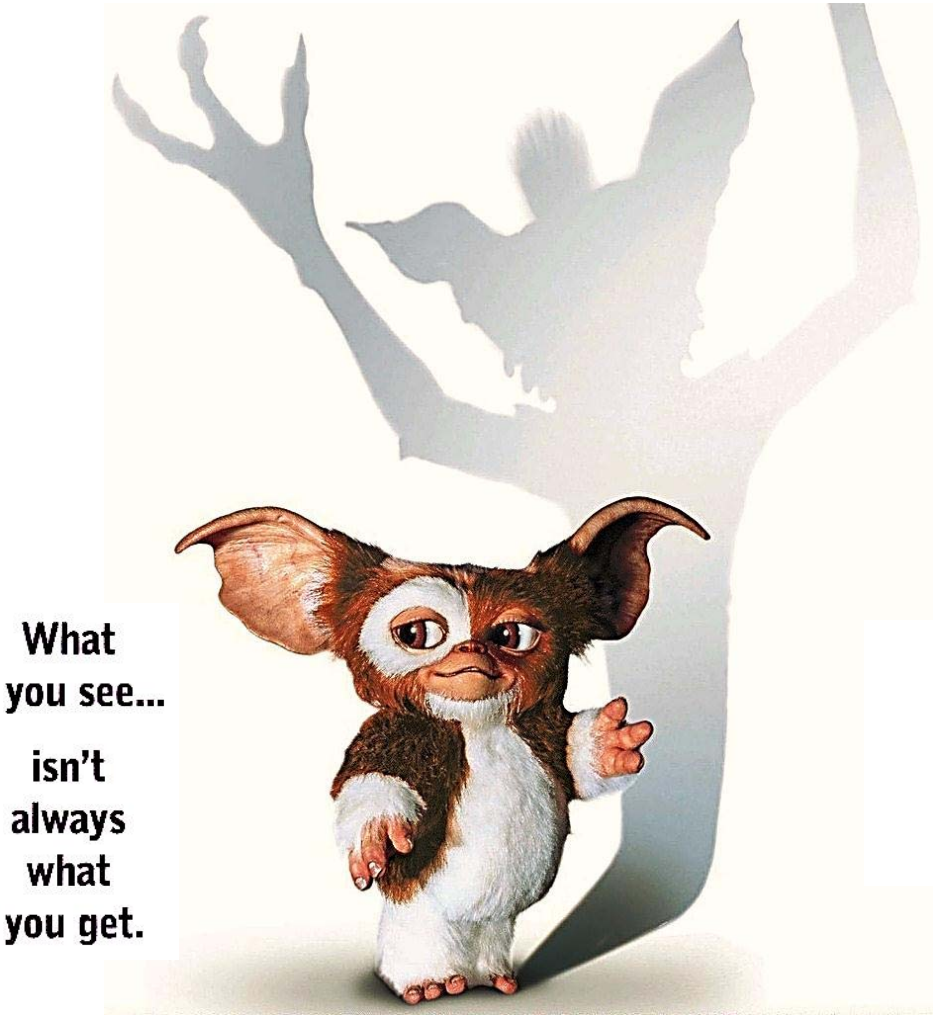
What lurks below the last plateau

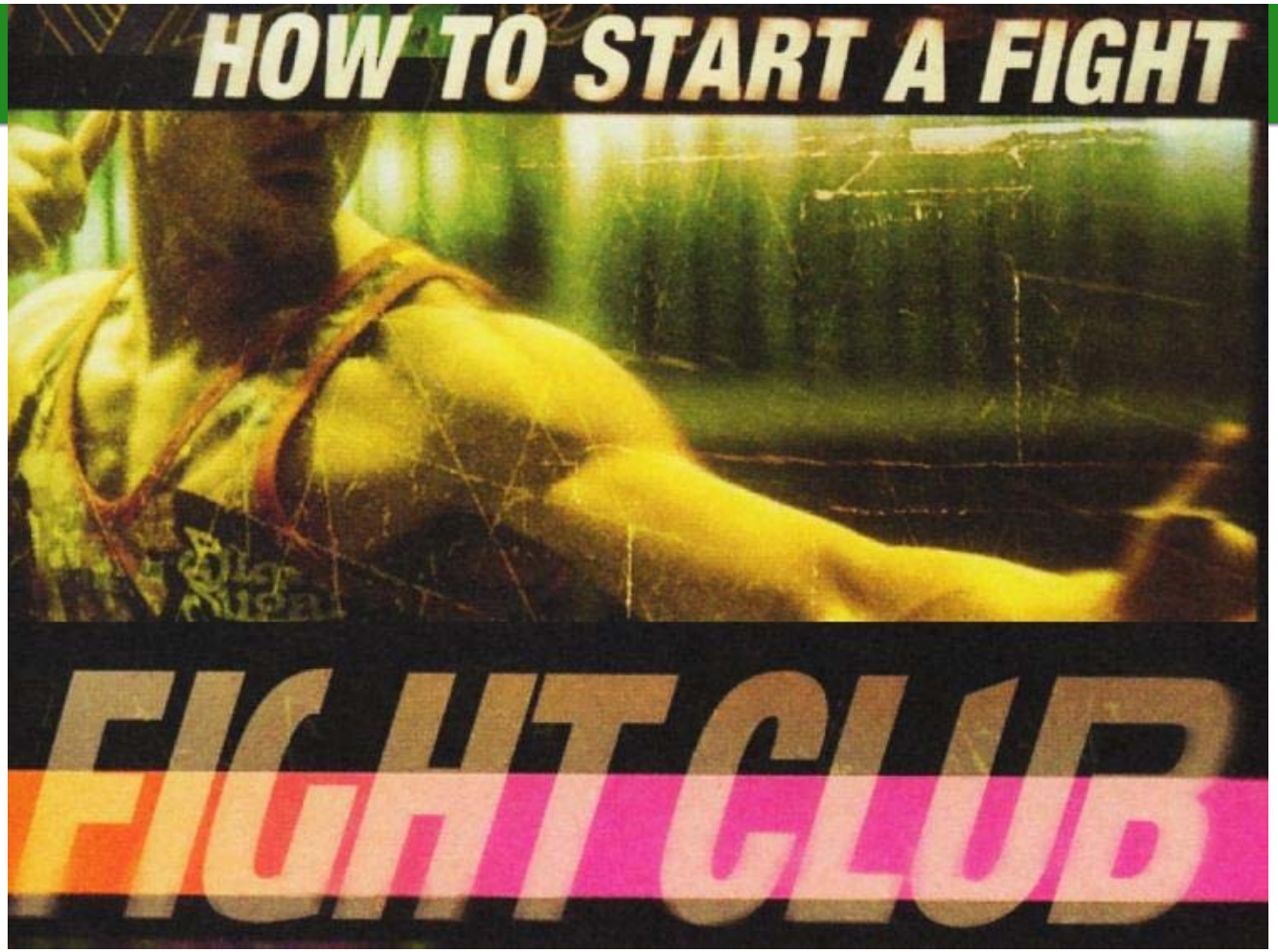
15+ years of 0.7: What have we learned and where to next?



Lecture 4: Bound-states and electron organization in QPCs

Adam Micolich

Nanoelectronics Group
School of Physics, UNSW.

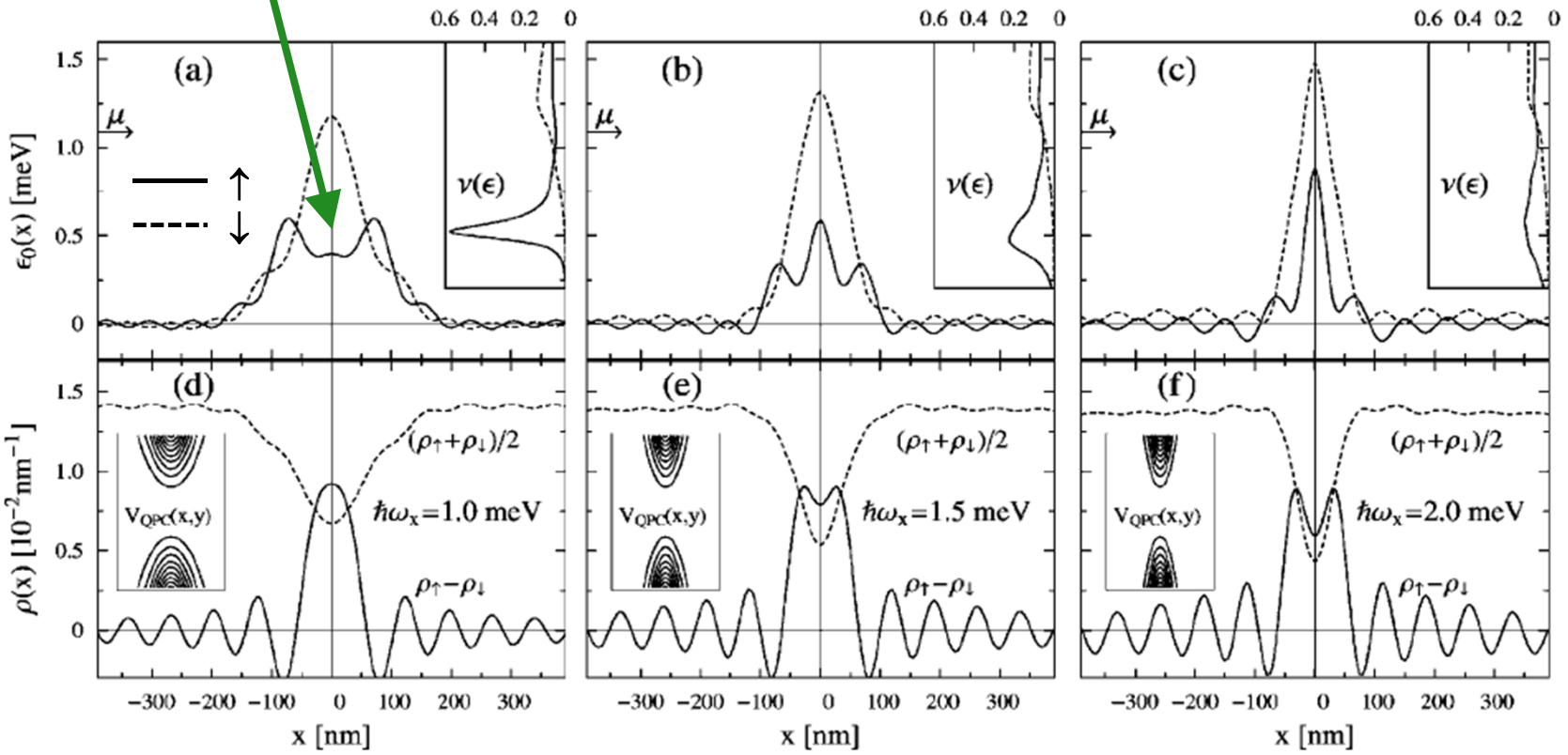


- **There are two areas of significant debate when it comes to Kondo and QPCs:**
  1. **Does the data really behave like you'd expect it to for Kondo? – Yesterday**
  2. **Is there really a bound-state inside a QPC? – Today**

Let's start again with spin DFT

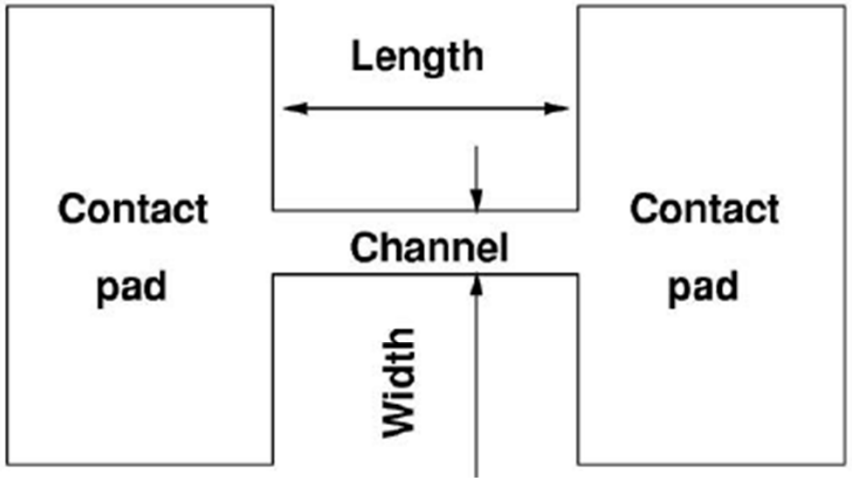
In some instances there is even a minimum at the centre of the QPC for ↑

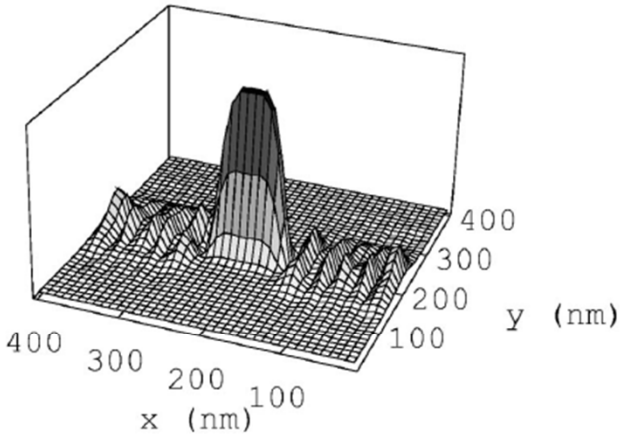
But it's not a robust outcome of SDFT calculations…

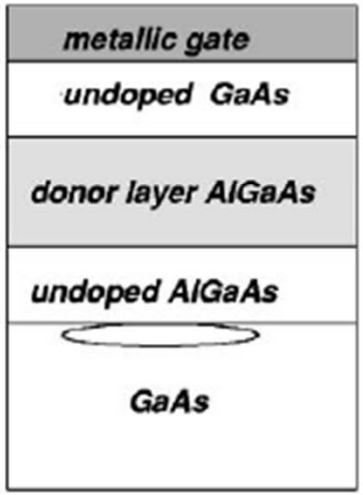


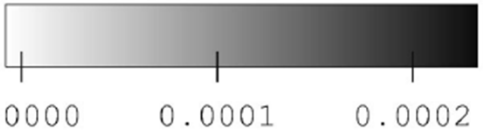
K. Hirose *et al*., PRL 90, 026804 (2003).

SDFT: Bound-states or not?





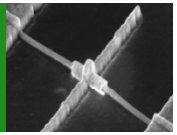




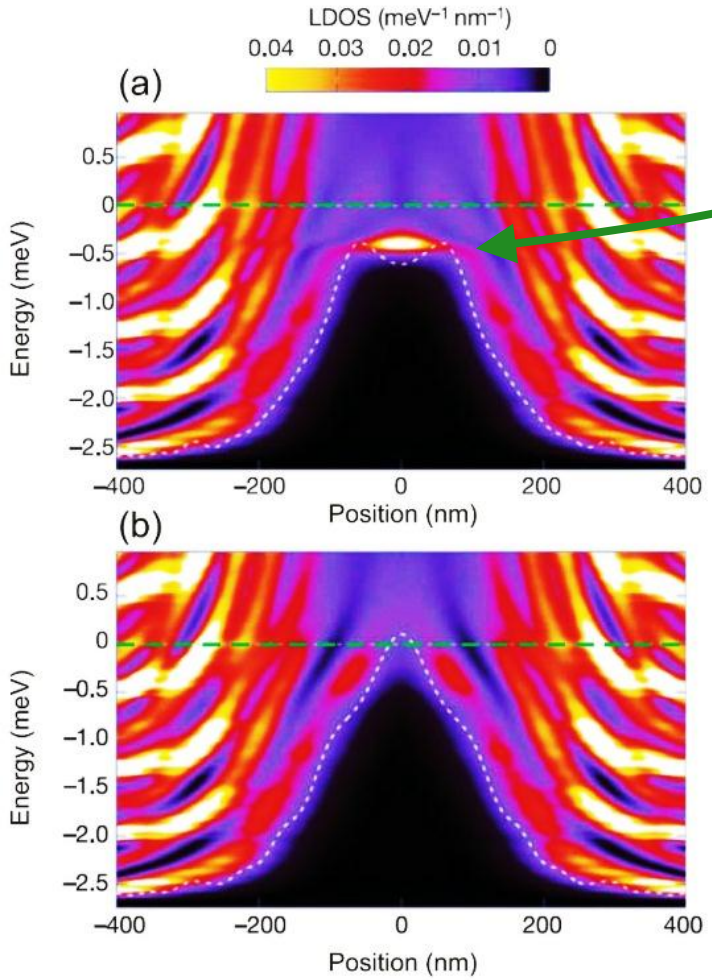
Spin polarization p per square nanometer

Note added in proof: Recently, Y. Meir, K. Hirose, and N.S. Wingreen have reported on a Kondo model for the "0.7 anomaly" (cond-mat/0207044). As in our work these authors also find local magnetization in the QPC using the Kohn-Sham equations. However, we do not recover their quasi-bound states, which might depend on different geometries.

K.-F. Berggren & I.I. Yakimenko, PRB 66, 085323 (2002).



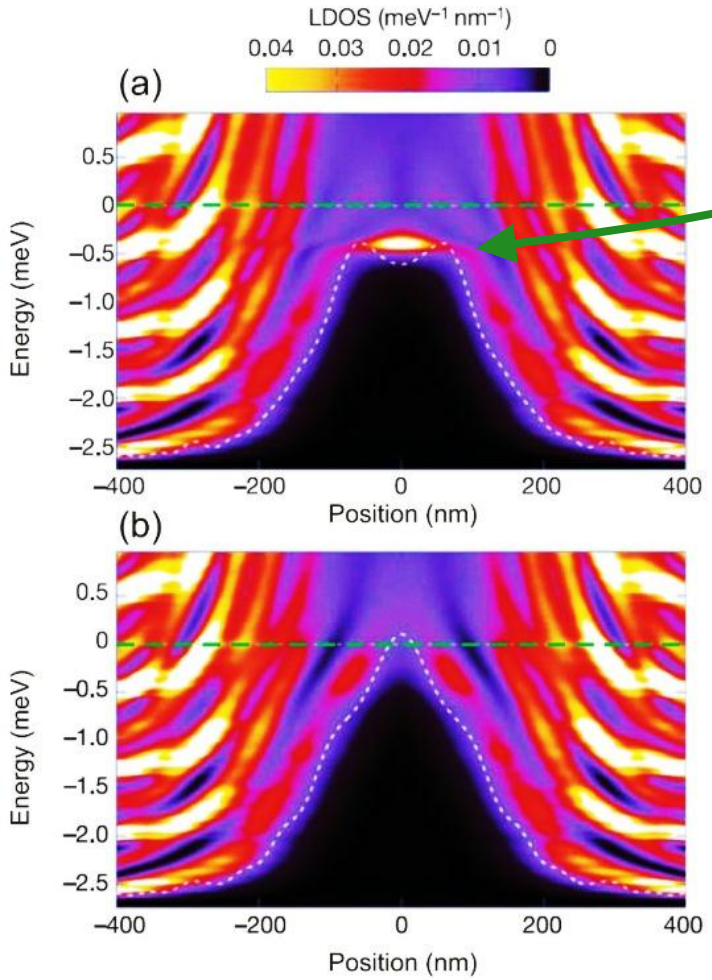
Spin-DFT: Interesting and controversial…



Bound state for ↑ but not ↓

T. Rejec & Y. Meir, Nature 442, 900 (2006).

Spin-DFT: Bound-states or not?



T. Rejec & Y. Meir, Nature 442, 900 (2006).

Bound state for ↑ but not ↓

"We have not found any indication of such [bound] states…"

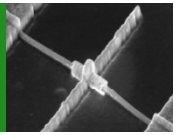
K.-F. Berggren *et al.*, JPCM 20, 164203 (2008).

