# Quantum Hall Transitions and Quantum Number Fractionalization in Trapped Cold Atom Systems

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# Outline

- Brief introduction of quantum control in cold atom systems and some advertisements.
- A quantum Hall transition near an s-wave Feshbach resonance in a rotating fermion gas, and emergent quantum particles obeying semionic statistics at the quantum phase transition. (KY and H. Zhai, PRL 08).
- A quantum Hall transition near a p-wave Feshbach resonance in a rotating fermion gas, and fractionalization via a $Z_2$ gauge field or visons. (Y. Barlas and KY PRL 11).
- Summary.

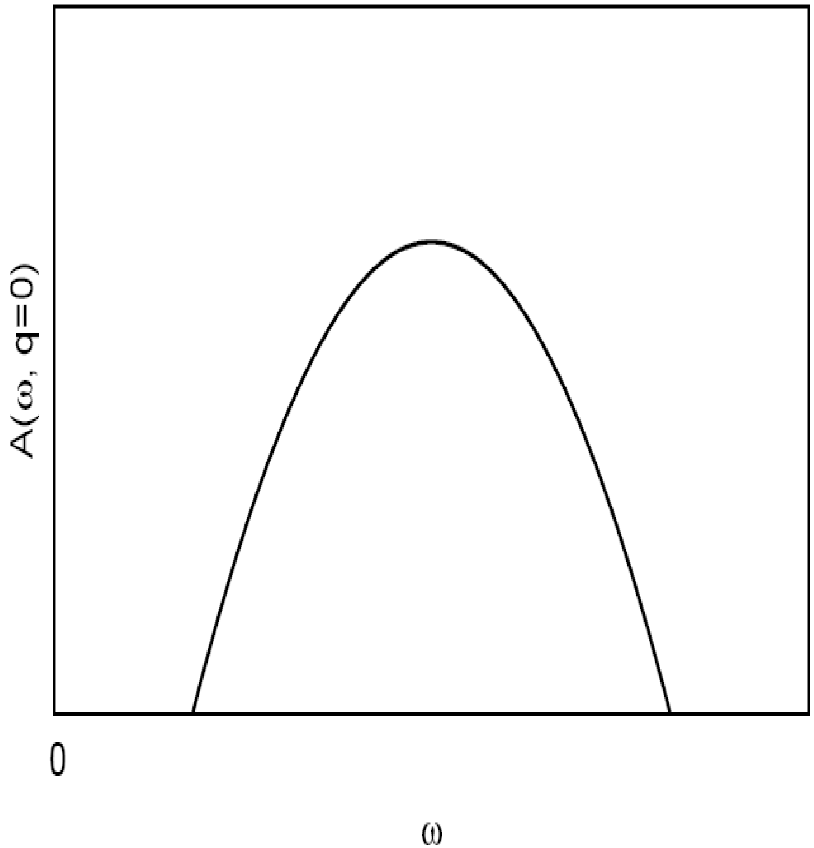
# Quantum Control in Cold Atom Systems

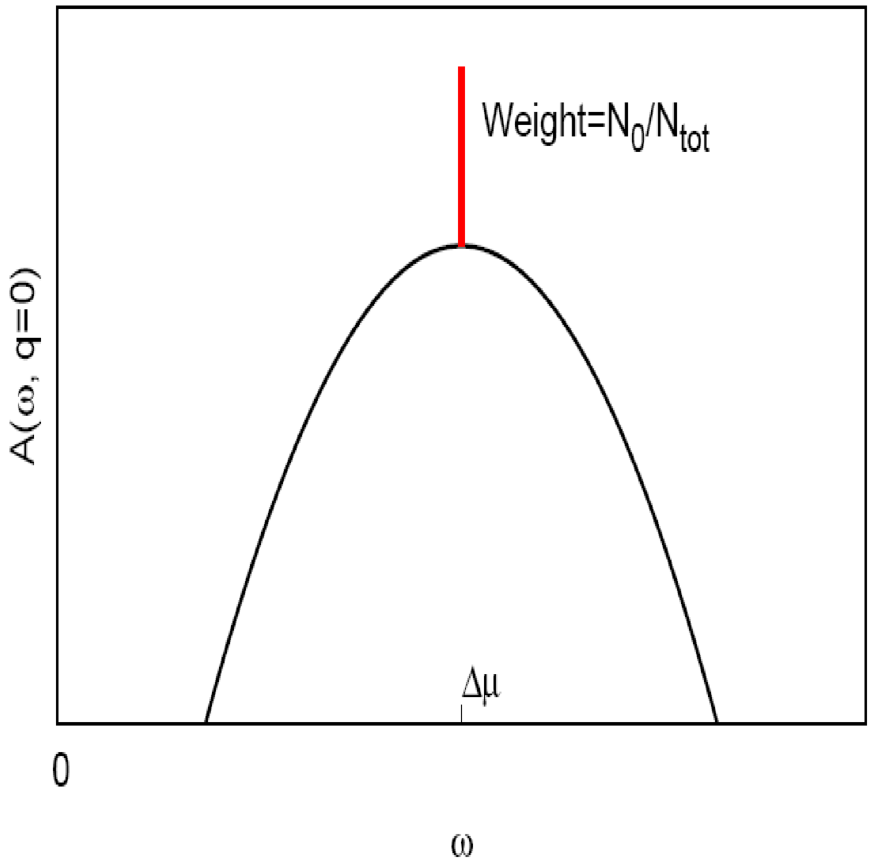
- Atoms/molecules with both Fermi and Bose statistics, and in particular their mixtures are available (can be used to realize supersymmetry! Y. Yu and KY, PRLs 08 and 10).
- Optical lattice potential can be controlled through frequency and intensity of light; band structure/effective mass controllable (achieve boson superfluid-insulator transition; possible new universality class in the presence of fermions. KY PRB 08).
- "Magnetic field" controlled through rotation of optical lattice/trap (Coriolis' force " $\approx$ " Lorentz force), or synthetic gauge field (Spielmann).
- Interaction strength (and sign!) controlled via (real) magnetic field tuned Feshbach resonances.

# Supersymmetry in Bose-Fermi Mixtures

- Bose-Fermi mixtures (like mixture of $^6$Li and $^7$Li) have been realized experimentally.
- Through quantum control, one can tune parameters such that the bosons and fermions have same dispersion and interaction, to realize supersymmetry.
- Supersymmetry always broken, either spontaneously or explicitly, resulting in a **fermionic** Goldstone mode called **Goldstino**.
- **Goldstino detectable experimentally! Thus supersymmetry and its breaking can be studied in cold atom labs!** (in addition to billion $ accelerators)

