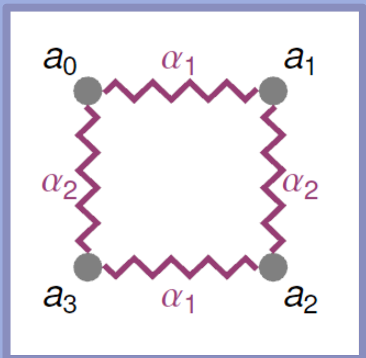
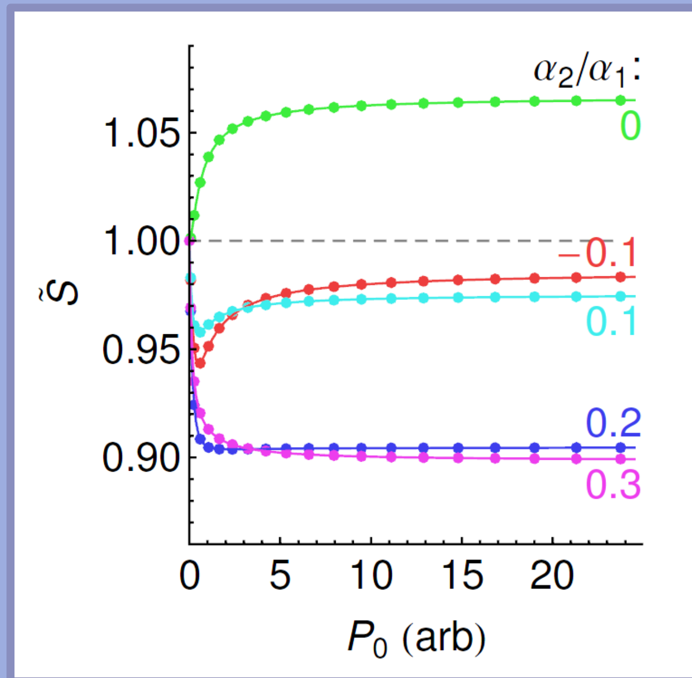
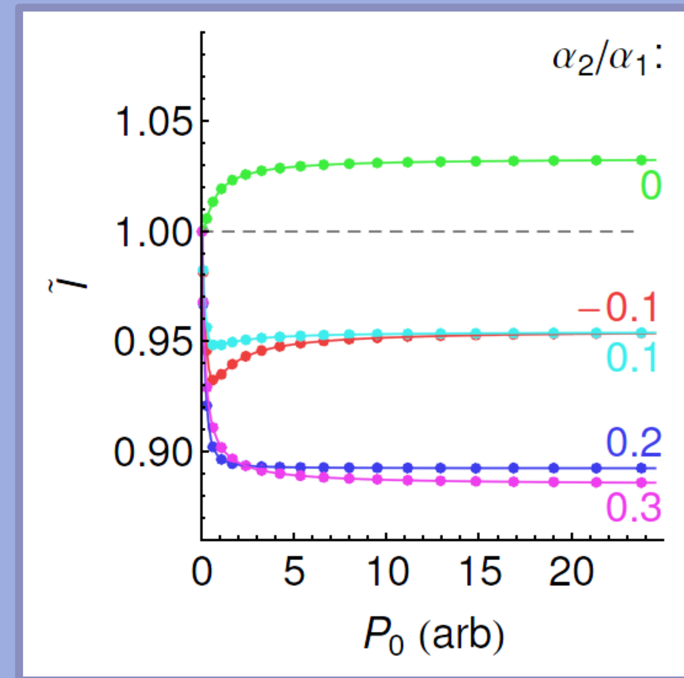
Entanglement between pairs of modes





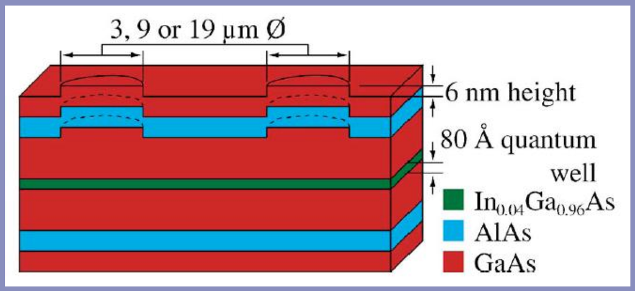


Conditions for Quadripartite Entanglement:

$$I_1 = V(\hat{p}_0 - \hat{p}_1) + V(\hat{q}_0 + \hat{q}_1 + g_2\hat{q}_2 + g_3\hat{q}_3) \geq 1,$$
$$I_2 = V(\hat{p}_1 - \hat{p}_2) + V(g_0\hat{q}_0 + \hat{q}_1 + \hat{q}_2 + g_3\hat{q}_3) \geq 1,$$
$$I_3 = V(\hat{p}_2 - \hat{p}_3) + V(g_0\hat{q}_0 + g_1\hat{q}_1 + \hat{q}_2 + \hat{q}_3) \geq 1,$$