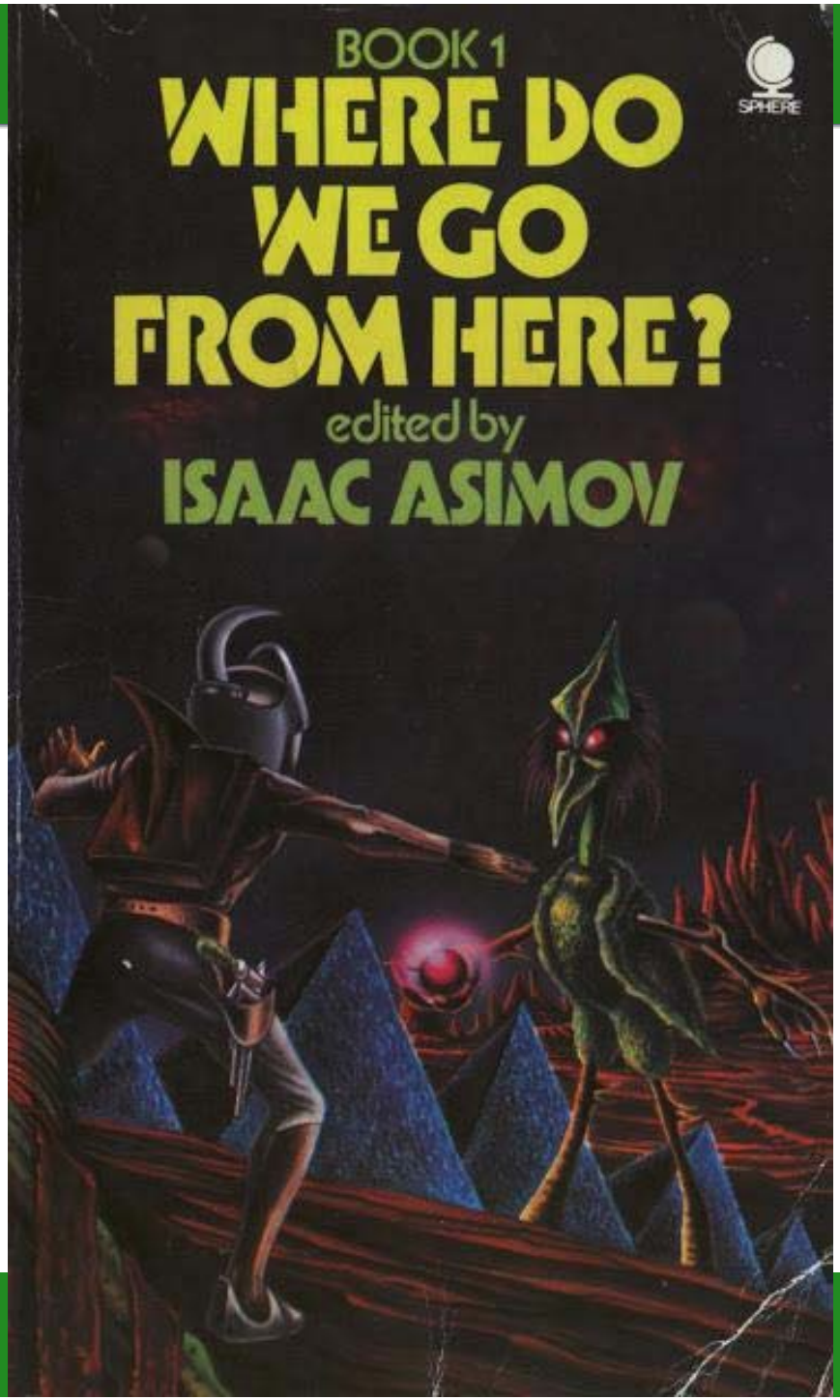
"The features of the evolution and formation of the localized spin-polarized quasi-bound states in the QPC agree well with the results reported by Hirose et al and Rejec and Meir."

I.V. Zozoulenko *et al.*, JPCM 20, 164217 (2008).

"The feature we have found can be best described as weakly bound."

R. Akis & D.K. Ferry, JPCM 20, 164201 (2008).



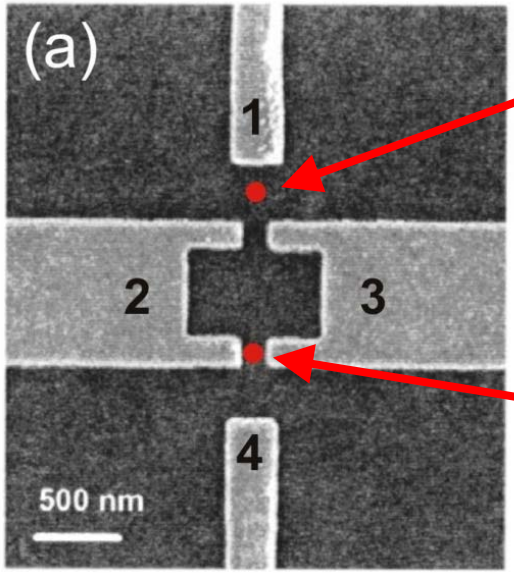


What other evidence do we have for bound-state formation in QPCs?

Experimental evidence for bound states in QPCs

- To get some insight, we need to go to more complex devices.

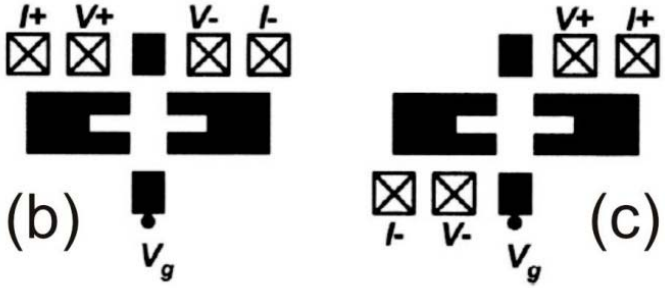
Gates 1, 2 and 3 held at –1.2V while gate 4 is swept from 0 to –2V.



The upper QPC separates reservoirs in top-left and top-right corners, and couples via the side into the quantum dot.

The lower QPC is the dot opening, and it is controlled with gate 4.

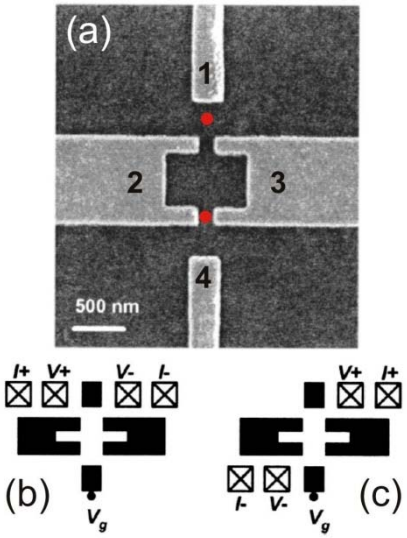
A set of 8 contacts allow two different measurement configurations to be established.

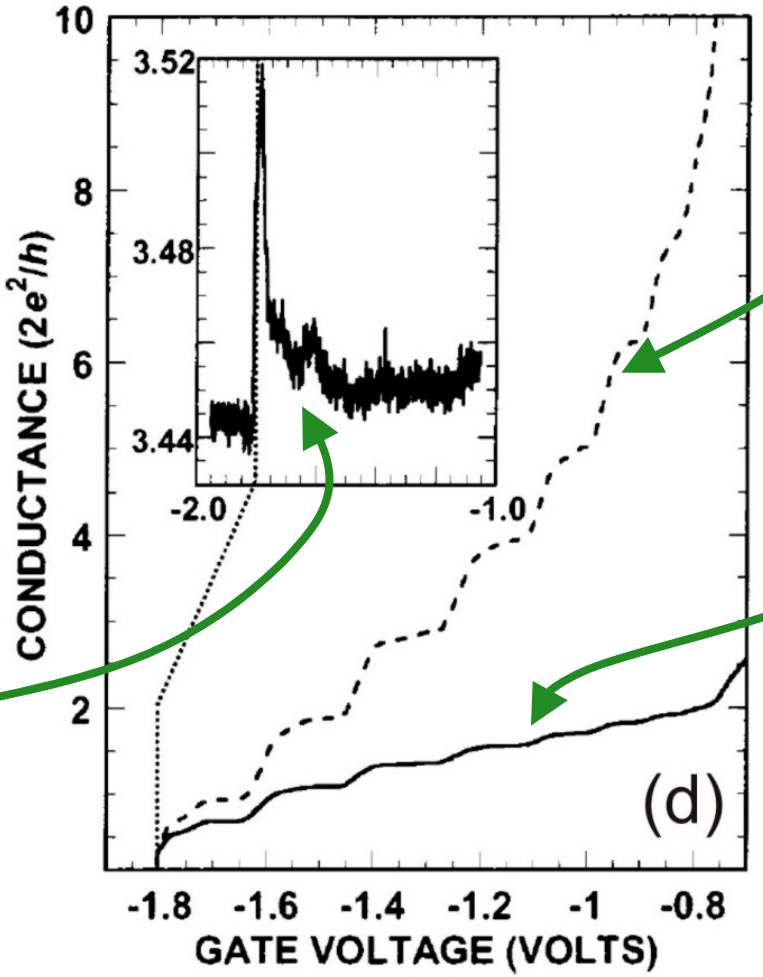


Rationale: Use the upper QPC as a non-local probe of the lower QPC as its driven to pinch-off.

T. Morimoto *et al.*, APL 82, 3952 (2003).

Experimental evidence for bound states in QPCs





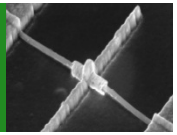
Data in configuration (c) with 4747 Ω series resistance removed

Data in configuration (c) without correction

Inset is data in configuration (b)

Note the big spike as the lower QPC begins to populate.

T. Morimoto *et al.*, APL 82, 3952 (2003).



Support from Anderson model calculations

- Puller *et al*. use an Anderson Hamiltonian that's a bit more complex than usual.

Regular Anderson terms for localized state in swept QPC

Free electrons in fixed QPC

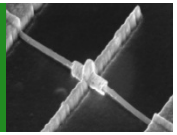
$$\hat{H} = \sum_{\sigma} \epsilon_{\sigma} n_{\sigma} + \frac{1}{2} U \sum_{\sigma} n_{\sigma} n_{\bar{\sigma}} + \sum_{q,\sigma} \epsilon_{q\sigma} n_{q\sigma} + \sum_{k,\sigma} E_{k\sigma} c^{+}_{k\sigma} c_{k\sigma}$$
$$+ \sum_{k,\sigma} (V_{k\sigma} c^{+}_{k\sigma} a_{\sigma} + V^{*}_{k\sigma} a^{+}_{\sigma} c_{k\sigma})$$
$$+ \sum_{k,q,\sigma} (v_{kq\sigma} c^{+}_{k\sigma} a_{q\sigma} + v^{*}_{kq\sigma} a^{+}_{q\sigma} c_{k\sigma}).$$

Electrons in the dot mediating QPC interaction

Tunnel coupling swept QPC to dot

Tunnel coupling fixed QPC to dot

V.I. Puller *et al.*, PRL 92, 096802 (2004).



Support from Anderson model calculations

- **The mathematics gets complex (i.e., read the paper) but ultimately, they calculate the conductance for the upper QPC using the Landauer formalism:**

$$g = \frac{e^2}{h} \sum_{\sigma} \int d\epsilon [-f'(\epsilon)] \Gamma_{\sigma} \rho_{\sigma}(\epsilon).$$

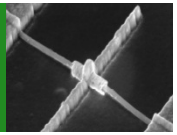
where $f(\varepsilon)$ is the Fermi distribution, Γ_σ is the coupling of the upper QPC to its leads and $\rho_\sigma(\varepsilon)$ is the density of states per spin in the upper QPC. The coupling via the preceding Hamiltonian affects this density of states, inducing a correction to the upper QPC conductance:

$$g = \bar{g} + \Delta g$$

where:
$$\Delta g = \frac{2e^2}{\hbar} \frac{|T|^2}{4} \sum_{n=0}^{N-1} \frac{1}{E_F - \epsilon_0 - n\hbar\omega_y} \times \sum_{\sigma} \frac{E_F - \epsilon_\sigma - U\langle n_{\bar{\sigma}} \rangle}{(E_F - \epsilon_\sigma - U\langle n_{\bar{\sigma}} \rangle)^2 + (\Delta_\sigma + \pi_\sigma)^2}$$

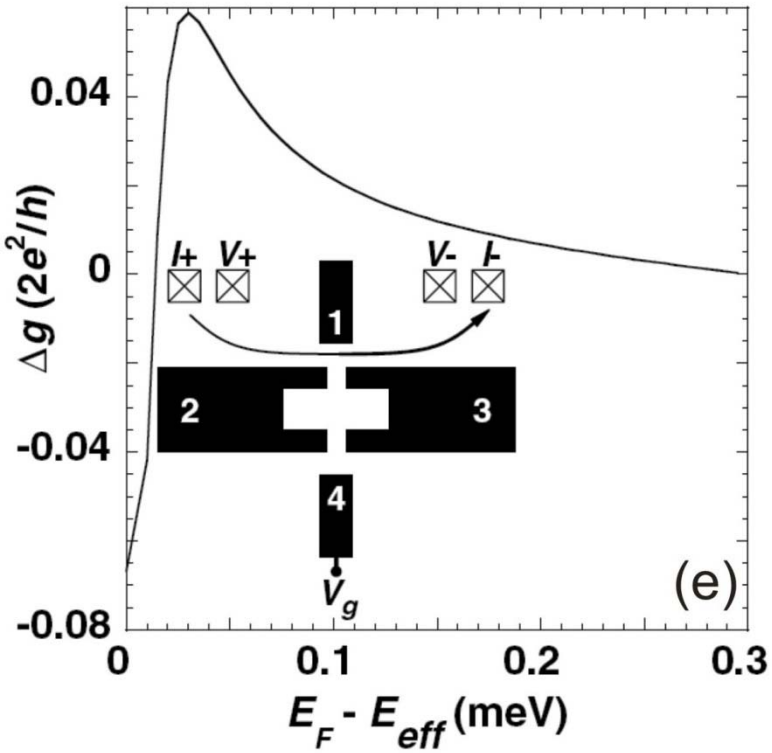
If you plot this…

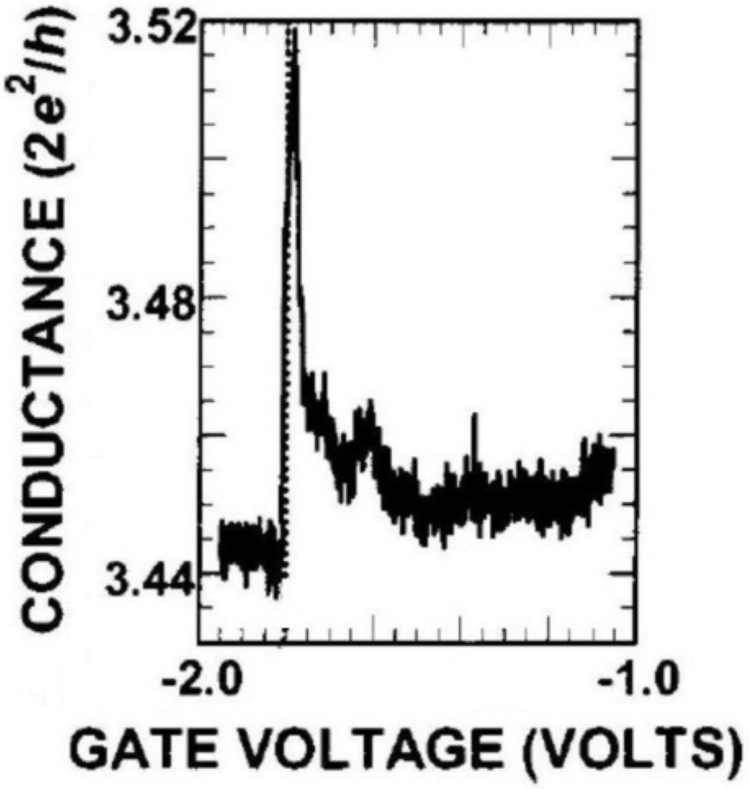
V.I. Puller *et al.*, PRL 92, 096802 (2004).



Support from Anderson model calculations

- You get a structure looking much like that seen in the experimental data.

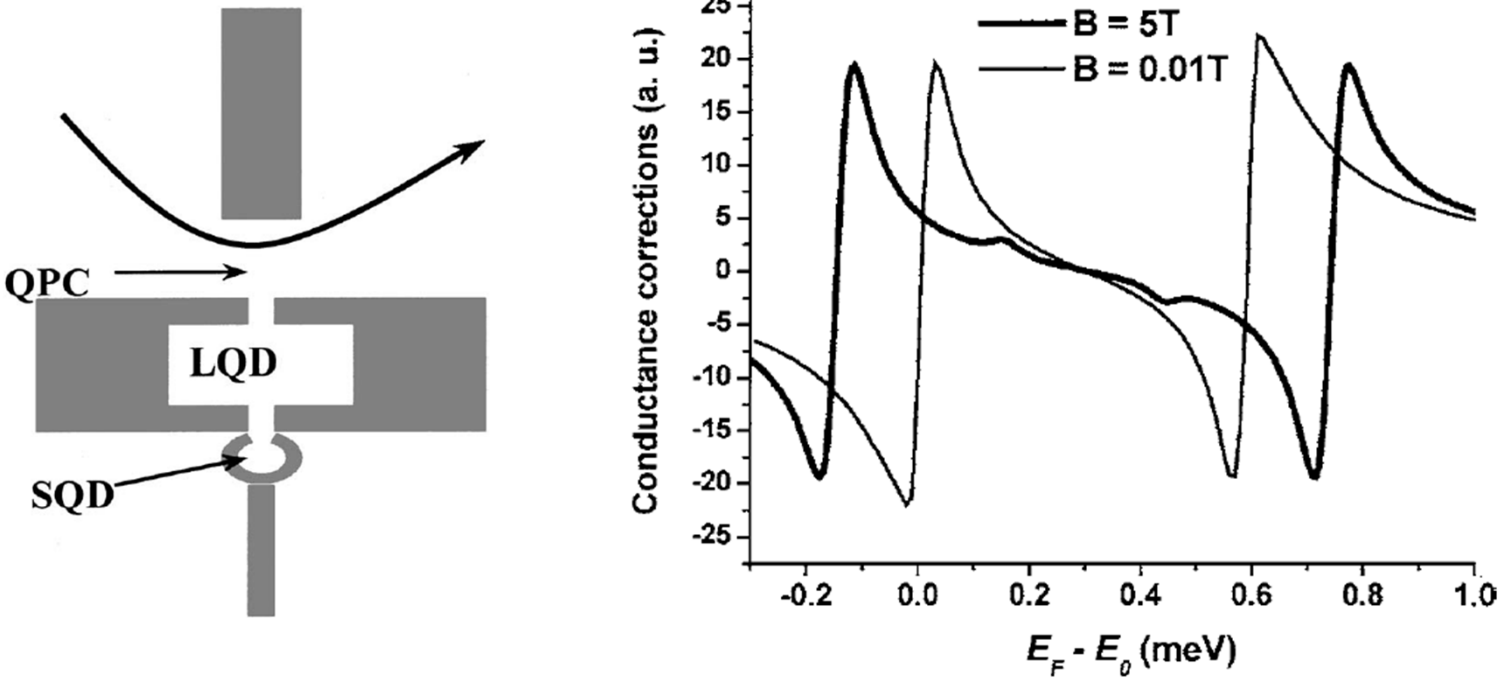




V.I. Puller *et al.*, PRL 92, 096802 (2004).

Is the signal in the upper QPC a Fano resonance?

- **Follow-up work suggested that the conductance correction is actually a Fano resonance…**

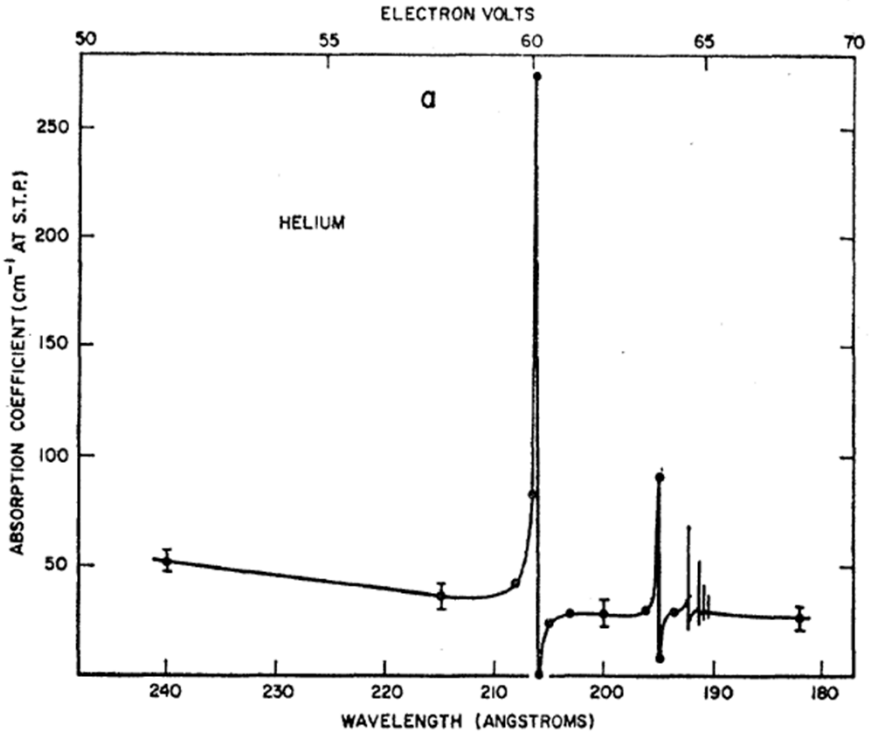


L.G. Mourokh *et al.,* Appl. Phys. Lett. 87, 192501 (2005).

