Trial states for the Quantum Hall Effect: entanglement and field theory

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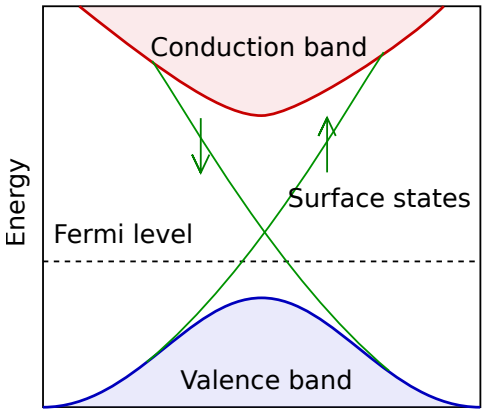
with Nick Read and Ed Rezayi (see [arXiv:1207.7119])

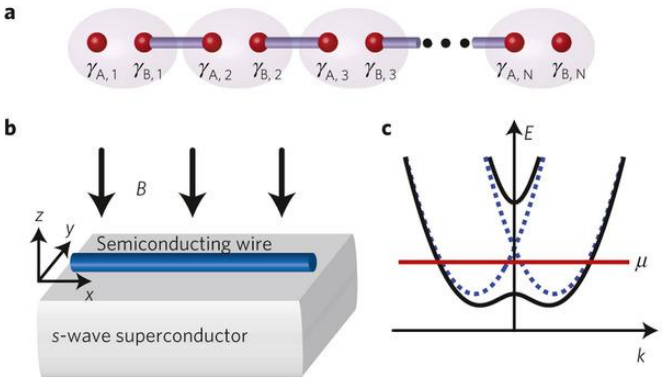
Nordita - Stockholm - August 23, 2012

"topological phase"	$\leftrightarrow$	gapped phase $E_{exc.} > 0$ (massive)
"topological" quantity	$=$	invariant as long as $E_{exc.} > 0$

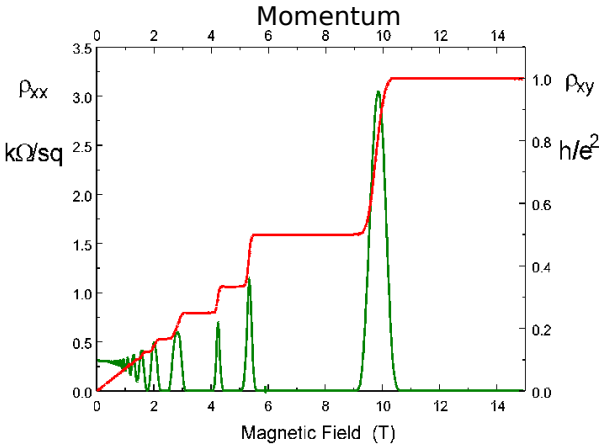
Examples (electronic):



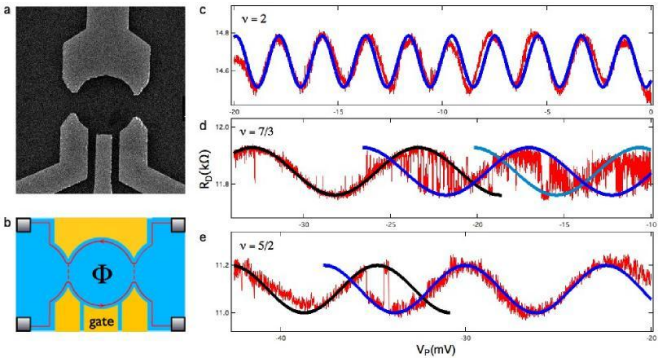
insulator



super-conductor



integer QHE



fractional QHE

Topologically non-trivial phases:

free particles + topology

(topological insulators+superconductors)

quadratic Hamiltonian +topology (+perturbations)

$\longrightarrow$ well-known methods are available

interacting particles

(FQHE, ???)

difficult problem

$\longrightarrow$ requires new methods

~~Hamiltonian with interactions~~

~~$\longrightarrow$ diagonalization~~

possible (difficult) numerically

privileged method (at least for FQHE):

