Quantum Braiding Statistics and Quantum Hall Interferometry

Steven H. Simon

UNIVERSITY OF OXFORD

Funding: EPSRC Oxford Theory Grant and EPSRC TOPNES Grant

Outline

Fractional Braiding Statistics

Experiments in Fractional Quantum Hall Interferometry

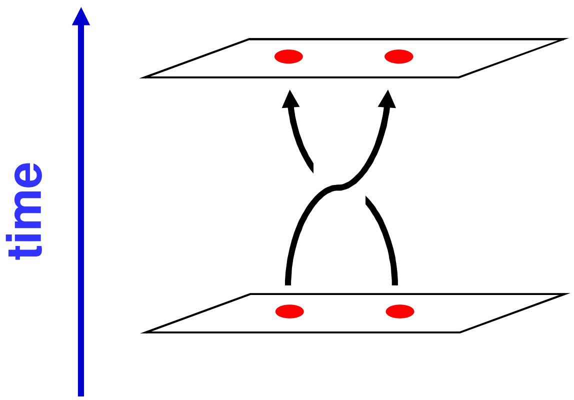
Nonabelian Braiding Statistics

(why we are so interested)

Same experiment but much more complicated

Leinaas+Myrheim 1977
Wilczek 1982

Fractional Statistics



W = Topological Winding Number of Braid

$$\Psi_f = e^{iW\alpha}\,\Psi_i$$

α = "Statistical Angle"

Bosons: $\alpha = 0$

Fermions: $\alpha = \pi$

Anyons: other α

Quasiparticle Excitations of "Simple" Fractional Quantum Hall States Really Are Anyons!

Halperin 1984
Schrieffer+Arovas+Wilczek 1984

Experimental Proof?

Braiding of Abelian and Non-Abelian Anyons in the Fractional Quantum Hall Effect

Sanghun An[1], P. Jiang[1,2], H. Choi[1], W. Kang[1], S.H. Simon[3], L.N. Pfeiffer[4], K.W. West[4], and K.W. Baldwin[4]

[1] *James Franck Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637*

[2] *Department of Physics, National Taiwan Normal University, Taipei, Taiwan*

[3] *Rudolf Peierls Centre for Theoretical Physics, 1 Keble Road, Oxford University, OX1 3NP, UK*

[4] *Department of Electrical Engineering, Princeton University, Princeton, NJ 08544*

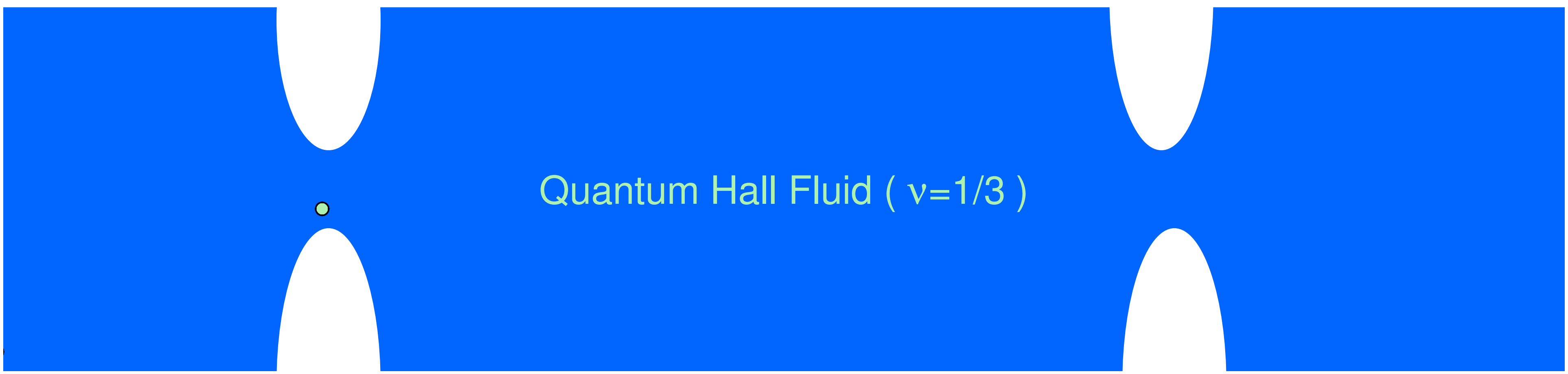
arXiv:1112.3400
Under Review at Science

The Quantum Hall Fabry-Perot Interferometer

Hoping to prove fractional statistics

Theory: Chamon, Wen, et al 1997 + many many others since

Experiment: Goldman Group; Willett Group; Kang Group; Marcus Group; Heiblum Group



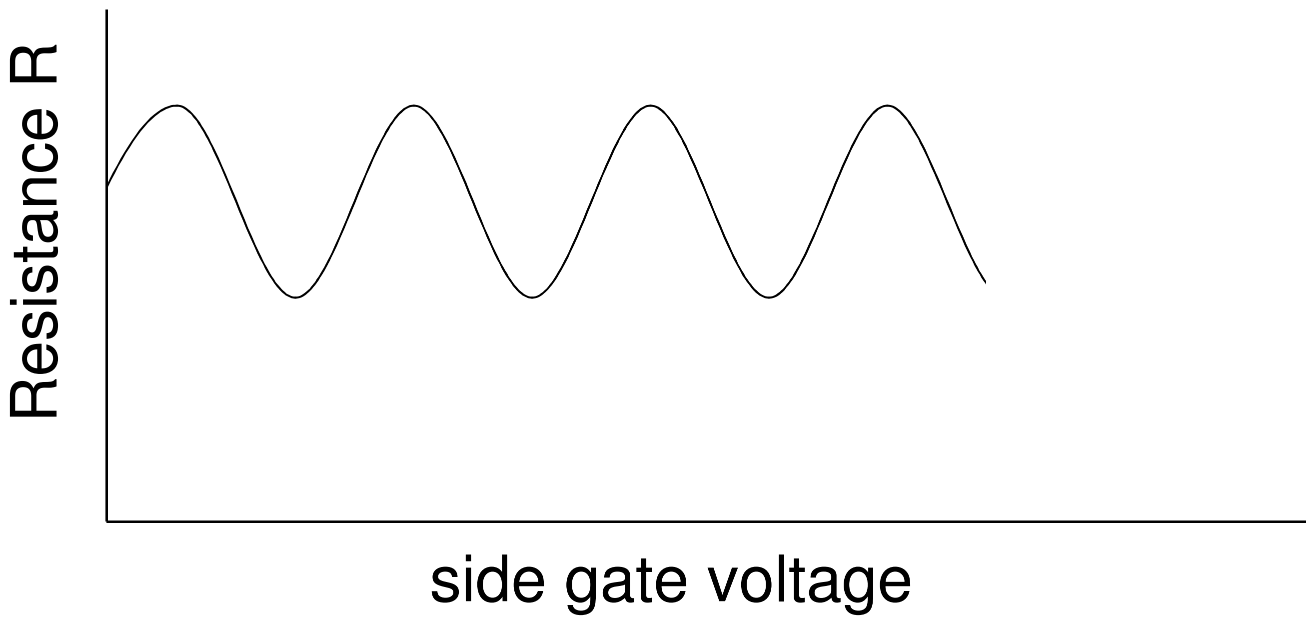
Beam Splitter

Mirror

interference of two partial waves

Conventional Quantum Hall States ($\nu=1/3$)

Side gate changes phase



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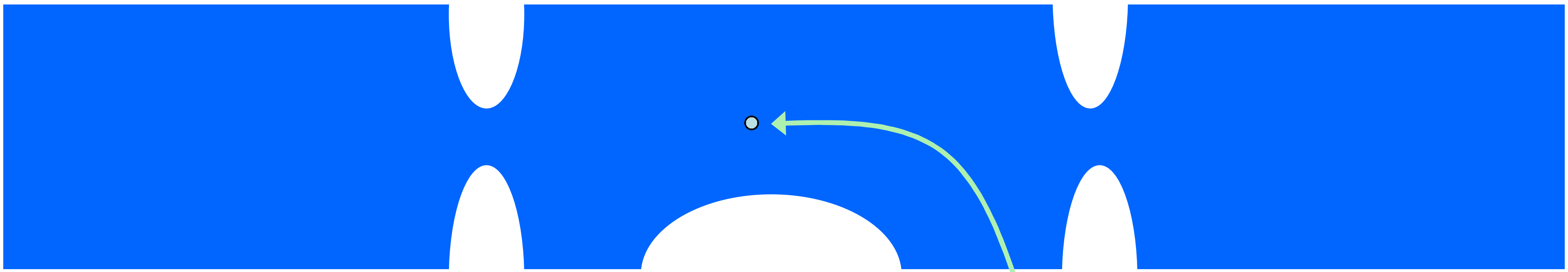
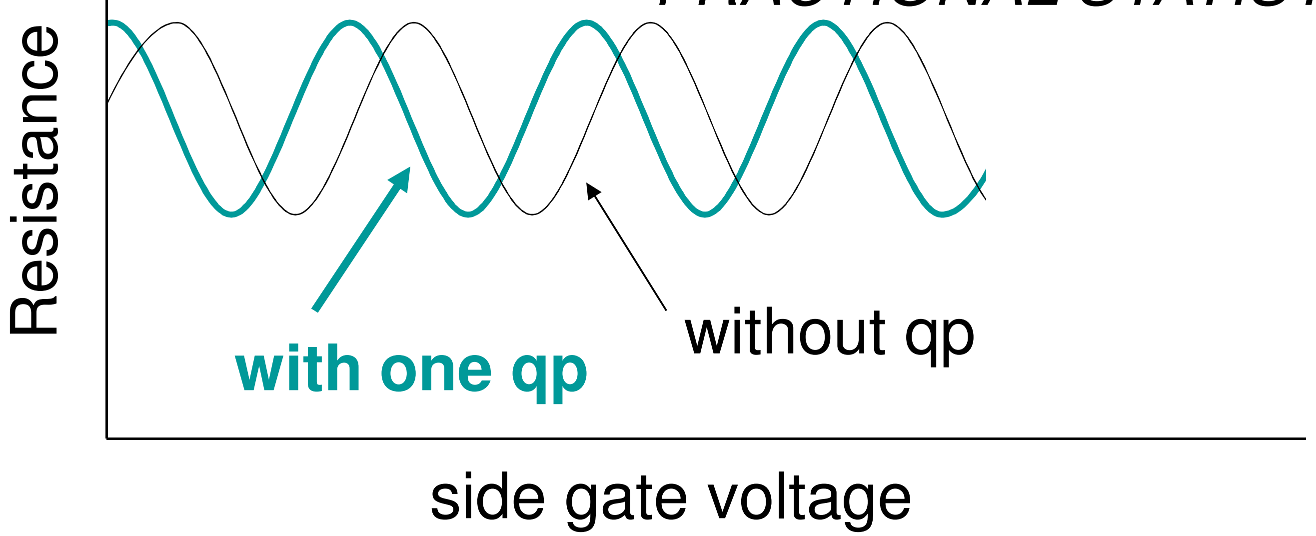


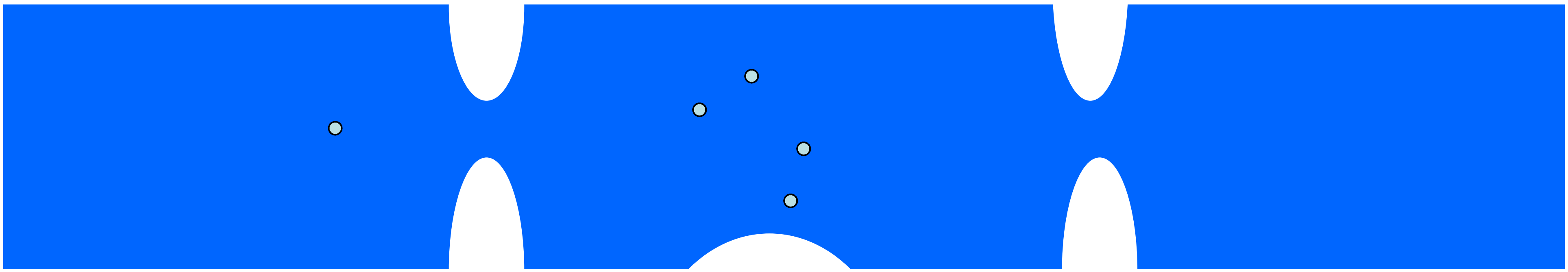
Adding 1 quasiparticle
shifts interference pattern by 2π / 3

