Magnetic field induced Fabry-Pérot resonances in helical edge states

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Outline

1. Introduction
2. Edge state Hamiltonian and $\vec{B}$-field
3. Fabry-Pérot Resonances
4. Conditions for experimental realization

1 Introduction

2 Edge state Hamiltonian and $\vec{B}$-field

3 Fabry-Pérot Resonances

4 Conditions for experimental realization

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- Whether a material is metal or insulator can be explained by simple band theory of noninteracting electrons.
- However, a new class of materials by name Topological Insulators emerged in the last decade which insulating in the bulk but metallic on the surface.

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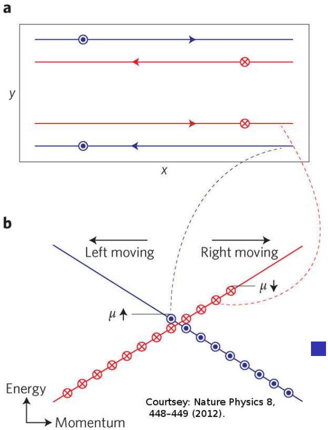

Courtsey: Nature Physics 8, 448–449 (2012).

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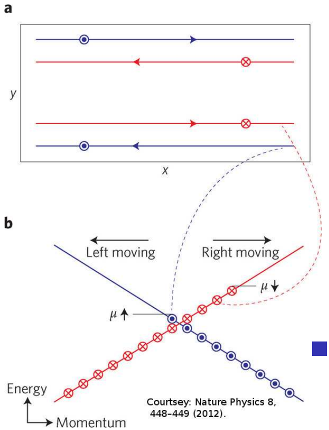

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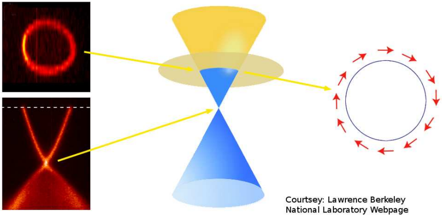

Courtsey: Lawrence Berkeley National Laboratory Webpage

Band-structures of 2D (left) and 3D (right) Topological Insulators.

- The edge/surface states are helical protected by Time-reversal symmetric.

- Following the seminal work of Kane and Mele (2005), Bernevig-Hughes-Zhang proposed that HgTe/CdTe quantum wells (that have strong spin orbit coupling) can be 2D TI (Science-2006).
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- At a given edge the states are helical.
- Described by 1D massless Dirac equation, edge states are robust against static (Time-reversal-invariant) disorder due to Klein tunneling.
- Magnetic field applied to these ballistic channels can produce backscattering.
- We show that magnetic field Zeeman-coupled with these helical 1D-channels over a patch can be used to tune the transmission and shows resonances.

- The edge state Hamiltonian is given by-

$$H_0 = -i\hbar v_F \int dx\ \psi^\dagger(x)\sigma_z \partial_x \psi(x), \quad \text{where } \psi = [\psi_\uparrow\ \psi_\downarrow]^T$$

- Note spin-momentum locking.

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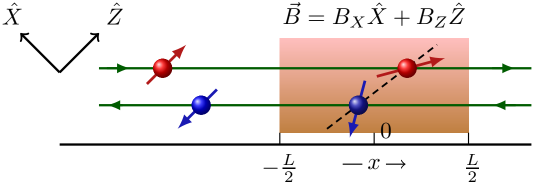
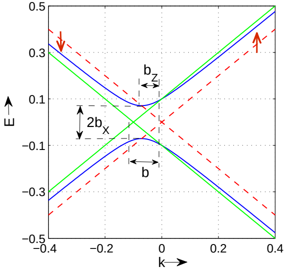
- Note spin-momentum locking.
- Magnetic field is introduced by Zeeman-coupling over a region $\Delta_L(x)$

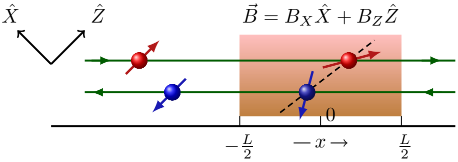
$$H_B = g\mu_B \int dx\ \Delta_L(x)\vec{S}(x)\cdot\vec{B}$$





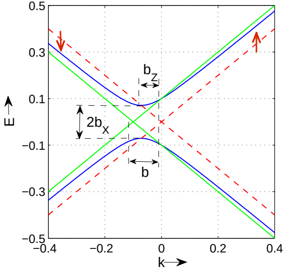
Right: Dispersion for various $\vec{B}$-field configurations.

- In the $\vec{B}$-field patch, dispersion changes. B_X opens up a gap in the spectrum.



Above: Schematic of the set-up.

Right: Dispersion for various $\vec{B}$-field configurations.



- In the $\vec{B}$-field patch, dispersion changes. B_X opens up a gap in the spectrum.
- Also, the spin orientation of the L/R-modes in the patch get twisted and are no more orthogonal.

