Screening Properties and Scaling Exponents of Quantum Hall States

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Topological States of Matter Workshop - Nordita

August 20, 2012

work done in collaboration with:

A. Bernevig and N. Regnault, arXiv:1207.3305

Introduction

Laughlin (1983)

- quantum Hall states are quantum liquids
- trial wavefunctions are useful:
  - represent universality classes
  - help guide our understanding
- plasma mapping allows derivation of important universal properties from trial wavefunctions (charge, statistics, exponents)

Introduction

Moore and Read (1991)

- CFT can be used to generate/describe candidate QH states/universality classes
- describes the edge theory – "bulk-edge correspondence"
- universal properties are easy to predict but not necessarily easy to derive

Introduction

Many candidate QH states have been proposed:

- Laughlin
- HH hierarchy/CF
- MR Pfaffian
- Read-Rezayi
- NASS
- BS hierarchy
- Haldane-Rezayi
- Gaffnian
- Jacks
- Hermanns hierarchy
- minimal model states
- …

Introduction

Main questions:

1. How do we know whether a candidate is a “good” QH state?

2. How do we extract the edge theory?

- Can we exhibit the problems with the Gaffnian?

Introduction

Partial Answer: “Plasma analogy”

- quasiparticle charge and statistics

  Laughlin (1983)
  Arovas, Schrieffer, and Wilczek (1984)
  Bonderson, Gurarie, and Nayak (2011)

- edge excitation scaling exponents

  Wen (1992)
  Bernevig *et al.* (unpublished)

- more detailed edge theory can also be obtained from generalized screening assumption

  Dubail, Read, and Rezayi (2012)

Introduction

Partial Answer: "Plasma analogy"

- not all candidates have a plasma mapping
- for those that do, not all the plasmas are well-understood (screening properties)
  - perfect for Laughlin and some hierarchy states
  - great for MR, BS, and M(5,4) states
  - not fully understood for Gf
  - unknown for others (including RR)
- need another, more general method…

Consider a trial wavefunction for N particles

- in the planar disk geometry, with coordinates $z_i = x_i + iy_i$
- with a quasiparticle at η
- possibly other quasiparticles (at 0 for convenience)

$$\Psi(\eta; z_i) = \sum_{a=0}^{n_\phi} \eta^a P_a(z_1, \ldots, z_N)\, e^{-\frac{1}{4\ell^2}\sum_{i=1}^{N}|z_i|^2}$$

n_ϕ, the highest power of η, depends on the state

$P_a(z_i)$ are symmetric polynomials for bosons
antisymmetric polynomials for fermions

Define the inner product

$$\Gamma(\bar{\eta}, \eta') = \int \prod_{j=1}^{N} d^2 z_j \, \overline{\Psi(\eta; z_i)} \Psi(\eta'; z_i)$$

$$= \sum_{a=0}^{n_\phi} (\bar{\eta}\eta')^a \mathcal{N}_a$$

where

$$\mathcal{N}_a \equiv \int \prod_{j=1}^{N} d^2 z_j \, |P_a(z_1, \ldots, z_N)|^2 \; e^{-\frac{1}{2\ell^2} \sum_{i=1}^{N} |z_i|^2}$$

(P_a are orthogonal)

Let the quasiparticle coordinate be located outside the Hall droplet, i.e. $|\eta| > R$ QH disk radius

For a properly screening state, the norm should take the form (in thermodynamic limit)

$$\|\Psi\|^2 = \Gamma(\bar{\eta}, \eta) \sim C\, |\eta|^{2n_\phi} \left(1 - \frac{R^2}{|\eta|^2}\right)^{-g}$$

and by analytic continuation

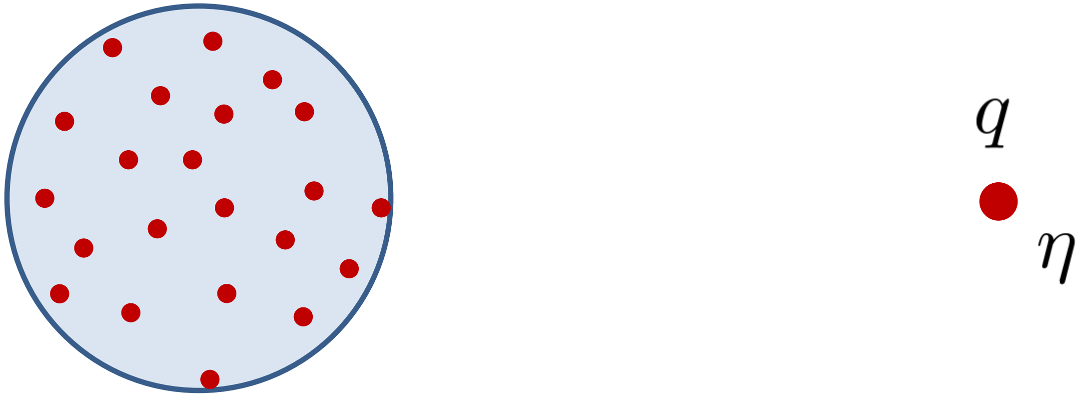
$$\Gamma(\bar{\eta}, \eta') \sim C\, (\bar{\eta}\eta')^{n_\phi} \left(1 - \frac{R^2}{\bar{\eta}\eta'}\right)^{-g}$$

$$\simeq C\, (\bar{\eta}\eta')^{n_\phi} \sum_{n=0}^{n_\phi} \begin{pmatrix} g+n-1 \\ n \end{pmatrix} \left(\frac{R^2}{\bar{\eta}\eta'}\right)^{n}$$

