Topological liquid nucleation induced by vortex-vortex interactions in Kitaev's honeycomb model

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Topological liquid nucleation:

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A mechanism where a new topological phase emerges ("is nucleated") when interacting non-Abelian anyons (e.g. localized Majorana zero modes) are arranged on lattices

Ludwig et al., NJP 13, 045014 (2011)

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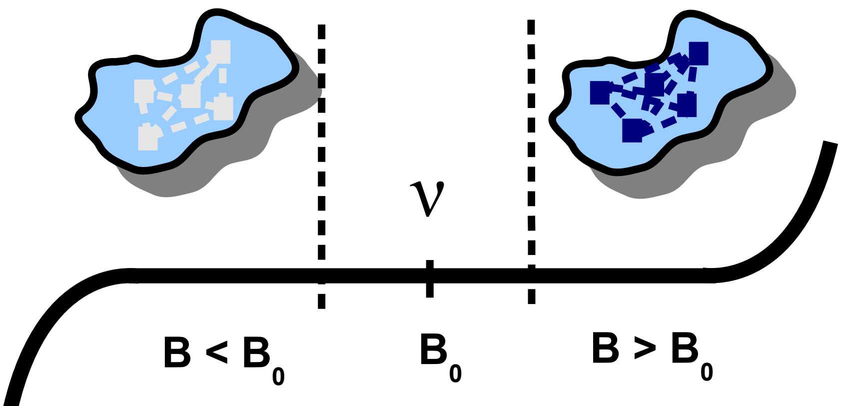
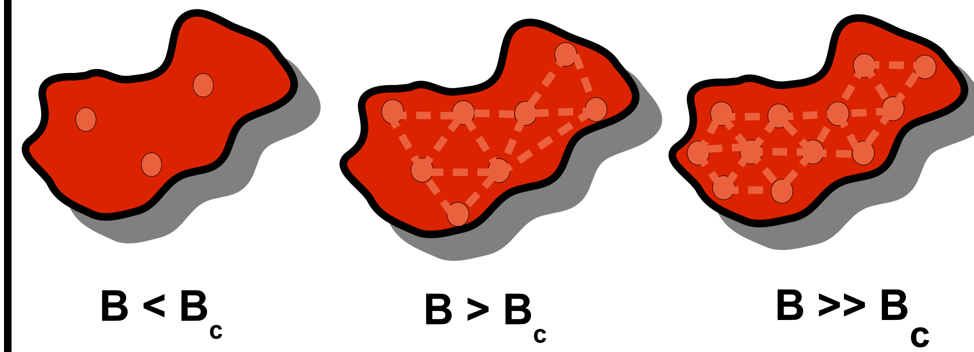
Motivation:
Nucleation can be relevant in all topological systems

- Wigner crystals in FQHE liquids



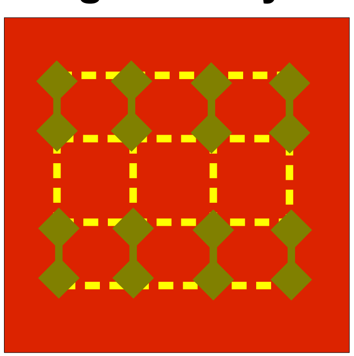
Interacting? Yes.

- Abrikosov lattices in topological SCs



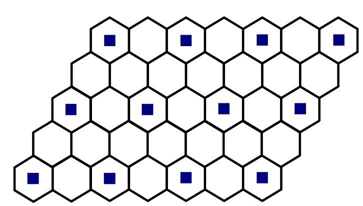
Interacting? Yes.

- Engineering 2D arrays of nanowires?



Interacting? Yes.

- Superlattices in spin lattice models

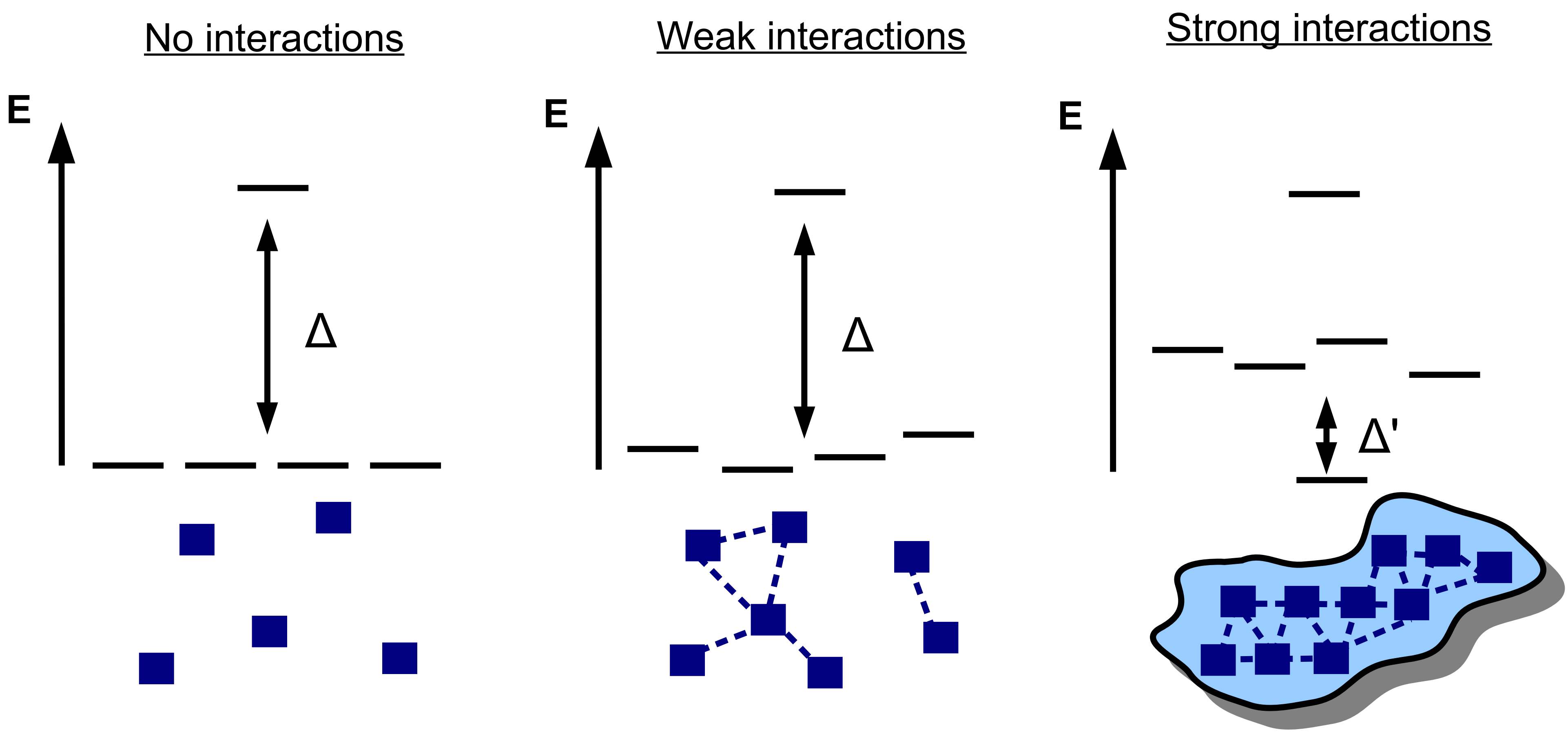


  - Kitaev's honeycomb model and its generalizations
  - Quantum double models

Interacting? Yes.

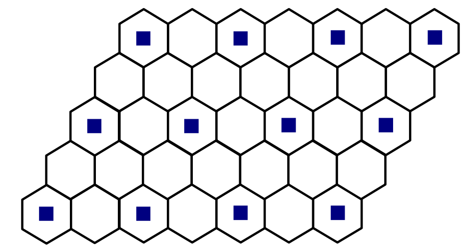
Motivation: Nucleation is destructive for TQC

- Topological quantum computing requires in general the manipulation of many (interacting) anyons in a finite system
- Nucleation can destroy any topological quantum computing scheme based on non-Abelian anyons even when the temperature is below the energy gap



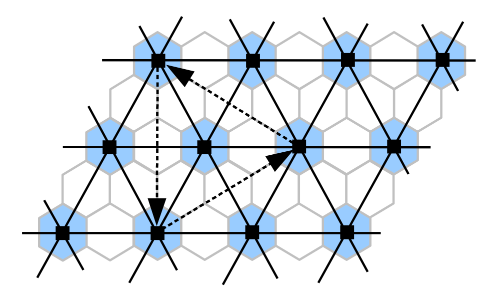
Outline

<u>“Laboratory”: Kitaev's honeycomb lattice model</u>



- New Abelian phases emerge in the presence of superlattices of non-Abelian vortices
- Mechanism: Band structure hybridization due to vortex-vortex interactions

<u>“Theory”: Majorana model on the vortex lattice</u>



- Majorana fermions with nearest and next to nearest neighbour tunneling on the vortex lattice
- Relate the free parameters of this effective model to the physical observables of the honeycomb model

<u>Main result:</u>
The character of the nucleated *many-vortex* state is fully determined by the *pairwise* microscopic vortex interactions!

