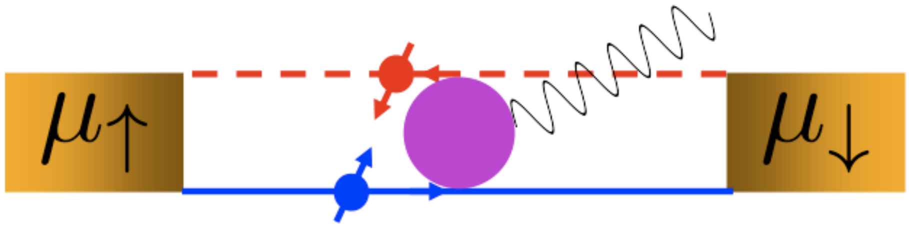
Nordita-workshop, 30.07.12, Stockholm

Transport properties of helical edge states



Patrik Recher
Institute of Mathematical Physics
TU Braunschweig

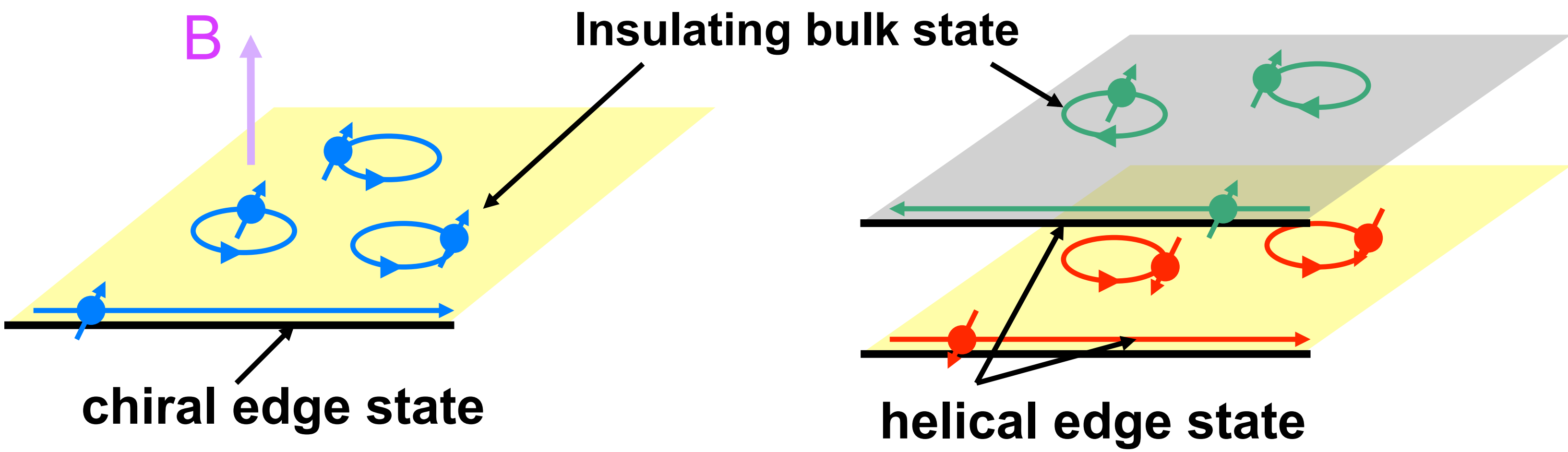
Outline

- What are helical edge states?
- Backscattering at a helical edge
  - the role of phonons
  - the role of electron-electron interaction
- Proximity effect with superconductors

QSH effect and helical edge state

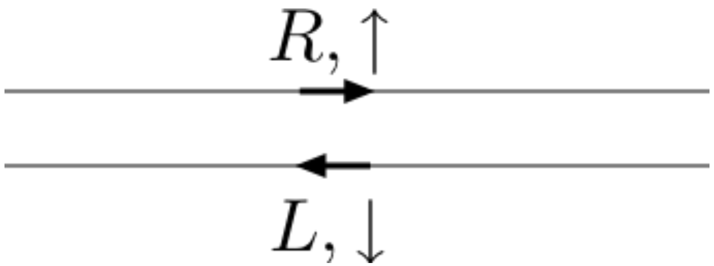
- QSH state: two copies of quantum Hall state with time reversal (TR) symmetry

Quantum Hall Effect　　　　Quantum Spin Hall Effect



Break TR, magnetic field　　　　Preserve TR, SOC

Luttinger-liquid description of helical edge states



helical edge state

kinetic energy:

$$H_0 = -v_F \int dx \left(\psi^\dagger_{R\uparrow} i\partial_x \psi_{R\uparrow} - \psi^\dagger_{L\downarrow} i\partial_x \psi_{L\downarrow} \right)$$

forward scattering Coulomb interaction terms:

$$H_{int} = \frac{\lambda_4}{2} \int dx \left(\rho^2_{R\uparrow}(x) + \rho^2_{L\downarrow}(x) \right) + \lambda_2 \int dx \rho_{R\uparrow}(x) \rho_{L\downarrow}(x)$$

$$\rho_{R\uparrow(L\downarrow)}(x) = \psi^\dagger_{R\uparrow(L\downarrow)}(x) \psi_{R\uparrow(L\downarrow)}(x)$$ chiral charge density

bosonization (like for spinless LL):

$$\psi_{R\uparrow(L\downarrow)} \propto \eta_{R\uparrow(L\downarrow)} e^{\mp i\sqrt{4\pi}\phi_{R\uparrow(L\downarrow)}}$$

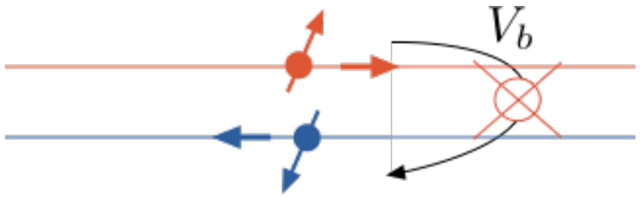
$$\boxed{H = \frac{v}{2}\int dx \left(g(\partial_x\theta)^2 + \frac{1}{g}(\partial_x\varphi)^2 \right)}$$

conjugate momentum:

$$\Pi_\varphi = -\partial_x\theta$$

$$v = v_F\sqrt{\left(1+\frac{\lambda_4}{2\pi v_F}\right)^2 - \left(\frac{\lambda_2}{2\pi v_F}\right)^2} \qquad g = \sqrt{\frac{2\pi v_F + \lambda_4 - \lambda_2}{2\pi v_F + \lambda_4 + \lambda_2}}$$

$$\varphi = \phi_{R\uparrow} + \phi_{L\downarrow} \qquad \theta = \phi_{R\uparrow} - \phi_{L\downarrow}$$ non-chiral bosonic phase fields



usual backscattering absent $\Rightarrow$ ballistic

$V_b\psi_\downarrow^\dagger\psi_\uparrow + h.c.$ (not allowed by TRS)

Xu and Moore, PRB (2006)
Wu et al., PRL (2006)

