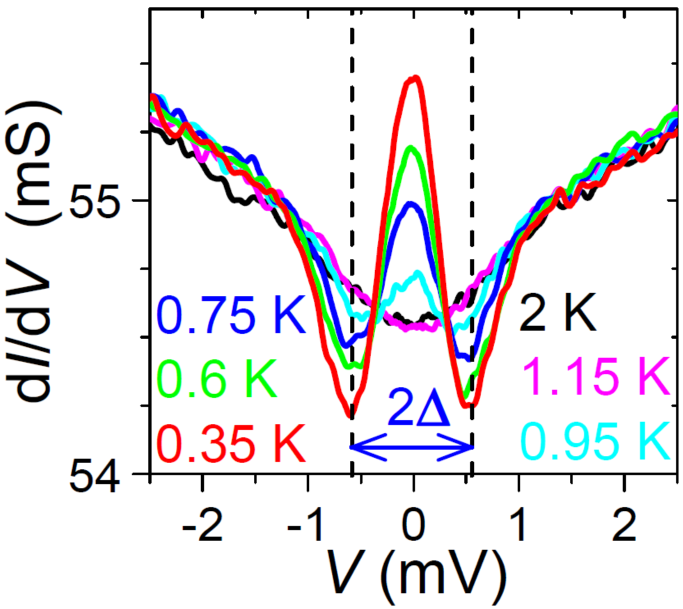
Wray et al.,
Nat. Phys. (2010)

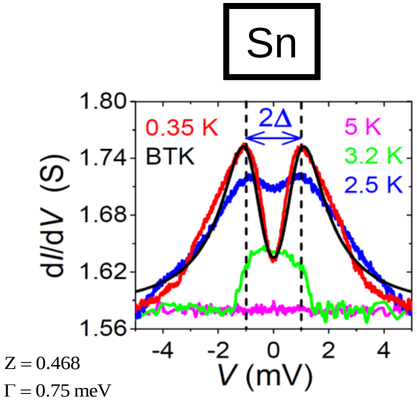
Superconducting Topological Insulator (STI)

Recently measurement of tunneling conductance has been done for this STI.

[Sasaki, Kriener, Segawa, Yada, Tanaka, MS, Ando PRL (11)]







Robust zero-bias peak appears in the tunneling conductance

"Evidence" of Majorana fermions on the surface

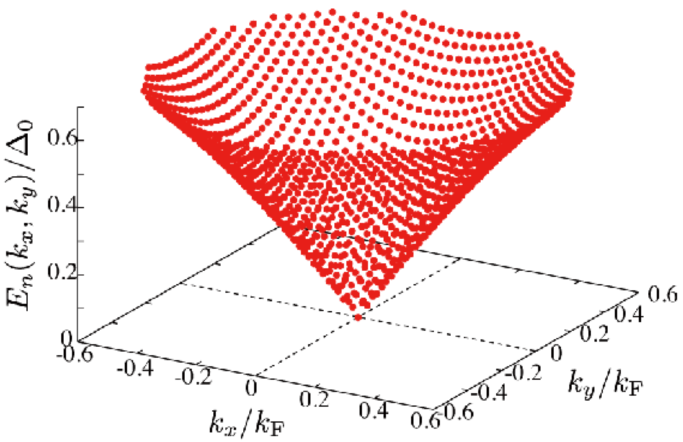
But the story is not so simple …

cf.) Superconducting analogue of 3He B-phase

$$\hat{\Delta}(\boldsymbol{k}) = i\boldsymbol{d}(\boldsymbol{k}) \cdot \boldsymbol{\sigma}\sigma_2 \quad \text{with} \quad \boldsymbol{d}(\boldsymbol{k}) = \Delta_0 \boldsymbol{k}$$

