Kondo effect in mesoscopic and nanoscopic systems

Pablo S. Cornaglia

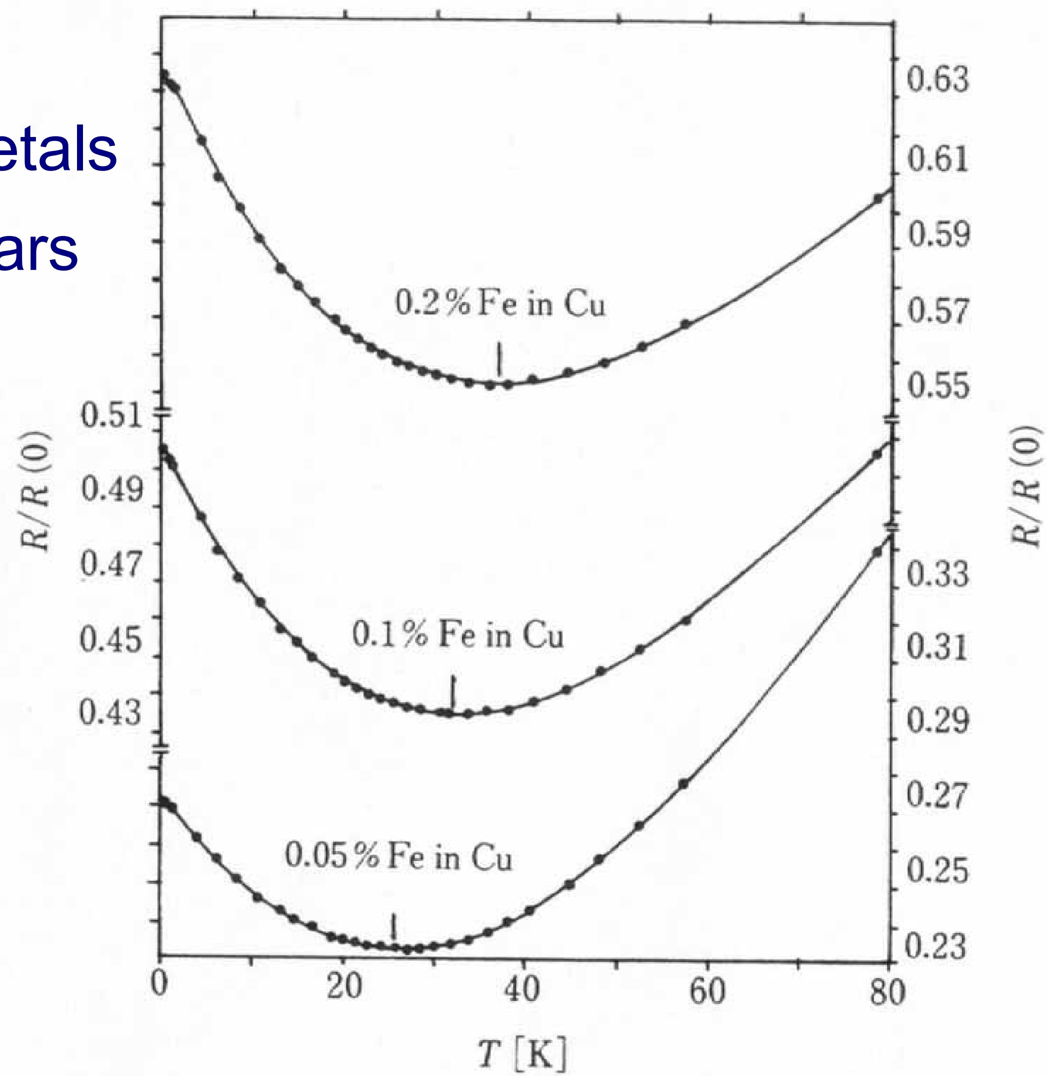
Centro Atómico Bariloche and Instituto Balseiro,

Comisión Nacional de Energía Atómica, Bariloche, Argentina

Consejo Nacional de Investigaciones Científicas y Técnicas (CONICET), Argentina

Kondo problem

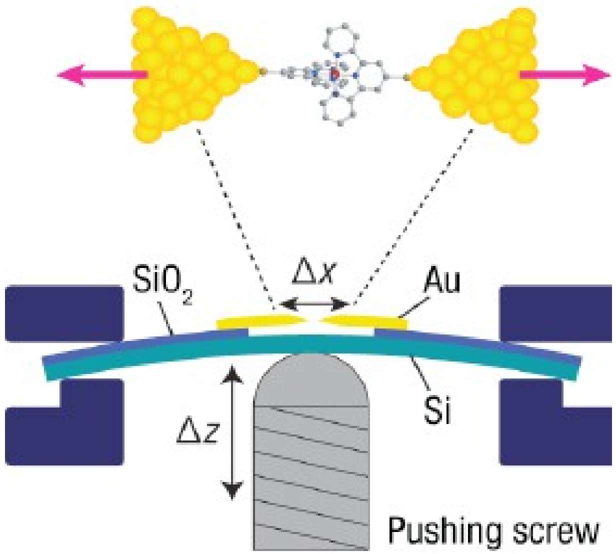
- Resistivity minimum in metals
- Origin unknown for 30 years
- Local magnetic moment coupled to a metal
- Lifting of a degeneracy by the coupling to a Fermi sea



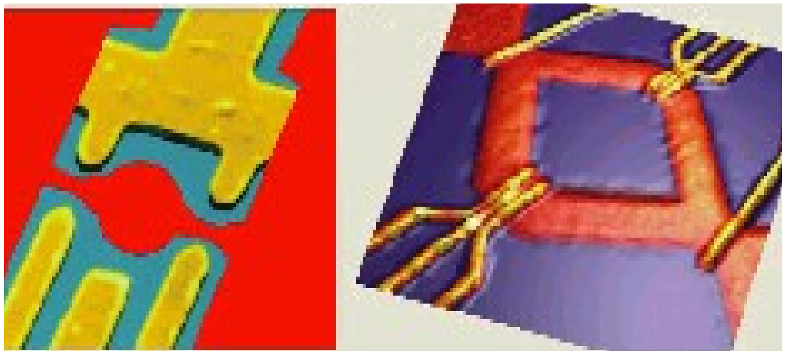
[Fe in Cu] J. P. Franck *et al.* Proc. R. Soc. Lond. **263** 494-507 (1961)

[Fe in Au] W.J. de Haas *et al.* , Physica, **1**, 1115 (1934)

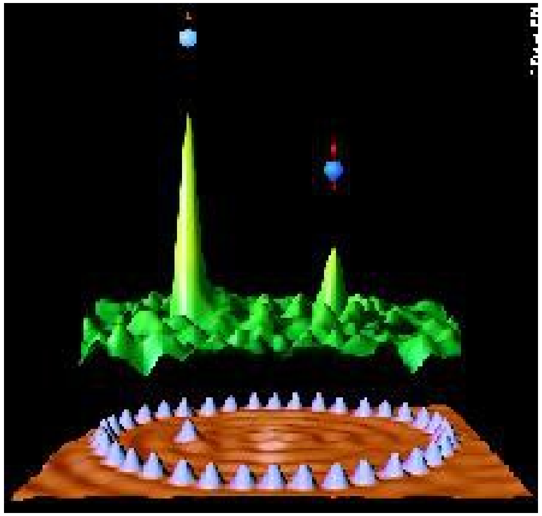
Kondo effect



Molecular junctions



Quantum dots



Magnetic Impurities

Quantum impurity problems as the Kondo problem are at the core of the dynamical mean field theory (DMFT) approach to strongly correlated materials

References

- A. C. Hewson, *The Kondo problem to Heavy Fermions* [book]
- P.W. Anderson, Nobel lecture (1977)
- P. W. Anderson, Phys. Rev. 124 (1961)
- J. Kondo Prog. Theor. Phys. **32** 37 (1964)
- K.G. Wilson, Rev. Mod. Phys. **47** 773 (1975)
- P. W. Anderson J. Phys. C: Solid State Phys. **3** 2436 (1970)

Program

- Lecture 1:
  - Models and methods: Anderson model, Hartree-Fock, Schrieffer-Wolff transformations, poor man's scaling.
- Lecture 2:
  - Methods: Numerical renormalization, Nozières Fermi liquid, Slave bosons
- Lecture 3:
  - Transport through nanostructures
  - Molecular transistors: effect of vibrations
- Lecture 4
  - Double quantum dots, S=1 molecules
  - Future trends: out of equilibrium, exotic states

Outline of lecture 1

- Introduction to the Kondo effect
- Anderson model
- Magnetic moment formation
- Schrieffer-Wolff transformations
    - From Anderson model to Kondo model
- Jun Kondo's explanation of the resistivity minumun in metals.
- Anderson's Poor man's scaling

Anderson model

One of my strongest stylistic prejudices in science is that many of the facts Nature confronts us with are so implausible given the simplicities of non-relativistic quantum mechanics and statistical mechanics, that the mere demonstration of a reasonable mechanism leaves no doubt of the correct explanation. This is so especially if it also correctly predicts unexpected facts

(...)

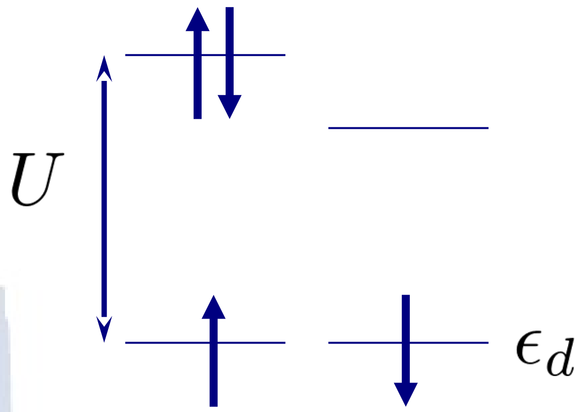
Very often such, a simplified model throws more light on the real workings of nature than any number of "ab initio" calculations of individual situations, which even where correct often contain so much detail as to conceal rather than reveal reality. It can be a disadvantage rather than an advantage to be able to compute or to measure too accurately, since often what one measures or computes is irrelevant in terms of mechanism. After all, the perfect computation simply reproduces Nature, does not explain her.