Topological protection in formulae

$$|\psi\rangle = T|\phi\rangle \qquad [H,T]=0$$

$$\langle\psi|H|\phi\rangle = \langle\phi|H|\psi\rangle^* = \langle T\phi|TH|\psi\rangle =$$
$$\langle\psi|HT|\psi\rangle = \langle\psi|HT^2|\phi\rangle = -\langle\psi|H|\phi\rangle$$

$$\Rightarrow \langle\psi|H|\phi\rangle = 0 \quad \square$$

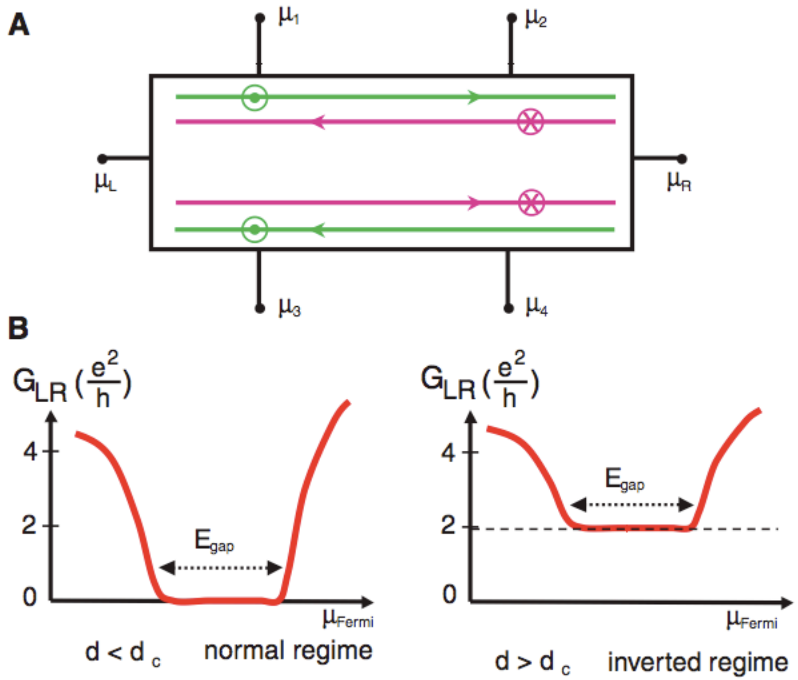
Xu and Moore, PRB (2006)
Wu et al., PRL (2006)

Protection against backscattering for...

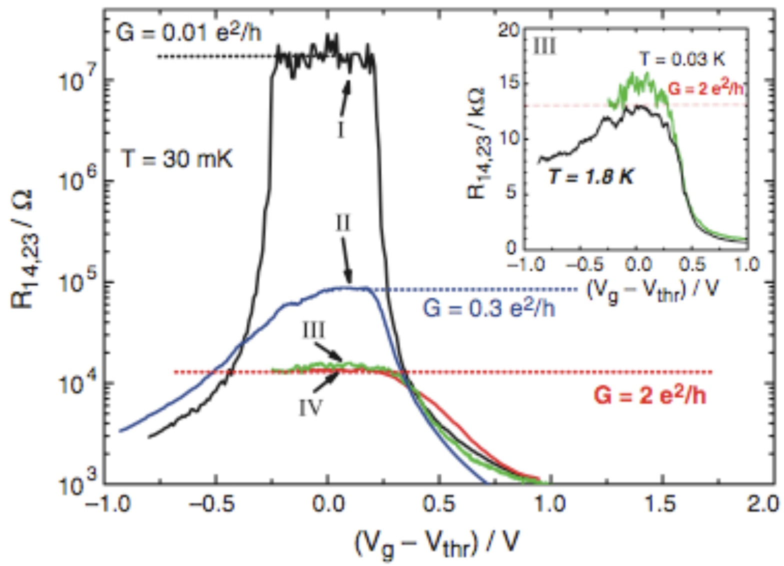
- TRS preserving single particle Hamiltonians
- Elastic processes since Kramers partners degenerate

Experimental evidence of edge states

prediction:



observation:

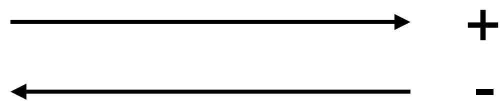


Bernevig et al., Science 2006

Koenig et al., Science 2007

Presence of Rashba Hamiltonian in the helical liquid

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix}$$



$$H_R = \frac{1}{2}\int \Psi^\dagger \left(\{\alpha_1, p\} + \{\alpha_3, p^3\}\right) \sigma_y \Psi$$

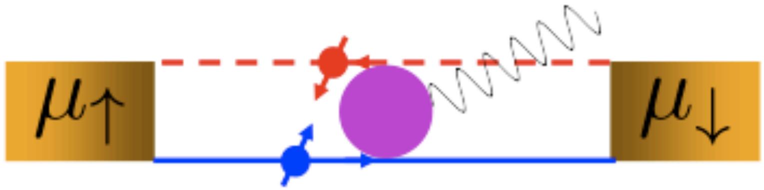
linear Rashba: α_1

qubic Rashba: α_3

$[H_R, T] = 0 \quad \Longrightarrow$ no elastic single-particle scattering

Can Rashba-interaction do anything ?

HLL in presence of Rashba-impurity and phonon-bath



J.C. Budich, F. Dolcini, PR, B. Trauzettel, PRL 2012

The helical wire with Rashba SOC and phonons

$$H_{\rm hl} = \int \Psi^\dagger p \sigma_z \Psi$$

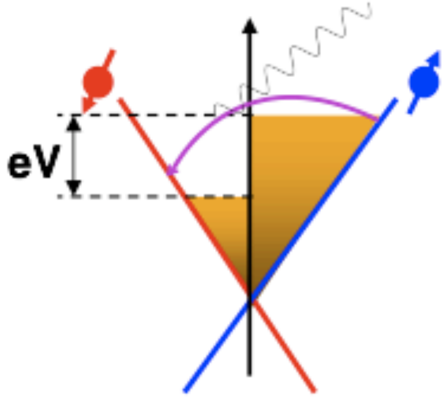
$$H_R = \frac{1}{2} \int \Psi^\dagger \left(\{\alpha_1, p\} + \{\alpha_3, p^3\} \right) \sigma_y \Psi$$

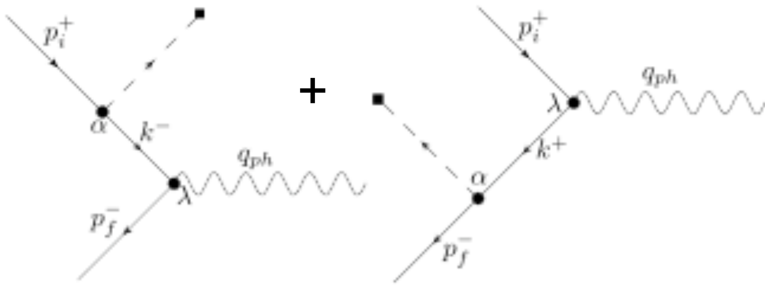
$$H_p = \frac{1}{2} \int \Pi_d^2 + c^2 d'^2$$

$$H_{\rm ep} = \lambda \int \Psi^\dagger \sigma_0 \Psi d'$$

$$H = H_{\rm hl} + H_p + H_R + H_{\rm ep} = H_0 + H_I$$

Results for inelastic single electron backscattering





Interference of two scattering processes

Results up to second order in H_I

$$M_{\mathrm{if}} = \langle p_f -, q_{\mathrm{ph}} | H_I G_0 H_I | p_i + \rangle$$

$M_{\mathrm{if}} = 0$ due to linear Rashba +HLL-dispersion for α_1-Rashba

$$|M_{\mathrm{if}}|^2 = \frac{\lambda^2 c}{16\pi} \tilde{\alpha}_3^2 \left(q_{\mathrm{ph}} + p_f^- - p_i^+ \right) |q_{\mathrm{ph}}|^5$$ for α_3-Rashba

$$I_{\mathrm{BS}} = \frac{\alpha_3^2 \lambda^2 e}{672\pi^2 c^5} V^7$$ with Golden-Rule approach at T=0

We have shown that . . .

- inelastic single particle backscattering not forbidden by symmetry
- most relevant contribution vanishes
- robustness of quantized conductivity beyond topological protection

We have shown that ...

