I - Introduction to Semiconductor Microcavities

- Structure
- Light-matter coupling → Exciton-Polaritons
- Schrödinger Equation for spatial dynamics

II - Spin & Linear Effects - Spin structure & Pseudospin Representation

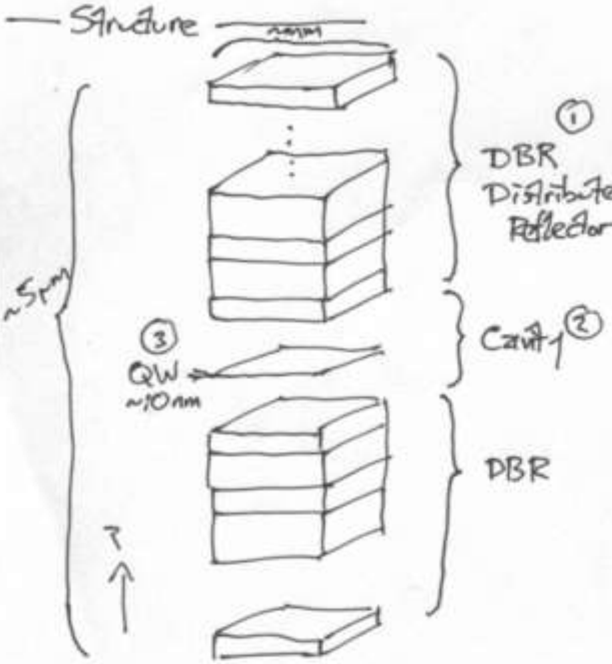
- TE-TM splitting
- OSHE
- Rayleigh Scattering

III - Nonlinear Interactions

IV - Bistability & related devices

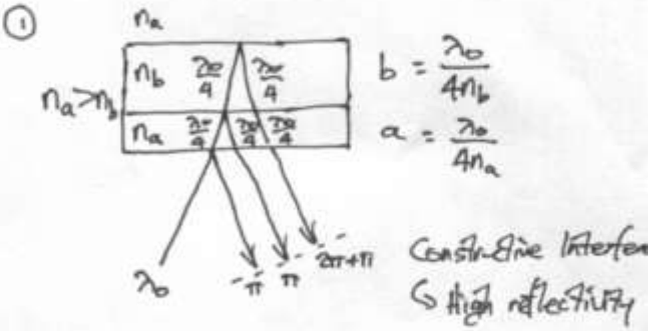
IV - Quantum Entanglement of spin degrees of freedom

— Structure —



Optical path length = length × refractive index

①

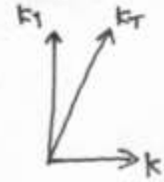


$b = \frac{\lambda_0}{4n_b}$

$a = \frac{\lambda_0}{4n_a}$

Constructive Interference
↳ High reflectivity

②



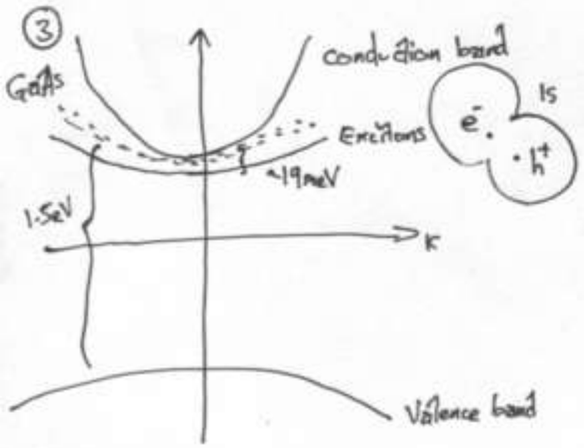
k_z is quantized!

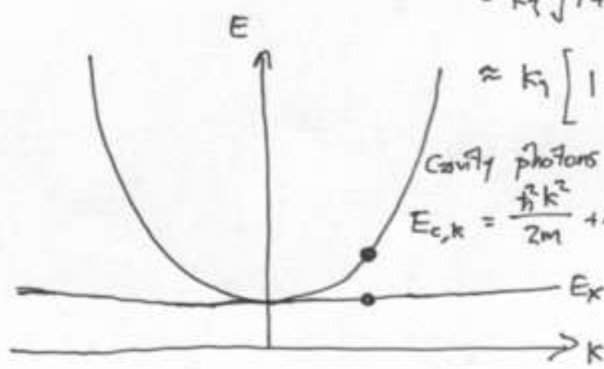
For small cavity, k_z is large

$$E = \frac{\hbar c}{n} k_T$$

$$k_T = \sqrt{k_z^2 + k^2} = k_z\sqrt{1+\left(\frac{k}{k_z}\right)^2} \approx k_z\left[1 + \frac{1}{2}\left(\frac{k}{k_z}\right)^2\right]$$