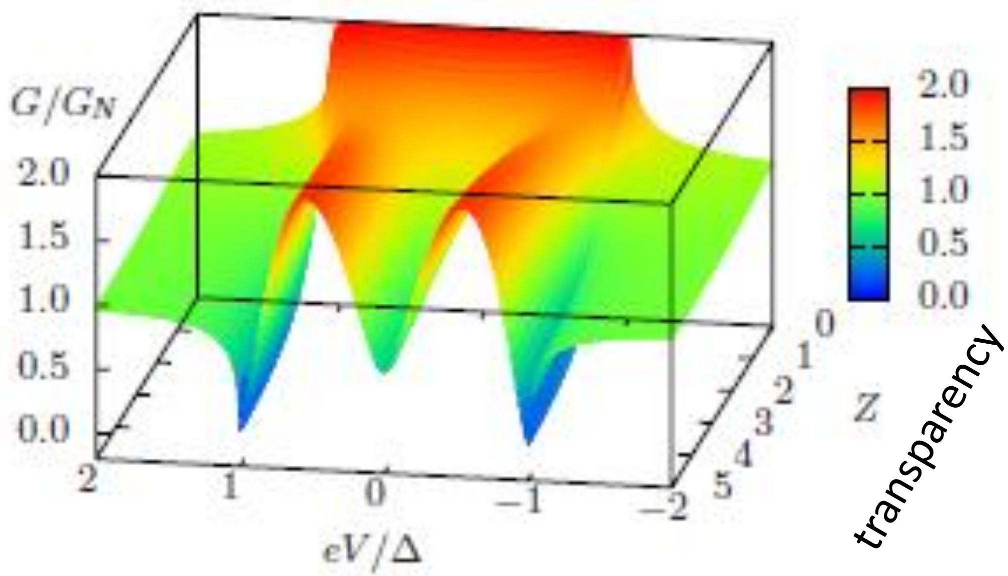
Spin-triplet

- The simplest 3D TRI topological SC
  [Schnyder et al. (08), Qi et al. (09), …]
- It supports helical Majorana fermions on its surface



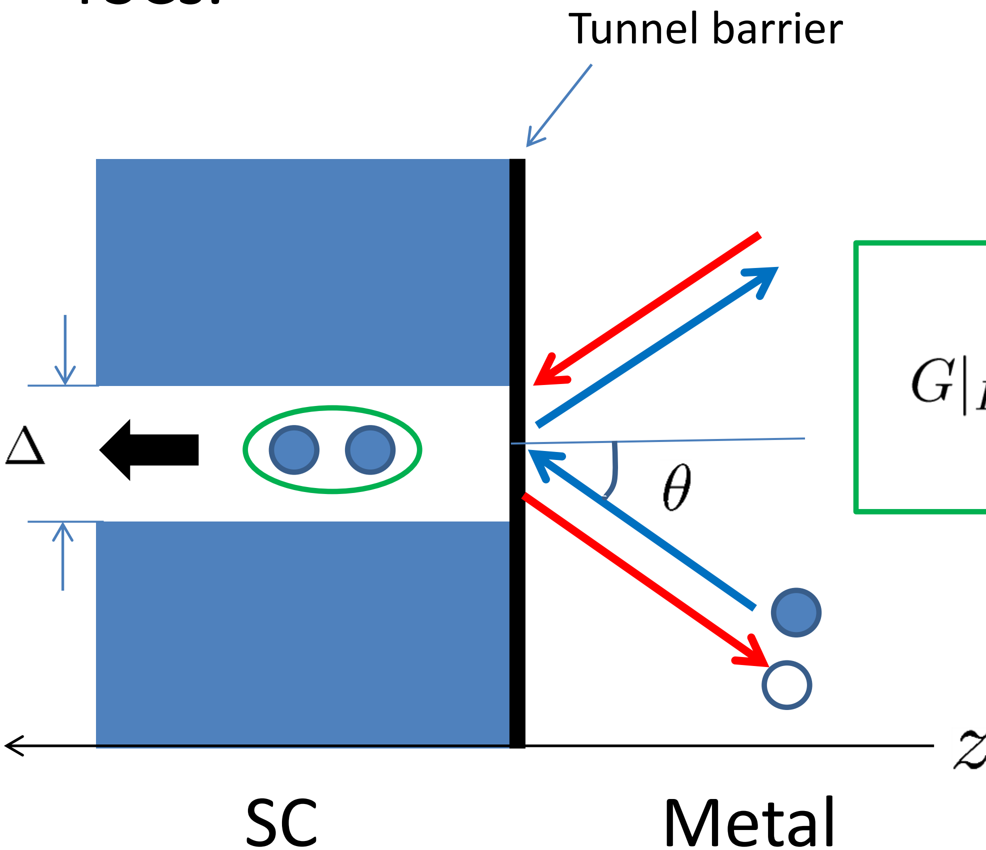
Tunneling conductance for the superconducting analogue of 3He-B

[Y.Asano et al. PRB (03)]



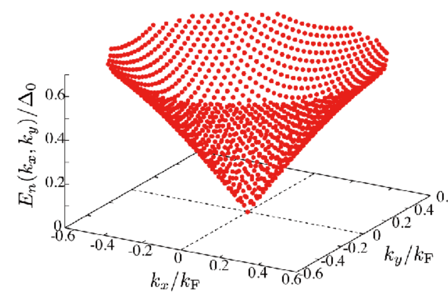
- There exist surface **helical Majorana fermions**.
- But the tunneling conductance shows **no single peak structure**

Actually, the zero-bias **dip** is likely a general feature of 3D TSCs.