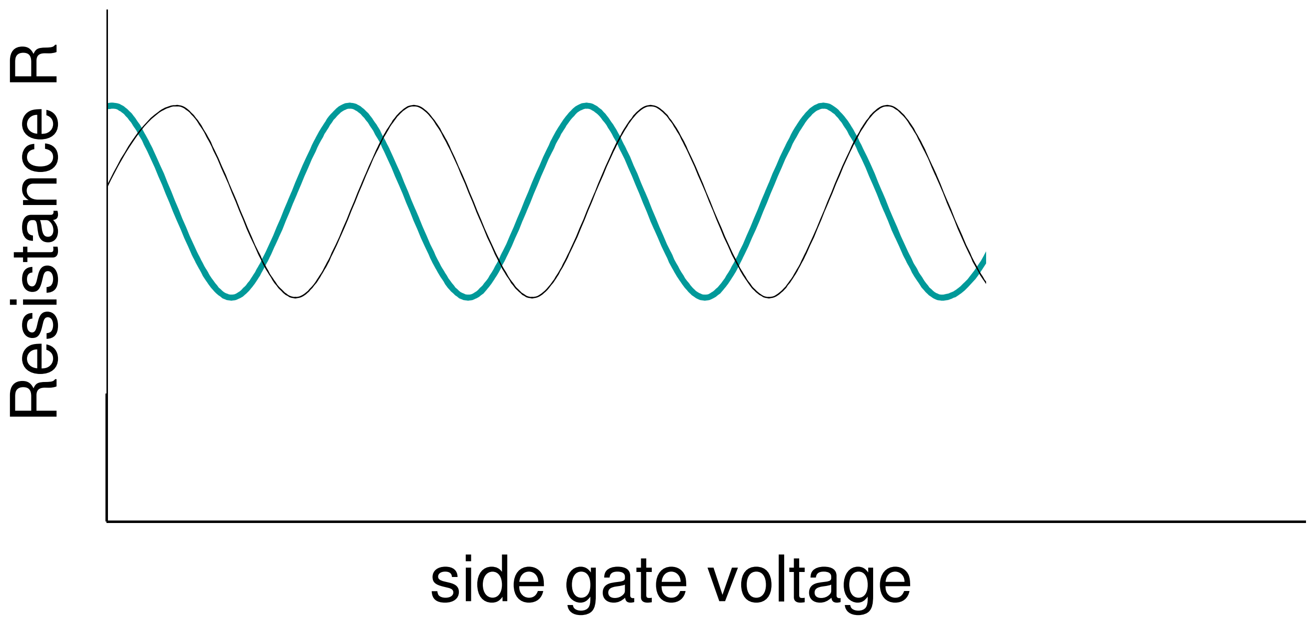
FRACTIONAL STATISTICS



Telegraph Noise

Slowish time scale = caused by glassy motion of dopant impurities





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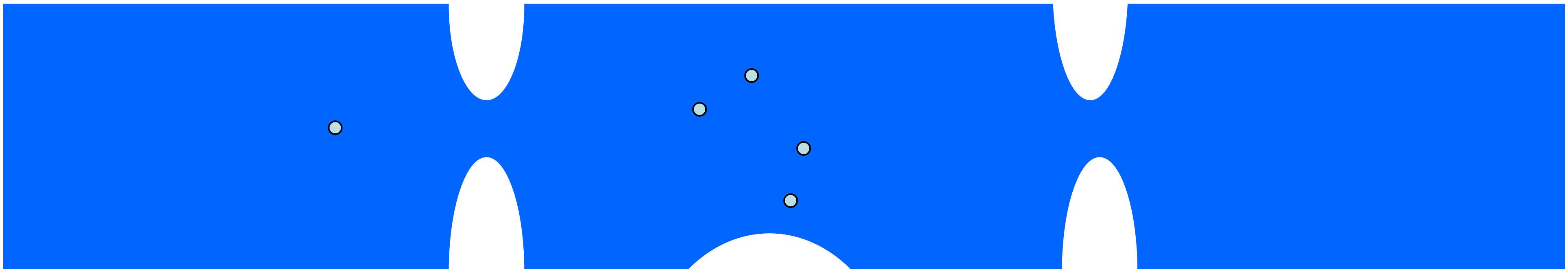
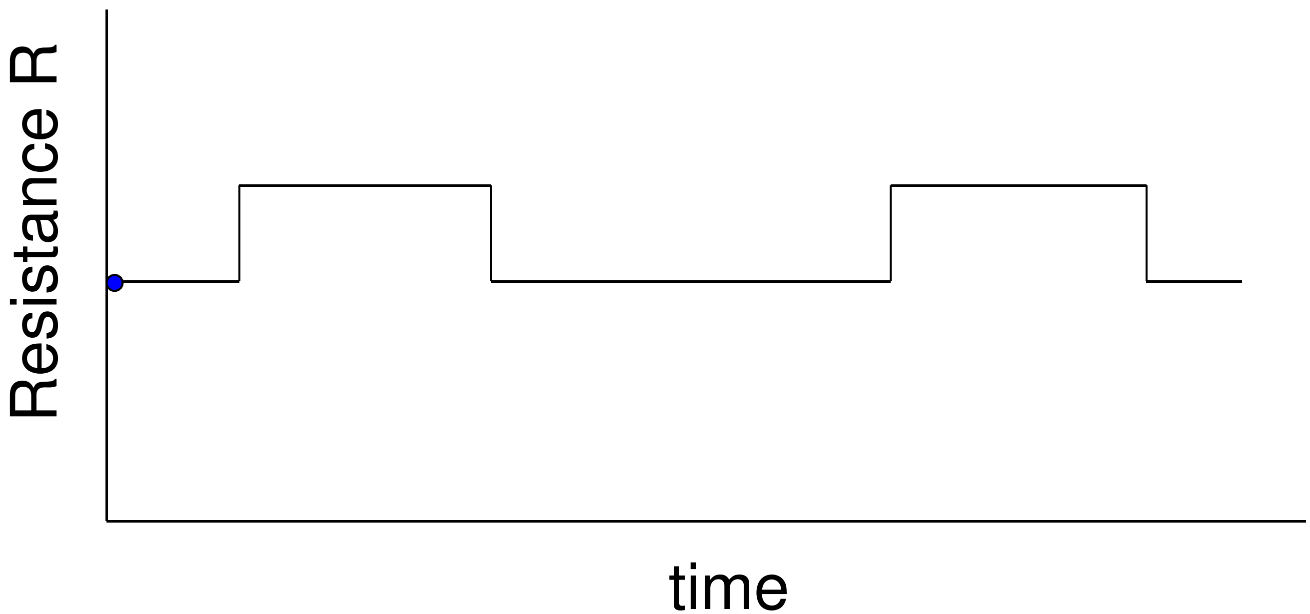
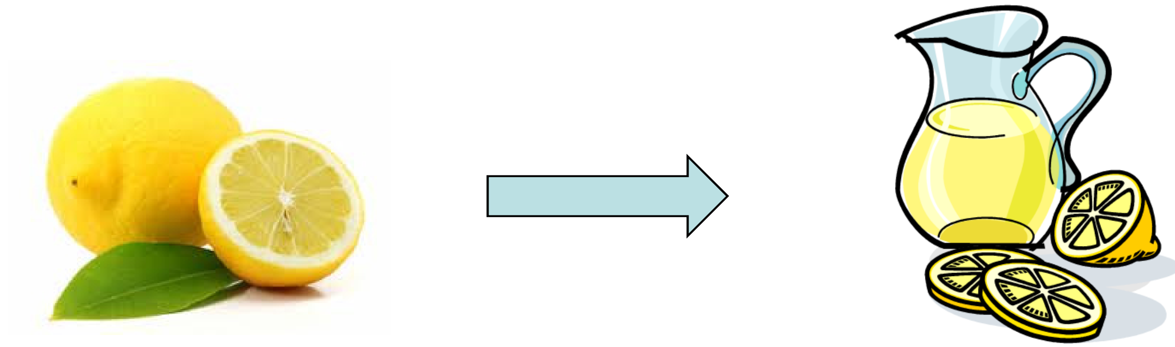


Fixed Side Gate Voltage



Telegraph Noise



PRL **96,** 226803 (2006) PHYSICAL REVIEW LETTERS week ending 9 JUNE 2006

Switching Noise as a Probe of Statistics in the Fractional Quantum Hall Effect

Eytan Grosfeld,[1] Steven H. Simon,[2] and Ady Stern[1]

Telegraph noise and the Fabry-Perot quantum Hall interferometer

B. Rosenow[1] and Steven H. Simon[2]

arXiv:1111.6475

Braiding of Abelian and Non-Abelian Anyons in the Fractional Quantum Hall Effect

Sanghun An[1], P. Jiang[1,2], H. Choi[1], W. Kang[1], S.H. Simon[3], L.N. Pfeiffer[4], K.W. West[4], and K.W. Baldwin[4]

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ν=7/3

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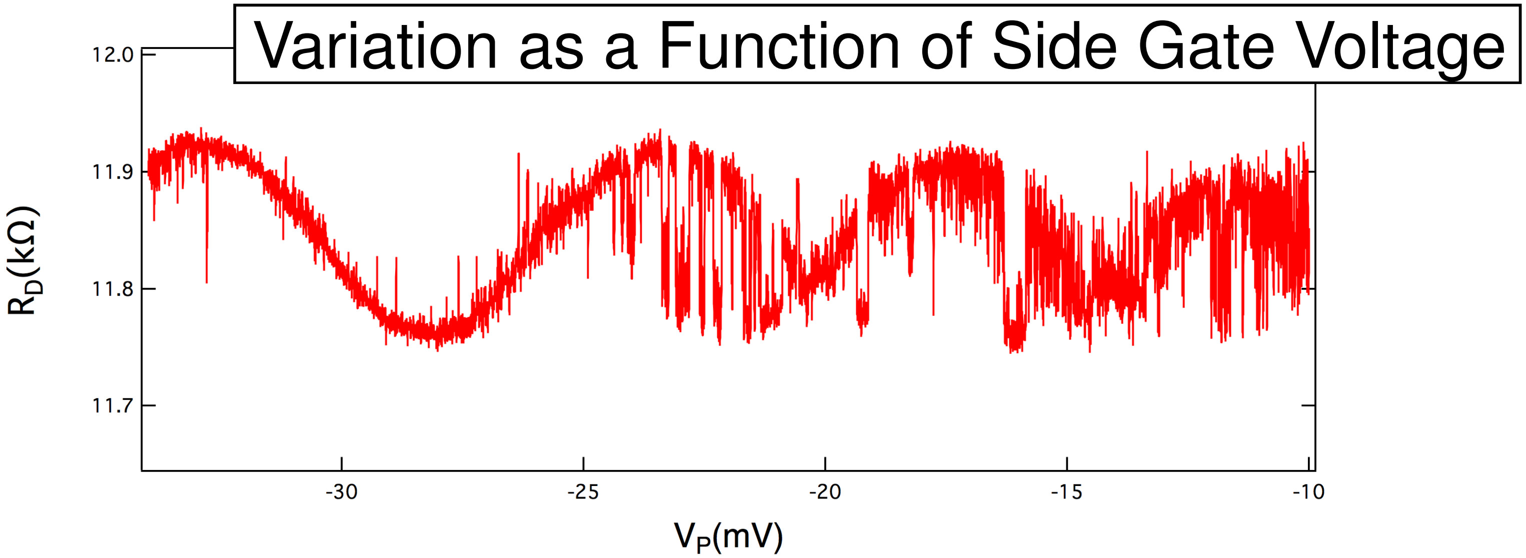

Telegraph noise seen at some ranges of side gate voltage

ν=7/3

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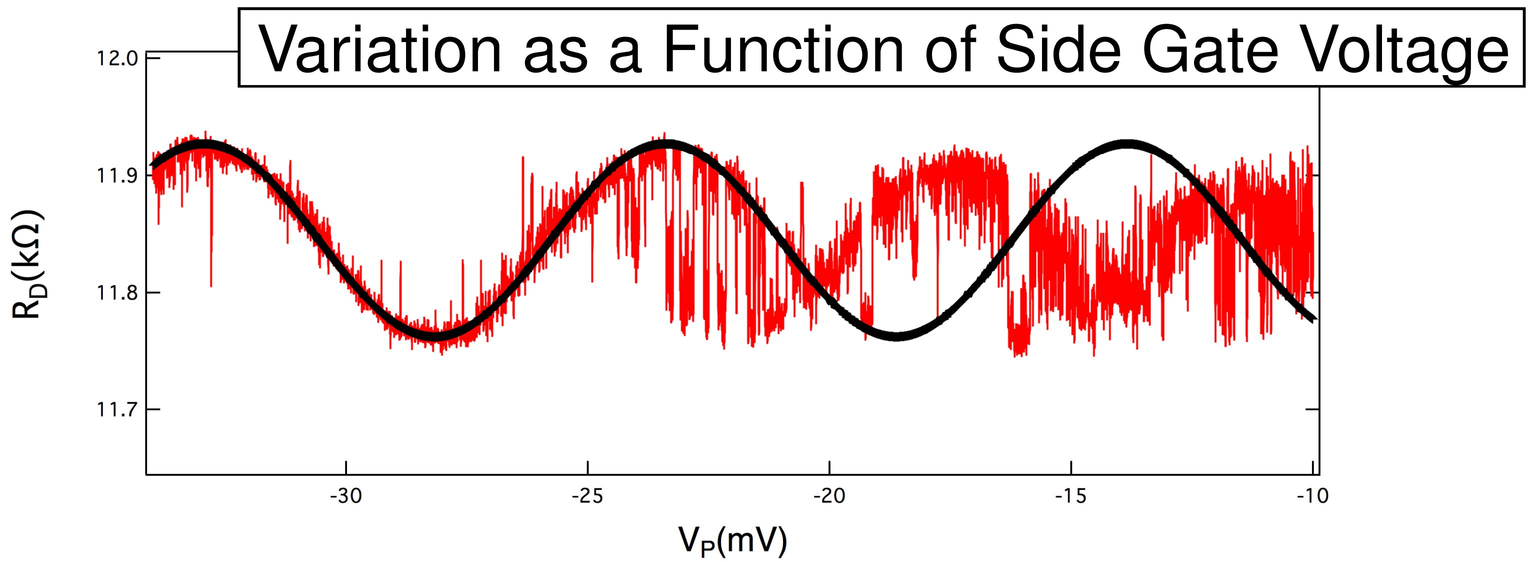

$$R_D = R_0 + A\cos\left(2\pi\frac{V_p}{\Delta V} + \phi_0\right)$$

ν=7/3

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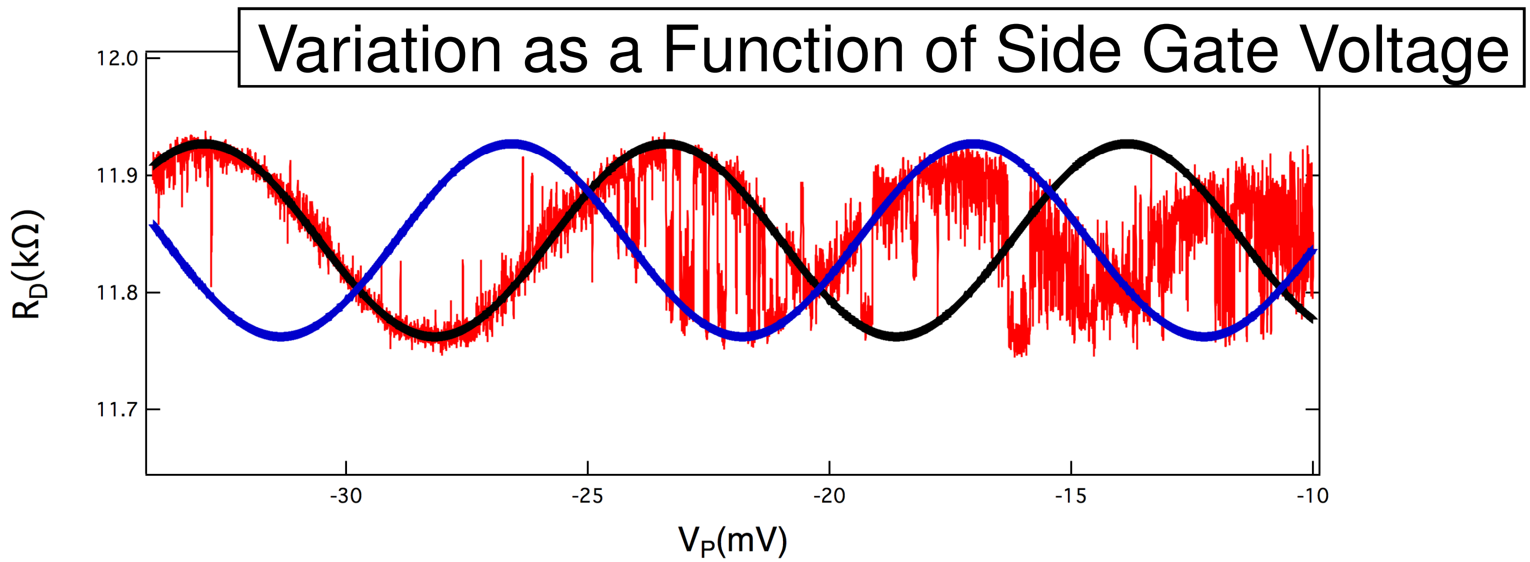