- **Before considering Fano resonances in QPCs further, let's digress briefly to look at the Fano effect.**

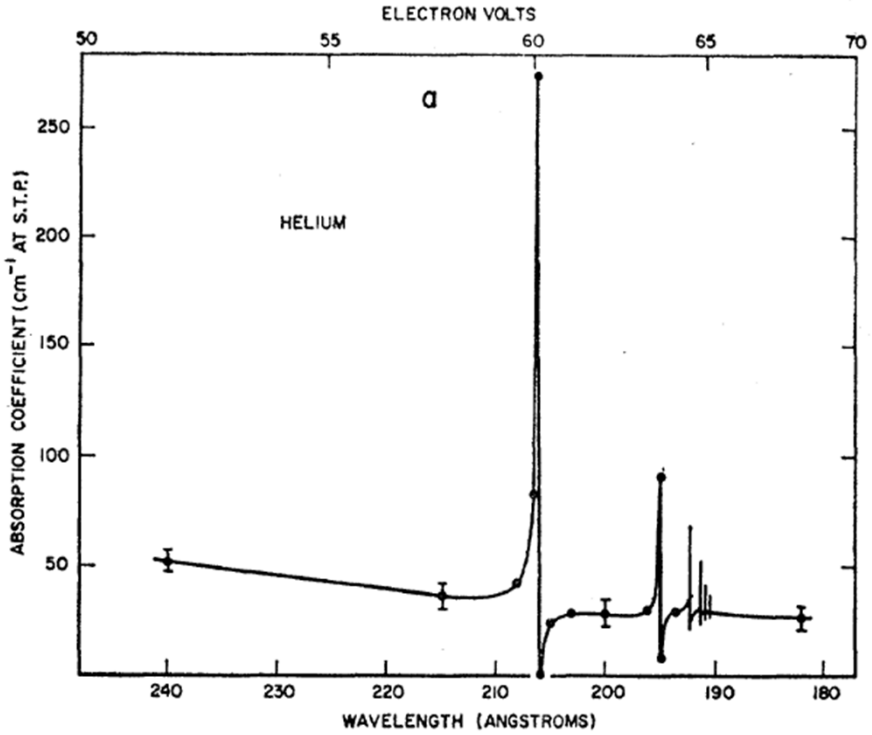
The Fano effect

- The Fano effect is named after Ugo Fano, who in 1935 explained the strange highly-asymmetrical lineshapes observed in the optical absorption spectra of noble gases.
- The lineshape occurs when a discrete state in the spectrum interferes with the continuum of states amongst which it resides.



U. Fano, Nuovo Cimento 12, 154 (1935); U. Fano & J.W. Cooper, Phys. Rev. 137, A1364 (1965).

The Fano effect



These lineshapes form a family described by the Fano lineshape formula:

$$\sigma(\varepsilon) = \sigma_a[(q + \varepsilon)^2/(1 + \varepsilon^2)] + \sigma_b$$

where $\sigma(\varepsilon)$ is the absorption cross-section for incident photons of energy E and $\varepsilon = E - E_r / \frac{1}{2}\Gamma$ is the separation between E and a resonance energy E_r due to a discrete auto-ionizing level in the atom, with Γ being the lifetime broadening of this level.

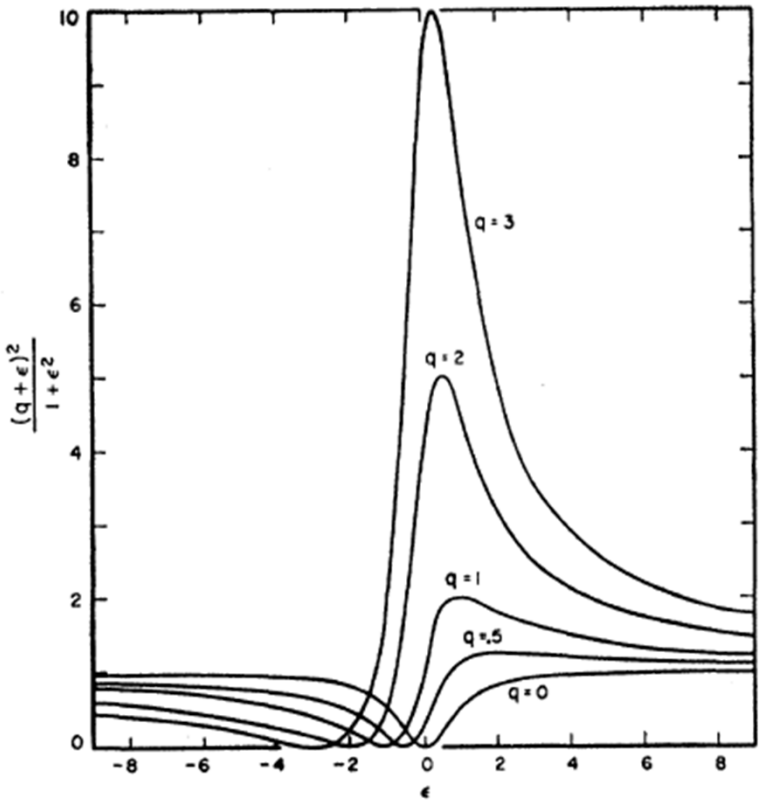
The prefactors σ_a and σ_b represent the two components of the spectrum corresponding to states in the continuum that do and do not interact with the discrete level.

The last detail is the Fano factor q, which controls the lineshape…

U. Fano & J.W. Cooper, Phys. Rev. 137, A1364 (1965).

The Fano effect

- **The Fano factor *q* controls the lineshape, which starts out as a symmetric minima for *q* = 0, becomes highly asymmetric at *q* = 1, and becomes the Lorentzian Breit-Wigner lineshape at $q \rightarrow \infty$.**



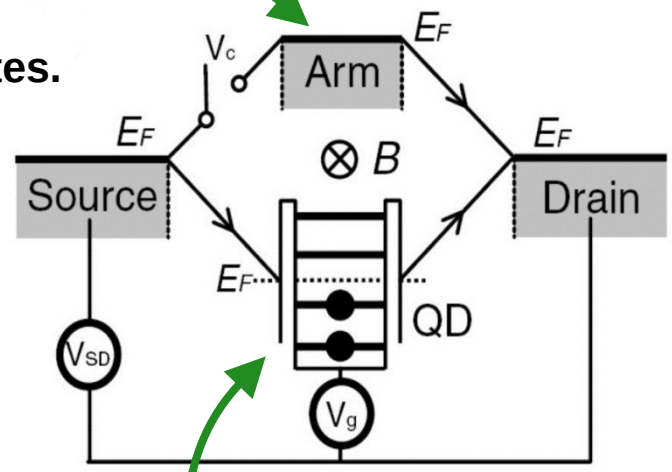
U. Fano, Phys. Rev. 124, 1866 (1961).

The important thing about Fano is its…

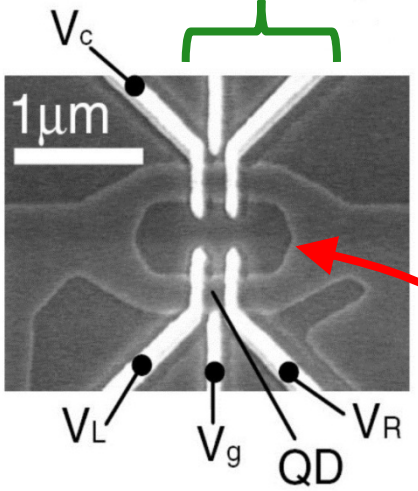
Taking Fano into mesoscopic devices…

- At its simplest, all the Fano effect requires is interference between a discrete state and a continuum of states. You don't need atoms for this, you can do it with mesoscopic devices too.

One arm is dot-free, providing a continuum of states.



These two gates aren't used.



The other has a quantum dot in CB to provide the discrete states.

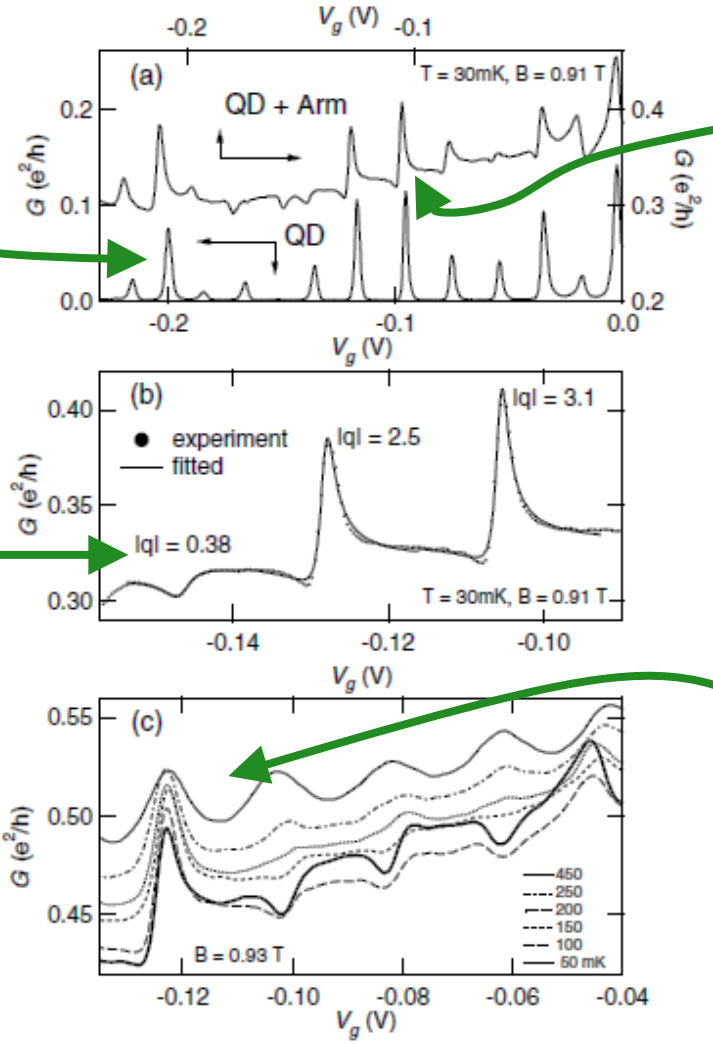
Aharonov-Bohm interferometer geometry provides the interference

K. Kobayashi *et al*., PRL 88, 256806 (2002).

Taking Fano into mesoscopic devices…

Dot alone gives the usual CB oscillations

Can fit the conductance to $G(\varepsilon) \propto (q + \varepsilon)^2/(1 + \varepsilon^2)$, with the resulting q giving insight into the strength of the discrete-continuum interference.



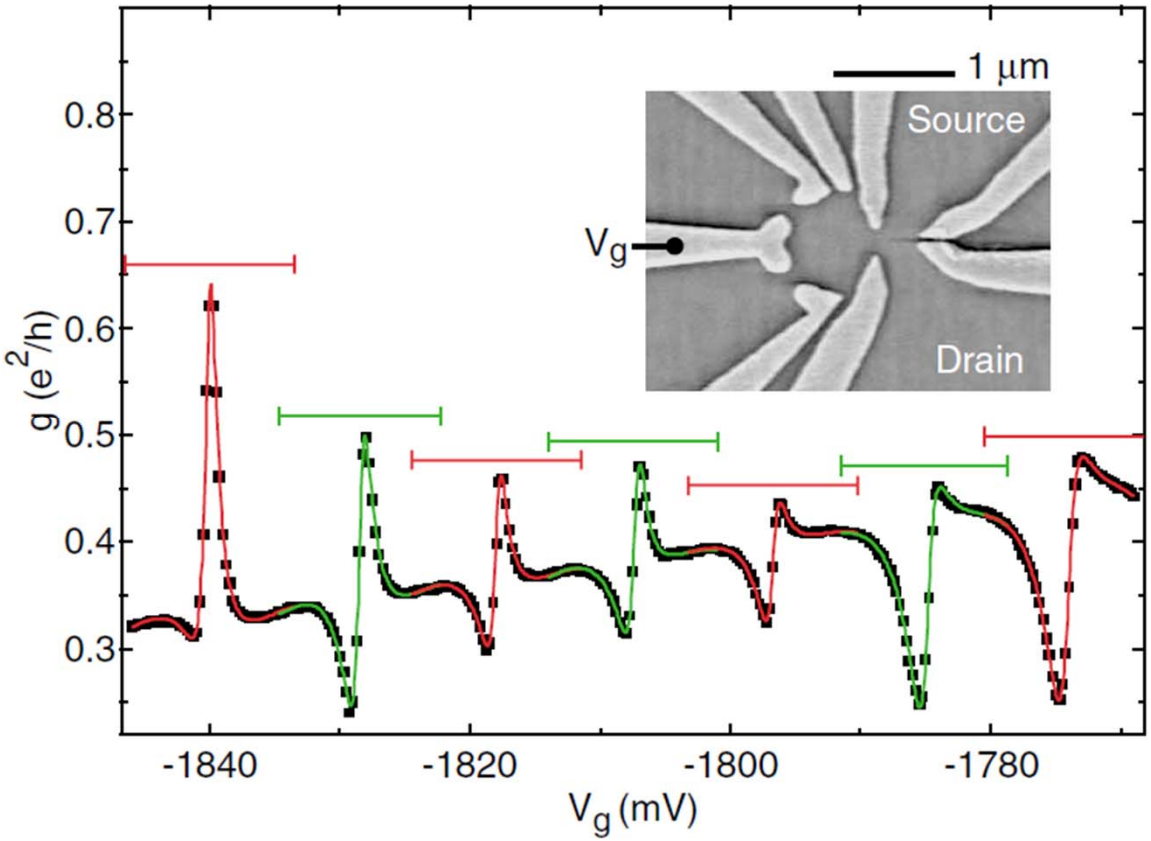
Dot and arm in parallel gives Fano resonances.

Increasing *T* causes dephasing, driving $q \to \infty$ and turning the asymmetric resonances into Lorentzian peaks.

K. Kobayashi *et al*., PRL 88, 256806 (2002).

Taking Fano into mesoscopic devices…

- **Can also be found in the conductance of a QPC side-coupled to a quantum dot.**

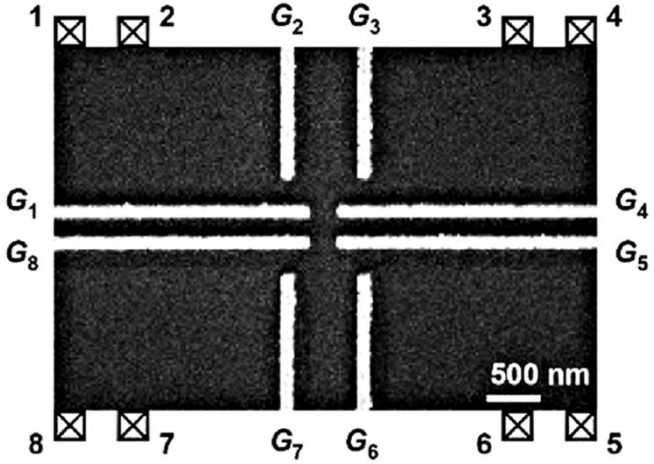


- **Seems to be ideal for detecting a bound state in two coupled QPCs.**

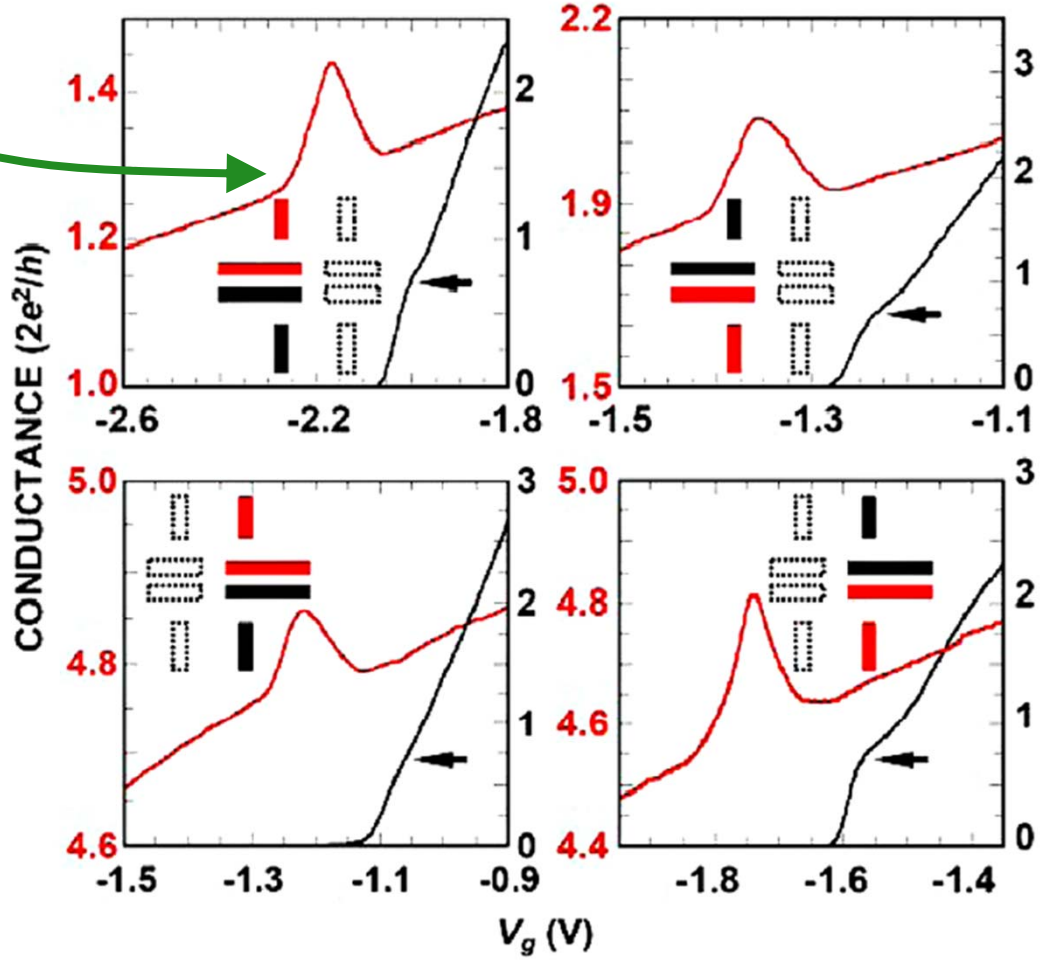
A.C. Johnson *et al*., PRL 93, 106803 (2004).

Fano resonance in coupled QPCs

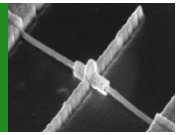
Pinch-off results in a peak in the detector conductance, independent of device.



Black = swept QPC **Red = detector**

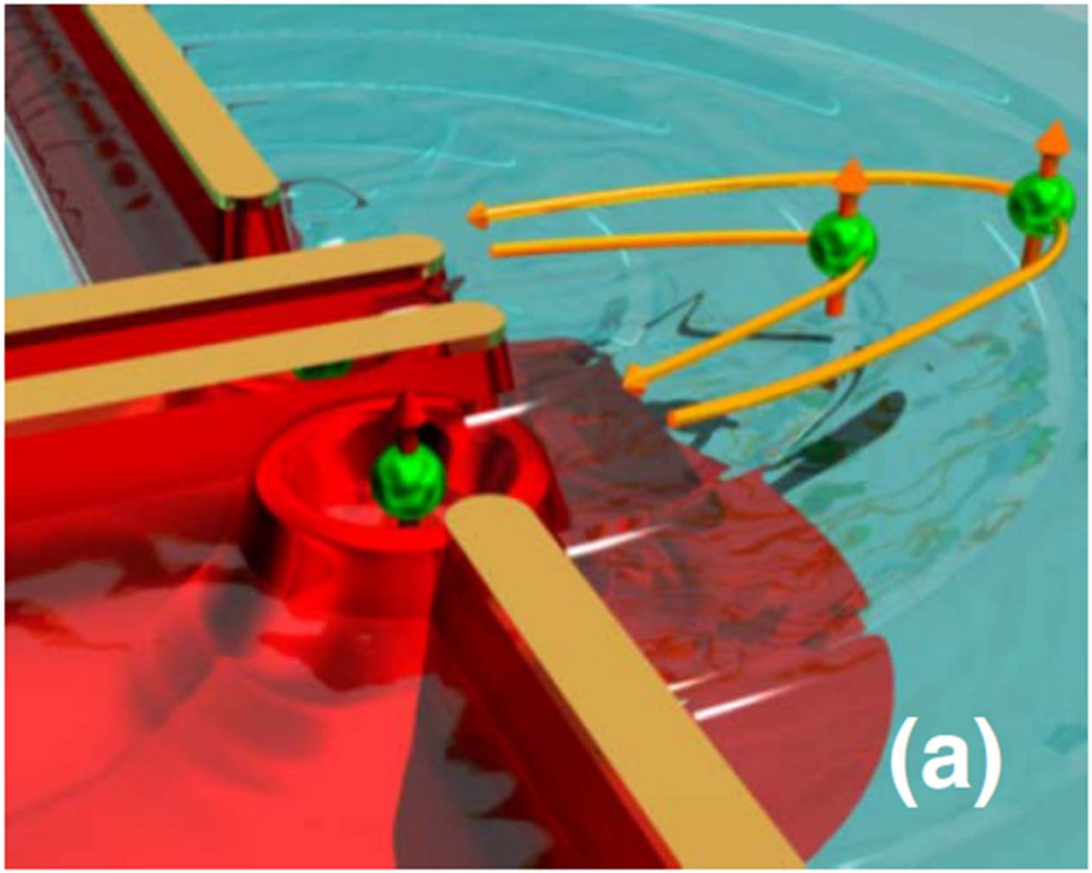


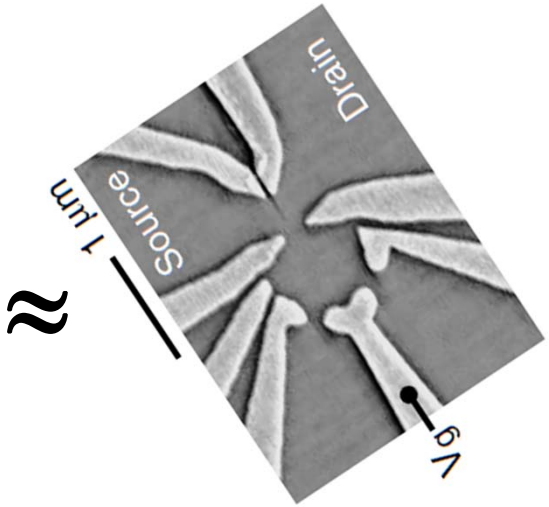
Y. Yoon *et al*., PRL 99, 136805 (2007).



