

# Coherent spin dynamics in semiconductor low-dimensional systems

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Spin-related phenomena in mesoscopic transport  
17<sup>th</sup> of September, 2012



Electron spin plays major role in various phenomena. Most importantly, the spin of electrons results in the Pauli exclusion principle, which in turn underlies the periodic table of chemical elements and makes our existence possible.

It is needless to explain in this audience the manifestations of spin in the electron transport. **Hence, the aim of my talk is to demonstrate alternative, in particular, optical means to access electron spin in low-dimensional systems.**



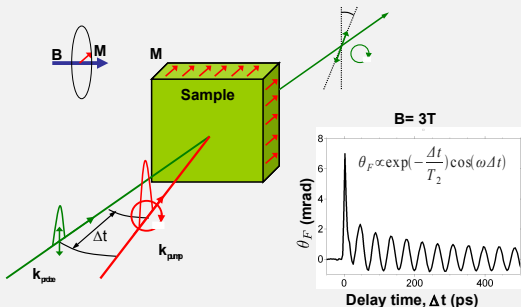
time-resolved,  $t \ll \tau_{\text{rad}}$ ,  
“non-destructive”

## DICHROISM AND OPTICAL ANISOTROPY OF MEDIA WITH ORIENTED SPINS OF FREE ELECTRONS

A. G. Aronov and E. L. Ivchenko

Sov. Phys. Solid State  
(1973)

A. F. Ioffe Physicotechnical Institute,  
Academy of Sciences of the USSR, Leningrad



Faraday rotation



Ellipticity



**Pump:** circularly polarized pulse  
orients carrier spins

$$S_z \neq 0 \Rightarrow \begin{cases} n_+ \neq n_- \\ \alpha_+ \neq \alpha_- \end{cases}$$

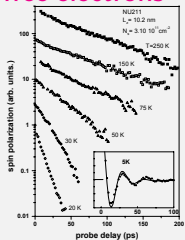
Faraday rotation

$$\Theta \propto S_z$$

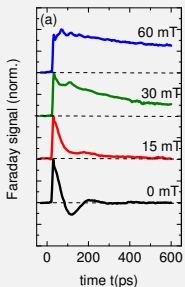
- **Faraday Effect** (polarization plane rotation)
- **Ellipticity Effect** (appearance of the circular polarization)
- **Kerr Effect** (polarization plane rotation of reflected light)



## Quantum wells, free electrons



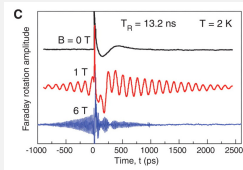
R.T. Harley et al. (2007)



T. Korn et al. (2009)

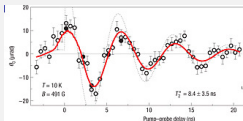
M.M. Glazov

## Quantum dot ensembles



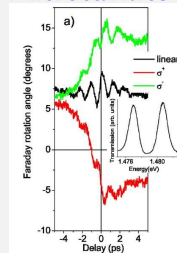
A. Grelich et al. (2006)

## Single quantum dots



M.H. Mikkelsen et al. (2007)

## Microcavities



A. Brunetti et al. (2006)

Pump-probe is a widespread technique to measure relaxation times, precession frequencies

- What can we learn from “microscopics” of signals?
- What about signal amplitudes?

# We focus here on quantum dots:



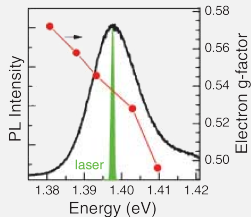
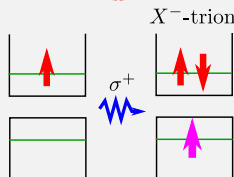
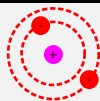
*n*-type **single quantum dots** and quantum dot arrays

- Carrier localization leads to the spin relaxation **slowdown**
- Possibility to address resonantly **singlet trion**
- Hyperfine** interaction of electron and nuclear spins is effective



Similar physics for quantum wires, wells and bulk materials with low,  $n_d a_B^d \ll 1$ , density electrons/holes

Other systems: Phys. Solid State **54**, 1 (2012)

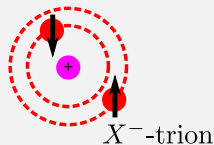
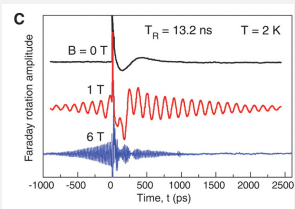


Element	<sup>27</sup> Al	<sup>69(71)</sup> Ga	<sup>75</sup> As	<sup>115</sup> In
<i>Z</i>	13	31	33	49
<i>I</i>	5/2	3/2	3/2	9/2

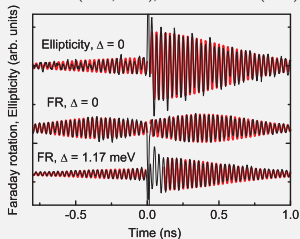


## Spin pump-probe signals for InAs $n$ -type quantum dots array

### Experiments:



A. Greulich et al. (2006, 2007); S. Carter et al. (2009)



MMG et al. (2010)

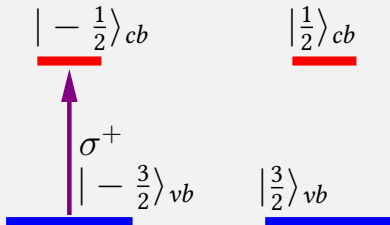
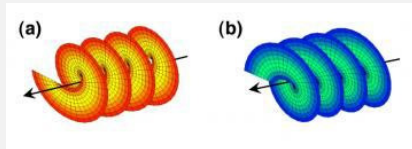
- 1 Origin of the long-living signal at singlet ( $S = 0$ ) trion excitation?
- 2 Signals at negative delays, i.e. before the next pump pulse?
- 3 Growth of Faraday rotation signal with time?