# Fermion Spectral Function at q=0



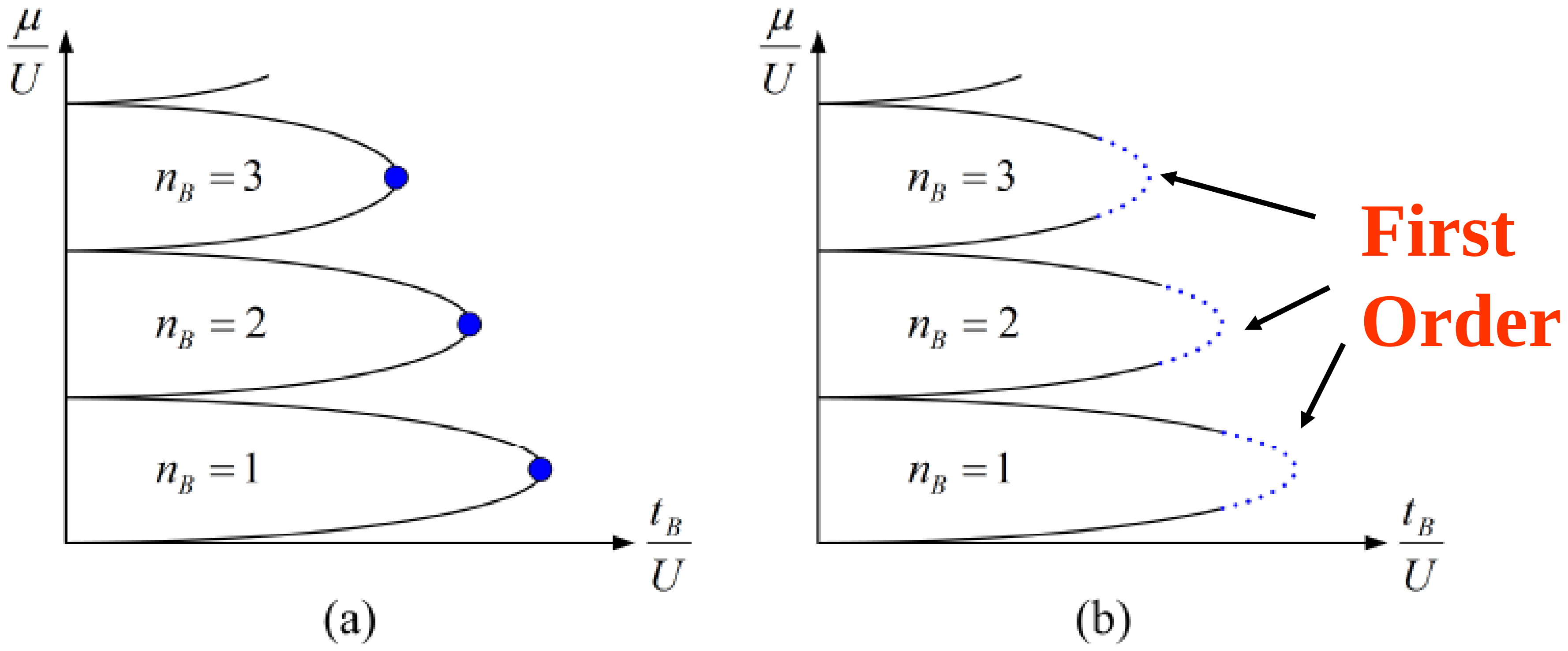

Without Supersymmetry **With Supersymmetry**

**(Y. Yu and KY, PRL 08; see also http://physicsworld.com/cws/article/news/33425)**

By arranging Dirac spectrum for the fermions, possible to have (emergent) relativistic/space-time supersymmetry (that enlarges Lorentz symmetry);
for realization of Wess-Zumino model with (emergent) space time SUSY, see Y. Yu and KY, PRL 10.