③





Consider cavity photons & excitons at some given in-plane wavevector

— Light-matter coupling —

Second quantized Hamiltonian:

Independent of k

$$\hat{\mathcal{H}}_k = E_x \hat{\chi}_k^+ \hat{\chi}_k + E_{c,k} \hat{\phi}_k^+ \hat{\phi}_k + V\left(\hat{\chi}_k^+ \hat{\phi}_k + \hat{\phi}^+ \hat{\chi}_k\right)$$

$$\hat{\mathcal{H}}_k = E_k \hat{\psi}_k^+ \hat{\psi}_k$$

$$\uparrow$$

$$\hat{\psi}_k = A_k \hat{\chi}_k + B_k \hat{\phi}_k \qquad |A_k|^2 + |B_k|^2 = 1$$

Commutation Rules: $[\hat{\phi}_k, \hat{\phi}_{k'}^+] = \delta_{kk'}$ Excitons, photons & polaritons have integer spin

$$[\hat{\psi}_k, \hat{\mathcal{H}}_k] = E_k \Big(\underbrace{\hat{\psi}_k \hat{\psi}_k^+ \hat{\psi}_k - \hat{\psi}_k^+ \hat{\psi}_k \hat{\psi}_k}_{\hat{\psi}_k}\Big) = E_k \left(A_k \hat{\chi}_k + B_k \hat{\phi}_k\right)$$

$$[\hat{\psi}_k, \hat{\mathcal{H}}_k] = (A_k \hat{\chi}_k + B_k \hat{\phi}_k)\left[E_x \hat{\chi}_k^+ \hat{\chi}_k + E_{c,k} \hat{\phi}_k^+ \hat{\phi}_k + V\left(\hat{\chi}_k^+ \hat{\phi}_k + \hat{\phi}_k^+ \hat{\chi}_k\right)\right]$$

$$- \left[E_x \hat{\chi}_k^+ \hat{\chi}_k + E_{c,k} \hat{\phi}_k^+ \hat{\phi}_k + V\left(\hat{\chi}_k^+ \hat{\phi}_k + \hat{\phi}_k^+ \hat{\chi}_k\right)\right](A_k \hat{\chi}_k + B_k \hat{\phi}_k)$$

$$= A_k E_x \overbrace{(\hat{\chi}_k \hat{\chi}_k^+ - \hat{\chi}_k^+ \hat{\chi}_k)}^{1} \hat{\chi}_k + B_k E_{c,k} \overbrace{(\hat{\phi}_k \hat{\phi}_k^+ - \hat{\phi}_k^+ \hat{\phi}_k)}^{1} \hat{\phi}_k$$

$$+ A_k V \underbrace{(\hat{\chi}_k \hat{\chi}_k^+ - \hat{\chi}_k^+ \hat{\chi}_k)}_{1} \hat{\phi}_k + B_k V \underbrace{(\hat{\phi}_k \hat{\phi}_k^+ - \hat{\phi}_k^+ \hat{\phi}_k)}_{1} \hat{\chi}_k$$

$$= A_k E_x \hat{\chi}_k + B_k E_{c,k} \hat{\phi}_k + V(A_k \hat{\phi}_k + B_k \hat{\chi}_k)$$

Compare coefficients of $\hat{\phi}_k$ & $\hat{\chi}_k$

$$\boxed{\begin{aligned} A_k E_k &= A_k E_x + V B_k \\ B_k E_k &= B_k E_{c,k} + V A_k \end{aligned}}$$