- 1 Introduction. Questions to theory
- 2 Interaction of light with spins
  - Optical orientation via trions
  - Detection of spin polarization
- 3 Spin mode-locking effect
  - Phase synchronization condition & passive mode-locking
  - Nuclei-induced active mode-locking
  - Emergence of Faraday rotation
- 4 Spin dynamics in equilibrium
- 5 Conclusions



**Optical orientation** is a transformation of the photon angular momentum to **the system of electron spins**



(normal light incidence)

Semiconductor quantum well or self-organized quantum dot:

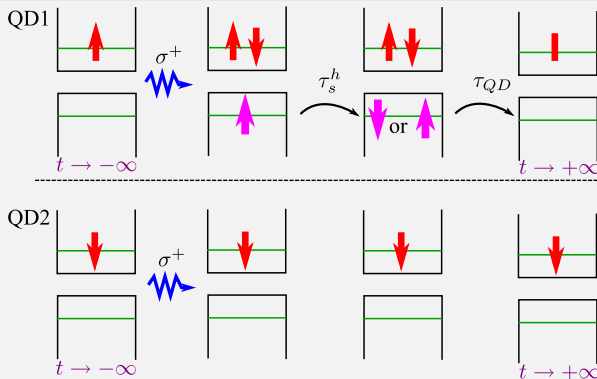
- at  $\sigma^+$  pump  $-\frac{3}{2} + 1 = -\frac{1}{2}$   
i.e.  $(e, hh) = (-1/2, 3/2)$
- at  $\sigma^-$  pump  $\frac{3}{2} + (-1) = \frac{1}{2}$   
i.e.  $(e, hh) = (1/2, -3/2)$

Absorption of circularly polarized light

generates spin-polarized electrons and holes



# Long-living electron spin polarization



$\sigma^+$  pump  $\Rightarrow (-1/2e, 3/2h)$

The carriers with the spins opposite to those of photo-created electrons are picked out

Hole spin relaxation/fast spin precession (@  $B \neq 0$ )  $\Rightarrow$  spin of returning electron is negligible

Resident electrons become spin polarized after recombination

Shabaev, Efros, Gammon, Merkulov (2003); Grelich et al (2006); Zhukov, Yakovlev, Bayer, MMG, Ivchenko, Karczewski, Kossut (2007)

# Spin pumping in quantum dots



Only QDs with a **certain spin projection** interact with the **circularly polarized light**

## Technical details:

- Short pump pulse:  $\tau_p \ll \tau_{QD}, 2\pi/\Omega_L$
- Four level model: two ground states  $\psi_{\pm 1/2}$ , two excited states  $\psi_{\pm 3/2}$
- Returning electron is depolarized

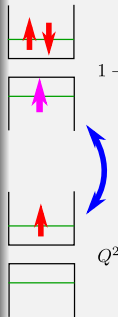
Under  $\sigma^+$  pump the wave function transforms as

$$\psi_{1/2}(t \rightarrow +\infty) = Q e^{i\Phi} \psi_{1/2}(t \rightarrow -\infty)$$

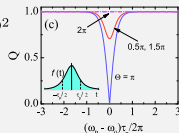
$$\psi_{-1/2}(t \rightarrow +\infty) = \psi_{-1/2}(t \rightarrow -\infty)$$

Transformation of the wave function means transformation of the spin

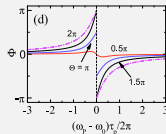
Similar to N. Rosen & C. Zener (1932)



$1 - Q^2$



$Q^2$



Parameters  $Q$  and  $\Phi$  are determined by the pump pulse **shape, area, and detuning from resonance**

# Spin pumping in quantum dots



Only QDs with a **certain spin projection** interact with the **circularly polarized light**

$$\text{Electron spin } \mathbf{S} = \frac{1}{2} \langle \psi | \boldsymbol{\sigma} | \psi \rangle$$

$$S_z^+ = \frac{Q^2 + 1}{2} S_z^- + \frac{Q^2 - 1}{4}$$

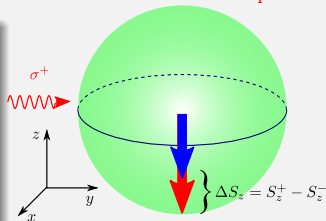
$$S_y^+ = Q \cos \Phi S_y^- - Q \sin \Phi S_x^-$$

$$S_x^+ = Q \cos \Phi S_x^- + Q \sin \Phi S_y^-$$

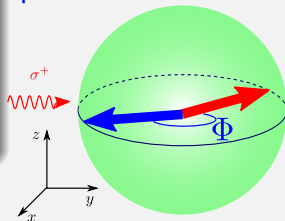
- Before pump pulse  $\mathbf{S}^-$ , after  $\mathbf{S}^+$
- An increase of spin z-component
- In-plane components rotation

Yugova, MMG, Ivchenko, Efros (2009)

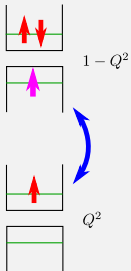
Spin orientation  $\sim \frac{Q^2 - 1}{4}$



Spin rotation  $\sim \Phi$



# Spin rotation by optical pulse



## Circular pulse

- generates spin coherence (optical orientation)
- rotates spin (inverse Faraday effect)

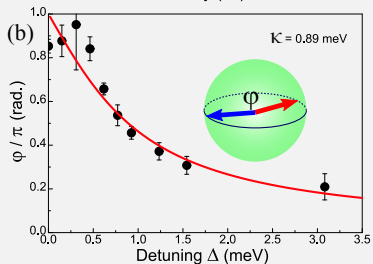
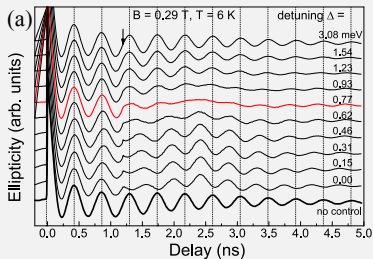
$$S_z^+ = \frac{Q^2 + 1}{2} S_z^- + \frac{Q^2 - 1}{4}$$

$$S_y^+ = Q \cos \Phi S_y^- - Q \sin \Phi S_x^-$$

$$S_x^+ = Q \cos \Phi S_x^- + Q \sin \Phi S_y^-$$

Phys. Rev. B **80**, 104436 (2009)