Fano resonance in coupled QPCs

- **The QPC detects a bound-state in a second QPC in the same way that it detects a deliberately coupled quantum dot.**

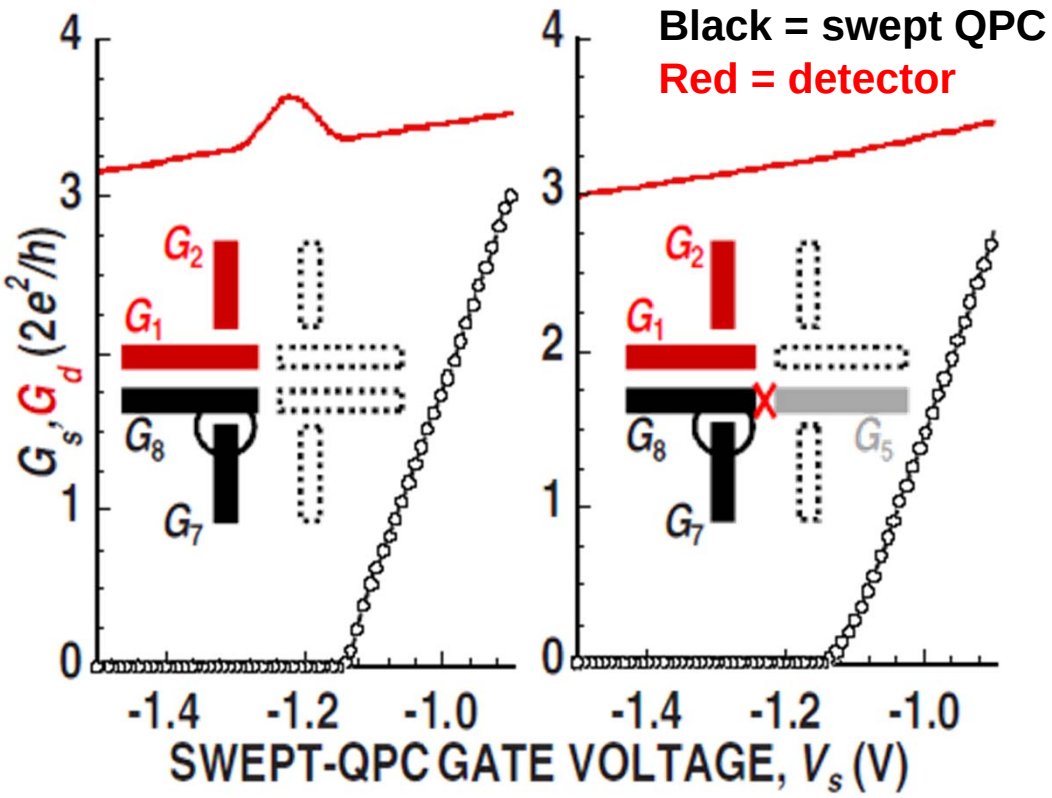




Y. Yoon *et al*., PRB 79, 121304 (2009).

Fano resonance in coupled QPCs

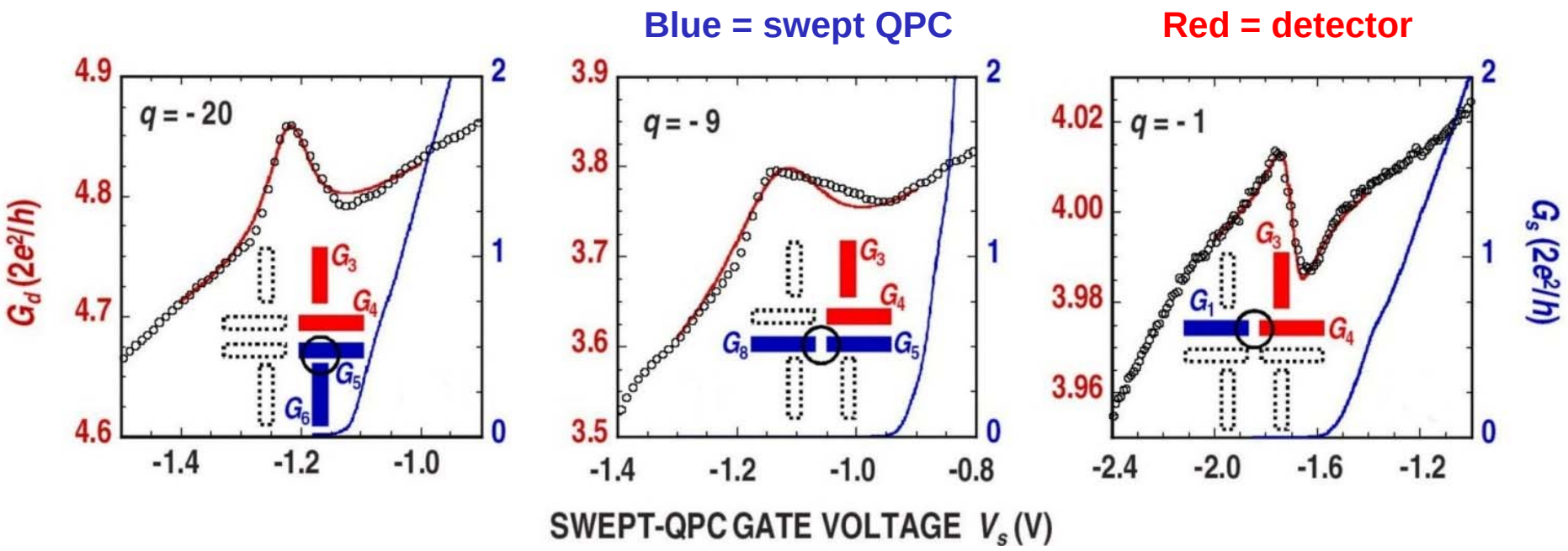
- **If you cut off the connection between the detector and the swept QPC, then the Fano resonances go away.**



Y. Yoon *et al*., PRB 79, 121304 (2009).

Fano resonance in coupled QPCs

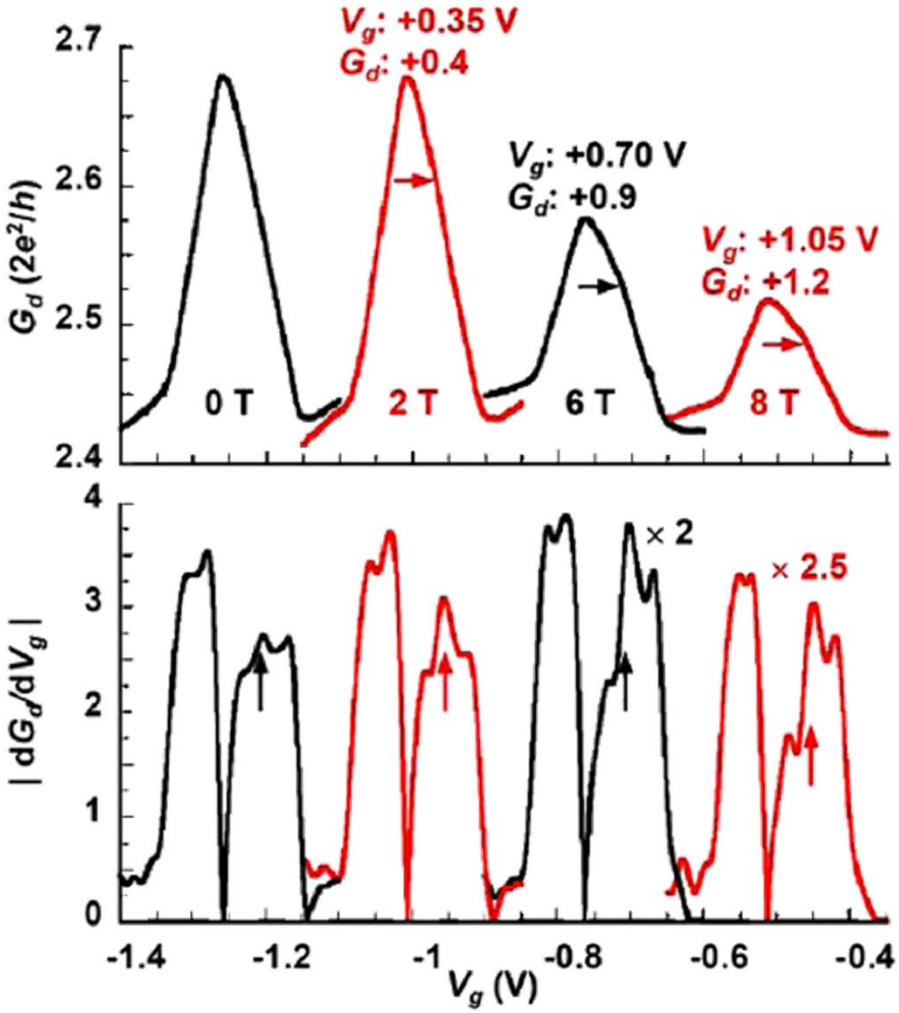
- **If you change the separation between the detector and swept QPC, then the Fano factor changes, with q becoming larger, indicating a weaker discrete-continuum interference, as the two QPCs are moved apart.**



Y. Yoon *et al*., PRB 79, 121304 (2009).

Looking at the field dependence

A second peak starts to emerge as the in-plane magnetic field is increased.



There is a corresponding feature that emerges in the derivative for the peak above.

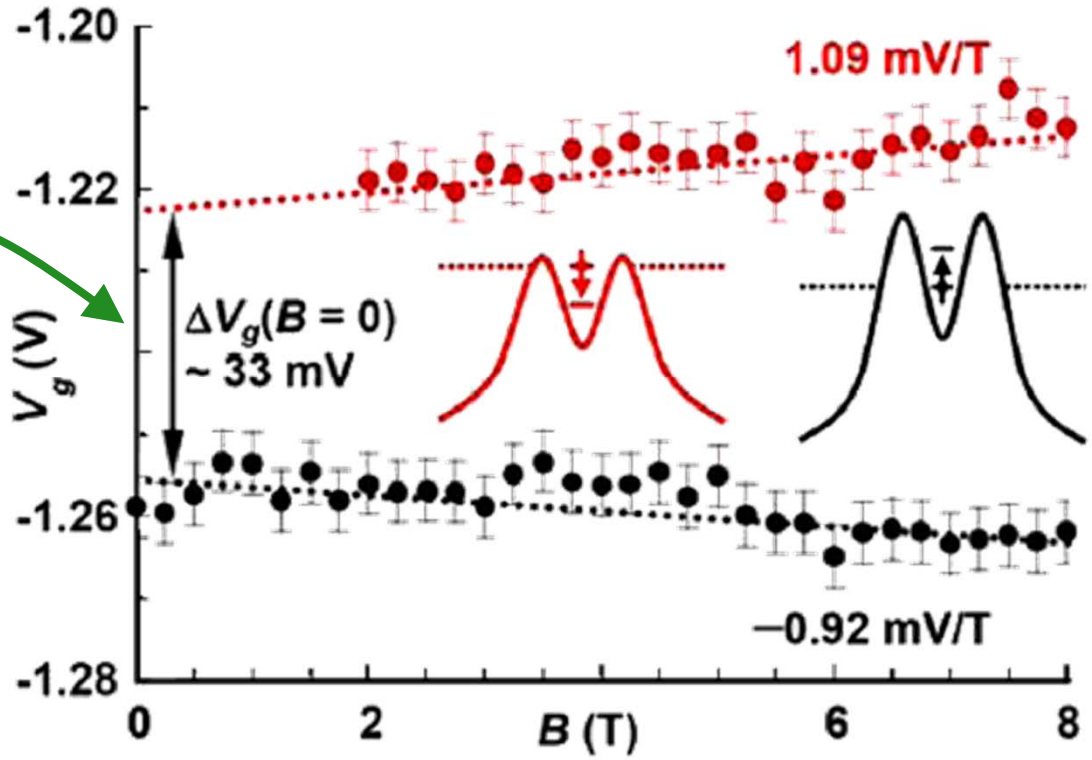
Y. Yoon *et al*., PRL 99, 136805 (2007).

Looking at the field dependence

- This peak splitting is interpreted as Zeeman splitting of a bound-state within the QPC.

Remarkably, the peak splitting does not extrapolate to zero at $B = 0$T

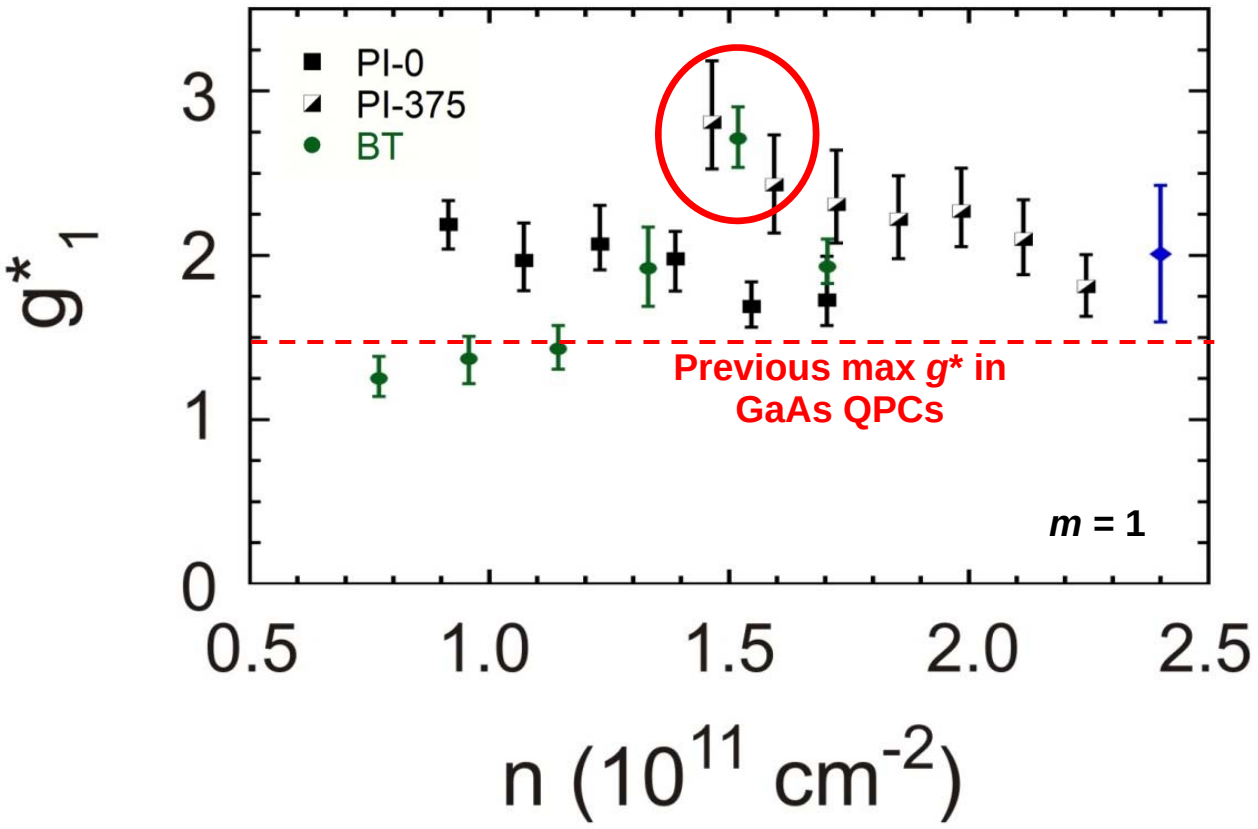
Suggests a remnant static spin-polarization, which is inconsistent with a Kondo scenario.



Y. Yoon *et al*., PRL 99, 136805 (2007).

Is there a bound-state formed in a QPC?

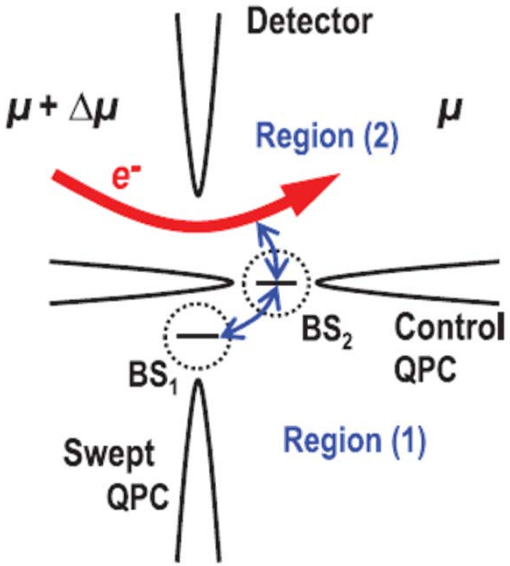
- **Bound-state formation in a QPC is consistent with recent *g** measurements. Exchange effects and *g** are highly sensitive to shape, size, etc in quantum dots.**

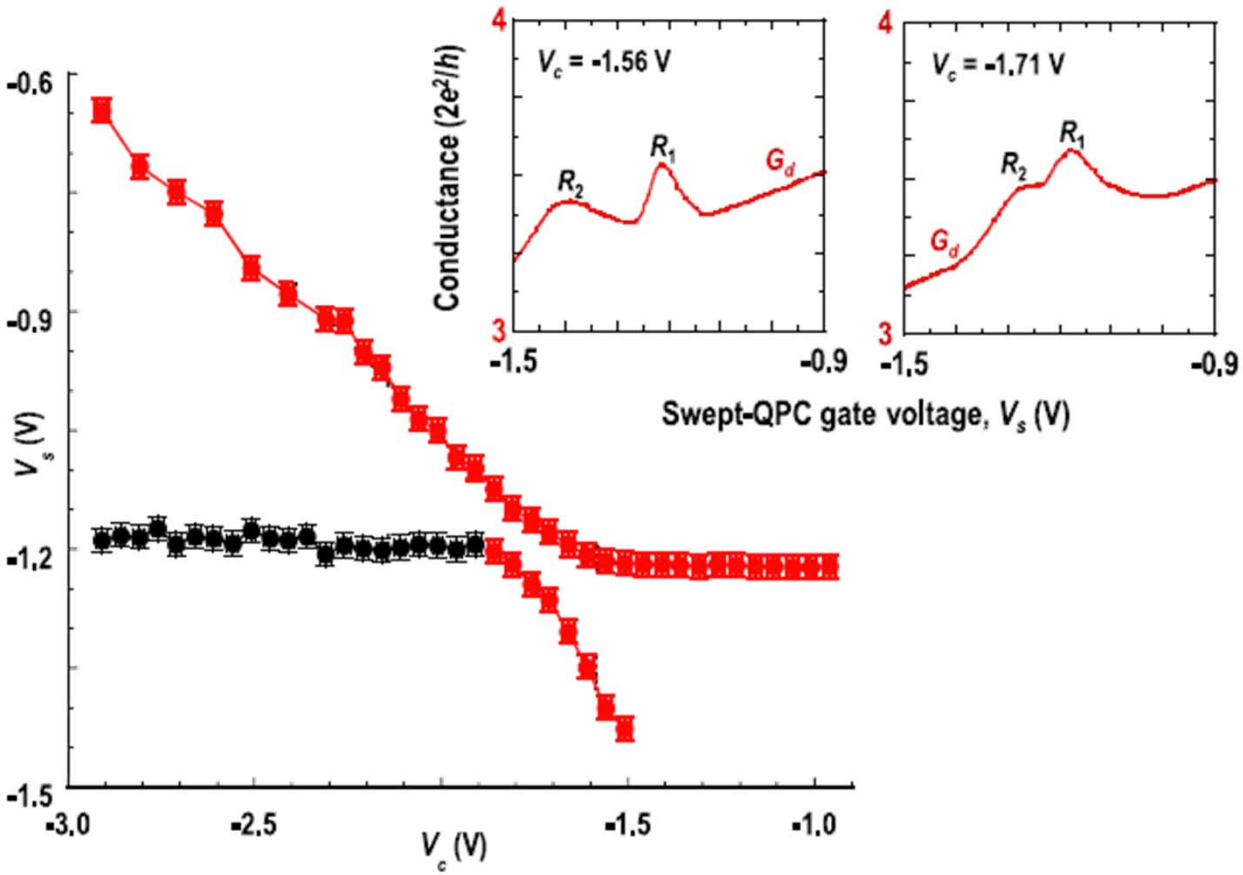


A.M. Burke *et al.*, Nano Lett. *in press*. doi: 10.1021/nl301566d

More recently…

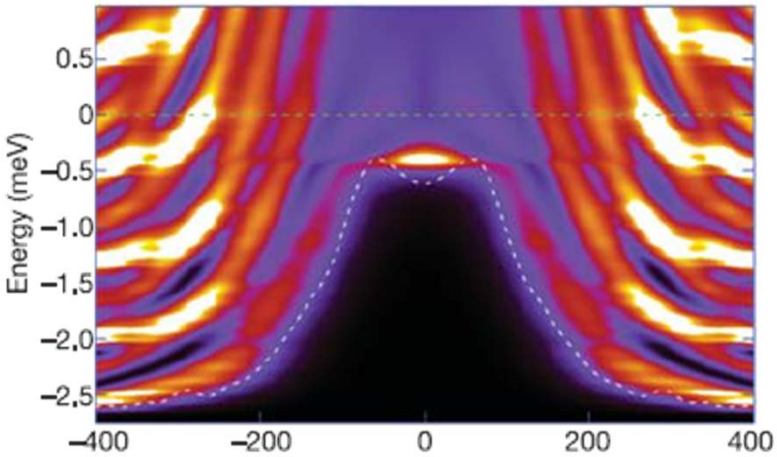
- **Work has focussed on interactions between bound-states in two adjacent QPCs.**