P.W. Anderson, Nobel lecture (1977)

Anderson model

- Fe impurity in a simple metal like Cu
- Impurity:
  - Localized *d* level in the magnetic impurity
  - Ignore orbital degeneracy and consider a single level with *spin-degeneracy.*
  - Take into account local electron-electron interaction.
- Metallic host
  - Described within Landau's Fermi-liquid theory.
  - Free electrons with renormalized parameters.

Anderson model



$$H = H_d + H_m + H_V$$

$$H_d = \epsilon_d(\hat{n}_\uparrow + \hat{n}_\downarrow) + U\hat{n}_\uparrow\hat{n}_\downarrow$$

$$\hat{n}_\sigma = d^\dagger_\sigma d_\sigma$$

$$H_m = \sum_k \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} \qquad \mathrm{E}_F = 0$$

$$H_V = \sum_k V_k c^\dagger_{k\sigma} d_\sigma + V^*_k d^\dagger_\sigma c_{k\sigma}$$

Isolated level:

$$E_0 = 0$$

$$E_1 = \epsilon_d$$

$$E_2 = 2\epsilon_d + U$$

P. W. Anderson, Phys. Rev. 124 (1961)

Definitions

- *Correlation function*

$$\langle\langle A, B\rangle\rangle_{t-t'} = -i\theta(t-t')\langle[A(t), B(t')]_+\rangle\varrho$$

$$\langle\langle A, B\rangle\rangle_E = \int d(t-t')\langle\langle A, B\rangle\rangle_{t-t'} e^{iE(t-t')}$$

$$\varrho(T) = \frac{1}{Z(T)}\sum_i e^{-E_i/T}|i\rangle\langle i| \qquad Z(T) = \sum_i e^{-E_i/T}$$

- *Green's function*

$$G_\sigma(E) = \langle\langle d_\sigma, d_\sigma^\dagger\rangle\rangle_E$$

- *d* level spectral density

$$A_{d\sigma}(E) = -\frac{1}{\pi}\mathrm{Im}[G_\sigma(E+i0)]$$

$$A_{d\sigma}(E, T=0) = \frac{1}{N_f}\sum_i \left[|\langle\Psi_0|d^\dagger|\Psi_i\rangle|^2\delta(E+E_i-E_0) + \langle\Psi_0|d|\Psi_i\rangle|^2\delta(E-(E_i-E_0))\right]$$

Anderson model (U=0)

- The Green's function is obtained using the equation of motion:

$$\frac{dG_\sigma(t-t')}{dt} = \frac{d}{dt}\left(-i\theta(t-t')\langle[\hat{d}_\sigma(t),\hat{d}_\sigma^\dagger(t')]_+\rangle\varrho\right)$$

$$\frac{d}{dt}\hat{d}_\sigma(t) = i[H,\hat{d}_\sigma(t)]$$

$$(E-\epsilon_d)G_\sigma(E) - \sum_k V_k\langle\langle c_{k\sigma}, d_\sigma^\dagger\rangle\rangle = 1$$

$$(E-\epsilon_k)\langle\langle c_{k\sigma}, d_\sigma^\dagger\rangle\rangle - V_k G_\sigma(E) = 0$$

$$G_\sigma(E) = \frac{1}{E-\epsilon_d-\mathrm{Re}\Gamma(E) - i\mathrm{Im}\Gamma(E)} \qquad \Gamma(E) = \sum_k \frac{|V_k|^2}{E-\epsilon_k}$$

Anderson model (U=0)

- The localized level is broadened by the coupling to the electron bath. The broadening is determined by the hybridization function:

$$\Delta(E) = \mathrm{Im}[\Gamma(E)] = \pi \sum_k |V_k|^2 \delta(E - \epsilon_k)$$

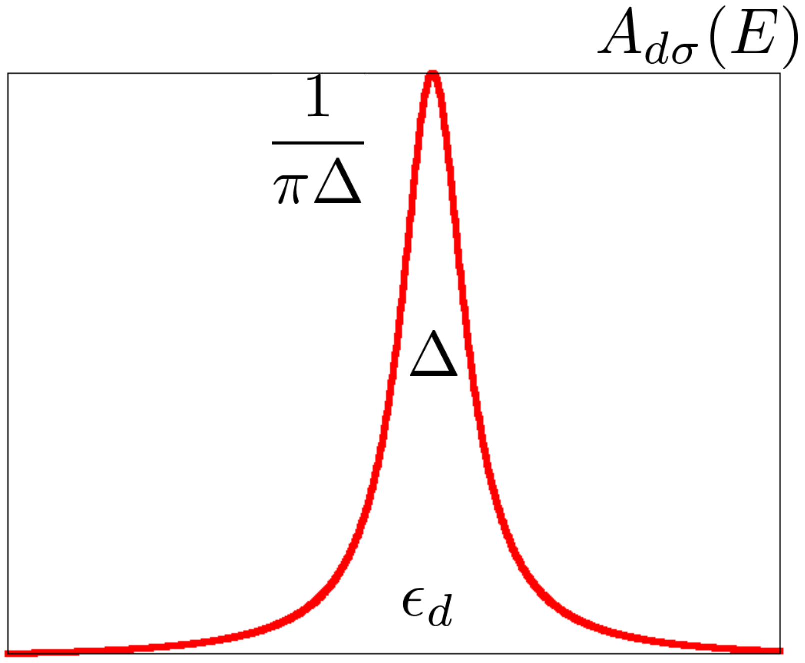
- Density of states

$$\rho(E) = \sum_k \delta(E - \epsilon_k)$$

- *d*-level density of states:

$$\rho(E) = \rho_0 \qquad V_k = V \qquad \Delta = \pi \rho_0 V^2$$

$$\delta(E - \epsilon_d) \rightarrow \sim \frac{\Delta/\pi}{(E - \epsilon_d)^2 + \Delta^2}$$



Anderson model (U=0,T=0)

Single particle resonance:

$$|\phi_{i\sigma}\rangle = (\alpha_i d_\sigma^\dagger + \sum_k \beta_{ik} c_{k\sigma}^\dagger)|0\rangle \qquad H|\phi_{i\sigma}\rangle = E_i|\phi_{i\sigma}\rangle$$

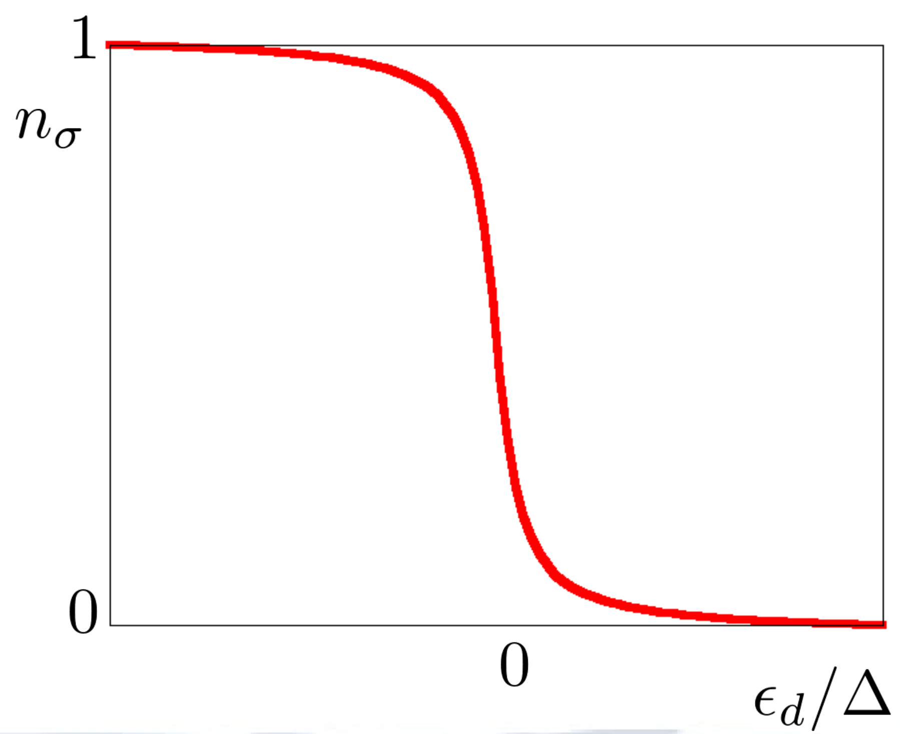
$$A_{d\sigma}(E, T=0, U=0) = \sum_i |\alpha_i|^2 \delta(E - (E_i - E_F))$$

$$n_\sigma \equiv \langle \hat{n}_\sigma \rangle = \int_{-D}^{E_F} dE A_{d\sigma}(E)$$

$$n_\sigma = \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\epsilon_d - E_F}{\Delta}\right)$$

$$\epsilon_d \ll \Delta \;\; \rightarrow \;\; \text{double occupancy}$$

$$\epsilon_d \gg \Delta \;\; \rightarrow \;\; \text{empty level}$$



Hartree-Fock solution

- Approximating:

$$\hat{n}_\uparrow \hat{n}_\downarrow \to \langle \hat{n}_\uparrow \rangle \hat{n}_\downarrow + \hat{n}_\uparrow \langle \hat{n}_\downarrow \rangle - \langle \hat{n}_\uparrow \rangle \langle \hat{n}_\downarrow \rangle$$

$$H_d \to H_d^{HF} = (\epsilon_d + U\langle \hat{n}_\downarrow \rangle)\hat{n}_\uparrow + (\epsilon_d + U\langle \hat{n}_\uparrow \rangle)\hat{n}_\downarrow$$