Parameters  $Q$  and  $\Phi$  are determined by the pump pulse intensity, duration and detuning from the resonance ( $\Phi$ )



A. Greilich et al. (2009): InGaAs QDs  
C. Phelps et al. (2009): CdTe QWs



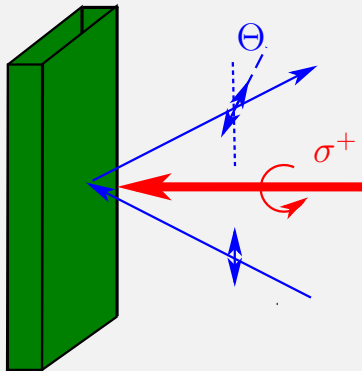
**Circularly polarized pulses being resonant with the singlet trion transition orient electron spins in quantum dots**

**Slightly off-resonant pulse rotates spins**

# Spin coherence detection



QW or ensemble of QDs, ...



**Probe:** transmission/reflection of weak linearly-polarized pulse  
weak=does not affect spin coherence



$r_+ \neq r_- \Rightarrow$  rotation of the linear polarization plane of the probe pulse and appearance of its ellipticity

Ellipticity and Faraday rotation

$$\mathcal{E} + i\mathcal{F} \propto r_+ - r_-$$

Resonance trion, exciton, ...

$$r_{\pm}(\omega) = \frac{i\Gamma_{0,\pm}}{\omega_{0,\pm} - \omega - i(\Gamma_{0,\pm} + \Gamma_{\pm})}$$

$$t_{\pm} = 1 + r_{\pm}$$

Zhukov, Yakovlev, Bayer, MMG, Ivchenko, et al. (2007)

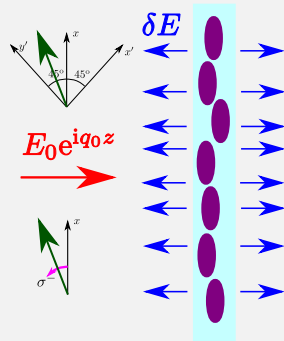


- **Faraday Effect** (polarization plane rotation)

$$\mathcal{F} = \lim_{z \rightarrow +\infty} \int_{-\infty}^{\infty} \left[ |E_{x'}^{(t)}(z, t)|^2 - |E_{y'}^{(t)}(z, t)|^2 \right] dt$$

- **Ellipticity Effect** (appearance of the circular polarization)

$$\mathcal{E} = \lim_{z \rightarrow +\infty} \int_{-\infty}^{\infty} \left[ |E_{\sigma^-}^{(t)}(z, t)|^2 - |E_{\sigma^+}^{(t)}(z, t)|^2 \right] dt$$



## Probe induced field

$$\delta \mathbf{E}(t) = -4\pi \left( \frac{\omega_{\text{pr}}}{c} \right)^2 \frac{i e^{iq|z|}}{2q} N_{\text{QD}}^{2d} \mathbf{\Pi}(t)$$

$$\Pi_x \propto (n_e - n_{tr}) E_x^{\text{probe}}$$

$$\Pi_y \propto (S_z - J_z) E_x^{\text{probe}}$$

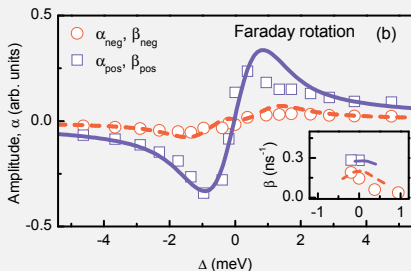
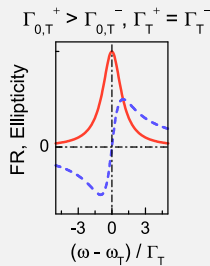
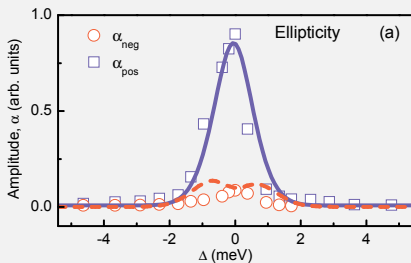
## Spectral function

$$\mathcal{E} + i\mathcal{F} \propto$$

$$\int_{-\infty}^{\infty} dt \int_{-\infty}^t dt' f(t) f(t') e^{i\Delta(t-t')}$$

$$\Delta = \omega_{\text{pr}} - \omega_0; \mathbf{E}_{\text{pr}} \propto f(t) e^{-i\omega t}$$

# Electron spin coherence detection



**Linearly polarized pulse can readout spin polarization**

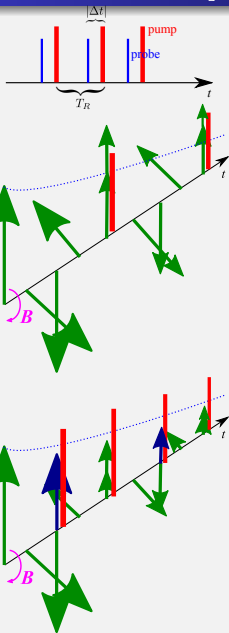
**Correct description is based on reflection/transmission rather than on dielectric constant**

MMG et al. (2010)





- 1 Introduction. Questions to theory
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  - Phase synchronization condition & passive mode-locking
  - Nuclei-induced active mode-locking
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- 4 Spin dynamics in equilibrium
- 5 Conclusions

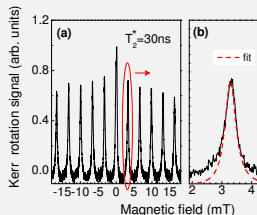


Signal dependence on  $B$  at a fixed delay  $\Delta t$

Resonant spin amplification is determined by commensurability of spin precession and pump repetition periods

$$\Omega_L T_{\text{rep}} = 2\pi$$

Kikkawa, Awschalom (1998)



$$\Omega_L T_{\text{rep}} = \pi$$

Astakhov, MMG, Yakovlev, Zhukov, Ossau, Molenkamp, Bayer (2008)