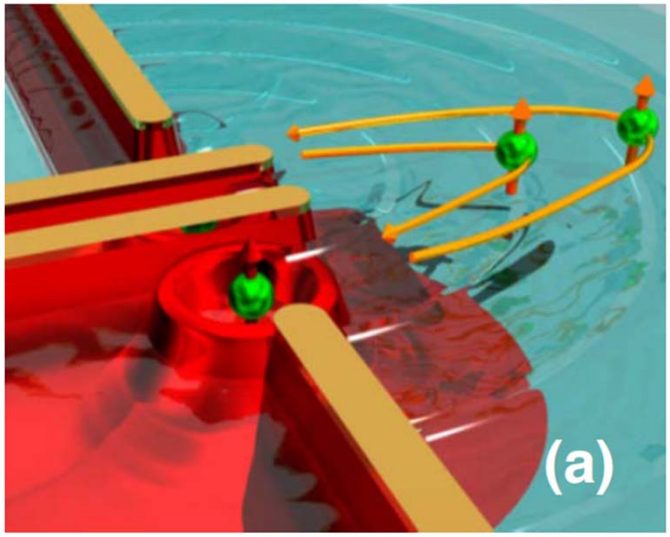


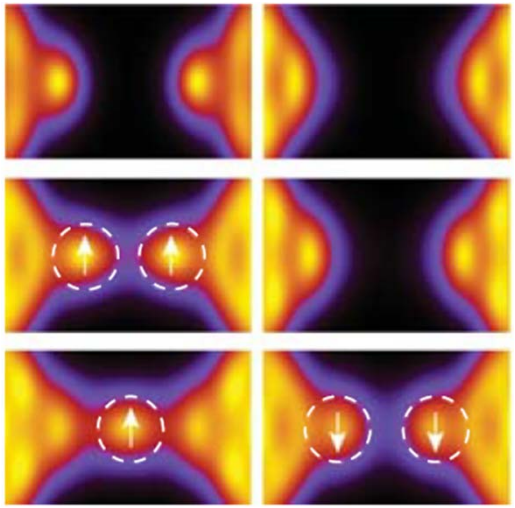


Y. Yoon *et al*., PRX 2, 021003 (2012).

Towards spontaneous ordering of electrons



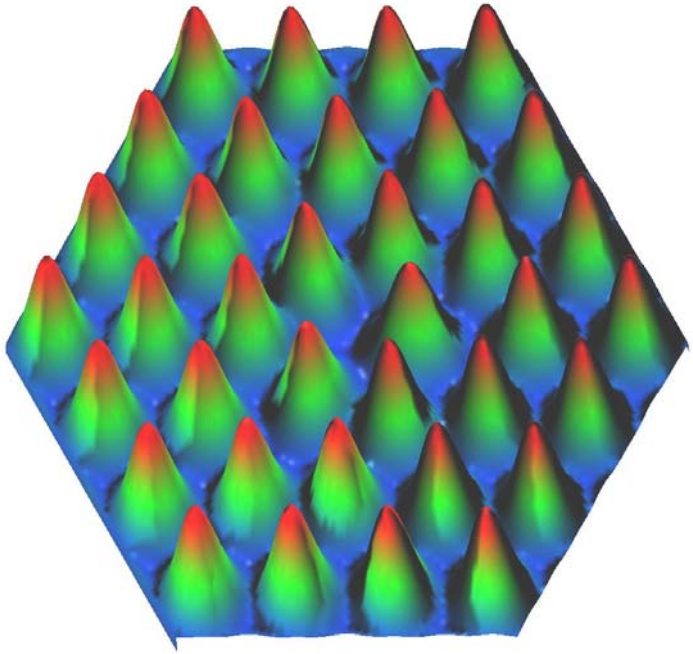




If electrons subject to exchange and correlation can reorganize to form bound-states, is there scope for larger organized electron structures?

The Wigner crystal

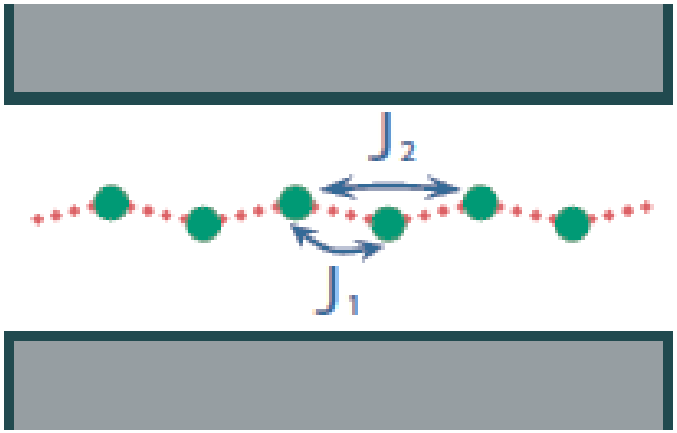
- **In the low density limit, the potential energy dominates over the kinetic energy for a gas of electrons. This drives the electrons to form an ordered crystalline lattice to minimise the potential energy.**



- **Not only should this happen in 2D, but it can happen in 1D too. And in crystallizing, the electrons need to account for their exchange energy too…**

1D spin chain models

- **The simplest model is a chain of electrons in a quantum wire with a transverse confining potential $V_{conf}(y) = m\Omega^2 y^2/2$, where Ω is the harmonic oscillator frequency.**



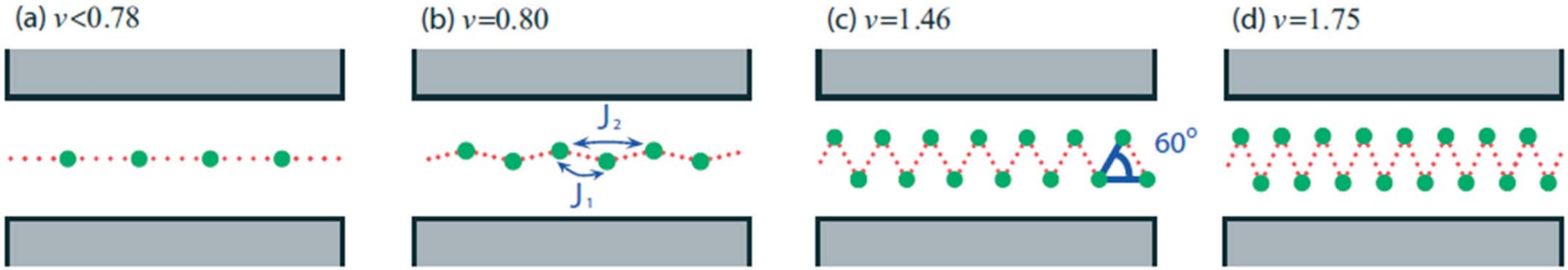
- **The Hamiltonian contains the usual terms (e.g., kinetic & potential energy due to confinement and electron-electron interaction) along with an exchange contribution described by the Heisenberg Hamiltonian:**

$$H_{12} = \sum_j (J_1 \boldsymbol{S}_j \boldsymbol{S}_{j+1} + J_2 \boldsymbol{S}_j \boldsymbol{S}_{j+2})$$

where J_1 and J_2 are exchange coupling parameters for the nearest neighbour and next-nearest neighbour spin interactions.

1D spin chain models

- At low density, the electrons form an ordered 1D lattice, but on increasing the density, a zig-zag lattice forms once the electron separation becomes less than a characteristic length scale for the 1D confinement $r_0 = (2e^2/\varepsilon m\Omega^2)^{1/3}$.



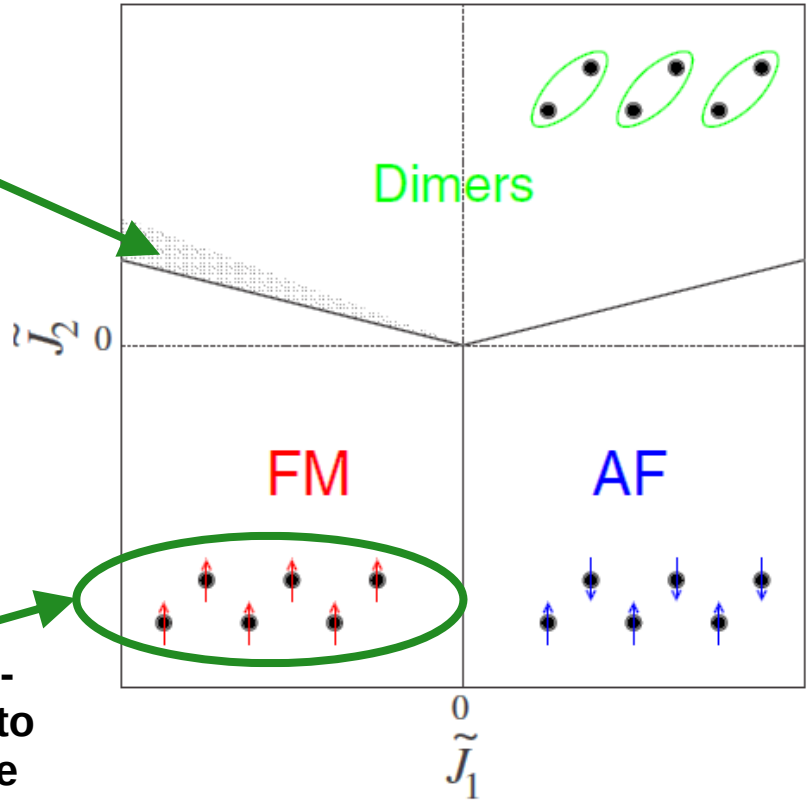
- Defining a dimensionless density $\nu = nr_0$, where n is the 1D density in the quantum wire, the linear crystal is found to be stable for $\nu < 0.78$, with the zig-zag chain occurring for $0.78 < \nu < 1.75$.

A.D. Klironomos *et al.*, Europhys. Lett 74, 679 (2006).

1D spin chain models

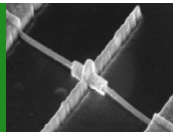
- The spin state of the system depends on the sign of the two exchange coupling parameters J_1 and J_2 (and the number of exchange terms included).

Exotic spin phases (e.g., chiral nematic phase)



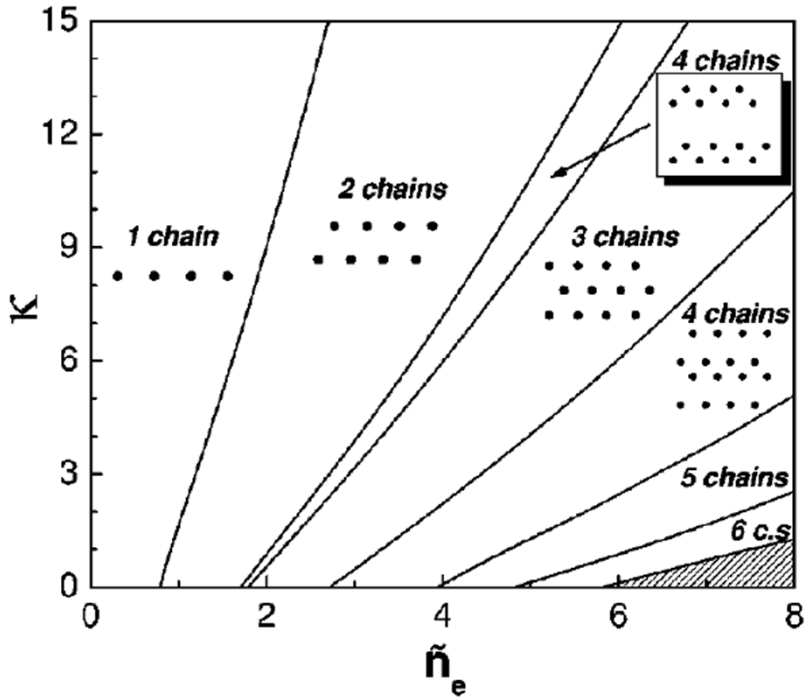
Doesn't cause problems with Lieb-Mattis theorem due to zig-zag breaking the one-dimensionality

A.D. Klironomos *et al.*, PRB 76, 075302 (2007).



1D spin chain models

- At higher density, structures with three or more rows can be obtained, with discrete transitions between phases with different numbers of chains.



- The parameters for the phase diagram are $\kappa = r_0 l/\lambda$, where λ is the screening length, and $n_e = l r_0 l a$ where l is the number of chains and a is the separation between two adjacent particles in the same chain. r_0 has the same meaning as previously.

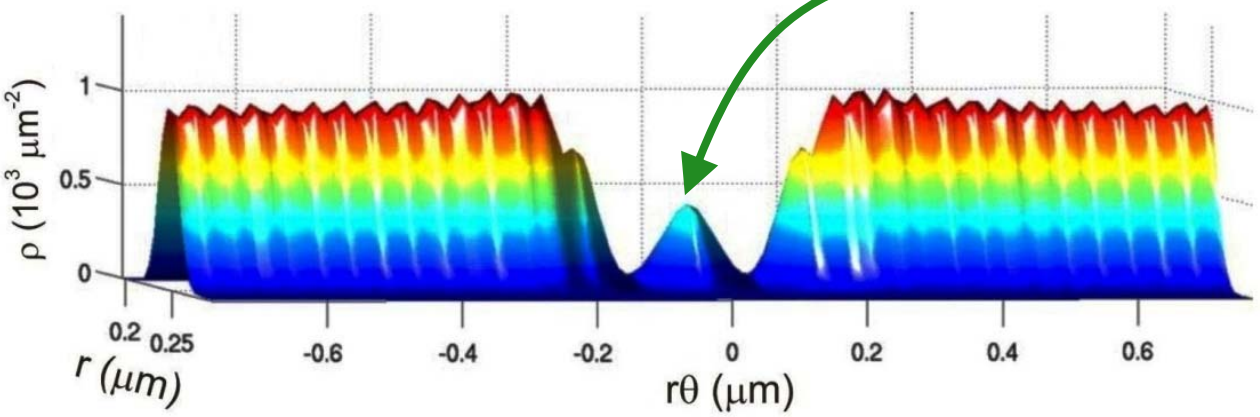
G. Piacente *et al.*, PRB 69, 045324 (2004).

Numerical studies of electron ordering

- **Güçlü *et al*. performed quantum Monte Carlo simulations to study a narrow 2D quantum ring with a constriction in it. The system is described by:**

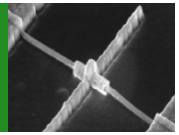
$$H = -\frac{1}{2}\sum_{i}^{N}\nabla_i^2 + \frac{1}{2}\sum_{i}^{N}\omega^2(r_i - r_0)^2 + \sum_{i<j}^{N}\frac{1}{r_{ij}} + V_g\{\tanh[s(\theta_i + \theta_0)] - \tanh[s(\theta_i - \theta_0)]\}$$

where the parabolicity ω controls the width of the ring, r_0 is its radius, V_g is the gate voltage controlling the constriction, and *s* and θ are the sharpness and length of the gate potential defining the constriction.



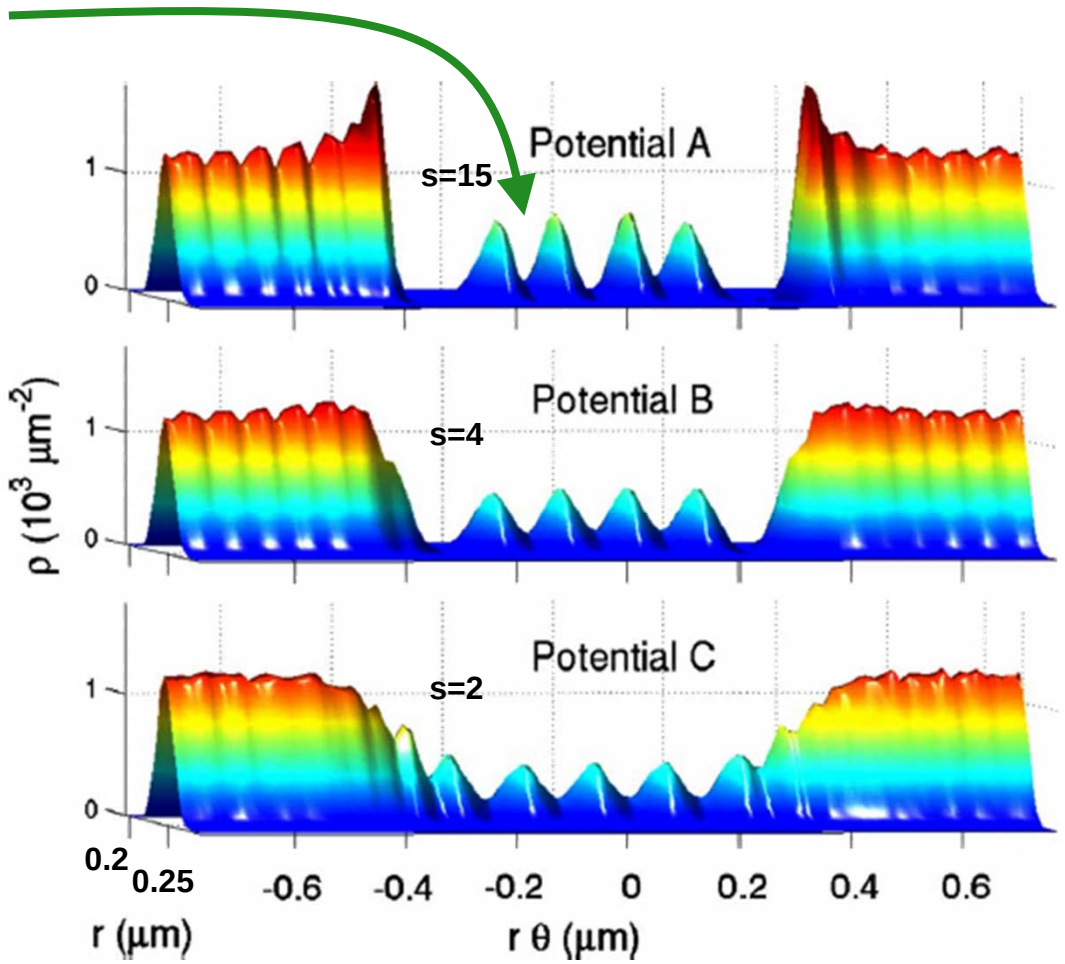
For a short constriction, a single localized electron is obtained.

A.D. Güçlü *et al.*, PRB 80, 201302 (2009).



Numerical studies of electron ordering

Localized electrons in a low density region separated from the 'liquid' leads

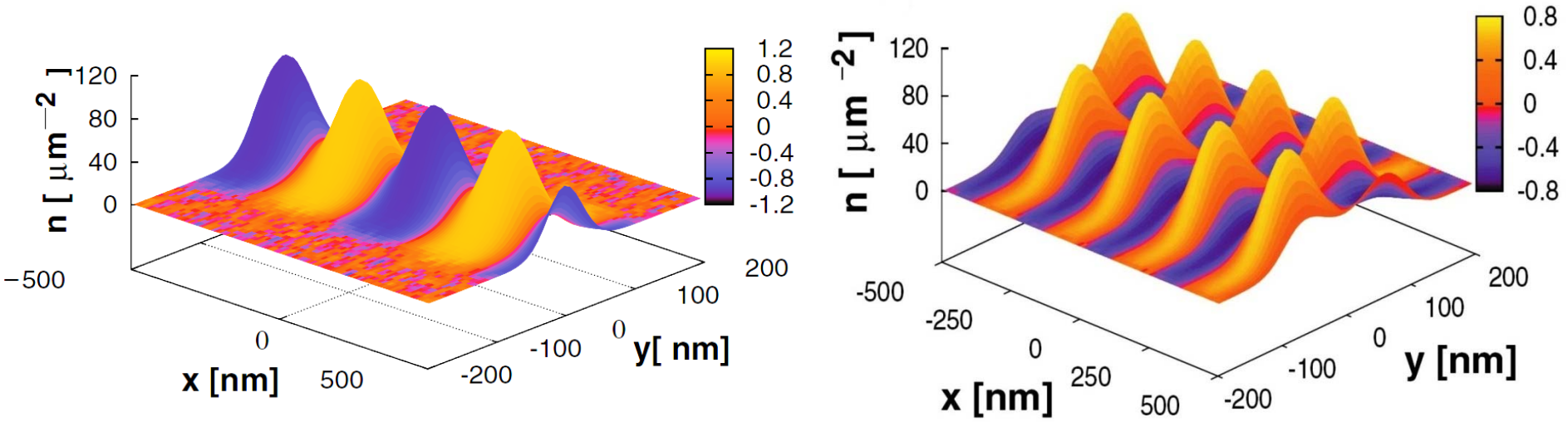


Abrupt barrier with flat plateau enhances localization and gap between liquid and crystal regions.

A.D. Güçlü *et al.*, PRB 80, 201302 (2009).