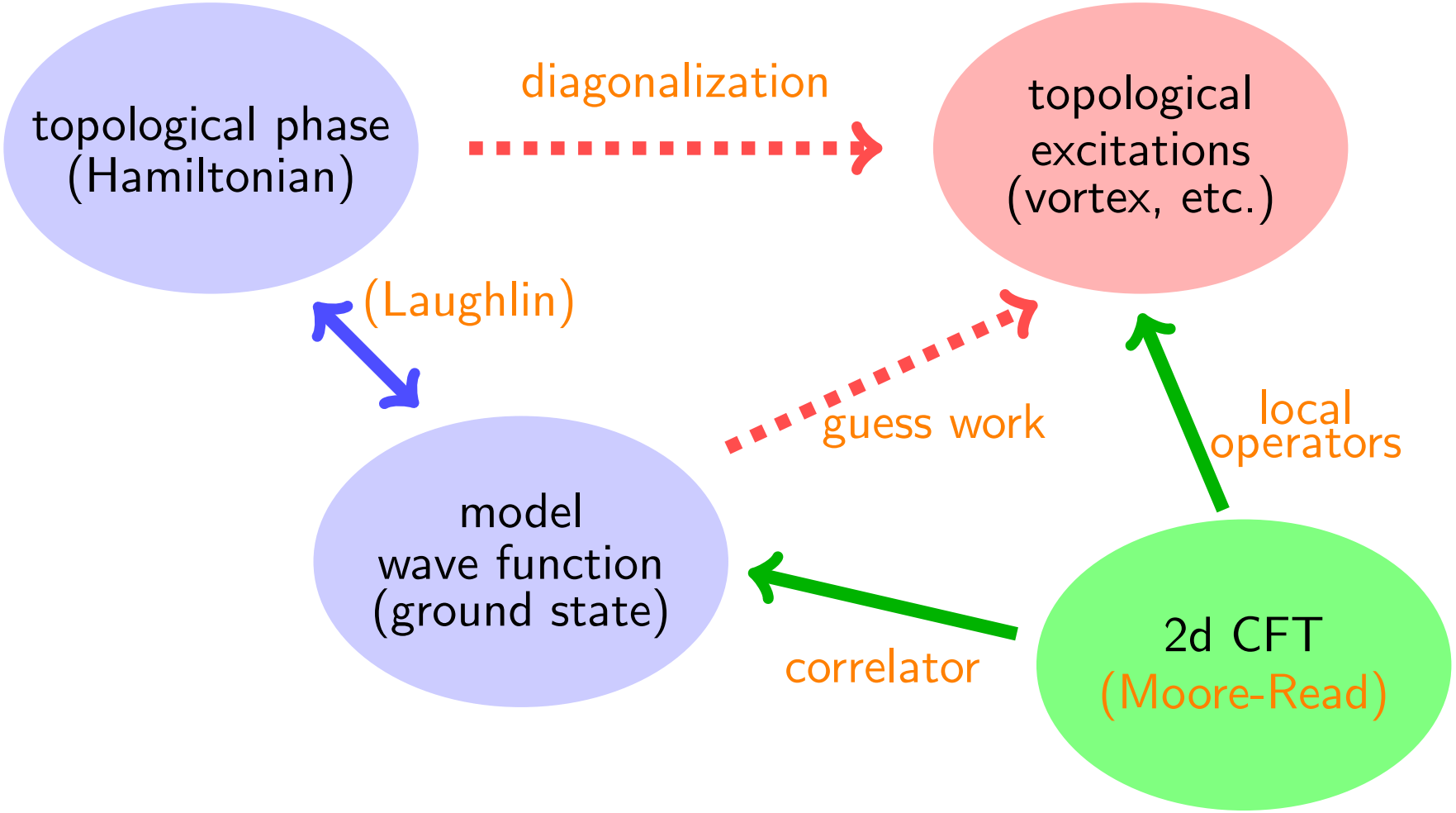
model wave function

ground state + excitations

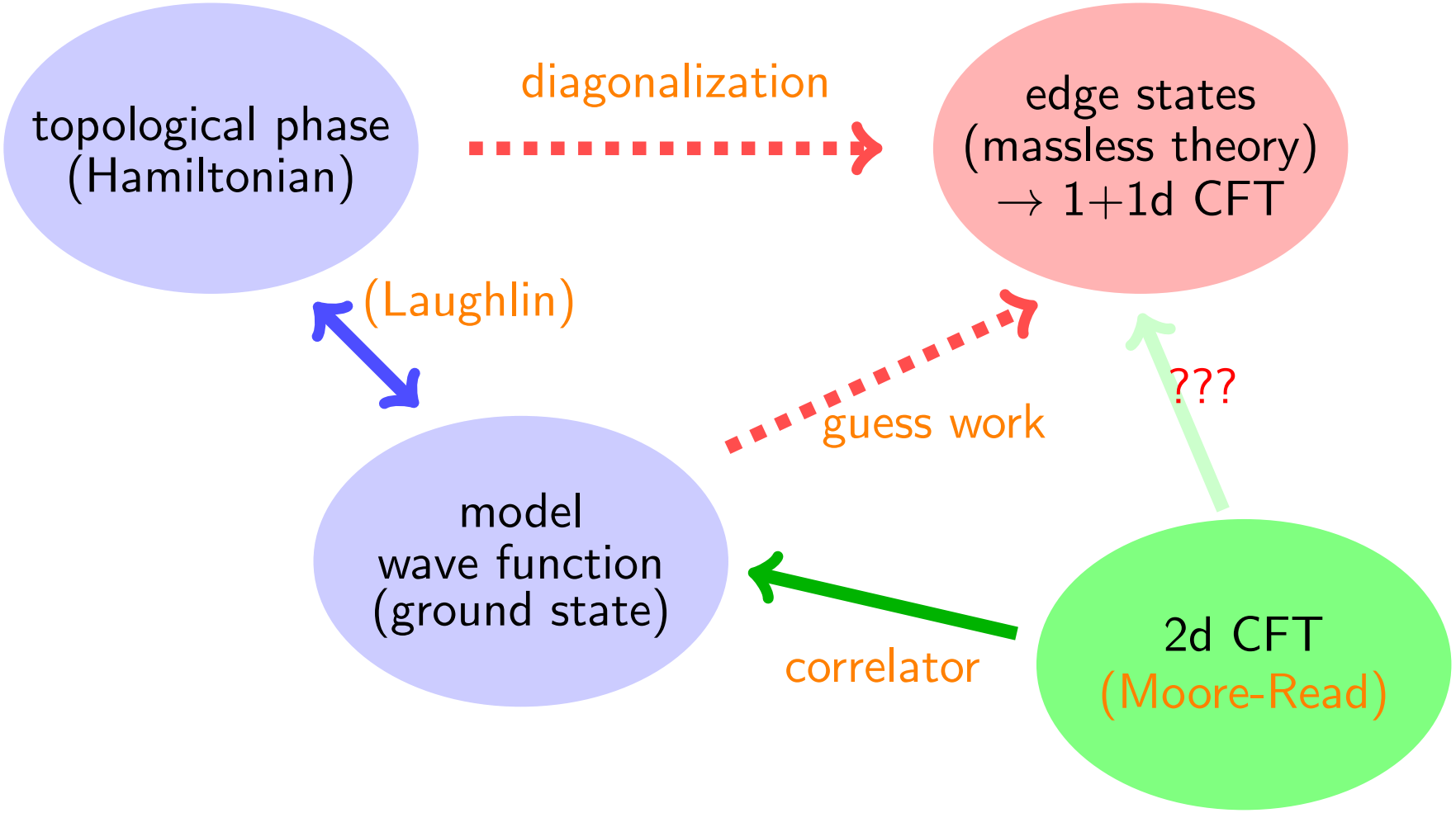
[Laughlin 1983]

$\longrightarrow$ a powerful Ansatz: use conformal blocks as w.f.
[Moore and Read, 1991]

Moore-Read construction



Moore-Read construction: what about the edge excitations?



Plan of the talk

- the space of edge excitations, and consequence of the short rangeness of bulk correlations (*a.k.a* "generalized screening hypothesis")
- real-space entanglement spectrum
- some remarks about trial wavefunctions

Part I. Edge state inner products

FQHE: trial wave functions as conformal blocks

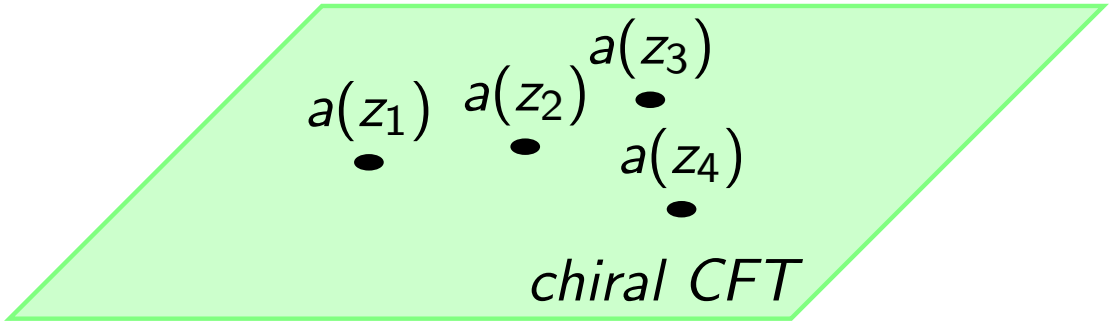
- N particles in the LLL

$$\psi(z_1, \ldots, z_N)$$

analytic in the z_i's, and antisymmetric (Fermi statistics)

- Moore-Read: trial wave function given by conformal block

$$\psi(z_1, \ldots, z_N) = \langle a(z_N) \ldots a(z_1) \rangle$$



Edge states (I)

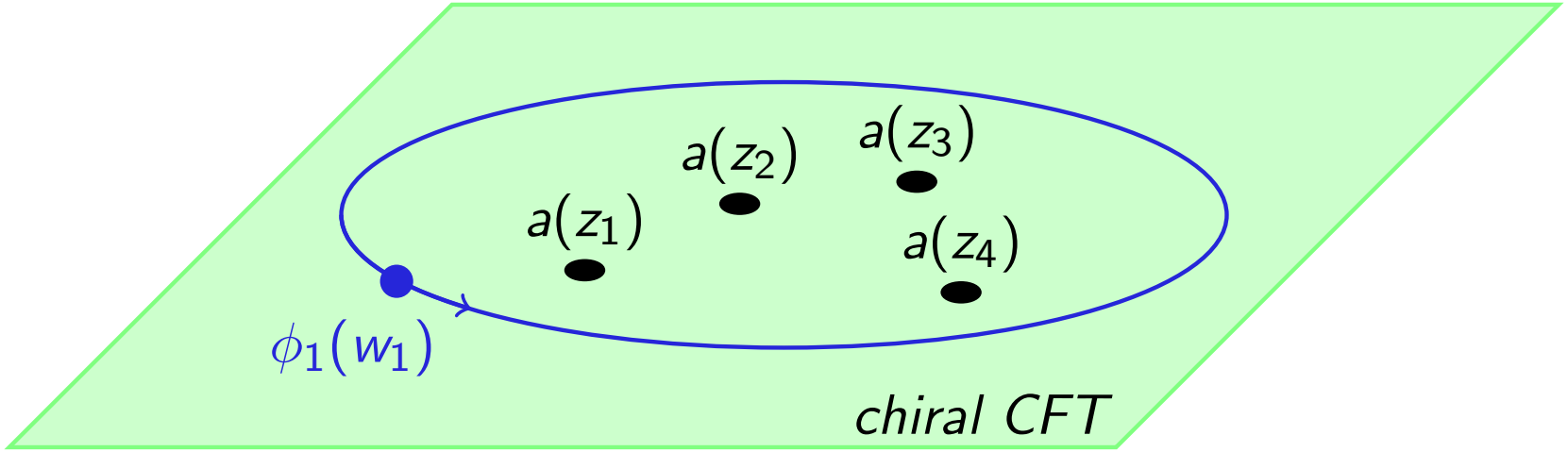
- the ground state w.f. is defined as

$$\psi_{g.s}(z_1, \ldots, z_N) = \langle a(z_1) \ldots a(z_N) \rangle$$

- to construct the edge states w.f., we insert contour integrals

$$\left\langle \oint w_1^{n_1} \phi(w_1) dw_1 \ldots \oint w_k^{n_k} \phi(w_k) dw_k \; a(z_1) \ldots a(z_N) \right\rangle$$

