The Role of Spin in Some Fractional Quantum Hall Ground States

Simon C. Davenport

with Steven H. Simon

University of Oxford

NORDITA, August 20, 2012

Why might spin play a role in quantum Hall physics?

- FQHE observed at high fields (several T) low temperatures ($\sim$ 0.5K), and in high mobility samples ($\sim 10^6 \text{cm}^2/\text{Vs}$) e.g. GaAs heterostructures.
- Might expect electron spins are polarised due to high magnetic field.
- But there are *two* competing energy scales: *Zeeman* energy and *Coulomb* energy (Halperin 1983).
- In GaAs at 10T, Zeeman energy $\sim$ 3K and Coulomb energy $\sim$ 170K.
- **Can minimise combined energy with a non-polarised ground state: decrease in Coulomb energy can compensate for increase in Zeeman energy.**

Outline

- Experiments: Measuring the Degree of Spin Polarization
  - Kukushkin's Experiment: Optical measurements
- Theoretical perspective
  - Composite fermion theory
  - Numerical calculation
  - Our results for $\nu = 2/3, 3/5$ and $4/7$
  - Including finite-thickness corrections
  - Predictions for the 2nd LL
- Engineering the NASS state
  - Comparison Between CF and NASS at $\nu = \frac{4}{7}$ and $\nu = 2 + \frac{4}{7}$
- Conclusions

Experiments: Measuring the Degree of Spin Polarization

Vary: applied field strength, keeping ν fixed.
Measure: "degree of spin polarization"

$$\gamma_e = \frac{N^e_{1/2} - N^e_{-1/2}}{N^e_{1/2} + N^e_{-1/2}}.$$

- $\gamma_e = 1 \rightarrow$ Polarised.
- $0 < \gamma_e < 1 \rightarrow$ Non-polarised.

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How:

- Tilted-fields (Du *et al.* ~1990s)
- Optical measurements (Kukushkin *et al.* ~1990s)
- NMR (Bar-Joseph *et al.* ~2000s, 2010s)
- Activation gaps (Eisenstein *et al.* ~1990s; Pan *et al.* ~2010s)
- Also: hysteresis; g-factor tuning; compressibility measurements

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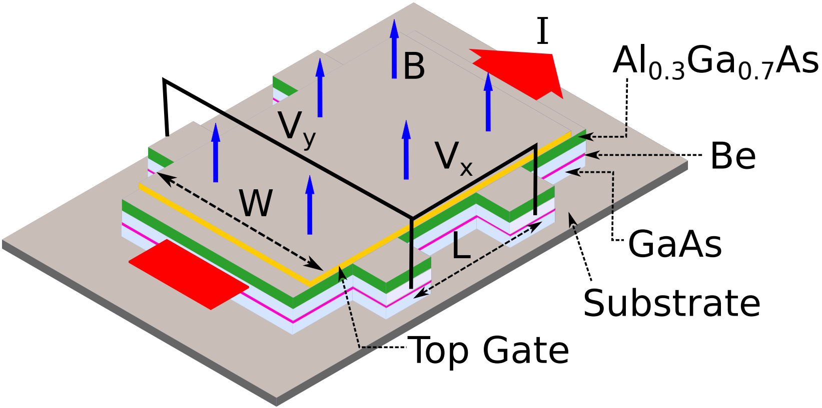
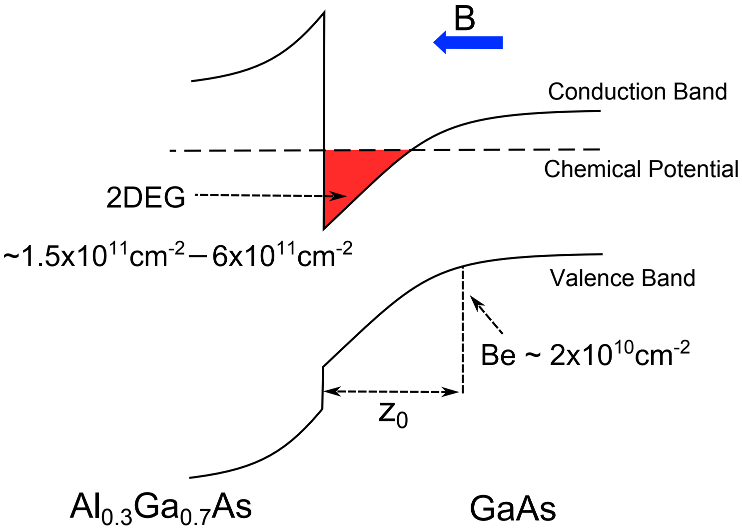
Kukushkin's Experiment (PRL **82** 3665 1999)