- inelastic single particle backscattering not forbidden by symmetry
- most relevant contribution vanishes
- robustness of quantized conductivity beyond topological protection

Integrating out phonons exactly will change φ propagator

$$\left(G_e^{-1}(k,\omega)\right)_{\varphi\varphi} \to \left(G_e^{-1}(k,\omega)\right)_{\varphi\varphi} - \frac{\lambda^2 k^4}{\omega^2 - c^2 k^2}$$

Martin and Loss, PRB (1994) in imaginary-time

Doing a calculation on Keldysh contour shows absence of leading order backscattering in α_1-Rashba even with Coulomb interactions

J.C. Budich, F. Dolcini, PR, B. Trauzettel, PRL 2012

RG treatment of the Rashba impurity

F. Crepin, J.C. Budich, F. Dolcini, PR, B. Trauzettel, arXiv:1205.0374

Helical liquid:

$$\mathcal{H}_0 = \frac{v}{2\pi}\int dx \left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right]$$

$K \leq 1$ LL-Parameter

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$$\mathcal{H}_0 = \frac{v}{2\pi}\int dx\,\left[K(\partial_x\theta)^2 + \frac{1}{K}(\partial_x\phi)^2\right] \qquad K \leq 1 \quad \text{LL-Parameter}$$

Linear Rashba-term:

$$H_R = \int dx\,\alpha(x)\left[(\partial_x\Psi_+^\dagger)\Psi_- - \Psi_+^\dagger(\partial_x\Psi_-)\right](x) + \text{H.c.}$$

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Linear Rashba-term:

$$H_R = \int dx\, \alpha(x)\left[(\partial_x\Psi_+^\dagger)\Psi_- - \Psi_+^\dagger(\partial_x\Psi_-)\right](x) + \mathrm{H.c.}$$

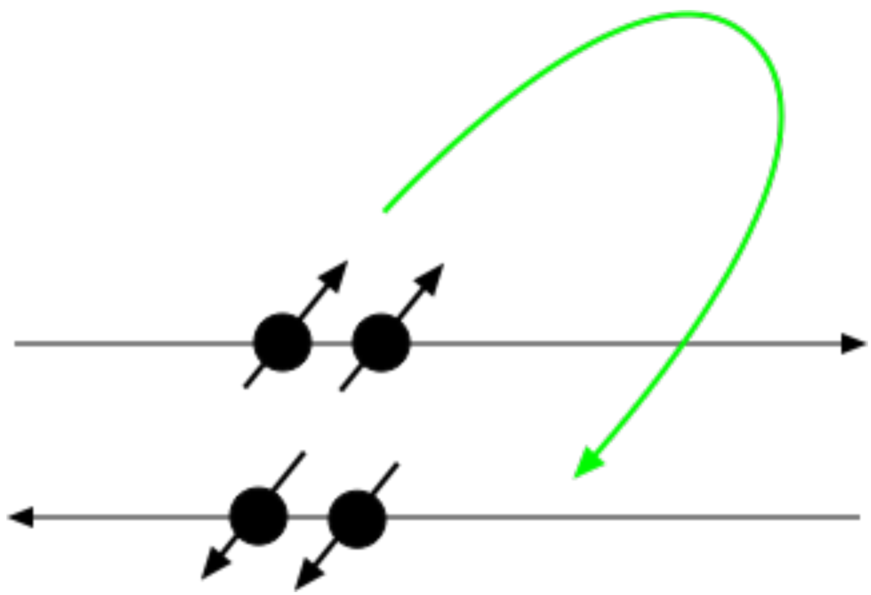
$\Longrightarrow$ bosonize

$$H_R = i\kappa_+\kappa_- \int dx \frac{\alpha(x)}{\pi a}\left(\frac{2\pi a}{L}\right)^K \times$$

$$\times :\partial_x\theta(x)\left(:e^{-i2\phi(x)}:e^{i2k_F x} + :e^{i2\phi(x)}:e^{-i2k_F x}\right):$$

Generated two-particle backscattering term (in a point):

$$H_{2\mathrm{p}}^{\mathrm{in}} = \gamma_{2\mathrm{p}}^{\mathrm{in}} \left[(\partial_x \Psi_+^\dagger) \Psi_+^\dagger (\partial_x \Psi_-) \Psi_- \right] (x_0) + \mathrm{H.c.}$$



generic form (most relevant):

Wu et al. PRL 2006
Ström et al. PRL 2010

- conserves time-reversal symmetry

Expand partition function to second-order in Rashba and first order in two-particle term with coupling $\gamma_{\mathrm{2p}}^{\mathrm{in}}$

$$\mathcal{Z} = \mathcal{Z}_0 \left[1 - \int_0^\beta d\tau \, \langle \hat{\mathcal{H}}_R(\tau_1) \rangle_0 + \frac{1}{2} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \, \langle \mathcal{T} \hat{\mathcal{H}}_R(\tau_1) \hat{\mathcal{H}}_R(\tau_2) \rangle_0 + o(\alpha^3) \right]$$

Expand partition function to second-order in Rashba and first order in two-particle term with coupling γ_{2p}^{in}

$$\mathcal{Z} = \mathcal{Z}_0 \left[1 - \int_0^\beta d\tau \, \langle \hat{\mathcal{H}}_R(\tau_1) \rangle_0 + \frac{1}{2} \int_0^\beta d\tau_1 \int_0^\beta d\tau_2 \, \langle \mathcal{T} \hat{\mathcal{H}}_R(\tau_1) \hat{\mathcal{H}}_R(\tau_2) \rangle_0 + o(\alpha^3) \right]$$

Flow-equation based on perturbative RG approach:

$a \;\rightarrow\; a' \;=\; (1 + d\ell) a$ + scale Invariance of partition function

$$\frac{d\tilde{\alpha}}{d\ell} = -K \tilde{\alpha}(\ell)$$

$$\frac{d\tilde{\gamma}_{2p}^{in}}{d\ell} = (1 - 4K) \tilde{\gamma}_{2p}^{in}(\ell) + \left(1 - \frac{1}{K} \right) (1 - 2K) \tilde{\alpha}(\ell)^2$$

$\tilde{\alpha}(0) = \tilde{\alpha}$

$\tilde{\gamma}_{2p}^{in}(0) = 0$

Expand partition function to second-order in Rashba and first order in two-particle term with coupling γ_{2p}^{in}

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Scaling dimension of operators

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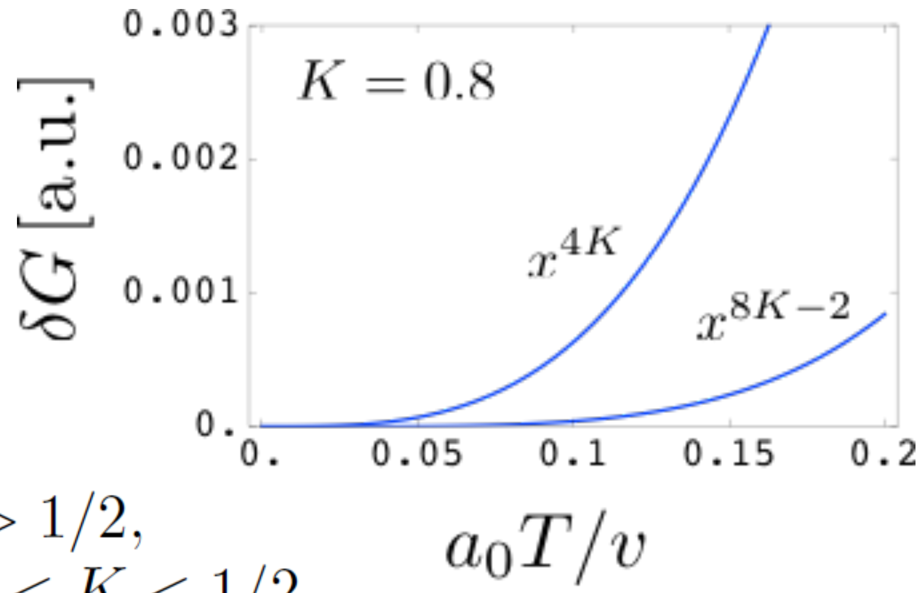
Generated in second-order perturbation theory