Supersymmetry unbroken in this model, but still detectable.

# Possible New Phase Diagrams for Boson Superfluid-Insulator Trantion in 3D in the Presence of Fermions

**KY, PRB 08**

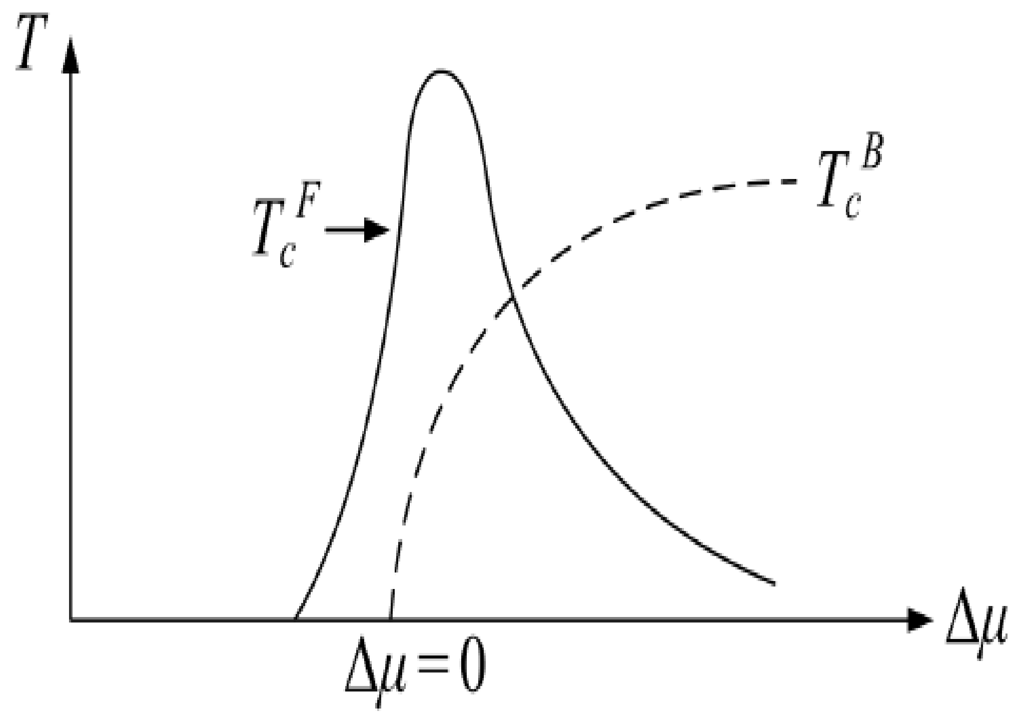


# Quantum Critical Fluctuation Mediated Fermion p-wave Pairing (KY, PRB 08)

$$V_{eff}(\mathbf{q},\omega) = V^2\chi(\mathbf{q},\omega) \approx -\frac{V^2\kappa}{1+|\mathbf{q}|^2\xi^2}$$