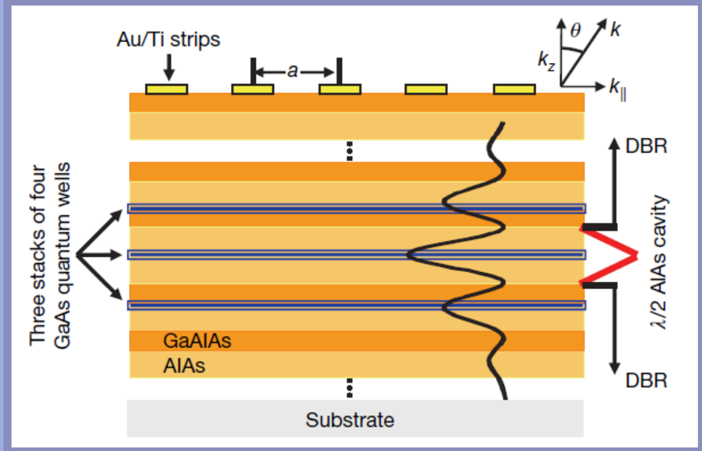
P van Loock & A Furusawa, PRA, 67, 052315 (2003)

Coupled Polariton Boxes



Pattern Cavity Thickness

R Idrissi Kaitouni, et al., PRB, 74, 155311 (2006)



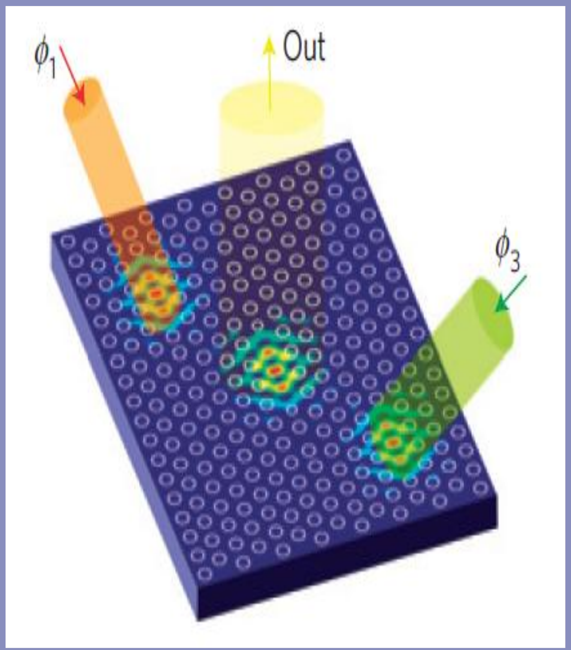
Metal surface pattern

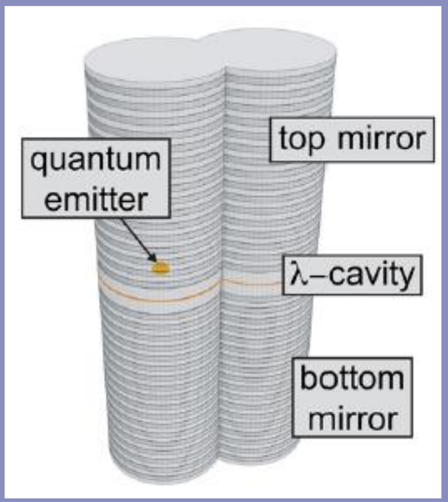
C W Lai, et al., Nature, 450, 529 (2007)

S Utsunomiya, et. al., Nat. Phys., 4, 700 (2008)

Apply Stress

R Balili, et al., Science, 316, 1007 (2007)





Optically Induced Potential

A Amo, et al., PRB, 82, 081301(R) (2010)

Coupled Micropillar Cavities

S Michaelis de Vasconcellos, et al., APL, 99, 101103 (2011)

M Galbiati, et al., Phys. Rev. Lett., 208, 126403 (2012)