Justified (partly) by plasma mapping arguments

$$f(|\eta|)\Gamma(\bar{\eta},\eta) = e^{-F}$$

F is the free energy of a 2D classical plasma with a test charge q at position η



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$$f(|\eta|)\Gamma(\bar{\eta},\eta) = e^{-F}$$

In its screening phase, the plasma behave like a metal

η dependence of F given by the charging energy of a test charge.

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η dependence of F given by the charging energy of a test charge.

By method of images: $E = \frac{1}{2}q^2 \log\left(1 - \frac{R^2}{|\eta|^2}\right)$

Justified (partly) by an edge CFT

$$\left\langle \Phi^\dagger \left(w'\right) \Phi \left(w\right) \right\rangle = \left(w' - w\right)^{-2h}$$

Φ is the quasiparticle's edge excitation operator,
i.e. a primary field with conformal weight h

Taking the quasiparticles to the edge $\eta = Re^{i\theta}$ $\eta' = Re^{i\theta'}$
the inner product should match (up to phases)

$$\Gamma\left(\bar{\eta}, \eta'\right) \sim \left\langle \Phi^\dagger \left(w'\right) \Phi \left(w\right) \right\rangle$$

$$w = e^{\frac{t}{R} + i\theta} \quad w' = e^{\frac{t}{R} + i\theta'}$$

$$\Rightarrow \quad g = 2h$$

Matching the expressions

$$\Gamma(\bar{\eta}, \eta') = \sum_{a=0}^{n_\phi} (\bar{\eta}\eta')^a \mathcal{N}_a$$

$$\simeq C\,(\bar{\eta}\eta')^{n_\phi} \sum_{n=0}^{n_\phi} \left(\begin{array}{c} g+n-1 \\ n \end{array} \right) \left(\frac{R^2}{\bar{\eta}\eta'}\right)^n$$

by powers gives (for n small)

$$\mathcal{N}_{n_\phi - n} \simeq C \left(\begin{array}{c} g+n-1 \\ n \end{array} \right) R^{2n}$$

Defines a sequence of approximations for g

$$\binom{g^{(n)}+n-1}{n} = \frac{\mathcal{N}_{n_\phi - n}}{R^{2n}\mathcal{N}_{n_\phi}}$$

Just need to compute the norms $\mathcal{N}_a$

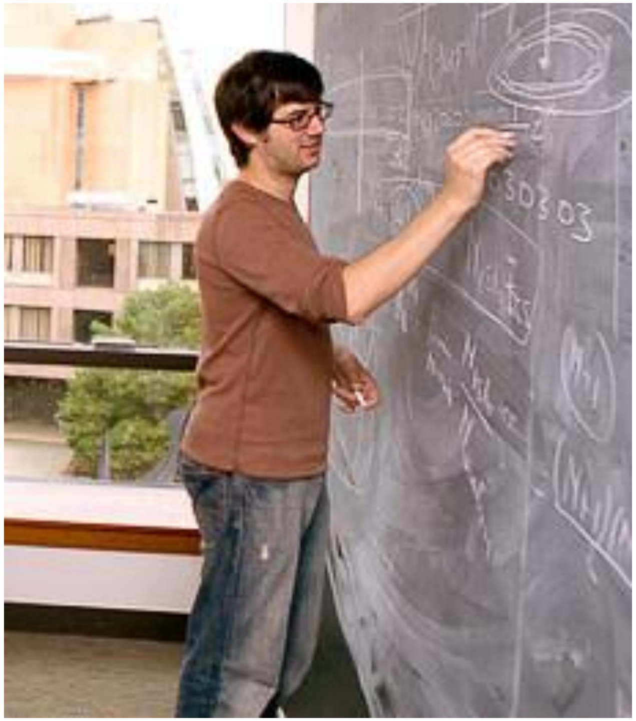
n small are most accurate, so we focus on

$$g^{(1)} = \frac{\mathcal{N}_{n_\phi - 1}}{R^2 \mathcal{N}_{n_\phi}}$$

$$g^{(2)} = \left[2\frac{\mathcal{N}_{n_\phi - 2}}{R^4 \mathcal{N}_{n_\phi}} + \frac{1}{4}\right]^{\frac{1}{2}} - \frac{1}{2}$$

With powerful Jack polynomial machinery, these are easier to compute for some states.