- backscattering conductance:

$$\frac{\delta G}{G_0} = -b\,\tilde{\gamma}_{\rm 2p}^{\rm in}(\ell^*)^2,$$



At low temperatures:

$$\delta G/G_0 \sim \begin{cases} (a_0 T/v)^{4K} & \text{if } K > 1/2, \\ (a_0 T/v)^{8K-2} & \text{if } 1/4 < K < 1/2 \end{cases}$$

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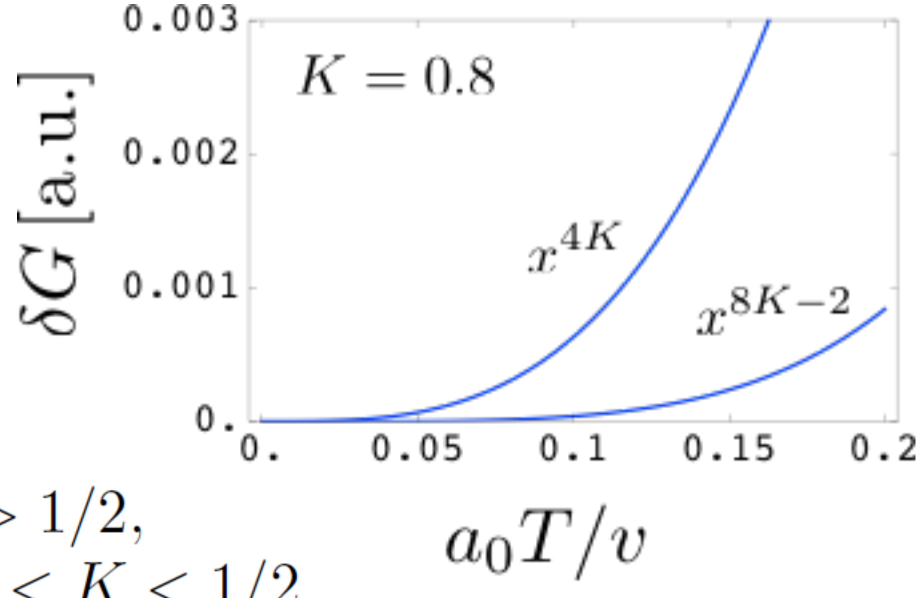
$$\frac{\delta G}{G_0} = -b\,\tilde{\gamma}_{2\mathrm{p}}^{\mathrm{in}}(\ell^*)^2,$$



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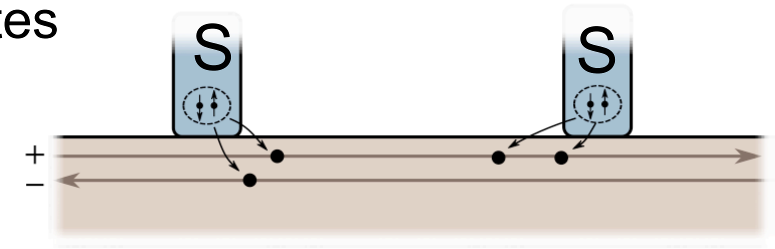
Conclusions:

- Two-particle backscattering generated by Rashba interaction in 2nd order
- Term becomes relevant if $K < 1/4$, Wu et al. 2006, Ström et al. 2010
- As K→ 1, $\delta G \sim T^4$

Homogeneous Rashba-interaction in the HLL

- Spin-rotation in HgTe-based edge states



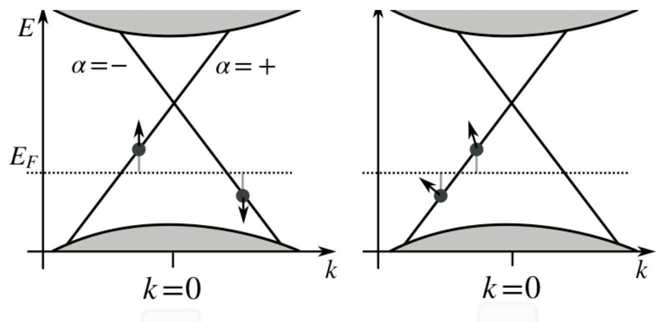
Superconducting proximity effect

2D-treatment of edge states
within 4-band model of HgTe QWs

Bernevig et al., Science (2006), Rothe et al., NJP (2010)

$$H = H_0 + \sum_{\alpha\beta=\pm} \int \mathrm{d}x\, \Gamma_{\alpha\beta}(x)\psi_\alpha(x)\psi_\beta(x+a) + \mathrm{h.c.}$$

P. Virtanen, PR, PRB 2012



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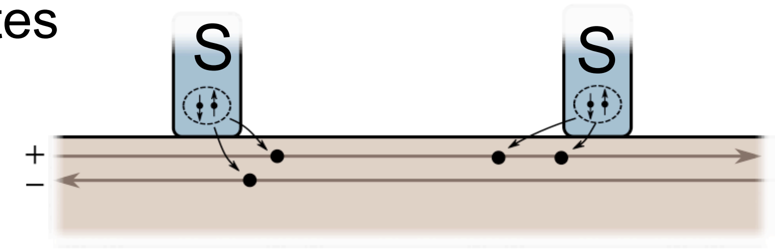

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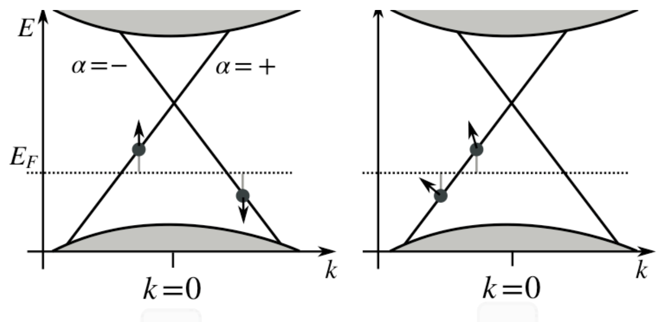


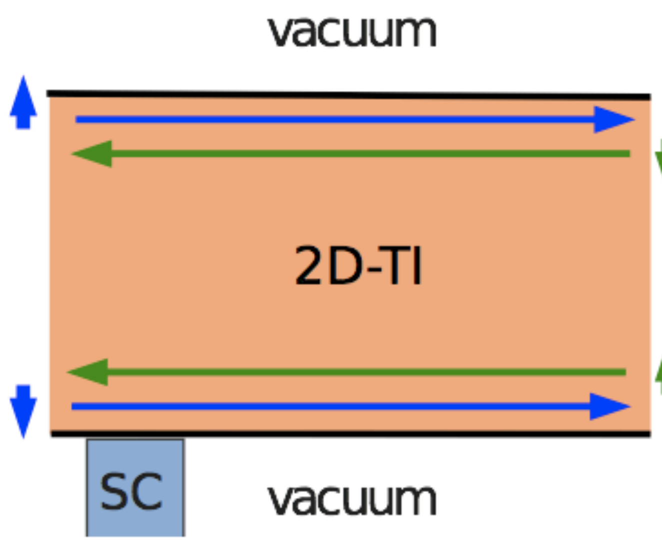
Scattering at scalar potential $V_{bs} = (\rho_\uparrow + \rho_\downarrow)V$