$$\xi \sim \sqrt{t_b/\Delta\mu},$$

$$\kappa \approx 1/U$$



# Preliminary Evidence of FQH-like Correlation Reported Recently in Rotating Optical Lattices!

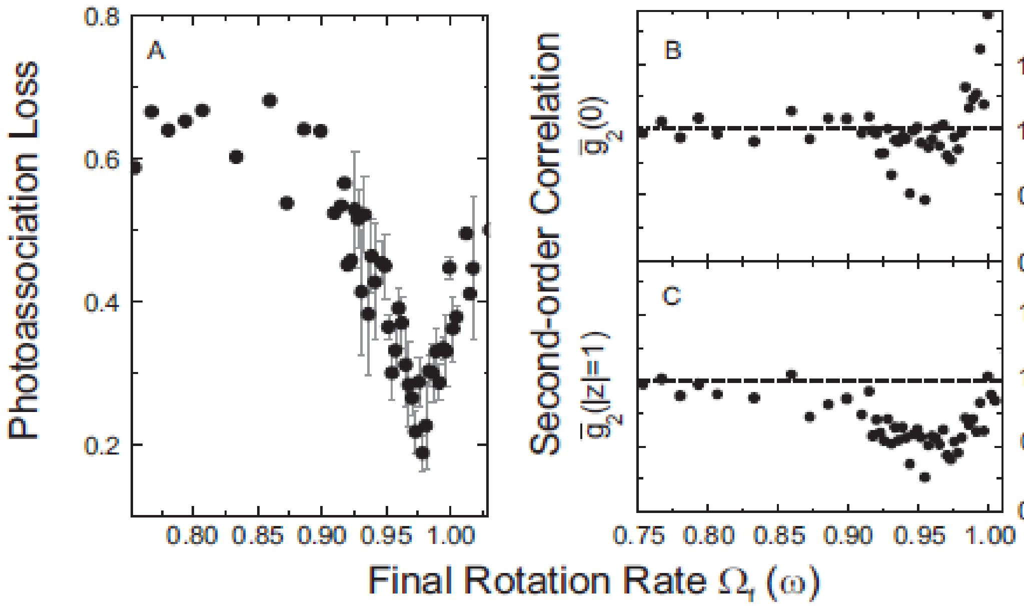



Figure 4: Probing atomic correlation with photoassociation. (A) Shows fraction $(N_{npa} - N_{pa})/N_{npa}$ of atoms lost after rotation sequence and short pulse of photoassociation light. The strong suppression at the centrifugal limit is indicative of strong correlation. (B,C) Measurement of density profile with and without photoassociation allows extraction of the second-order correlation function $g_2(z)$ as defined in the text. The depression near the centrifugal limit is most apparent at nonzero radius.