Volume factor

$$G|_{E=0} = \int d\theta d\phi \sin\theta \, T(k_x, k_y)|_{E=0}$$



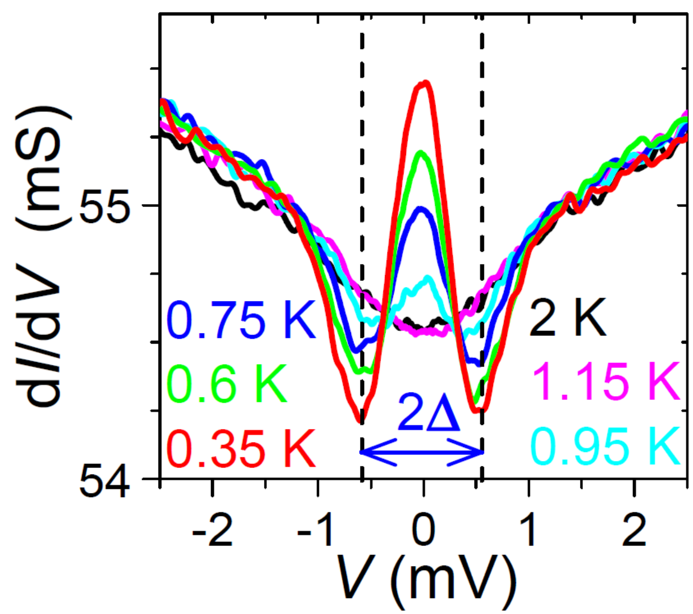
When the tunnel barrier is high,

$$T(k_x, k_y)|_{E=0} \propto N(k_x, k_y)|_{E=0}$$

Surface DOS

Nonzero only at $\theta = 0$

Is the single peak structure of the tunneling conductance really explained by helical Majorana fermions ?



Our answer: **Yes**

Two possible solutions

	gap type	parity	energy gap structure
Δ_1	$\Delta^{11}_{\uparrow\downarrow} = -\Delta^{11}_{\downarrow\uparrow} = \Delta^{22}_{\uparrow\downarrow} = -\Delta^{22}_{\downarrow\uparrow}$ $\Delta^{11}_{\uparrow\downarrow} = -\Delta^{11}_{\downarrow\uparrow} = -\Delta^{22}_{\uparrow\downarrow} = \Delta^{22}_{\downarrow\uparrow}$	even	full gap
Δ_2	$\Delta^{12}_{\uparrow\downarrow} = -\Delta^{12}_{\downarrow\uparrow} = \Delta^{21}_{\uparrow\downarrow} = -\Delta^{21}_{\downarrow\uparrow}$	odd	full gap
Δ_3	$\Delta^{12}_{\uparrow\downarrow} = \Delta^{12}_{\downarrow\uparrow} = -\Delta^{21}_{\uparrow\downarrow} = -\Delta^{21}_{\downarrow\uparrow}$	odd	point node
Δ_4	$\Delta^{12}_{\uparrow\uparrow} = \Delta^{12}_{\downarrow\downarrow} = -\Delta^{21}_{\uparrow\uparrow} = -\Delta^{21}_{\downarrow\downarrow}$ $\Delta^{12}_{\uparrow\uparrow} = -\Delta^{12}_{\downarrow\downarrow} = -\Delta^{21}_{\uparrow\uparrow} = \Delta^{21}_{\downarrow\downarrow}$	odd	point node

[Fu-Berg (10)]

1. Deformed helical Majorana fermion with flat band.

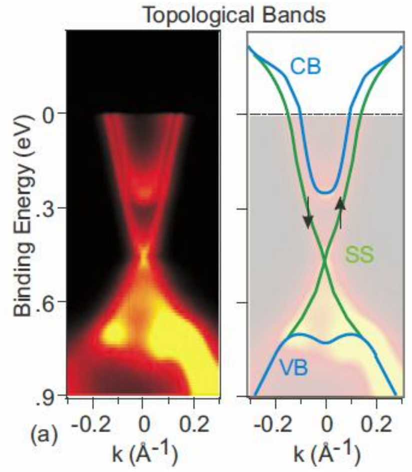
Sasaki, MS el al PRL (11)

2. Structural transition of helical Majorana fermion

Yamakage, Yada, MS, Tanaka PRB (12)

Model Hamiltonian

parent topological insulator

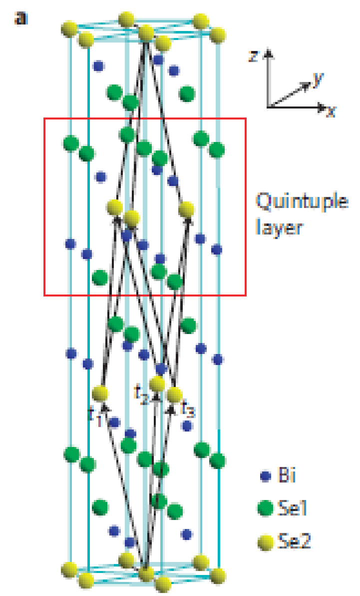


$$H_{\mathrm{TI}}(\boldsymbol{k}) = m(\boldsymbol{k})\sigma_x + v_z k_z \sigma_y + v\sigma_z(k_x s_y - k_y s_x)$$

$$m(\boldsymbol{k}) = m_0 + m_1 k_z^2 + m_2(k_x^2 + k_y^2), \quad (m_1 m_2 > 0)$$