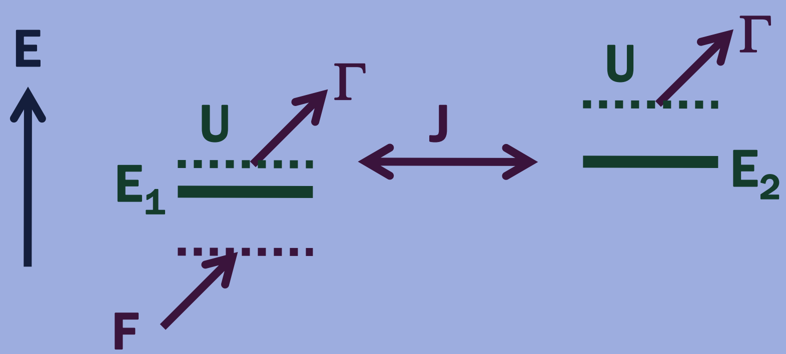
Coupled Photonic Crystal Cavities

D Gerace, et al., Nature Phys., 5, 281 (2009)

Theory

Hamiltonian:

$$\hat{\mathcal{H}} = E_1\hat{a}_1^\dagger\hat{a}_1 + E_2\hat{a}_2^\dagger\hat{a}_2 + U\left(\hat{a}_1^\dagger\hat{a}_1^\dagger\hat{a}_1\hat{a}_1 + \hat{a}_2^\dagger\hat{a}_2^\dagger\hat{a}_2\hat{a}_2\right) - J(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1) + F\hat{a}_1^\dagger + F^*\hat{a}_1$$



J=0.5 meV
(3um boxes, 1um apart)

U=0.012 meV (3um size)
[J Kasprzak, et al., PRB, 75, 045326]

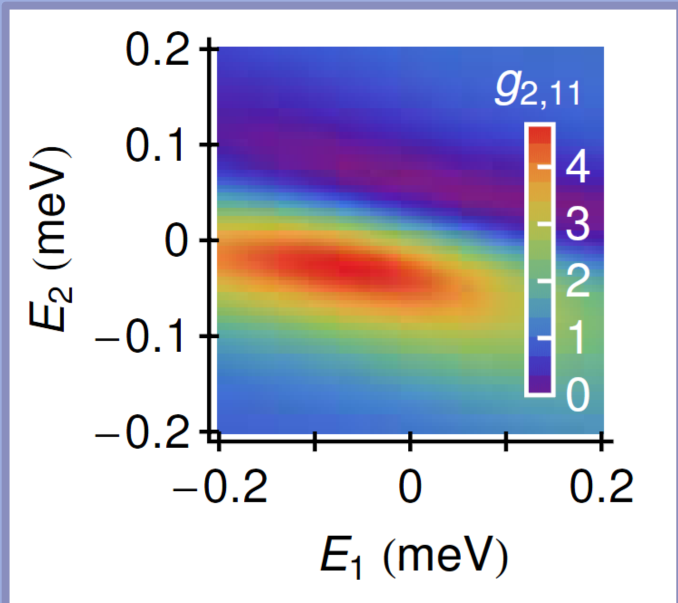
Γ=0.2 meV (3.3 ps)

Master Equation:

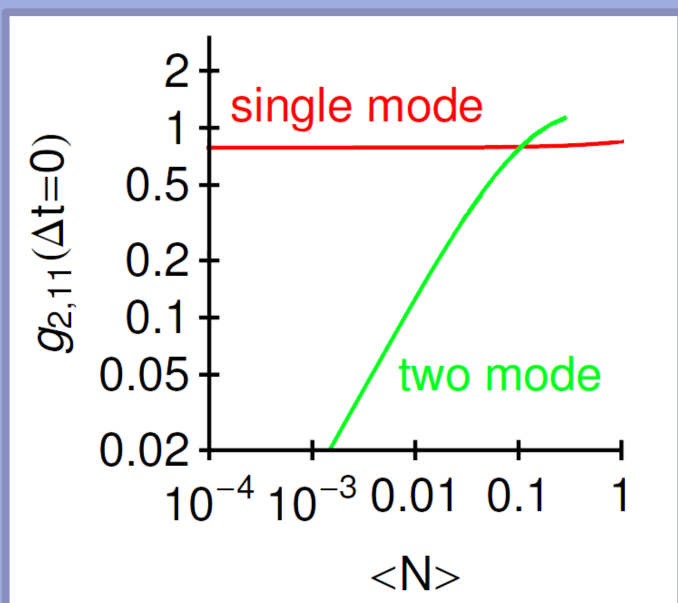
$$i\hbar\frac{d\boldsymbol{\rho}}{dt} = \left[\hat{\mathcal{H}}, \boldsymbol{\rho}\right] + i\frac{\Gamma}{2}\sum_{n=1}^{2}\left(2\hat{a}_n\boldsymbol{\rho}\hat{a}_n^\dagger - \hat{a}_n^\dagger\hat{a}_n\boldsymbol{\rho} - \boldsymbol{\rho}\hat{a}_n^\dagger\hat{a}_n\right)$$