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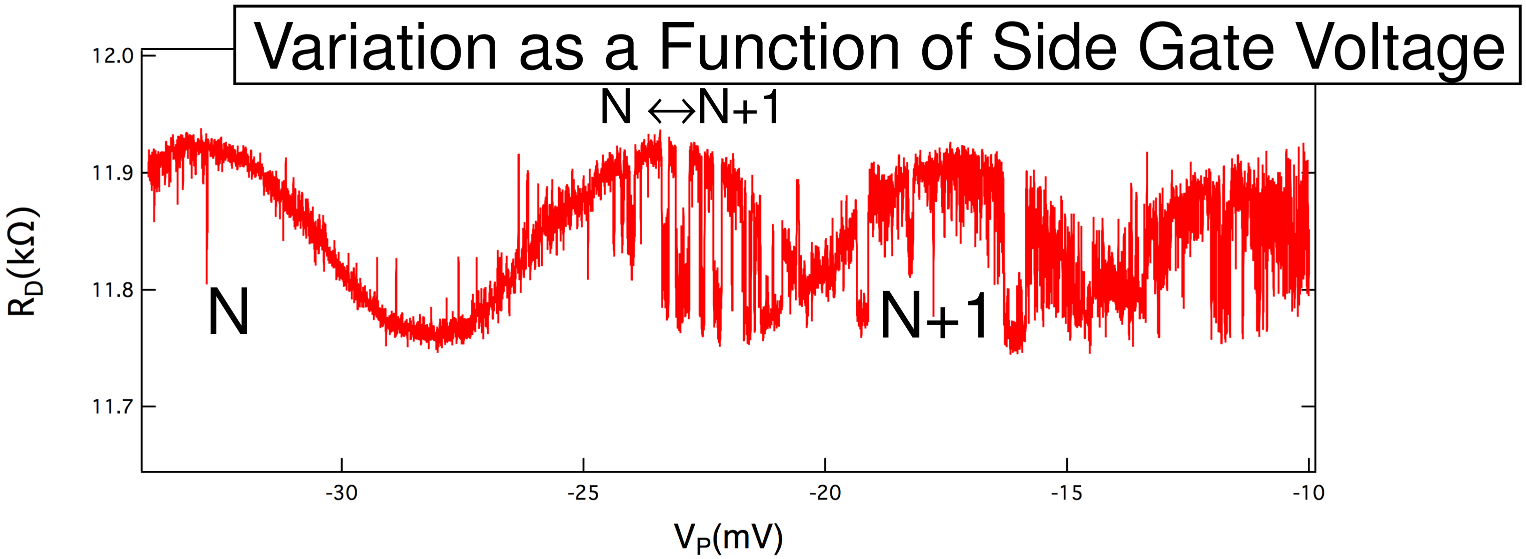

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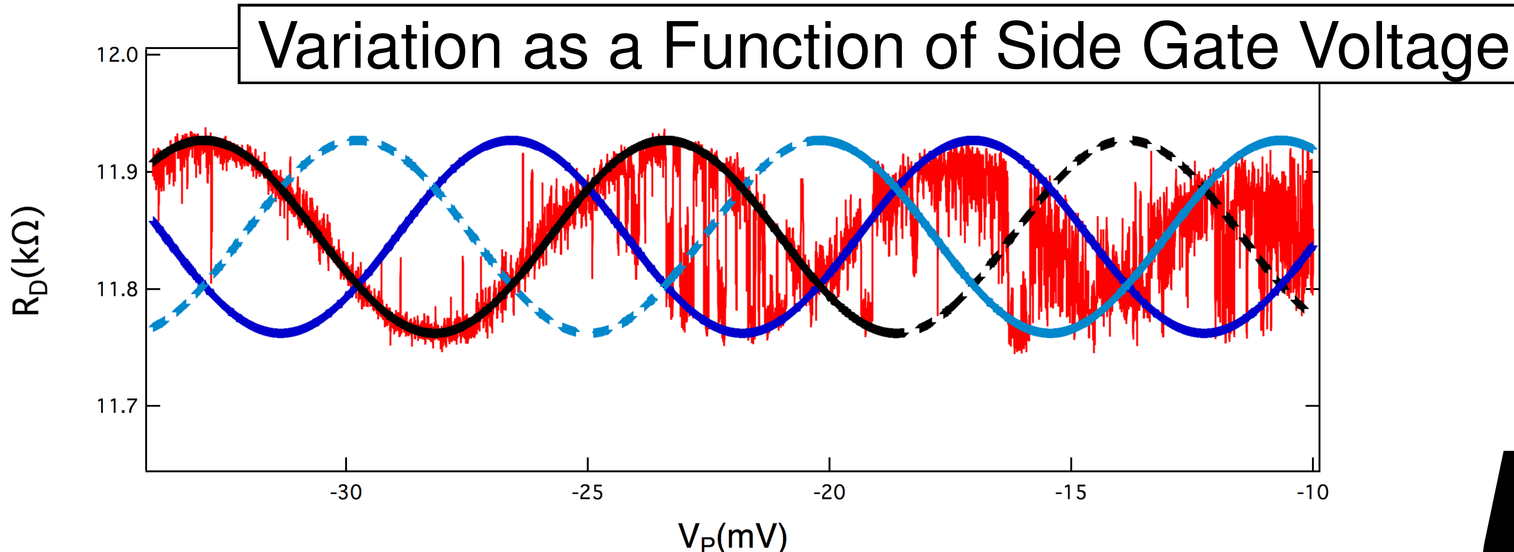

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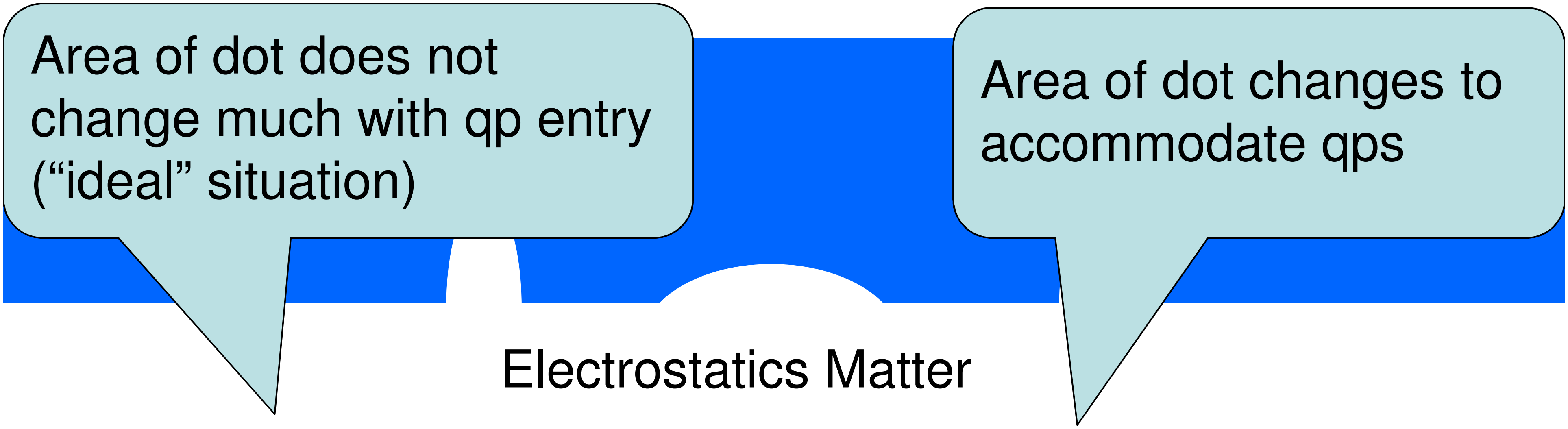
$$R_D = R_0 + A\cos\left(2\pi\frac{V_p}{\Delta V} + \phi_0 + \frac{4\pi}{3}\right)$$

!

Assuming Reproducible

Can a qp really jump in without deforming the “dot”



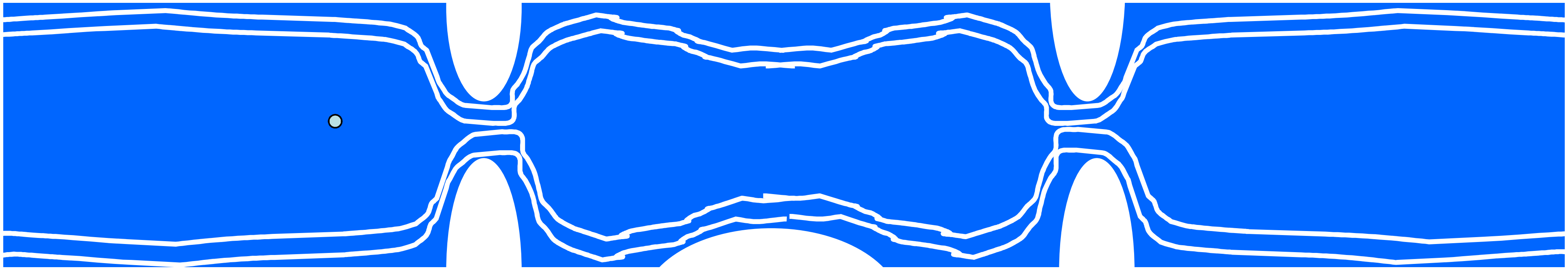
Electrostatics Matter

Aharonov-Bohm Regime vs. *Coulomb Dominated Regime*

Theory: Rosenow, Halperin 06; Halperin, Stern, Neder, Rosenow 10

Exp: Y. Zhang, **Marcus** et al 09; N. Ofek, **Heiblum** et al 10;
Godfrey, **Kang**, Simon, et al arXiv:0708.2448 ;
Choi, Jiang, Godfrey, **Kang**, Simon, … et al , New J. Phys, 2010.

Multiple Edge States



When qp enters, where does the charge leave from?

Can a qp really jump in without deforming the “dot”

Area of dot does not change much with qp entry (“ideal” situation)

Area of dot changes to accommodate qps

Electrostatics Matter

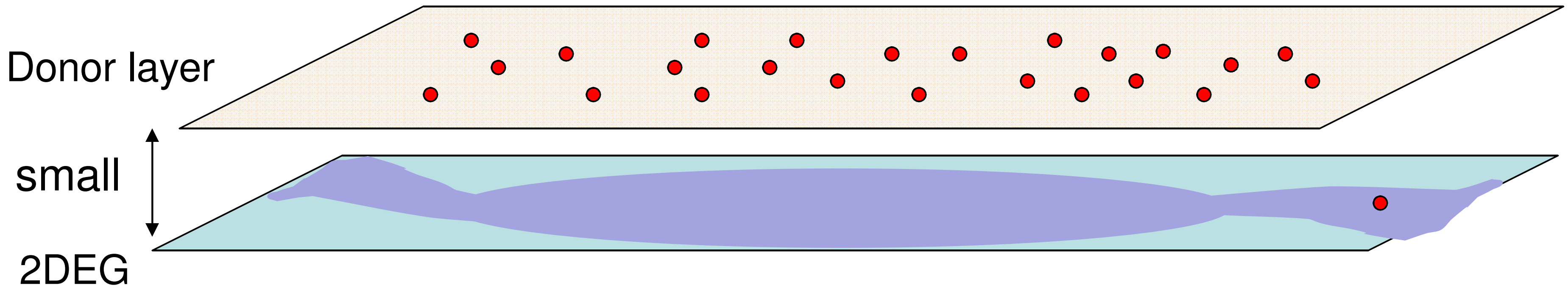
Aharonov-Bohm Regime vs. *Coulomb Dominated Regime*

Woowon’s Device shows signs of being Coulomb Dominated…
… so why do we measure the “ideal” phase slip?

- Those signs are wrong… OR
- Fine tuned : qp to edge interaction roughly canceled by dopant to edge interaction.

Rosenow, Simon arXiv:1111.6475, PRB 2012

Correlated Motion of Impurities and Quasiparticles



BUT… actually donor charge = 1
quasiparticle charge = 1/3

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Fractional Statistics

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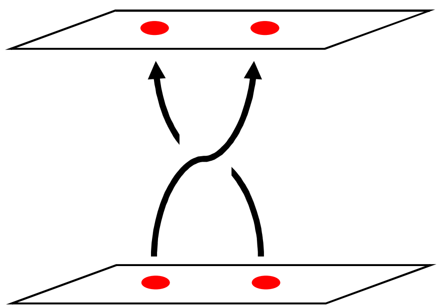
Wilczek 1982



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Winding Number of Braid

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Halperin 1984
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Experimental Proof?