$$\tilde{\epsilon}_{d\sigma} = \epsilon_d - U\langle n_{\bar{\sigma}} \rangle$$

- Effective non-interacting problem

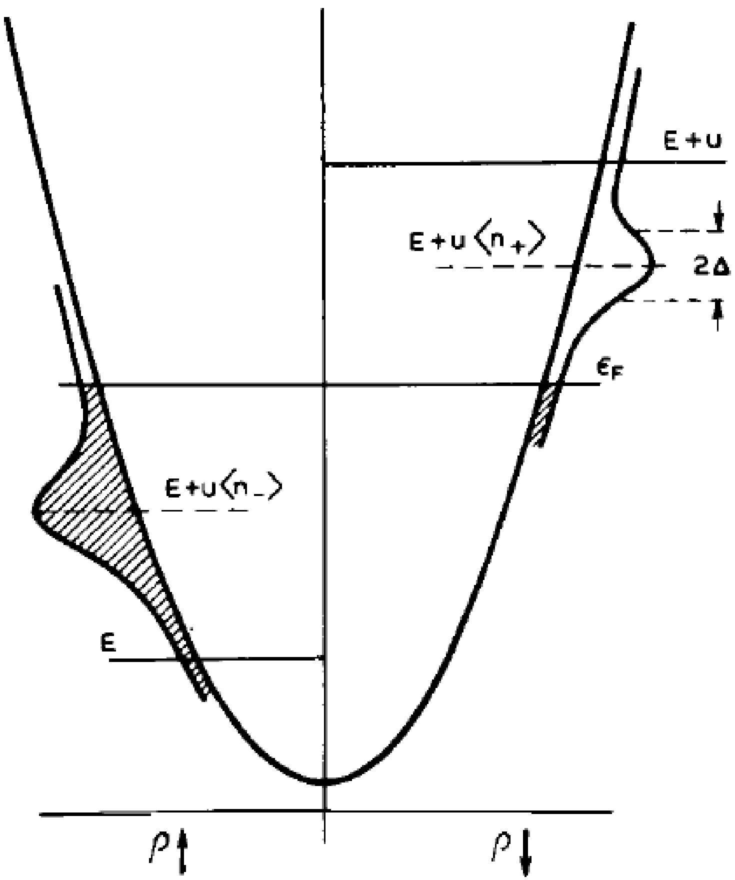
$$G_\sigma(E + i0) = \frac{1}{E - \epsilon_d - U\langle n_{\bar{\sigma}} \rangle + i\Delta} = \frac{1}{E - \tilde{\epsilon}_{d\sigma} + i\Delta}$$

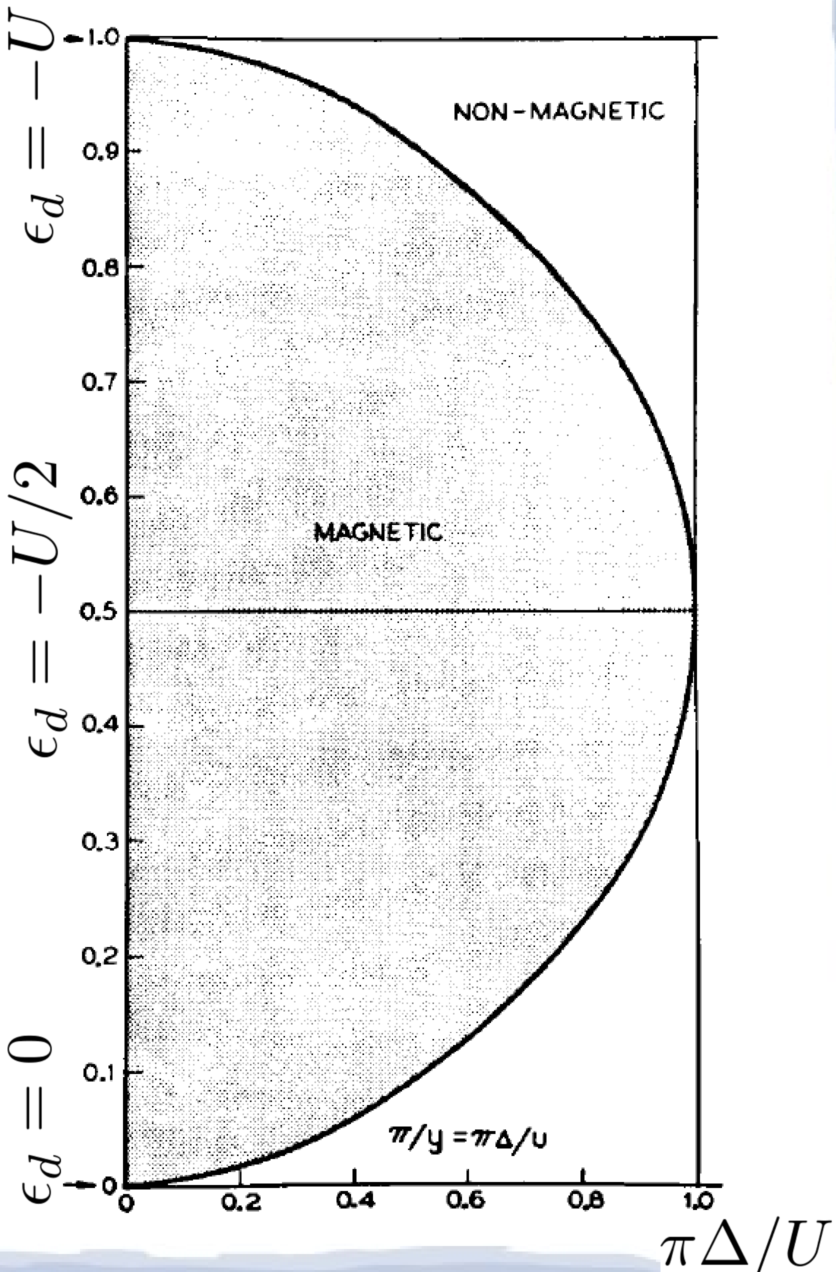
- The occupations $\langle \hat{n}_\sigma \rangle$ are calculated self-consistently

$$n_\downarrow = \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\tilde{\epsilon}_{d\uparrow}}{\Delta}\right) \qquad n_\uparrow = \frac{1}{2} - \frac{1}{\pi} \arctan\left(\frac{\tilde{\epsilon}_{d\downarrow}}{\Delta}\right)$$

Hartree-Fock solution

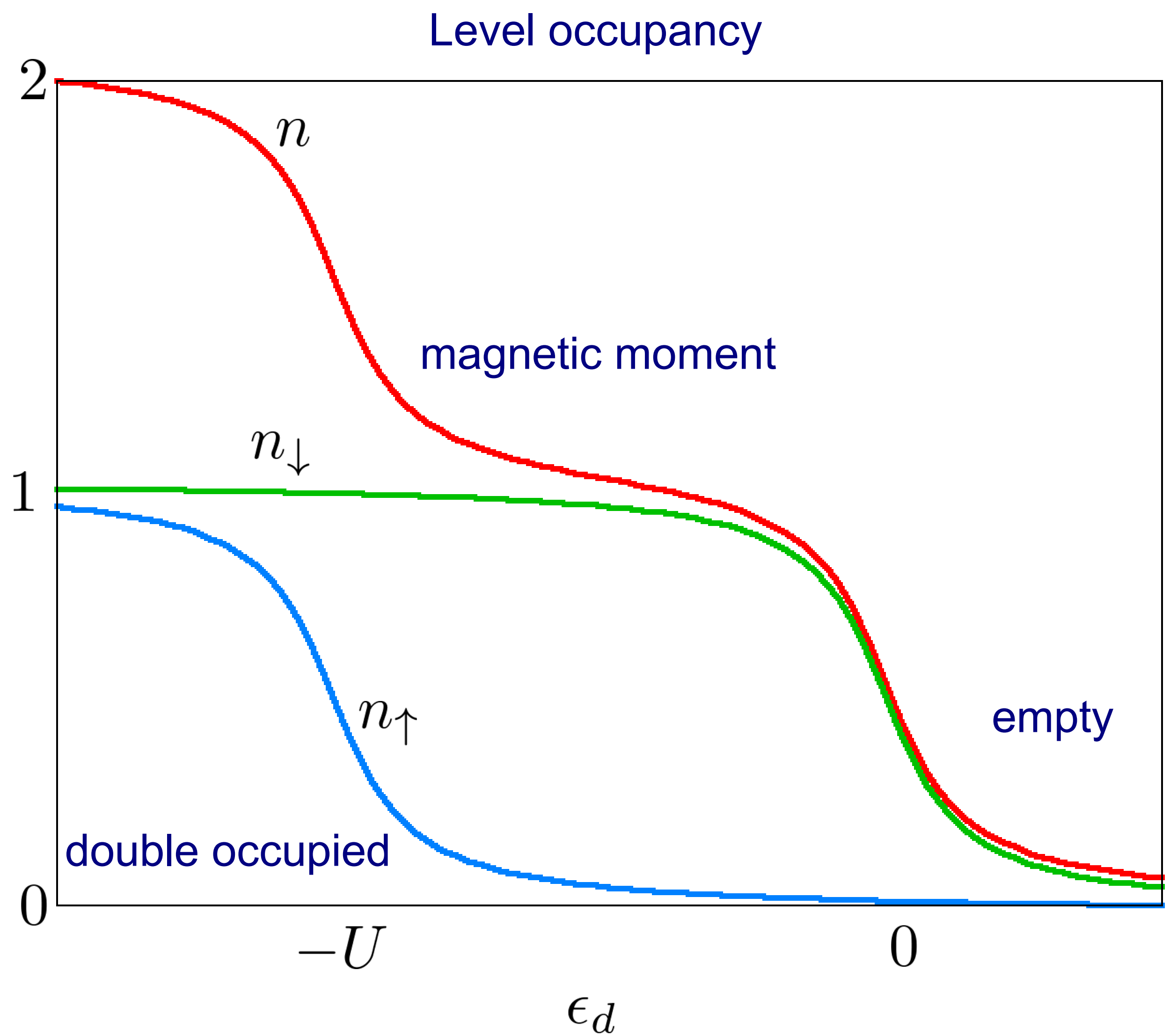
- Magnetic solutions for: $\pi\Delta < U$





