III - Nonlinear Interactions

- Nonlinear Schrödinger Equation for spatial dynamics (Gross-Pitaevskii Equation)
- Review of recent work on nonlinear spatial dynamics of exciton-polaritons
- Nonlinear effects
  - Polarization Inversion ⟶ Polarization Sensitive Optical Gate
  - Self-Induced Larmor Precession ⟶ Optically Controlled Spin Transistor
  - Bistability ⟶ Memory Elements
    ⟶ Polariton Neurons for Optical Circuits

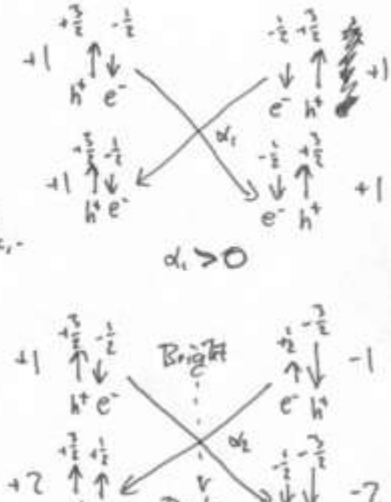
Nonlinear Schrödinger Equation

- local interaction ($q \ll \frac{1}{a_B}$)
- Spin conserving

$\sigma = \pm$

$$\hat{\mathcal{H}}_{int} = \frac{\alpha_1}{2}\sum_{x,\sigma} \hat{\Psi}^+_{x,\sigma}\hat{\Psi}^+_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma} + \alpha_2 \sum_x \hat{\Psi}^+_{x,+}\hat{\Psi}^+_{x,-}\hat{\Psi}_{x,+}\hat{\Psi}_{x,-}$$

Exchange Interaction



$\alpha_1 > 0$

$|\alpha_2| \ll \alpha_1$

$\alpha_2 = \pm$

$$i\hbar\frac{\partial\hat{\Psi}_{x,\sigma}}{\partial t} = \underbrace{\left[\hat{\Psi}_{x,\sigma}, \hat{\mathcal{H}}_{int}\right]}_{} + \underset{\text{terms in Schrödinger Equation}}{(SE)}$$

$$= \frac{\alpha_1}{2}\left(\overbrace{\hat{\Psi}_{x,\sigma}\hat{\Psi}^+_{x,\sigma}\hat{\Psi}^+_{x,\sigma}}^{\hat{\Psi}^+_{x,\sigma}\hat{\Psi}^+_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma} + 2\hat{\Psi}^+_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma}}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma} - \hat{\Psi}^+_{x,\sigma}\hat{\Psi}^+_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma}\right)$$

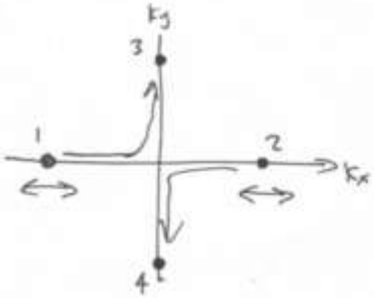
$$+ \alpha_2\underbrace{\left(\hat{\Psi}_{x,\sigma}\hat{\Psi}^+_{x,\sigma}\hat{\Psi}_{x,\sigma} - \hat{\Psi}^+_{x,\sigma}\hat{\Psi}_{x,\sigma}\hat{\Psi}_{x,\sigma}\right)}_{\hat{\Psi}_{x,\sigma}}\hat{\Psi}^+_{x,-\sigma}\hat{\Psi}_{x,-\sigma}$$

mean-field approx: $\hat{\Psi}_{x,\sigma} \to \langle\hat{\Psi}_{x,\sigma}\rangle$

$$i\hbar\frac{\partial\langle\hat{\Psi}_{x,\sigma}\rangle}{\partial t} = \left(\hat{E} - \frac{i\Gamma}{2} + V_x + \alpha_1|\langle\Psi_{x,\sigma}\rangle|^2 + \alpha_2|\langle\Psi_{x,-\sigma}\rangle|^2\right)\langle\Psi_{x,\sigma}\rangle + \frac{\hbar\Omega_0}{2}\left(i\frac{\partial}{\partial x} + \sigma\frac{\partial}{\partial y}\right)^2\langle\Psi_{x,-\sigma}\rangle + F_{x,\sigma}e^{-i\omega t}$$

$$\left.\begin{aligned} i\hbar\frac{\partial\Psi_\sigma(x)}{\partial t} = &\left[\hat{E} - \frac{i\Gamma}{2} + V(x) + \alpha_1|\Psi_\sigma(x)|^2 + \alpha_2|\Psi_{-\sigma}(x)|^2\right]\Psi_\sigma(x) \\ &+ \frac{\hbar\Omega_0}{2}\left(i\frac{\partial}{\partial x} + \sigma\frac{\partial}{\partial y}\right)^2\Psi_{-\sigma}(x) + F_\sigma(x)e^{-i\omega t}\end{aligned}\right\}\text{Gross-Pitaevskii Equation}$$