2012 ?

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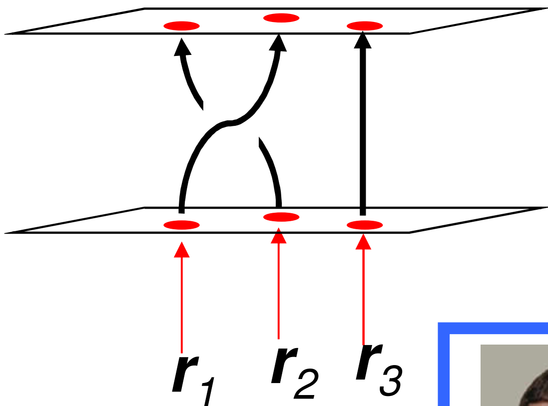
Nonabelian Braiding Statistics

(why we are so interested)

Same experiment but much more complicated

What if there is a multiply degenerate ground state?

Suppose 2 Degenerate Orthogonal States $|\psi_1\rangle$, $|\psi_2\rangle$



Vector Represents State

$$\Psi_i = a_1|\psi_1\rangle + a_2|\psi_2\rangle = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

$$\Psi_f = \tilde{a}_1|\psi_1\rangle + \tilde{a}_2|\psi_2\rangle = \begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix}$$

Makes a great Qubit !!

Kitaev. Freedman

Topological Quantum Computation

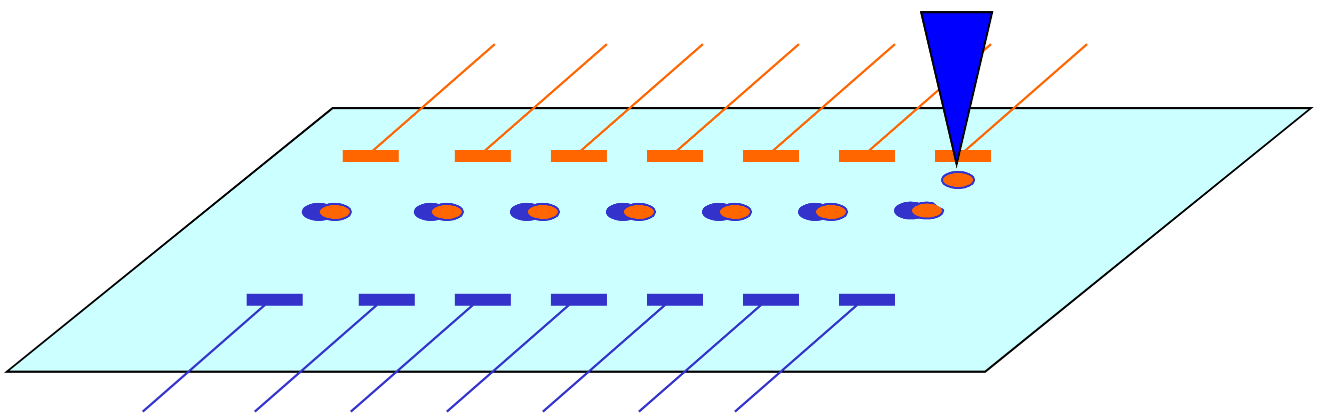
$$\begin{pmatrix} \tilde{a}_1 \\ \tilde{a}_2 \end{pmatrix} = U \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Unitary Matrix Represents Braiding Operation

“Non-Abelian” Statistics
(Matrices U don’t commute)

Froelich, Witten, Seiberg, Moore
TQFTs, 1980s
Moore+Read 1991

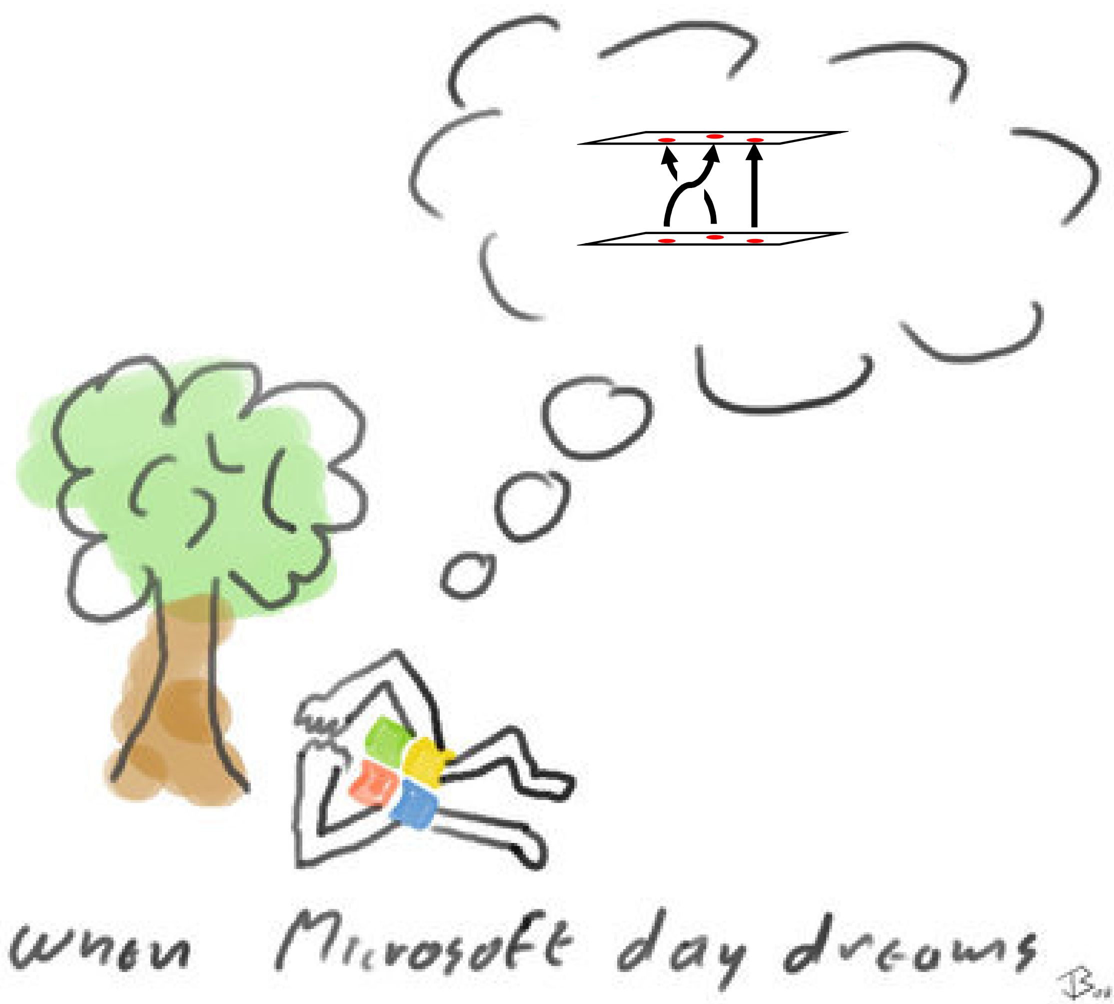
"Artist's conception" of a
Topological Quantum Computing Device



Ref: *Non-Abelian Anyons and Topological Quantum Computation*
C. Nayak, S.H. Simon, A. Stern, M. Freedman, S. DasSarma, Rev Mod Phys 2008

How close are we to achieving this dream?

First Step: Need to Show Non-Abelian Statistics Exist



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Same Experiment… but with quantum Hall state of matter

$\nu=5/2$

… a likely candidate for showing nonabelian statistics

arXiv:1112.3400
Under Review at Science

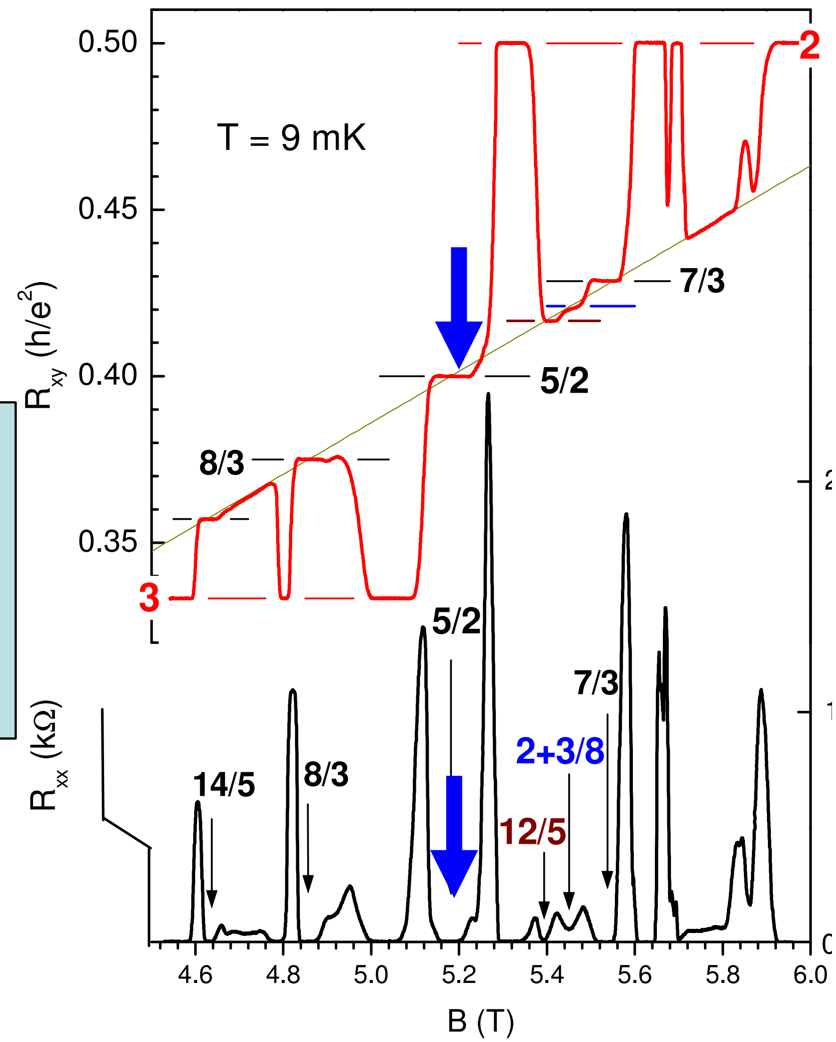
What phase of
matter is this plateau?

Numerics points strongly to Moore-Read Pfaffian
or it's particle-hole conjugate

(For our purposes these are essentially the same)

Can we see the effect of
nonabelian statistics in experiment?

Quantum Hall State
at ν=5/2 Exists



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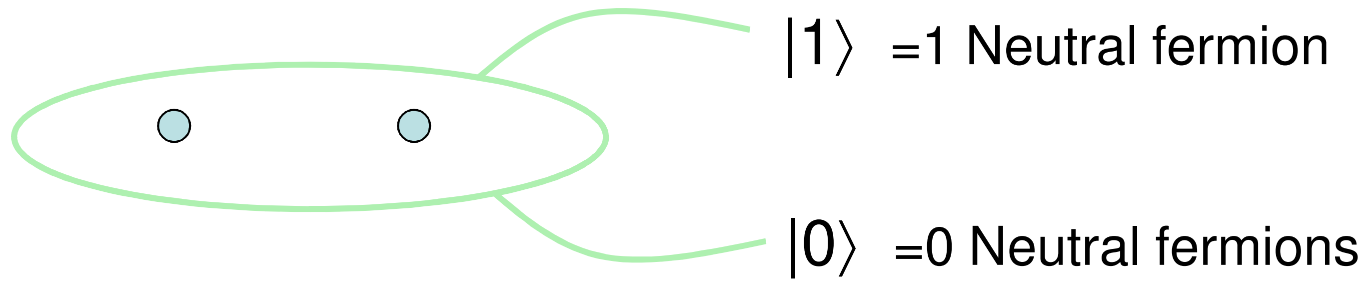
The Fundamental Principles of 5/2 Nonabelions



- For each *pair* of e/4 qps there is a single *two state system*.
  called: a “neutral (dirac) fermion” or a “qubit”

  (i.e, each qp associated with a majorana)

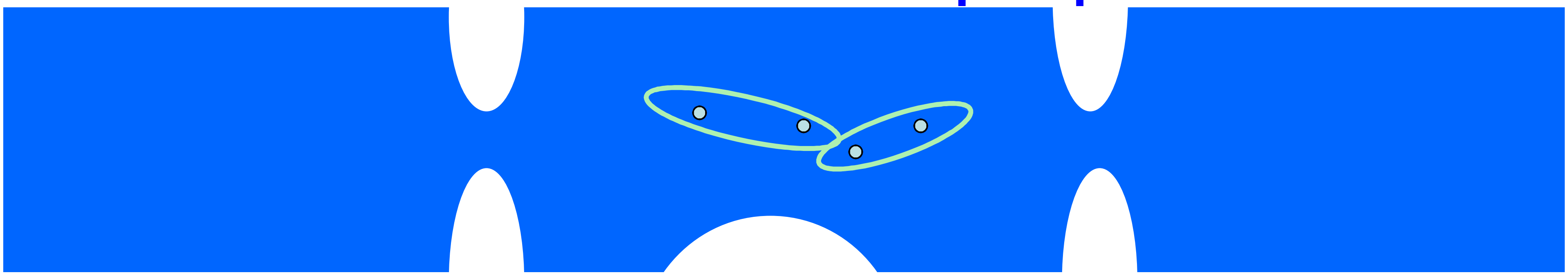
- Braiding a third qp through the two flips the state of the qubit