**Nathan Gemelke, Edina Sarajlic, and Steven Chu, arxiv1007**

# Vortices observed in a synthetic gauge field (Lin et al., Nature 10)

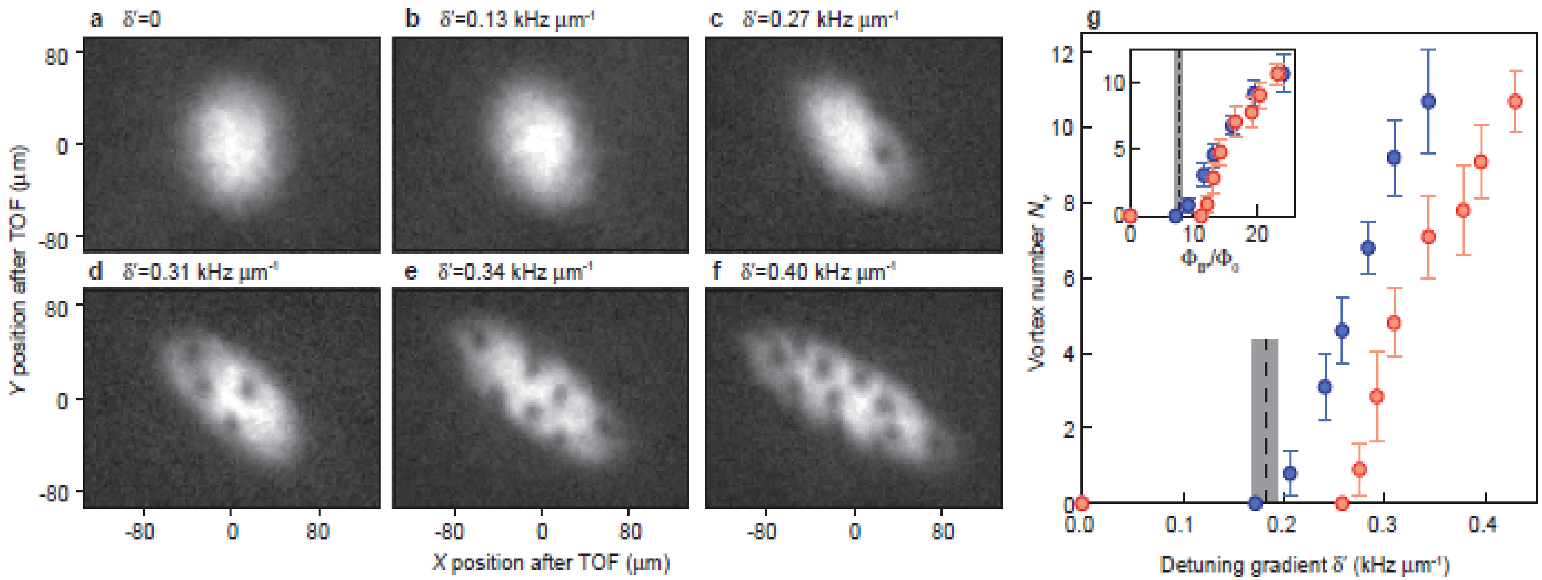



FIG. 2: **Appearance of vortices at different detuning gradients.** Data was taken for $N = 1.4 \times 10^5$ atoms at hold time $t_h = 0.57$ s. **a-f**, Images of the $|m_F = 0\rangle$ component of the dressed state after a 25.1 ms TOF with detuning gradient $\delta'$ from 0 to 0.43 kHz $\mu m^{-1}$ at Raman coupling $\hbar\Omega_R = 8.20\ E_L$. **g**, Vortex number $N_v$ versus $\delta'$ at $\hbar\Omega_R = 5.85\ E_L$ (blue circles), and 8.20 $E_L$ (red circles). Each data point is averaged over $\geq 20$ experimental realizations, and the uncertainties represent standard deviations ($\sigma$). The inset displays $N_v$ versus the synthetic magnetic flux $\Phi_{B^*}/\Phi_0 = Aq^*\langle B^*\rangle/h$ in the BEC. The dashed lines indicate $\delta'$ below which vortices become energetically unfavorable according to our GPE computation, and the shaded regions show the 1-$\sigma$ uncertainty from experimental parameters.

# Quantum Hall States in a Rotating Trap

- Integer quantum Hall (IQH) states: fully-filled Landau levels, filling factor $\nu = m$; fermions only.
- Fractional quantum Hall (FQH) states: Laughlin type of states with $\nu = 1/m$; even $m$ for boson and odd $m$ for fermion.
- Starting with spin-1/2 fermionic atom IQH state at $\nu = 2$; turning on strong s-wave attraction➔ bosonic molecule FQH state at $\nu = ½$! (**Haldane+Rezayi 04**)