Polarization Inversion & the Optical Gate



$$\mathcal{H}_{scatt} = \frac{\alpha_1}{2}\left(\hat{\psi}^{+}_{3+}\hat{\psi}^{+}_{4+}\hat{\psi}_{1+}\hat{\psi}_{2+} + \hat{\psi}^{+}_{3-}\hat{\psi}^{+}_{4-}\hat{\psi}_{1-}\hat{\psi}_{2-}\right)$$
$$+ \alpha_2\left[\left(\hat{\psi}^{+}_{3+}\hat{\psi}^{+}_{4-} + \hat{\psi}^{+}_{3-}\hat{\psi}^{+}_{4+}\right)\left(\hat{\psi}_{1+}\hat{\psi}_{2-} + \hat{\psi}_{1-}\hat{\psi}_{2+}\right)\right]$$
$$+ h.c.$$

change to linear polarization basis

$$\psi_{n,+} = \frac{\psi_{n,x} + i\psi_{n,y}}{\sqrt{2}}$$
$$\psi_{n,-} = \frac{\psi_{n,x} - i\psi_{n,y}}{\sqrt{2}}$$

$$\mathcal{H}_{scatt} = \frac{\alpha_1}{8}\Big[\left(\psi^{+}_{3x} - i\psi^{+}_{3y}\right)\left(\psi^{+}_{4x} - i\psi^{+}_{4y}\right)\left(\psi_{1x} + \cancel{i\psi_{1y}}\right)\left(\psi_{2x} + \cancel{i\psi_{2y}}\right)$$
$$+ \left(\psi^{+}_{3x} + i\psi^{+}_{3y}\right)\left(\psi^{+}_{4x} + i\psi^{+}_{4y}\right)\left(\psi_{1x} - \cancel{i\psi_{1y}}\right)\left(\psi_{2x} - \cancel{i\psi_{2y}}\right)\Big]$$
$$+ \frac{\alpha_2}{4}\Big[\left(\psi^{+}_{3x} - i\psi^{+}_{3y}\right)\left(\psi^{+}_{4x} + i\psi^{+}_{4y}\right) + \left(\psi^{+}_{3x} + i\psi^{+}_{3y}\right)\left(\psi^{+}_{4x} - i\psi^{+}_{4y}\right)\Big]$$
$$\times \Big[\left(\psi_{1x} + \cancel{i\psi_{1y}}\right)\left(\psi_{2x} - \cancel{i\psi_{2y}}\right) + \left(\psi_{1x} - \cancel{i\psi_{1y}}\right)\left(\psi_{2x} + \cancel{i\psi_{2y}}\right)\Big] + h.c.$$

assume pump x-polarized.

$$= \frac{\alpha_1}{4}\left(\psi^{+}_{3x}\psi^{+}_{4x} - \psi^{+}_{3y}\psi^{+}_{4y}\right)\psi_{1x}\psi_{2x} + \frac{\alpha_2}{4}\left(\psi^{+}_{3x}\psi^{+}_{4x} + \psi^{+}_{3y}\psi^{+}_{4y}\right)\psi_{1x}\psi_{2x}$$

$$= \underbrace{\frac{\alpha_1+\alpha_2}{4}\psi^{+}_{3x}\psi^{+}_{4x}\psi_{1x}\psi_{2x}}_{\text{polarization conserving}} + \underbrace{\frac{\alpha_2-\alpha_1}{4}\psi^{+}_{3y}\psi^{+}_{4y}\psi_{1x}\psi_{2x}}_{\text{polarization inversion}}$$

transition rates: $x \to x \quad \dot{N}_{3,x} \sim |\alpha_1+\alpha_2|^2$

$x \to y \quad \dot{N}_{3,y} \sim |\alpha_1-\alpha_2|^2$

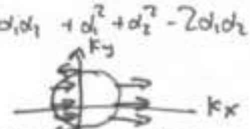
linear polarization degree: $\rho_{3,x} = \dfrac{N_{3,x} - N_{3,y}}{N_{3,x} + N_{3,y}}$ $\qquad \dot{\rho}_{3x} \sim \dfrac{|\alpha_1+\alpha_2|^2 - |\alpha_1-\alpha_2|^2}{|\alpha_1+\alpha_2|^2 + |\alpha_1-\alpha_2|^2}$

$$\dot{\rho}_{3,x} \sim \frac{\alpha_1^2+\alpha_2^2 + 2\alpha_1\alpha_2 - \alpha_1^2 - \alpha_2^2 + 2\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2 + 2\alpha_1\alpha_2 + \alpha_1^2+\alpha_2^2 - 2\alpha_1\alpha_2} = \frac{2\alpha_1\alpha_2}{\alpha_1^2+\alpha_2^2}$$