P. W. Anderson, Phys. Rev. 124 (1961)

Hartree-Fock solution



$U > \pi\Delta$

Hartree-Fock solution

- Two degenerate solutions:
  - $n_\uparrow < n_\downarrow$
  - $n_\uparrow > n_\downarrow$
- + Gives a mechanism for the formation of local magnetic moments and a criterion for their existence.
  - $\epsilon_d < -\Delta, \;\; \epsilon_d + U > \Delta$
- - But: breaks spin symmetry and polarization axis depends on the choice of quantization axis.
- Second order perturbation theory on H_V connects the two solutions

Schrieffer Wolff transformation

- Deep in the local moment regime

$$\epsilon_d \ll -\Delta, \;\; \epsilon_d + U \gg \Delta$$

- Canonical transformation (second order perturbation in H_V)

$$H = H_d + H_V + H_m$$

$$\tilde{H} = e^{iS} H e^{-iS} = (1 + iS - S^2/2 + \cdots)H(1 - iS - S^2/2 + \cdots)$$

$$\tilde{H} = H + i[S, H] - \frac{1}{2}[S, [S, H]] + \mathcal{O}(S^3)$$

$$\tilde{H} = H_d + H_m + H_V + i[S, H_d + H_m] + i[S, H_V] - \frac{1}{2}[S, [S, H]] + \mathcal{O}(S^3)$$

- Select S to eliminate linear term in H_V

$$H_V + i[S, H_d + H_m] = 0$$

Schrieffer Wolff transformation

- Select S to eliminate linear term in H_V

$$H_V + i[S, H_d + H_m] = 0$$

$$\tilde{H} = H_d + H_m + i[S, H_V] - \frac{1}{2}[S, iH_V + [S, H_V]] + \mathcal{O}(S^3)$$

$$\tilde{H} = H_d + H_m + \frac{i}{2}[S, H_V] + \mathcal{O}(S^3)$$

- Matrix elements of $\tilde{H}$ in a basis of eigenstates of $H_d + H_m$

$$(H_d + H_m)|\mu, k\rangle = (E_\mu + \epsilon_k)|\mu, k\rangle$$

$$\langle \mu, k|S|\mu', k'\rangle = \frac{-i\langle \mu, k|H_V|\mu', k'\rangle}{E_\mu + \epsilon_k - E_\mu' - \epsilon_{k'}}$$

Schrieffer Wolff transformation

$$\tilde{H} = H_d + H_m + \frac{i}{2}[S, H_V] + \mathcal{O}(S^3)$$

$$\langle \mu, k | S | \mu', k' \rangle = \frac{-i \langle \mu, k | H_V | \mu', k' \rangle}{E_\mu + \epsilon_k - E_\mu{}' - \epsilon_{k'}}$$

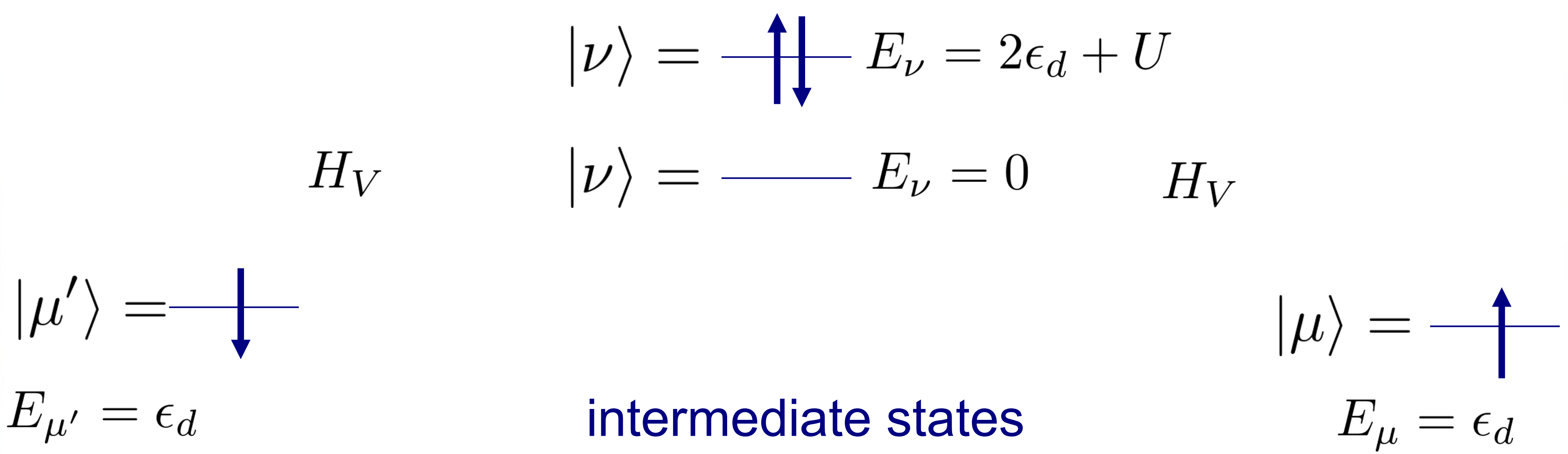
$$\langle \mu, k | \tilde{H} | \mu', k' \rangle \simeq E_k + E_\mu + \frac{i}{2} \langle \mu, k | S H_V | \mu', k' \rangle - \frac{i}{2} \langle \mu, k | H_V S | \mu', k' \rangle$$

Insert identity: $\mathcal{I} = \sum_{\nu q} |\nu q\rangle \langle \nu q|$

Schrieffer Wolff transformation

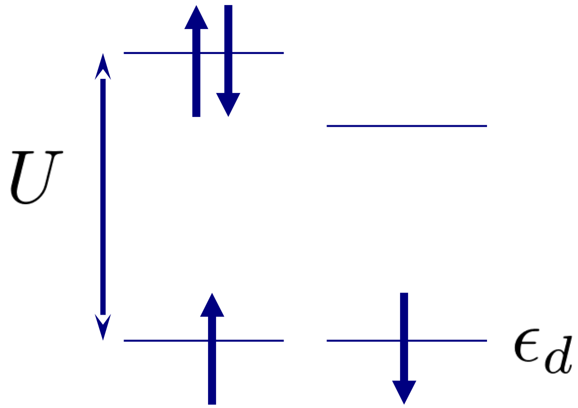
$$
\begin{aligned}
\langle \mu, k | \tilde{H} | \mu', k' \rangle &= E_k + E_\mu + \frac{1}{2} \sum_{q\nu} \frac{\langle \mu, k | H_V | \nu, q \rangle \langle \nu, q | H_V | \mu', k' \rangle}{E_\mu + \epsilon_k - E_\nu - \epsilon_q} \\
&- \frac{1}{2} \sum_{q\nu} \frac{\langle \mu, k | H_V | \nu, q \rangle \langle \nu, q | H_V | \mu', k' \rangle}{E_\nu + \epsilon_q - E_{\mu'} - \epsilon_{k'}} + \mathcal{O}(H_V^3)
\end{aligned}
$$

- Different terms: example spin flip



Kondo model

- At low enough energies and temperatures discard high energy empty and double occupied states.
- Assume *k* independent *d* level - bath coupling: $V_k = V$
- Keep terms up to second order in H_V
- Assume: $|\epsilon_k| \ll U, |\epsilon_d|$



Spin ½:

$$\vec{S} = \frac{1}{2} \begin{pmatrix} d^\dagger_\uparrow & d^\dagger_\downarrow \end{pmatrix} \vec{\sigma} \begin{pmatrix} d_\uparrow \\ d_\downarrow \end{pmatrix}$$

$$\vec{\sigma} = \left(\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right)$$

Kondo model

- Exchange term:

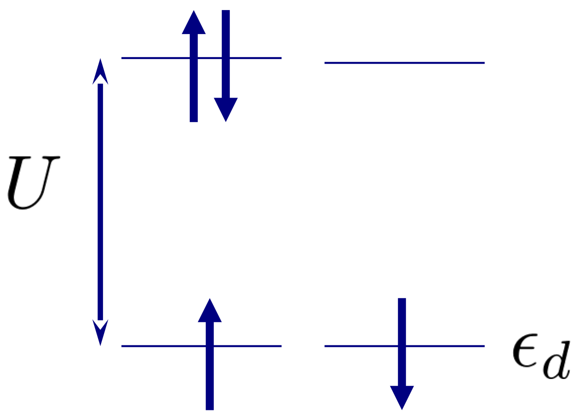
$$H_K = \sum_{kk'} J_{kk'} (S^+ c^\dagger_{k\downarrow} c_{k'\uparrow} + S^- c^\dagger_{k\uparrow} c_{k'\downarrow} + S_z (c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k\downarrow} c_{k'\downarrow}))$$

$$J_{kk'} \simeq V_k^* V_{k'} \left(\frac{1}{-\epsilon_d} + \frac{1}{\epsilon_d + U} \right)$$