- Now, consider the $\vec{B}$-field over a finite region on the helical edge states.

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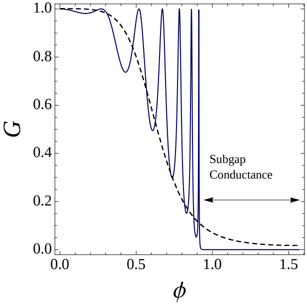
- Due to interference between the left-moving and right moving modes, one can expect resonance.

The wavefunction will look like

$$\psi = \begin{cases} |\uparrow\rangle\, e^{ik_i x} + r_{k_i} |\downarrow\rangle\, e^{-ik_i x} \\ A_R\, \psi_{E_i\, R}\, e^{ik_R x} + A_L\, \psi_{E_i\, L}\, e^{-ik_L x} \\ t_{k_i}\, e^{ik_i x} |\uparrow\rangle \end{cases}$$

for $x < -L/2$, $|x| < L/2$ and $x > L/2$ respectively.



dI/dV **vs the angle** ϕ **between** $\vec{B}$ **and** $\hat{Z}$**.**

- Look at the total phase acquired by the electron in traversing one complete cycle back and forth in the patch.

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$$(k_R + k_L)L = n \cdot 2\pi$$

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- Number of peaks can be found from the above condition to be- greatest integer less than $k_i L/\pi$.
- Evenescent modes do not contribute to resonances. The wavefunction in the patch decays exponentially for these modes.

- The spin component $\langle S_Z(x)\rangle$ is a conserved quantity and is same everywhere.
- But $\langle S_X(x)\rangle$ and $\langle S_Y(x)\rangle$ show a helical texture across the length of the patch.

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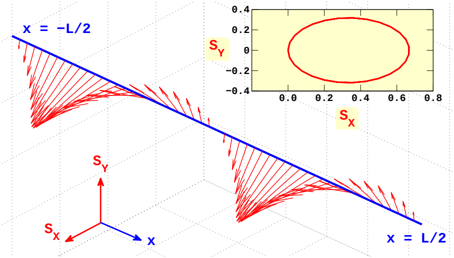
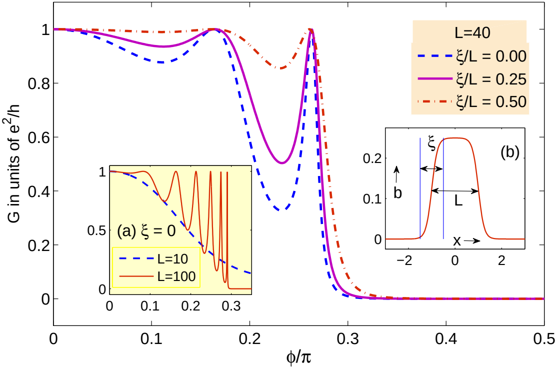
- Resonance is when $\langle\vec{S}\rangle$ makes an integer number of complete precessions.



These resonances persist even when the $\vec{B}$-field changes smoothly on the edge.



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- The static impurity can be modelled as a rectangular potential barrier/well charecterised by a height η and width ℓ positioned at x_ℓ in the patch ($|x_\ell| < L/2$).

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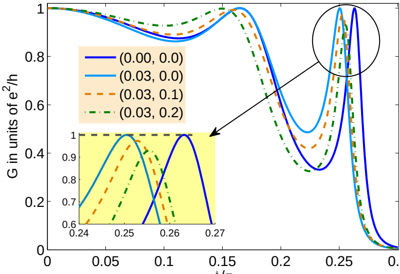
- In a good sample we expect such impurity to be weak and sparsely spaced.

  Parameters: $\xi/L = 0.05, \ell/L = 0.1$. The legend shows $(\eta, x_\ell/L)$ for different curves.



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- The gap opened up by the $\vec{B}$-field should be less than the bulk-gap and the Fermi energy should be well within the bulk-gap.
- Coherence: A realistic sample can have inelastic backscatterings (König et al-2007) on the edge which causes spin decoherence. It is essential that spin decoherence length - $l_d \gg L$.

Summary and Conclusions

- We have studied what happens to the helical edge states in presence of a magnetic field ($\vec{B}$) Zeeman-coupled to the edge states.

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- Hence this is a concrete proposal for a spin-transistor.
- In a realistic sample, the resonances also give an idea of the spin decoherence length which is important in spintronic applications.

Important References

- Review on Topological Insulators:
  X-L Qi and S-C Zhang, Rev. Mod. Phys. **83**, 1057 (2011);
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- Our work:
  A. Soori, S. Das and S. Rao, arXiv: 1112.5400.
- Some recent interesting papers:
  Delplace p., Li J., Buttiker, arXiv: 1207.2400
  Braunecker B., Störm A., Japaridze G. I., arXiv: 1206.5844.

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- Thanks to audience.