- A phase of π is accumulated going around a neutral fermion

5/2 state interference experiment

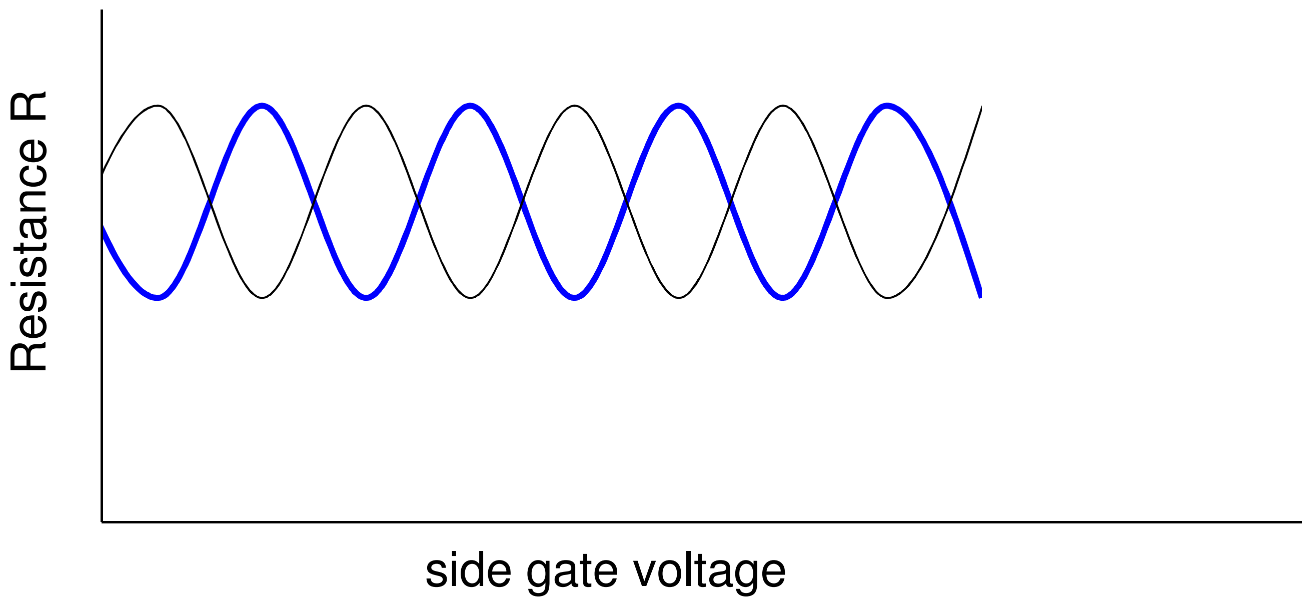
Fradkin et al 98; Stern, Halperin 06; Bonderson, Shtengel, Kitaev 06; Das Sarma, Nayak, Freedman 05

With even number of quasiparticles



Can get π phase shift

Depending on even/odd neutral fermions



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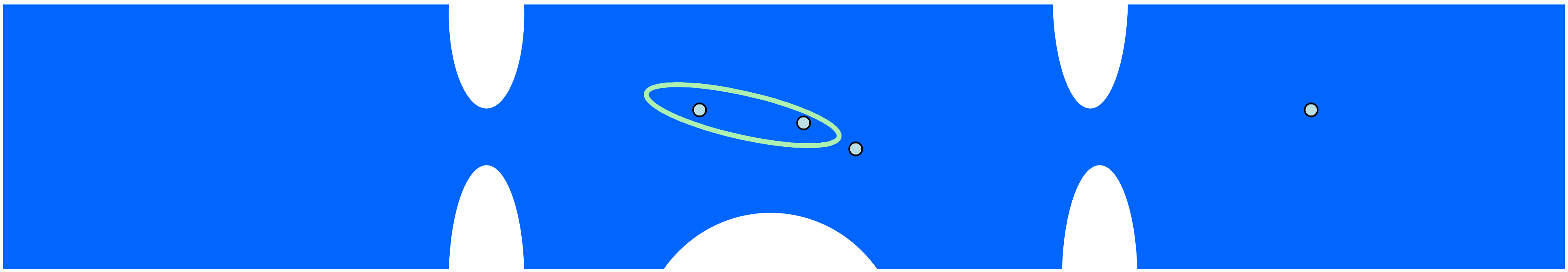
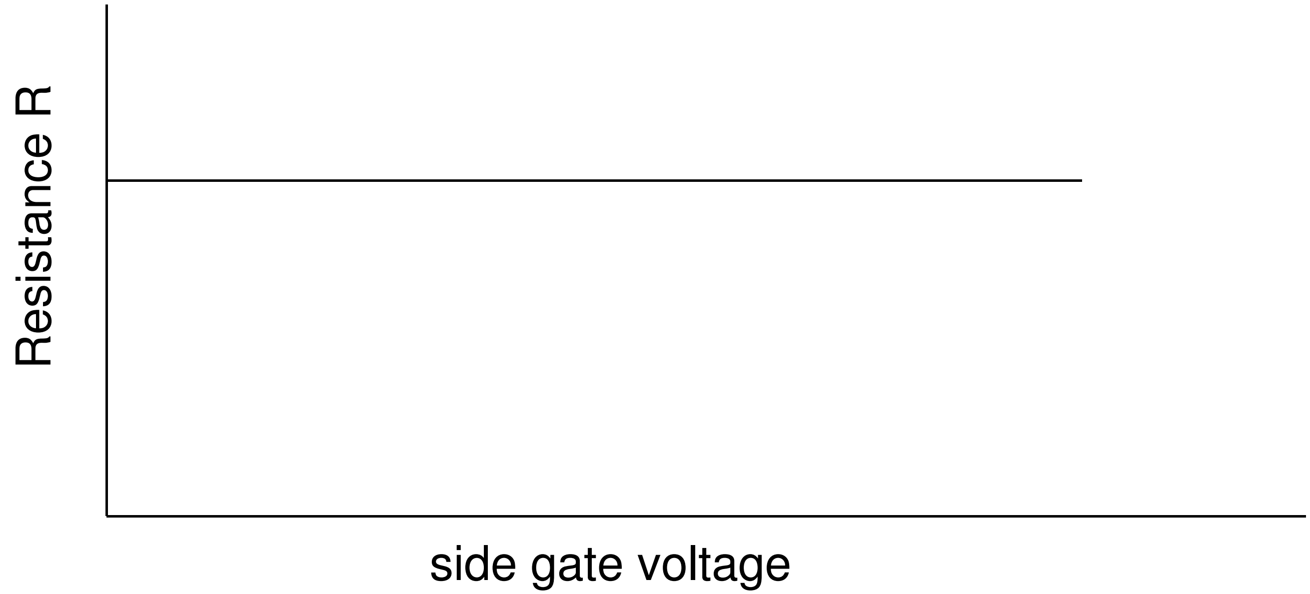
With odd number of quasiparticles



No Interference!



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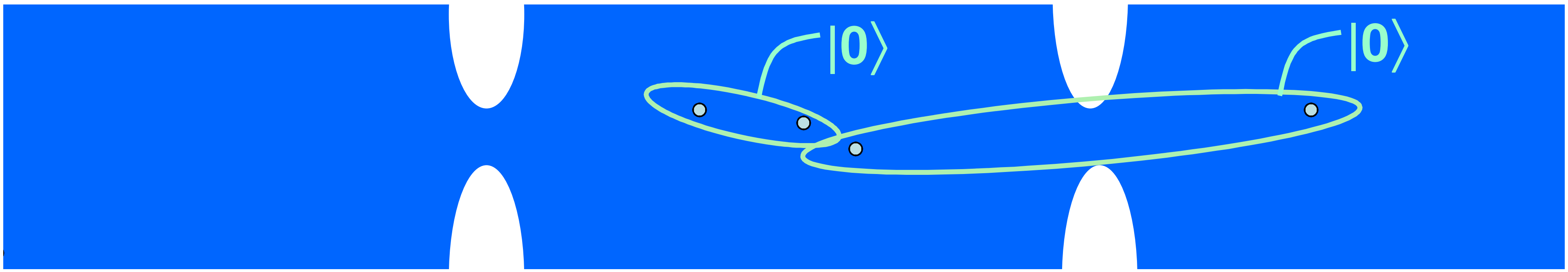
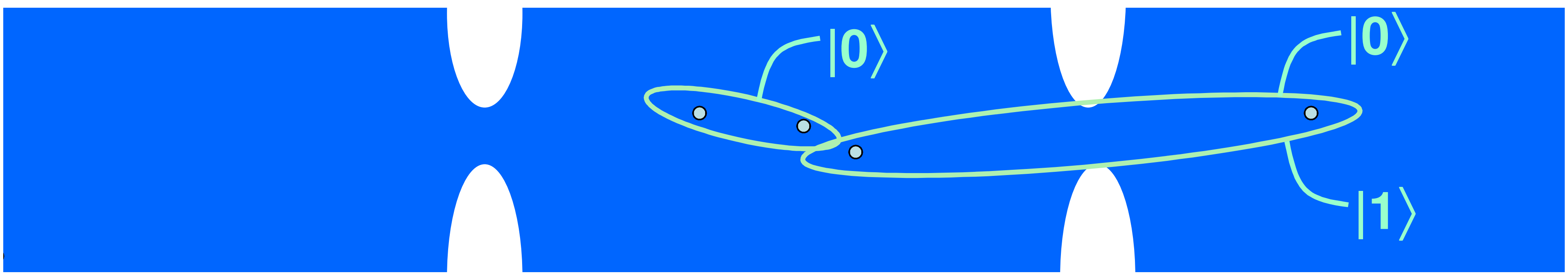


1st partial wave



2nd partial wave

Partial Waves are Orthogonal ⇒ No Interference!

5/2 state interference experiment

Summary of Orthodox Theory:

- If an odd # of qps are in the interferometer, no interference
- If an even # of qps are in the interferometer, yes interference

Phase = 0 if even # of neutral fermions
Phase = π if odd # of neutral fermions

Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW B **82**, 205301 (2010)

Alternation and interchange of e/4 and e/2 period interference oscillations consistent with filling factor 5/2 non-Abelian quasiparticles

R. L. Willett,* L. N. Pfeiffer, and K. W. West

Claims of experimental data in agreement with Orthodox Theory

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Selected for a Viewpoint in *Physics*

PHYSICAL REVIEW B **80**, 155303 (2009)

Interferometric signature of non-Abelian anyons

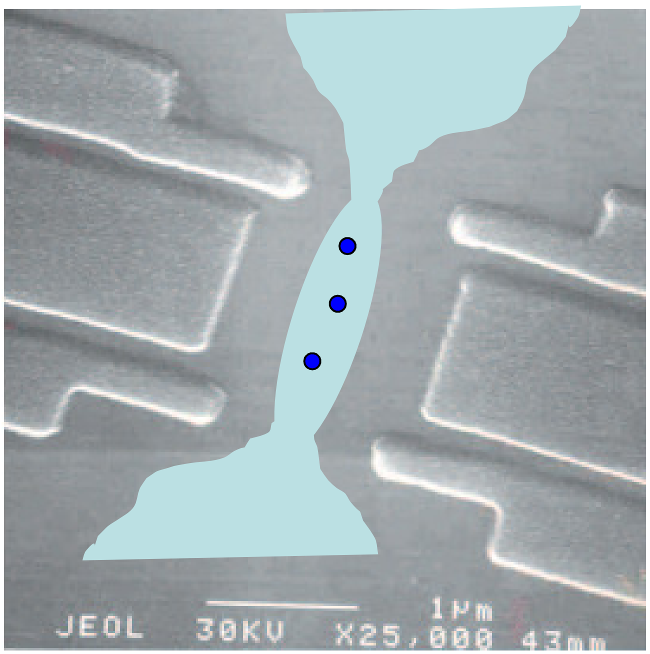
Waheb Bishara,[1] Parsa Bonderson,[2] Chetan Nayak,[2,3] Kirill Shtengel,[4,5] and J. K. Slingerland[6,7]

+ Theoretical Blessing

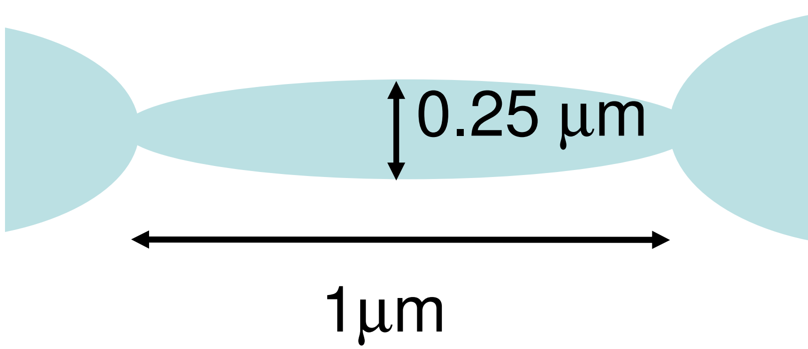
But then…. it turned out that

… the orthodox theory should not apply to this experiment *!*

R. L. Willett, L. N. Pfeiffer, and K. W. West
Phys. Rev. B **82**, 205301 (2010)



Area Estimate = 0.2 μm²



PROBLEM =
DEVICE IS SMALL….

qps (qubits) in the dot must be strongly coupled to each other, and to the edge… by majorana coupling !

VERY UNLIKE
ORTHODOX THEORY

Energy Scales

(1) T ≈ 20 mK

e*V [for 1n Amp current] ≈ .01 meV ≈ 100mK

(2) qp-edge majorana coupling

(3) qp-qp majorana coupling

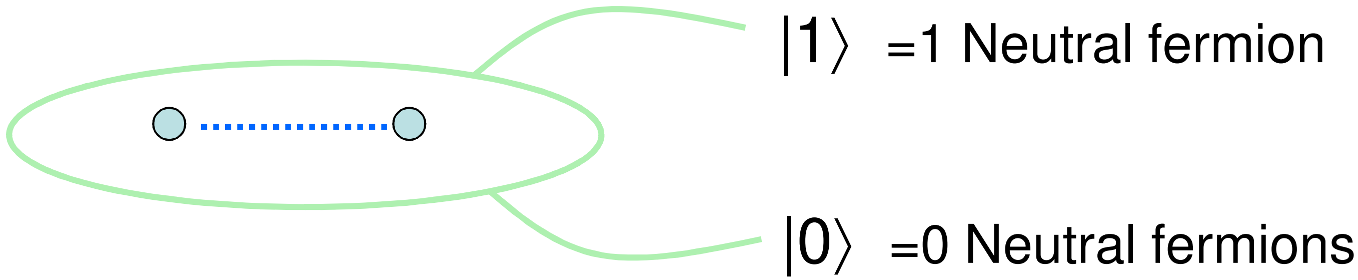
All Similar Order

What is this coupling?