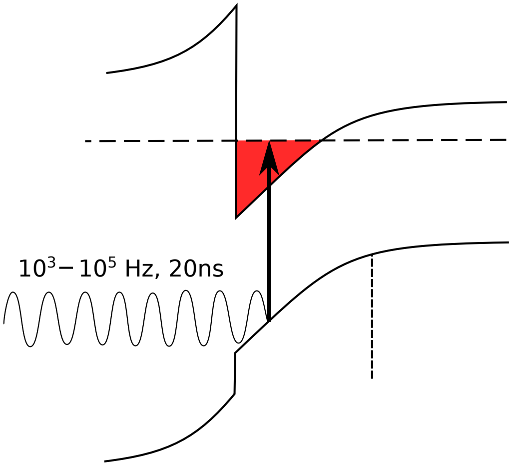
GaAs-Al$_x$Ga$_{1-x}$As single heterojunction with δ doped layer of Be.



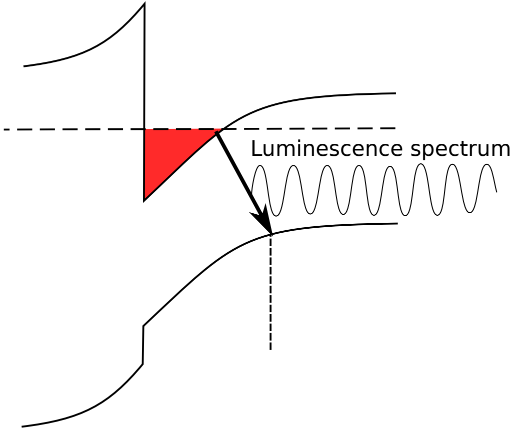
Band structure near the interface.



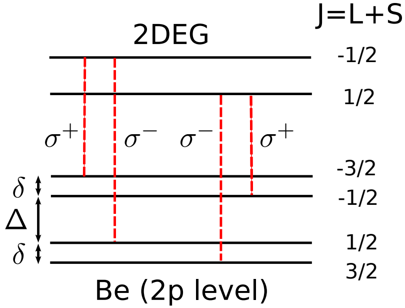
Optical measurements I: Photo-excitation



Optical measurements II: Radiative recombination



Optical measurements III: Selection rules for optical transitions



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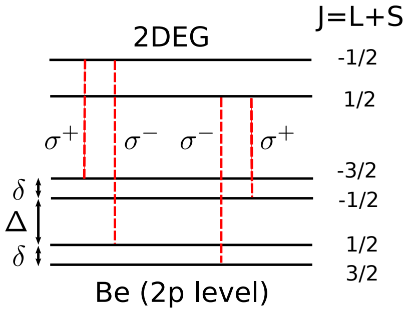
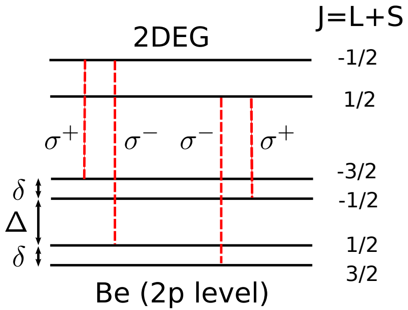


Want to find: $N^e_{1/2}$ and $N^e_{-1/2}$ in the 2DEG. **Assumptions:**

- Transition probabilities only depend on populations of electrons and holes in contributing levels.
- Thermal population of holes in Be levels: $N^h_J \propto \exp(-E_J/T)$

Optical measurements III: Selection rules for optical transitions



Intensities for left/right circularly-polarised light:

$$I_+ = 3N^e_{-1/2}N^h_{-3/2} + N^e_{1/2}N^h_{-1/2}$$

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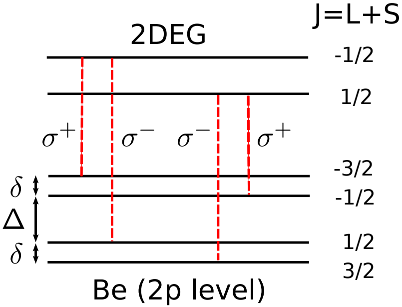