Berevig *et al.* (2007-…)



(1,2) Jack state = Laughlin $\nu = 1/2$

(2,2) Jack state = MR $\nu = 1$

(k,2) Jack state = Z_k–RR $\nu = 3/2$

(2,3) Jack state = Gf $\nu = 2/3$

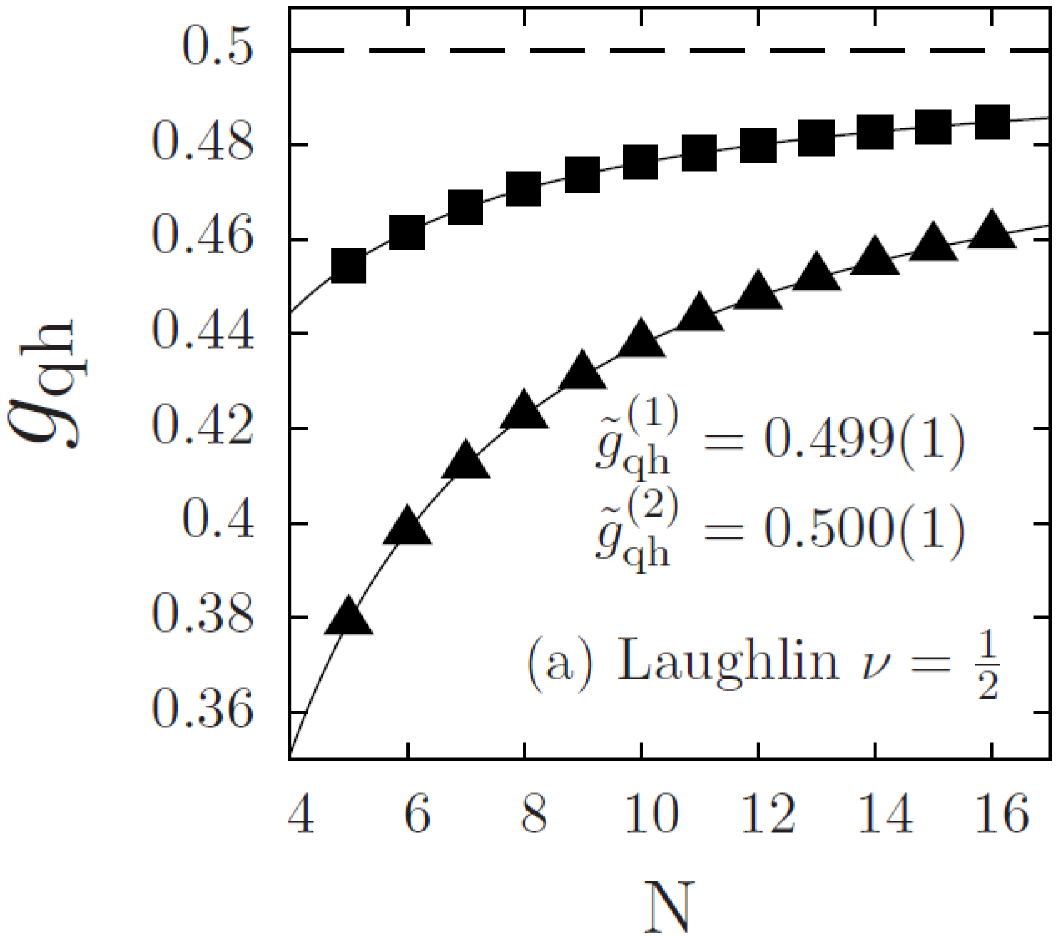
Consider a (k,m) Jack state at ν = k/m
with a fundamental (flux 1/k) quasihole at η
and a flux (k-1)/k quasiparticle at 0

$$n_\phi = \frac{N}{k} \qquad\qquad R = \sqrt{2\left(\nu^{-1}N+1\right)}\ell$$

$$P_a(z_i) = (-k)^{\frac{N}{k}-a} J^{\alpha}_{\mu_a}(z_i) \qquad\qquad \alpha = -\tfrac{k+1}{m-1}$$

$$\mu_a = \left[\left(1, k-1, 0^{m-2}\right)^a, 0, \left(k, 0^{m-1}\right)^{\frac{N}{k}-a}\right]$$