In inhomogeneous systems

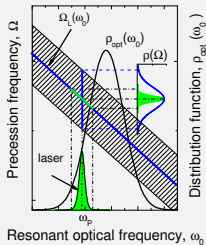
certain electrons become polarized stronger than others (**mode-locking**)

# Electron spin precession mode-locking



Train of pump and probe pulses,  $T_R$  is the repetition period

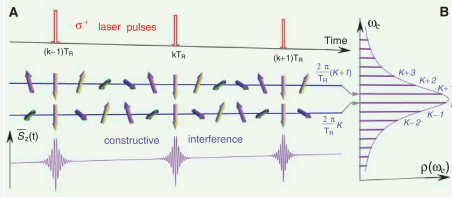
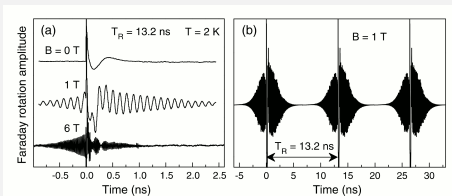
- Large spread of electron  $g$ -factors  $\Rightarrow$  fast dephasing
- Signal reappears before the next pump pulse arrival



$$\hbar/\tau_p \sim 1 \text{ meV}$$

$$\Omega T_R = 2\pi K$$

Constructive  
interference  
exp.:  $K \sim 10^2$

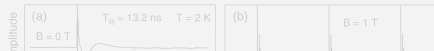
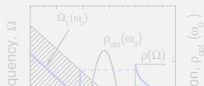


# Electron spin precession mode-locking



Train of pump and probe pulses,  $T_R$  is the repetition period

- Large spread of electron g-factors  $\Rightarrow$  fast dephasing
- Signal reappears before the next pump pulse arrival



“Passive” mode-locking:

all synchronized modes have same initial phases owing to the pump

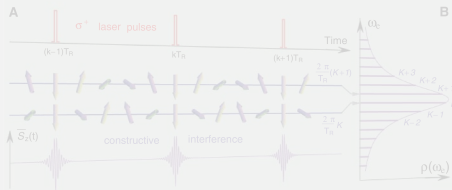


Resonant optical frequency,  $\omega_0$

$$\hbar/\tau_p \sim 1 \text{ meV}$$

$$\Omega T_R = 2\pi K$$

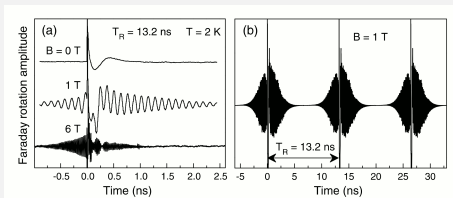
Constructive interference  
exp.:  $K \sim 10^2$



# Mode-locking is much stronger than expected:

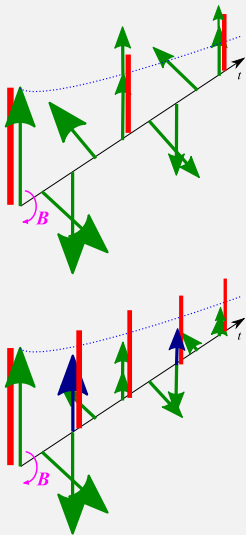


Where are **other spins** with precession frequencies  $\Omega \neq \frac{2\pi K}{T_R}$ ?





Very high amplitude at negative delays

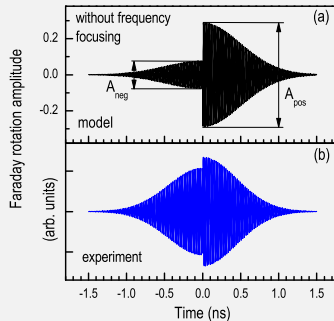


$$\Omega_{\text{eff}} T_R = 2\pi$$

↑  
Nuclei-  
induced  
frequency  
focusing



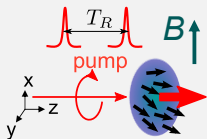
$$\Omega_{\text{eff}} T_R = \pi$$



$$\Omega_{\text{eff}} = \Omega_L + \Omega_{\text{nucl}}$$

Feedback of nuclei is necessary  
Nuclei-induced frequency focusing  
takes place

Greilich, Shabaev, Yakovlev, Efros, Yugova, Reuter, Wieck, Bayer (2007)



Nuclear spin polarization

$$\mathbf{m} = \sum_i \mathbf{I}_i$$

Spin precession between the pulses

$$\frac{d\mathbf{m}}{dt} = [\alpha \mathbf{S}(t) \times \mathbf{m}(t)] + [\boldsymbol{\omega} \times \mathbf{m}(t)],$$

$$\frac{d\mathbf{S}}{dt} = [\alpha \mathbf{m}(t) \times \mathbf{S}(t)] + [\boldsymbol{\Omega} \times \mathbf{S}(t)],$$

$$(n-1)T_R < t < nT_R$$

Pump pulse action:  $\mathbf{m}^+ = \mathbf{m}^-$

$$S_z^+ = \frac{Q^2 + 1}{2} S_z^- + \frac{Q^2 - 1}{4}$$

$$S_y^+ = QS_y^-, \quad S_x^+ = QS_x^-$$

**Time scales @  $B = 1$  T**

- 1 Electron spin precession

$$\frac{2\pi}{\Omega} \lesssim 0.1 \text{ ns}$$

- 2 Precession in nuclear field

$$\frac{2\pi}{\alpha m} \sim 10 \text{ ns}$$

- 3 Pulse repetition period

$$T_R \sim 10 \text{ ns}$$

- 4 Nuclear spin precession

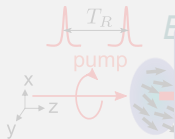
$$\frac{2\pi}{\omega} \lesssim 100 \text{ ns}$$

- 5 Precession in electron field

$$\frac{2\pi}{\alpha S} \gtrsim 10^3 \text{ ns}$$

MMG, Yugova, Efros (2012)

# Classical origin of the focusing



Nuclear spin polarization

$$\frac{m_x(t)}{m} \approx \frac{t}{\tau_{nf}}, \quad \frac{1}{\tau_{nf}} = \frac{\alpha^3 m T_R}{\omega \Omega^2} f(\Omega T_R, \Theta)$$