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$$I_+ = 3N^e_{-1/2}N^h_{-3/2} + N^e_{1/2}N^h_{-1/2} \propto 3N^e_{-1/2}e^{-(2\delta+\Delta)/T} + N^e_{1/2}e^{-(\delta+\Delta)/T}$$

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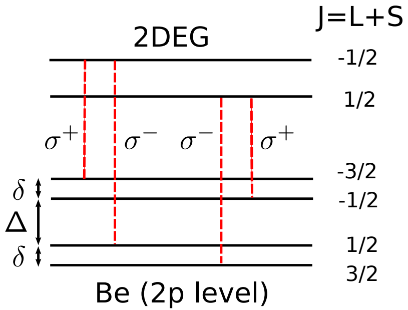

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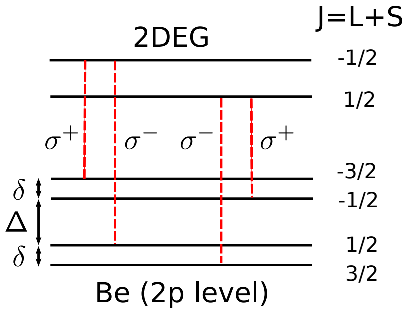

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Degree of circular polarisation given by

$$\gamma_{\text{circ.}} = \frac{I_+ - I_-}{I_+ + I_-}.$$

With

$$I_+ \propto 3N^e_{-1/2} e^{-(2\delta+\Delta)/T} + N^e_{1/2} e^{-(\delta+\Delta)/T},$$

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Determine δ, Δ by measurement of special cases for small B/T.

- Polarised case ($N^e_{-1/2} = 0$ for $\nu = 3, 5...$): $\gamma_{\text{circ.}} \approx \frac{3\delta}{4T} + \frac{\Delta}{2T}$.
- Filled LL case ($N^e_{-1/2} = N^e_{1/2}$ for $\nu = 2$): $\gamma_{\text{circ.}} \approx \frac{1}{2} + \frac{3(\Delta+\delta)}{8T}$.

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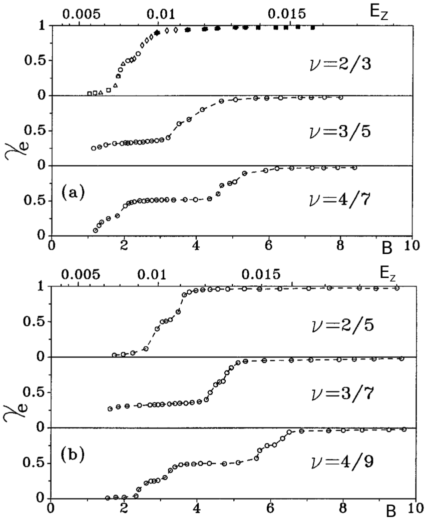
In general: Measure $\gamma_{\text{circ.}}$ and use known δ, Δ, B and T to calculate $N^e_{-1/2}/N^e_{1/2}$, and hence γ_e.

Results from Kukushkin et al. PRL 82, 3665 (1999).



Theoretical perspective

FQHE wavefunctions ψ are not known exactly, so we need variational trial wavefunctions:

- only consider ground state wavefunctions.
- require wavefunctions where spin can be incorporated.
- wavefunctions correspond to an ensemble of electron configurations $\rho_N(\{\mathbf{r}_i\}) \propto |\psi\{\mathbf{r}_i\}|^2$ that minimises the combined energy (Coulomb + Zeeman).

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Composite fermion (CF) theory is one possible framework

J. K. Jain, *Composite Fermions*, (CUP, 2007).
K. Park and J. K. Jain, Solid State Commun. **119**, 291 (2001).

CF theory overview

- Take an IQHE wavefunction for N fermions in n filled LLs: A Slater determinant of single-particle orbitals $\Phi_j^n(z_i)$

$$\mathrm{Det}[\Phi_j^n(z_i)].$$

- Turn it into a FQHE trial wavefunction by multiplying by a Jastrow factor

$$\prod_{l<m}(z_l - z_m)^2 \times \mathrm{Det}[\Phi_j^n(z_i)].$$

- But, this is **not** an analytic function. Obtain LLL trial wavefunction by projecting out the analytic part

$${}^2\mathrm{CF}_n = \hat{P}_{\mathrm{LLL}}\left\{\prod_{l<m}^{N}(z_l - z_m)^2 \mathrm{Det}[\Phi_j^n(z_i)]\right\}.$$