Numerical studies of electron ordering

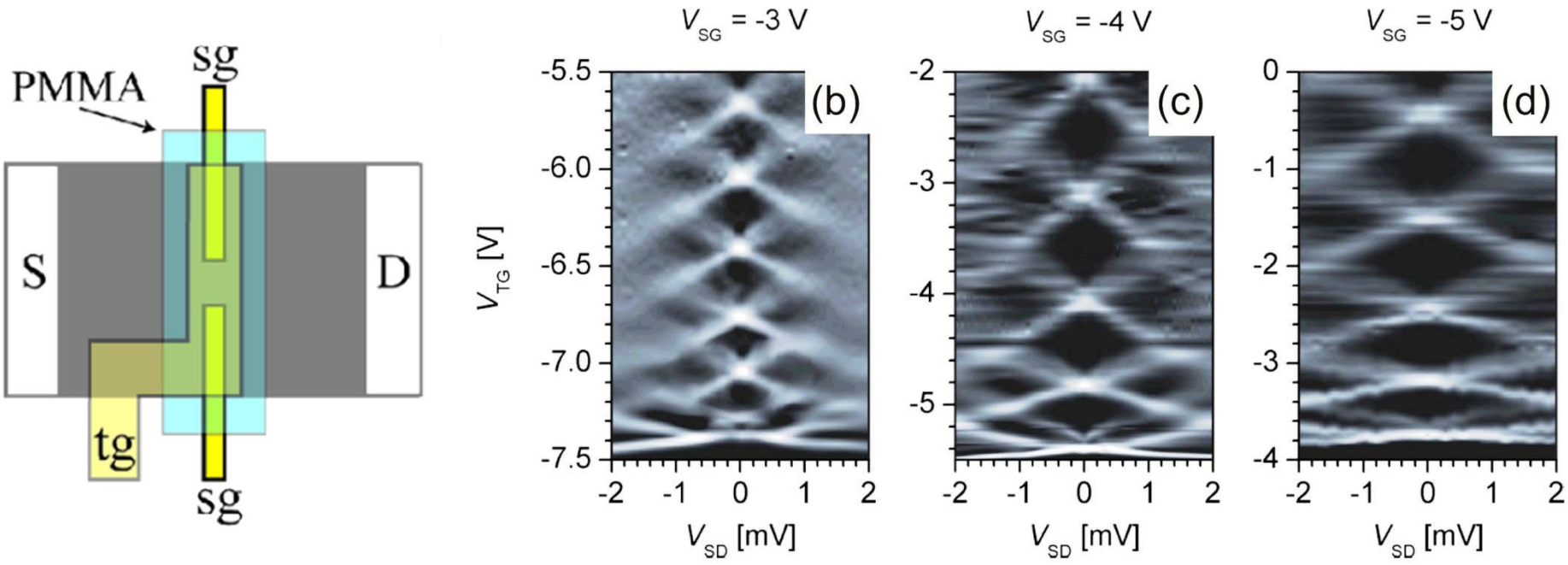
- **Similar structures emerge from spin density functional theory.**



E. Welander *et al.*, PRB 82, 073307 (2010).

Experiments on row coupling

- **Studies performed using a top-gated QPC. The device is operated a little differently to a normal QPC. The side-gates are held fixed and the top-gate is swept.**

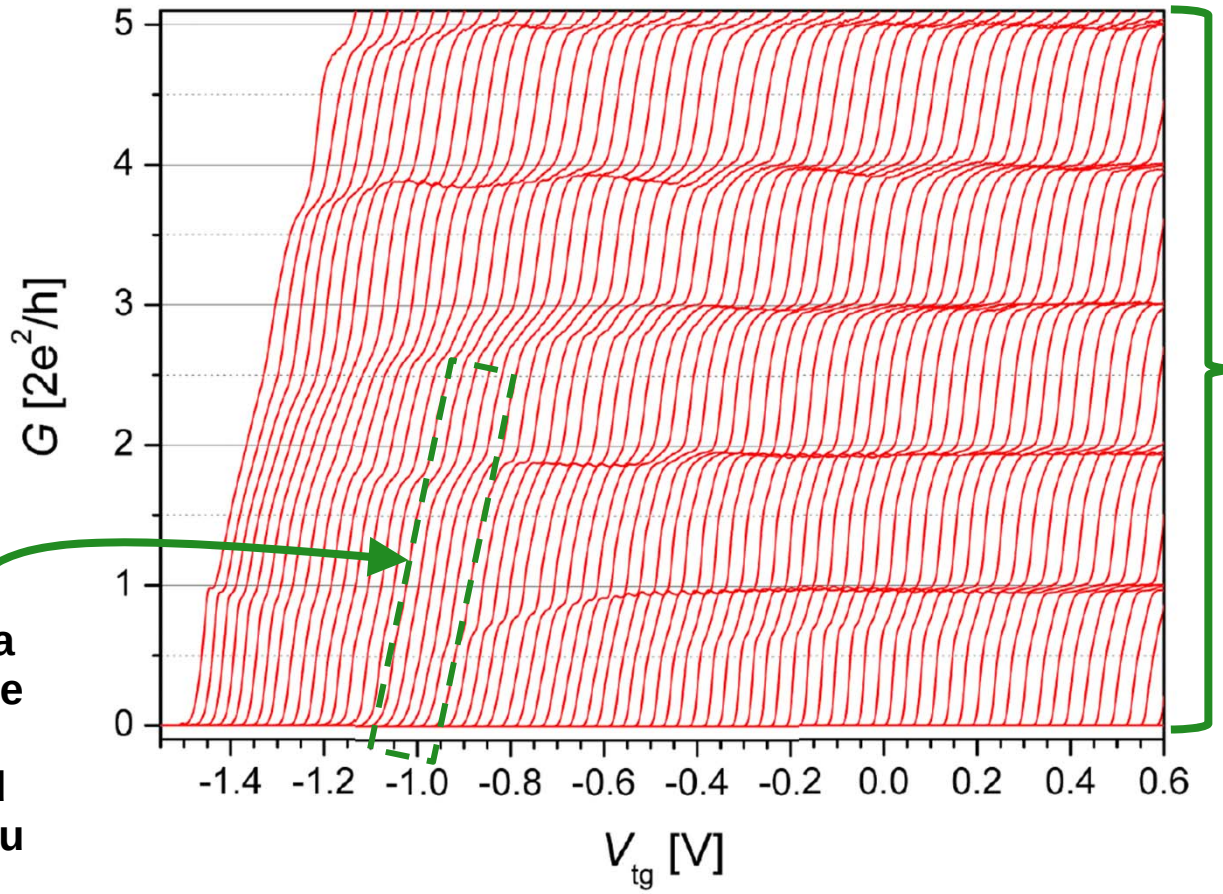


- **The strength of the 1D confinement is controlled by V_{sg}, as measured by $\Delta E_{n,n,+1}$**

W.K. Hew, Ph.D. Thesis, Cambridge University (2009).

Experiments on row coupling

- A zig-zag phase should act like two channels, giving a first plateau at $2G_0$ not G_0.



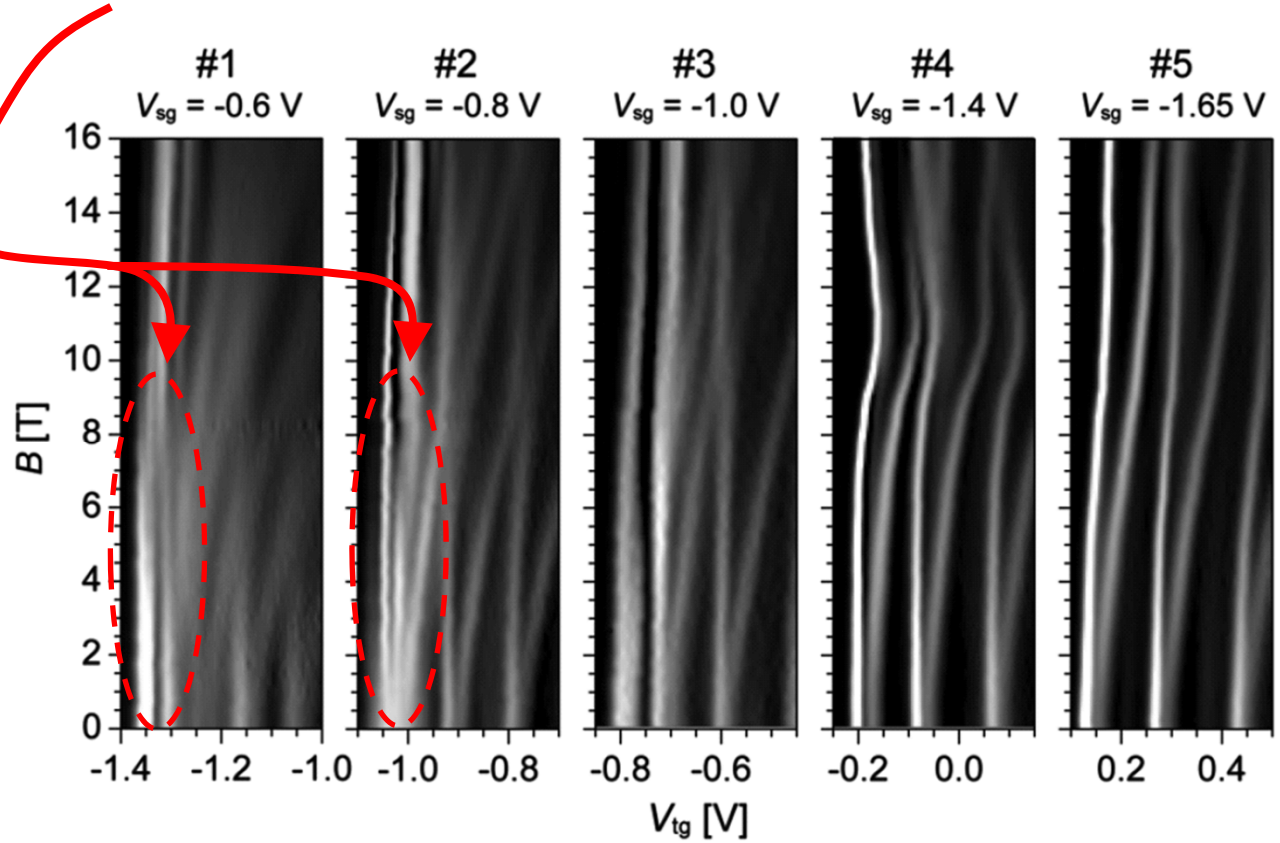
There comes a point where the G_0 plateau vanishes, and the first plateau is at $2G_0$.

In the strong confinement regime, the device behaves like a standard QPC with quantized conductance.

W.K. Hew *et al.*, PRL 102, 056804 (2009).

Experiments on row coupling

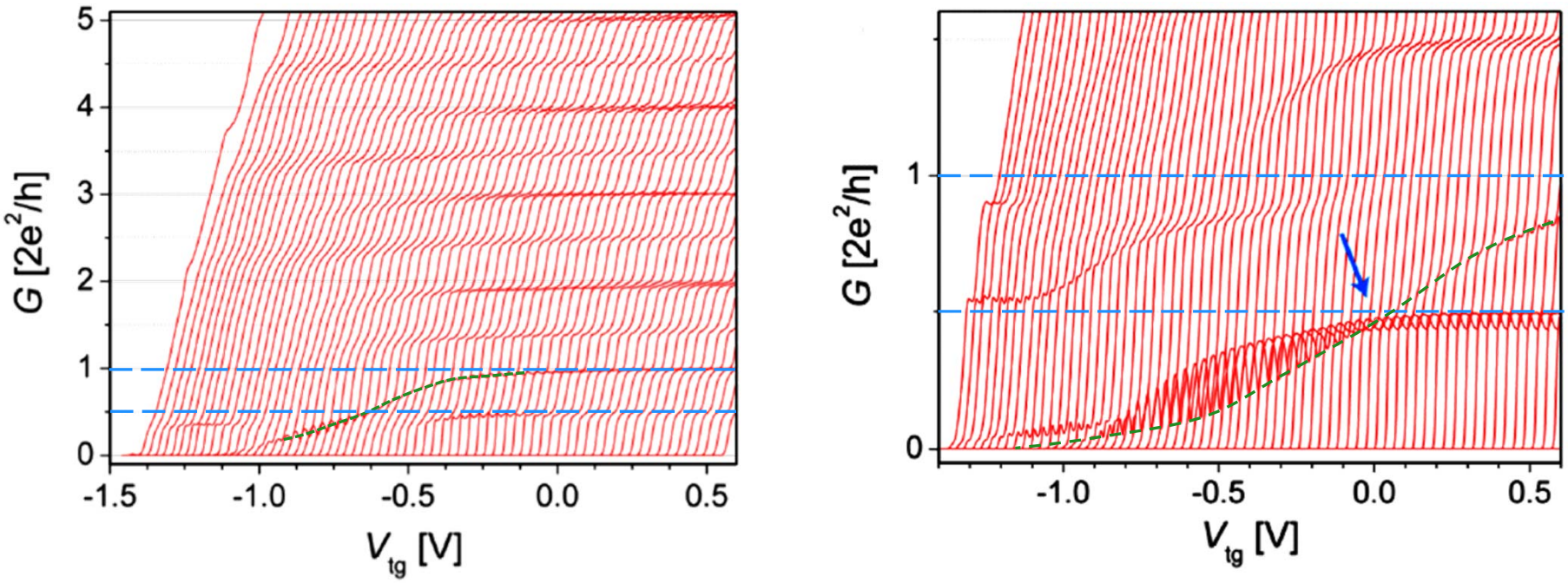
Unlike other plateaus, the weak-confinement $2G_0$ plateau appears to spin-split into more than two components.



W.K. Hew *et al.*, PRL 102, 056804 (2009).

Experiments on row coupling

- Measurements with $B_{||}$ = 7T (left) and 16T (right) reveal a G_0 plateau that crosses, and at the same time destroys, the ½G_0 plateau, which recovers thereafter.

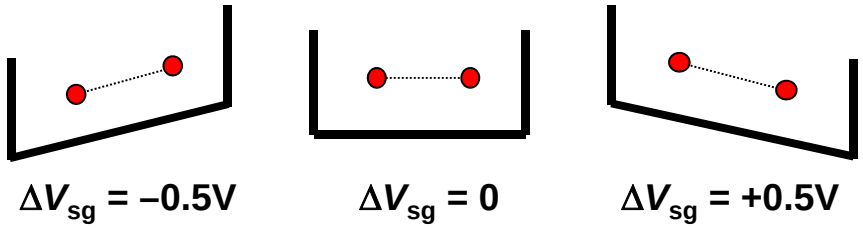


- This cannot occur in a non-interacting single wire, it is attributed to the bifurcation of a single channel wire in the strongly confined limit to a zig-zag configuration in the weakly-confined limit.

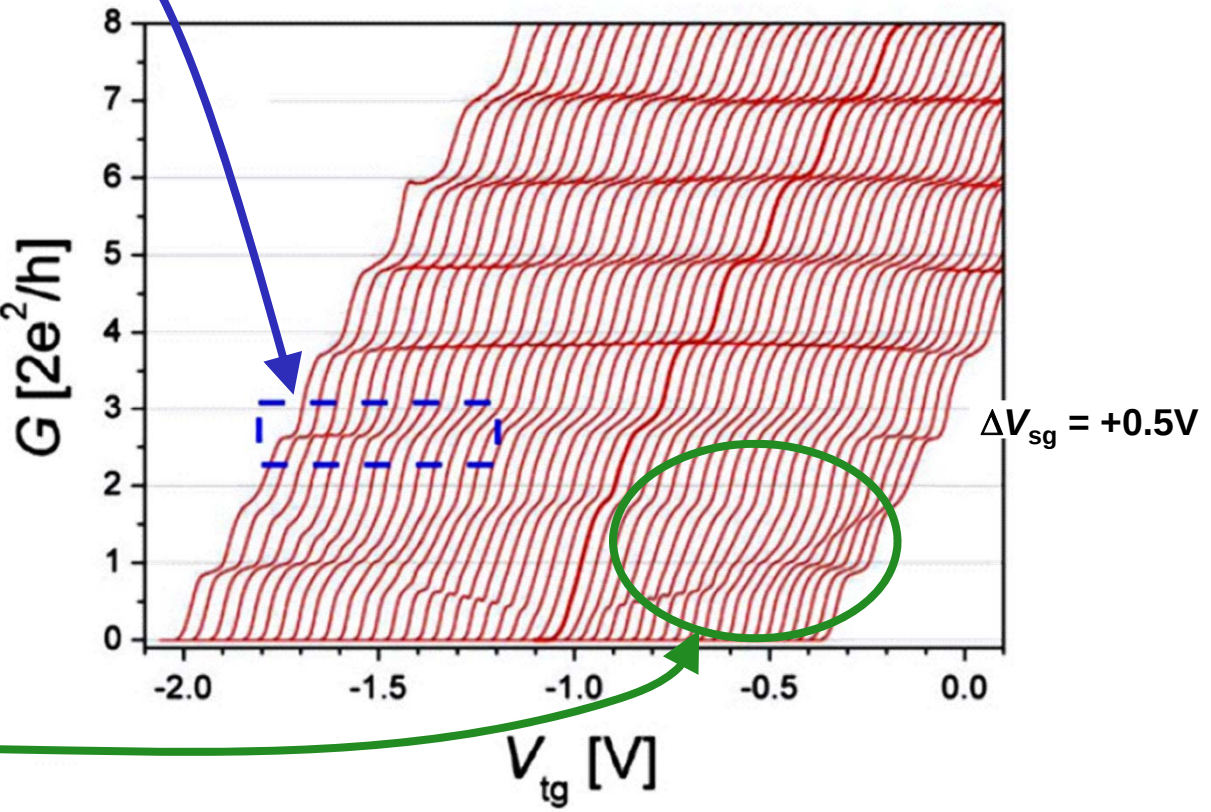
W.K. Hew *et al.*, PRL 102, 056804 (2009).

To the edge of understanding

Breaking the channel's transverse symmetry looks to have effects right up into the 3rd subband.

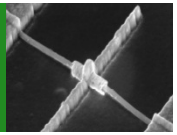


ΔV_{sg} = –0.5V ΔV_{sg} = 0 ΔV_{sg} = +0.5V

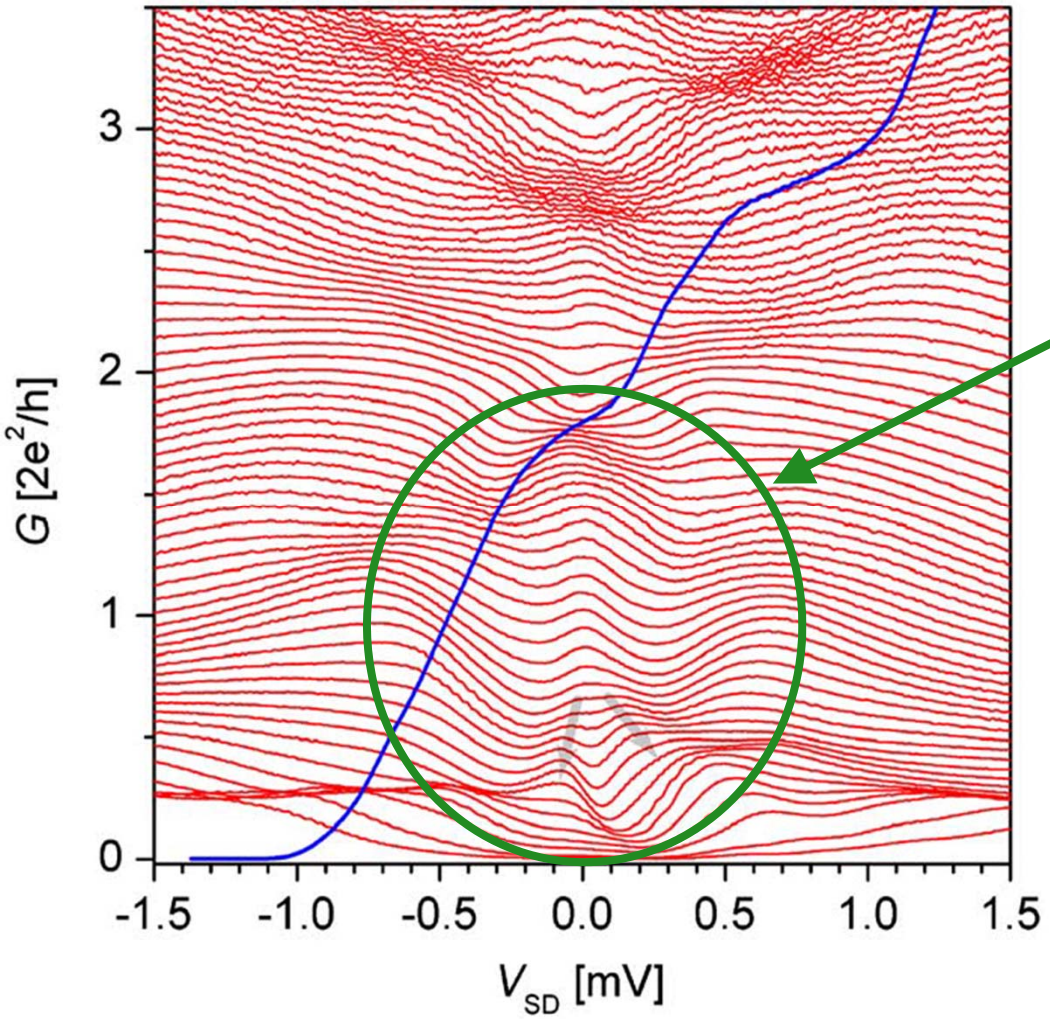


Potentially more interesting crossing behaviours

W.K. Hew *et al.*, Physica E 42, 1118 (2010).