$\phi_j(w_j)$ is in the chiral algebra generated by $a(w)$, $a^\dagger(w)$



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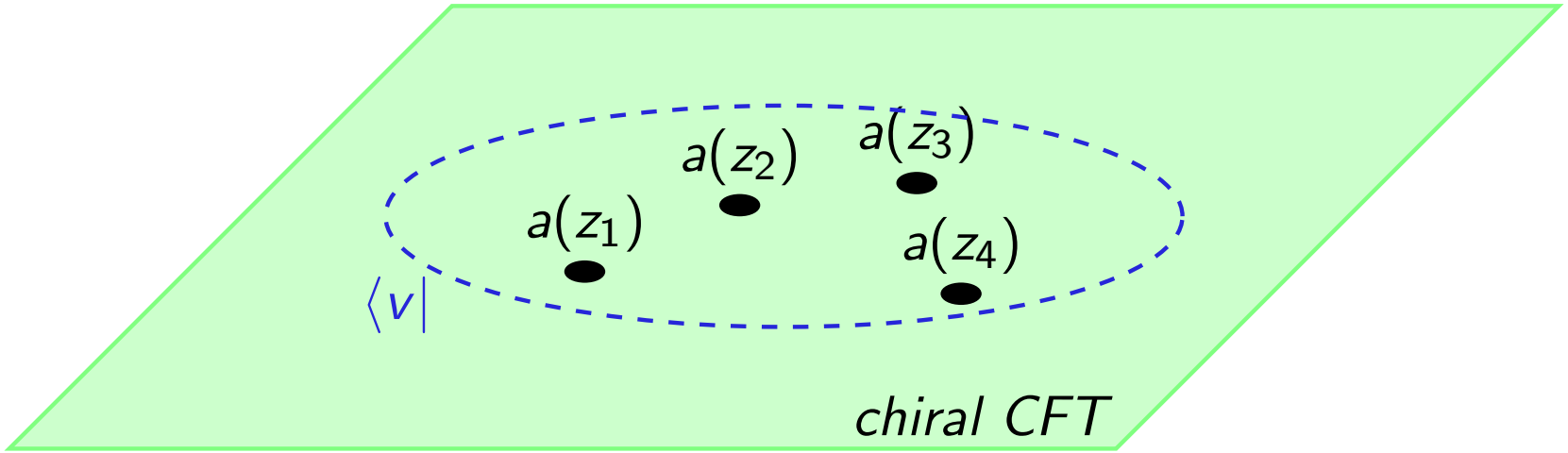
- in other words (radial quantization):

(dual) CFT Hilbert sp.	$\longrightarrow$	*edge theory Hilbert sp.*
$\langle v \vert$	$\longmapsto$	$\psi_{\langle v \vert} = \langle v \vert a(z_1) \ldots a(z_N) \vert 0 \rangle$



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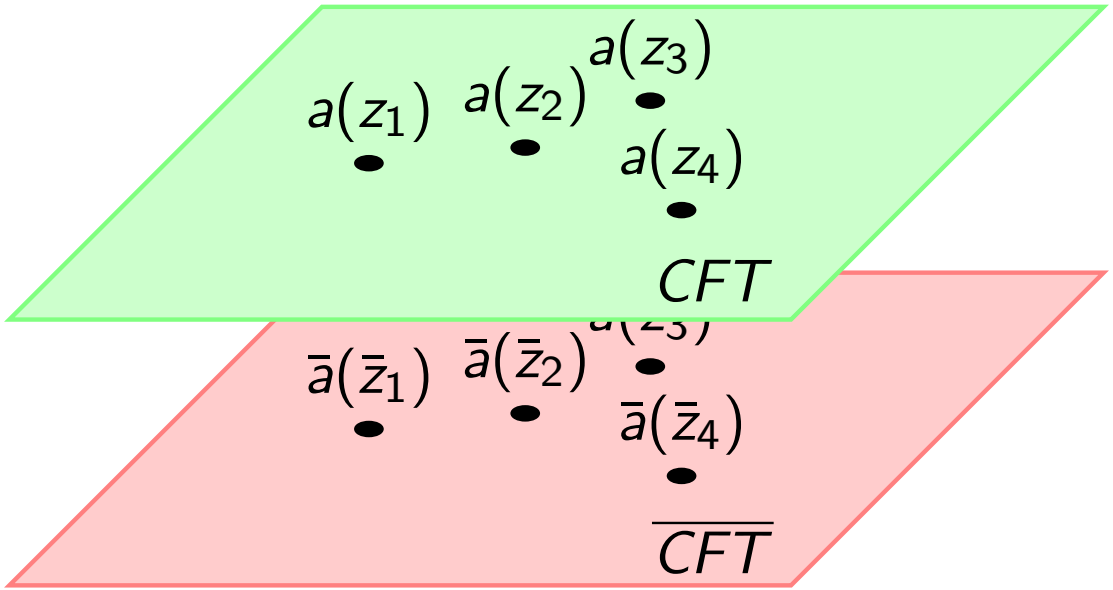
- straightforward generalization of the previous constructions of edge states for specific w.f.
  an example (Laughlin):

$$\psi_{\langle J_{n_1} J_{n_2} \vert}(\{z_i\}) \;\propto\; \sum_i z_i^{n_1} \sum_j z_j^{n_2} \;\times \prod_{k<l} (z_k - z_l)^{1/\nu}$$

Screening hypothesis (I)

$$Z_N = \frac{1}{N!}\int_{\mathbb{C}^N}\prod_{i=1}^{N} e^{V(z_i,\bar{z}_i)}d^2z_i\ |\psi(z_1,\ldots,z_N)|^2$$

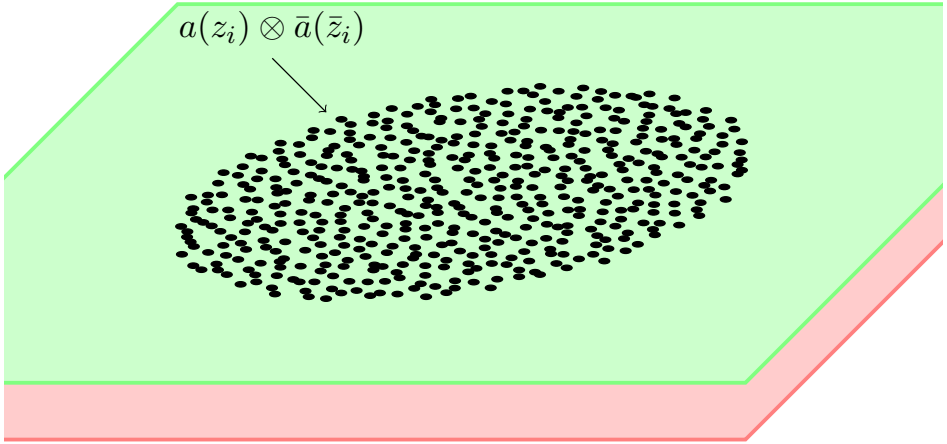
$$|\psi(z_1,\ldots,z_N)|^2 = \langle a(z_1)\ldots a(z_N)\rangle \times \langle \bar{a}(\bar{z}_1)\ldots \bar{a}(\bar{z}_N)\rangle$$



Screening hypothesis (II)

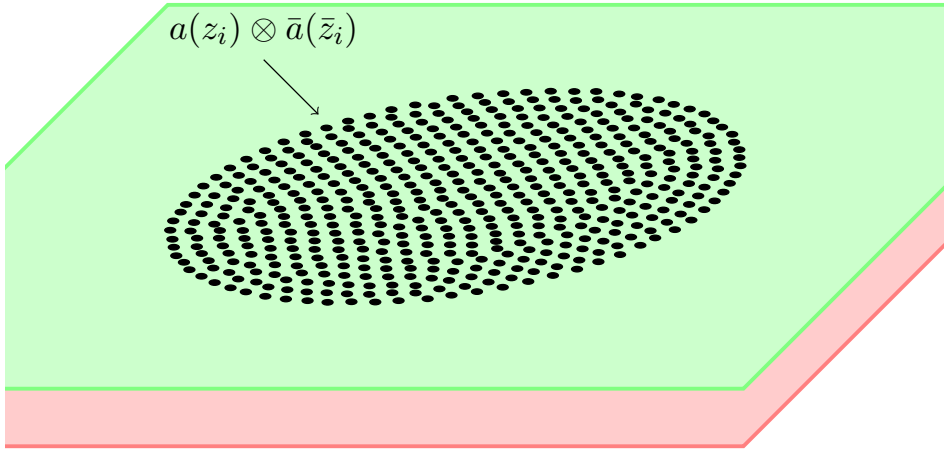
$$Z_N = \frac{1}{N!}\int \prod e^{V(z_i,\bar{z}_i)} d^2 z_i \quad \langle a(z_1)\dots a(z_N)\rangle \times \langle \bar{a}(\bar{z}_1)\dots \bar{a}(\bar{z}_N)\rangle$$

Screening



short-range correl. only

No screening



some long-range correl.
$\Rightarrow$ NOT a gapped phase of matter

Screening hypothesis (II-bis)