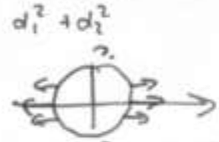
if $\alpha_2 > 0$ polarization conservation

$\alpha_2 < 0$ inversion

pseudospin symmetry:



co-polarized scattering



← suppressed since symmetry is not broken

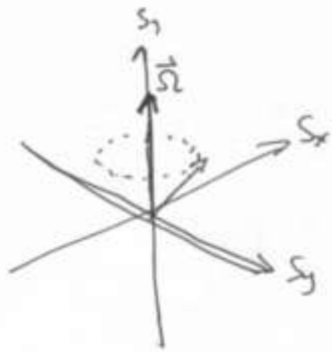
cross-polarized scattering

Self-Induced Larmor Precession

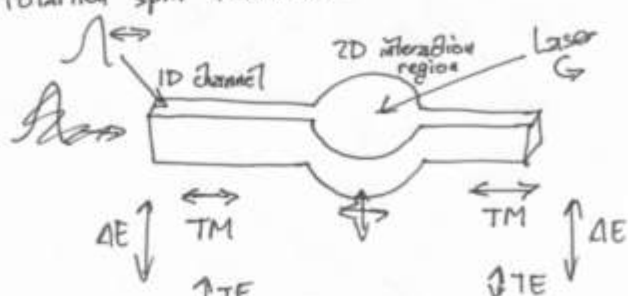
$$i\hbar\frac{\partial}{\partial t}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} = \begin{pmatrix}\alpha_1\overbrace{|\psi_+|^2}^{=n_+} + \alpha_2\overbrace{|\psi_-|^2}^{=n_-} & 0\\ 0 & \alpha_1|\psi_-|^2 + \alpha_2|\psi_+|^2\end{pmatrix}\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix} + \cdots$$

$$= \Bigg[\underbrace{\frac{(\alpha_1+\alpha_2)(n_+ + n_-)}{2}\begin{pmatrix}1&0\\0&1\end{pmatrix}}_{\text{polarization independent blueshift}} + \underbrace{\frac{(\alpha_1-\alpha_2)(n_+ - n_-)}{2}\overbrace{\begin{pmatrix}1&0\\0&-1\end{pmatrix}}^{\hat{\sigma}_z}}_{\Omega_z\cdot\hat{\sigma}}\Bigg]\begin{pmatrix}\psi_+\\ \psi_-\end{pmatrix}$$