How big is it?

What does it do?

The Fundamental Principles of 5/2 Nonabelions



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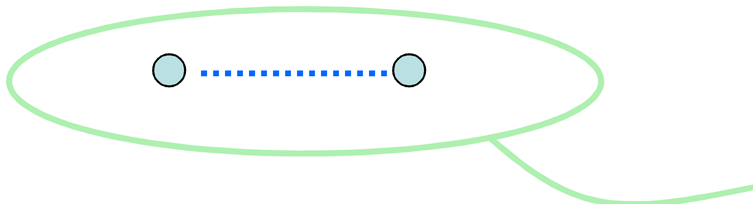
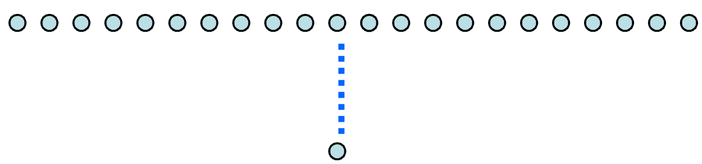
If qps are far apart, qubit states are degenerate in energy.

If qps are close together, majoranas couple – *splits qubit states*.

The Fundamental Principles of 5/2 Nonabelions





Similar coupling for qp-edge

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(2) qp-edge majorana coupling

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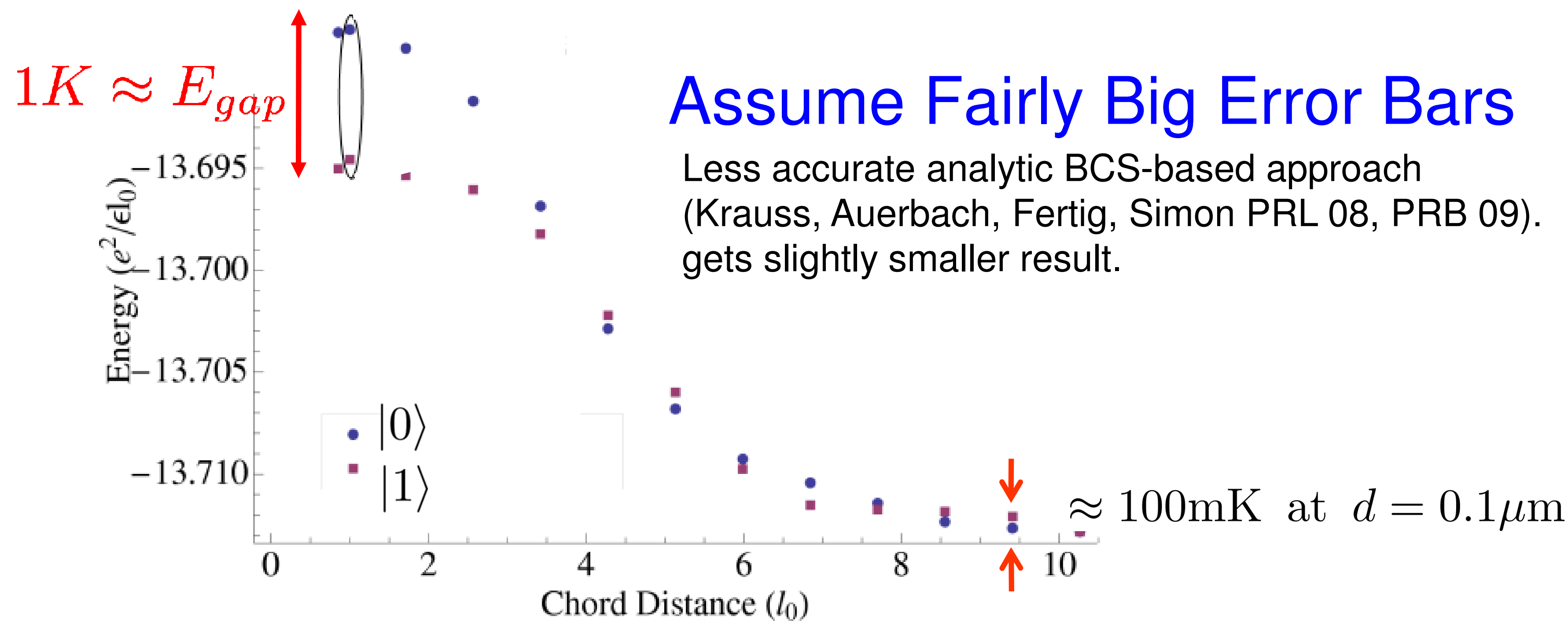
All Similar Order

What is this coupling?
How big is it?
What does it do?

Estimate from Trial Wavefunction Monte-Carlo for tunneling

Baraban, Zikos, Bonesteel, Simon, PRL 09

Two qps a distance d apart (4 qps in the calculation=2 fusion channels)



Distance between qps

Assume Fairly Big Error Bars

Less accurate analytic BCS-based approach (Krauss, Auerbach, Fertig, Simon PRL 08, PRB 09). gets slightly smaller result.

Energy Scales

(1) T ≈ 20 mK
e*V [for 1n Amp current] ≈ .01 meV ≈ 100mK

(2) qp-edge majorana coupling
allows qubit flipping !

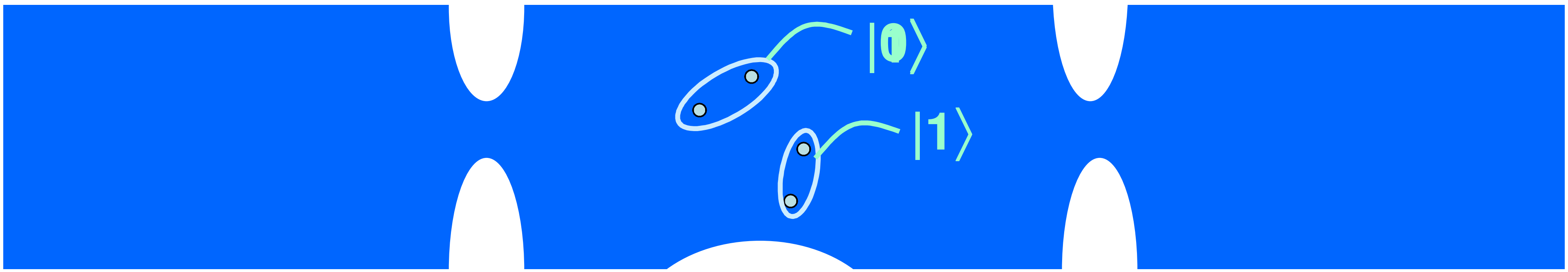
(3) qp-qp majorana coupling

All Similar Order

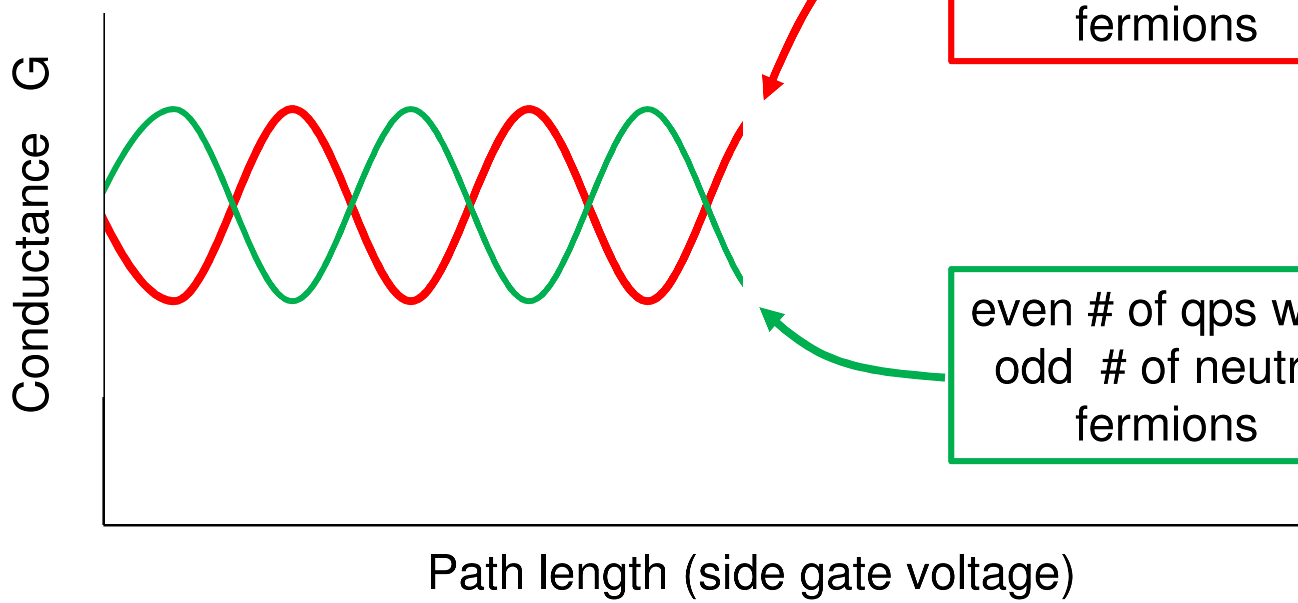
What is this coupling?
How big is it?
What does it do?

qp-edge coupling allows qubit flipping

Overbosch and Wen arXive 2008; Rosenow, Halperin, Simon, Stern PRL 2008, PRB 2009 ; Bishara and Nayak PRB 2009.



Can kill the interference signal!



even # of qps with even # of neutral fermions

even # of qps with odd # of neutral fermions

Energy Scales

(1) T ≈ 20 mK
e*V [for 1n Amp current] ≈ .01 meV ≈ 100mK

(2) qp-edge majorana coupling
allows qubit flipping !

(3) qp-qp majorana coupling ⬅
splits qubit states.
qubits can "freeze" into one of the two values

} All Similar Order

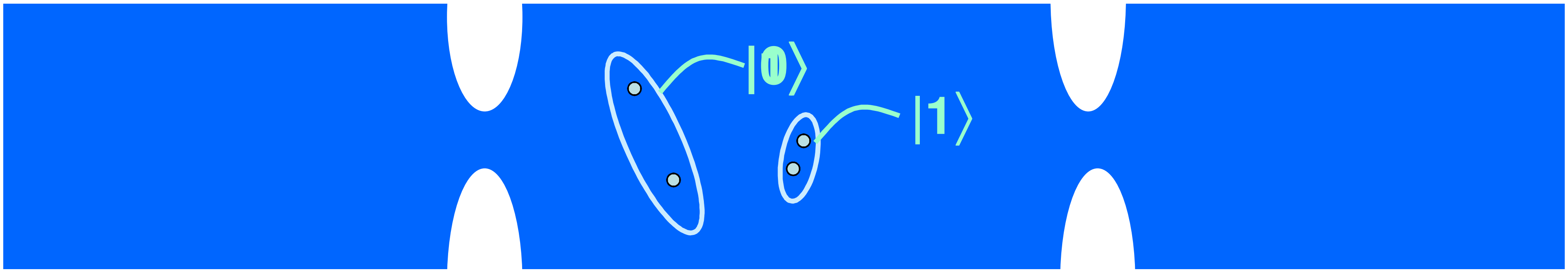
To observe interference we must assume 3 > 1,2

What do these couplings do?

Prediction 1

Expect π phase slips *!!*

– slips without adding a qp!



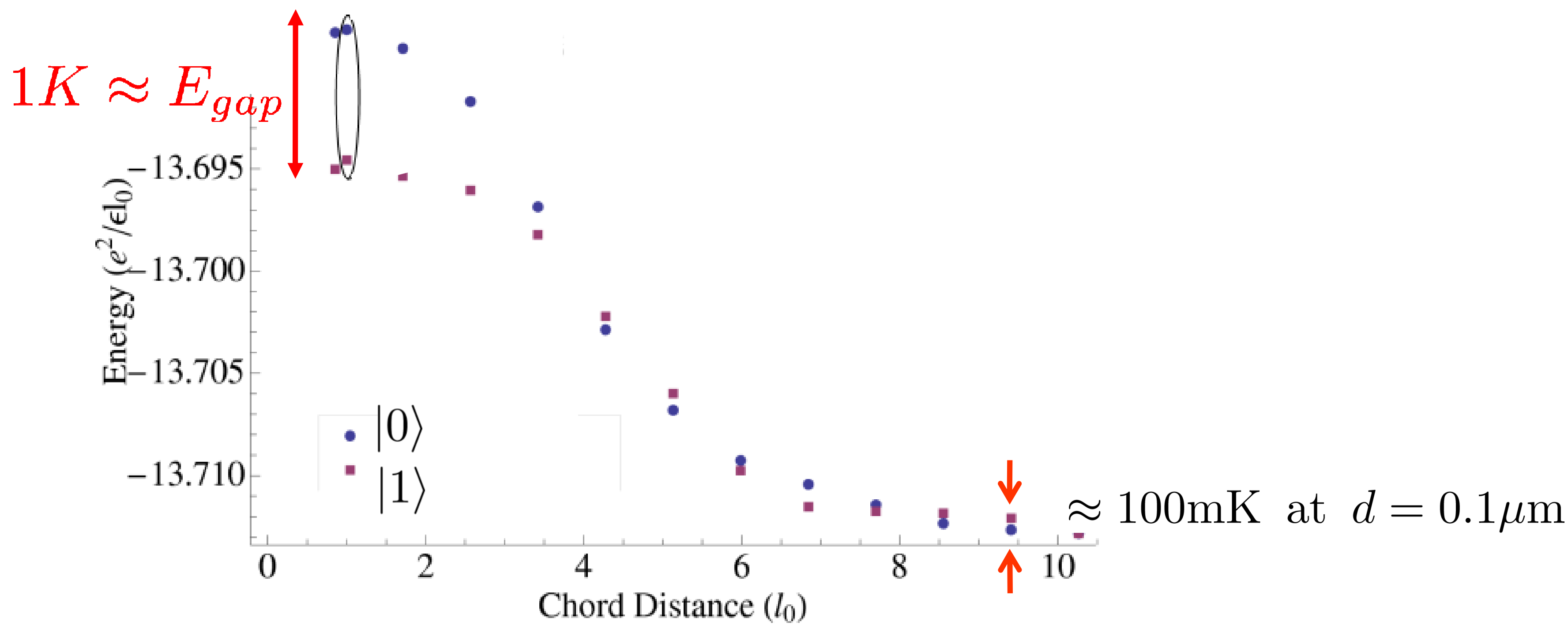
Which is lower energy (|0⟩ or |1⟩) depends on the detailed configuration of qps in the dot.