→ Energies of new modes

→ Exciton & Photon fractions of new modes

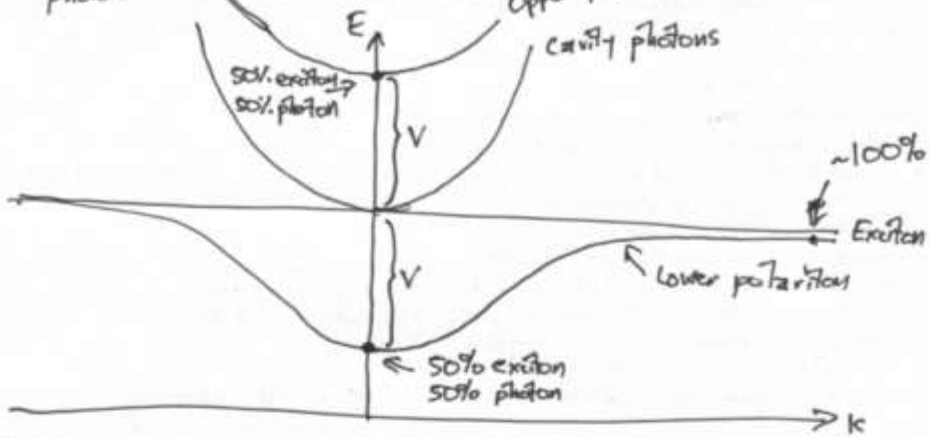
Energies:

$$\begin{pmatrix} E_x - E_k & V \\ V & E_{c,k} - E_k \end{pmatrix} \begin{pmatrix} A_k \\ B_k \end{pmatrix} = 0$$

$\downarrow$ Det $= 0$

$$E_k^2 - E_k(E_x + E_{c,k}) + E_x E_{c,k} - V^2 = 0$$

$$E_k = \frac{E_x + E_{c,k}}{2} \pm \frac{1}{2}\sqrt{\underbrace{(E_x + E_{c,k})^2 + 4V^2 - E_x E_{c,k}}_{(E_x - E_{c,k})^2 + 4V^2}}$$



Exciton & Photon fractions

$$A_k(E_k - E_x) = V\sqrt{1 - A_k^2}$$

Hopfield coefficients

fractions:

$$\therefore\ A_k^2(E_k - E_x)^2 = V^2(1 - A_k^2) \longrightarrow A_k = \frac{V}{\sqrt{(E_k - E_x)^2 + V^2}} \qquad |A_k|^2$$

Similiarly:

$$B_k = \frac{V}{\sqrt{(E_k - E_c)^2 + V^2}} \qquad |B_k|^2$$

Properties of Exciton-Polaritons

photons $\longrightarrow$ Exciton-Polaritons $\longleftarrow$ Excitons

- Light effective mass ($\sim 10^{-4} - 10^{-5}\, m_e$)
- Ballistic Propagation ~ few μm/ps or 1%c
- Direct coupling to External Optics
- Fast response times (ps scale dynamics)
- Long dephasing time (ns scale)

- Non-linear Interactions ($50\ \mathrm{W\,cm^{-2}}$ multistability)
- Control & manipulations by Electric/Magnetic Fields

+ Compact, micron-sized devices
+ Spin degree of freedom

— Schrödinger Equation for spatial dynamics —

$$\hat{\mathcal{H}} = \sum_k E_k \hat{\psi}_k^+ \hat{\psi}_k + \sum_x V_x \hat{\psi}_x^+ \hat{\psi}_x + \sum_k (\hat{\psi}_k^+ + \hat{\psi}_k) F_k e^{i\omega t}$$

① ② ③

Fourier transforms: $\hat{\psi}_k = \frac{1}{\sqrt{N}} \sum_x \hat{\psi}_x e^{-ikx}$

$\hat{\psi}_x = \frac{1}{\sqrt{N}} \sum_k \hat{\psi}_k e^{ikx}$

Commutator:

$$[\hat{\psi}_x, \hat{\psi}_{x'}^+] = \frac{1}{N} \sum_{kk'} \underbrace{(\psi_k \psi_{k'}^+ - \psi_{k'}^+ \psi_k)}_{\delta_{kk'}} e^{i(kx - k'x')} = \frac{1}{N} \overbrace{\sum_k e^{ik(x-x')}}^{N\delta_{xx'}}$$

$$= \delta_{xx'}$$

Master Equation / Von Neumann / Liouville Equation:

$$i\hbar \frac{\partial \rho}{\partial t} = [\hat{\mathcal{H}}, \rho] + \underbrace{\frac{i\Gamma}{2} \sum_k (2\hat{\psi}_k \rho \hat{\psi}_k^+ - \hat{\psi}_k^+ \hat{\psi}_k \rho - \rho \hat{\psi}_k^+ \hat{\psi}_k)}_{\text{④}}$$

Lindblad type dissipation term
(theory of open quantum systems)