Effective magnetic field in z-direction

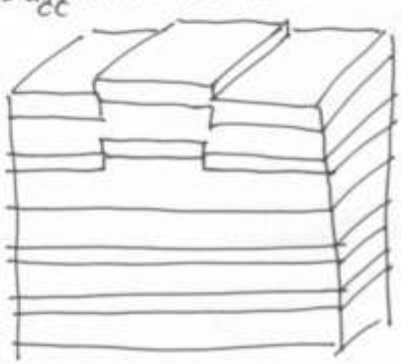


Polariton spin transistor



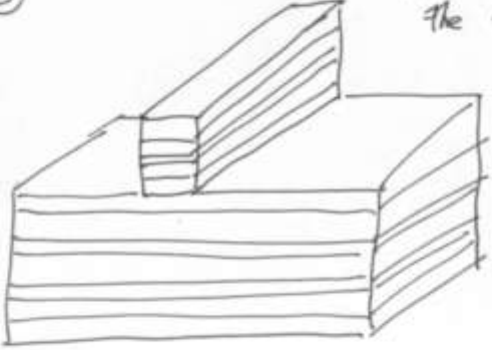
potential patterning

① Bragg mirror thickness variation

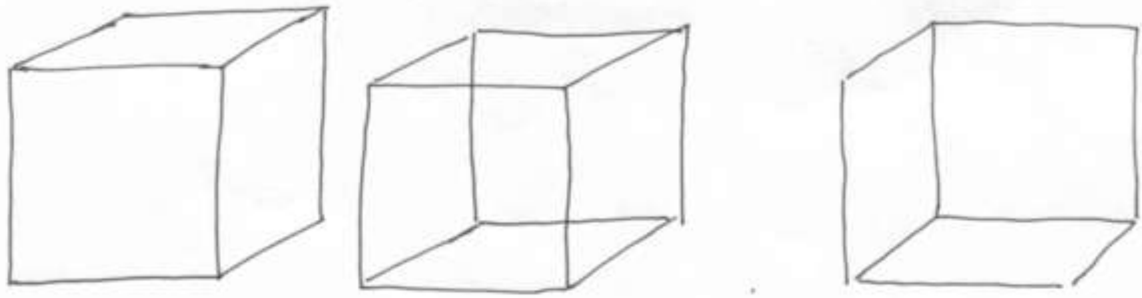


② • Deposit metal on microcavity surface

③ • Etch away parts of the structure

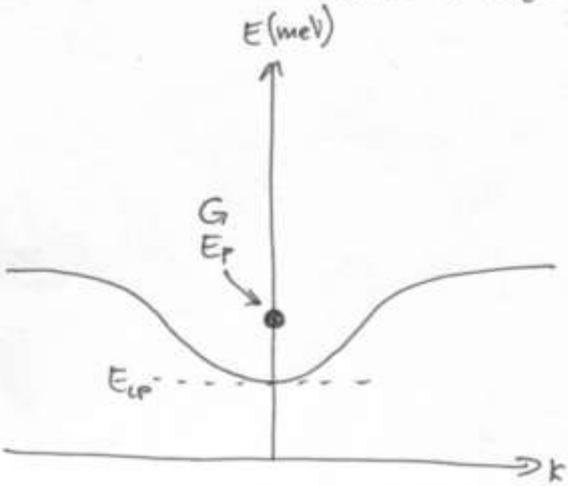


Bistability



(Bistable perception)

Consider a single pumped mode in k-space (e.g., $k=0$)



$$i\hbar\frac{\partial\psi_+}{\partial t} = \left(E_{LP} - \frac{i\Gamma}{2} + \alpha_1|\psi_+|^2 + \cancelto{0}{\alpha_2|\psi_-|^2}\right)\psi_+ + f_+ e^{-iE_p t/\hbar}$$

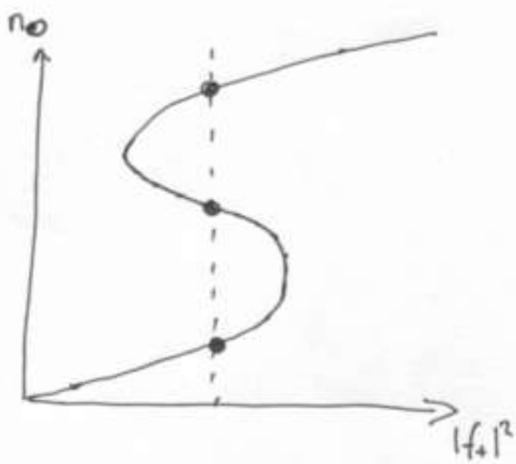
$$\psi_+ = \psi_0 e^{-iE_p t/\hbar}$$

$$E_p\psi_0 = \left(E_{LP} - \frac{i\Gamma}{2} + \alpha_1\overbrace{|\psi_0|^2}^{n_0}\right)\psi_0 + f_+$$

$$\therefore \left(E_p - E_{LP} - \alpha_1 n_0 + \frac{i\Gamma}{2}\right)\psi_0 = f_+$$

$$\left[\left(E_p - E_{LP} - \alpha_1 n_0\right)^2 + \frac{\Gamma^2}{4}\right]n_0 = \underset{\substack{\uparrow \\ \text{pump intensity}}}{|f_+|^2}$$

$$E_p - E_{LP} > \frac{\sqrt{3}\hbar}{\tau} = \sqrt{3}\Gamma$$



Stability Analysis

Examine behavior of perturbed solutions of the form:

$$\psi_+ = e^{-iE_p t/\hbar}\Big(\psi_0 + \underbrace{u e^{-i(kx - Et/\hbar)} + v^* e^{i(kx - E^*t/\hbar)}}_{\text{perturbation}}\Big)$$

$\downarrow$ substitute into ~~[illegible]~~ Gross-Pitaevskii equation

$$\overbrace{E_p\psi_0}^{\text{0th order}} + (E_p - E)u e^{-i(kx-Et/\hbar)} + (E_p + E^*)v^* e^{i(kx - E^*t/\hbar)}$$

$$= \left[E_{LP} - \frac{i\Gamma}{2} + \alpha_1\left|\psi_0 + u e^{-i(kx-Et/\hbar)} + v^* e^{i(kx-E^*t/\hbar)}\right|^2\right]\left(\psi_0 + u e^{-i(kx-Et/\hbar)} + v^* e^{i(kx-E^*t/\hbar)}\right)$$

$$+ f_+$$

keep terms linear in u & v^*:

$$= \overbrace{\left(E_{LP} - \frac{i\Gamma}{2} + \alpha_1|\psi_0|^2\right)\psi_0 + f_+}^{\text{0th order}}$$

$$+ \left(E_{LP} - \frac{i\Gamma}{2} + \alpha_1|\psi_0|^2\right)\left(u e^{-i(kx-Et/\hbar)} + v^* e^{i(kx-E^*t/\hbar)}\right)$$

$$+ \alpha_1|\psi_0|^2\left(u e^{-i(kx-Et/\hbar)} + v^* e^{i(kx-E^*t/\hbar)}\right)$$

$$+ \alpha_1\psi_0^2\left(u^* e^{i(kx-E^*t/\hbar)} + v e^{-i(kx-Et/\hbar)}\right)$$