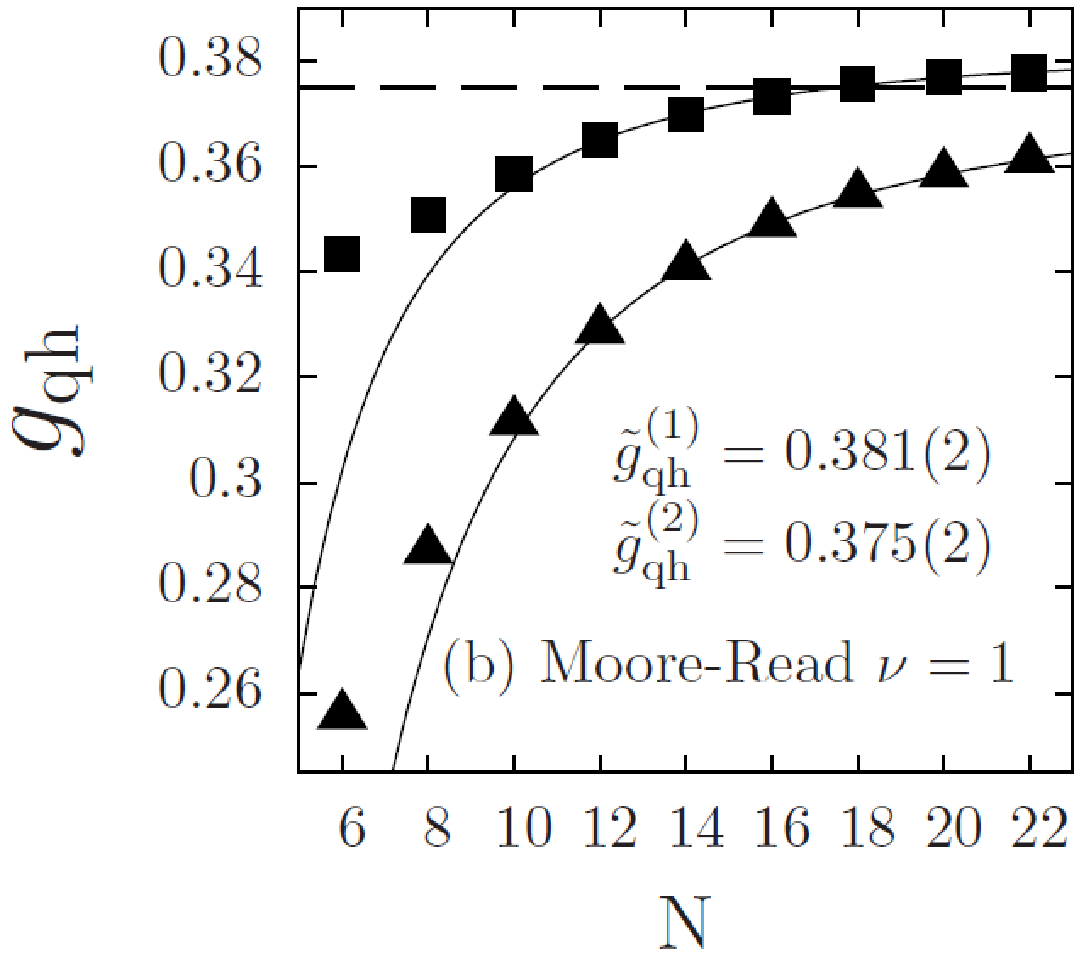
Laughlin State Quasihole



CFT: $g_{qh} = \frac{1}{2}$

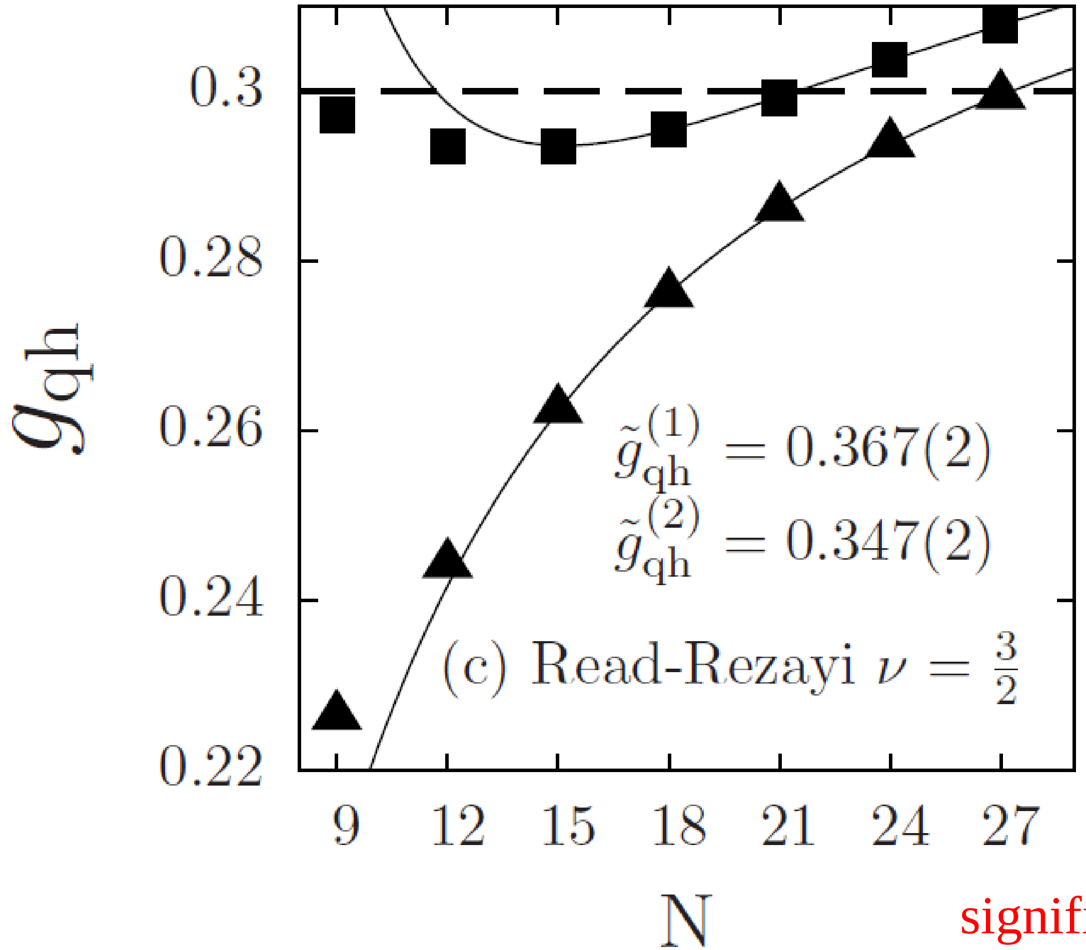
$$g^{(1)}_{qh} = \frac{1}{2+\frac{1}{N}}$$