σ_μ: Pauli matrices in orbital space (two p_z-orbitals of Se)

s_μ: Pauli matrices in spin space



Z2 invariant

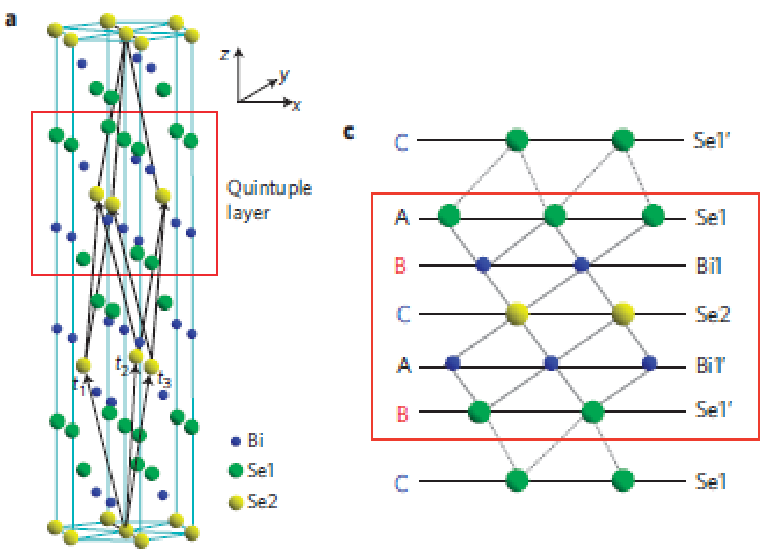
$$(-1)^\nu = \mathrm{sgn}(m_0 m_1)$$

$m_0 m_1 < 0$ Surface Dirac Fermion

$$H_D(\boldsymbol{k}) = v(k_x s_y - k_y s_x)$$

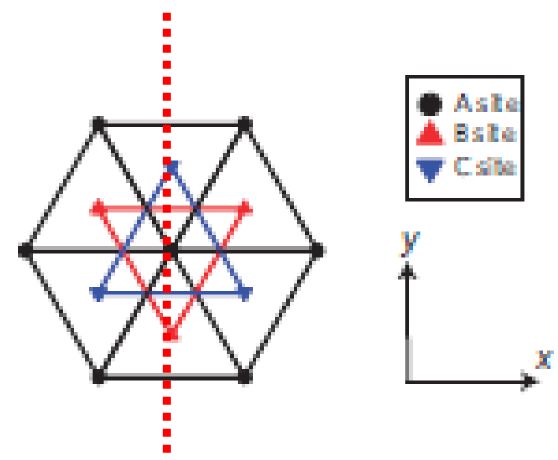
Symmetry

- Time-reversal invariance
- Inversion symmetry
- Mirror symmetry $\quad x \rightarrow -x, \quad s_y \rightarrow -s_y, \quad s_z \rightarrow -s_z$
- (Discrete) rotational symmetry in xy-plane



Side view

mirror



Top view

Superconducting TI

$(\psi_{\sigma\uparrow}, \psi_{\sigma\downarrow}, -\psi^{\dagger}_{\sigma\downarrow}, \psi^{\dagger}_{\sigma\uparrow})$ Nambu rep.

$$\boxed{H_{\mathrm{STI}}(\boldsymbol{k}) = \begin{pmatrix} H_{\mathrm{TI}}(\boldsymbol{k}) - \mu & \hat{\Delta} \\ \hat{\Delta}^{\dagger} & -H_{\mathrm{TI}}(\boldsymbol{k}) + \mu \end{pmatrix}}$$

Our assumption

The gap function is a constant matrix

The pairing int. is short-range and attractive

	gap type	parity	energy gap structure
Δ_1	$\Delta^{11}_{\uparrow\downarrow} = -\Delta^{11}_{\downarrow\uparrow} = \Delta^{22}_{\uparrow\downarrow} = -\Delta^{22}_{\downarrow\uparrow}$ $\Delta^{11}_{\uparrow\downarrow} = -\Delta^{11}_{\downarrow\uparrow} = -\Delta^{22}_{\uparrow\downarrow} = \Delta^{22}_{\downarrow\uparrow}$	even	full gap
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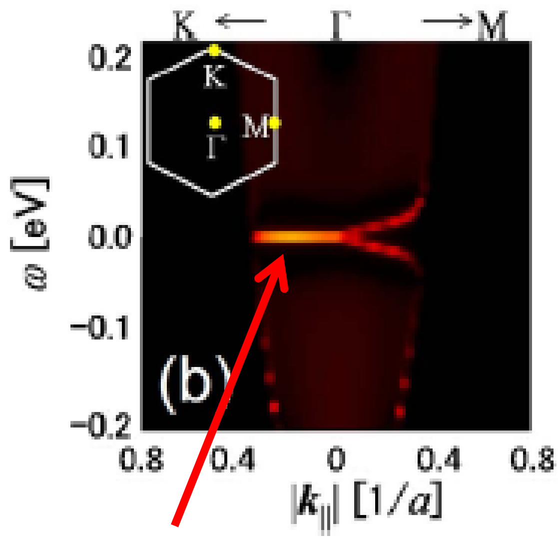
[Fu-Berg (10)]