Collect terms in $e^{-i(kx-Et/\hbar)}$ & $e^{i(kx-E^*t/\hbar)}$

$$(E_p - E)u = \left(E_{LP} - \frac{i\Gamma}{2} + 2\alpha_1|\psi_0|^2\right)u + \alpha_1\psi_0^2 v$$

$$(E_p + E^*)v^* = \left(E_{LP} - \frac{i\Gamma}{2} + 2\alpha_1|\psi_0|^2\right)v^* + \alpha_1\psi_0^2 u^*$$

$\downarrow$

$$\begin{pmatrix} E_{LP} - E_p + E - \frac{i\Gamma}{2} + 2\alpha_1|\psi_0|^2 & \alpha_1\psi_0^2 \\ \alpha_1\psi_0^{*2} & E_{LP} - E_p - E^* + \frac{i\Gamma}{2} + 2\alpha_1|\psi_0|^2 \end{pmatrix}\begin{pmatrix} u \\ v \end{pmatrix} = 0$$

$\downarrow$ Det $= 0$

$$\left(E_{LP} - E_p + E - \frac{i\Gamma}{2} + 2\alpha_1\underbrace{|\psi_0|^2}_{n_0}\right)\left(E_{LP} - E_p - E^* + \frac{i\Gamma}{2} + 2\alpha_1\underbrace{|\psi_0|^2}_{n_0}\right) - \alpha_1^2\underbrace{|\psi_0|^4}_{n_0^2}$$

$$\therefore \quad (E_{LP} - E_p + 2\alpha_1 n_0)^2 - E^2 + iE\Gamma + \frac{\Gamma^2}{4} - \alpha_1^2 n_0^2 = 0$$