Spin precession between the pulses

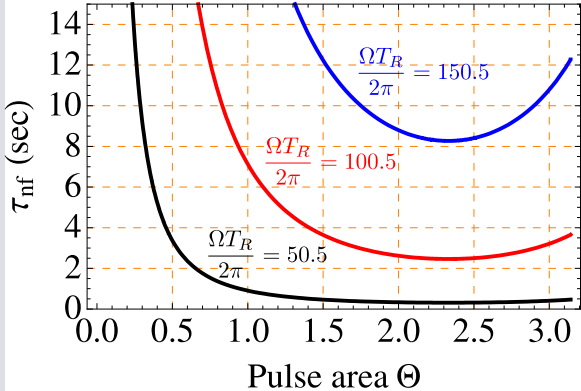
$$\frac{dm}{dt} = [\alpha S$$

$$\frac{dS}{dt} = [\alpha m$$

Pump pulse

$$S_z^+ =$$

$$S_y^+ =$$



1 T  
recession

0.1 ns

2 Precession in nuclear field

10 ns

period

0 ns

recession

100 ns

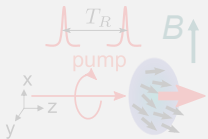
electron field

10<sup>3</sup> ns

G, Yugova, Efros (2012)



# Classical origin of the focusing



Nuclear spin polarization

$$m = \sum_i I_i$$

Spin precession between the pulses

$$\frac{dm}{dt} = [\gamma S(t) \times m(t)] + [\mu_N \times m(t)]$$

“Active” mode-locking:

spin precession frequencies are synchronized in all dots **thanks to nuclei**

$\frac{dm}{dt}$

$$(n-1)T_R < t < nT_R$$

Pump pulse action:  $m^+ = m^-$

$$S_z^+ = \frac{Q^2 + 1}{2} S_z^- + \frac{Q^2 - 1}{4}$$

$$S_y^+ = QS_y^-, \quad S_x^+ = QS_x^-$$

**Time scales @  $B = 1$  T**

- 1 Electron spin precession

$$\frac{2\pi}{\Omega} \lesssim 0.1 \text{ ns}$$

- 2 Precession in nuclear field

$$2\pi \lesssim 10 \text{ ns}$$

- 4 Nuclear spin precession

$$\frac{2\pi}{\omega} \lesssim 100 \text{ ns}$$

- 5 Precession in electron field

$$\frac{2\pi}{\alpha S} \gtrsim 10^3 \text{ ns}$$

MMG, Yugova, Efros (2012)



## Random nuclear spin dynamics

A. Grelich et al (2006)

Random nuclear spin flips driven by precessing electron spin:

$$\frac{1}{\tau_n} = \frac{\alpha^2 \tau_c}{1 + (\Omega \tau_c)^2} \sim \frac{\alpha^2}{\Omega^2 \tau_c}$$

M.I. Dyakonov and V.I. Perel' (1973)

Phenomenological electron correlation time  $\tau_c \propto 1/W_{tr}$ , where  $W_{tr}$  is the trion creation probability. At PSC nuclear spin flips stop.

✓ Our approach shows that the nuclear dynamics is **directed!**

## Dynamical nuclear polarization

V. Korenev (2010)

Equilibrium approach/detuned pump:

$$\frac{dm_x}{dt} = -\frac{m_x - qS_x(m_x)}{T_{1e}} - \frac{m_x}{T_1}$$

A.W. Overhauser (1953)

PSC is generally not stable!?

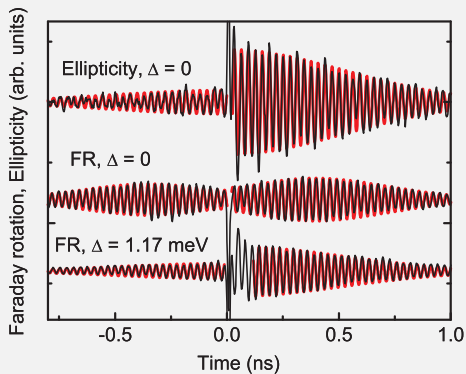
✓ **Nonequilibrium** regime for the **pump-probe** conditions!

✓ Tuning is possible for **resonant pump!**

# Puzzle of spin-Faraday effect



Nuclei induce effective tuning of electron spin precession frequencies to the synchronous with pump repetition period values



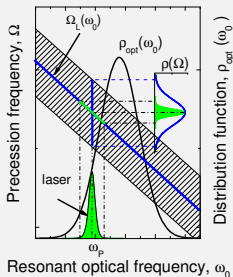
MMG et al. (2010)

Why does spin Faraday signal amplitude grow with time?



## Inhomogeneous array:

- Spread of resonant frequencies
- Related spread of g-factors
- Random nuclear fields



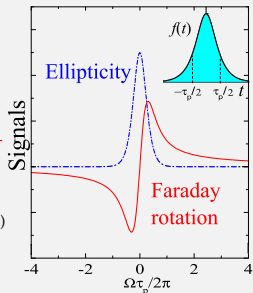
## Pump-probe signal calculation

QDs array, correlated distribution of resonant frequencies,  $\omega_0$ , and g-factors,  $p(\omega_0, g)$ :

$$S(\Delta t) \propto \int d\omega_0 dg p(\omega_0, g) S_z(\omega_0, g, \Delta t) \times \left\{ \begin{array}{l} \mathcal{F}(\omega_0 - \omega_{pr}) \\ \mathcal{E}(\omega_0 - \omega_{pr}) \end{array} \right\}$$

$$\sigma_g \approx 2 - \frac{4|p_{cv}|^2}{3m_0} \frac{\Delta}{E_g^{QD}(E_g^{QD} + \Delta)}$$