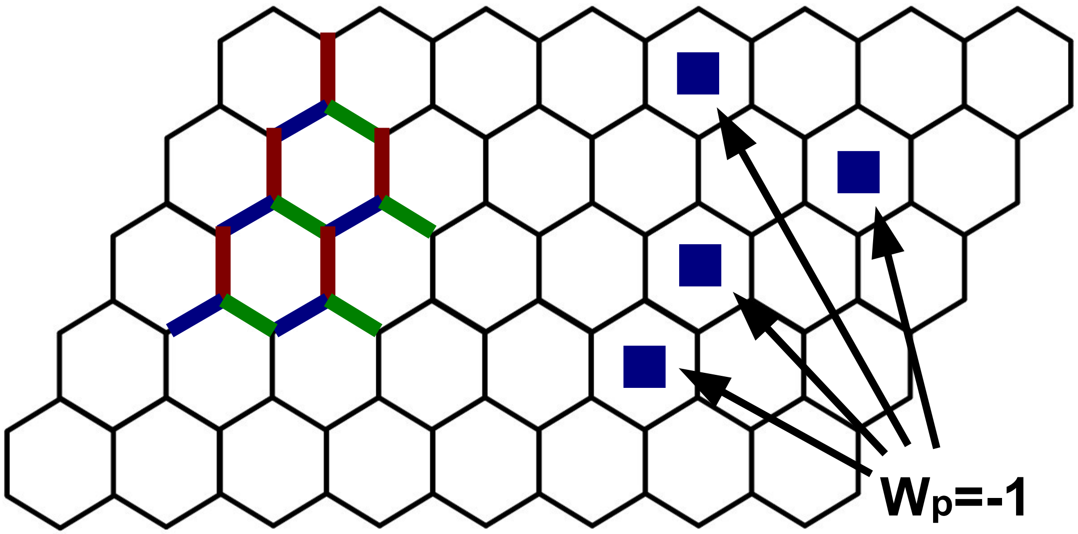
“Laboratory”:
Kitaev's honeycomb lattice model
–
A local spin lattice model that is adiabatically equivalent to a *p*-wave superconductor

Laboratory: Kitaev's honeycomb lattice model

Spin ½-particles on the vertices of a honeycomb lattice interacting according to the Hamiltonian:

$$H(J_x, J_y, J_z, K, \{W_p\})$$

J_α: Nearest neighbour spin-spin interactions
K: TRS breaking three spin term
$\{W_p\}$: Local Z_2 symmetry at every plaquette p



To solve the model:

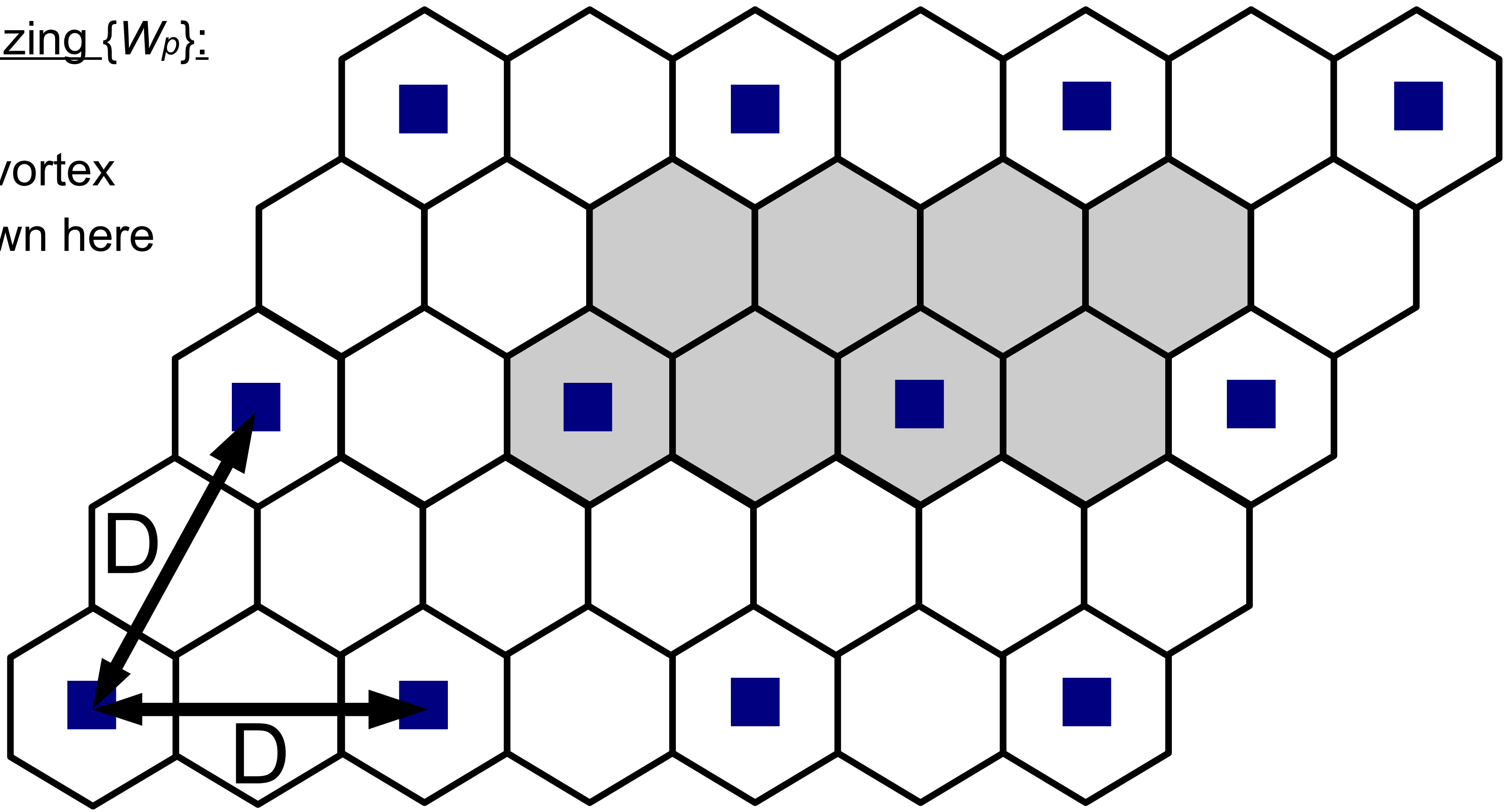
- Fix vortex sector $\{W_p\}$
- Map the model into free Majorana fermions coupled to a Z_2 gauge field
  - → W_p become Wilson loops
- Study how the fermionic spectrum on each sector and the Chern number for the ground state depend on J_α, K and $\{W_p\}$

Kitaev, Ann. Phys. 321,2 (2006)

Laboratory: homogenous vortex superlattices

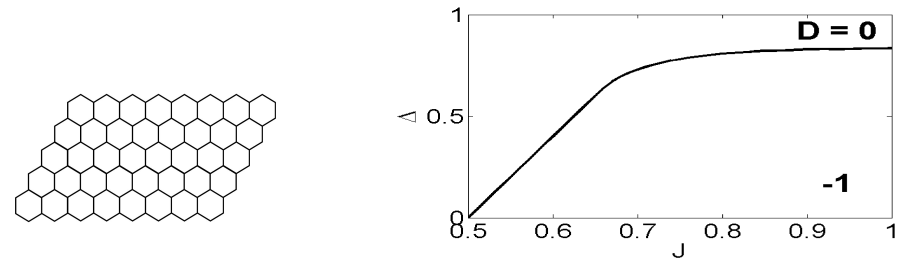
<u>Parametrizing $\{W_p\}$:</u>

D=1: full-vortex
D=2: shown here
....



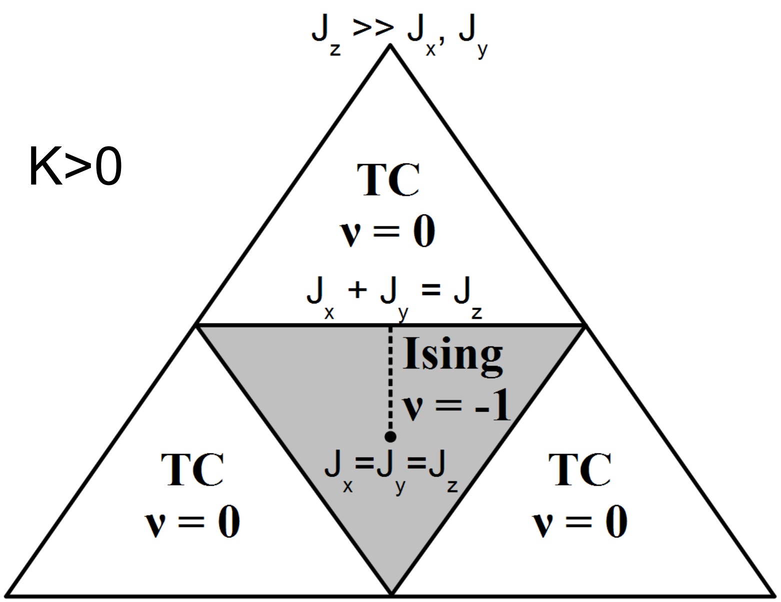
Vortex lattice of sparsity D $\leftrightarrow$ Bloch Hamiltonian of linear size $4D^2$