- Def:

$$f^\dagger_{0\sigma} = \frac{1}{V} \sum_k V_k^* c^\dagger_{k\sigma}$$

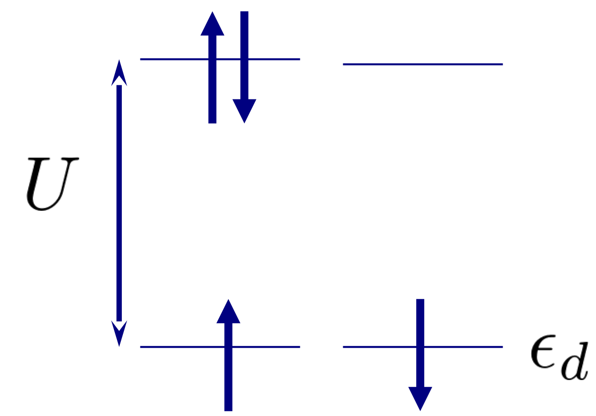
$$V = (\sum_k |V_k|^2)^{1/2}$$



Kondo model

- Potential scattering:

$$H_{pot} = \sum_{kk'\sigma\sigma'} K_{kk'} c^{\dagger}_{k\sigma} c_{k'\sigma'}$$



$$|\epsilon_k| \ll U, |\epsilon_d|$$

$$K_{kk'} \simeq V_k^* V_{k'} \left(\frac{1}{-\epsilon_d} - \frac{1}{\epsilon_d + U} \right)$$

cancels for: $\epsilon_d = -U/2$

This term can usually be disregarded

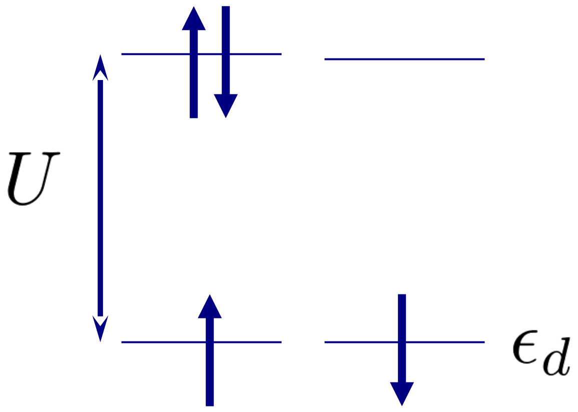
Kondo model

- Exchange term:

$$H_K = J(S^+ f^\dagger_{0\downarrow} f_{0\uparrow} + S^- f^\dagger_{0\uparrow} f_{0\downarrow} + S_z(f^\dagger_{0\uparrow} f_{0\uparrow} - f^\dagger_{0\downarrow} f_{0\downarrow}))$$

$$J = V^2 \left(\frac{1}{-\epsilon_d} + \frac{1}{\epsilon_d + U} \right) > 0$$

antiferromagnetic



- Can be written as:

$$H_K = 2J\vec{S} \cdot \vec{S}_0$$

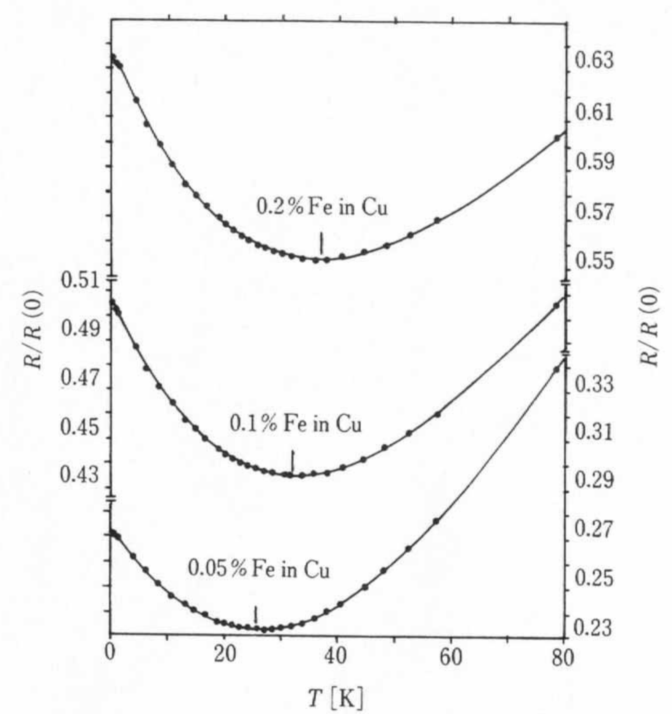
$$\vec{S}_0 = \frac{1}{2} \begin{pmatrix} f^\dagger_{0\uparrow} & f^\dagger_{0\downarrow} \end{pmatrix} \vec{\sigma} \begin{pmatrix} f_{0\uparrow} \\ f_{0\downarrow} \end{pmatrix}$$

Jun Kondo explanation of the resistance minimum

- Scattering rate of Kondo impurities: perturbation theory in J

$$R_{\text{imp}}(T) \propto c_{\text{imp}} J^2 (1 - aJ \log (k_B T/D)) + \mathcal{O}(J^4)$$

$$R_{\text{ph}} \simeq \alpha T^5$$



- + Correctly describes $\mathrm{T}_{\min} \propto c^{1/5}$
- - Diverges for $T \to 0$
- Logarithmic dependence on the high energy cutoff *D* !

[J. Kondo Prog. Theor. Phys. **32** 37 (1964)]

Jun Kondo explanation of the resistance minimum

- Logarithms appear in thermodynamic properties like the specific heat and the magnetic susceptibility.
- Higher order terms in perturbation in *J* do not cure the problem
- Infinite summations lead to divergence at finite temperature

$$T_K = De^{-1/\rho_0 J}$$

- Logarithms come from integrals of the form

$$\int_{k_B T}^{D} \frac{dE}{E} = \ln(D/k_B T)$$

all energy scales make the same contribution: $\int_{E_0}^{2E_0} = \ln 2$

- What is the ground state?

Renormalization group

- Problems where there are many relevant length or energy scales and associated divergences:
  - Field theory: diagrammatic calculations in quantum electrodynamics, *dressed* particles
  - Near phase transitions: e.g. Ising model ferromagnetic/paramagnetic transition
  - Many body problems: Kondo problem

K.G. Wilson, Rev. Mod. Phys. **47** 773 (1975)

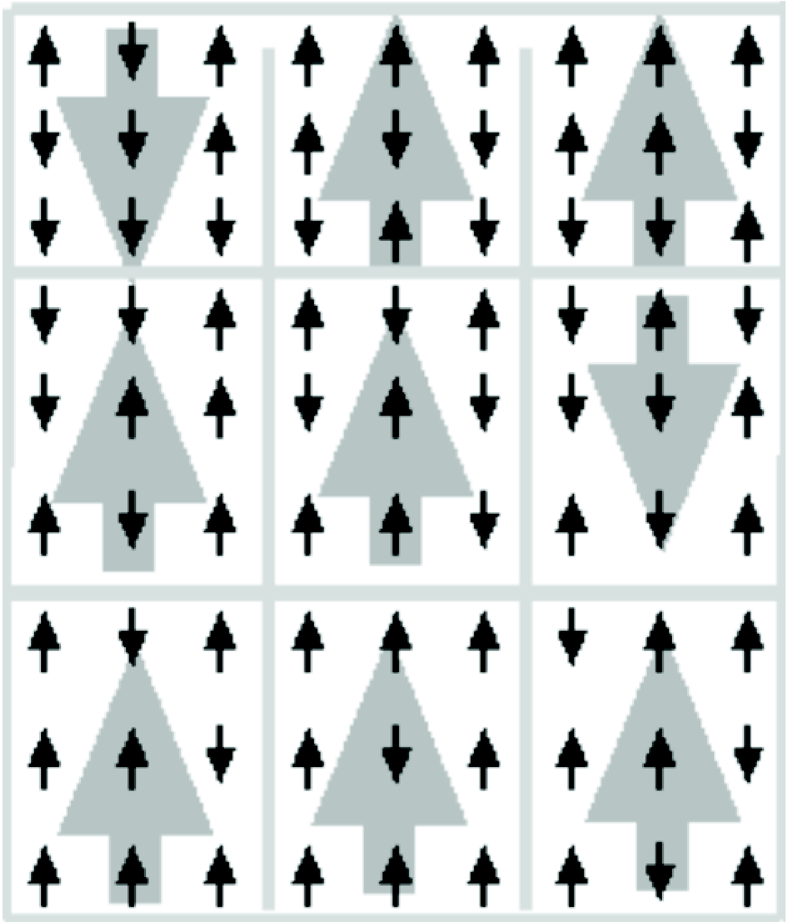
Kadanoff block spin renormalization

$$H_{\text{ising}} = J \sum_{\langle i,j \rangle} s_i s_j$$

Partition function

$$Z = \sum_{\{\mathbf{s}\}} \exp(K \sum_{\langle i,j \rangle} s_i s_j)$$

$$K = -J/k_B T$$



Partial sum: new partition function

$$Z \sim \sum_{\{\tilde{\mathbf{s}}\}} \exp(\tilde{K} \sum_{\langle i,j \rangle} \tilde{s}_i \tilde{s}_j)$$

$$K \to \tilde{K}$$

L. P. Kadanoff, Physics **2** 263 (1966).

Fixed points

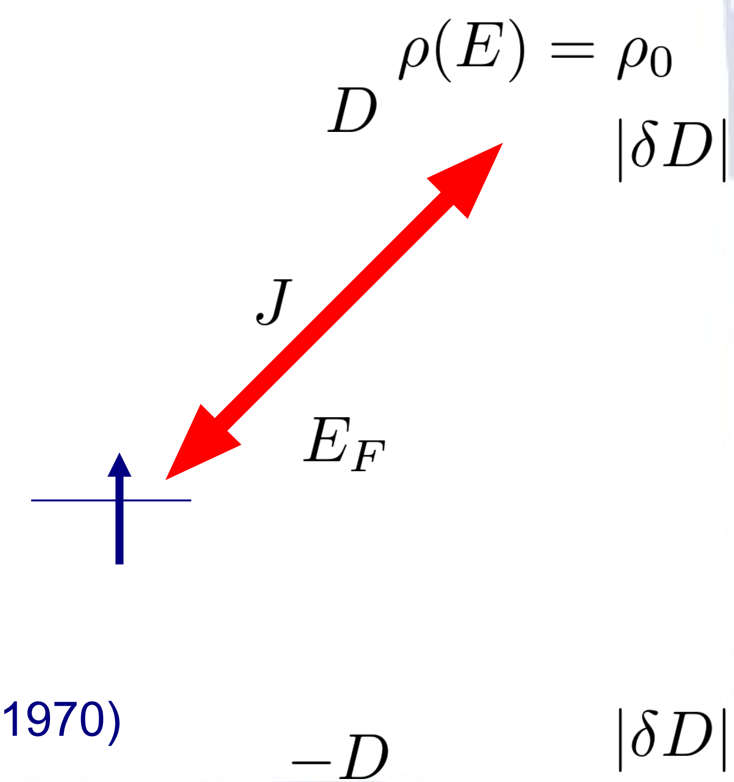
- Continue renormalization (changing length scale) until there are no more changes in the couplings (fixed point)
- Types of fixed points:
  - **Weak coupling**, e.g. paramagnet, vapor: $\tilde{K} \to 0$
  - **Strong coupling**, e.g. ferromagnet, liquid: $\tilde{K} \to \infty$
  - **Critical**, critical point: $\tilde{K} = K_c$
- Different fixed points are associated with different behavior.