MR State Quasihole



CFT: $g_{qh} = \frac{3}{8}$

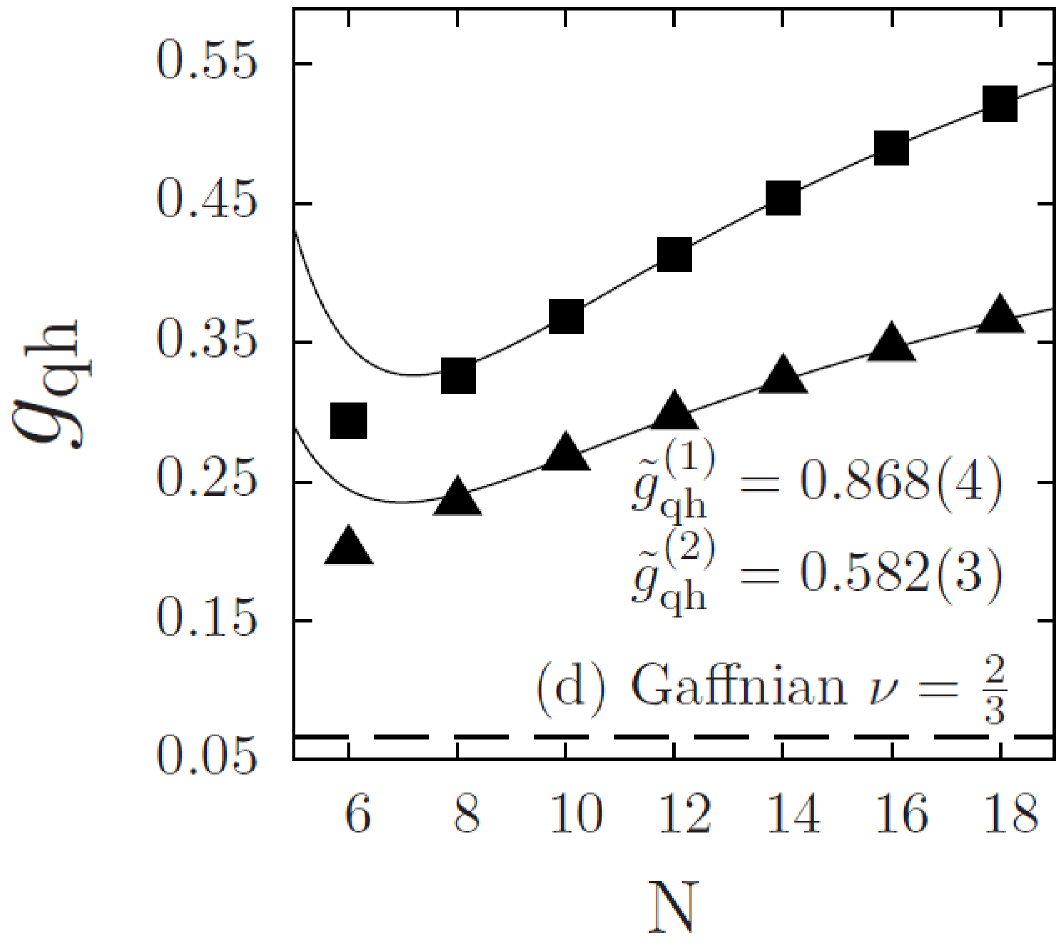
RR State Quasihole



CFT: $g_{qh} = \frac{3}{10}$

significant finite size effects
due to large quasiparticles

Gf State Quasihole



Diverges!