Laboratory: Phase diagram and vortex lattices



Vortex-free sector

K>0



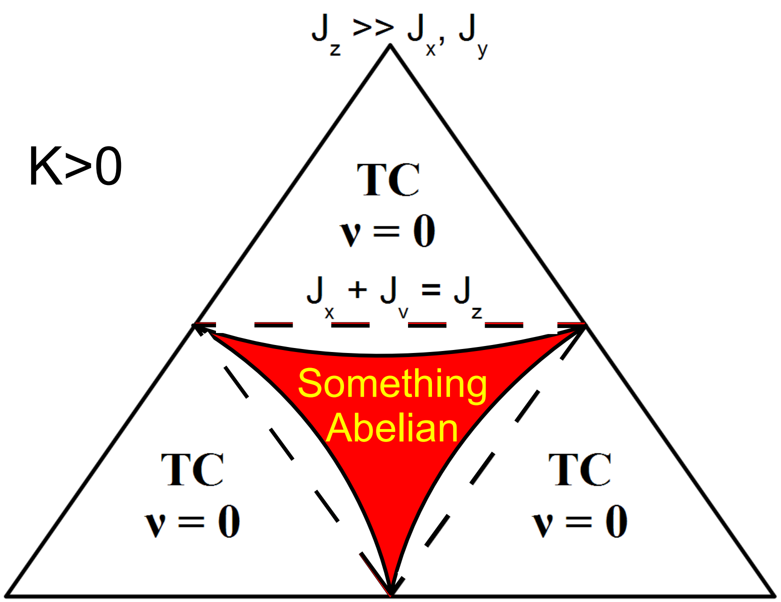
In p-wave language:

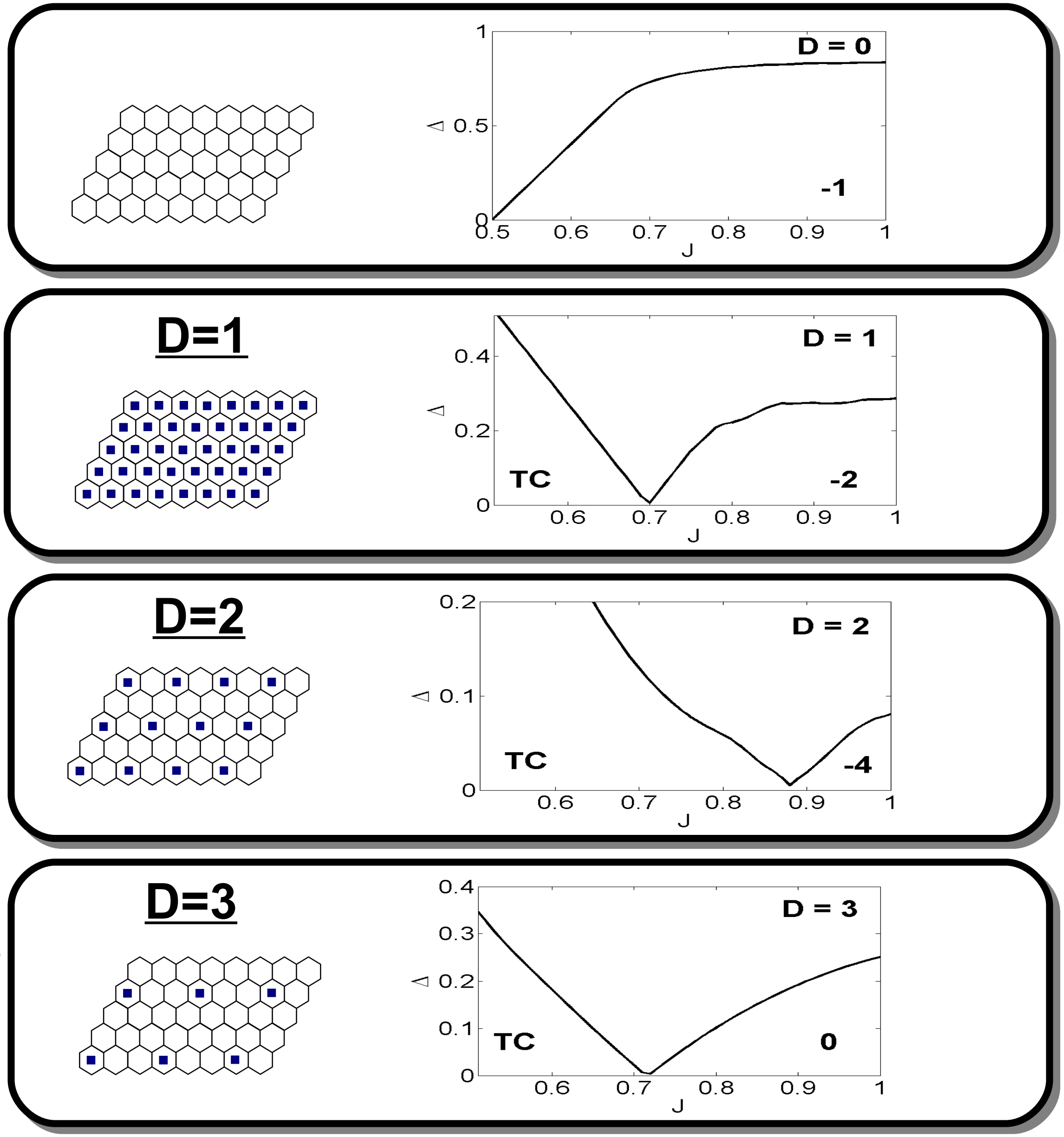
- ν = -1: non-Abelian weak pairing phase
- ν = 0 : Abelian strong pairing phase

Laboratory: Phase diagram and vortex lattices

Vortex-lattice sectors

K>0





In p-wave language:

- ν = -1: non-Abelian weak pairing phase
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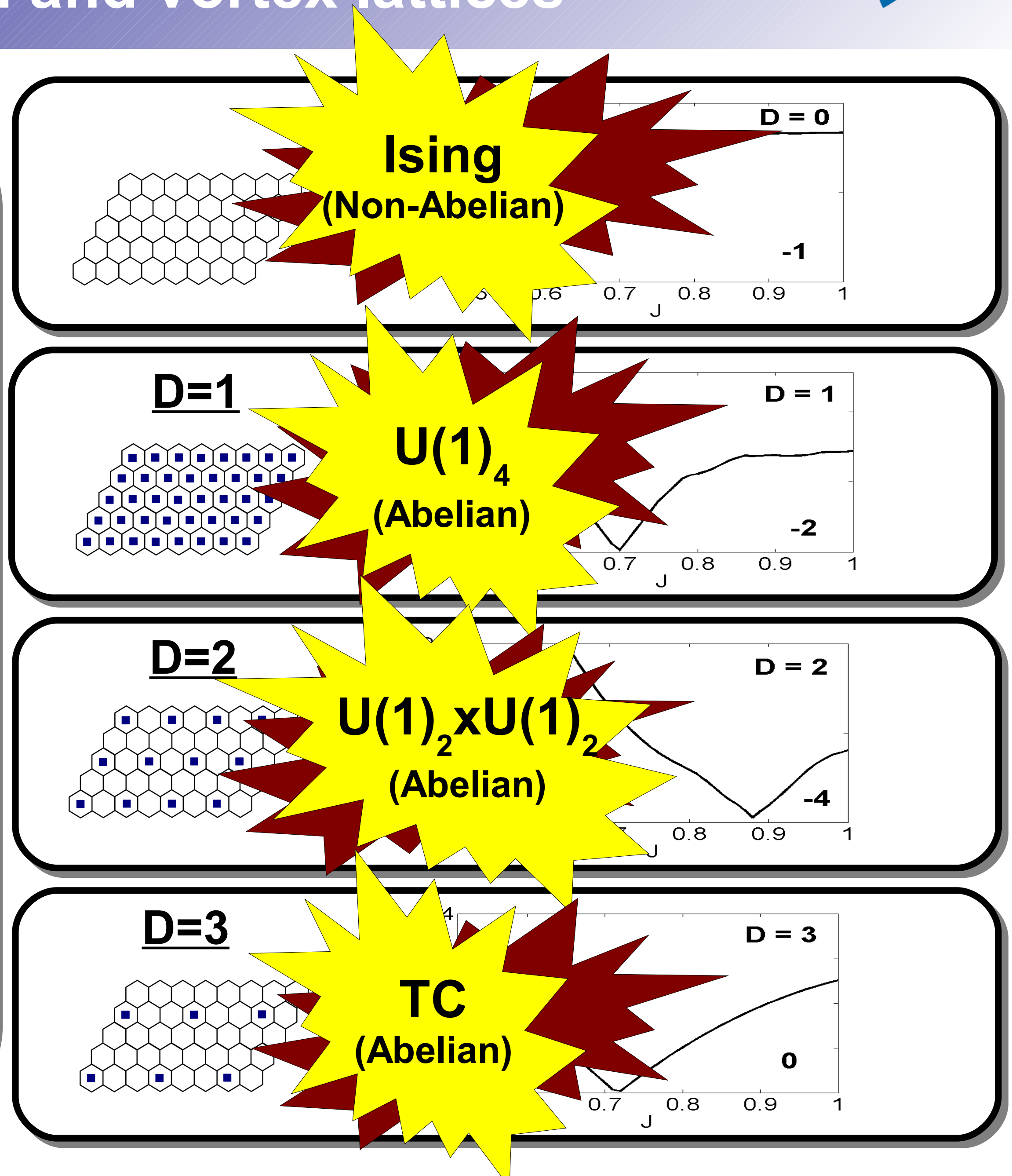
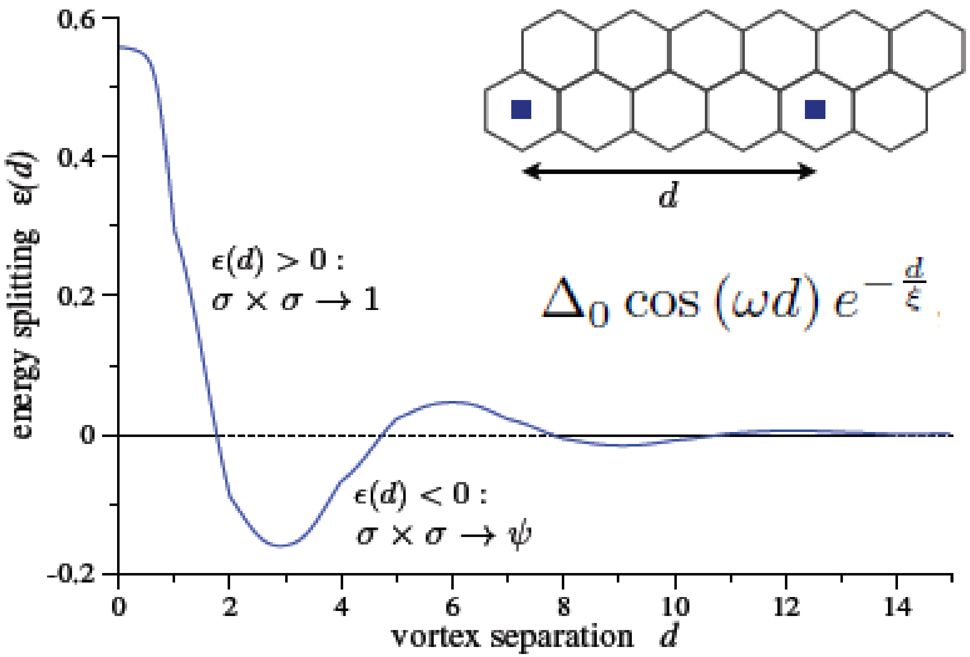
Laboratory: Phase diagram and vortex lattices