**Same quantized Hall conductance (e*: atom "charge"):**

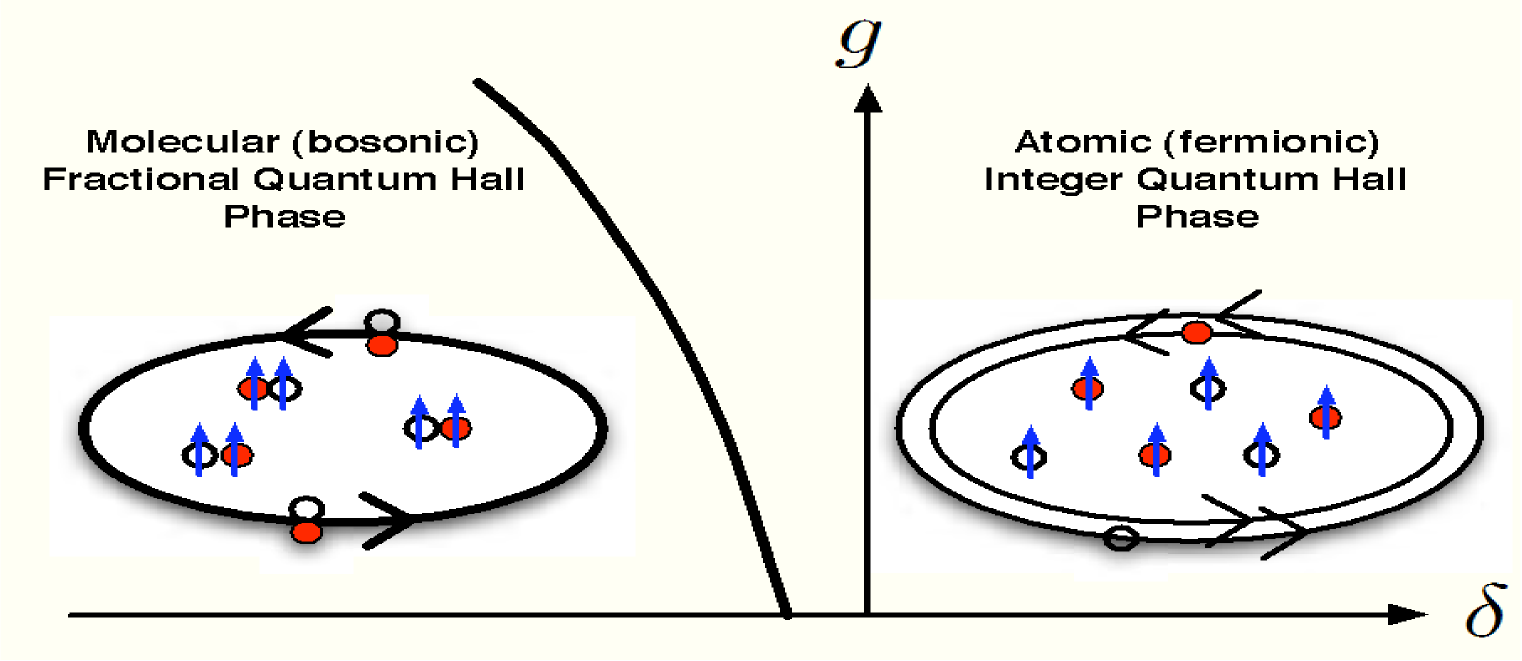
$$\sigma_{xy} = 2(e^*)^2/\hbar = \frac{1}{2}[(2e^*)^2/\hbar]$$

**Same quasiparticle charge: e*=2e*/2**

# There must be a quantum phase transition between boson FQH and fermion IQH states! (G. Moller and N. Cooper, PRL 07)

- Spin insulator.
- Quasiparticles are spinless semions.
- One edge mode.

- Spin QH state.
- Quasiparticles are spin-1/2 fermions.
- Two edge modes.




KY and H. Zhai, PRL 08

# Chern-Simons-Ginzburg-Landau (CSGL) Theory

$$
\begin{aligned}
L &= L_\uparrow + L_\downarrow + L_{\rm m} + L_{\rm cs} + g(\overline{\phi}\psi_\uparrow\psi_\downarrow + \phi\overline{\psi}_\uparrow\overline{\psi}_\downarrow) \\
L_\sigma &= \overline{\psi}_\sigma(\partial_\tau - a_0^\sigma)\psi_\sigma - \mu_\sigma|\psi_\sigma|^2 \\
&+ \frac{1}{2m_\sigma}|(-i\nabla - \mathbf{A} - \mathbf{a}^\sigma)\psi_\sigma|^2 + \cdots, \\
L_{\rm m} &= \overline{\phi}(\partial_\tau - a_0^\uparrow - a_0^\downarrow)\phi - (\mu_\uparrow + \mu_\downarrow - \delta)|\phi|^2 \\
&+ \frac{1}{2M}|(-i\nabla - 2\mathbf{A} - \mathbf{a}^\uparrow - \mathbf{a}^\downarrow)\phi|^2 + \cdots, \\
L_{\rm cs} &= L_{\rm cs}^\uparrow + L_{\rm cs}^\downarrow = \frac{1}{4\pi}\frac{\pi}{\theta}\epsilon^{\mu\nu\lambda}\left[a_\mu^\uparrow\partial_\nu a_\lambda^\uparrow + a_\mu^\downarrow\partial_\nu a_\lambda^\downarrow\right]
\end{aligned}
$$

Conserved charges: $N_\sigma = \int d^2\mathbf{r}(|\psi_\sigma(\mathbf{r})|^2 + |\phi(\mathbf{r})|^2).$

Constraints from CS term: $\nabla \times \mathbf{a}^\sigma = 2\theta(|\psi_\sigma|^2 + |\phi|^2)$

Mean-field approximation: $\nabla \times (\langle \mathbf{a}^\sigma \rangle + \mathbf{A}) = 0$