Interference signals can flip by π if a qp moves

Estimate from Trial Wavefunction Monte-Carlo for tunneling

Baraban, Zikos, Bonesteel, Simon, PRL 09

Two qps a distance d apart (4 qps in the calculation=2 fusion channels)



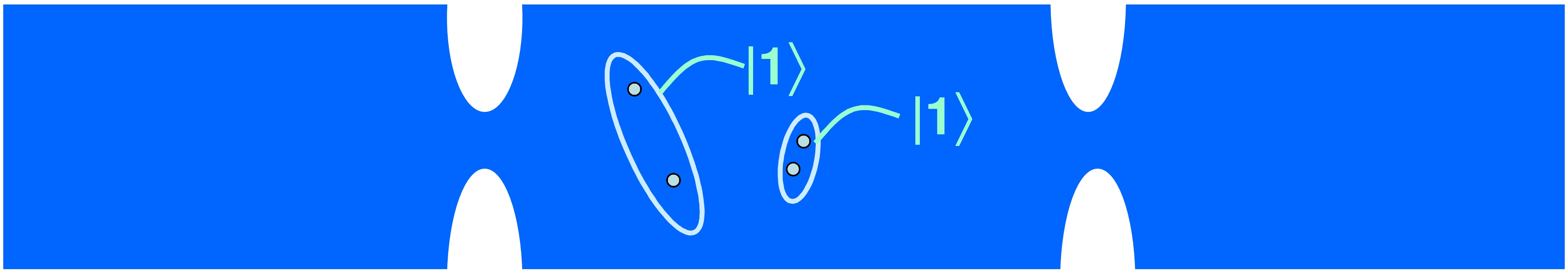
Which state of qubit is lower depends sensitively on distance!

(Friedel-like oscillations)

Prediction 1

Expect **π phase slips** *!!*

– slips without adding a qp!



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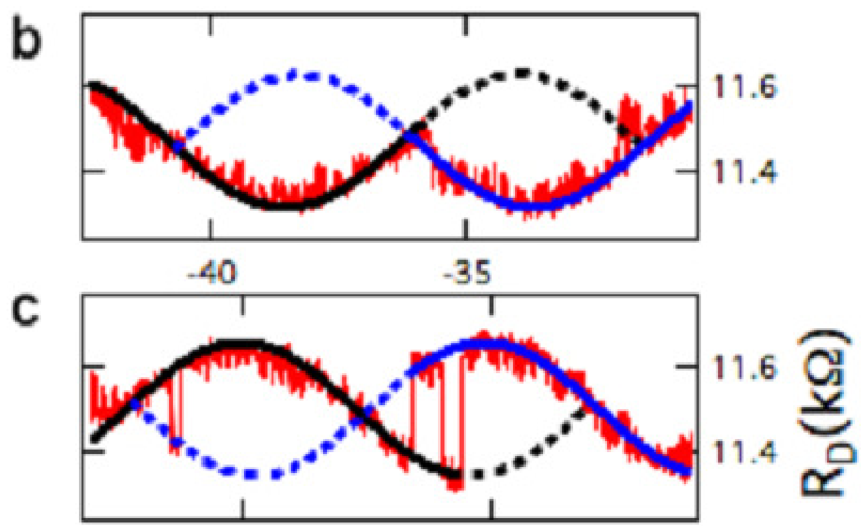
Braiding of Abelian and Non-Abelian Anyons in the Fractional Quantum Hall Effect

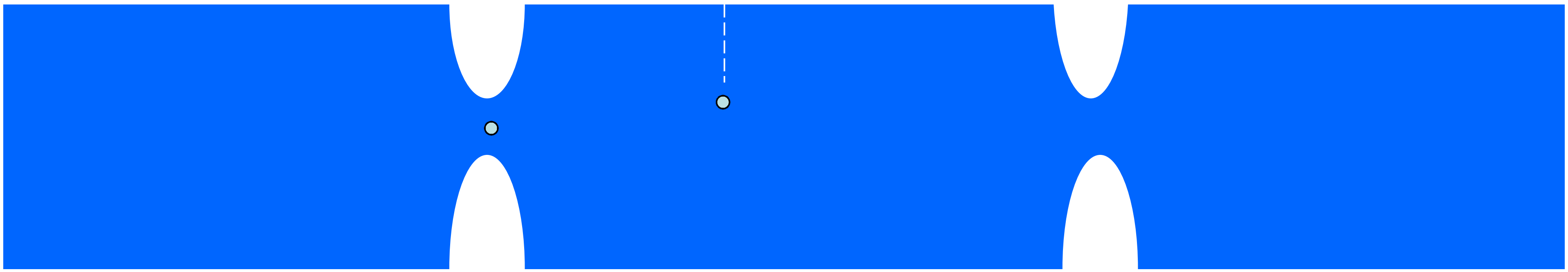


Evidence of Qubit Flipping!

What about the even-odd effect?

Overbosch and Wen arXive 2008; Rosenow, Halperin, Simon, Stern PRL 2008, PRB 2009 ; Bishara and Nayak PRB 2009.

For “odd” to kill interference, lone qp must be decoupled from edge



To see even-odd effect, must have

qp-edge coupling << e*V, T

If a majorana is coupled strongly to the edge,

it becomes part of the edge $\Rightarrow$ *Nothing encircles it*

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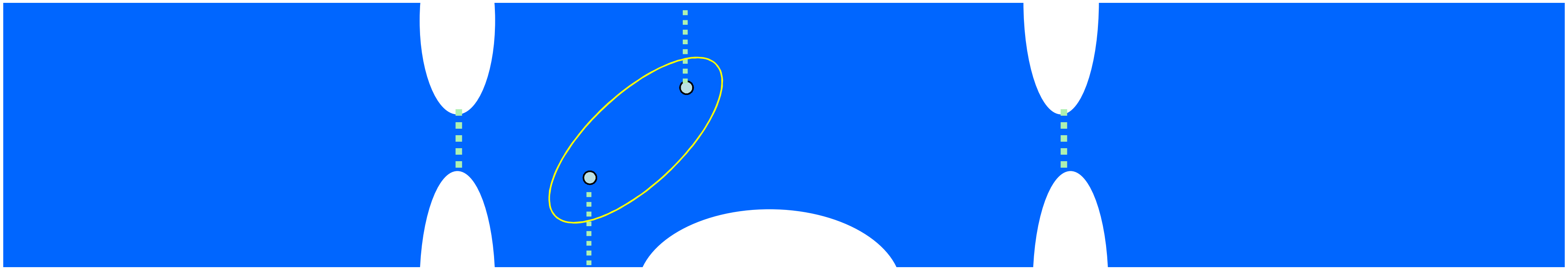
For “odd” to kill interference, lone qp must be decoupled from edge

A qp strongly coupled to the edge forms a bound state and decouples from the problem

Crossover behavior can be exactly calculated…



$$\mathcal{L}_{charge} = \frac{1}{4\pi\nu}\, \partial_x\varphi(v_c\partial_x \pm i\partial_\tau)\varphi$$ Edge charge mode (bosons)

$$\mathcal{L}_{neutral} = \psi(v_n\partial_x \pm \partial_t)\psi$$ Edge neutral (majorana) fermi mode

$$\mathcal{L}_{qps} = \Gamma_\alpha\, \partial_\tau \Gamma_\alpha$$ Vortex core (majorana) modes

$$\mathcal{L}_{edge-qp} = \lambda \Gamma_\alpha \psi(x_\alpha)$$ Edge to bulk coupling

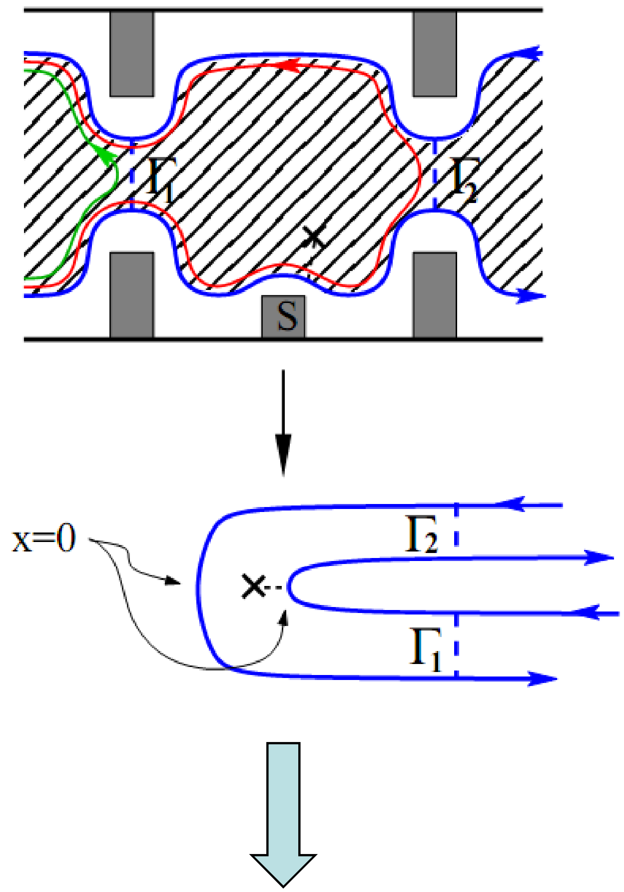
$$\hat{T}(x) = t\ \underbrace{\sigma_u(x)\sigma_d(x)}_{\text{Edge neutral fermion mode}}\ \underbrace{\left[e^{\frac{i}{\sqrt{8}}(\varphi_u - \varphi_d)} + h.c.\right]}_{\text{Moves charge across}}$$ Point Contacts

Moves charge across

Edge neutral fermion mode

Interference Term $= \mathrm{Re}\int_\tau e^{iV\tau}\ \langle \hat{T}_1(\tau)\ \hat{T}_2(0)\rangle$

Bishara-Nayak

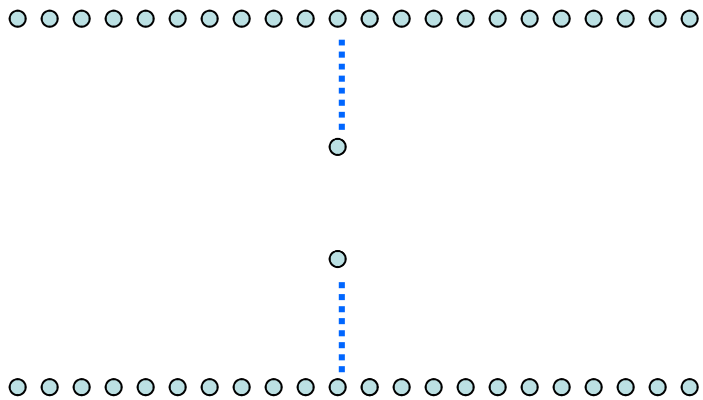


Ising CFT with Boundary

=

Rosenow, Halperin, Simon, Stern

Rewrite edge as continuum limit of coupled majoranas plus an impurity.



This becomes a quadratic 1d Hamiltonian (of Hilbert type) and with some (substantial) work, can be essentially solved.

What about the even-odd effect?

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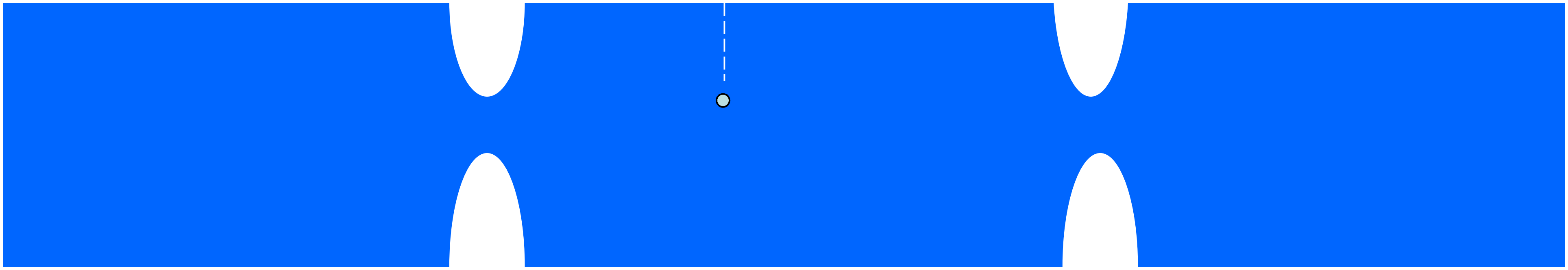