To the edge of understanding

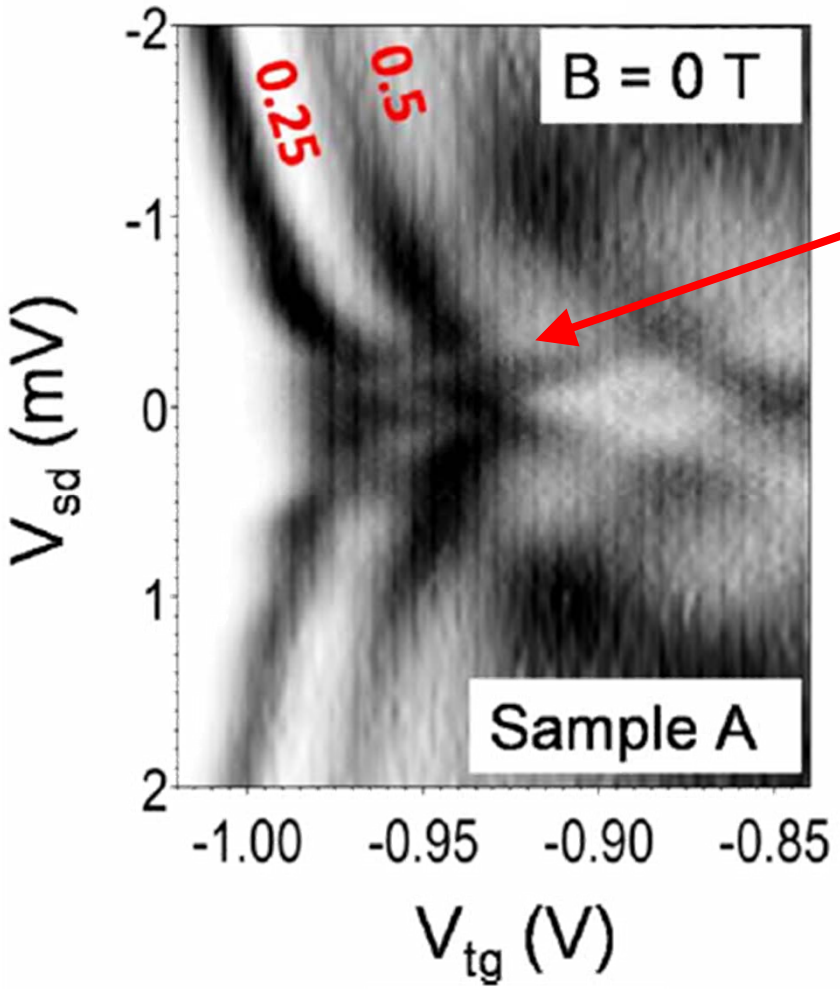


The zero-bias peak extends up to roughly $2G_0$, but most interestingly, it appears to split at lower *G*.

The physics behind this is an open question.

W.K. Hew *et al.*, Physica E 42, 1118 (2010).

To the edge of understanding



The normal source-drain bias structure for the first plateau is duplicated.

This is currently not understood either.

L.W. Smith *et al.*, PRB 80, 041306 (2009).

Throwing a cat amongst the pidgeons

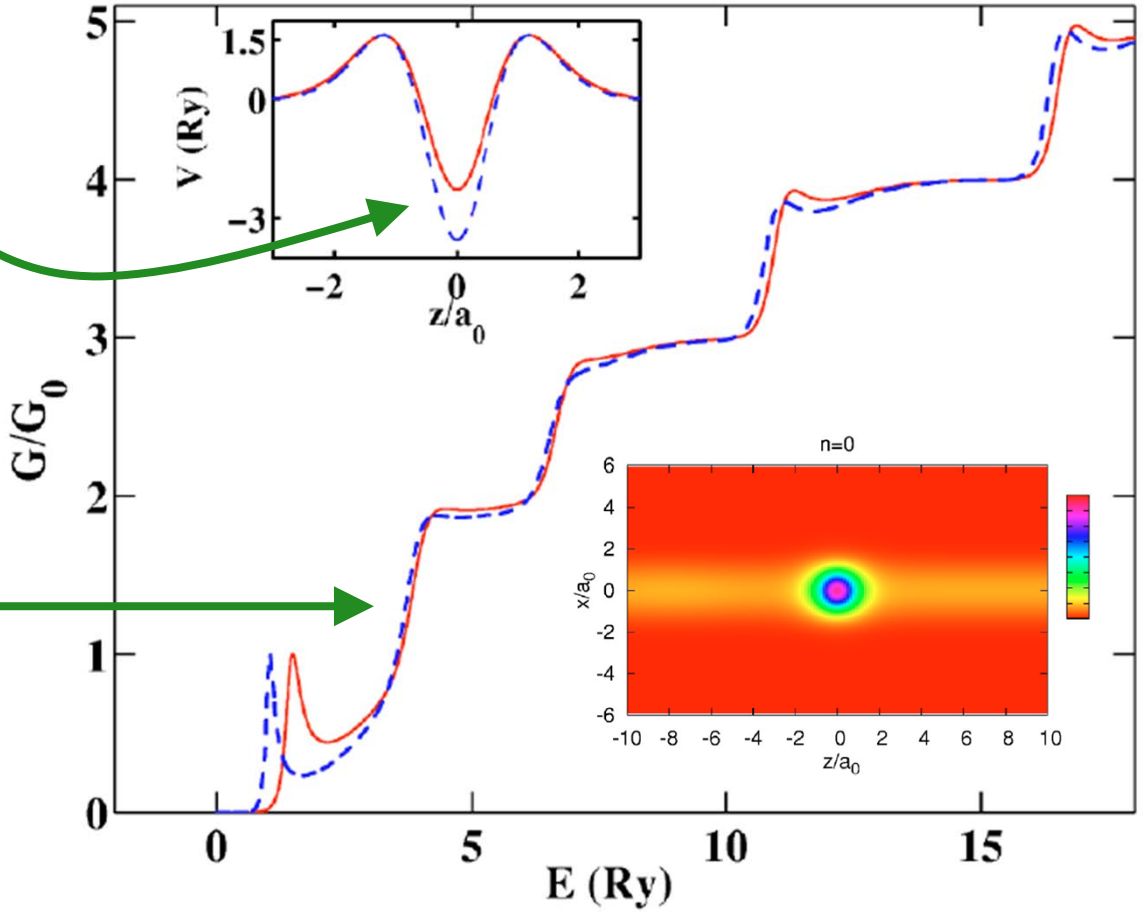
- **We may be conveniently forgetting something very important here…**

An alternate explanation

- It is worth noting that loss of the G_0 plateau can also arise from an attractive bound-state within a quantum wire.

The *T*-matrix Lippmann-Schwinger approach is used for transport in a quantum wire with a double Gaussian potential inside.

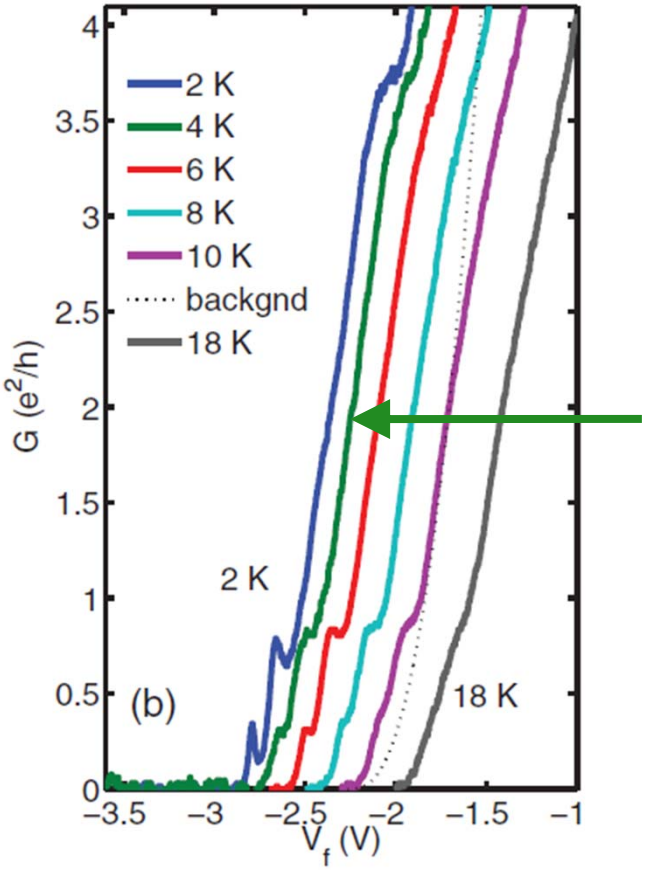
Results in destruction of the G_0 plateau, first plateau is at $2G_0$, with a resonance below.



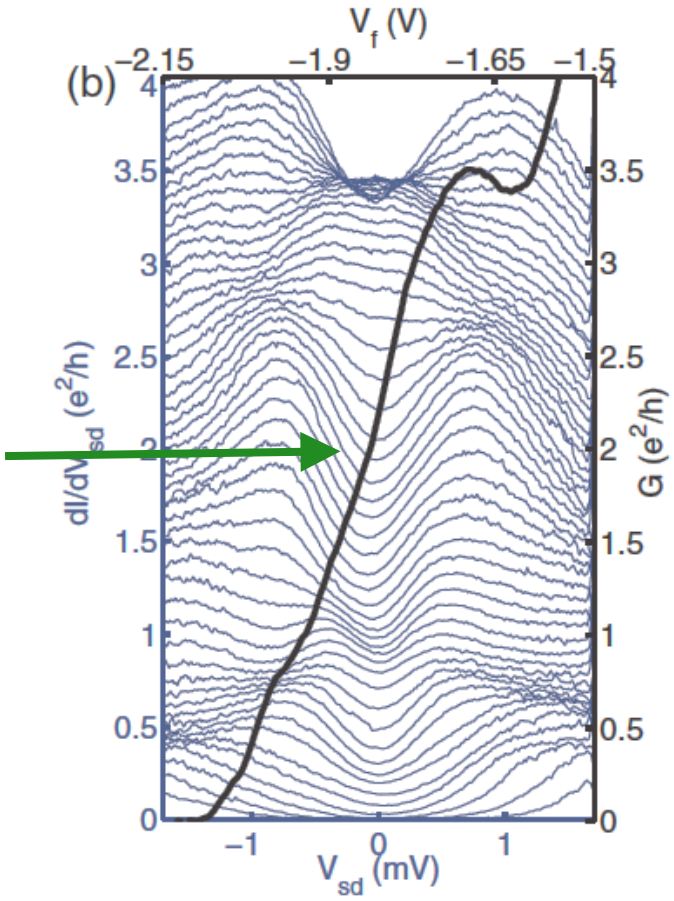
J.H. Bardarson *et al.*, PRB 70, 245308 (2004).

An alternate explanation

- The loss of the G_0 plateau is seen experimentally, without ‘weak confinement’, etc.



In both cases, the G_0 plateau is missing, the first plateau is up near $2G_0$.



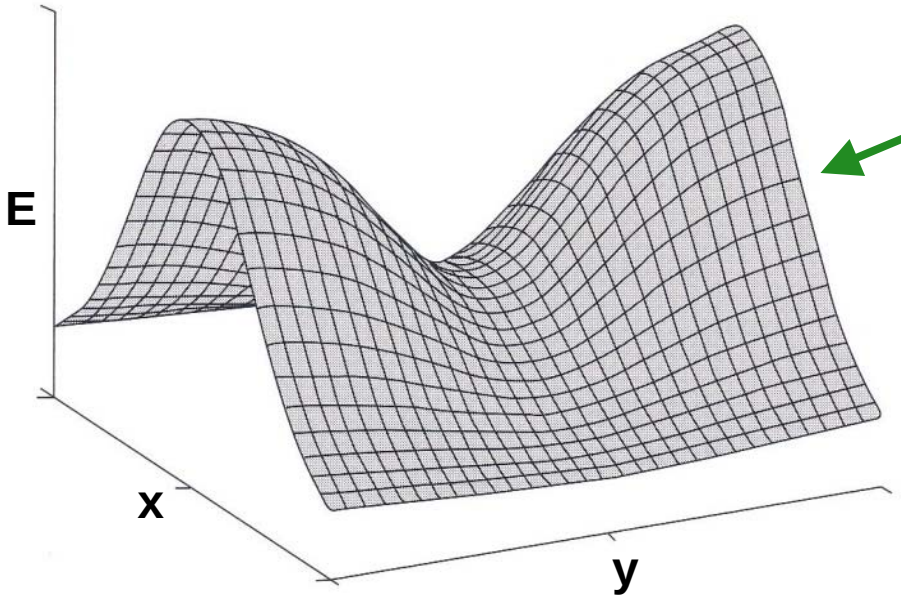
P.M. Wu *et al.*, PRB 85, 085305 (2012).

The skeleton in the closet

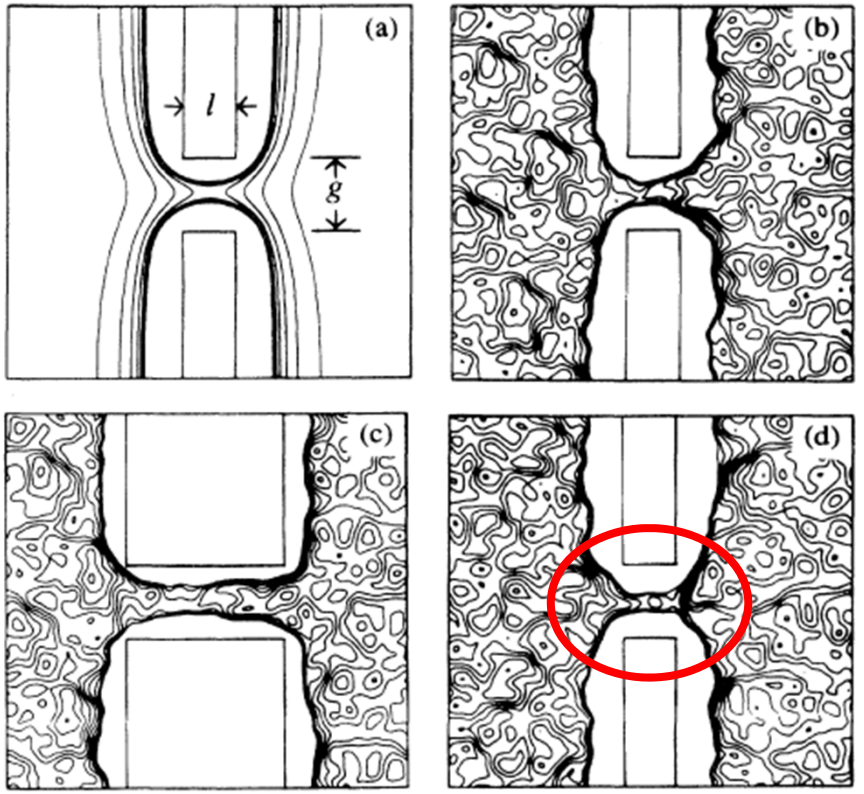
- A common criticism levelled in studies of QPCs is one of disorder. It is often used to dismiss data (both one's own, and that of others).

- A common strategy to dodge the disorder attack is channel shifting by asymmetric biasing of the QPC gates. I want to finish with a scrutinizing look at this.

An additional consideration -- Disorder



We really can't keep assuming a QPC looks like this…

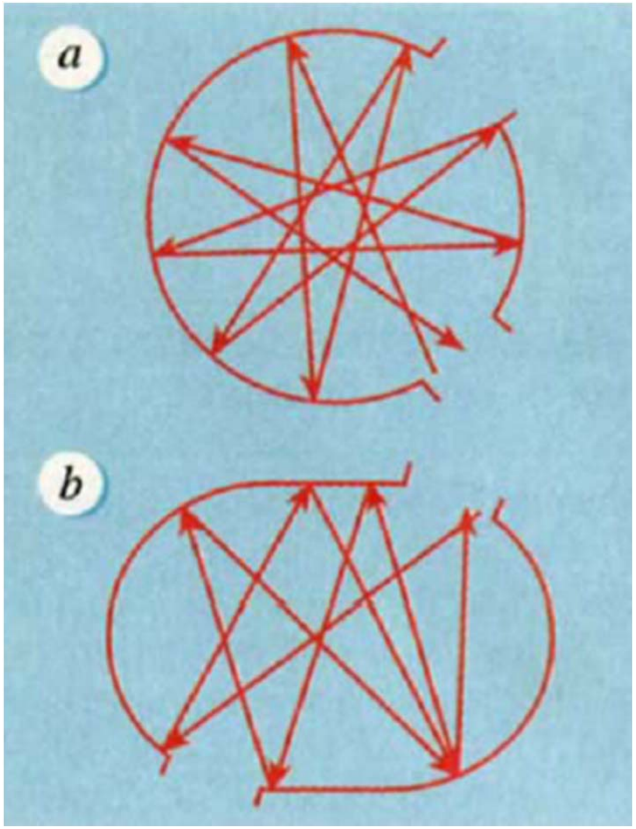


… when we know it looks like this.

J.A. Nixon *et al*., PRB 43, 12638 (1991).

The ballistic transport myth

- **To give one example about the danger in ignoring disorder in mesoscopic devices, consider ballistic transport in quantum dots.**

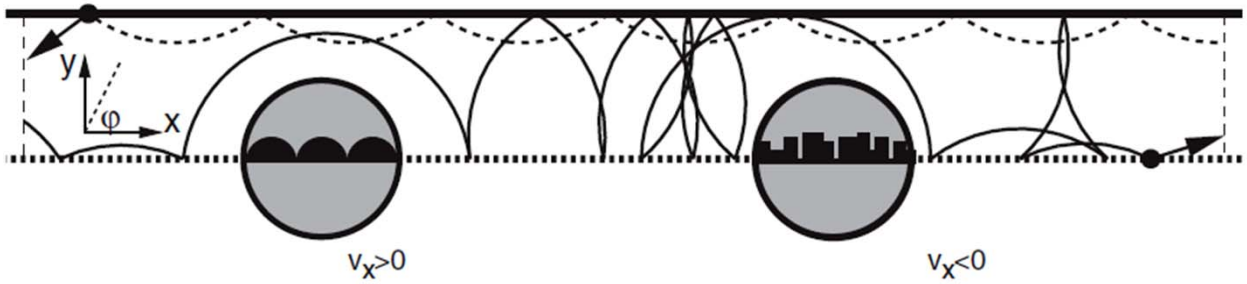


"Advances in semiconductor fabrication have made it possible to study the electrical properties of two-dimensional, solid-state devices so small (~ 1 μm) and cold (~ 0.05 K) that their length scales are much smaller than the mean free paths for impurity scattering of electrons[2]. In this 'ballistic transport regime' the electrical resistance arises from the elastic scattering of the electrons from the sides of the microstructure, so the conductance properties of these small solid-state devices are strongly dependent on their shapes."

R.V. Jensen, Nature 373, 16 (1995).

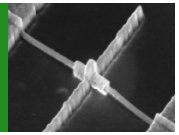
The ballistic transport myth

- **This line of thinking still holds today, across a wide range of mesoscopic device work.**



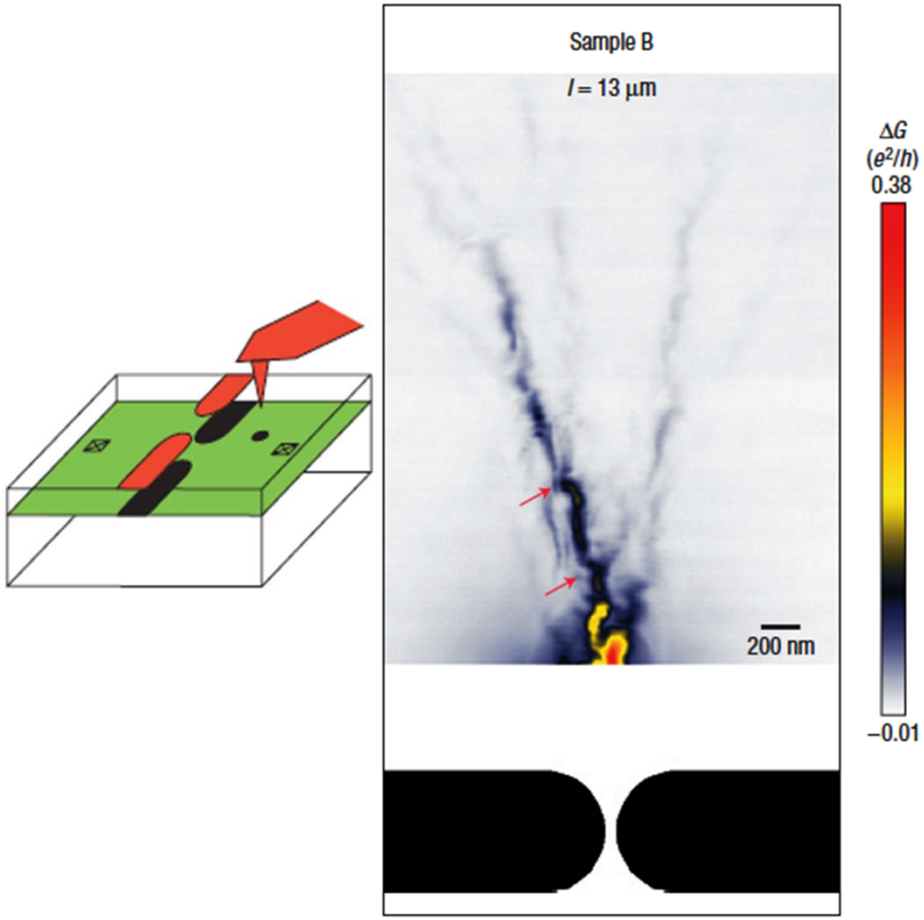
"Therefore, it is important to understand the consequences of nondiffusive electron dynamics on the electronic conductance or other transport properties. This question has been studied in much detail for semiconductor nanostructures in which the motion of electrons is ballistic rather than diffusive [1–4]. In such systems, disorder is negligible, and, consequently, all transport properties are determined by the shape of the sample, as in a billiard model."