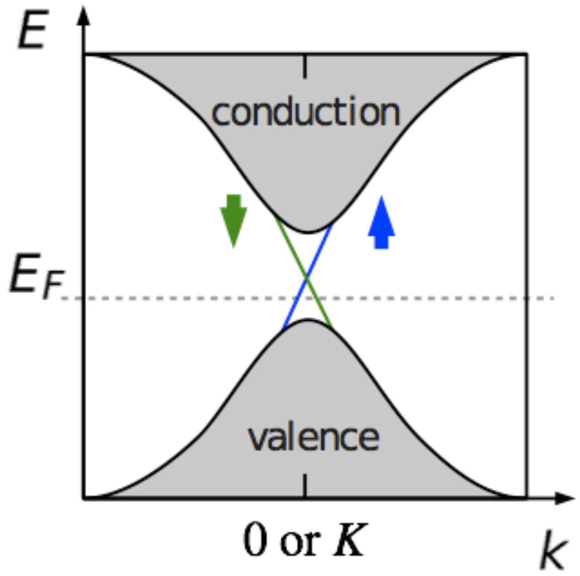
e-e interaction only perturbatively

$\delta G \sim T^4$ *T. Schmidt et al., PRL (2012)*

2D-TI edge states & s-wave superconductor

P. Virtanen, PR, PRB 2012





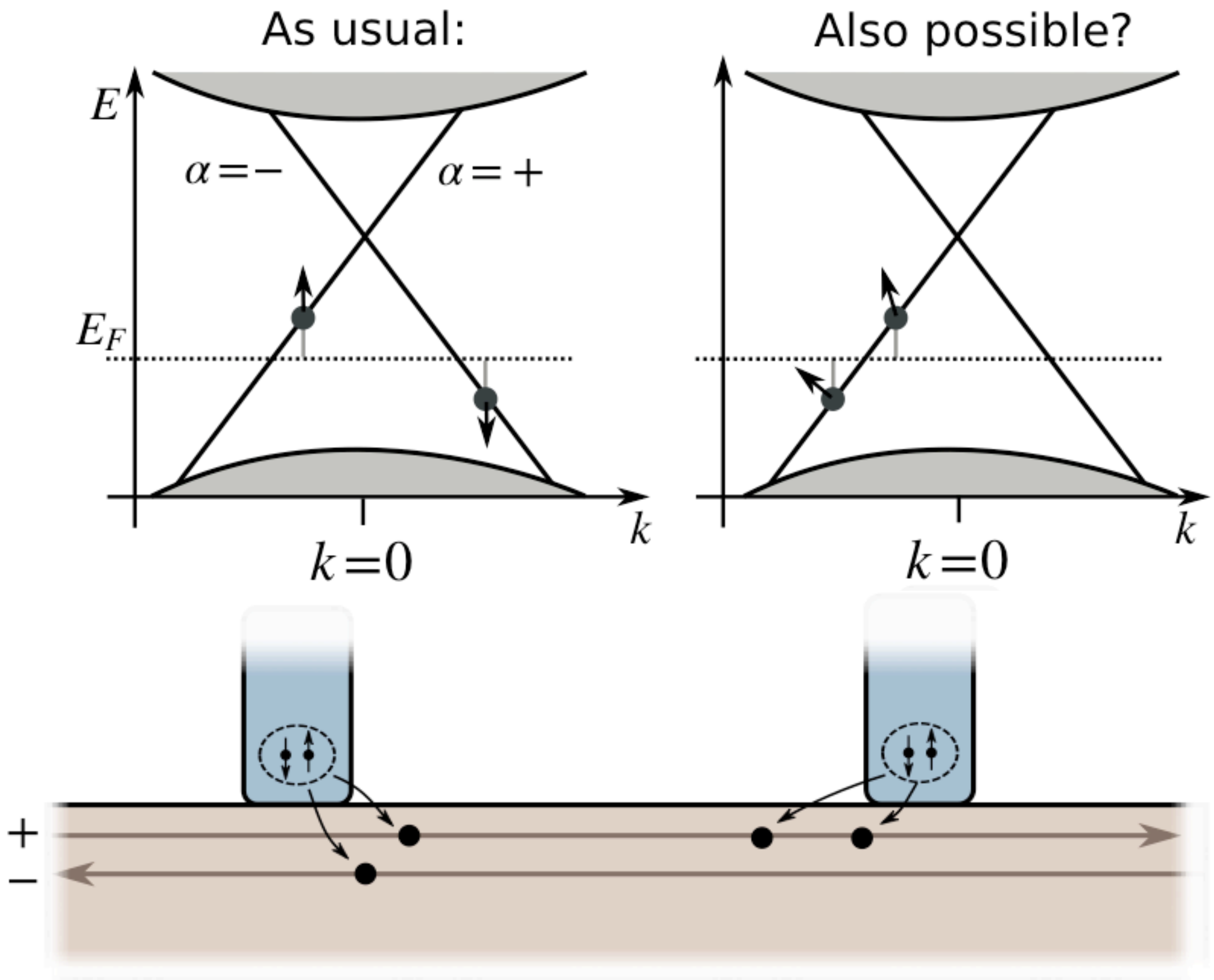
[Kane & Mele, *PRL* (2005); Bernevig et al., *Science* (2006); König et al., *Science* (2007)]

Basic *s*-wave SC coupling [Fu and Kane, *PRL* (2008)] A.M. Black-Schaffer, PRB 2011

$$H = \int \mathrm{d}x \, [v_F(\psi_+^\dagger \partial_x \psi_+ - \psi_-^\dagger \partial_x \psi_-) + \Gamma_{+-}(x)\psi_+(x)\psi_-(x) + \mathrm{h.c.}]$$

Q: Effects in addition to Γ_{+-} (and can you observe them)?

Cooper pair injection to the same mode?



Effective Hamiltonian: Perturbative RG

$$H_{\text{eff}} = H_0 + H_T + H_{T2}$$

$$H_T = \sum_{\alpha=\pm,\sigma'=\uparrow,\downarrow} \int \mathrm{d}x\, \mathrm{d}^3r'\, t_{\alpha\sigma'}(x,\vec{r}')\psi^\dagger_\alpha(x)\psi_{S\sigma'}(\vec{r}') + \mathrm{h.c.}$$

$$H_{T2} = \sum_{\alpha\beta} \int \mathrm{d}x\, \Gamma_{\alpha\beta}(x)\psi_\alpha(x)\psi_\beta(x+a_0) + \mathrm{h.c.}$$