1. Deformed Majorana fermion with flat dispersion

[Sasaki, MS el al PRL (11)]

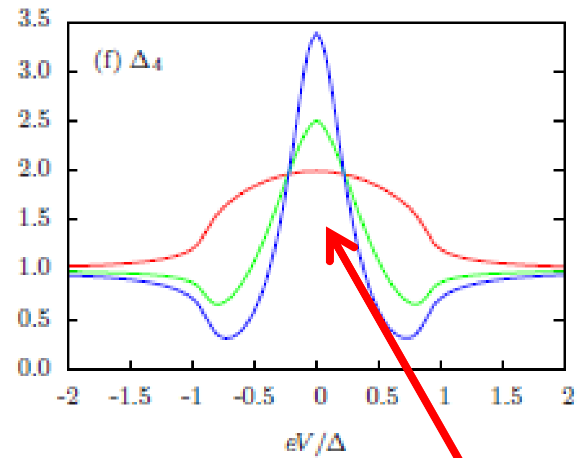
Δ_4 : nodal but topological

(111) Surface state



Tunneling conductance

[Yamakage, MS el al PRL (12)]



Deformed Majorana fermion → **Single peak**

Enhance SDOS at *E=0*

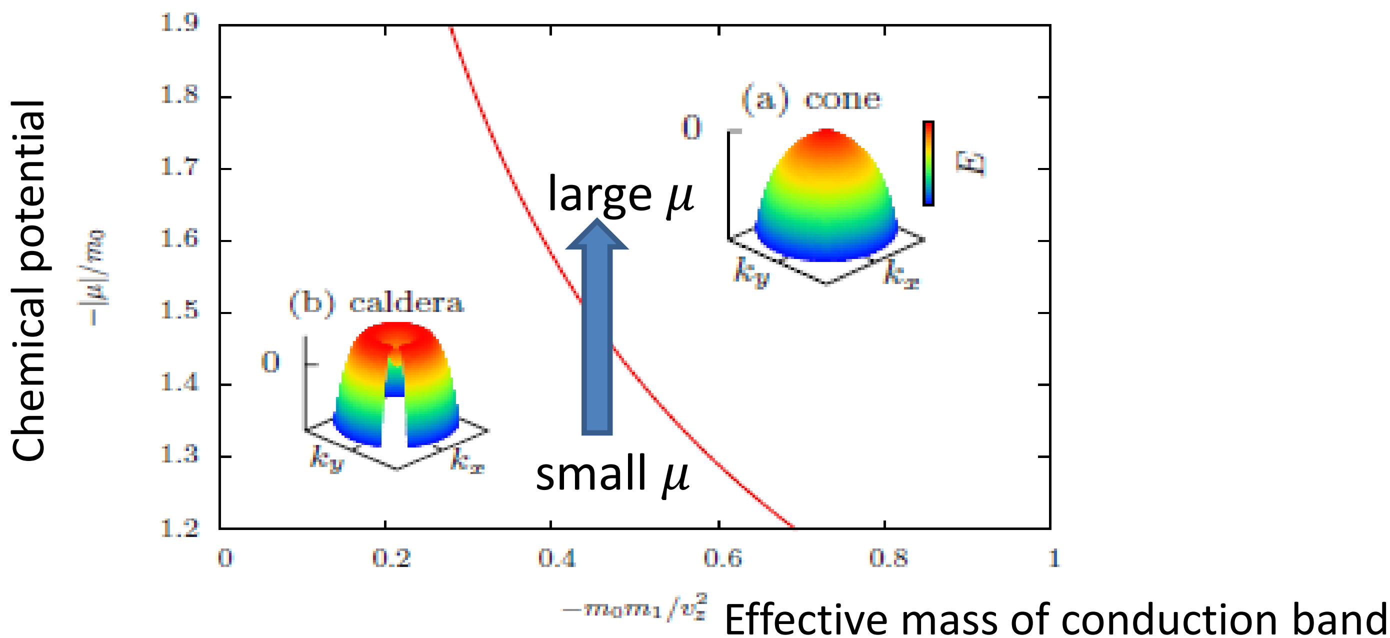
The MF in the nodal SC is characterized by the parity of 3d winding number (= mod 2 winding number) $(-1)^{\nu_{\mathrm{w}}}$

MS-Fujimot (10), Sasaki, MS el al PRL (11)