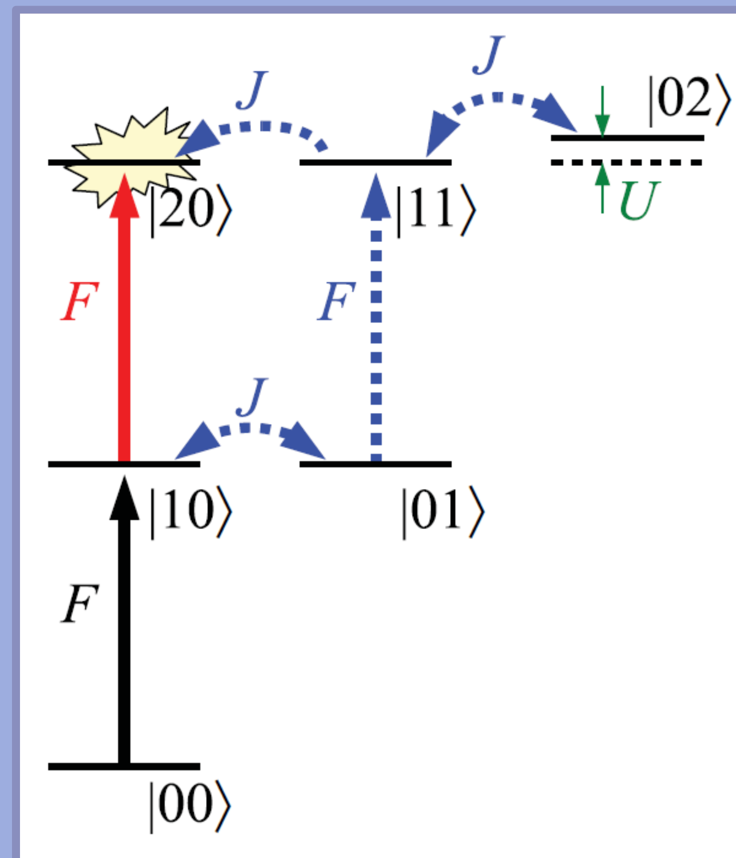
Quantum Interference

$$g_2 = \frac{\langle \hat{a}^\dagger \hat{a}^\dagger \hat{a} \hat{a} \rangle}{\langle \hat{a}^\dagger \hat{a} \rangle^2}$$



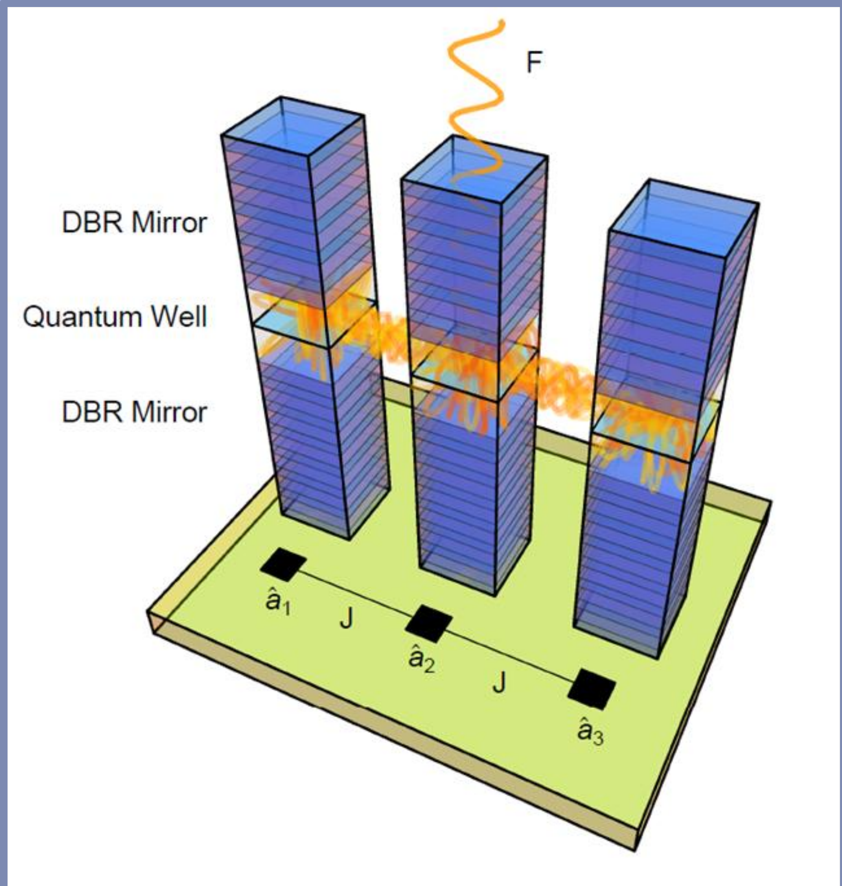
T C H Liew & V Savona,
PRL, 104, 183601 (2010).