L. Roth (1964); E.L. Ivchenko, A.A. Kiselev (1992)

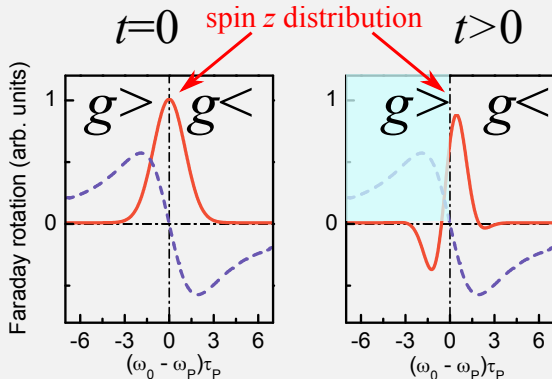
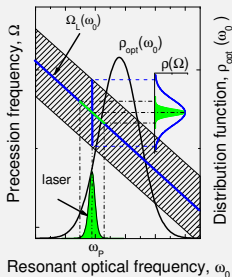


# Emergence of Faraday rotation



## Inhomogeneous array:

- Spread of resonant frequencies
- Related spread of  $g$ -factors
- Random nuclear fields



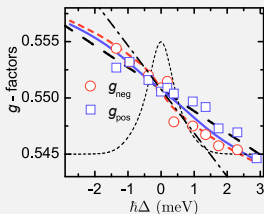
No Faraday rotation signal at  $t = 0$

Faraday rotation signal appears at  $t \neq 0$



## Inhomogeneous array:

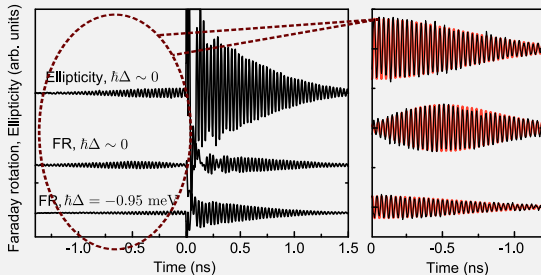
- Spread of resonant frequencies
- Related spread of  $g$ -factors
- Random nuclear fields



## Simple model

$$\mathcal{F}(t) = \frac{1}{2} \sqrt{\frac{\pi}{2\tau_p^2}} \exp \left[ \frac{-\Delta^2 \tau_p^2 / 2 - (\Omega' t)^2}{8\tau_p^2} \right] \times \left[ 2\Delta \tau_p \cos(\tilde{\Omega}_0 t) + \frac{\Omega' t}{\tau_p} \sin(\tilde{\Omega}_0 t) \right]$$

## Experiment



Faraday rotation does not reflect “averaged” spin dynamics of electron ensemble!

MMG et al., (2010)



## Inhomogenous array:

- Spread of

Faraday and ellipticity are formed by different ensembles of spins

- Different temporal behavior
- Faraday rotation does not reflect “averaged” spin dynamics of electron ensemble

• Random nuclear fields

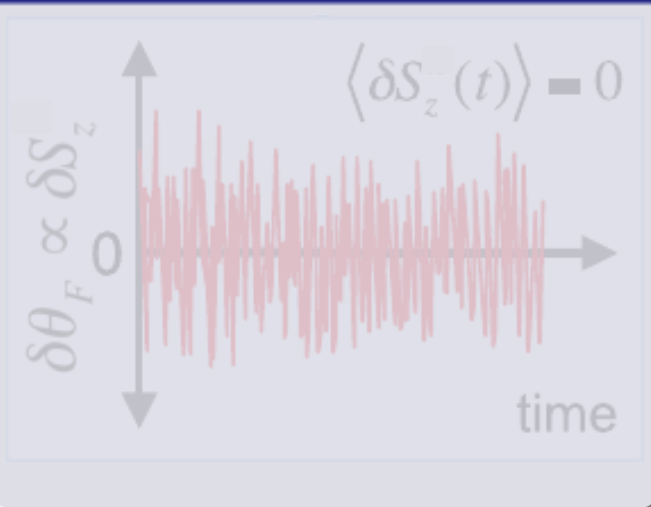


- 1 Introduction. Questions to theory
- 2 Interaction of light with spins
  - Optical orientation via trions
  - Detection of spin polarization
- 3 Spin mode-locking effect
  - Phase synchronization condition & passive mode-locking
  - Nuclei-induced active mode-locking
  - Emergence of Faraday rotation
- 4 Spin dynamics in equilibrium
- 5 Conclusions



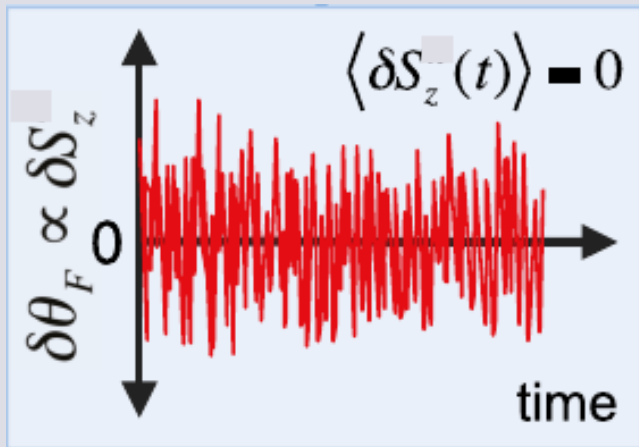


What would happen, if one measures Faraday or Kerr effect in the absence of pump?



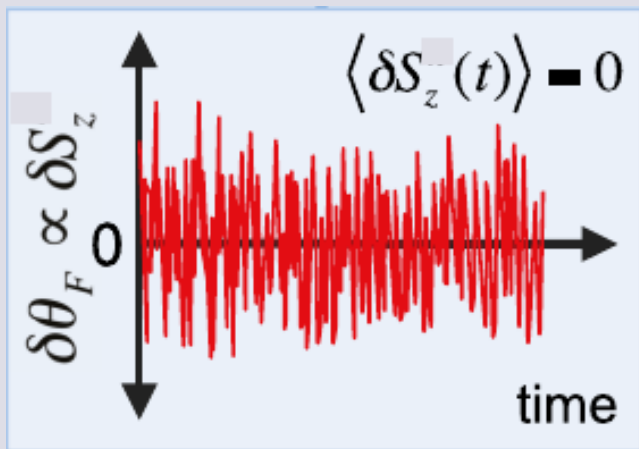


What would happen, if one measures Faraday or Kerr effect in the absence of pump?