- Filling factor is $\nu = n/(2n \pm 1) \quad n \in \mathbb{Z}.$

CFs with spin

- $N_\uparrow$ spin-up and $N_\downarrow$ spin-down fermions could occupy $n_\uparrow$ and $n_\downarrow$ independent effective LLs

$$^2\mathrm{CF}_{(n_\uparrow:n_\downarrow)} = \hat{P}_{\mathrm{LLL}} \left\{ \prod_{l<m}^{N} (z_l - z_m)^2 \mathrm{Det}[\Phi_j^{n_\uparrow}(z_i^\uparrow)]\mathrm{Det}[\Phi_j^{n_\downarrow}(z_i^\downarrow)] \right\}.$$

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- $^2\mathrm{CF}_{(n_\uparrow:n_\downarrow)}$ has "degree of spin polarisation"

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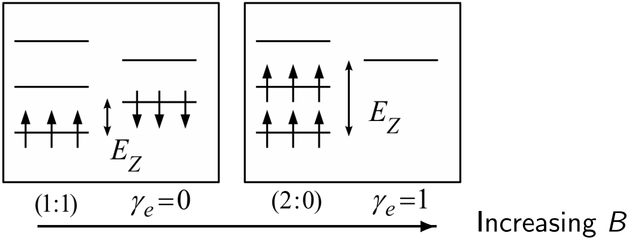
- Trial wavefunctions with total of $n = n_\uparrow + n_\downarrow$ filled effective LLs describe the **same** ν, but with **different** possible net spin polarisations.

Compare CF trial wavefunctions at the same ν, e.g. at $\nu = 2/3$:



- Each CF wavefunction associated with a different Coulomb energy. Difference in Coulomb energy between two states is ΔE_C per electron.
- Difference in Zeeman energy per electron = proportion flipped spins $\times$ Zeeman energy per spin E_Z.

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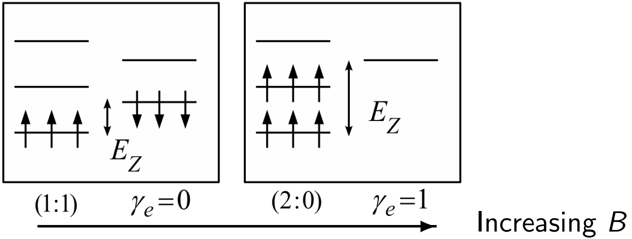

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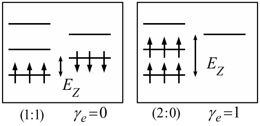
- Difference in Zeeman energy per electron = proportion flipped spins $\times$ Zeeman energy per spin E_Z.
- Condition for a spin transition to occur gives **critical Zeeman energy**:

$$E_Z^{\text{Crit}} = n\Delta E_C.$$

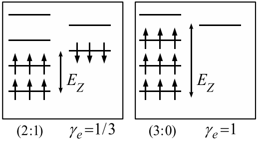
Summary of CF theory predictions

As B changes expect states with fixed ν and different γ_e.

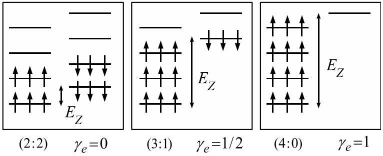
$n = 2$ $(\nu = \frac{2}{3}$ or $\frac{2}{5})$



$n = 3$ $(\nu = \frac{3}{5}$ or $\frac{3}{7})$

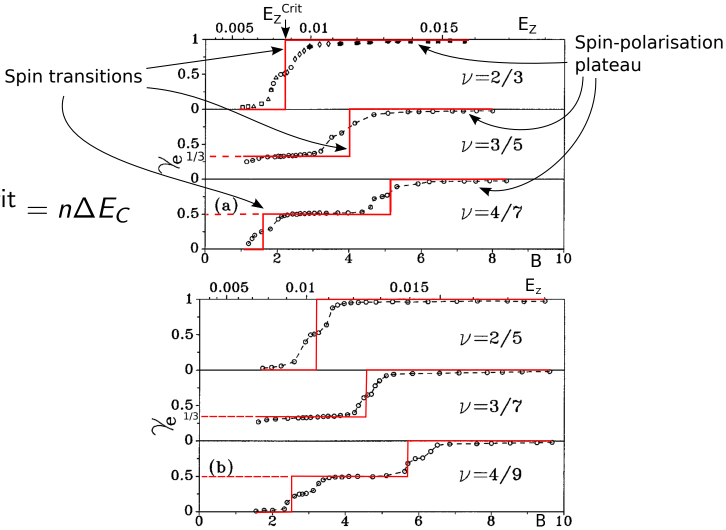


$n = 4$ $(\nu = \frac{4}{7}$ or $\frac{4}{9})$



Qualitative interpretation...