Poor man's scaling

- The local spin couples to the conduction band at all energy scales:

$$H_K = J \sum_{kk'} (S^+ c^\dagger_{k\downarrow} c_{k'\uparrow} + S^- c^\dagger_{k\uparrow} c_{k'\downarrow} + S_z (c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k\downarrow} c_{k'\downarrow}))$$

- Energy space renormalizacion: Construct effective low energy Hamiltonian eliminating high energy degrees of freedom in the conduction band successively.



P. W. Anderson J. Phys. C: Solid State Phys. **3** 2436 (1970)

Poor man's scaling

- Consider the effect of the high energy states (conduction states at the edge of the band) at second order perturbation theory on the coupling .
  - The coupling needs to be small: $\rho_0 J \ll 1$
- The low energy Hamiltonian has the same form as the original one with renormalized parameters:

$$H(D,J) \rightarrow \mathcal{H}(D+\delta D) = H(D+\delta D, J+\delta J)$$
$$T \rightarrow T'$$

P. W. Anderson J. Phys. C: Solid State Phys. **3** 2436 (1970)

Poor man's scaling

- Anderson considered an anisotropic Kondo Hamiltonian:

$$H_K = \sum_{\epsilon_{k'}=-D}^{D} \sum_{\epsilon_k=-D}^{D} H_{k,k'} + \sum_{k\sigma} \epsilon_k c^{\dagger}_{k\sigma} c_{k\sigma}$$

$$H_{kk'} = J_{\perp}(S^{+} c^{\dagger}_{k\downarrow} c_{k'\uparrow} + S^{-} c^{\dagger}_{k\uparrow} c_{k'\downarrow}) + J_{\|} S_z (c^{\dagger}_{k\uparrow} c_{k'\uparrow} - c^{\dagger}_{k\downarrow} c_{k'\downarrow})$$

- And a metal with a symmetric band of half-bandwidth D and an energy independent density of states ρ_0.

P. W. Anderson J. Phys. C: Solid State Phys. **3** 2436 (1970)

Poor man's scaling

$$\text{hi} = [-D, -D + |\delta D|] \cup [D - |\delta D|, D]$$

$$\text{lo} = (-D + |\delta D|, D - |\delta D|)$$

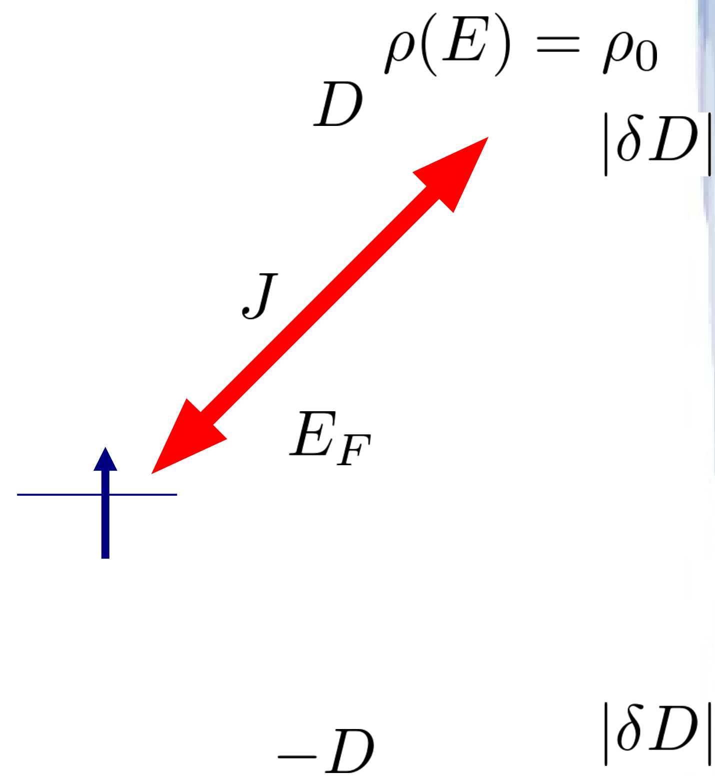
$$H_K = \sum_{\epsilon_{k'}=-D}^{D} \sum_{\epsilon_k=-D}^{D} H_{k,k'} + \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$

$$H_K = H_0^K + H_V^K + H_h^K$$

$$H_0^K = \sum_{\epsilon_{k'}\in\text{lo}} \sum_{\epsilon_k\in\text{lo}} H_{k,k'} + \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$

$$H_h^K = \sum_{\epsilon_{q'}\in\text{hi}} \sum_{\epsilon_q\in\text{hi}} H_{q,q'}$$

$$H_V^K = \sum_{\epsilon_{k'}\in\text{lo}} \sum_{\epsilon_q\in\text{hi}} H_{q,k'} + \sum_{\epsilon_q\in\text{hi}} \sum_{\epsilon_k\in\text{lo}} H_{k,q}$$



$$H_{kk'} = J_\perp (S^+ c^\dagger_{k\downarrow} c_{k'\uparrow} + S^- c^\dagger_{k\uparrow} c_{k'\downarrow}) + J_\parallel S_z (c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k\downarrow} c_{k'\downarrow})$$

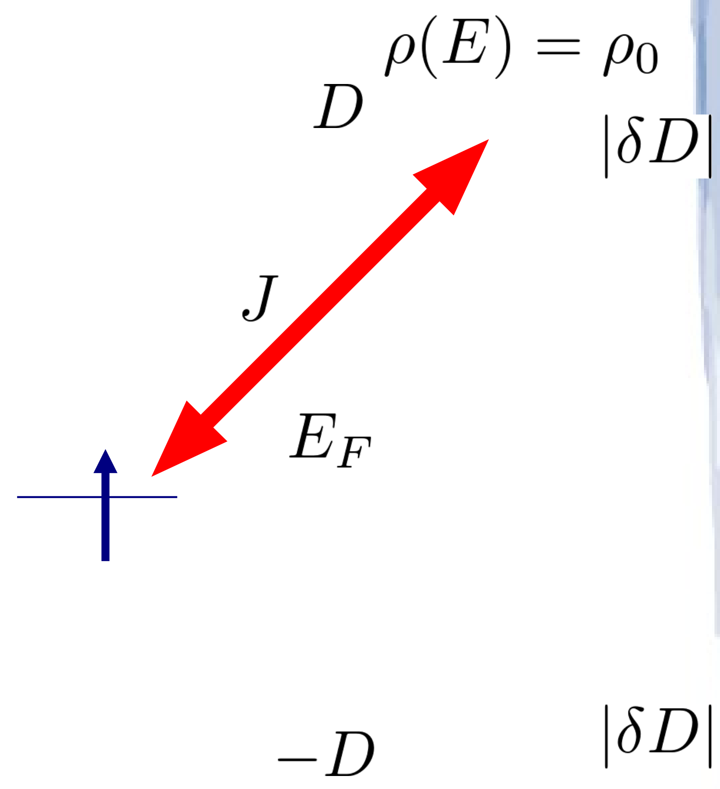
Poor man's scaling

$$H_K = H_0^K + H_V^K + H_h^K$$

$$H_0^K = \sum_{\epsilon_{k'} \in \mathrm{lo}} \sum_{\epsilon_k \in \mathrm{lo}} H_{k,k'} + \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$

$$H_V^K = \sum_{\epsilon_{k'} \in \mathrm{lo}} \sum_{\epsilon_k \in \mathrm{hi}} H_{k,k'} + \sum_{\epsilon_{k'} \in \mathrm{hi}} \sum_{\epsilon_k \in \mathrm{lo}} H_{k,k'}$$

$$H_h^K = \sum_{\epsilon_{q'} \in \mathrm{hi}} \sum_{\epsilon_q \in \mathrm{hi}} H_{q,q'}$$



- Canonical transformation to eliminate high energy states as done going from Anderson to Kondo model.
- Keep terms up to second order in H_V^K

Poor man's scaling

- Transformed Hamiltonian: $\epsilon_k, \ \epsilon_{k'} \ll D$

$$\begin{aligned}\langle \mu, k | \tilde{H}_K | \mu', k' \rangle &= \langle \mu, k | H_0^K | \mu', k' \rangle \\ &+ \frac{1}{2} \sum_{\epsilon_q \in \mathrm{hi}, \nu} \frac{\langle \mu, k | H_V^K | \nu, q \rangle \langle \nu, q | H_V^K | \mu', k' \rangle}{\epsilon_k - \epsilon_q} \\ &- \frac{1}{2} \sum_{\epsilon_q \in \mathrm{hi}, \nu} \frac{\langle \mu, k | H_V^K | \nu, q \rangle \langle \nu, q | H_V^K | \mu', k' \rangle}{\epsilon_q - \epsilon_{k'}} + \mathcal{O}(H_V^3)\end{aligned}$$

- The perturbation has the same form as the original Hamiltonian with an extra:
  - shift in energy
  - weak potential scattering (zero at the Fermi energy)