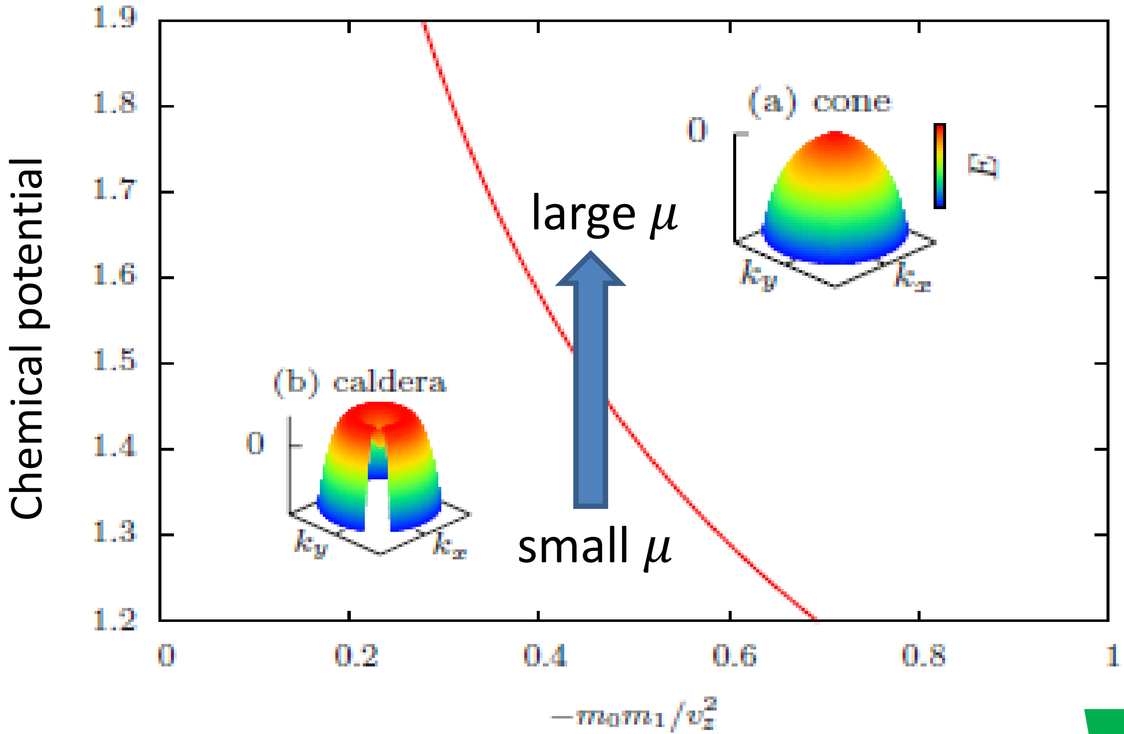
2. Structural transition of helical Majorana fermions

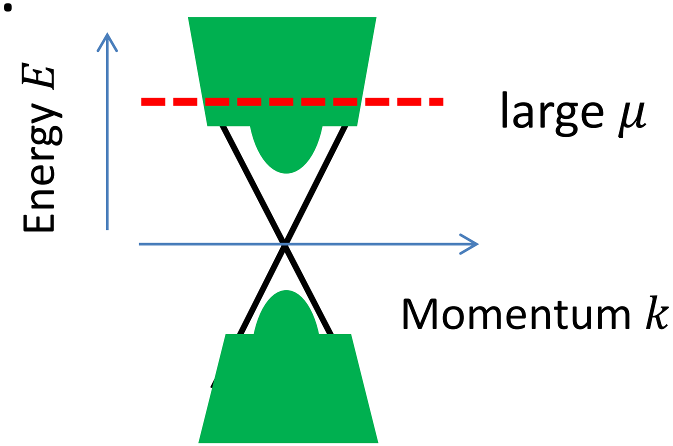
Yamakage, Yada, MS, Tanaka PRB (12)

- For superconducting TI, the helical Majorana fermion has a structural transition in the energy dispersion.
- The transition make it possible to have a robust zero bias peak

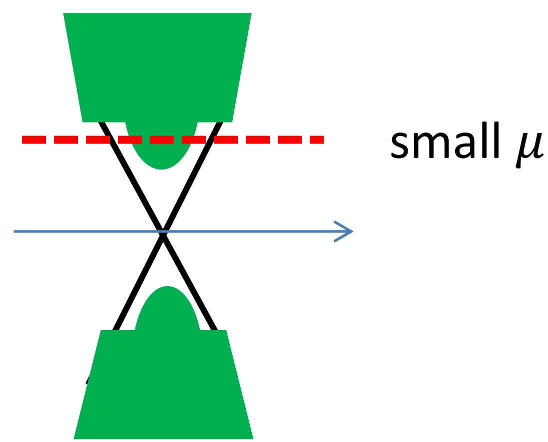


Why the structural transition occurs ?



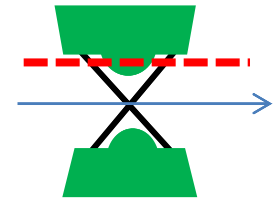


No Dirac fermion
near the Fermi surface



Surface Dirac fermion near the Fermi surface !