$$E_Z^{\mathrm{Crit}} = n\Delta E_C$$



How do we calculate ΔE_C?

- Calculate Coulomb energy for *each* trial wavefunction at the same ν e.g. at $\nu = 2/3$ for $^2\mathrm{CF}_{-2}$ and for $^2\mathrm{CF}_{(-1,-1)}$.
- Difference between the two values gives ΔE_C.
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Coulomb energy associated with an electron configuration:

$$\langle\psi|V|\psi\rangle = \int d\mathbf{r}_1 ... d\mathbf{r}_N \, |\psi(\{\mathbf{r}_i\})|^2 \, V(\mathbf{r}_1, ... \mathbf{r}_N)$$

where $V(\mathbf{r}_1, ... \mathbf{r}_N)$ Coulomb interaction + neutralising background.

Calculate Coulomb energy by Monte Carlo integration, with sample configurations $\rho_N(\{\mathbf{r}_i\})$.

- Evaluate for finite sized systems
- Extrapolate to the thermodynamic limit.
- Eliminate boundary effects by using spherical geometry.

Relation to previous work

- K. Park and J. K. Jain 2001: Calculated ΔE_C for “positive effective field” states at $\nu = 2/5, 3/7$ and $4/9$.
- G. Möller and S. H. Simon 2005: Technique to project the “negative effective field” CF wavefunctions.
- Problem: “Negative effective field” states at $\nu = 2/3, 3/5$ and $4/7$ are an order of magnitude *more difficult* to evaluate.

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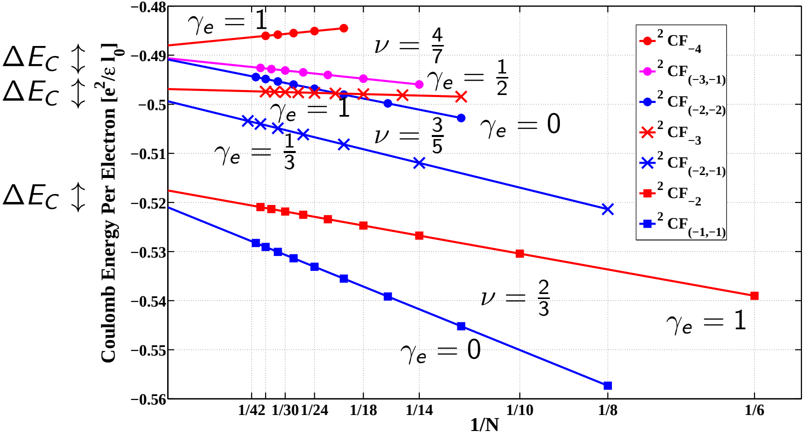
Our work: PRB 85, 245303 (2012)

- We designed an efficient algorithm to evaluate the "negative effective field" wavefunctions numerically.
- We calculated ΔE_C for the "negative effective field" case.

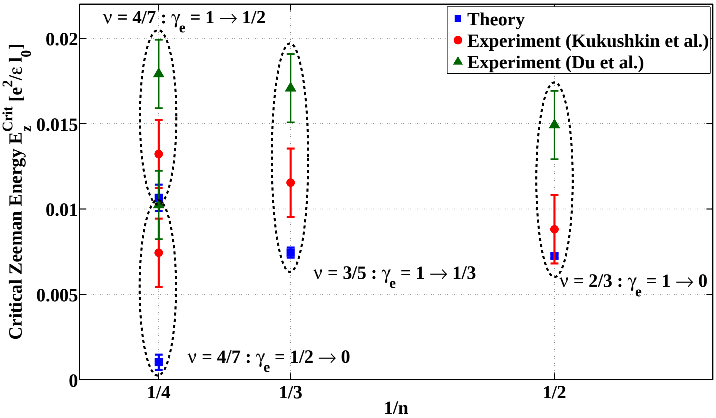
Our results for $\nu = 2/3, 3/5$ and $4/7$