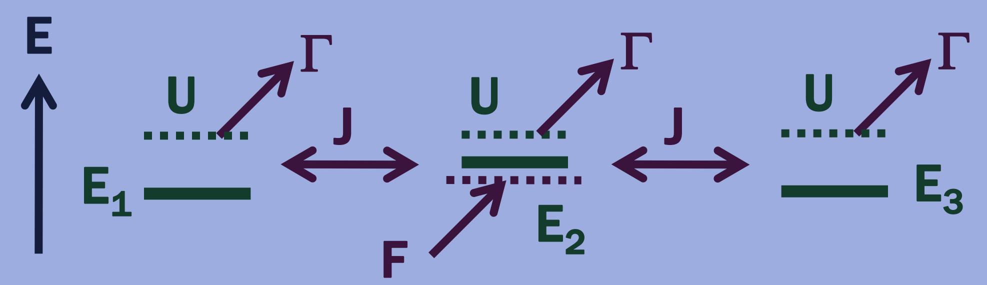


M Bamba, A Imamoglu, I Carusotto,
C Ciuti, PRA, 83, 021802 (2011)

Scheme for Two-Party Entanglement



$$\hat{H} = \sum_n \left(E_n \hat{a}_n^\dagger \hat{a}_n + U \hat{a}_n^\dagger \hat{a}_n^\dagger \hat{a}_n \hat{a}_n \right) + J\left(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_3 + \hat{a}_3^\dagger \hat{a}_2 \right) + F\left(\hat{a}_2^\dagger + \hat{a}_2 \right)$$



J=0.5 meV (3um boxes, 1um apart)

U=0.012 meV (3um size)

[J Kasprzak, et al., PRB, 75, 045326]

Γ=0.044 meV (15 ps)

T C H Liew & V Savona, PRA, 85, 050301(R) (2012).

$$i\hbar \frac{d\boldsymbol{\rho}}{dt} = \left[\hat{\mathcal{H}}, \boldsymbol{\rho} \right] + i\frac{\Gamma}{2} \sum_n \left(2\hat{a}_n \boldsymbol{\rho} \hat{a}_n^\dagger - \hat{a}_n^\dagger \hat{a}_n \boldsymbol{\rho} - \boldsymbol{\rho} \hat{a}_n^\dagger \hat{a}_n \right)$$

Two-Party Entanglement

No Qubits! Use continuous variables:

$$\hat{p}_n = \frac{\hat{a}_n + \hat{a}_n^\dagger}{2}, \qquad \hat{q}_n = \frac{\hat{a}_n - \hat{a}_n^\dagger}{2i}$$

Violation of "Bell" inequality:

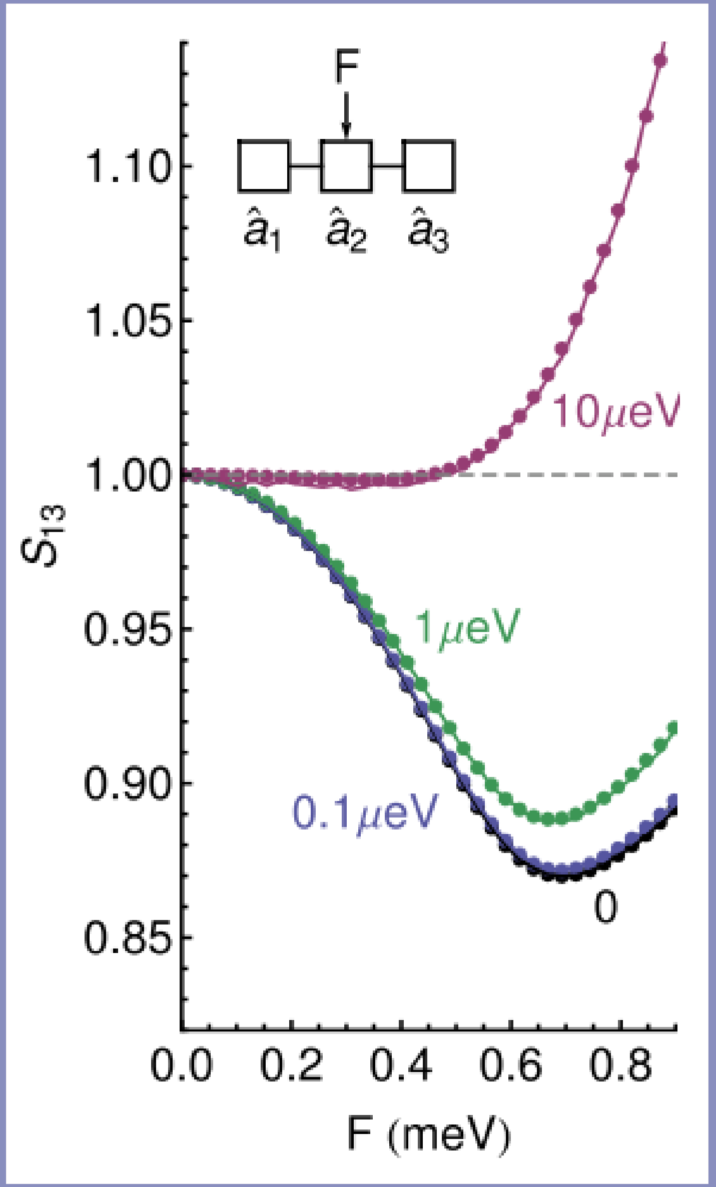
$$1 \leq S_{nm} = V(\hat{p}_n - \hat{p}_m) + V(\hat{q}_n + \hat{q}_m)$$