More details

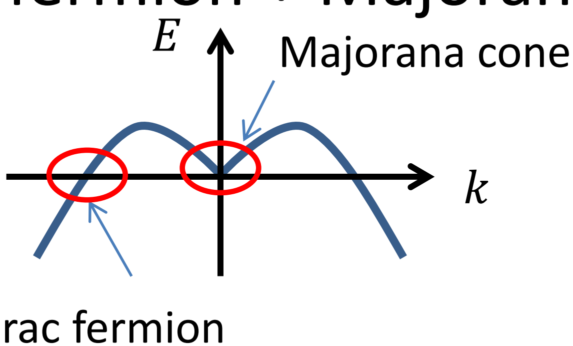


- For small μ, the surface Dirac fermion exists near the FS.
- The surface Dirac fermion remains gapless even in the superconducting state since no s-wave gap function is induced.

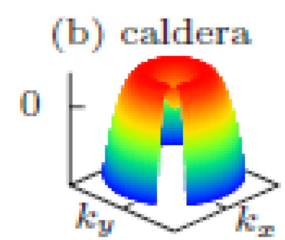
$$H_{\mathrm{Dirac}}(\boldsymbol{k}) = \begin{pmatrix} v(s_x k_y - s_y k_x) - \mu & \cancel{\Delta} \\ \cancel{\Delta} & -v(s_x k_y - s_y) + \mu \end{pmatrix}$$

s-wave gap function is not possible, since the induced pairing has the symmetry of Δ_2.

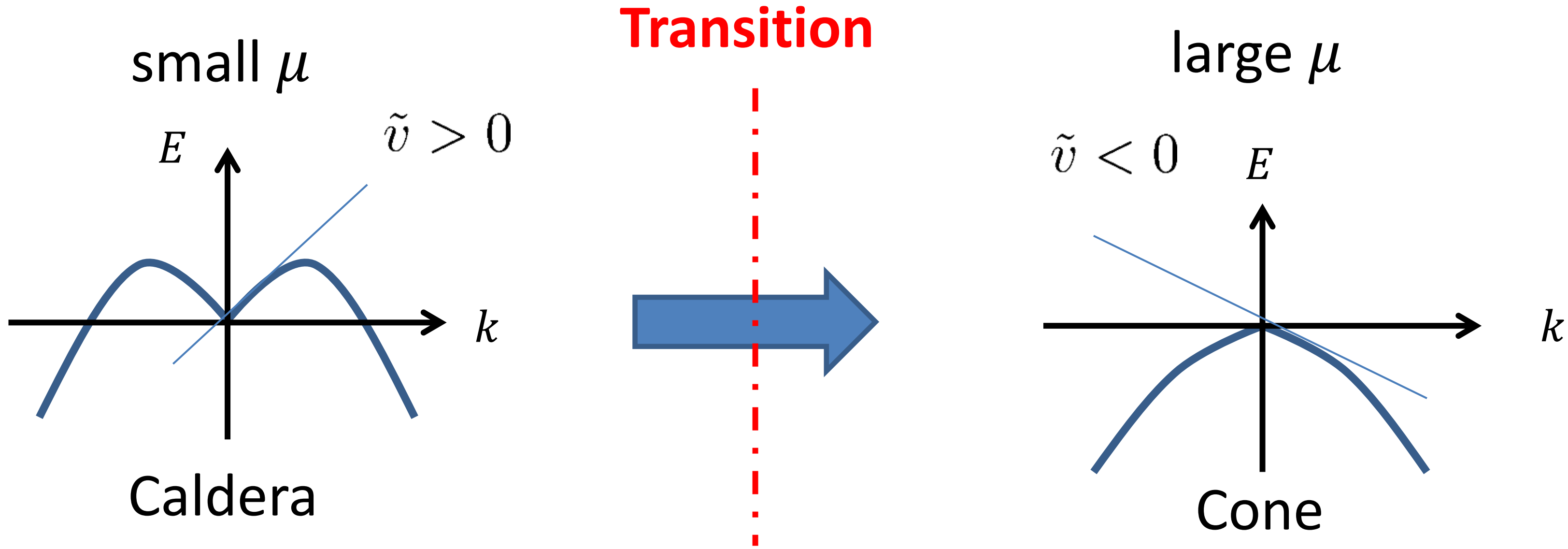
- Dirac fermion + Majorana cone = Caldera shape



$\approx$



Hao and Lee, PRB (11)
Hsieh and Fu, PRL (12)
Yamakage et al PRB (12)