- Only Abelian phases emerge in the presence of homogenous triangular vortex lattices
- The CFT based theory of Ludwig et al. predicts that the non-Abelian phase is lost and that either $U(1)_4$ or TC anyons should emerge
- Are vortex-vortex interactions really responsible for the novel phases and if so, why does the $U(1)_2 x U(1)_2$ phase emerge?



Laboratory:
The vortices are interacting non-Abelian anyons

Interactions split topological degeneracy



- Ising anyons can combine into two distinct states

$$\sigma \times \sigma = 1 + \Psi$$

VL, NJP 13, 075009 (2011)
Cheng et al. PRL 103, 107001 (2009)

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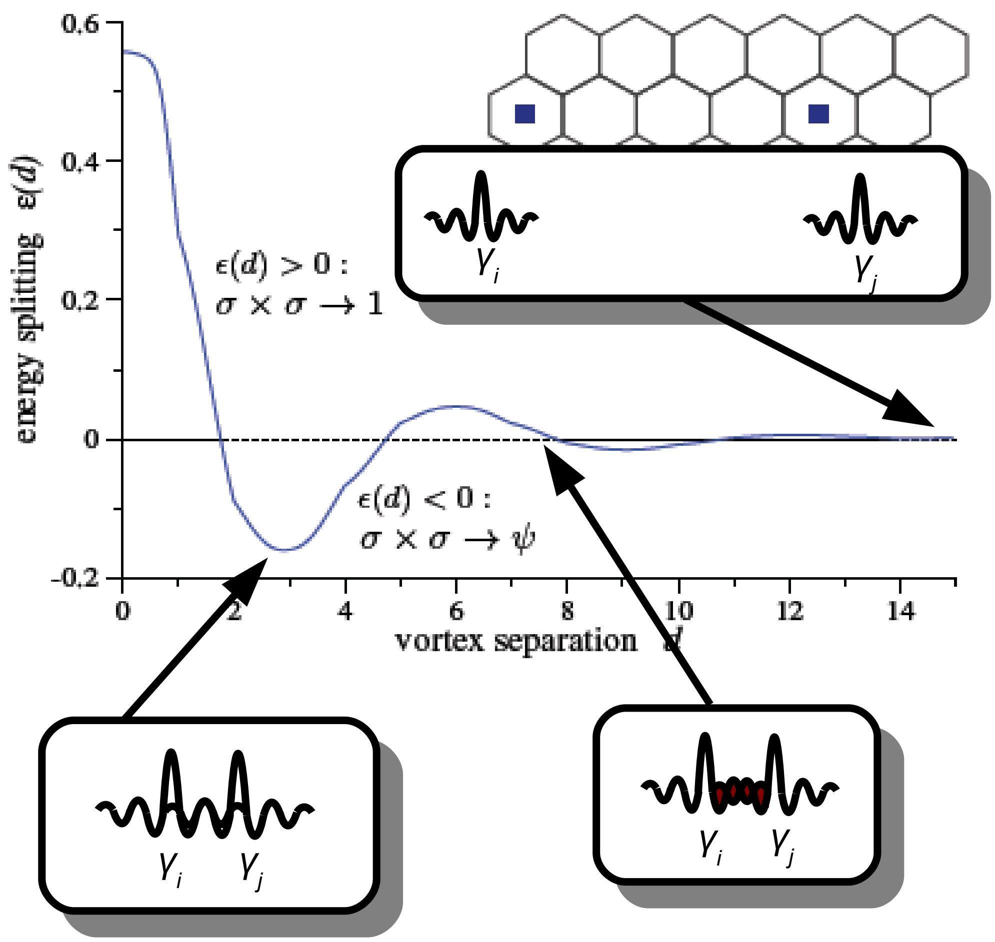


- Ising anyons can combine into two distinct states

$$\sigma \times \sigma = 1 + \Psi$$

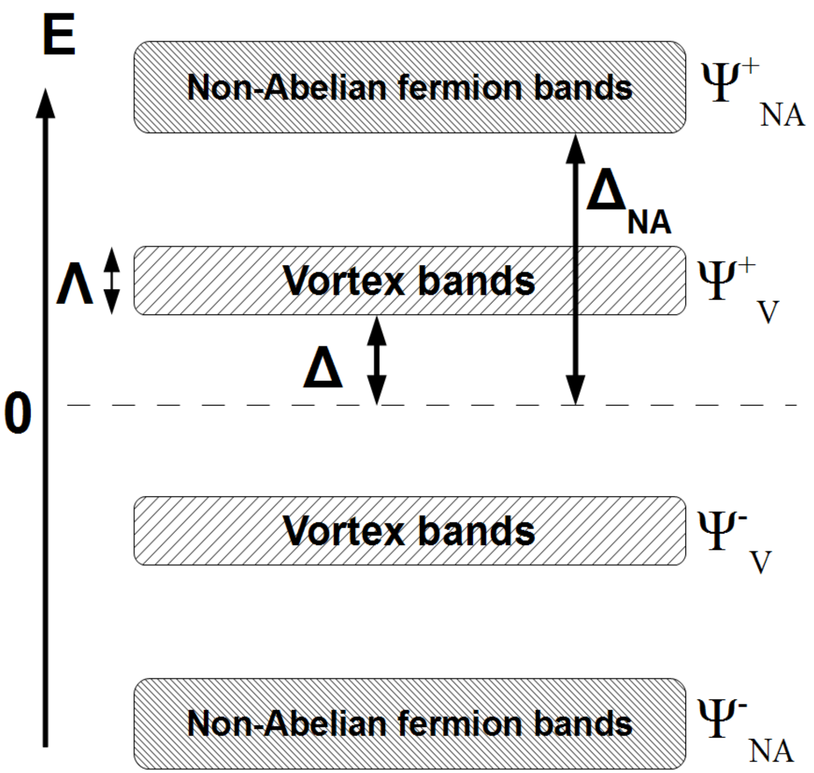
Think in terms of Majoranas:

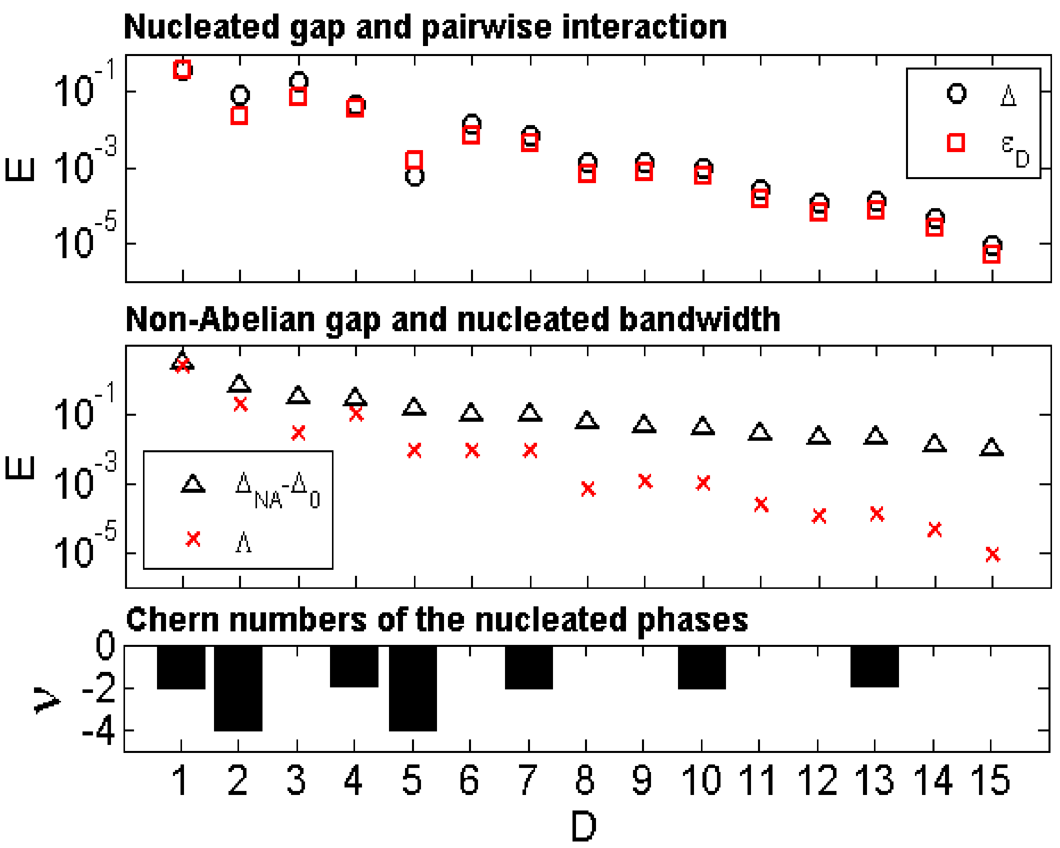
- 2^n-fold degeneracy for *2n* well separated vortices
  - *2n* localized Majorana modes
- Interactions lift degeneracy when vortices are nearby
  - Majorana modes tunnel

VL, NJP 13, 075009 (2011)
Cheng et al. PRL 103, 107001 (2009)

Laboratory:
Band hybridization due to the interactions

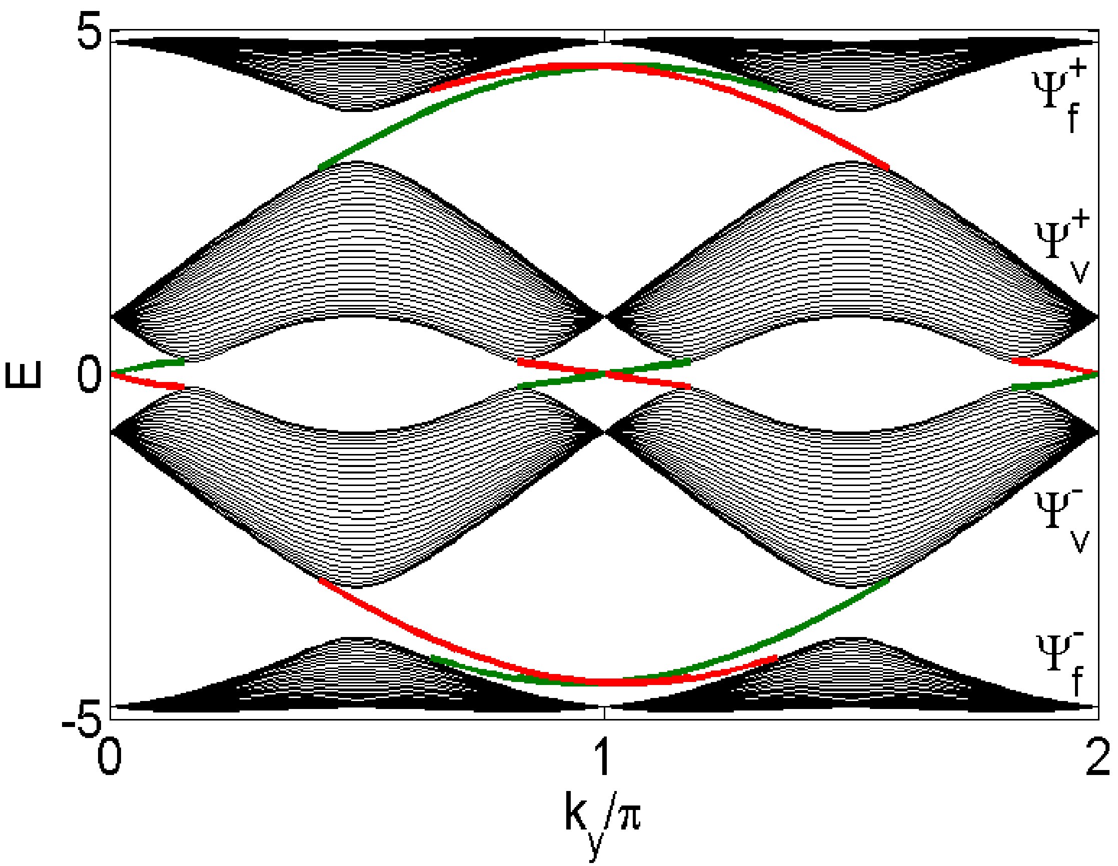
Nucleated band structure





Laboratory:
Band hybridization due to the interactions

Full-vortex spectrum ($\nu = -2$) on cylinder



The Chern number in the vortex lattice phases

$$\nu = \nu_f + \nu_v$$

- $\nu_f = -1$ always for K>0
- ν_v depends on the vortex lattice D

Can we find a model explaining how ν_v changes with D?

“Theory”:
Majorana model on the vortex lattice with longer range hopping

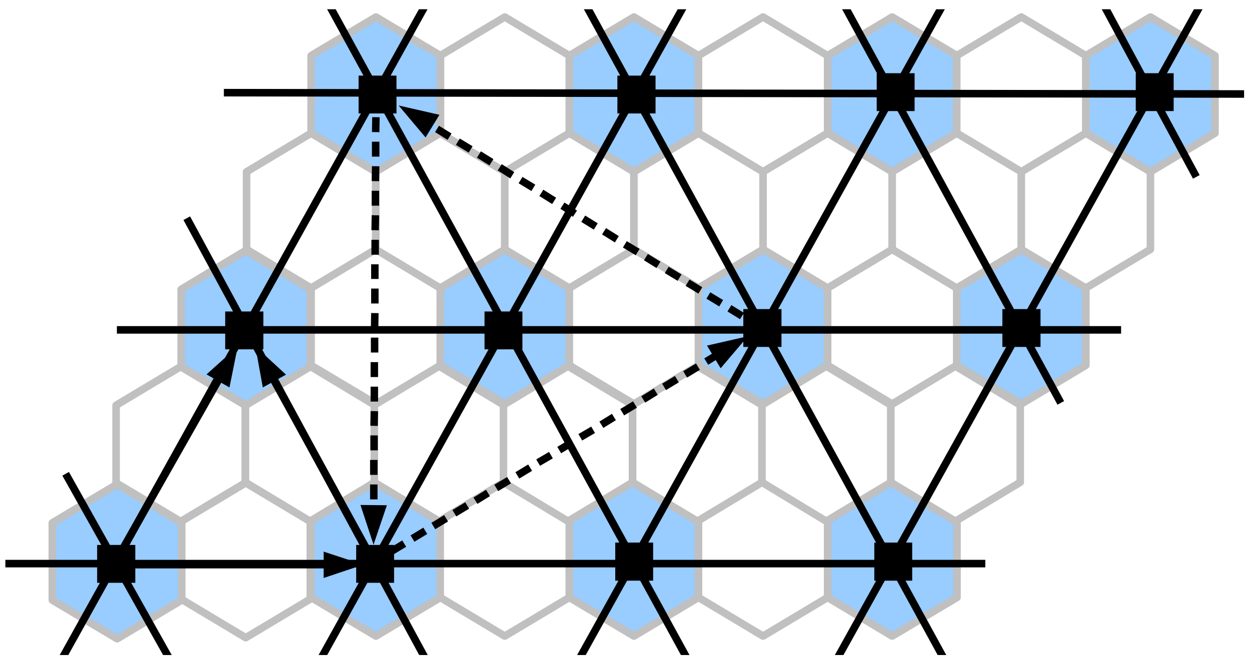
–

If tunneling of localized Majorana modes gives rise to pairwise energy splitting, perhaps a 2D Majorana lattice model would then describe the hybridized band?