What would happen, if one measures Faraday or Kerr effect in the absence of pump?



Just noise? **Spin noise!**

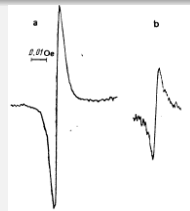
## Magnetic resonance in the Faraday-rotation noise spectrum

E. B. Aleksandrov and V. S. Zapasskiĭ

(Submitted 23 January 1981)

Zh. Eksp. Teor. Fiz. **81**, 132–138 (July 1981)

A maximum at the magnetic resonance frequency of sodium atoms in the ground state is observed near the 5896 Å absorption line in the fluctuation spectrum of the azimuth of the polarization plane of light crossing a magnetic field in sodium vapor. The experiment is a demonstration of a new EPR method which does not require in principle magnetic polarization of the investigated medium, nor the use of high-frequency or microwave fields to induce the resonance.



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Invited review

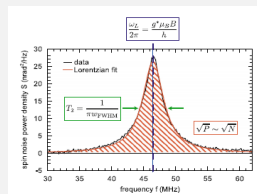
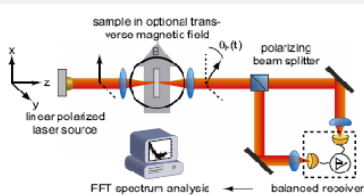
Semiconductor spin noise spectroscopy: Fundamentals, accomplishments, and challenges

Georg M. Müller, Michael Oestreich, Michael Römer, Jens Hübner\*

Institut für Festkörperphysik, Leibniz Universität Hannover, Appellstraße 2, D-30167 Hannover, Germany

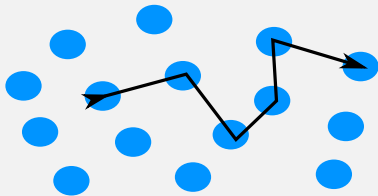
$$\langle \vartheta_F(t) \vartheta_F(t') \rangle,$$

$$\langle \vartheta_K(t) \vartheta_K(t') \rangle \propto \langle S_z(t) S_z(t') \rangle$$





**Transport**  $\delta \mathbf{v}(t) = \delta \mathbf{v}(0) e^{-t/\tau_p}$



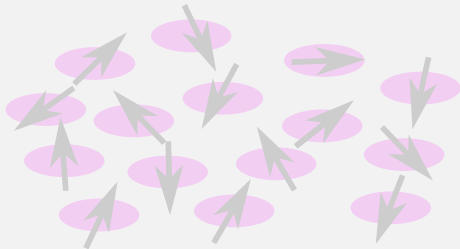
Electron **velocity noise**:

$$\langle v_x(t) v_x(0) \rangle = \frac{v_F^2}{2} e^{-t/\tau_p}$$

Diffusion coefficient

$$D = \int_0^\infty \langle v_x(t) v_x(0) \rangle dt = \frac{v_F^2 \tau_p}{2}$$

**Spin**  $\delta \mathbf{s}(t) = \delta \mathbf{s}(0) e^{-t/\tau_s}$



Electron **spin noise**:

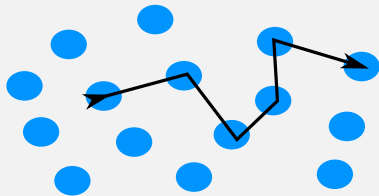
$$\langle s_z(t) s_z(0) \rangle = \langle s_z^2 \rangle e^{-t/\tau_s}$$

Magnetic susceptibility

$$\mu_{zz}(\omega) \propto \int_0^\infty \langle s_z(t) s_z(0) \rangle e^{i\omega t} dt$$



**Transport**  $\delta \mathbf{v}(t) = \delta \mathbf{v}(0) e^{-t/\tau_p}$



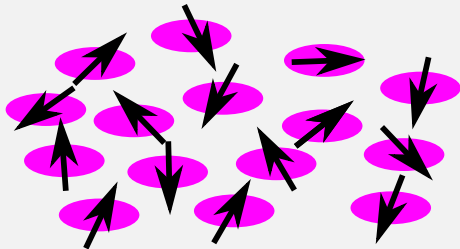
Electron velocity noise:

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Diffusion coefficient

$$D = \int_0^\infty \langle v_x(t) v_x(0) \rangle dt = \frac{v_F^2 \tau_p}{2}$$

**Spin**  $\delta \mathbf{s}(t) = \delta \mathbf{s}(0) e^{-t/\tau_s}$



Electron spin noise:

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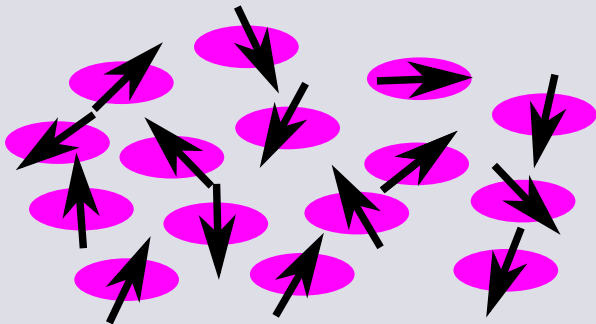


## Single spin

$$\langle s_x \rangle = \langle s_y \rangle = \langle s_z \rangle = 0 \quad \text{but} \quad \langle s_x^2 \rangle = \langle s_y^2 \rangle = \langle s_z^2 \rangle = \frac{1}{3} \times \frac{1}{2} \left( 1 + \frac{1}{2} \right)$$

## Spin ensemble

$$\sqrt{\langle S_i^2 \rangle} = \sqrt{N} \sqrt{\langle s_i^2 \rangle} = \frac{\sqrt{N}}{2}$$



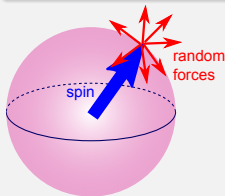


Spin fluctuation  $\delta\mathbf{s}$  in a dot

$$\frac{\partial \delta\mathbf{s}(t)}{\partial t} + \frac{\delta\mathbf{s}(t)}{\tau_s} + \delta\mathbf{s}(t) \times (\boldsymbol{\Omega}_B + \boldsymbol{\Omega}_N) = \boldsymbol{\xi}(t)$$