# Mean-field Description of Phases

Within mean-field treatment of statistical flux attached to particles, Quantum Hall Effect = Bose condensation of Chern-Simons bosons!
(Zhang, Hansson and Kivelson 89; Read 89)

Atomic IQH phase: two condensates and both U(1) symmetries broken: $\langle\psi_\uparrow\rangle \neq 0; \langle\psi_\downarrow\rangle \neq 0 \Longrightarrow \langle\phi\rangle \neq 0.$

Molecular FQH phase: one condensate and only one U(1) symmetry broken: $\langle\psi_\uparrow\rangle = \langle\psi_\downarrow\rangle = 0; \quad \langle\phi\rangle \neq 0.$

# Mean-field Description of Phase Transition

$$\tilde{L} = L_\uparrow[\mathbf{a}^\sigma, \mathbf{A}=0] + L_\downarrow[\mathbf{a}^\sigma, \mathbf{A}=0] + h(\psi_\uparrow\psi_\downarrow + \overline{\psi}_\uparrow\overline{\psi}_\downarrow)$$

Bogoliubov transformation:

$$\psi_+ = (\psi_\uparrow + \overline{\psi}_\downarrow)/\sqrt{2}; \qquad \psi_- = (\psi_\uparrow - \overline{\psi}_\downarrow)/\sqrt{2}$$

$$\begin{aligned}\tilde{L} &= \overline{\psi}_+\partial_\tau\psi_- + \overline{\psi}_-\partial_\tau\psi_+ - (\mu+h)|\psi_-|^2 + (h-\mu)|\psi_+|^2 \\ &+ \frac{1}{2m}\left[|\nabla\psi_-|^2 + |\nabla\psi_+|^2\right] + \cdots\end{aligned}$$

Integrate out massive field $\psi_+$ :

$$\begin{aligned}L_{\text{eff}}[\psi_-] &= \frac{|\partial_\tau\psi_-|^2}{h-\mu} + \frac{|\nabla\psi_-|^2}{2m} - (\mu+h)|\psi_-|^2 \\ &+ U|\psi_-|^4 + \cdots,\end{aligned}$$

Effective theory is that of 2+1D XY transition with Lorentz invariance! But coupling to CS field must be included.

# Full Theory of QH Phase Transition

Perform a similar transformation on the CS gauge field:

$$a_\mu^+ = (a_\mu^\uparrow + a_\mu^\downarrow)/2, \qquad a_\mu^- = (a_\mu^\uparrow - a_\mu^\downarrow)/2$$

$$L_{\rm cs} \;=\; \frac{1}{4\pi}\frac{\pi}{\theta'}\epsilon^{\mu\nu\lambda}\left[a_\mu^+\partial_\nu a_\lambda^+ + a_\mu^-\partial_\nu a_\lambda^-\right] \qquad \theta' = \theta/2 = \pi/2$$

Integrate out massive field $\psi_+$ :

$$L_{\rm eff}[\psi_-, a^-] = \frac{|(\partial_\tau - a_0^-)\psi_-|^2}{h-\mu} + \frac{|(\nabla - i\mathbf{a}^-)\psi_-|^2}{2m}$$
$$-\;\;(\mu + h)|\psi_-|^2 + U|\psi_-|^4 + \frac{1}{4\theta'}\epsilon^{\mu\nu\lambda}a_\mu^-\partial_\nu a_\lambda^- + \cdots$$

**Theory describes condensation of <u>emergent</u> particles with zero charge, spin-1/2, and semion statistics!**

# Comments on the Effective Field Theory

- Properties (or quantum numbers) of emergent particle expected from **difference** between two phases.
- Same types of theory studied earlier in context of QH-Insulator transition **on a lattice**; nature of transition controversial: Large-N limit suggests 2$^{nd}$ order transition with θ'-dependent critical behavior (Wen and Wu 92, Chen, Fisher and Wu 93); while Pryadko and Zhang (94) found fluctuation-driven 1$^{st}$ order transition in certain parameter range.
- Order of transition resolvable in cold atom system by measuring spin-gap using RF absorption, or numerical simulation (Haldane and Rezayi). Not (yet) realizable in electronic systems.

# A Simpler (but More Interesting) Case

Starting with *spinless* fermionic atom IQH state at $\nu = 1$; turning on strong **p-wave** pairing interaction ➔ bosonic molecule FQH state at $\nu = 1/4$ !