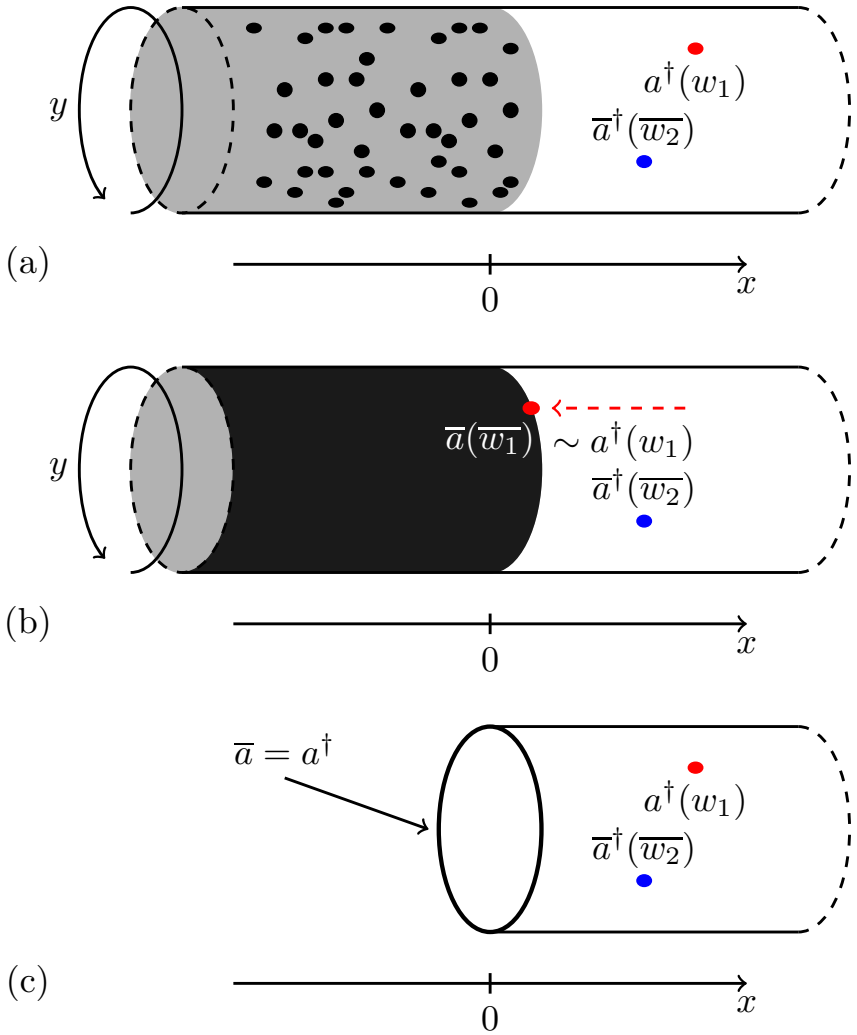
- plasma mapping [Laughlin, 1983]
- fractional statistics [Arovas, Schrieffer, Wilczek, 1984]
- screening hypothesis and massive fixed points [Nayak, Wilczek, 1996]
- discussions about Haldane-Rezayi state, hints about long range correlations [Read and Green, 2000]

more recently:

- generalized screening $\Rightarrow$ "holonomy = monodromy" [Read, 2009]
- construction of a plasma mapping for the MR (Pfaffian) w.f. [Bonderson, Gurarie, Nayak, 2011] and other states [Bonderson et al., unpublished] + numerical evidence for screening phase
- in this talk: assume short-rangeness of bulk correlations, and deduce nice properties

Screening hypothesis (II-bis-bis)

$$\left\langle a^\dagger(w_1)\ \overline{a}^\dagger(\overline{w_2})\ e^{\int_{gray} a(z)\otimes\overline{a}(\overline{z})d^2z}\ \ e^{i\rho_0\int_{gray}(\varphi(z)+\overline{\varphi(z)})d^2z}\right\rangle$$

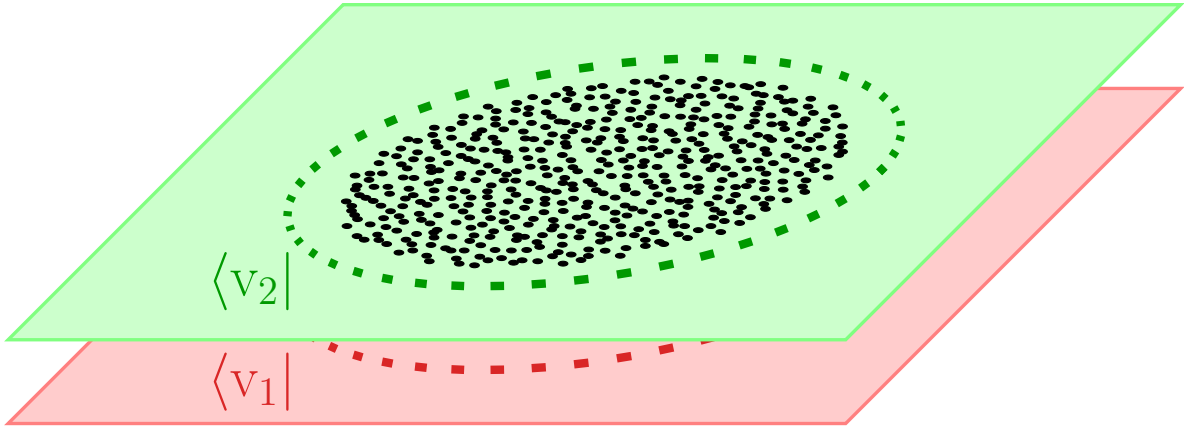


Edge states (II)

- quantum-mechanical inner product

$$\langle\langle\, \psi_{\langle v_1|} \,|\, \psi_{\langle v_2|} \,\rangle\rangle \;=\; \frac{1}{Z_N\, N!} \int_{\mathbb{C}^N} \prod_{i=1}^{N} e^{V} d^2 z_i \, \left[\psi_{\langle v_1|}(\{z_i\})\right]^* \psi_{\langle v_2|}(\{z_i\})$$

- $\psi_{\langle v_2|}(\{z_i\}) \left[\psi_{\langle v_1|}(\{z_i\})\right]^* = \langle v_2|\, a(z_1) \ldots |0\rangle \overline{\langle v_1|\, a(z_1) \ldots |0\rangle}$



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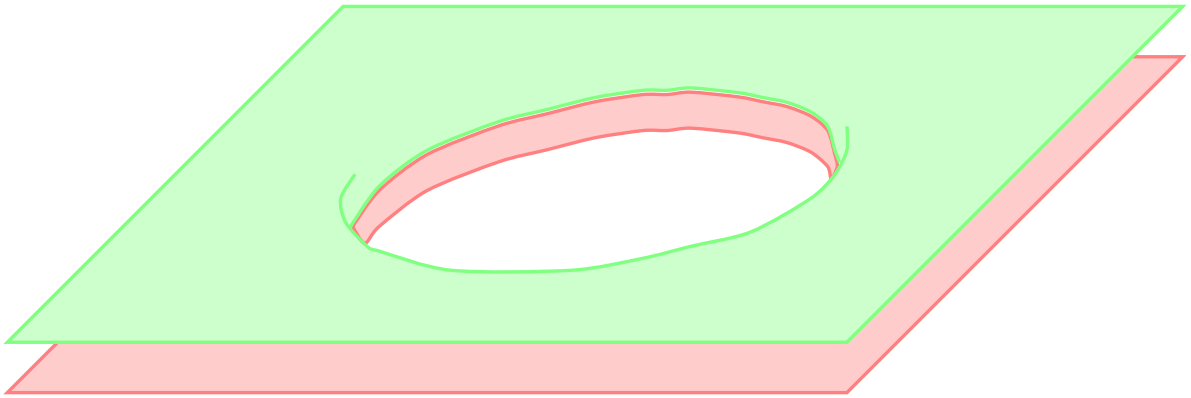

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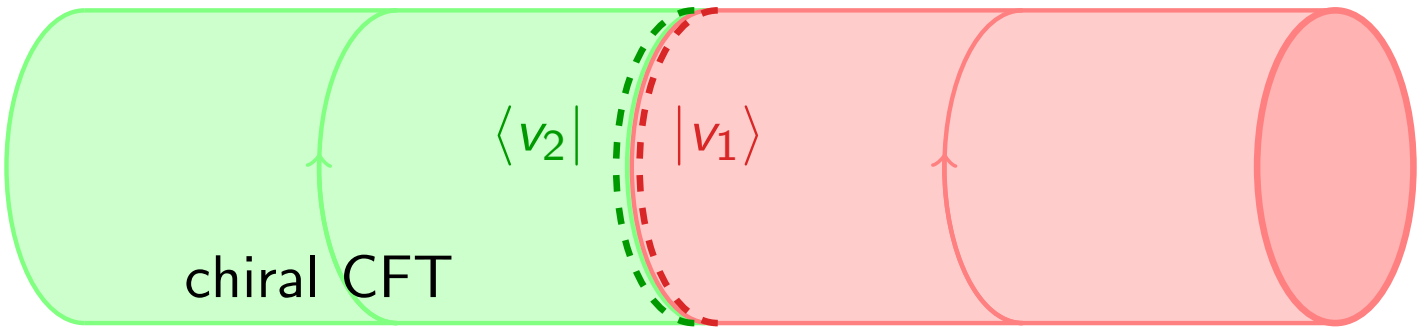


- Consequence: $\langle\langle\, \psi_{\langle v_1|} \,|\, \psi_{\langle v_2|} \,\rangle\rangle \;=\; \langle v_2 \,|v_1\rangle$ when $N \to \infty$.