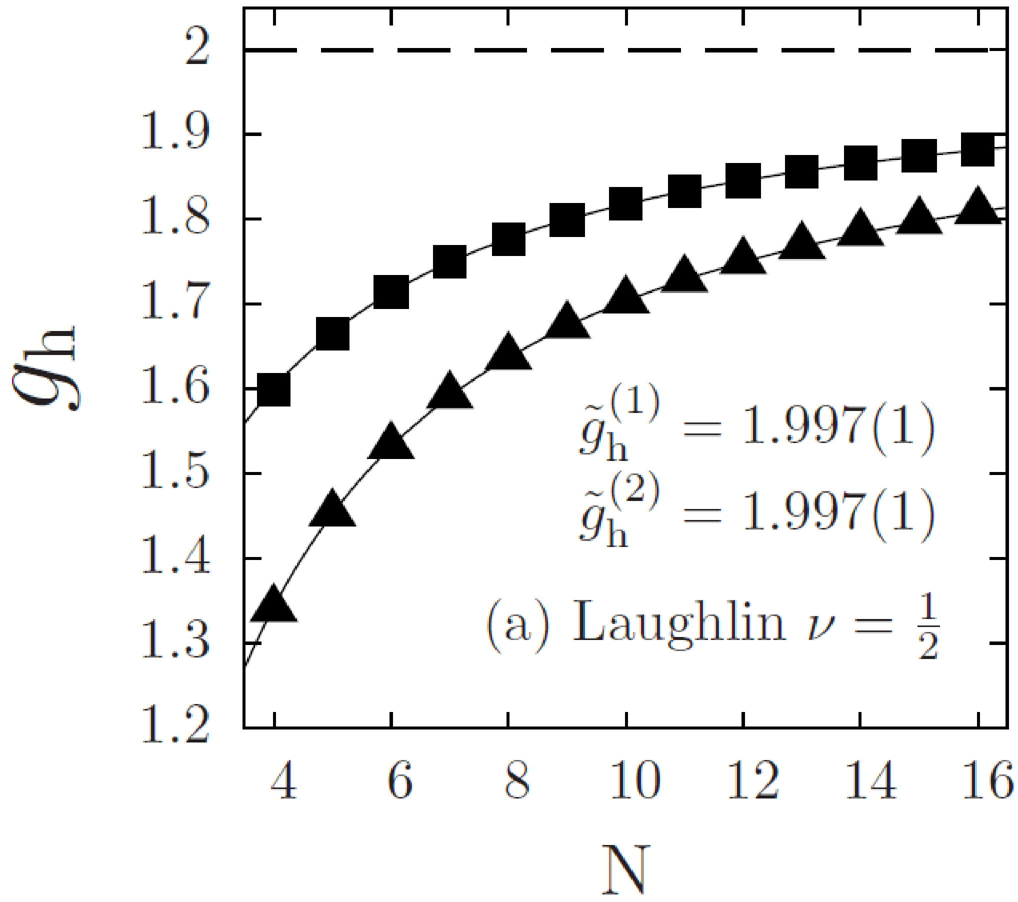
CFT: $g_{qh} = \frac{1}{15}$

Consider a (k,m) Jack state at ν = k/m
with a hole (flux m/k quasiparticle) at η

$$n_\phi = \frac{m}{k}(N+1) - m \qquad R = \sqrt{2\nu^{-1}(N+1)}\ell$$

$$\begin{aligned}
P_{n_\phi} &= J^{\alpha}_{\lambda_{n_\phi}}, \qquad P_{n_\phi-1} = -\frac{m}{k} J^{\alpha}_{\lambda_{n_\phi-1}}, \\
P_{n_\phi-2} &= a_1 J^{\alpha}_{\lambda^{(1)}_{n_\phi-2}} + a_2 J^{\alpha}_{\lambda^{(2)}_{n_\phi-2}}, \qquad \alpha = -\tfrac{k+1}{m-1} \\
\lambda_{n_\phi} &= \left[(k, 0^{m-1})^{\frac{N}{k}-1}, k-1\right], \\
\lambda_{n_\phi-1} &= \left[(k, 0^{m-1})^{\frac{N}{k}-2}, k-1, 1, 0^{m-2}, k-1\right], \\
\lambda^{(1)}_{n_\phi-2} &= \left[(k, 0^{m-1})^{\frac{N}{k}-2}, k-1, 0, 1, 0^{m-3}, k-1\right], \\
\lambda^{(2)}_{n_\phi-2} &= \left[(k, 0^{m-1})^{\frac{N}{k}-3}, (k-1, 1, 0^{m-2})^2, k-1\right]
\end{aligned}$$