Scaling equations, $x \mapsto xe^l$

$$t_{\alpha\sigma'}(l=0) = t^0_{\alpha\sigma'}\,, \qquad \frac{\mathrm{d}t_{\alpha\sigma'}}{\mathrm{d}l} = [2-\eta_1]t_{\alpha\sigma'}$$

$$\Gamma_{\alpha\beta}(l=0) = 0\,, \qquad \frac{\mathrm{d}\Gamma_{\alpha\beta}}{\mathrm{d}l} = [2-\eta_{2,\alpha\beta}]\Gamma_{\alpha\beta} + S_{\alpha\beta}(l)$$

$$\eta_1 = (g+g^{-1})/4 \quad \eta_{2,\alpha\alpha} = g + g^{-1}$$

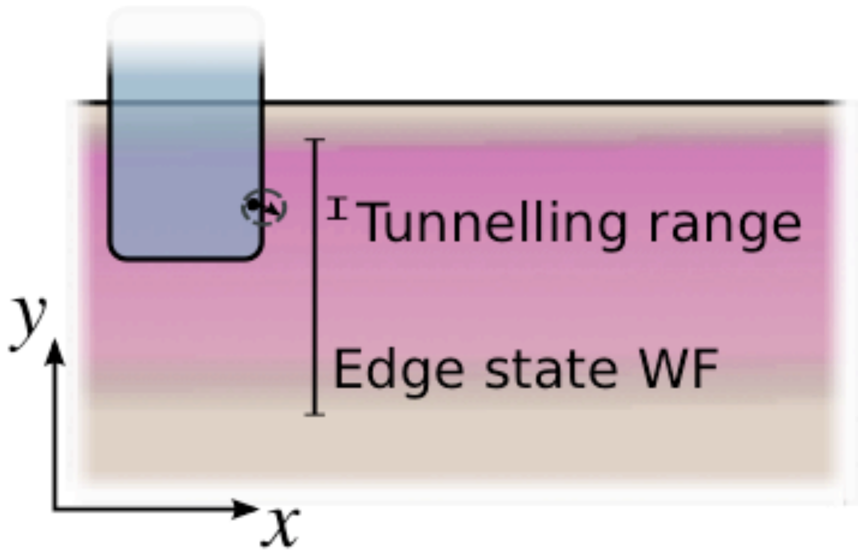
$$\eta_{2,\alpha,-\alpha} = g^{-1}$$

Tunnelling elements

$$t_{\alpha\sigma'}(x, \vec{r}') \equiv \langle \alpha, x | \hat{h}_T | \sigma', \vec{r}' \rangle$$

Tunnelling assumed to be ...

- Local (in real space; e.g. tight-binding model):
  $\langle \sigma \vec{r} | \hat{h}_T | \sigma' \vec{r}' \rangle \propto \delta(\vec{r} - \vec{r}')$
- Spin-conserving: $\langle \sigma \vec{r} | \hat{h}_T | \sigma' \vec{r}' \rangle \propto \delta_{\sigma\sigma'}$
- Time-reversal invariant: $\mathfrak{T} \hat{h}_T \mathfrak{T}^{-1} = \hat{h}_T$



Effective 2-particle tunnelling

Solution leads to an effective low-energy model.

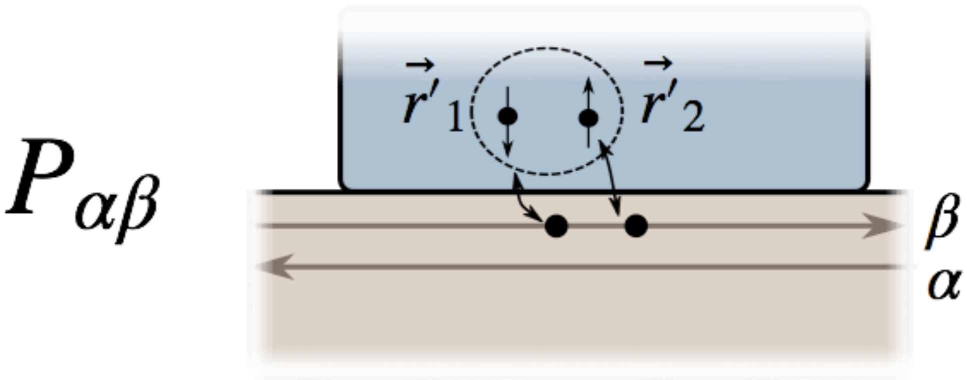
$$H_{T2} = \sum_{\alpha\beta} \int \mathrm{d}x\, \Gamma_{\alpha\beta}(x)\psi_\alpha(x)\psi_\beta(x+a) + \mathrm{h.c.}$$

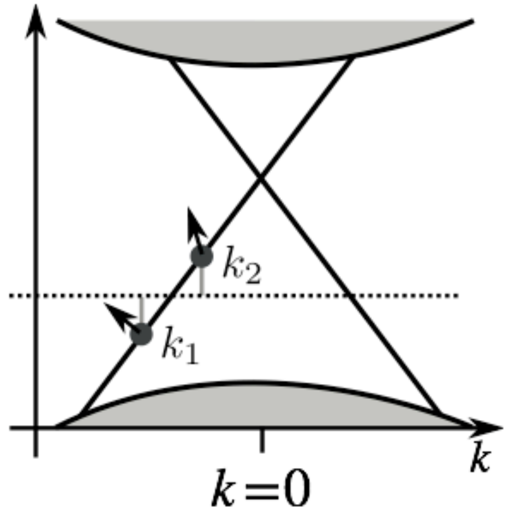
$$\Gamma_{\alpha\beta}(x) = \frac{1}{4}\int \mathrm{d}x'\mathrm{d}\tau'(a/a_0)^{1-\eta_{2,\alpha\beta}} u_{\alpha\beta}(x',\tau') \times \int \mathrm{d}^3 r_1' \,\mathrm{d}^3 r_2'\, F^\dagger(\vec{r}_1',\tau';\vec{r}_2',0) P_{\alpha\beta}(x+\frac{x'}{2},\vec{r}_1'; x-\frac{x'}{2},\vec{r}_2')$$

- New cutoff $a \gtrsim v_F \Delta^{-1}$
- Known: $\psi_1(z)\psi_2(z') \propto :e^{i\phi_1(z)+i\phi_2(z)}: u(z-z')$;
  $F^\dagger(\vec{r}_1,\tau_1;\vec{r}_2,\tau_2) = \langle T[\psi^\dagger_{S\downarrow}(\vec{r}_1,\tau_1)\psi^\dagger_{S\uparrow}(\vec{r}_2,\tau_2)]\rangle$
- Need to find out: $P_{\alpha\beta}$ (contains tunnel amplitudes $t_{\alpha\sigma}(x,\vec{r}'))$

$$P_{\alpha\beta}(x_1,\vec{r}_1';x_2,\vec{r}_2') = t_{\alpha\downarrow}(x_1,\vec{r}_1')t_{\beta\uparrow}(x_2,\vec{r}_2') - t_{\alpha\uparrow}(x_1,\vec{r}_1')t_{\beta\downarrow}(x_2,\vec{r}_2') + [\vec{r}_1' \leftrightarrow \vec{r}_2']$$

Singlet pair tunnelling amplitudes?





$$P_{++}(k_1, \vec{r}_1'; k_2, \vec{r}_2') = \langle +, -k_1 | \hat{Z}(\vec{r}_1', \vec{r}_2') \mathfrak{T} | +, -k_2 \rangle$$

Z: some TR-invariant spin-conserving operator describing tunnelling

Estimating amplitudes

Rewrite using TR symmetry (and Fourier transform):

$$P_{\alpha_1\alpha_2}(k_1,\vec{r}_1';k_2,\vec{r}_2') = \langle \alpha_1, -k_1|\hat{Z}(\vec{r}_1',\vec{r}_2')\mathfrak{T}|\alpha_2,-k_2\rangle$$

$$\hat{Z}(\vec{r}_1',\vec{r}_2') \equiv \hat{h}_T[1_\sigma \otimes (|\vec{r}_1'\rangle\langle\vec{r}_2'| + |\vec{r}_2'\rangle\langle\vec{r}_1'|)]\hat{h}_T\,,\ \ t_{\alpha\sigma'}(k,\vec{r}') \equiv \langle \alpha, -k|\hat{h}_T|\sigma', \vec{r}'\rangle$$