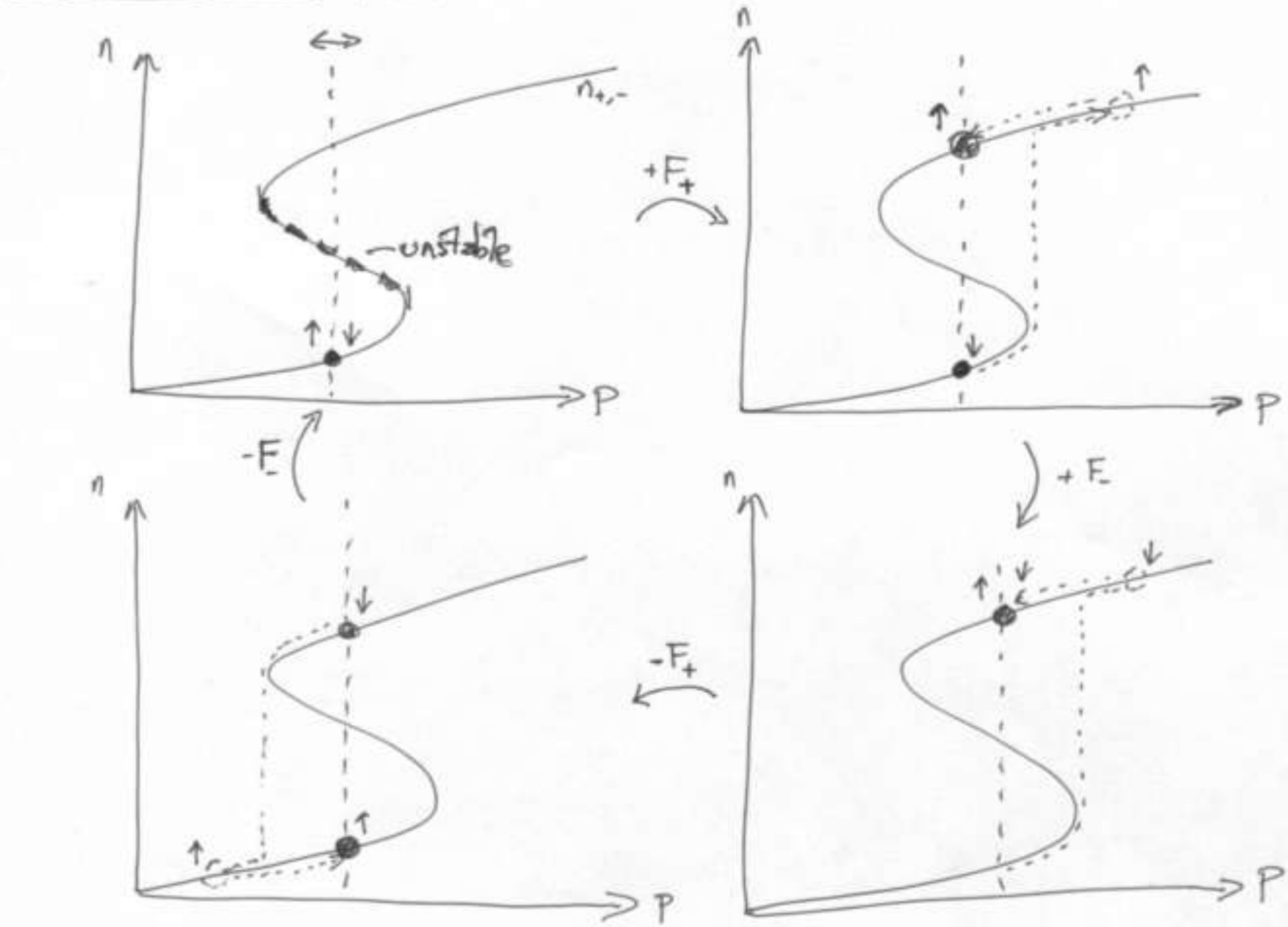
$$\downarrow$$

$$E = \frac{i\Gamma}{2} \pm \underbrace{\frac{1}{2}\sqrt{\cancel{-\Gamma^2} + 4\left[(E_{LP} - E_p + 2\alpha_1 n_0)^2 + \cancel{\frac{\Gamma^2}{4}} - \alpha_1^2 n_0^2\right]}}_{\pm\sqrt{(E_{LP} - E_p + 2\alpha_1 n_0)^2 - \alpha_1^2 n_0^2}}$$

if $\mathrm{Im}\{E\} > 0$ perturbation decays $\rightarrow$ stable state

$\mathrm{Im}\{E\} \leq 0$ perturbation grows $\rightarrow$ unstable state

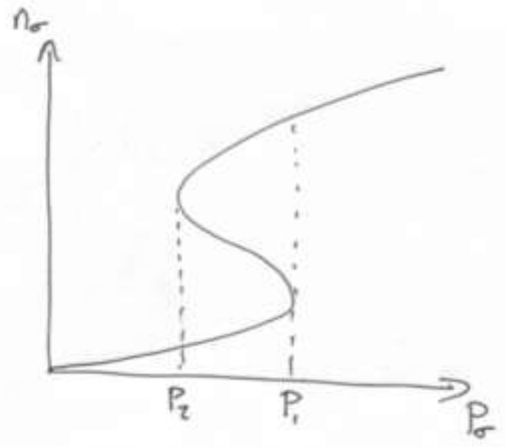
Polarization Multistability & Spin Memory