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All Similar Order

To observe interference we must assume $3 > 1,2$

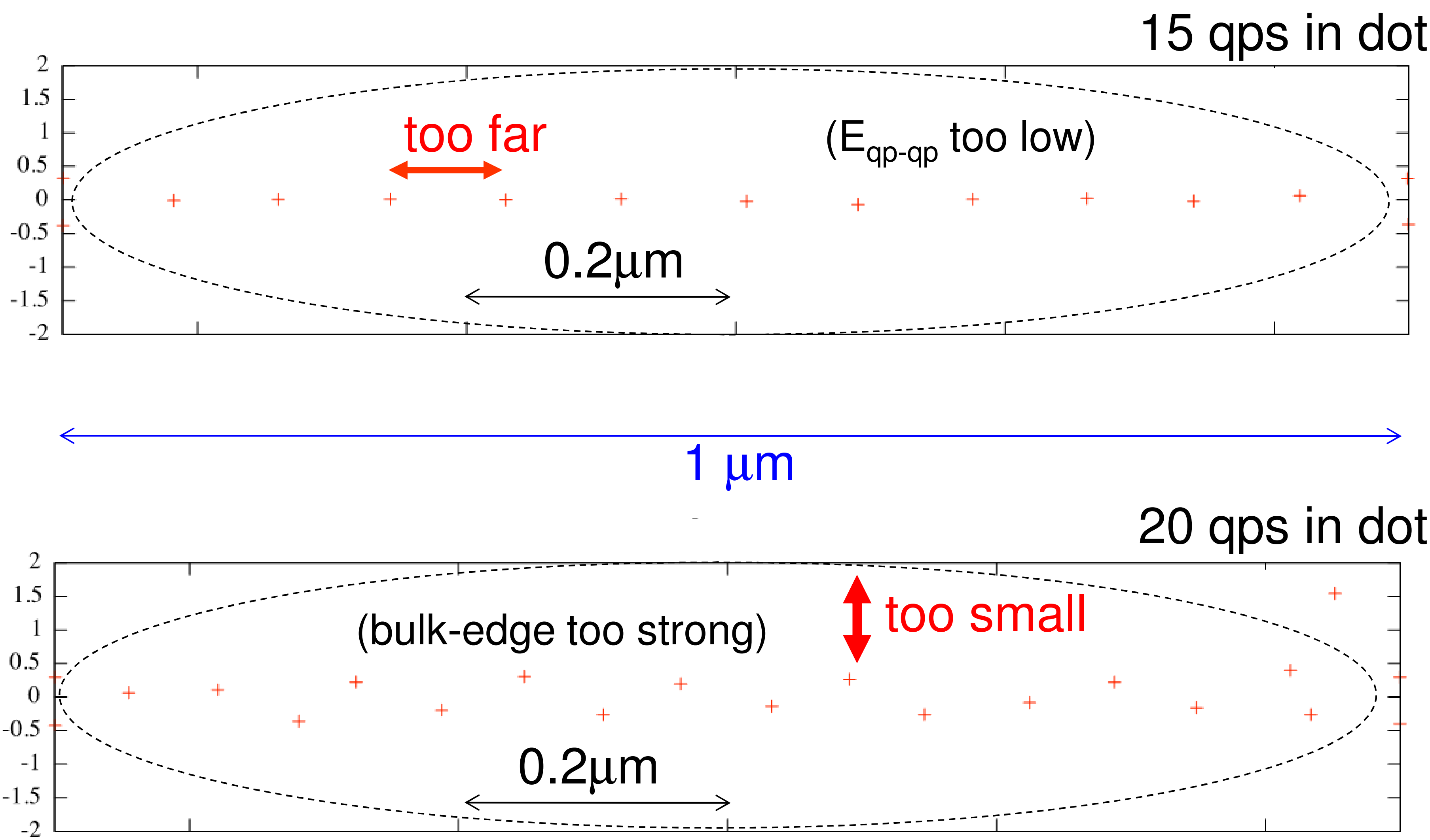
To see even-odd effect, must have

qp-edge coupling $\ll$ e^*V, T

$3 > 1 \gg 2$

This is probably impossible since 3, 2 are about the same size!

Detailed Electrostatic Simulation (w/ von Keyserlingk)



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All Similar Order

To observe interference we must assume 3 > 1,2

To see even-odd effect, must have

qp-edge coupling << e^*V, T

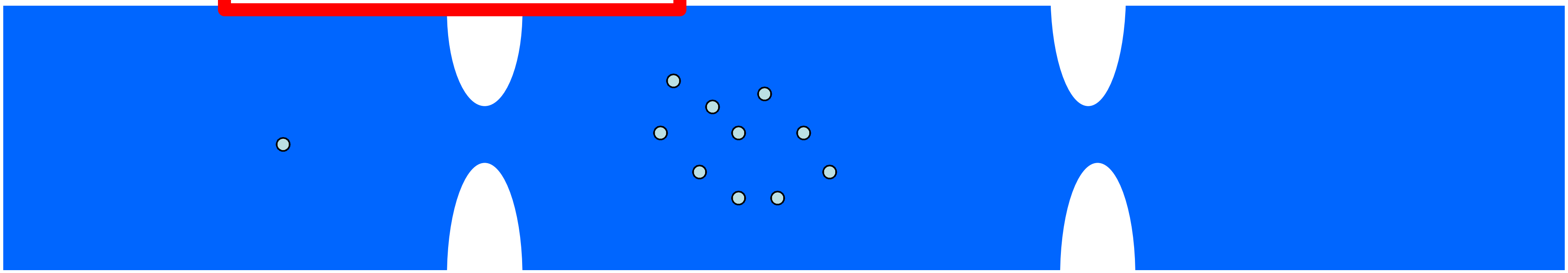
3 > 1 >> 2

This is probably impossible since 3, 2 are about the same size!

Prediction 2: **Even-odd effect should NOT be seen!**

Prediction 3

$\pm\,\pi/4$ phase slips – occur with qp/qh addition



Going from even to odd,

if $E_{edge\text{-}bulk} >> e^*V, T$ zero-mode majorana absorbed into edge

Only see phase slip ($\pm\,\pi / 4$) from abelian piece of the qp.

(rough argument:

$\nu=1/2$ implies statistical angle $2\pi/2$ for charge e/2 around e/2 .

e/4 quasiparticle is only half of this charge, hence quarter of the phase)

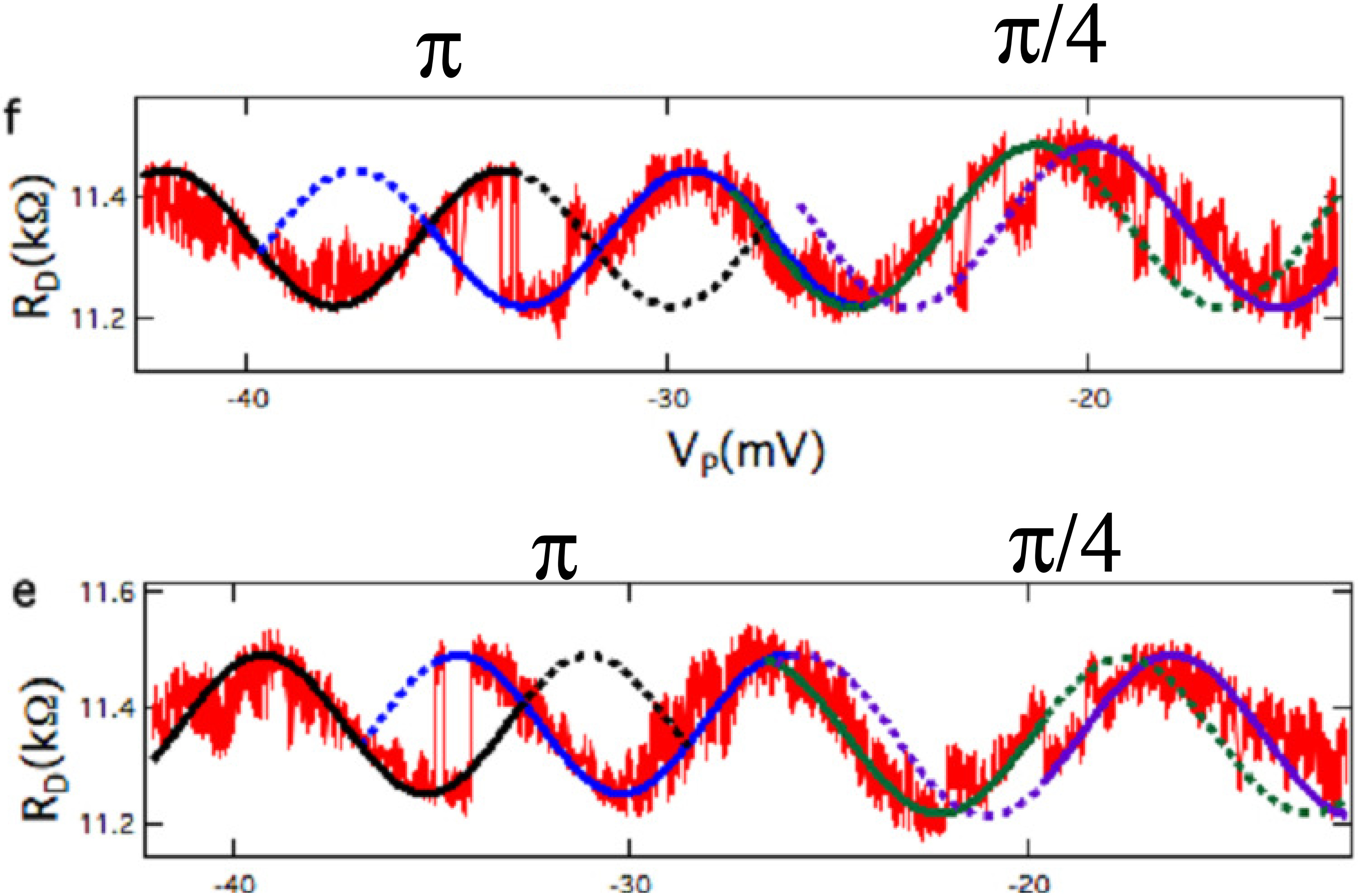
$E_{edge\text{-}bulk} \sim e^*V,\ T$ gives not quite $\pi / 4$

and less than full visibility of interference

Overbosch and Wen; Rosenow, Halperin, Simon, Stern ; Bishara and Nayak

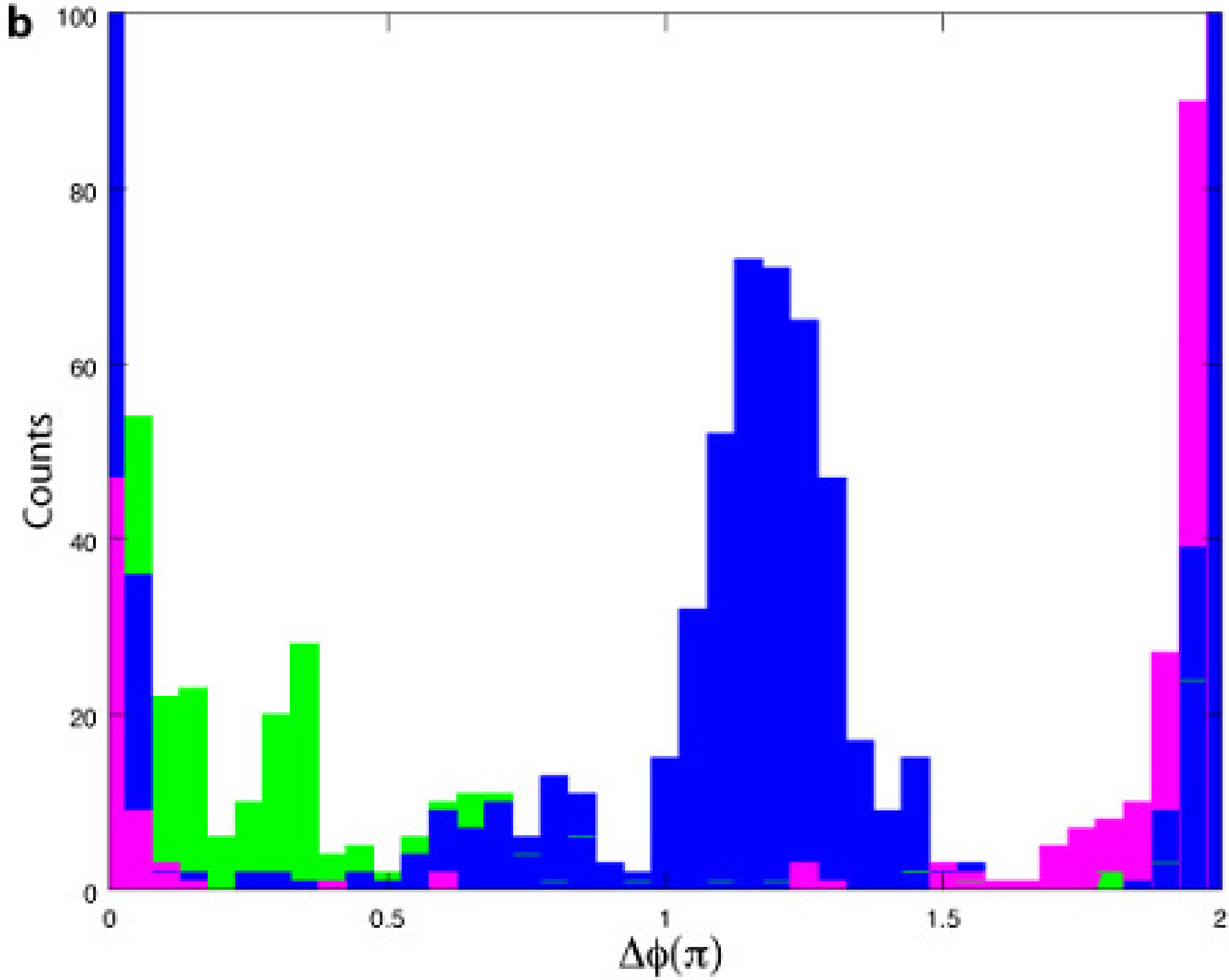
Prediction 3

± π/4 phase slips – occur with qp/qh addition



Predictions: Phase slips of π and π/4 (and 5π/4)

Histogram of measured phase slips:



Summary

- 7/3, Fractional statistics observed in phase slips

- 5/2 more complicated
  - Need qubit splitting to see any interference
  - Strong qp-edge coupling kills even-odd effect
  - Expect π slips associated with qubit flips
  - Expect $\pm\pi/4$ slips associated with qp addition

These results appear in agreement with Kang's data.

(In particular evidence for qubit flips looks fairly good)

Quantum Braiding Statistics and Quantum Hall Interferometry

Steven H. Simon
UNIVERSITY OF OXFORD

Thank you for Listening

My Funding: EPSRC Oxford Theory Grant and EPSRC TOPNES Grant