Example

Edge states with constant spin axis (eg. Kane–Mele model):

$$|+,k\rangle = |\uparrow\rangle \otimes |k\rangle_+\,, \quad |-,k\rangle = |\downarrow\rangle \otimes |k\rangle_-$$

so that

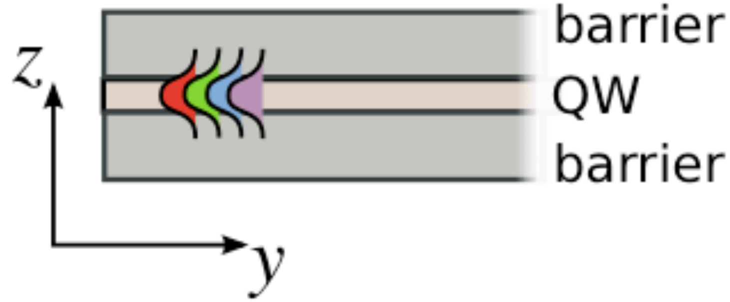
$$P_{++} = \langle +,-k_1|\hat{Z}\mathfrak{T}|+,-k_2\rangle = \langle\uparrow||\downarrow\rangle \times {}_+\langle -k_1|\hat{Z}|k_2\rangle_- = 0$$

$$P_{+-} = \langle +,-k_1|\hat{Z}\mathfrak{T}|-,-k_2\rangle = \langle\uparrow||\uparrow\rangle \times {}_+\langle -k_1|\hat{Z}|+k_2\rangle_+ \neq 0$$

Concrete model system: HgTe/CdTe quantum wells

4-band model

$$\langle \sigma, \vec{r} | | \alpha, k \rangle = \begin{pmatrix} \langle \sigma, \vec{r} | | E1+ \rangle \\ \langle \sigma, \vec{r} | | H1+ \rangle \\ \langle \sigma, \vec{r} | | E1- \rangle \\ \langle \sigma, \vec{r} | | H1- \rangle \end{pmatrix}^T \hat{\Psi}_{\alpha,k}(x, y)$$



in terms of quantum well $k_{x/y} = 0$ states: $|E1+\rangle$, $|E1-\rangle$, $|H1+\rangle$, $|H1-\rangle$.

4-band Hamiltonian

$$H_{BHZ} \hat{\Psi} = E \hat{\Psi} \,,$$

$$H_{BHZ} = \begin{pmatrix} h(k) & 0 \\ 0 & h(-k)^* \end{pmatrix} , \quad h(k) = \epsilon(k)\sigma_0 + \vec{d}(k) \cdot \vec{\sigma} \,,$$

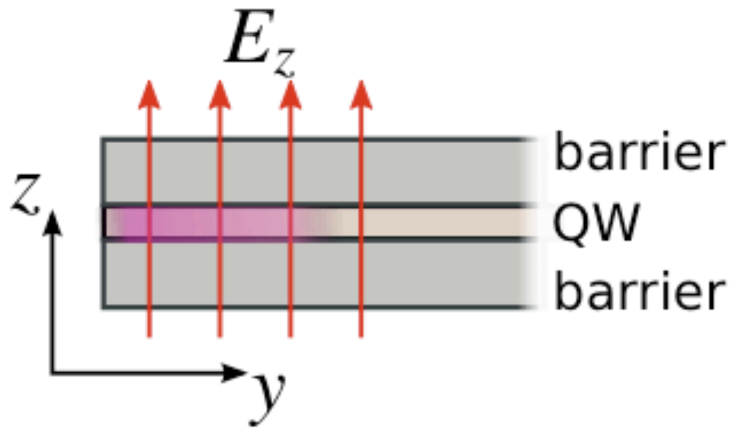
$$\epsilon(k) = C - Dk^2 \,, \vec{d}(k) = (Ak_x, -Ak_y, M - Bk^2) \,.$$

[Bernevig, Hughes, Zhang, *Science* (2006)]

Tunable spin rotation by Rashba spin orbit

$$H_{BHZ} \mapsto \begin{pmatrix} h(k) & h_R(k) \\ h_R(k)^\dagger & h(-k)^* \end{pmatrix}$$

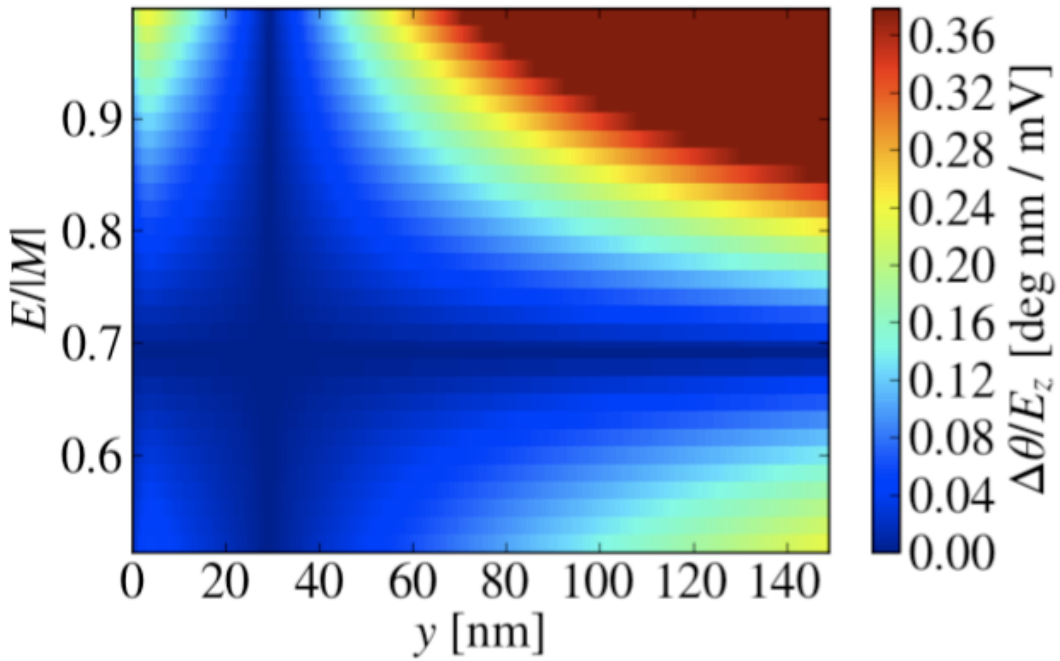
$$h_R = i \begin{pmatrix} -R_0(k_x - ik_y) & iS_0(k_x - ik_y)^2 \\ -iS_0(k_x - ik_y)^2 & T_0(k_x - ik_y)^3 \end{pmatrix}$$

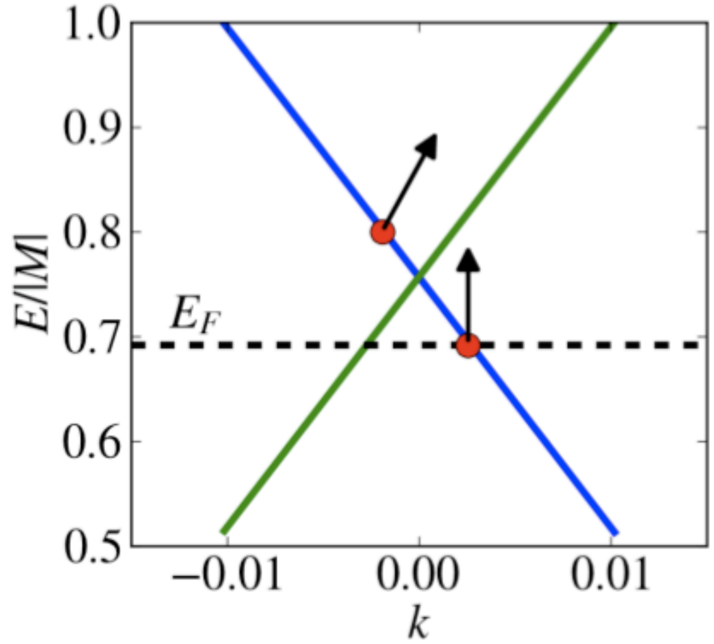


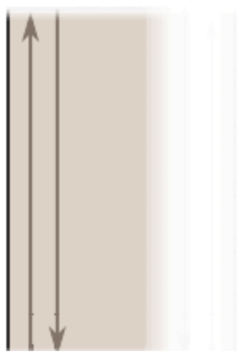
- h_R mixes the $+$ and $-$ Kramers blocks
- $R_0, S_0, T_0 \propto E_z$
- BIA can also be included