**Same quantized Hall conductance (e*: atom "charge"):**

$$\sigma_{xy} = (e^*)^2/\hbar = \frac{1}{4}[(2e^*)^2/\hbar]$$

**Different quasiparticle charge: e* ≠ 2e*/4=e*/2; charge fractionalization! Different statistcs as well. Same number of edge modes; similar edge physics!**

# CSGL Theory of Phases and Phase Transition

$$\mathcal{L} = \bar{\psi}(\partial_\tau - (A_0 + a_0))\psi + \frac{1}{2m}|(\partial_i - iA_j - ia_j)\psi|^2 - \mu|\psi|^2$$
$$+ \bar{\phi}(\partial_\tau - 2a_0)\phi + \frac{1}{4m}|(\partial_i - 2iA_j - 2ia_j)\phi|^2 - (2\mu - \delta)|\phi|^2$$
$$+ g(\phi\bar{\psi}\bar{\psi} + \bar{\phi}\psi\psi) + \frac{1}{4\theta}\epsilon_{\nu\mu\lambda}a_\nu\partial_\mu a_\lambda + \cdots$$

## Differences from previous case:

•Only one Chern-Simons Gauge Field (coupled to charge).

•Only one condensate in both phases!

•Gauge field has an Anderson-Higgs mass (overwhelming Chern-Simons term), and plays no-role at the transition!

•Reduces to an Ising model:

$$\mathcal{L} = \bar{\psi}\partial_\tau\psi + \frac{1}{2m}|\nabla\psi|^2 - \mu|\psi|^2 + \lambda|\psi|^4 + h(\bar{\psi}\bar{\psi} + \psi\psi)$$

Theory identical to that of transition between atomic and molecular BECs without rotation (Radzihovsky+Park +Weichman; Romans+Duine+Sachdev+Stoof, PRLs 04); **Ising transition!**

- Stable 2nd order transition; unlike transition between atomic and molecular BECs unstable due to collapse of system near Feshbach resonance.
- Critical properties known accurately: $z=1$, $\upsilon \approx 0.63$; neutral gap vanishing with exponent $\upsilon z$ near critical point.

- **However**: Looks like a conventional (Landau-type) transition; topological nature of phases involved and transition not explicit.
- Change of topological properties not manifested, like change of torus degeneracy, charge fractionalization etc.

# (Dual) Chern-Simons/$Z_2$ Gauge Theory

$$S = \frac{1}{4\pi} \sum_{i,\mu\nu\lambda} \epsilon_{\mu\nu\lambda} a_{i\mu} \Delta_\nu a_{i\lambda} + t_V \sum_{i,\mu} \sigma_{i\mu} \cos(\Delta_\mu \varphi_i - a_{i\mu}/2) + \beta \sum_{\square} \prod_{\square} \sigma$$

$$J_{i,\mu} = \frac{1}{2\pi} \epsilon_{\mu\nu\lambda} \Delta_\nu a_{i\lambda}$$ Conserved atom 3-current.

$\sigma = \pm 1$: $Z_2$ gauge field. $\varphi$: phase of molecular (half) vortex field.

**Phases and Phase Transition:**

•Small $\beta$: $Z_2$ vortices (or visons) condensed; molecular vortices confined; only atomic vortices present, which represent original charge-1, fermionic quasiparticles. IQH phase.

•Large $\beta$: $Z_2$ vortices (or visons) gapped; molecular vortices de-confined = Laughlin quasiparticle; fractionalization! FQH phase.

•At transition: vison gap closes; Ising criticality.

## Non-trivial checks:

- Torus degeneracy changes from 1 to 4 due to the phase transition in $Z_2$ sector. Topological nature of phases and phase transition explicitly manifested.
- **No** additional edge modes!

(See Y. Barlas and KY, PRL 11 for more details)

Story similar to, but simpler than Senthil-Fisher theory for spin-charge separation in cuprates.

# Summary/Conclusions

•Trapped cold atom systems offer opportunities to study strongly correlated systems **different** from those encountered in electronic systems.

•Discussed examples of quantum Hall phase transitions driven by **attractive** interactions, whose critical theories are relativistic massless semions, and massless visons in $Z_2$ gauge theories respectively. These transitions are very similar to quantum number fractionalization transitions discussed in other contexts, but have not yet found concrete experimental realizations there.

•Represent two large classes of Abelian QH transitions: with or without additional condensates/edge modes.

•**Outlook**: Possible transitions between Abelian and non-Abelian QH states, like $\upsilon$=4 fermions to $\upsilon$=1 bosons (Moore-Read).