The structural transition occurs when $\tilde{v} = 0$

$$\boxed{\mu_c^2 = \frac{v_z^2}{m_0^2}\left(\frac{1}{-m_0 m_1}\right)}$$

Positive only for TI

(Z2 non-trivial)

$$H_{\mathrm{TI}}(\boldsymbol{k}) = m\sigma_x + v_z k_z \sigma_y + v\sigma_z(k_x s_y - k_y s_x)$$

$$m = m_0 + m_1 k_z^2 + m_2(k_x^2 + k_y^2), \quad (m_1 m_2 > 0)$$

Near the transition, we have enhancement of the surface DOS at zero energy

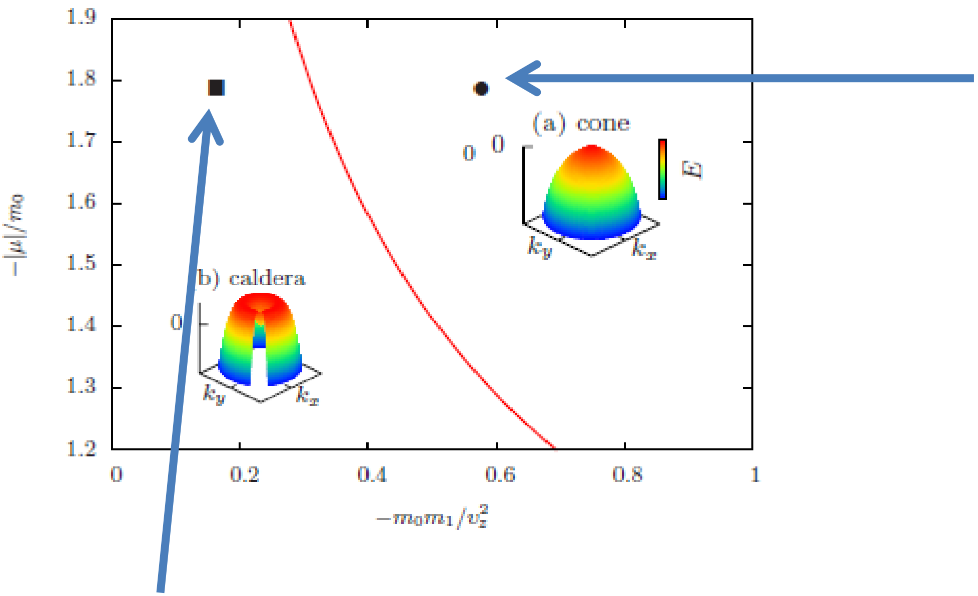
Surface density of states at E=0

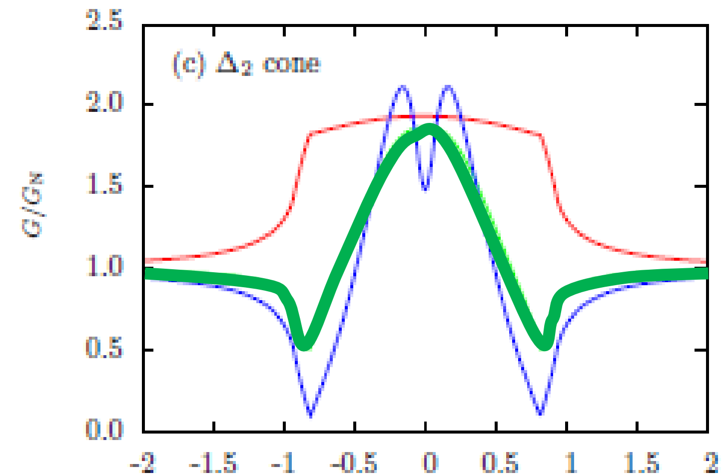
$$N(E=0) \sim \frac{1}{(\partial E/\partial k)|_{k=0}} = \frac{1}{\tilde{v}}$$

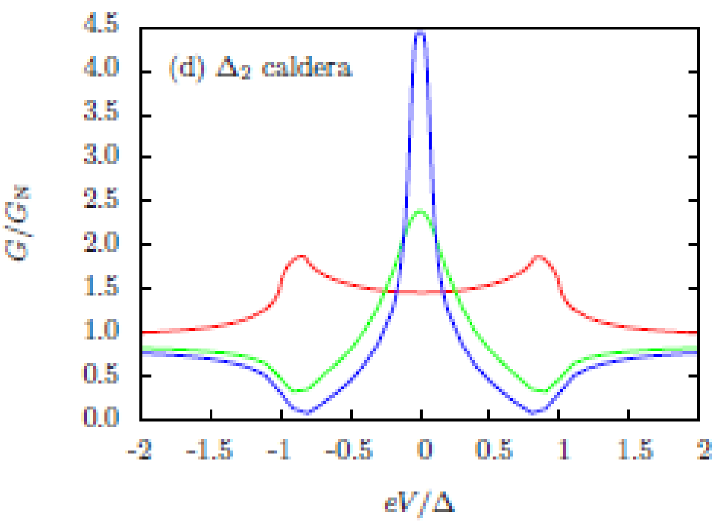
Singular at the critical chemical potential

⇒ Robust zero-bias peak of the tunneling conductance

Tunneling conductance near the transition







Robust single zero-bias peak !!

Summary

Recent measurements of tunneling conductance for the superconducting TI $Cu_xBi_2Se_3$ show a pronounced zero-bias conductance peak.

For originally 3D TSCs, helical Majorana fermions show zero bias dip rather than zero-bias peak in the tunneling conductance.

But the superconducting topological insulator $Cu_xBi_2Se_3$ may support helical Majorana fermions that are consistent with recent experiments of tunneling conductance.