[Rothe et al., *New J. Phys.* (2010)]

Tunable spinor rotation by Rashba spin orbit interaction







$$\sin\theta_{\mathrm{Rashba}}(E,y) \equiv \frac{\hat{\Psi}_{k,-}(E,y)^\dagger \mathfrak{T} \hat{\Psi}_-(E_F,y)}{\|\hat{\Psi}_-(E,y)\| \|\hat{\Psi}_-(E_F,y)\|} \sim P_{++}$$

Tunnel amplitudes from 4-band model

Amplitudes

$P_{\alpha_1\alpha_2}(k_1, \vec{r}_1'; k_2, \vec{r}_2') = \langle \alpha_1, -k_1 | \hat{Z}(\vec{r}_1', \vec{r}_2') \mathfrak{T} | \alpha_2, -k_2 \rangle$

Simplify: length scale separation & parameterize by symmetries

$$P_{\alpha_1\alpha_2} \propto \hat{\Psi}^{\dagger}_{\alpha_1,-k_1} \begin{pmatrix} \mathcal{A} & \mathcal{C} & 0 & \mathcal{D} \\ \mathcal{C}^* & \mathcal{B} & -\mathcal{D} & 0 \\ 0 & -\mathcal{D}^* & \mathcal{A} & \mathcal{C}^* \\ \mathcal{D}^* & 0 & \mathcal{C} & \mathcal{B} \end{pmatrix} \mathfrak{T} \hat{\Psi}_{\alpha_2,-k_2}$$

Order of magnitude (localized contacts):

$$P_{++}(k_1, k_2) \sim i(k_1 - k_2) \frac{z_0}{|M|} P_{+-} \, ,$$

$$z_0 = \text{Rashba strength} = 1\,\mathrm{eV} \times (E_z / 500\,\mathrm{mV/nm})$$

Low-energy Hamiltonian

Low-energy model for 2-particle tunnelling

$$H = H_0 + \sum_{\alpha\beta=\pm} \int \mathrm{d}x\, \Gamma_{\alpha\beta}(x)\psi_\alpha(x)\psi_\beta(x+a) + \mathrm{h.c.}$$

$$\Gamma_{+-} \simeq (a\Delta)^{\frac{g-1}{g}}(a_0\Delta)^{\frac{g+g^{-1}}{2}-1}\mathcal{K},$$

$$\Gamma_{++} \simeq i(a\Delta)^{\frac{g-g^2-1}{g}}(a_0\Delta)^{\frac{g+g^{-1}}{2}-1}\frac{z_0\Delta}{v_F|M|}\mathcal{K}$$

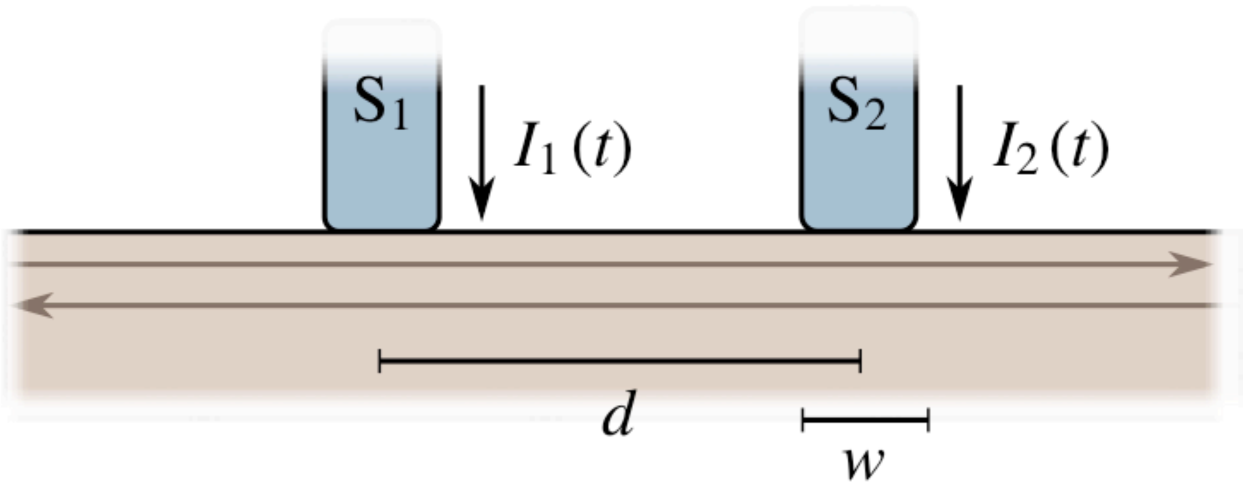
- a_0 interaction cutoff
- $a > \Delta^{-1}$ low-energy cutoff

Noninteracting case ($g = 1$):

$\Gamma_{+-} \sim \frac{\hbar v_F}{w}\frac{R_K}{4R_N}$, where w is contact length

$$z_0 = \mathrm{Rashba\,strength} \sim 1\,\mathrm{eV} \times (E_z/500\,\mathrm{mV/nm})$$

Transport features



- Assume no quasiparticle current (suppressed as $e^{-|\Delta|/T}$).

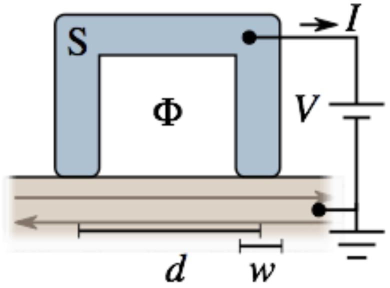
High-resistance contacts

Simple perturbation theory:

$$\partial_t \hat{N} = i[\hat{H}_{\text{eff}}, \hat{N}] = \hat{I}_1 + \hat{I}_2$$

$$\langle \hat{I}(t) \rangle \simeq \int^t \mathrm{d}t' \, i \langle [\hat{V}(t'), \hat{I}(t)] \rangle_0$$

DC conductance (oscillations)



Noninteracting case ($g = 1$): Component of DC current oscillating with $\varphi = 2\pi \frac{\Phi}{\Phi_0}$

Exclusion principle

$\propto V^2, T^2$: finite energy required in $++$ channel [cf. Fisher, *PRB* (1994)]
$|\Gamma_{++}|^2 (aT)^2 = (T/E_\mathrm{r})^2 |\Gamma_{+-}|^2, \qquad E_{\mathrm{r,HgTe-QW}} = |M| v_F / z_0$

Increasing contact separation d

- $++$ channel: $\propto \cos(2Vd + 4k_F d)$
- $+-$ channel: $\propto e^{-2\pi Td/v_F}$ [for low transparency $w|\Gamma_{+-}| < 1$]

$$\delta I_{NS} = \frac{2|w\Gamma_{++}|^2}{3\pi} V \cos(2Vd + 4k_F d) \cos(\varphi) [(aV)^2 + 4\pi^2 (aT)^2] + 4|w\Gamma_{+-}|^2 \sin(2Vd) \cos(\varphi) \frac{T}{v_F \sinh(2\pi Td/v_F)}$$

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- Combination of potential scattering + Rashba S.O. - SC proximity effect