$$V(\hat{\mathcal{O}}) = \langle \hat{\mathcal{O}}^2 \rangle - \langle \hat{\mathcal{O}} \rangle^2$$

L M Duan, et al., PRL, 84, 2722 (2000)
R Simon, PRL, 84, 2726 (2000)

$$\frac{\Gamma_P}{2} \sum_n \left(2\hat{a}_n^\dagger \hat{a}_n \boldsymbol{\rho} \hat{a}_n^\dagger \hat{a}_n - \hat{a}_n^\dagger \hat{a}_n \hat{a}_n^\dagger \hat{a}_n \boldsymbol{\rho} - \boldsymbol{\rho} \hat{a}_n^\dagger \hat{a}_n \hat{a}_n^\dagger \hat{a}_n\right)$$

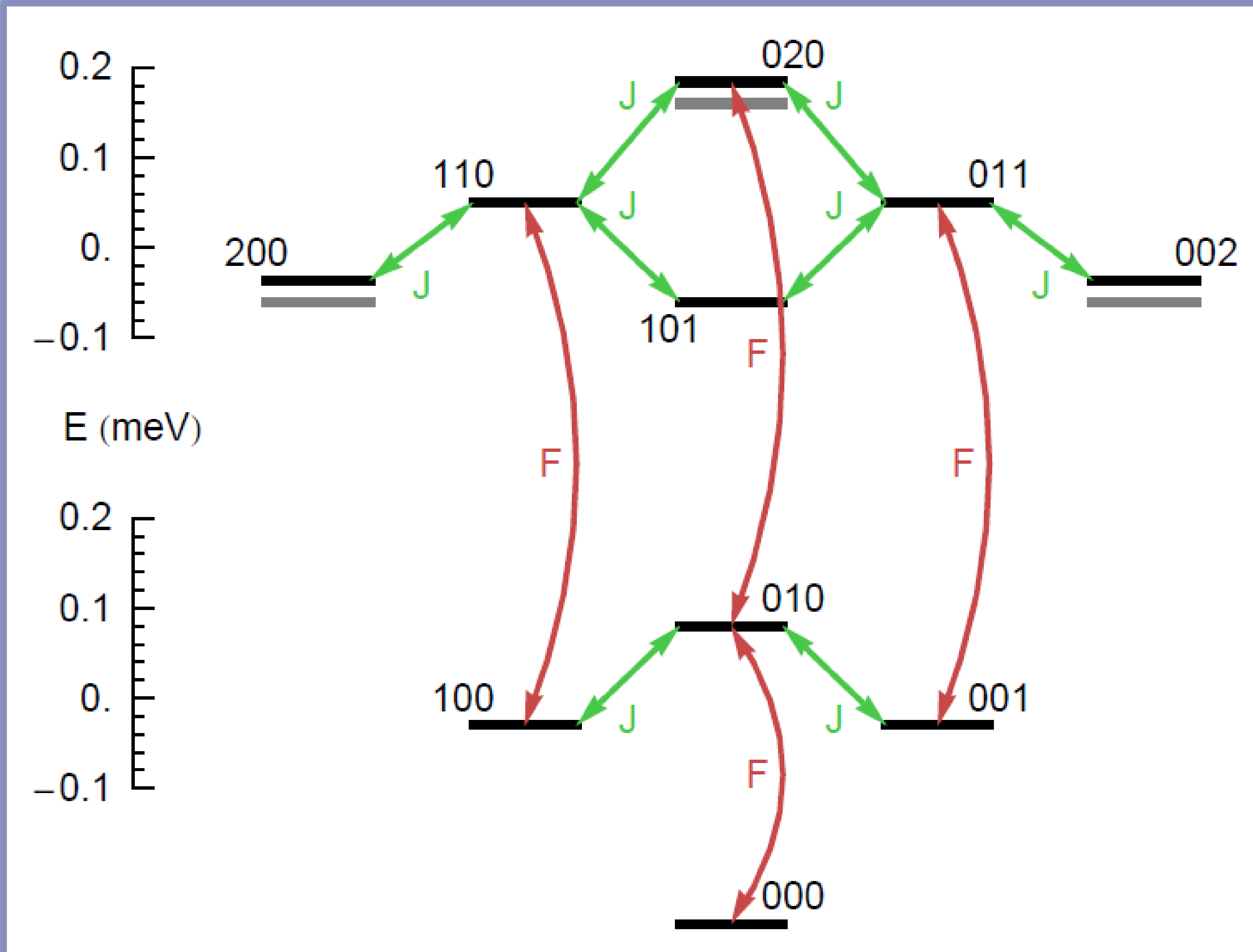
D F Walls, M J Collet, & G J Milburn,
Phys. Rev. D., 32, 3208 (1985).