Bulk/edge correspondence

Short-rangeness of bulk correlations implies that

(dual) CFT Hilbert sp. *inner product* $\langle . \,\vert . \rangle$	$\longrightarrow$	*edge theory Hilbert sp.* *inner product* $\langle\langle . \,\vert . \rangle\rangle$
$\langle v \vert$	$\longmapsto$	$\psi_{\langle v \vert} = \langle v \,\vert a(z_1) \ldots a(z_N) \vert\, 0 \rangle$

is an isometric isomorphism in the thermodynamic limit:

$$\langle\langle\, \psi_{\langle v_1 \vert} \,\vert\, \psi_{\langle v_2 \vert} \,\rangle\rangle \;=\; \langle v_2 \,\vert v_1 \rangle$$

(for the Laughlin w.f., similar argument appeared first in [Wen, 1992], which Parsa explained in his talk)

Isometric isomorphism: an example

MR (Pfaffian) wavefunction: $\prod_{i<j}(z_i - z_j)^2 \, Pf\left\{\frac{1}{z_k - z_l}\right\}$

- $U(1)$ excitations: multiplication by

$$S_n = \frac{1}{\sqrt{\nu}} \sum_{i=1}^{N} z_i^n$$

- fermionic excitations: replace $Pf\left\{\frac{1}{z_k - z_l}\right\}$ by

$$Pf \begin{pmatrix} 0 & 0 & z_1^{n_1-\frac{1}{2}} & z_2^{n_1-\frac{1}{2}} & \cdots & z_N^{n_1-\frac{1}{2}} \\ 0 & 0 & z_1^{n_2-\frac{1}{2}} & z_2^{n_2-\frac{1}{2}} & \cdots & z_N^{n_2-\frac{1}{2}} \\ -z_1^{n_1-\frac{1}{2}} & -z_1^{n_2-\frac{1}{2}} & 0 & \frac{1}{z_1-z_2} & \cdots & \frac{1}{z_1-z_N} \\ -z_2^{n_1-\frac{1}{2}} & -z_2^{n_2-\frac{1}{2}} & \frac{-1}{z_1-z_2} & 0 & \ddots & \\ \vdots & \vdots & \vdots & \ddots & \ddots & \frac{1}{z_{N-1}-z_N} \\ -z_N^{n_1-\frac{1}{2}} & -z_N^{n_2-\frac{1}{2}} & \frac{-1}{z_1-z_N} & & \frac{-1}{z_{N-1}-z_N} & 0 \end{pmatrix}$$

Isometric isomorphism: an example

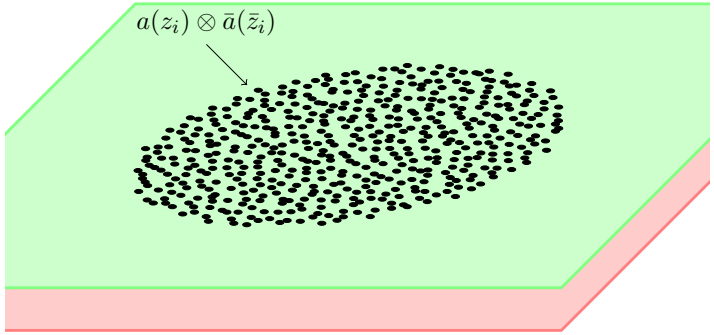
We compute the overlaps with Monte-Carlo: $U(1)$ sector:

	MC (N=100)	analytic
$\langle\langle\Psi \mid S_1^* S_1 \mid \Psi\rangle\rangle$	1.002	$\left\langle J_1 J_{-1}\right\rangle = 1$
$\langle\langle\Psi \mid S_2^* S_2 \mid \Psi\rangle\rangle$	1.993	$\left\langle J_2 J_{-2}\right\rangle = 2$
$\langle\langle\Psi \mid (S_1^*)^2 S_2 \mid \Psi\rangle\rangle$	0.005	$\left\langle J_2 J_{-1}^2\right\rangle = 0$
$\langle\langle\Psi \mid (S_1^*)^2 (S_1)^* \mid \Psi\rangle\rangle$	1.995	$\left\langle J_1^2 J_{-1}^2\right\rangle = 2$
$\langle\langle\Psi \mid (S_2^*)^2 (S_2)^2 \mid \Psi\rangle\rangle$	7.934	$\left\langle J_2^2 J_{-2}^2\right\rangle = 8$

fermionic sector:

	MC ($N = 100$)	analytic ($N \to \infty$)
$\langle\langle\Psi \mid F^*_{\frac{1}{2},\frac{3}{2}} F_{\frac{1}{2},\frac{3}{2}} \mid \Psi\rangle\rangle$	1.124	$\left\langle \psi_{\frac{1}{2}} \psi_{\frac{3}{2}} \psi_{-\frac{3}{2}} \psi_{-\frac{1}{2}} \right\rangle = 1$
$\langle\langle\Psi \mid F^*_{\frac{1}{2},\frac{5}{2}} F_{\frac{1}{2},\frac{5}{2}} \mid \Psi\rangle\rangle$	1.151	$\left\langle \psi_{\frac{1}{2}} \psi_{\frac{5}{2}} \psi_{-\frac{5}{2}} \psi_{-\frac{1}{2}} \right\rangle = 1$
$\langle\langle\Psi \mid F^*_{\frac{3}{2},\frac{5}{2}} F_{\frac{3}{2},\frac{5}{2}} \mid \Psi\rangle\rangle$	1.189	$\left\langle \psi_{\frac{3}{2}} \psi_{\frac{5}{2}} \psi_{-\frac{5}{2}} \psi_{-\frac{3}{2}} \right\rangle = 1$
$\langle\langle\Psi \mid F^*_{\frac{1}{2},\frac{7}{2}} F_{\frac{3}{2},\frac{5}{2}} \mid \Psi\rangle\rangle$	-0.001	$\left\langle \psi_{\frac{3}{2}} \psi_{\frac{5}{2}} \psi_{-\frac{7}{2}} \psi_{-\frac{1}{2}} \right\rangle = 0$
$\langle\langle\Psi \mid F^*_{\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}} F_{\frac{1}{2},\frac{3}{2},\frac{5}{2},\frac{7}{2}} \mid \Psi\rangle\rangle$	1.381	$\left\langle \psi_{\frac{1}{2}} \psi_{\frac{3}{2}} \psi_{\frac{5}{2}} \psi_{\frac{7}{2}} \psi_{-\frac{7}{2}} \psi_{-\frac{5}{2}} \psi_{-\frac{3}{2}} \psi_{-\frac{1}{2}} \right\rangle = 1$

Subleading corrections

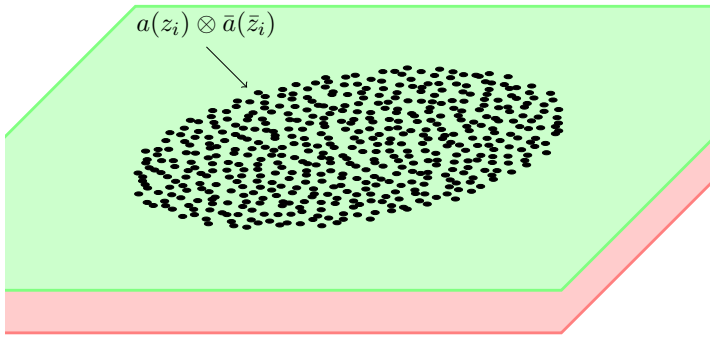


- for infinite density, conformal b.c. $\overline{a}(\overline{z}) = a^\dagger(z)$
- idea: at finite-density, one should perturb this b.c. by local operators

$$S_{CFT} \to S_{CFT} + \sum_a \lambda_a \int dz\ \phi_a(z)$$

- coupling scales like $\lambda_a \propto \ell_B^{h_a - 1}$
- relevant $h_a < 1$, marginal $h_a = 1$
- irrelevant $h_a > 1$