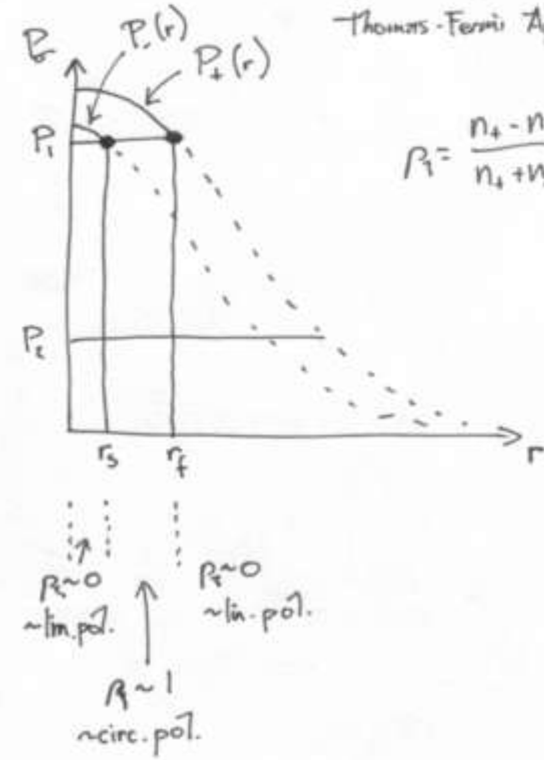


Spatial Effects of Polarization Multistability

ignore dispersion $\left(\frac{\hbar^2\nabla^2}{2m}\right)$

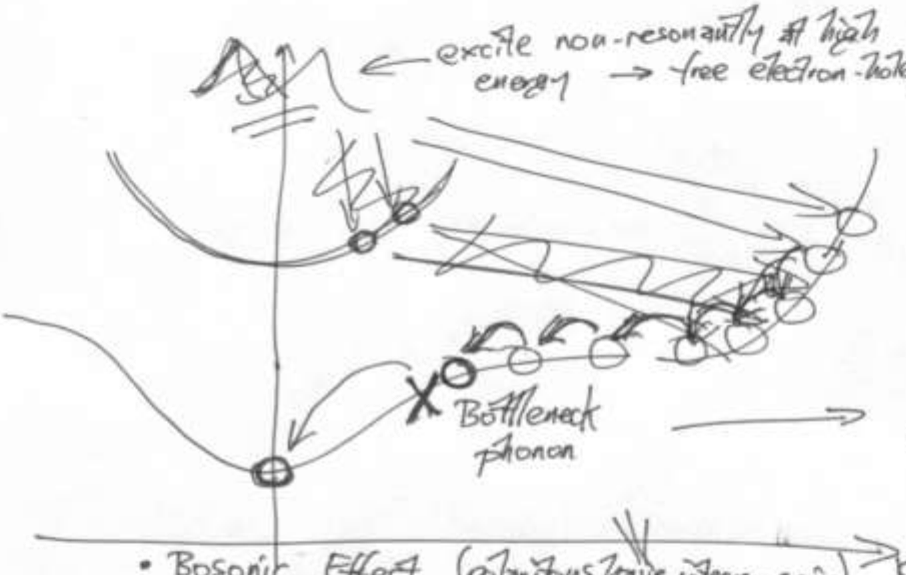
Thomas-Fermi Approx.





$$\rho_1 = \frac{n_+ - n_-}{n_+ + n_-}$$

Polariton Condensation



Stimulating scattering
High polariton densities
Long lifetime (high Q)
High interaction strength (CdTe, GaN)

new GaAs samples

- Bosonic Effect (polaritons have integer spin)
- Macroscopic occupation of lowest energy state
- Spontaneous coherence
- ~~Spontaneous ...~~
- Thermal Equilibrium ?

Ground States in a (real) magnetic field

E_{-} ——

E_{+} —— $\updownarrow 2\Omega$ $\uparrow B$

$$\hat{\mathcal{H}}_0 = -\frac{\Omega}{\cancel{2}}\left(\hat{\psi}_+^+\hat{\psi}_+ - \psi_-^+\psi_-\right)$$

$$\hat{\mathcal{H}}_{int} = \frac{\alpha_1}{2}\left(\hat{\psi}_+^+\hat{\psi}_+^+\hat{\psi}_+\hat{\psi}_+ + \hat{\psi}_-^+\hat{\psi}_-^+\hat{\psi}_-\hat{\psi}_-\right) + \alpha_2 \hat{\psi}_+^+\hat{\psi}_-^+\hat{\psi}_+\hat{\psi}_-$$

assume coherent state:

$$|\psi\rangle = \left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle$$

Interaction energy:

$$\hat{\psi}_\sigma\left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle = \psi_\sigma\left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle$$

$$U = \langle\psi|\hat{\mathcal{H}}_0|\psi\rangle + \langle\psi|\hat{\mathcal{H}}_{int}|\psi\rangle$$

$$\left\langle\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right|\hat{\psi}_\sigma^+ = \psi_\sigma^*\left\langle\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right|$$

$$\left\langle\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right| |\psi_+|^2 - |\psi_-|^2 \left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle$$

$$= -\frac{\Omega}{\cancel{2}}\left(|\psi_+|^2 - |\psi_-|^2\right)$$

$$\frac{\alpha_1}{2}\left(\left\langle\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right|\psi_+^*\psi_+^*\psi_+\psi_+\left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle + \left\langle\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right|\psi_-^*\psi_-^*\psi_-\psi_-\left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle\right)$$

$$+\alpha_2\left\langle\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right|\psi_+^*\psi_-^*\psi_+\psi_-\left|\begin{matrix}\psi_+\\ \psi_-\end{matrix}\right\rangle = \frac{\alpha_1}{2}\overbrace{\left(|\psi_+|^4 + |\psi_-|^4\right)}^{\frac{n^2+4S_T^2}{2}} + \alpha_2 \overbrace{|\psi_+|^2|\psi_-|^2}^{\frac{n^2-4S_T^2}{4}}$$