Γ_P~=0.1-0.3 μeV

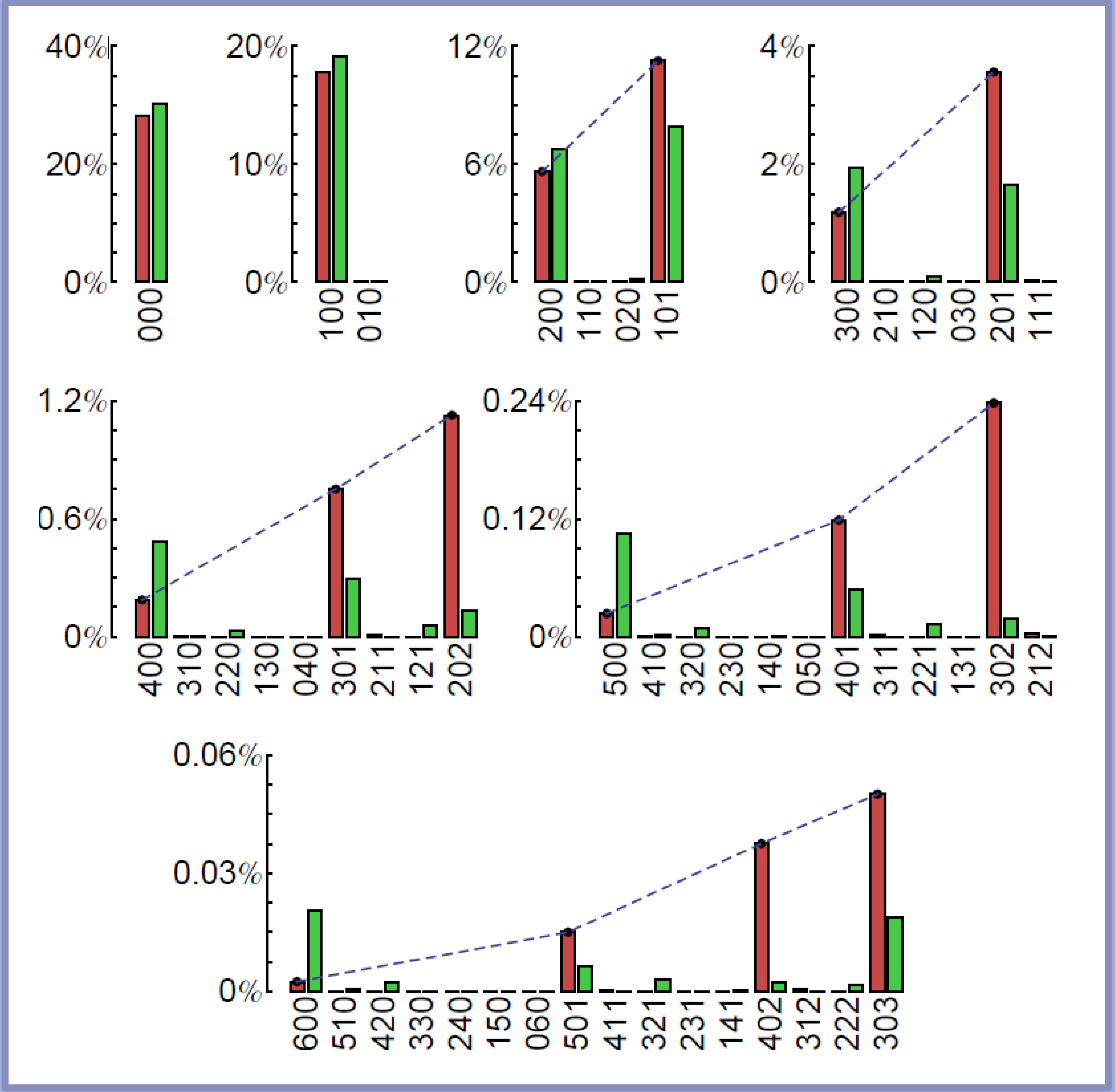
V Savona & C Piermarocchi, Phys. Status Solidi A, 164, 45 (1997).