Subleading corrections



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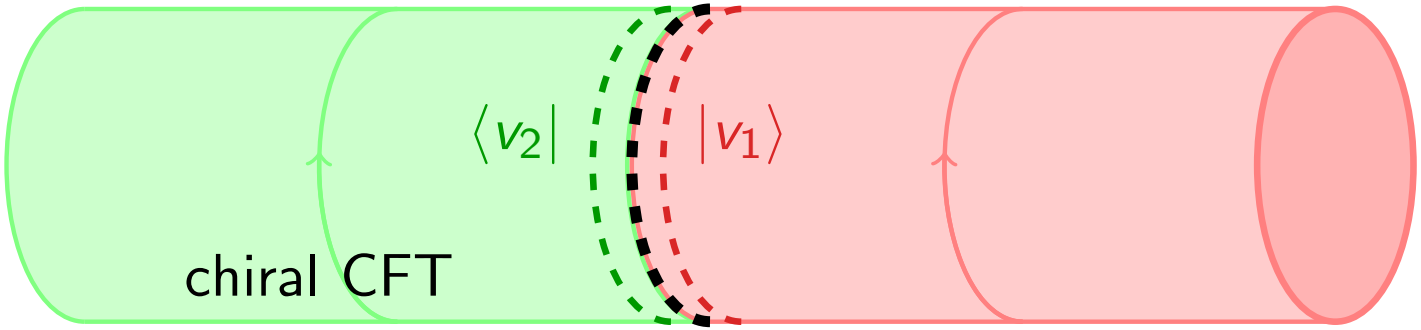


$$e^{-S_{\text{pert.}}} = e^{-\sum \lambda_a \int dz\ \phi_a(z)}$$

$$\langle\langle\, \psi_{\langle v_1|} \,|\, \psi_{\langle v_2|} \,\rangle\rangle \;=\; \langle v_2|\, e^{-S_{\text{pert.}}}\, |v_1\rangle$$

Subleading corrections

An example: for the Laughlin wavefunction

$$S_{\text{pert.}} = \frac{\lambda_3}{\sqrt{N}^2} \oint \frac{dz}{2\pi i} z^2 (i\partial\varphi)^3(z) + \frac{\lambda_4}{\sqrt{N}^3} \oint \frac{dz}{2\pi i} z^3 (i\partial^2\varphi)^2(z) + \dots$$

which gives the corrections to scaling

$$\begin{aligned} \left\langle\left\langle \psi_{\langle v_1|} \middle| \psi_{\langle v_2|} \right\rangle\right\rangle &= \langle v_2 | v_1 \rangle + \frac{\lambda_3}{N} \langle v_2 | \sum_{k,p>0} J_{-k} J_{-p} J_{k+p} | v_1 \rangle \\ &\quad + \frac{\lambda_4}{\sqrt{N}^{3/2}} \langle v_1 | \sum_{k>0} \left(k^2 - 1\right) J_{-k} J_k | v_2 \rangle + \dots \end{aligned}$$

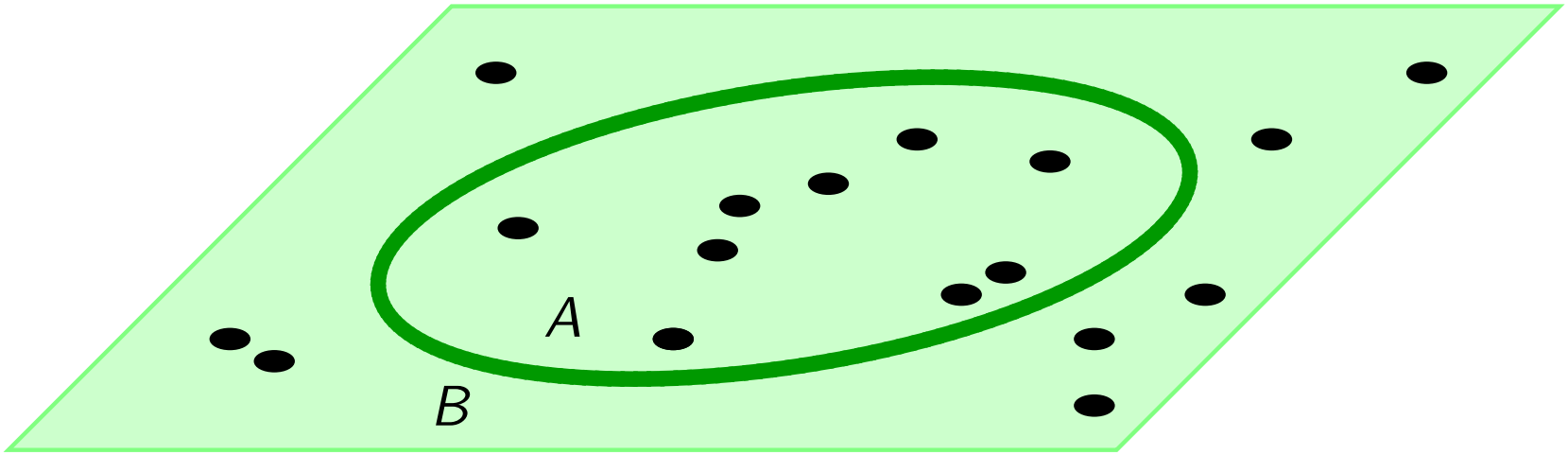
Note: using the machinery from [Zabrodin Wiegmann 2006], it is possible (although, painful) to recover this formula. Their interpretation of this result, however, seems to be different (the $c < 1$ story).

Part II. Entanglement spectrum

Application to the entanglement spectrum (I)

- what's the Schmidt decomp. of $\psi_{g.s}(z_1, \ldots, z_N)$?

$$|\psi_{g.s}\rangle\rangle = \sum_{N_A=0}^{N} \sum_{n} e^{-\xi_n/2} \left|\psi_n^A\right\rangle\rangle \otimes \left|\psi_n^B\right\rangle\rangle$$



- we focus on Real Space Partition

Application to the entanglement spectrum (II)

- divide the $N = N_A + N_B$ coordinates z_i's into two sets

$$z_1, \ldots, z_{N_A} \qquad\qquad w_1 = z_{N_A+1}, \ \ldots, \ w_{N_B} = z_{N_A+N_B}$$

- because $\psi_{g.s}$ is a conformal correlator, one has

$$\begin{aligned}
\psi_{g.s}(\{z_j\}, \{w_k\}) &= \langle a(z_1)..a(z_{N_A})\, a(w_1)..a(w_{N_B})\rangle \\
&= \sum_{|v_n\rangle} \langle\, a(z_1)..a(z_{N_A})|\, v_n\rangle\, \langle v_n\, |a(w_1)..a(w_{N_B})\,\rangle \\
&= \sum_{|v_n\rangle} \psi_{\langle v_n|}(\{z_j\})\, \tilde{\psi}_{\langle v_n|}(\{w_k\})
\end{aligned}$$

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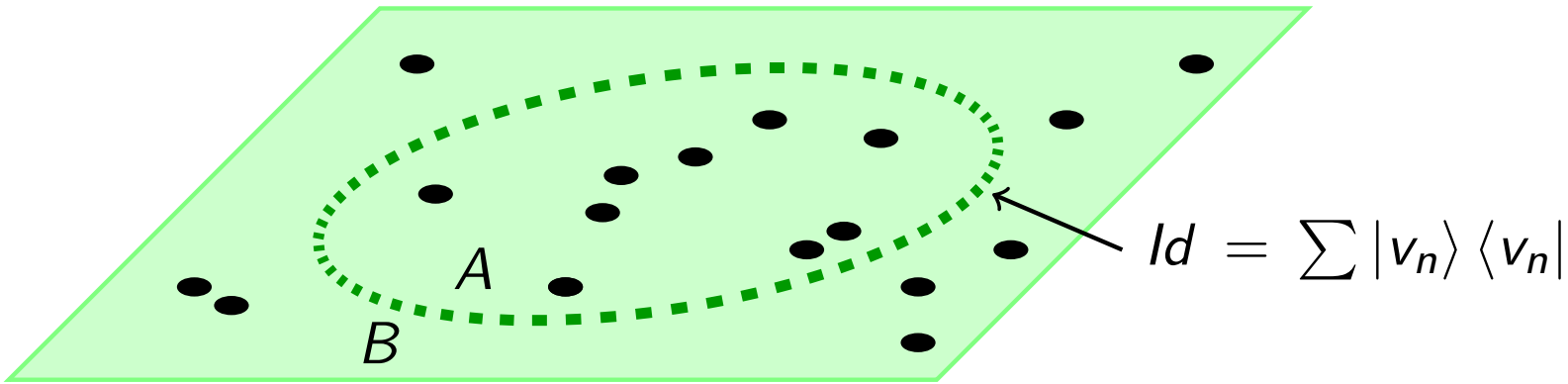