Theory: Majorana model on the vortex lattice

Hamiltonian with nearest t_1 (⟶) and next to nearest $t_{\sqrt{3}}$ (- - -▶) hopping:

$$H_M = \sum_{l=1,\sqrt{3}} H_l, \qquad H_l = i t_l \sum_{|i-j|=l} s^l_{ij} \gamma_i \gamma_j$$



Theory: Majorana model on the vortex lattice

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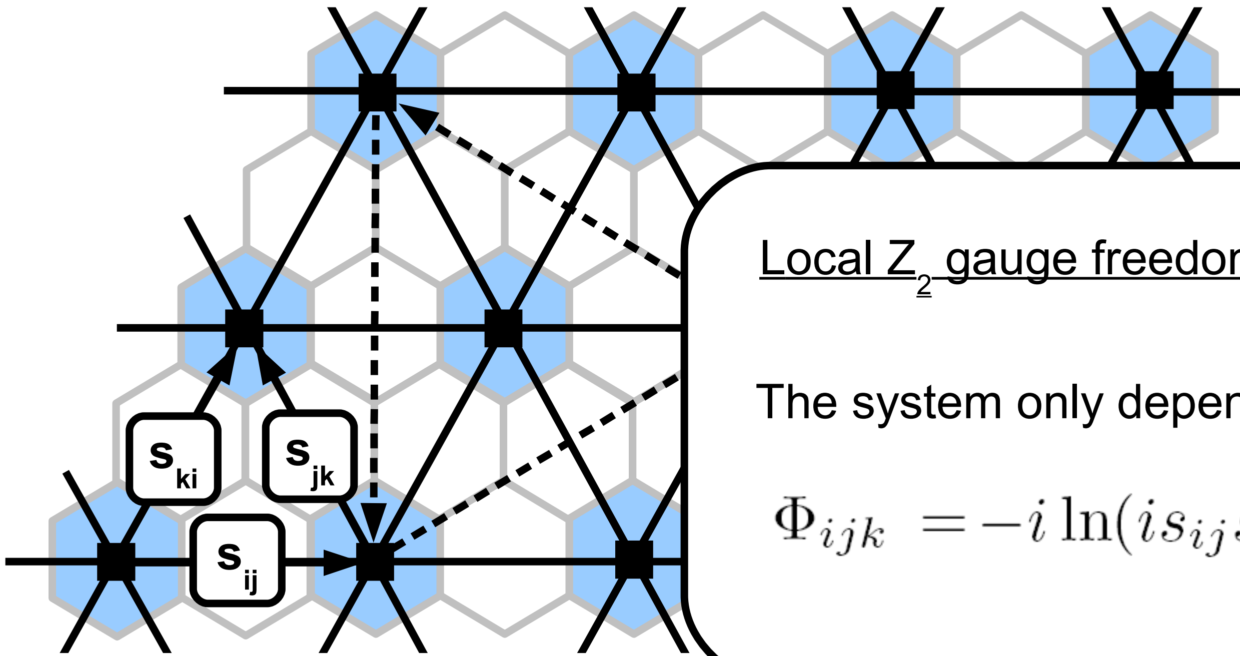

Local Z_2 gauge freedom on every link

The system only depends on the “flux”:

$$\Phi_{ijk} = -i \ln(i s_{ij} s_{jk} s_{ki}) = \pm \frac{\pi}{2}$$

Theory: Majorana model on the vortex lattice

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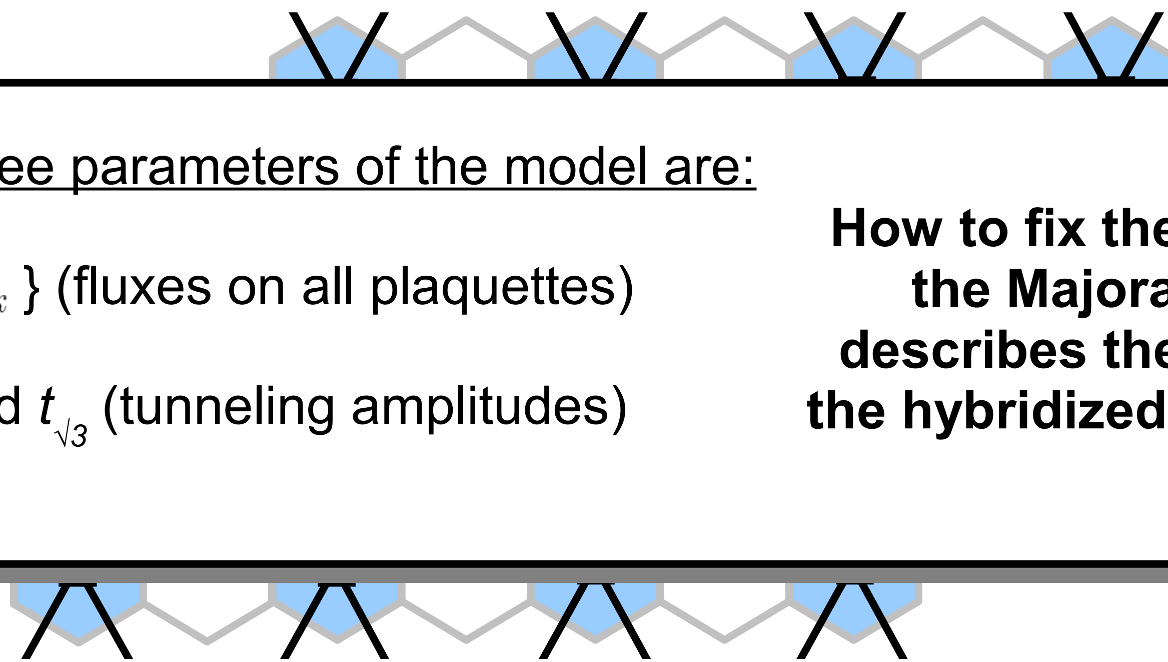


The free parameters of the model are:

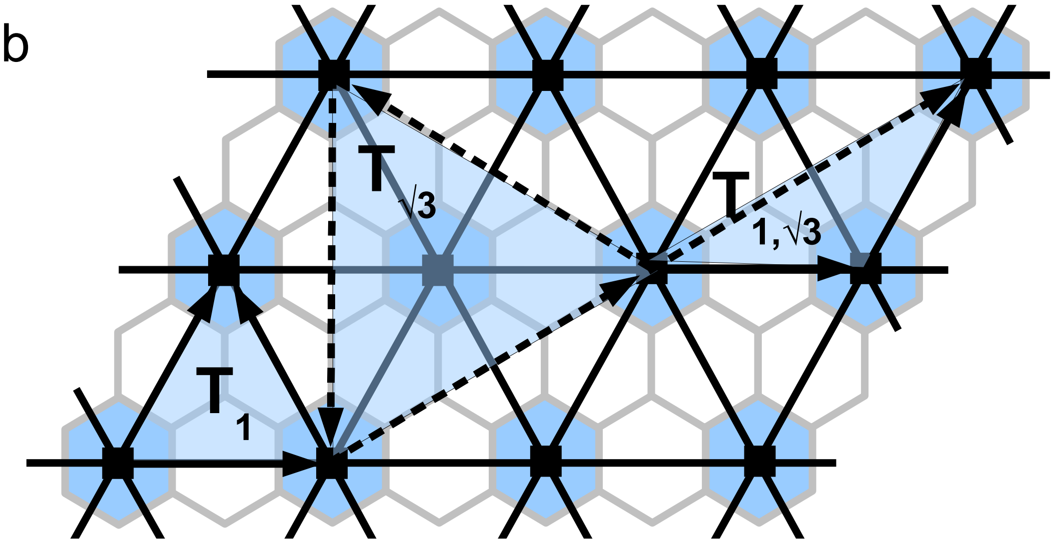
- { Φ_{ijk} } (fluxes on all plaquettes)
- t_1 and $t_{\sqrt{3}}$ (tunneling amplitudes)

How to fix these such that the Majorana model describes the behavior of the hybridized vortex band?

Theory: Fixing the fluxes

The vortices of the underlying honeycomb model carry Π-flux.

Assign flux on each Majorana plaquette equal to the flux enclosed by each vortex lattice plaquette.[1]



$$\Phi_{T_1} = \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{6} = \frac{\pi}{2}$$

$$\Phi_{T_{\sqrt{3}}} = \frac{\pi}{6} + \frac{\pi}{6} + \frac{\pi}{6} + \pi = \frac{3\pi}{2}$$

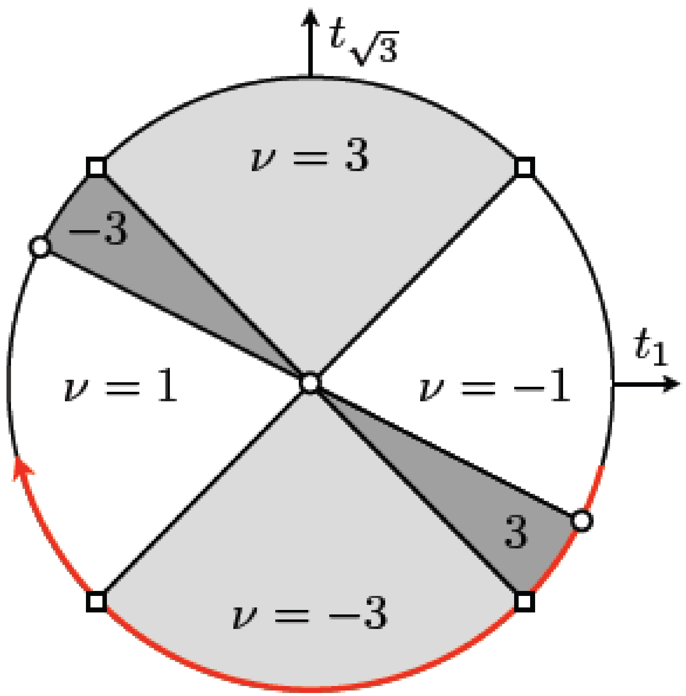
$$\Phi_{T_{1,\sqrt{3}}} = \frac{\pi}{12} + \frac{\pi}{12} + \frac{\pi}{3} = \frac{\pi}{2}$$