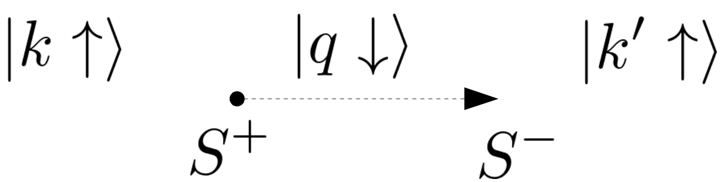
Poor man's scaling

$$H_{qk'} \quad = \quad J_\perp (S^+ c^\dagger_{q\downarrow} c_{k'\uparrow} + S^- c^\dagger_{q\uparrow} c_{k'\downarrow}) + J_{||} S_z (c^\dagger_{q\uparrow} c_{k'\uparrow} - c^\dagger_{q\downarrow} c_{k'\downarrow})$$

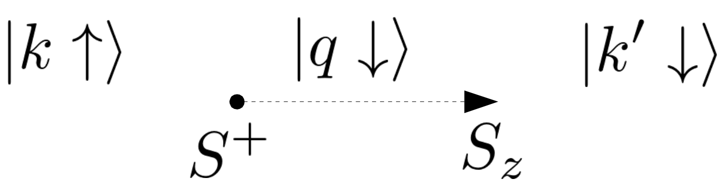
$$\langle \mu, k | H_V^K | \nu, q \rangle \langle \nu, q | H_V^K | \mu', k' \rangle$$

$$H_{kq} \quad = \quad J_\perp (S^+ c^\dagger_{k\downarrow} c_{q\uparrow} + S^- c^\dagger_{k\uparrow} c_{q\downarrow}) + J_{||} S_z (c^\dagger_{k\uparrow} c_{q\uparrow} - c^\dagger_{k\downarrow} c_{q\downarrow})$$

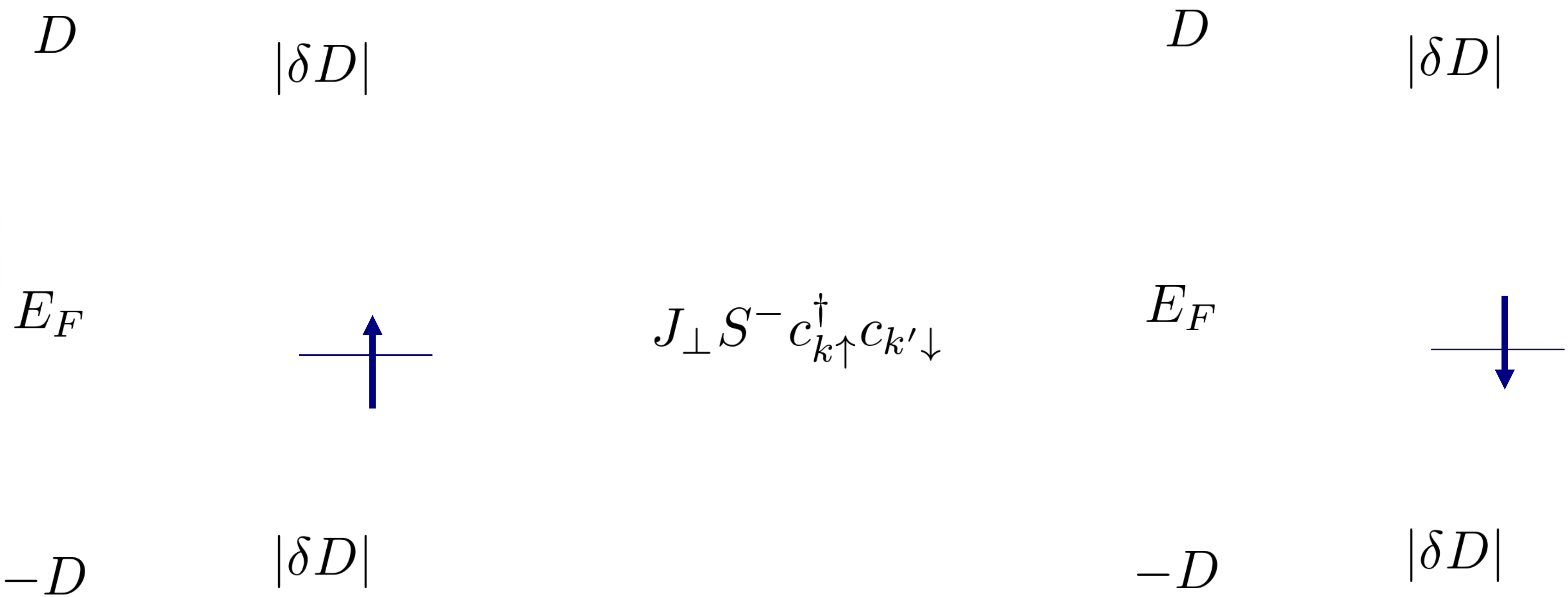
Terms of the form $J_\perp J_\perp$ contribute to $\delta J_{||}, \;\; J_{||} \rightarrow J_{||} + \delta J_{||}$



Terms of the form $J_\perp J_{||}$ contribute to $\delta J_\perp, \;\; J_\perp \rightarrow J_\perp + \delta J_\perp$



Low energy sector

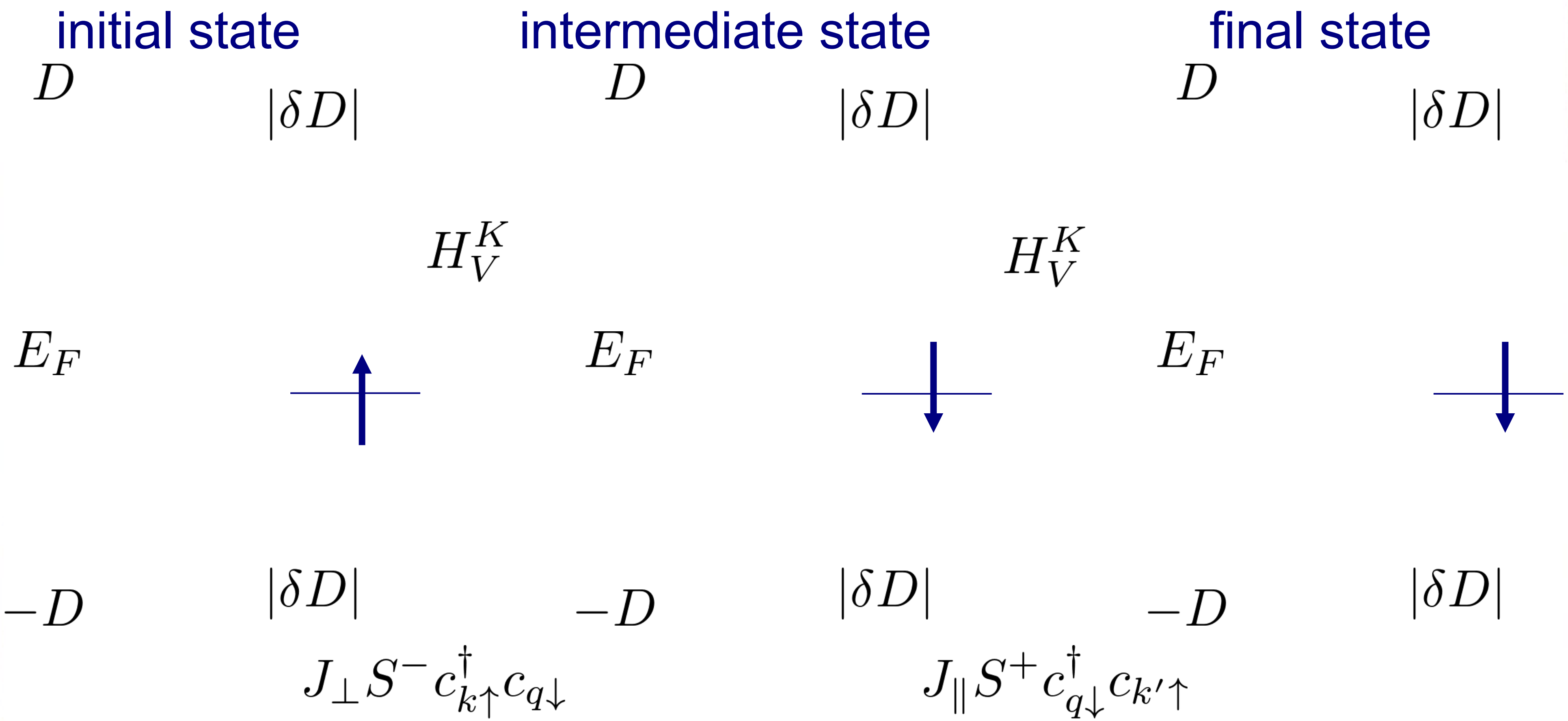


$$H_0^K = \sum_{\epsilon_{k'} \in \mathrm{lo}} \sum_{\epsilon_k \in \mathrm{lo}} H_{k,k'} + \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma}$$

$$H_{kk'} = J_\perp (S^+ c^\dagger_{k\downarrow} c_{k'\uparrow} + S^- c^\dagger_{k\uparrow} c_{k'\downarrow}) + J_\parallel S_z (c^\dagger_{k\uparrow} c_{k'\uparrow} - c^\dagger_{k\downarrow} c_{k'\downarrow})$$

Second order contribution $(H_V^K)^2$

$$H_V^K = \sum_{\epsilon_{k'} \in \text{lo}} \sum_{\epsilon_k \in \text{hi}} H_{k,k'} + \sum_{\epsilon_{k'} \in \text{hi}} \sum_{\epsilon_k \in \text{lo}} H_{k,k'}$$