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$$\begin{aligned}
\psi_{g.s}(\{z_j\}, \{w_k\}) &= \langle a(z_1)..a(z_{N_A})\, a(w_1)..a(w_{N_B})\rangle \\
&= \sum_{|v_n\rangle} \langle\, a(z_1)..a(z_{N_A})|\, v_n\rangle\, \langle v_n\, |a(w_1)..a(w_{N_B})\,\rangle \\
&= \sum_{|v_n\rangle} \psi_{\langle v_n|}(\{z_j\})\, \tilde{\psi}_{\langle v_n|}(\{w_k\})
\end{aligned}$$

- we get the entanglement spectrum using the overlaps

$$\left\langle\left\langle \psi_{\langle v_n|}\, \middle|\psi_{\langle v_m|}\right\rangle\right\rangle = \langle v_m|\, e^{-S_{\mathrm{pert.}}}\, |v_n\rangle$$

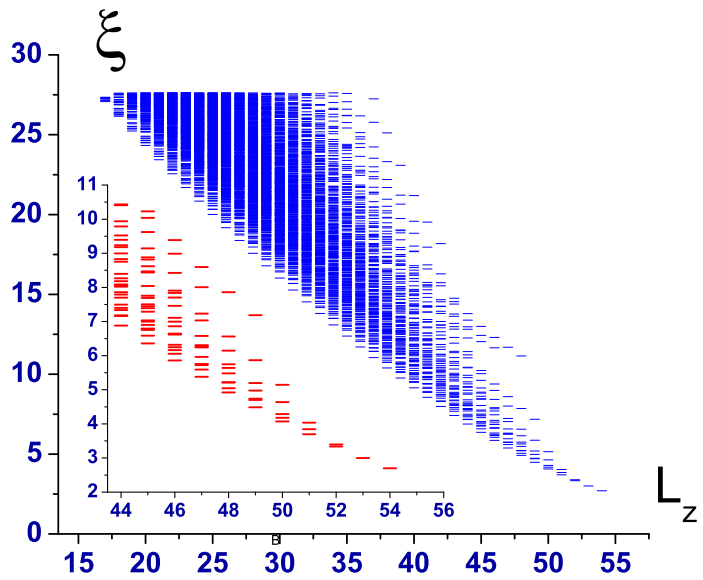
Application to the entanglement spectrum (III)

This proves the so-called "scaling property" [JD-Read-Rezayi 2012]

the entanglement spectrum is the spectrum of the Hamiltonian of a 1+1d CFT perturbed by local operators

$$-\log \rho_A \equiv \alpha R - \gamma_{\rm topo} + \frac{v_F}{R} L_0 + \sum_a \lambda_a \int_{|z|=R} \phi_a(z)|dz|$$

locality of the entanglement Hamiltonian $H_E - \log \rho_A$



Part III. Remarks: trial states, entanglement, and correlators in local field theories

Question:

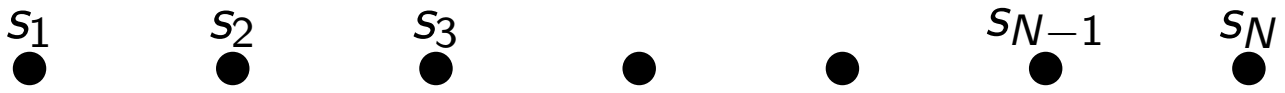
"Why should FQHE wavefunctions have anything to do with conformal blocks?"

Trial wave functions in 1d

In 1d, short-rangeness of correlations implies that the ground-state can be approximated by a Matrix Product State

$$\psi(s_1, \ldots, s_N) = \langle v| \, M(s_1) M(s_2) \ldots M(s_N) \, |u\rangle$$

where $M(s_i)$ is a $D \times D$ matrix, and $|u\rangle, |v\rangle$ are D-dim vectors



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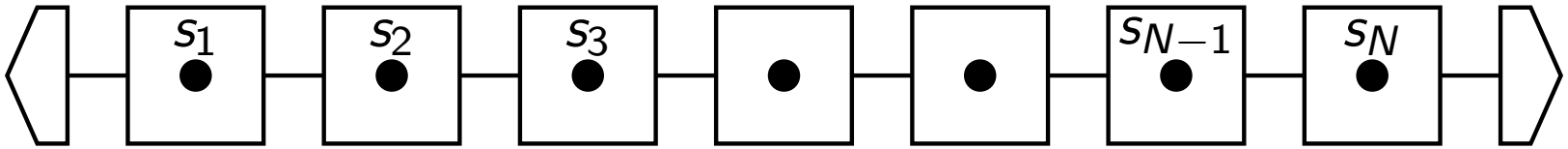
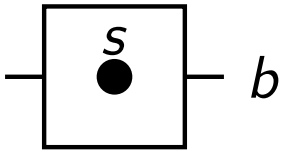
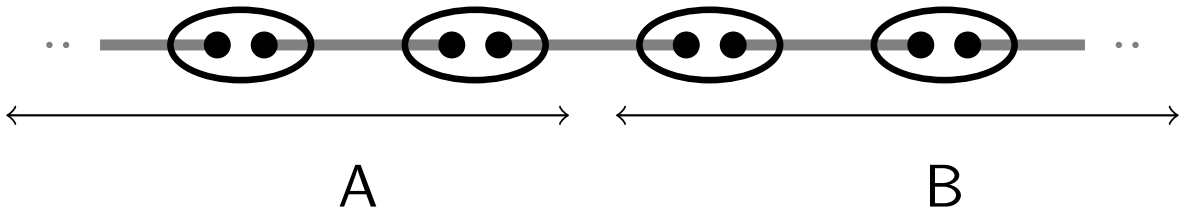


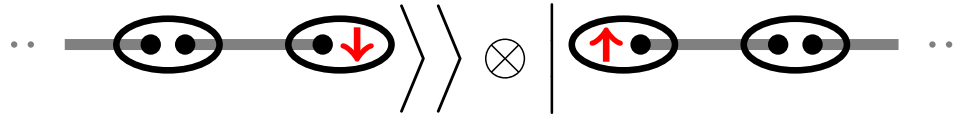
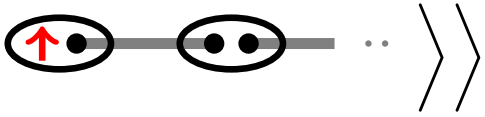
$[M(s)]_{ab} =$ 

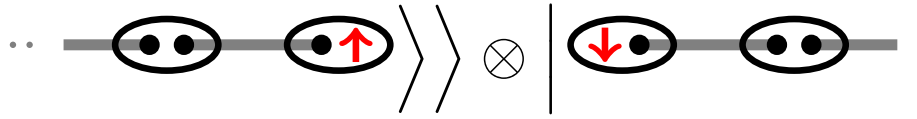
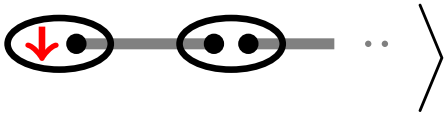
Entanglement

Example: AKLT chain (spin 1) is an MPS with $D = 2$



- Schmidt decomposition for bipartition $\mathcal{H}_A \otimes \mathcal{H}_B$

$|AKLT\rangle\rangle = \lambda_1 \Big|$  $\Big\rangle\rangle \otimes \Big|$  $\Big\rangle\rangle$

$+\lambda_2 \Big|$  $\Big\rangle\rangle \otimes \Big|$  $\Big\rangle\rangle$

Entanglement

- the Schmidt rank is always $\leq D$ for an MPS

$$|MPS\rangle\rangle = \sum_{i=1}^{D} \lambda_i \, |A\rangle\rangle \otimes |B\rangle\rangle$$

huge reduction compared to $min(dim\mathcal{H}_A, dim\mathcal{H}_B)$

- follows from inserting

$$1 = \sum_{i=1}^{D} |v_i\rangle \langle v_i|$$

in $\psi(s_1, \ldots, s_N) = \langle v| A(s_1) \ldots A(s_{N_A}) A(s_{N_A+1}) \ldots A(s_N) |u\rangle$

Matrix Product States (MPS)

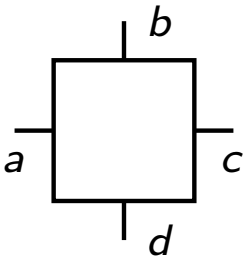
Ideas in the air, from the Quatum Information community:

- Hilbert space of a many-body system is huge
- however, physical states are far from generic: they have short-range correlations only (for gapped phases of matter)
- states with short-range correlations can be approximated with arbitray accuracy by MPS with controlable D
- MPS are optimal, in the sense that they are minimally entangled

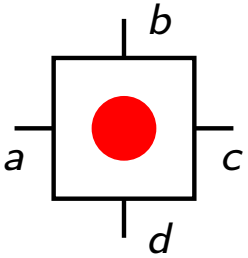
- these ideas are supported by several theorems [Hastings, . . .]