Analysis of Number States



Analogy with: M Bamba, A Imamoglu, I Carusotto, C Ciuti, PRA, 83, 021802 (2011)

Analysis of Number States



Quantum Monte Carlo

Non Hermitian Hamiltonian:

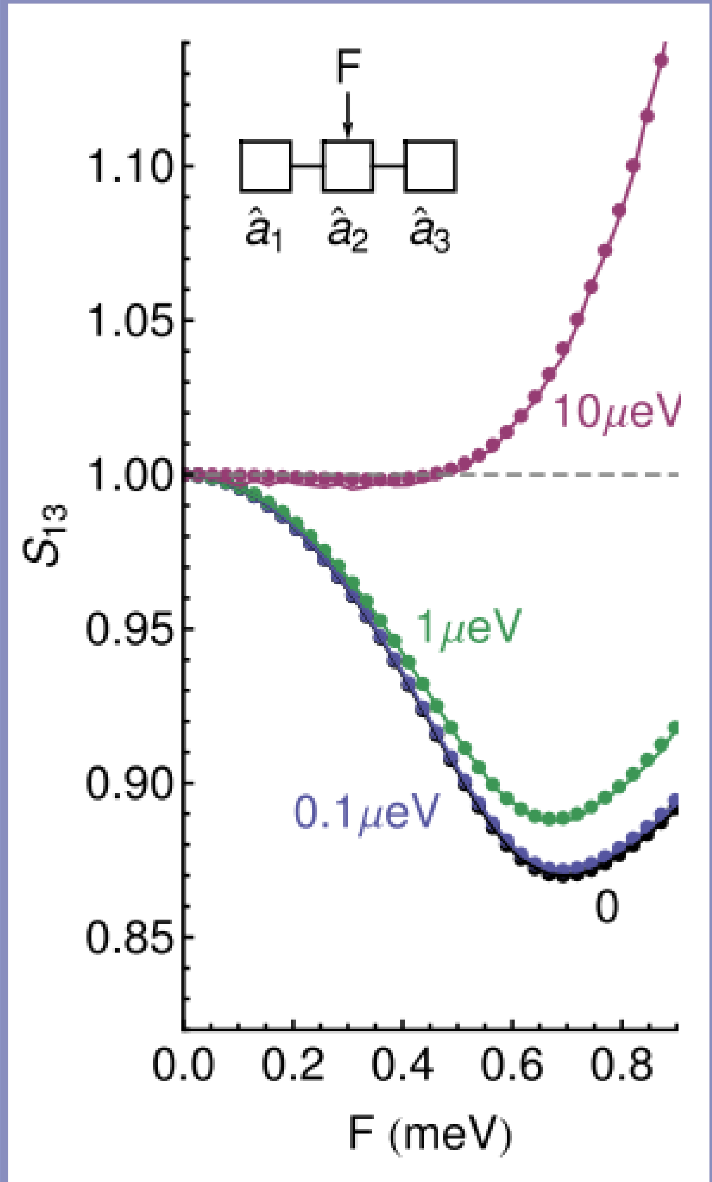
$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_S - \frac{i\hbar}{2}\Gamma \sum_n \hat{n}_n - \frac{i\hbar}{2}\Gamma_P \sum_n \hat{n}_n^2$$

Hamiltonian Evolution supplemented with random quantum Jumps:

$$|\psi(t+\delta t)\rangle = \hat{a}_m |\psi(t)\rangle$$
$$|\psi(t+\delta t)\rangle = \hat{n}_m |\psi(t)\rangle$$

Observables averaged over different wavefunction realizations:

$$\langle \mathcal{O} \rangle = \frac{1}{N_R} \sum_{i=1}^{N_R} \left\langle \psi^{(i)}(t) | \hat{\mathcal{O}} | \psi^{(i)}(t) \right\rangle$$



K Molmer, Y Castin, & J Dalibard, J. Opt. Soc. Am. B, 10, 524 (1993).

Quadripartite Entanglement