Free energy $\quad n_+ = |\psi_+|^2 \quad n_- = |\psi_-|^2 \quad n = n_+ + n_- \quad S_T = \frac{1}{2}(n_+ - n_-)$

$$F = -\mu n - 2\Omega S_T + \frac{\alpha_1+\alpha_2}{4}n^2 + (\alpha_1-\alpha_2)S_T^2$$

$$\frac{1}{2}\left(\frac{\alpha_1+\alpha_2}{2}\right)n^2 + \alpha_1-\alpha_2 S_T^2$$

$$= \frac{1}{4}(\alpha_1+\alpha_2)\left(n_+^2+n_-^2+2n_+n_-\right)$$

$$+\frac{\alpha_2}{4}\left(n_+^2+n_-^2-2n_+n_-\right)$$

① At zero magnetic field:
ground state minimum F corresponds to $S_T = 0$

↳ linearly polarized condensate. (assume $\alpha_1 > |\alpha_2|$)

↑↓↑↓↑↓↑↓↑↓

Spontaneously chosen orientation of linear pol.

(spontaneous symmetry breaking)

Chemical Potential ~~Look for~~ ground state.

minimize F with respect to n:

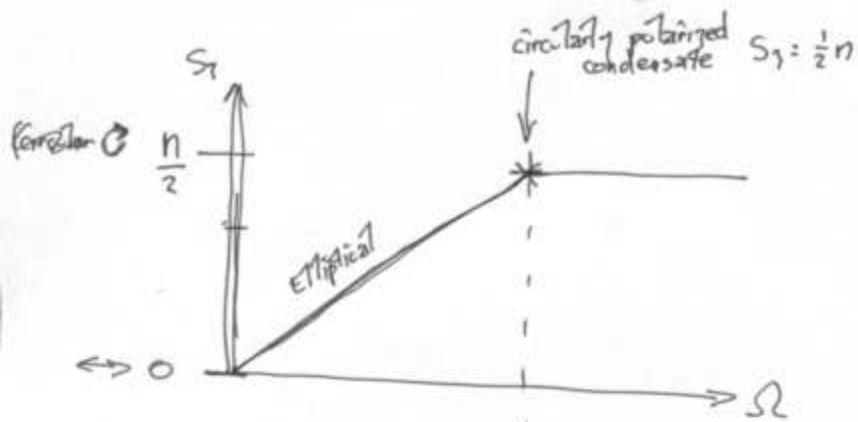
$$\frac{\partial F}{\partial n} = -\mu + \frac{\alpha_1+\alpha_2}{\cancel{4}2}n \qquad \xrightarrow{\left.\frac{\partial F}{\partial n}\right|_{\mu=\mu_0}=0} \cancel{\mu_0} \quad \mu_0 = \frac{\alpha_1+\alpha_2}{2}n$$

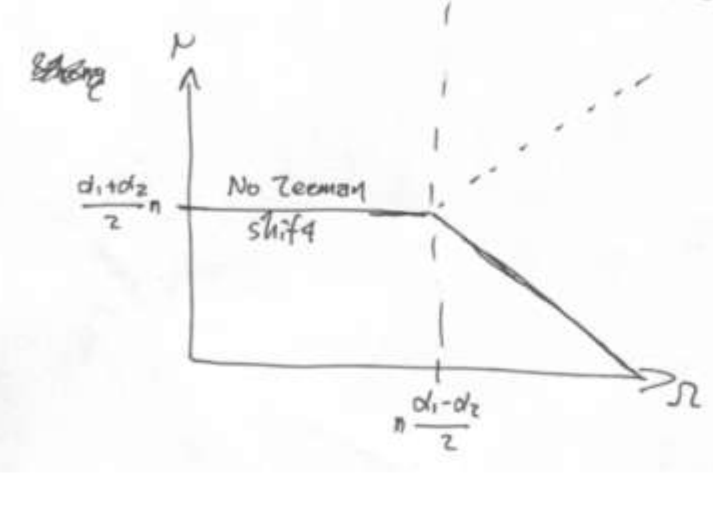
Polarization

~~Weak magnetic field: $\Omega \ll n\frac{\alpha_1-\alpha_2}{2}$~~

minimize F with respect to S_z:

$$\frac{\partial F}{\partial S_z} = -2\Omega + 2(\alpha_1-\alpha_2)S_z \quad \therefore \quad S_z = \frac{\Omega}{\alpha_1-\alpha_2}$$





Strong magnetic field

$$F = -\mu n - \Omega n + \underbrace{\frac{\alpha_1+\alpha_2}{4}n^2 + \frac{\alpha_1-\alpha_2}{4}n^2}_{\frac{\alpha_1}{2}n^2}$$

~~$\frac{\partial F}{\partial n} =$~~

$$\frac{\partial F}{\partial n} = -\mu - \Omega + \alpha_1 n \Rightarrow \mu_0 = \alpha_1 n - \Omega$$