Random (Langevin) forces

$$\langle \xi_\alpha(t') \xi_\beta(t) \rangle = \frac{1}{2\tau_s} \delta_{\alpha\beta} \delta(t' - t)$$



$$(\delta s_\alpha \delta s_\beta)_\omega =$$

$$\int_{-\infty}^{+\infty} \langle \delta s_\alpha(t+\tau) \delta s_\beta(t) \rangle e^{i\omega\tau} d\tau$$

MMG, Ivchenko (2012)

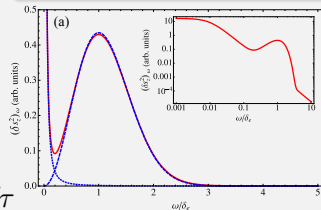
Zero field

Nuclei fluctuations

$$(\delta s_\alpha^2)_\omega = \frac{\pi}{6} \{ \Delta(\omega) + \int_0^\infty d\Omega_N F(\Omega_N) [\Delta(\omega - \Omega_N) + \Delta(\omega + \Omega_N)] \}$$

$$+ \int_0^\infty d\Omega_N F(\Omega_N)$$

$$[\Delta(\omega - \Omega_N) + \Delta(\omega + \Omega_N)] \}$$



- Zero field peak
- Peak at  $\Omega \sim \langle \sqrt{\Omega_N^2} \rangle$



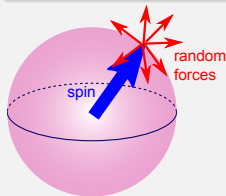


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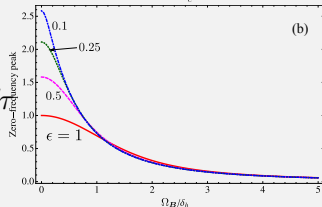
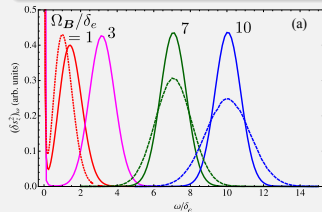
$$(\delta s_\alpha \delta s_\beta)_\omega =$$

$$\int_{-\infty}^{+\infty} \langle \delta s_\alpha(t+\tau) \delta s_\beta(t) \rangle e^{i\omega\tau} d\tau$$

MMG, Ivchenko (2012)

Transverse field

- Zero field peak is reduced
- Main peak shifts  $\sim \Omega_B$

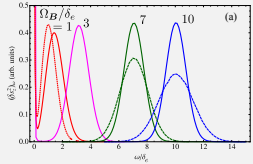
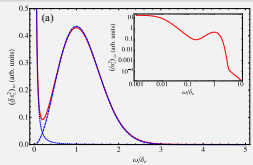
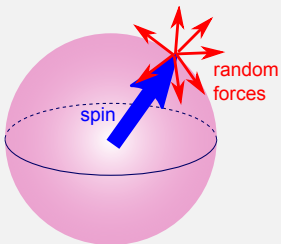




$$\frac{\partial \delta \mathbf{s}(t)}{\partial t} + \frac{\delta \mathbf{s}(t)}{\tau_s} + \delta \mathbf{s}(t) \times (\boldsymbol{\Omega}_B + \boldsymbol{\Omega}_N) = \boldsymbol{\xi}(t)$$

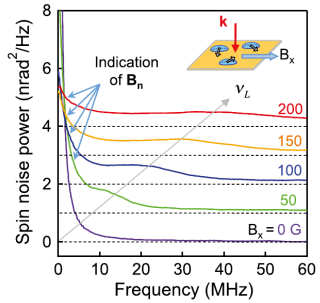
## Random (Langevin) forces

$$\langle \xi_\alpha(t') \xi_\beta(t) \rangle = \frac{1}{2\tau_s} \delta_{\alpha\beta} \delta(t' - t)$$



MMG, Ivchenko (2012)

## Experiment



Yan Li, Sinitsyn, Smith, Reuter, Wieck, Yakovlev, Bayer, Crooker (2012)

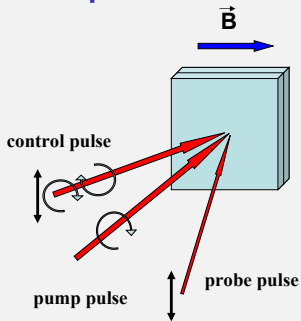


## Spin noise spectroscopy

provides information about spin precession and dephasing in **close-to-equilibrium** conditions



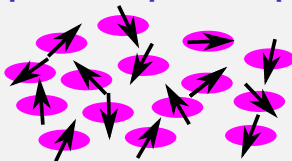
## Spin control



## 3 pulse technique

Chen, et al. (2004)  
Economou, Sham, Wu, Steel (2006)

## Spin noise spectroscopy

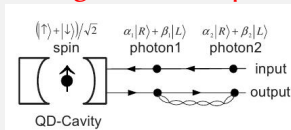


$$\langle S_i^2 \rangle = N \langle s_i^2 \rangle$$

monitoring  
Faraday/Kerr/Ellipticity  
fluctuations (in an  
absence of a pump)

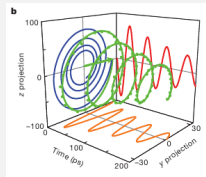
Aleksandrov, Zapasskii (1981)

## Spins in cavities entanglement via spin

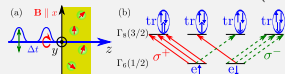


Hu, Munro, Rarity (2008)

## 3D spin tomography



Kosaka et al. (2009)



Smirnov, MMG (2012)



Faraday effect and ellipticity provide complementary information about dynamics of electron and nuclear spins in QDs

Coherent spin dynamics of electrons and excitons in nanostructures (a review)  
Phys. Solid State 54, 1 (2012)

## Plan

- 1 Introduction. Questions to theory
- 2 Interaction of light with spins
- 3 Spin mode-locking effect
- 4 Spin dynamics in equilibrium
- 5 Conclusions

## Co-authors

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~~SOLAB~~