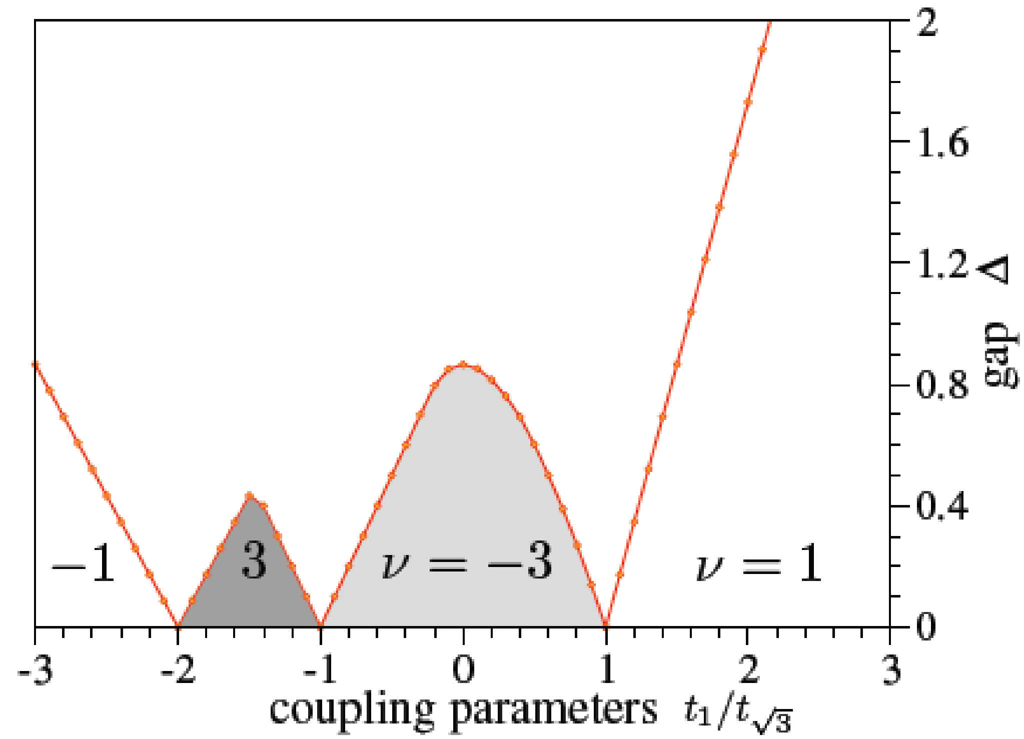
<u>We fix the Majorana model flux to:</u>

$$(\Phi_{T_1}, \Phi_{T_{\sqrt{3}}}, \Phi_{1,T_{\sqrt{3}}}) = (\tfrac{\pi}{2}, -\tfrac{\pi}{2}, \tfrac{\pi}{2})$$

[1] more rigorously: Grosfeld & Stern, PRB 73, 201303 (2006)

Theory: Phase diagram of the Majorana model





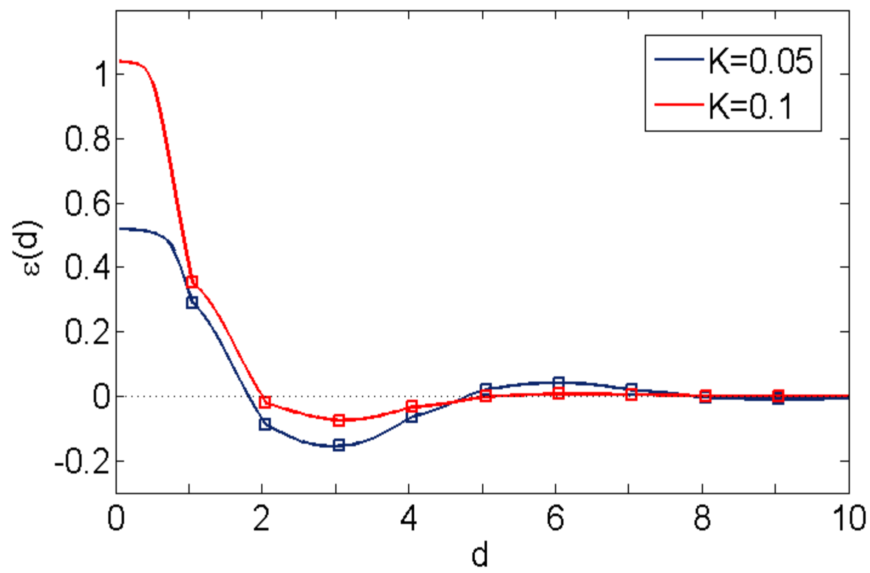
Since the Majorana model describes only the hybridized band, in the background of the constant ν_f = -1 band our model can predict phases with:

$$\nu = \nu_M - 1 = 0, \pm 2, -4$$

Promising!

Theory: Fixing the tunneling amplitudes

- The energy splitting is proportional to the overlap of the Majorana zero mode wave functions, which also gives the tunneling amplitudes
- The tunneling amplitudes are isotropic → we restrict to $J_x = J_y = J_z$ and vary only K
- K still has non-trivial effect on the energy splitting due to the oscillations

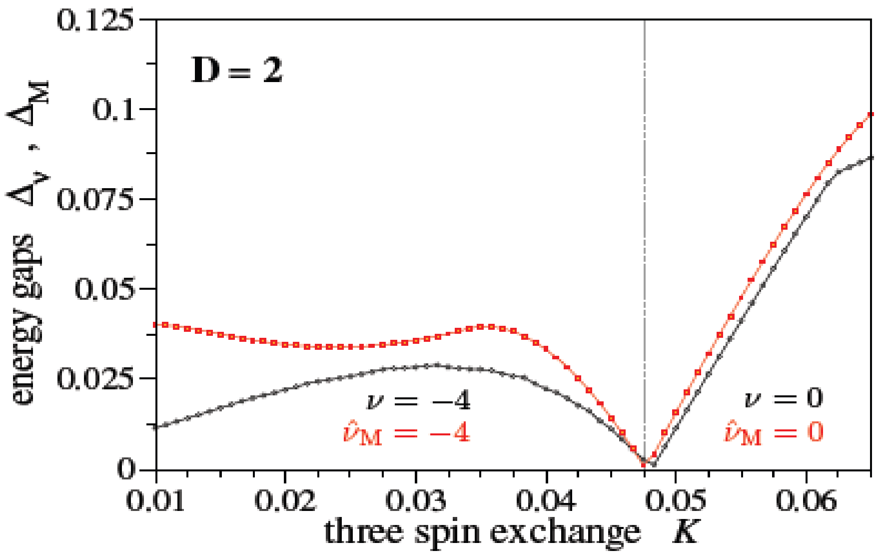


We assume the tunneling is bi-partite and use the ansatz:

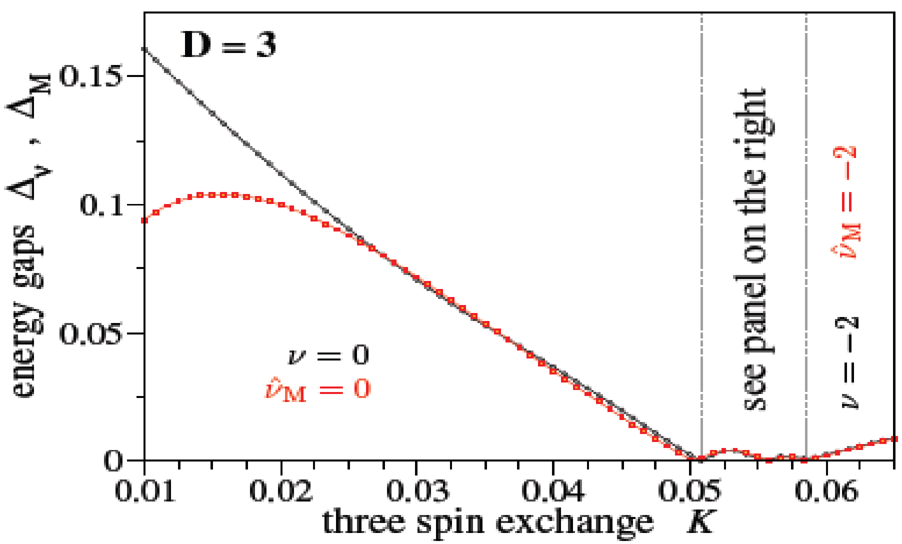
$$t_l(K) \leftrightarrow \varepsilon_{ID}(K)$$