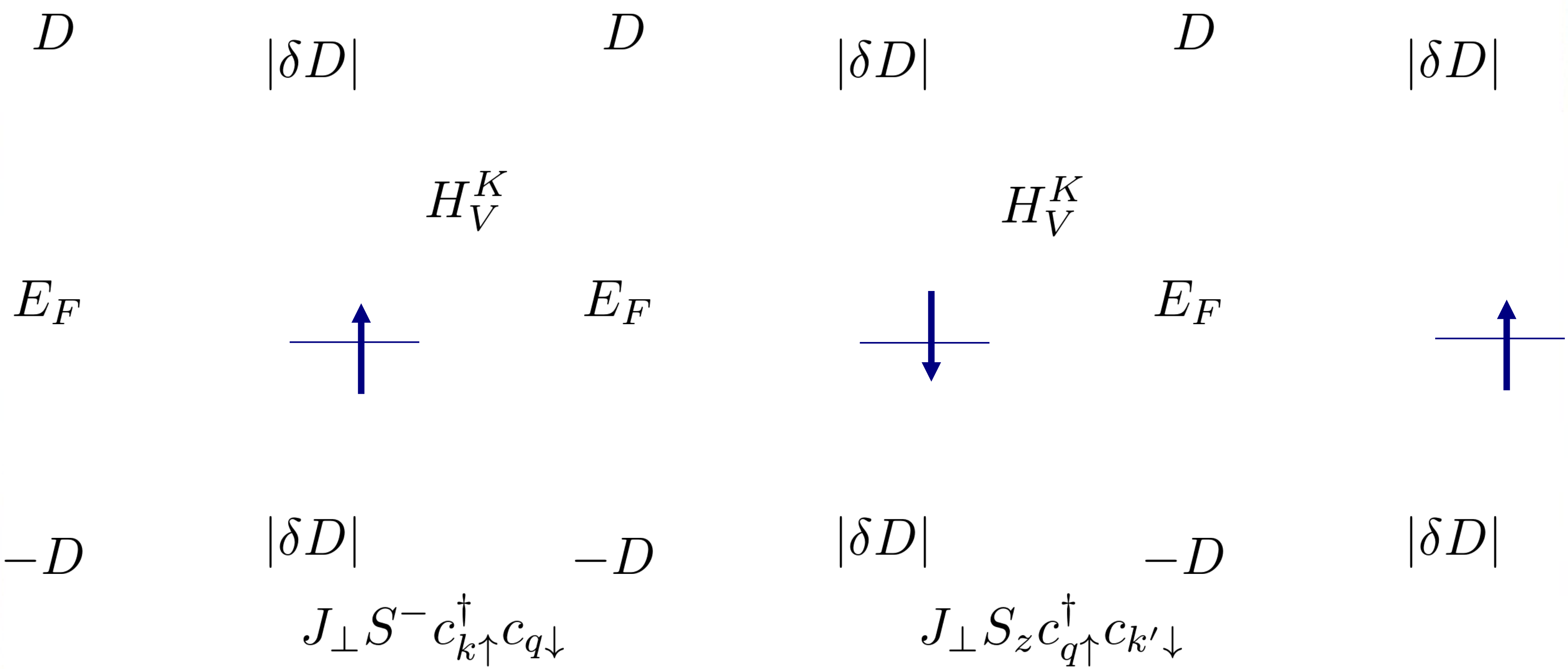


The effect of the second contribution term is equivalent to a low energy spin flip term $\propto S^- c_{k\uparrow}^\dagger c_{k'\downarrow} \to \delta J_\perp$

Second order contribution $(H_V^K)^2$

$$H_V^K = \sum_{\epsilon_{k'} \in \text{lo}} \sum_{\epsilon_k \in \text{hi}} H_{k,k'} + \sum_{\epsilon_{k'} \in \text{hi}} \sum_{\epsilon_k \in \text{lo}} H_{k,k'}$$

initial state intermediate state final state



The effect of the second contribution term is equivalent to a low energy parallel term $\propto S_z c_{k\uparrow}^\dagger c_{k'\uparrow} \to \delta J_{||}$

Poor man's scaling

$$\langle \mu, k | \delta H_0^K | \mu', k' \rangle = \frac{1}{2} \sum_{\epsilon_q \in \mathrm{hi}, \nu} \frac{\langle \mu, k | H_V^K | \nu, q \rangle \langle \nu, q | H_V^K | \mu', k' \rangle}{E_k - E_q} - \frac{1}{2} \sum_{\epsilon_q \in \mathrm{hi}, \nu} \frac{\langle \mu, k | H_V^K | \nu, q \rangle \langle \nu, q | H_V^K | \mu', k' \rangle}{E_q - E_{k'}}$$

$$\delta D \ll D \to E_q \sim D$$

$$\frac{1}{N_s} \sum_{\epsilon_q \in \mathrm{hi}} 1 \to \int d\epsilon_q \rho(\epsilon_q) \sim \rho_0 \delta D$$

Scaling equations:

$$\delta J_{\|} = -2\rho_0 J_{\perp}^2 \delta D / D > 0 \qquad \delta J_{\perp} = -2\rho_0 J_{\|} J_{\perp} \delta D / D$$

P. W. Anderson J. Phys. C: Solid State Phys. **3** 2436 (1970)

Poor man's scaling

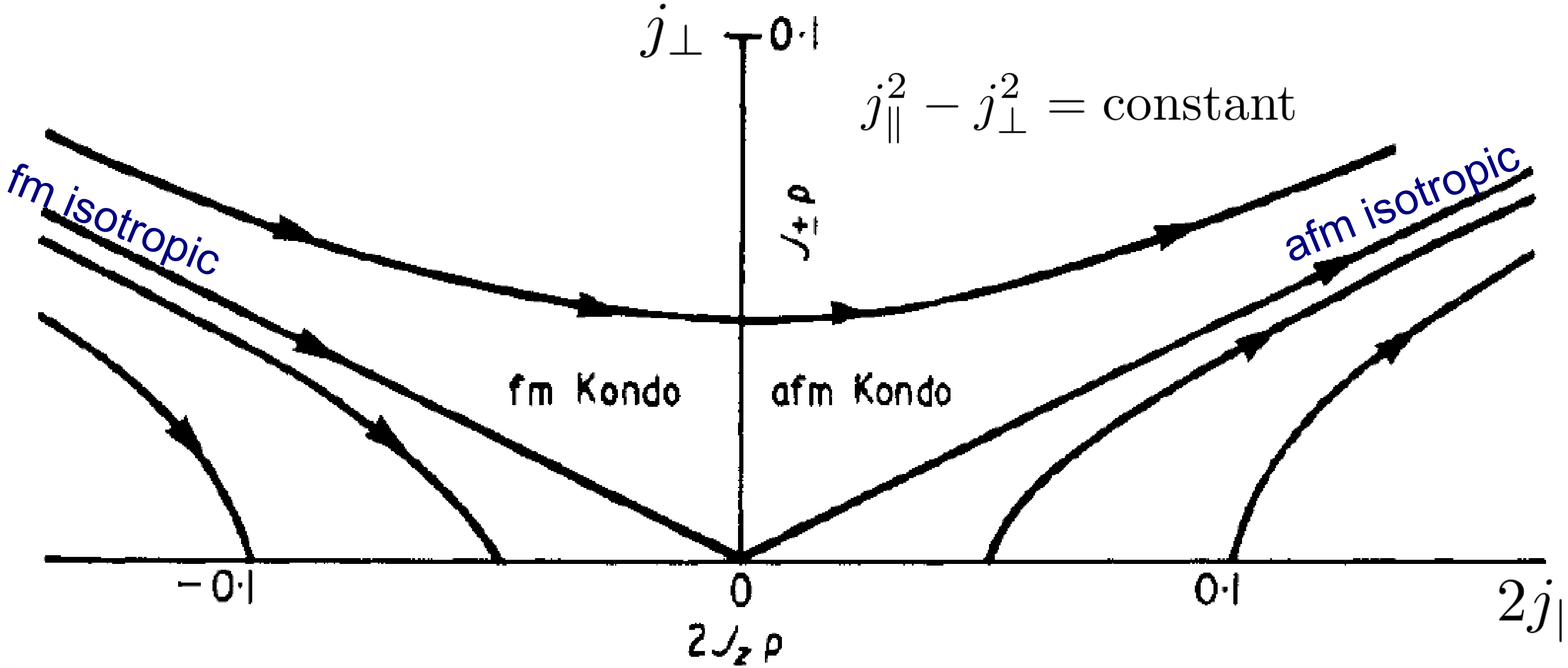
$$\delta J_{\parallel} = -2\rho_0 J_{\perp}^2 \delta D/D$$

$$\delta J_{\perp} = -2\rho_0 J_{\parallel} J_{\perp} \delta D/D$$

$$j_\alpha = \rho_0 J_\alpha$$

$$\frac{dj_{\parallel}}{d\ln D} = -2j_{\perp}^2$$

$$\frac{dj_{\perp}}{d\ln D} = -2j_{\parallel} j_{\perp}$$



Poor man's scaling

- Isotropic case weak coupling: $J = J_{\perp} = J_{\|}, \quad \rho_0 J \ll 1$
- Flow equation: $\dfrac{dJ}{d\ln D} = -2\rho_0 J^2$
- Integrating:

$$\frac{1}{\rho_0 \tilde{J}} - \frac{1}{\rho_0 J} = \ln(\tilde{D}/D)$$

- Scale invariant: $De^{-1/\rho_0 J} = \tilde{D}e^{-1/\rho_0 \tilde{J}}$

$$k_B T_K = De^{-1/\rho_0 J}$$

$$\tilde{D} \to k_B T_K, \quad \tilde{J} \to \infty$$

Poor man's scaling

- Ferromagnetic Kondo:
  - Flows to weak coupling fixed point $J \to 0$
  - The impurity spin is asymptotically free at low temperatures.
- Antiferromagnetic Kondo:
  - Flows to strong coupling fixed point $J \to \infty$
  - The approximations breakdown for $J \to 1$
  - Energy scale: $T_K = De^{-1/\rho_0 J}$
  - Singlet ground state?

Conclusion

- Resonance + local interactions lead to magnetic moment
- Anderson model spin couples antiferromagnetically to conduction electrons
- Perturbation theory explains resistance minimum but leads to logarithms on the temperature and the high energy cutoff.
- Scaling approach reveals Kondo scale and suggests singlet ground state
- Better methods needed to study low temperatures $T < T_K$
  - Next lecture:
    - Numerical renormalization group
    - Nozières Fermi liquid and slave bosons