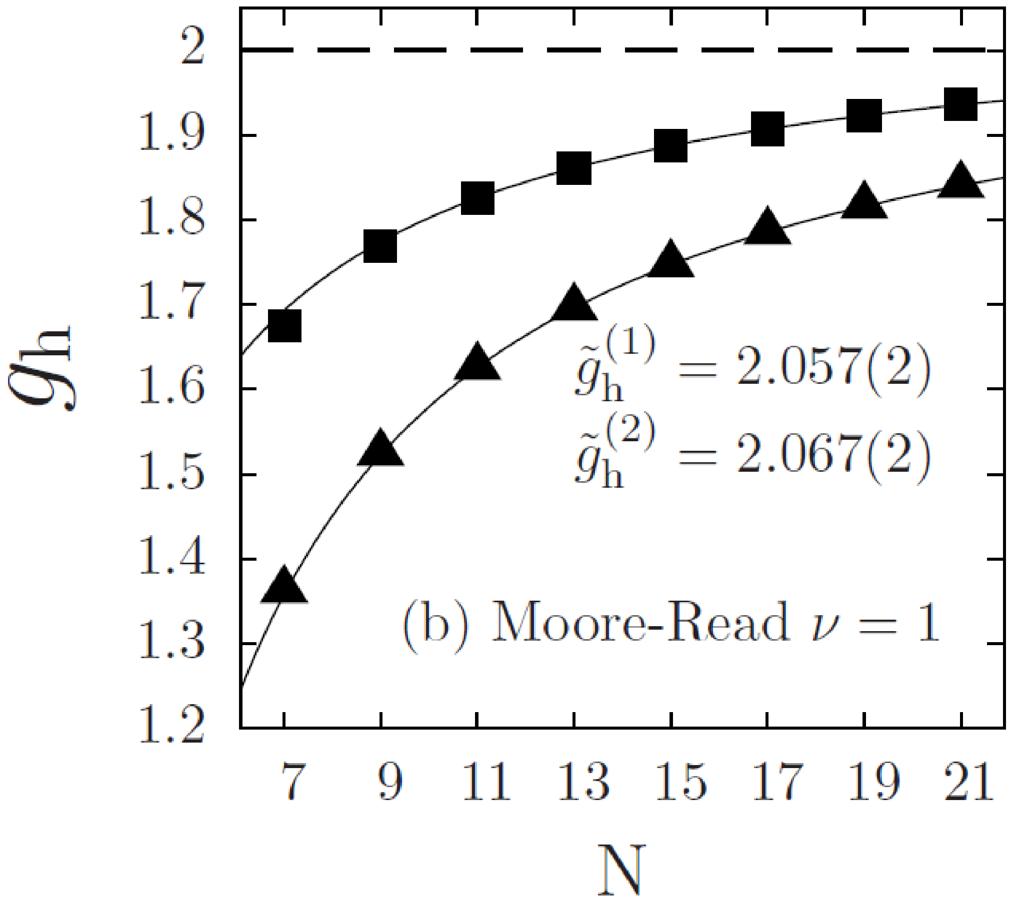
Laughlin State Hole/Particle



CFT: $g_h = 2$

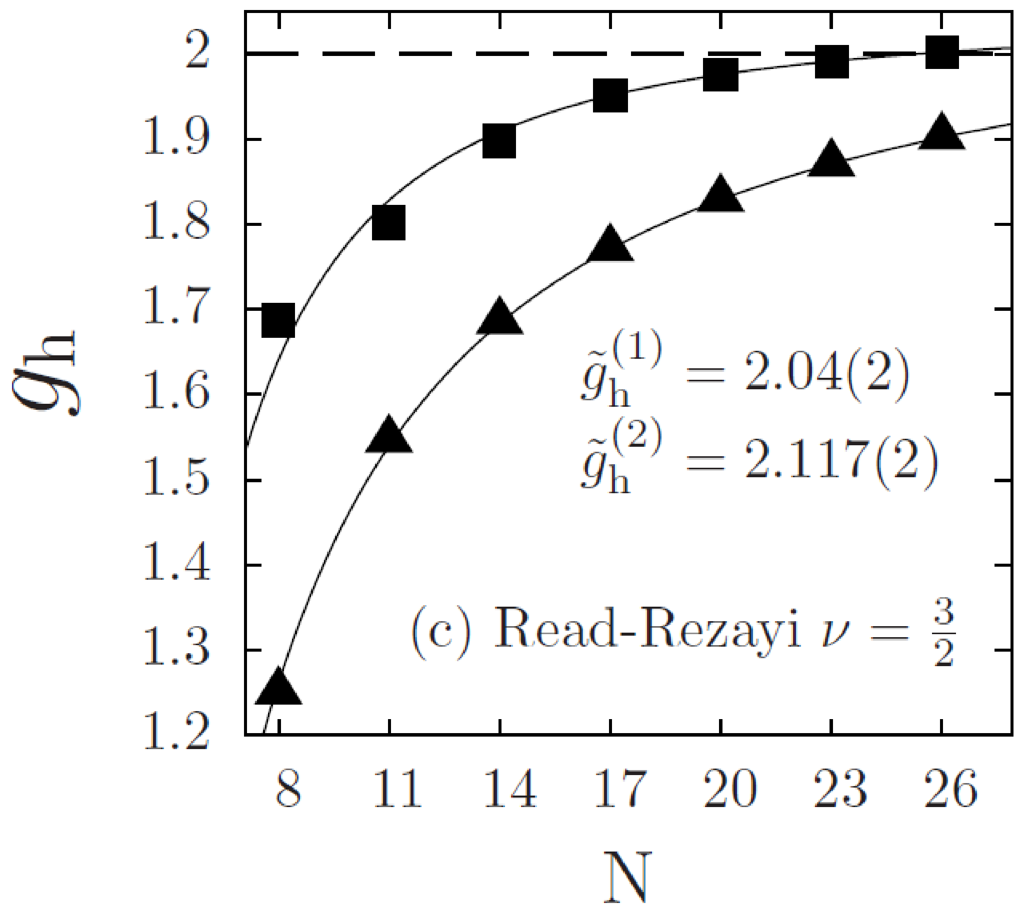
$$g_h^{(1)} = \frac{2}{1+\frac{1}{N}}$$

MR State Hole/Particle



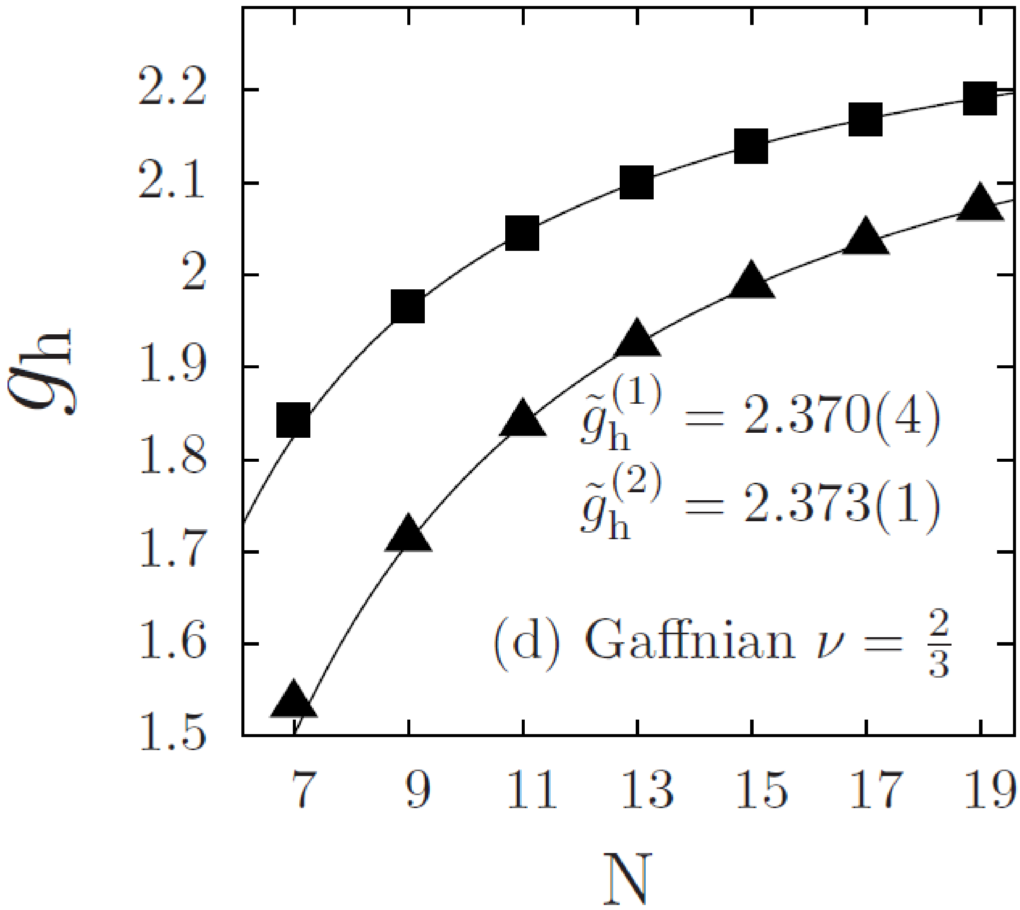
CFT: $g_h = 2$

RR State Hole/Particle



CFT: $g_h = 2$

Gf State Hole/Particle



CFT: $g_h = 3$

Converges, but to a nonsensical value!

Conclusion

- We have developed a new and general method to test candidate wavefunctions and extract the scaling exponents of their edge excitations.
- Confirms bulk-edge correspondence for MR!
- Not as good for RR, likely due to finite size.
- Exhibits the pathologies of the Gaffnian!
- Results for more states on the way…