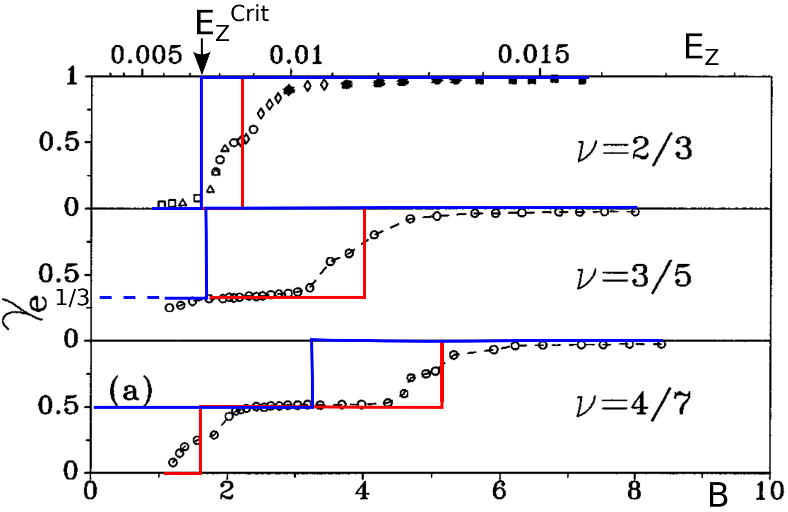
PRB **85**, 245303 (2012)



Comparison with experiments

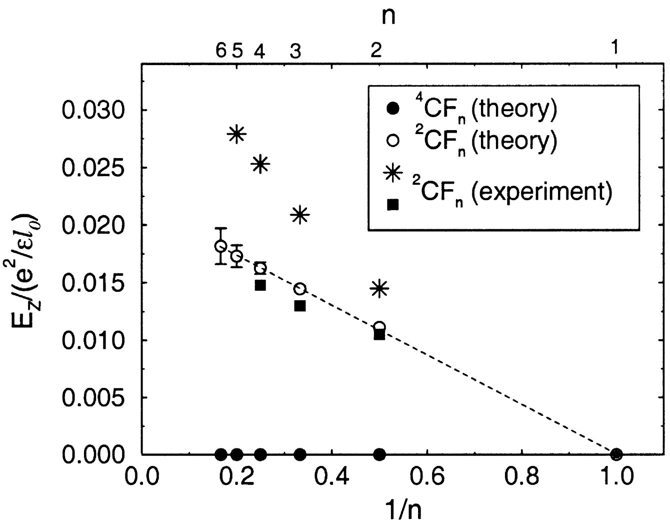
PRB **85**, 245303 (2012)




E_Z^{Crit}
0.005
0.01
0.015
E_Z
1
0.5
0
ν=2/3
ν=3/5
ν=4/7
γ_e
1/3
(a)
0
2
4
6
8
10
B

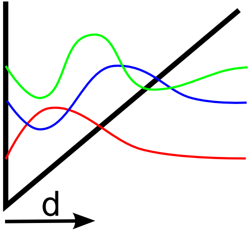
Park and Jain's Results for $\nu = 2/5, 3/7, 4/9...$

K. Park and J. K. Jain Solid State Commun. **119** 291, (2001)



Finite-thickness corrections

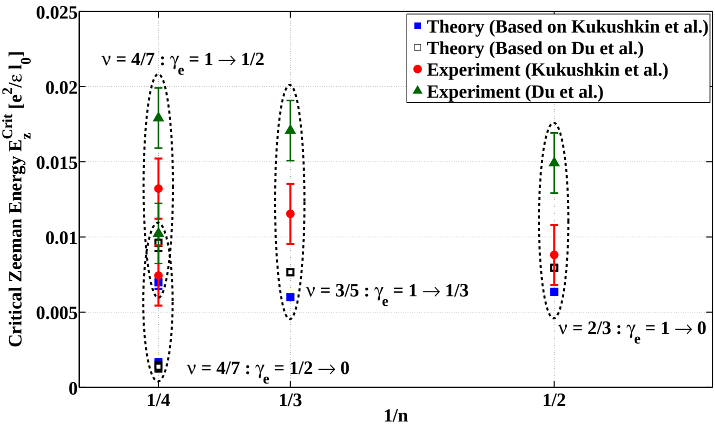
- In a real system the wavefunctions are not perfectly 2D! They have some finite extent, d, out of the plane.