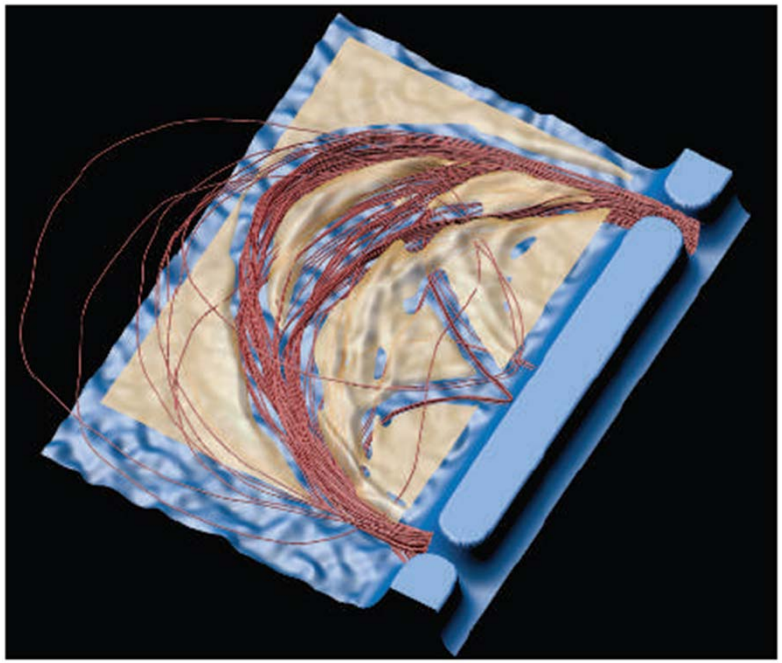
Prusty & Schanz, PRL 96, 130601 (2006).



We can see its not ballistic…

- **Deflections at the sub-100nm length scale despite ‘mean free paths’ of >10μm due to small-angle scattering from ionized impurities.**



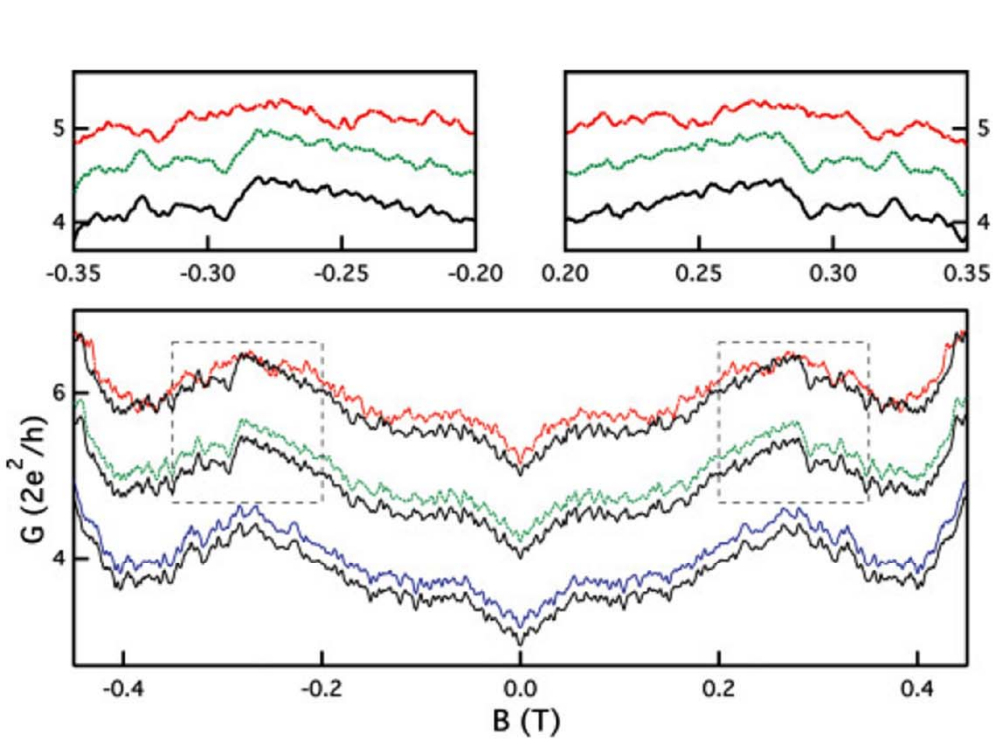


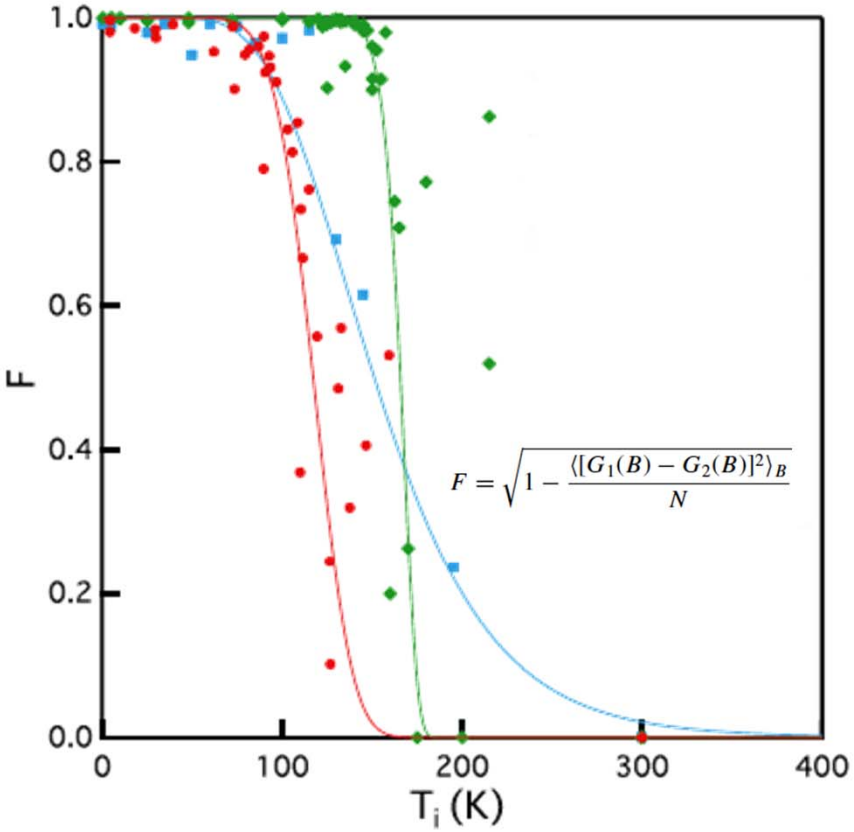
M.P. Jura *et al.*, Nat. Phys. 3, 841 (2007).

K.E. Aidala *et al.*, Nat. Phys. 3, 464 (2007).

… and we can measure it too.

- **Raising the temperature above 120K alters the ionized impurity potential, which in turn changes the interference. If the transport was really ballistic and determined only by the geometry, then this should not happen.**

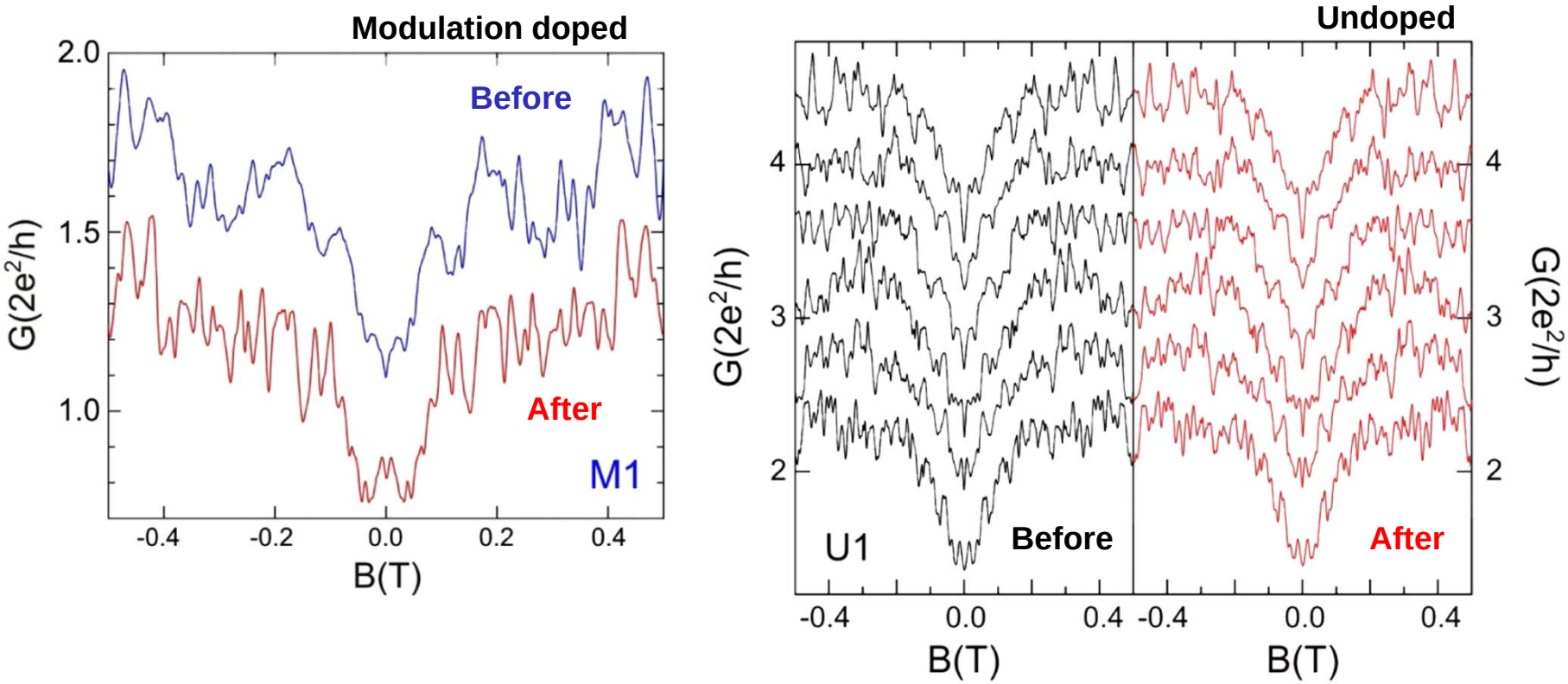




B.C. Scannell *et al.*, PRB 85, 195319 (2012).

… and we can measure it too.

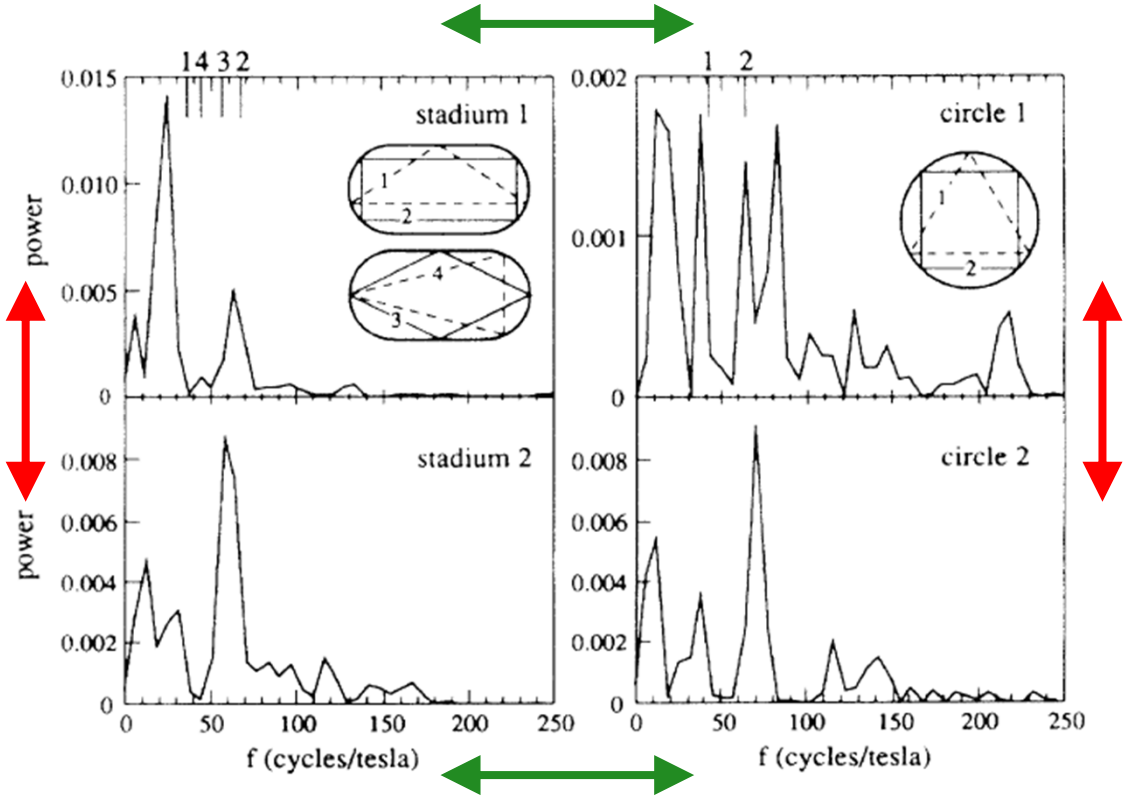
- If you remove the dopants, the conductance fluctuations become robust to room temperature thermal cycling… but that probably doesn't mean they're gone entirely.



A.M. See *et al.*, PRL 108, 196807 (2012).

Reconsidering the past

- These new results force us to reconsider our notions of ballistic transport.



For example, the horizontal differences are no more significant than the vertical differences comparing data panels.

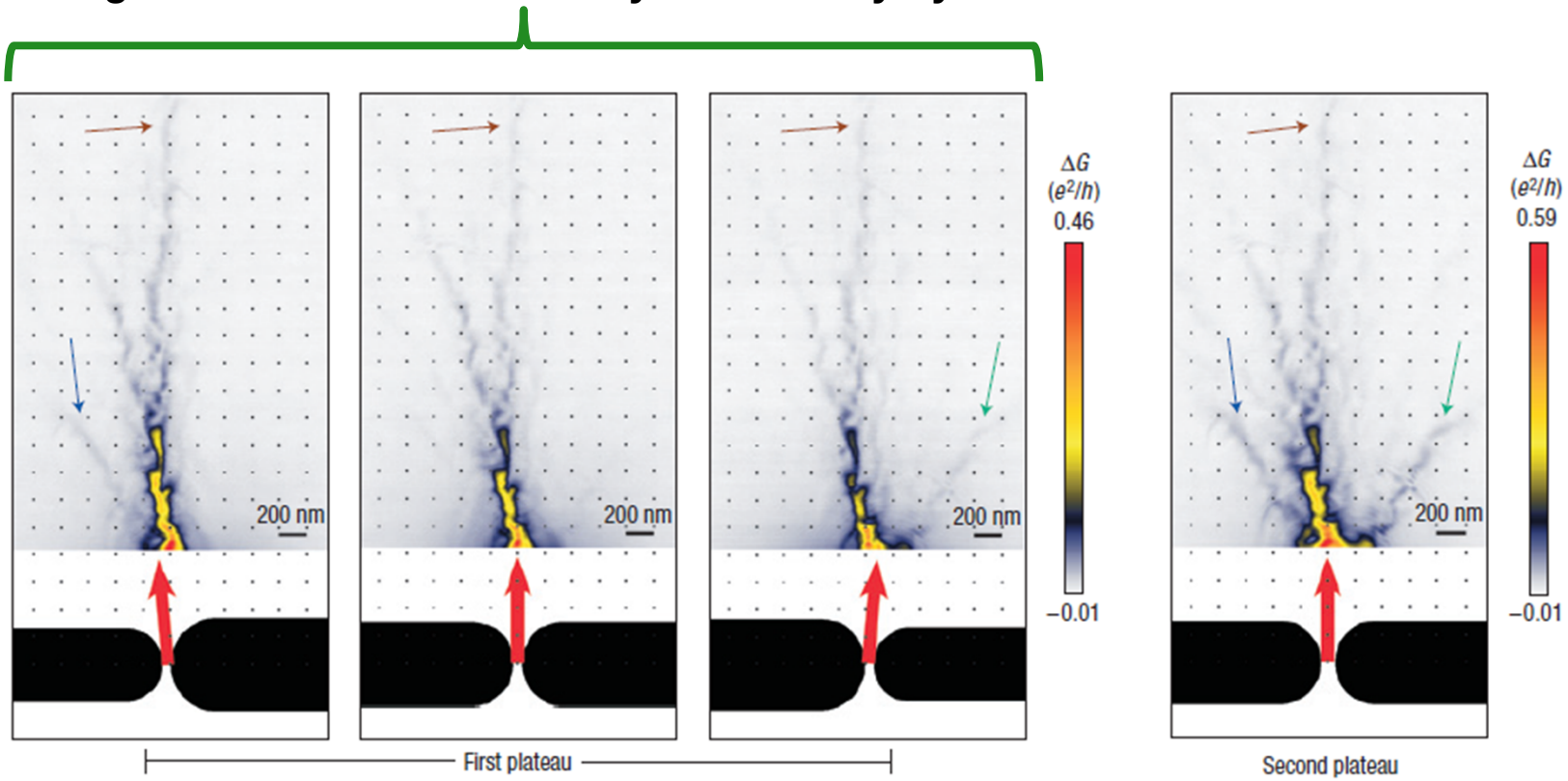
This can be explained by disorder rather than geometry dominating the transport (i.e., these 'ballistic' devices aren't really ballistic).

C.M. Marcus *et al.*, PRL 69, 506 (1992); for more, see A.P. Micolich *et al.*, doi: 10.1002/prop.201200081 (2012).

Back to QPCs

- **To give just one example where this might matter for QPCs…**

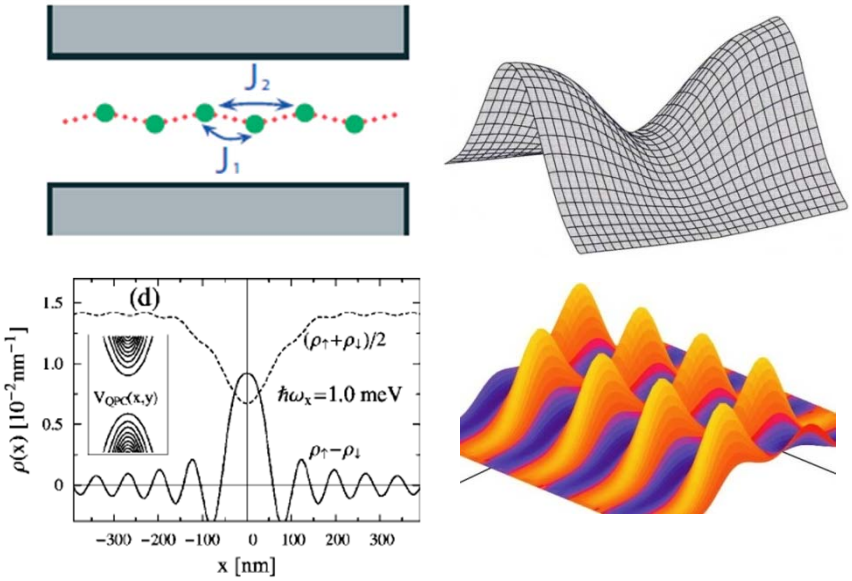
A QPC with an impurity doesn't get made into a clean QPC just by shifting the channel. The whole system is dirty by definition!



M.P. Jura *et al.*, Nat. Phys. 3, 841 (2007).

Perhaps a little controversial ☺, but…

Does indulging in pictures like:



=

?

- **Given this, we really have to seriously ask ourselves whether ‘elegant’ pictures of simple potentials, clean electron distributions and nice ideas like 1D Wigner crystal zig-zag chains really make sense? They’re useful, but do they explain reality?**

- **And if they do, how they survive the horrible mess of disorder in the experimental systems that we all work on is something in urgent need of understanding.**

What are the key questions?

1. If we go back to just having a disorder potential like in Nixon and Davies, but properly accounting for exchange, correlation, etc., do we get 0.7?

   In other words, how robust is 0.7 to disorder from a theoretical perspective?

2. Can more complex manifestations of Kondo explain some of the things we see in QPCs (e.g., strange zero-bias peak behaviour)? To what extent can disorder reliably produce such scenarios?

3. Is it really possible that a 1D Wigner crystal can survive and flow with disorder?

Thank you for listening

And best of luck if you decide to get involved in sorting out the mess that is 0.7…