Mean-field: ~~Approximation~~ $\langle \psi_k \rangle = \mathrm{tr}\{\rho \hat{\psi}_k\}$

$$i\hbar \frac{\partial \langle \psi_k \rangle}{\partial t} = \underbrace{\mathrm{tr}\left\{ E_k \left(\hat{\psi}_k^+ \hat{\psi}_k \rho \hat{\psi}_k - \rho \hat{\psi}_k^+ \hat{\psi}_k \hat{\psi}_k \right) \right\}}_{E_k \mathrm{tr}\{(\hat{\psi}_k \hat{\psi}_k^+ - \hat{\psi}_k^+ \hat{\psi}_k) \hat{\psi}_k \rho\} = E_k \langle \psi_k \rangle} + \ldots (②, ③, ④)$$

(skip derivation of terms ② ④ :

$$② : \frac{1}{\sqrt{N}} \mathrm{tr}\left\{ \sum_x V_x \hat{\psi}_x^+ \hat{\psi}_x \rho \sum_{x'} \hat{\psi}_{x'} e^{-ikx'} - \sum_x V_x \rho \hat{\psi}_x^+ \hat{\psi}_x \sum_{x'} \hat{\psi}_{x'} e^{-ikx'} \right\}$$

$$= \frac{1}{\sqrt{N}} \mathrm{tr}\left\{ \sum_{xx'} V_x \underbrace{(\hat{\psi}_{x'} \hat{\psi}_x^+ \hat{\psi}_x - \hat{\psi}_x^+ \hat{\psi}_x \hat{\psi}_{x'})}_{\psi_x \delta_{xx'}} e^{-ikx'} \rho \right\} = \frac{1}{\sqrt{N^3}} \mathrm{tr}\left\{ \underbrace{\sum_{xk'k''} V_{k'} e^{ik'x} \psi_{k''} e^{ik''x} e^{-ikx} \rho}_{\sum_{k'k''} V_{k'} \psi_{k''} \underbrace{\sum_x e^{i(k'+k''-k)x}}_{N\delta_{k'',k-k'}}} \right\}$$

$$= \frac{1}{\sqrt{N}} \sum_{k'} V_{k'} \langle \psi_{k-k'} \rangle$$

③: $$\mathrm{tr}\left\{ F_k e^{i\omega t} \left(\psi_k^{+} \rho \psi_k - \rho \psi_k^{+} \psi_k \right) \right\} = F_k e^{i\omega t}$$

④: $$\frac{i\Gamma}{2} \underbrace{\mathrm{tr}\left\{ 2\hat{\psi}_k \rho \hat{\psi}_k^{+} \hat{\psi}_k - \hat{\psi}_k^{+} \hat{\psi}_k \rho \hat{\psi}_k - \rho \hat{\psi}_k^{+} \hat{\psi}_k \hat{\psi}_k \right\}}_{\mathrm{tr}\left\{ \left(2\hat{\psi}_k^{+} \hat{\psi}_k \hat{\psi}_k - \hat{\psi}_k \hat{\psi}_k^{+} \hat{\psi}_k - \hat{\psi}_k^{+} \hat{\psi}_k \hat{\psi}_k \right) \rho \right\}} = -\frac{i\Gamma}{2} \langle \psi_k \rangle$$

k-space:

$$i\hbar \frac{\partial \langle \psi_k \rangle}{\partial t} = \left(E_k - \frac{i\Gamma}{2} \right) \langle \psi_k \rangle + \frac{1}{\sqrt{N}} \sum_{k'} V_{k'} \langle \psi_{k-k'} \rangle + F_k e^{-i\omega t}$$

x-space:

$$i\hbar \frac{\partial \langle \psi_x \rangle}{\partial t} = \frac{1}{\sqrt{N}} \sum_{x'} E_{x'} \langle \psi_{x-x'} \rangle + \left(V_x - \frac{i\Gamma}{2} \right) \langle \psi_x \rangle + F_x e^{-i\omega t}$$

$\downarrow$ Continuous space

$$i\hbar \frac{\partial \psi(x)}{\partial t} = \left(\hat{E} + V(x) - \frac{i\Gamma}{2} \right) \psi(x) + F(x) e^{-i\omega t}$$ ← Schrödinger Equation with modifications for pump & decay.

$$\hat{E} = -\frac{\hbar \hat{\nabla}^2}{2m}$$ ← Parabolic effective mass approx.