Theory: Results

D=2



D=3



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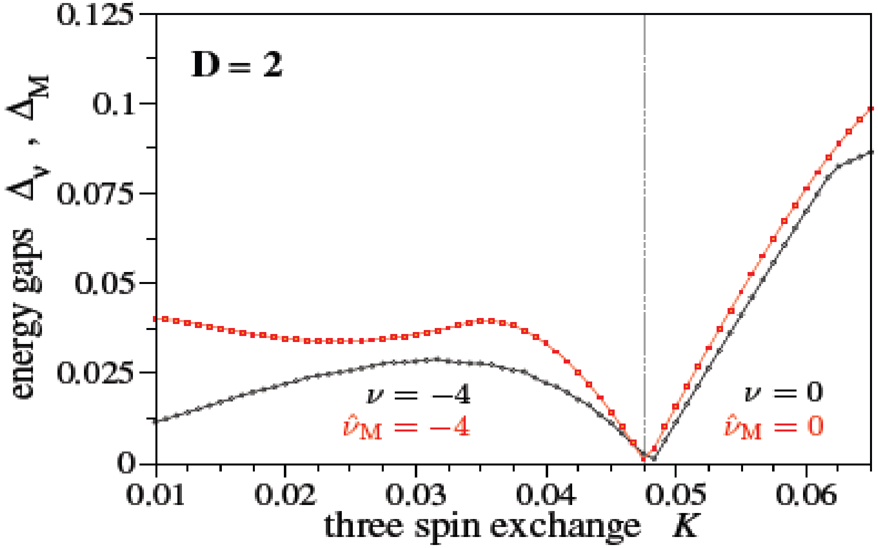

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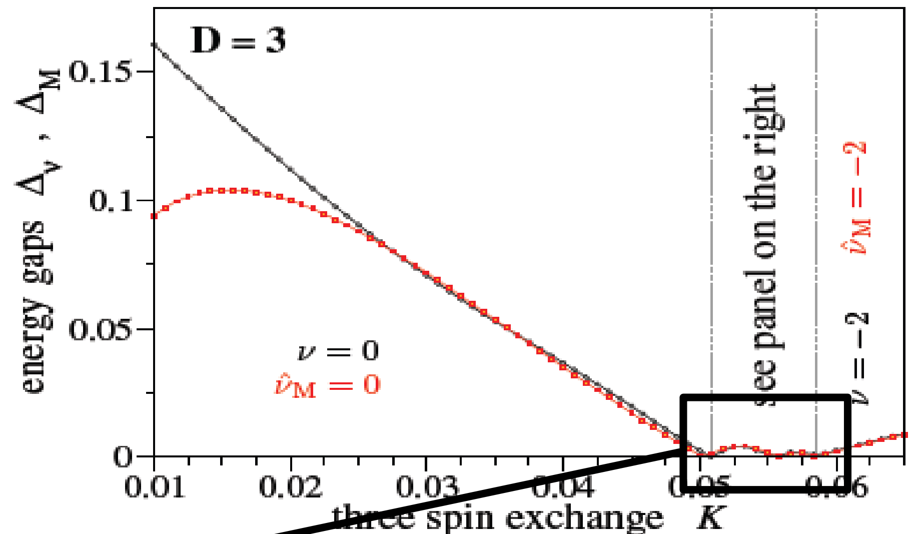

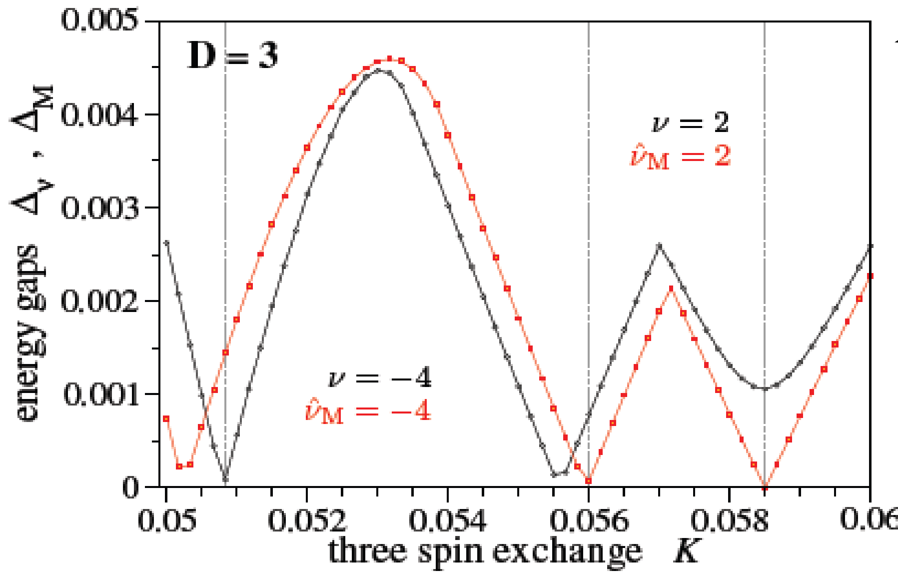

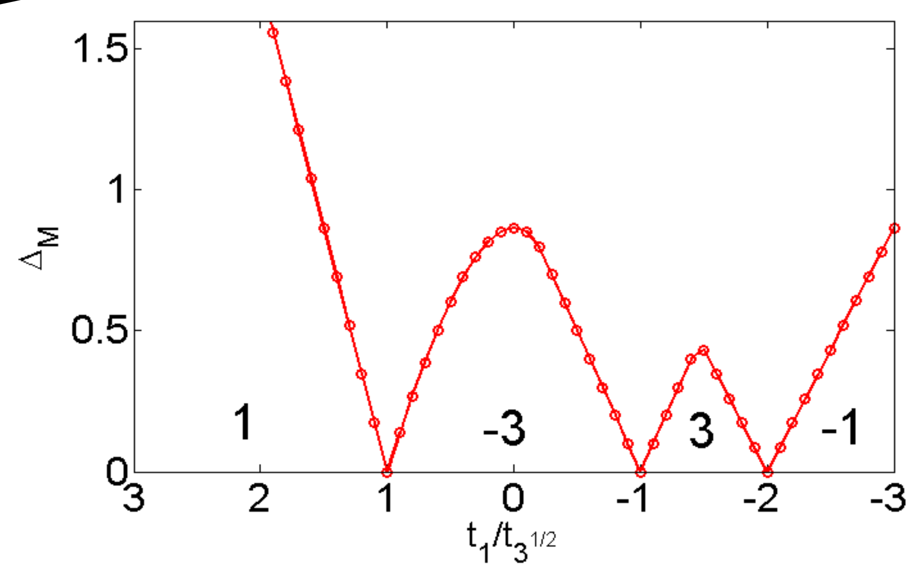

Theory: Results

- We have verified that **nucleation occurs and that the effective model works perfectly at least for spacings up to D ≤ 6**

- **Our model with nearest and next to nearest interactions is comprehensive.** If other Chern numbers were to appear (possible if even longer range interactions are relevant), they should have appeared for the tightest packed superlattices due to the exponential suppression of the interactions

- Too good? Well, there is one small thing...

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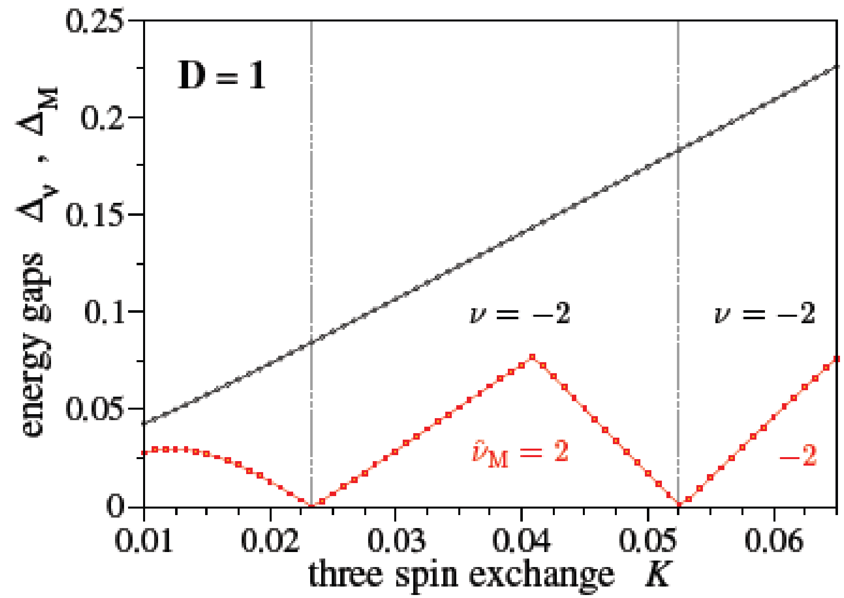
- Too good? Well, there is one small thing...

D=1 (full-vortex lattice)

- Agreement only in the K > 0.05 region
- For K < 0.05 the coherence length $\zeta \sim K^{-1}$ is larger than vortex lattice spacing D
  - Individual vortices no longer well defined
  - Tunneling amplitudes not captured by ε_{ID}
- **Our microscopic approach provides quantitative agreement when ζ < D.**



Conclusions

- **First demonstration of the topological liquid nucleation in the context of a microscopic model**
- **The Majorana model provides full description of the *many-anyon* state directly from the *pairwise* interactions without fine-tuning or fitting**
- **Microscopics are important – oscillations can cause longer range interactions to become dominant and determine the system's behavior**
- **Since the oscillating interactions are omnipresent, longer range interactions are likely to be relevant also in p-wave SCs and FQHE liquids**

Future work:

- Disordered vortex lattices and stability of the nucleated phases

References:

- Topological liquid nucleation: **arXiv: 1111.3296**
- The full-vortex lattice phase: **VL & JKP, PRB 81, 245132 (2010)**
- Interacting Ising vortices: **VL, NJP 13, 075009 (2011)**