- recently, MPS played a key role in classification of 1d topological phases [Fidkowski+Kitaev, Chen+Gu+Wen, Turner+Pollman+Berg, Schuch+Perez-Garcia+Cirac, 2011]

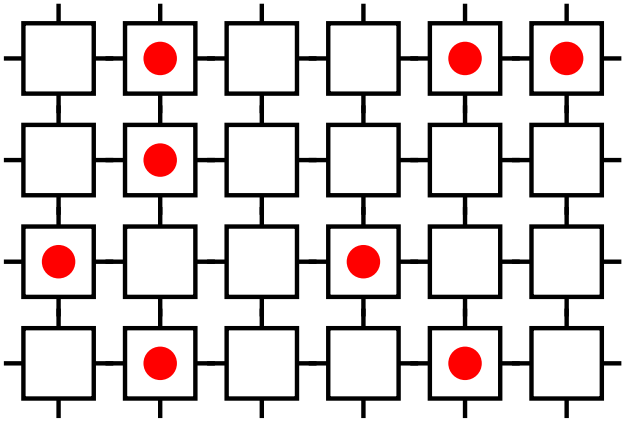
Trial states for mobile particles in 2d

tensor with 4 indices: $T^0_{abcd} =$ 

no particle

$T^1_{abcd} =$ 

one particle



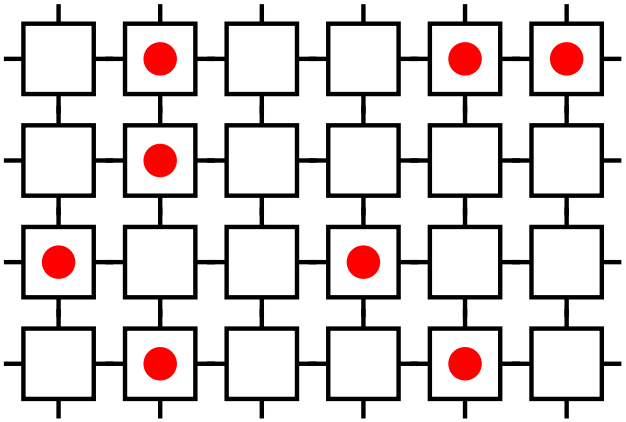
$$\psi(p_1, \ldots, p_N) = \sum_{edge\ ind.} \prod_{plaq.} T^{\epsilon_p}_{a_p b_p c_p d_p}$$

"Tensor Product State"

Trial states for mobile particles in 2d

Now,

- relabel the edge indices, $a, b, c, d \to \varphi(e)$
- $\sigma[p, \varphi] = \log T^0_{\varphi(e_1)\varphi(e_2)\varphi(e_3)\varphi(e_4)}$
- write $V[p, \varphi] = \frac{T^1_{\varphi(e_1)\varphi(e_2)\varphi(e_3)\varphi(e_4)}}{T^0_{\varphi(e_1)\varphi(e_2)\varphi(e_3)\varphi(e_4)}}$



$$\psi(p_1, \ldots, p_N) = \sum_{\varphi(e)\, conf.} V[p_1, \varphi] \ldots V[p_N, \varphi] \prod_{plaq.} e^{\sigma[}$$

Trial states for mobile particles in 2d

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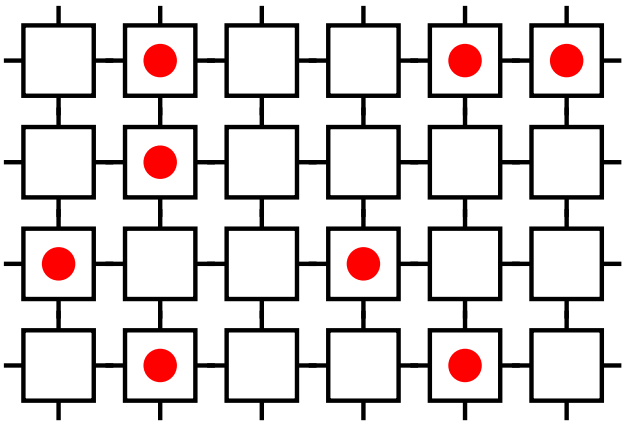
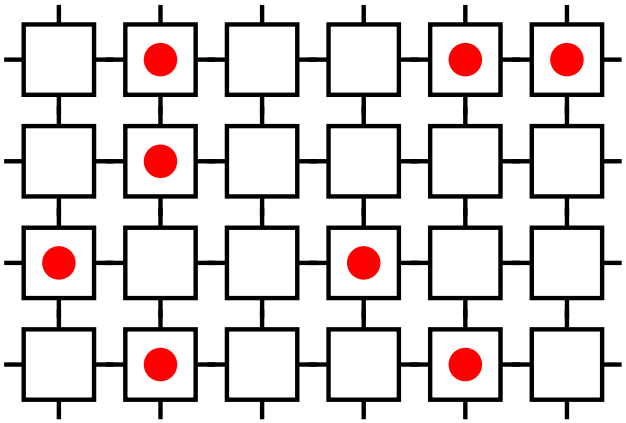


$$\psi(p_1, \ldots, p_N) = \sum_{\varphi(e)\, conf.} V[p_1, \varphi] \ldots V[p_N, \varphi] e^{S[\varphi]}$$

$$\text{with} \quad S[\varphi] = \sum_{plaq.} \sigma[p, \varphi]$$

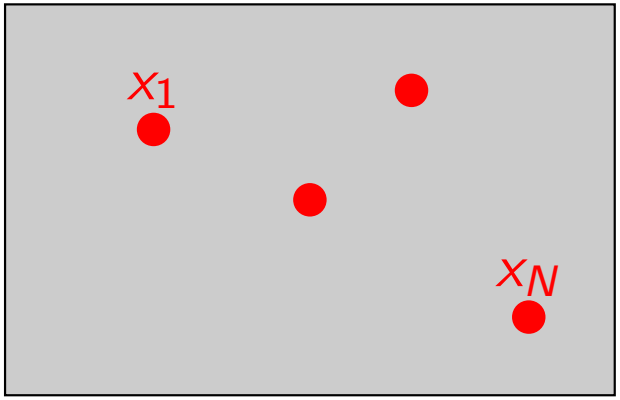
Trial states for mobile particles in 2d

Tensor Product State for particles on a lattice:



$$\psi(p_1, \ldots, p_N) = \sum_{\varphi(e)\, conf.} V[p_1, \varphi] \ldots V[p_N, \varphi] e^{S[\varphi]}$$

continuous version of a Tensor Product State:



$$\psi(x_1, \ldots, x_N) = \int \mathcal{D}[\varphi]\ V[\varphi(x_1)] \ldots V[\varphi(x_N)]\ e^{S[\varphi]}$$

Main message: a continuous Tensor Product State is a correlator of local operators in a local field theory.

Less locality in the field theory means more entanglement for the Tensor Product State. Looking at correlators of local operators in a scale-invariant theory is a way of finding optimally entangled trial states.

Summary

- The use of conformal correlators as wavefunctions is not weird. It is natural. Moreover, a scale-invariant theory (in particular, a CFT) leads to minimally entangled trial states.

- For these states, provided short-rangeness of bulk correlators, there is an isometry between the physical space of edge states and the auxiliary space when $N \to \infty$

$$\left\langle\left\langle \psi_{\langle v_1|} \,\middle|\, \psi_{\langle v_2|} \right\rangle\right\rangle \underset{N\to\infty}{\longrightarrow} \langle v_2 | v_1 \rangle$$

- This follows from the CFT outside the droplet, which has a conformal boundary condition (+local perturbations)

$$\overline{a}(\overline{z}) = a^\dagger(z)$$

- The short-rangeness of bulk correlators implies that the entanglement Hamiltonian is local along the cut.

Thank you.