- d depends on the particular experimental set-up. Also d=d(B) through magnetic length units.
- We model finite thickness effects with a modified potential for the 2D system; a softened Coulomb interaction e.g. Fang-Howard potential for a triangular well.

Our results

PRB **85**, 245303 (2012)



Predictions for the 2nd LL

- What about CF states at filling factor $\nu = \nu_{\mathrm{LLL}} + 2$ e.g. $8/3$?
- Calculate associated Coulomb energy using LLL trial wavefunctions and an effective potential
- **2nd LL with Coulomb = LLL with effective potential**

C. Töke *et al.* **72**, 125315 (2005)

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Predictions of CF theory

- Spin transition occurs for $\nu = 8/3$ with $E_Z^{\mathrm{Crit}} = 0.0048(6)\frac{e^2}{\epsilon l_0}$.
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Recent observation

- Spin transition observed at $\nu = 8/3$ with $E_Z^{\mathrm{Crit}} \sim 0.006\frac{e^2}{\epsilon l_0}$ (W. Pan *et al.* , PRL, 2012)

- Experiments: Measuring the Degree of Spin Polarization
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The Non-Abelian Spin Singlet (NASS) State

What other ground state trial wavefunctions have been proposed?

- Moore and Read: FQH trial wavefunctions can be written as conformal blocks. See Nucl. Phys. B 360, 362 (1991).
- E. Ardonne, and K. Schoutens: NASS state: a spin singlet state constructed from conformal blocks of Gepner parafermions. See PRL. **82**, 5096–5099 (1999).
- Conformal field theory properties can be used to predict behaviour of quasi-particle excitations: NASS quasi-particle excitations exhibit *non-abelian braiding statistics.*

- At filling $\nu = \frac{4}{7}$ the co-ordinate version of the NASS is

$$\psi_{\mathrm{NASS}} = \prod_{i<j} (z_i - z_j) \hat{S}_{z\uparrow, z\downarrow} [\phi]$$

where

$$\phi = \prod_{a=1}^{2} \left(\prod_{i<j}^{N/4} \left(z_i^{\uparrow a} - z_j^{\uparrow a} \right)^2 \prod_{i<j}^{N/4} \left(z_i^{\downarrow a} - z_j^{\downarrow a} \right)^2 \prod_{i,j}^{N/4} \left(z_i^{\downarrow a} - z_j^{\uparrow a} \right) \right)$$

K. Schoutens, E. Ardonne, and F.J.M van Lankvelt, cond-mat/0112379 (2001)

- Comparable CF trial wavefunction at $\nu = \frac{4}{7}$ is ${}^2CF_{(-2,-2)}$.
- Evaluate suitability of ψ_{NASS} against competing ${}^2CF_{(-2,-2)}$ by comparing Coulomb energy.

Comparison Between CF and NASS

Results so far... ${}^2CF_{(-2,-2)}$ is more energetically favourable for

- pure Coulomb interaction at $\nu = \frac{4}{7}$.
- Fang-Howard potential at $\nu = \frac{4}{7}$.
- pure Coulomb interaction at $\nu = 2 + \frac{4}{7}$.

Does not look promising!

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Results so far... ${}^{2}CF_{(-2,-2)}$ is more energetically favourable for

- pure Coulomb interaction at $\nu = \frac{4}{7}$.
- Fang-Howard potential at $\nu = \frac{4}{7}$.
- pure Coulomb interaction at $\nu = 2 + \frac{4}{7}$.

Does not look promising!

But there are other options to try...

- bilayer potentials.
- potential due to capacitor plates.
- take into account LL mixing.

Conclusions

- Established record of experimental evidence for significant role of spin in some FQHE ground states.
- CF theory conjectures trial wavefunctions describing FQH states with spin — qualitatively matches some experimental observations, but not all.
- CF theory prediction for the critical Zeeman energy compares moderately well with the experimental data — some aspects of the data remain unexplained.
- An observation of extensive non-polarized behaviour at $\nu = 2 + 2/3, 2 + 3/5$ or $2 + 4/7$ would suggest ground state wavefunctions not predicted by CF theory.
- At $\nu = 4/7$ and $\nu = 2 + 4/7$ the NASS state might not provide the most energetically favourable trial wavefunction.