Spin relaxation in low dimensions

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Plan of lectures

Lecture I:

1. Spin relaxation mechanisms (overview)
2. Spin relaxation (qualitative model)
3. Dyakonov-Perel spin relaxation

Lecture II:

1. Spin-orbit coupling fluctuations (crossover between Dyakonov-Perel and Elliott-Yafet mechanisms)
2. Spin decoherence in quantum dots

Spin relaxation - I

1. Spin relaxation mechanisms (overview)
   - Free electrons
   - Localized electrons
2. Spin relaxation (qualitative model)
   - Spin in a fluctuating magnetic field
3. Dyakonov-Perel spin relaxation
   - Spin splittings in nanostructures
   - Spin density matrix & kinetic equation
   - Electron-electron interaction
   - Spin relaxation anisotropy
   - Features of high-mobility structures

Spin relaxation mechanisms

Localized electrons:
See next lecture

Free electrons:

- Bir-Aronov-Pikus mechanism (electron-hole exchange interaction)
- Paramagnetic scattering (in magnetic systems)
- Elliott-Yafet mechanism (spin flip at scattering)
- Dyakonov-Perel mechanism (spin precession)

Driving force is the spin-orbit coupling:
Interaction of electron spin and its momentum

Spin relaxation mechanisms

Localized electrons:

- Spin-orbit mediated electron-phonon interaction
- Hyperfine interaction (electron-nuclei coupling)
- Inhomogeneous broadening (in magnetic field: *g*-factor spread)

Orbital motion is quenched, hence spin-orbit coupling plays minor role

Spin relaxation

Most general model: **spin precession in time-dependent magnetic field**

$$\mathcal{H}(t) = \frac{\hbar}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}(t)$$

τ_c is the correlation time of $\boldsymbol{\Omega}$

$$\frac{1}{\tau_s} \sim \langle \Omega^2 \rangle \tau_c$$

Spin splittings

Dyakonov-Perel spin relaxation is caused by the electron spin precession in the effective magnetic field arising from the **spin-orbit interaction**

Lack of an inversion center:

- Bulk inversion asymmetry
- Structure inversion asymmetry
- Interface inversion asymmetry

Spin splittings - I

Lack of an inversion center:

- Bulk Inversion Asymmetry (BIA)

$$\mathcal{H}_{so} = \gamma[\sigma_x k_x(k_y^2 - k_z^2) + \sigma_y k_y(k_z^2 - k_x^2) + \sigma_z k_z(k_x^2 - k_y^2)]$$

$$\mathcal{H}_D = \gamma\langle k_z^2\rangle(\sigma_y k_y - \sigma_x k_x) \qquad z||[001].$$

- Structure Inversion Asymmetry (SIA)

$$\mathcal{H}_{so} = \alpha[\boldsymbol{\sigma} \times \boldsymbol{k}] \cdot \boldsymbol{n}$$

- Interface Inversion Asymmetry (IIA)
  like BIA in (001) quantum wells

Spin splittings - II

Effective magnetic field

$$\mathcal{H} = \alpha_{ij}\sigma_i k_j + \frac{\hbar^2 k^2}{2m} + \gamma_{iklj}\sigma_i k_j k_k k_l$$

$$\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_{so} = \frac{\hbar^2 k^2}{2m} + \frac{\hbar}{2}(\boldsymbol{\sigma} \cdot \boldsymbol{\Omega}_{\boldsymbol{k}})$$

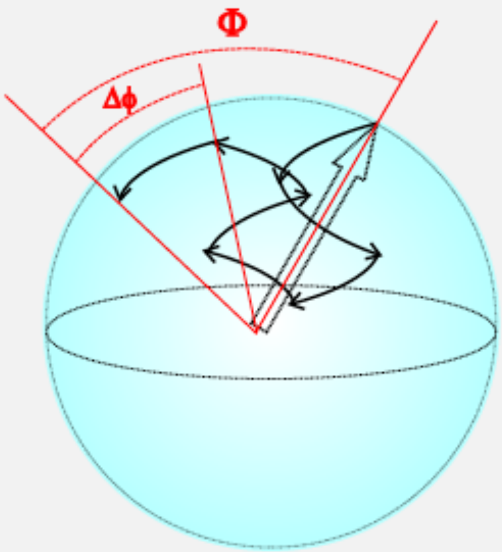
$$\Omega_{\boldsymbol{k},i} = \frac{2}{\hbar}\alpha_{ij}k_j + \frac{2}{\hbar}\gamma_{iklj}k_j k_k k_l$$

Estimate (10 nm QW):

$$\gamma\langle k_z^2\rangle \approx \gamma\frac{\pi^2}{a^2} \approx 20\text{eV}\text{Å}^3\frac{\pi^2}{(100\text{Å})^2} = 2\times10^{-2}\ \text{eVÅ}.$$

$$\gamma\langle k_z^2\rangle k_\parallel \approx 2\times10^{-2}\ \text{eVÅ}\times10^{-2}\ \text{Å}^{-1} = 0.2\ \text{meV}.$$

Spin relaxation (qualitative)



M.I. Dyakonov, V.I. Perel (1971)

Collision dominated regime:

$$\Delta\phi = \Omega\tau \ll 1$$

Spin is lost between the scattering events

$$\langle \Phi^2 \rangle = N(\Delta\phi)^2$$

Spin relaxation time $\langle \Phi^2 \rangle \sim 1 \Rightarrow N \sim (\Delta\phi)^{-2}$

$$\tau_s = N\tau \sim \frac{1}{\Omega^2\tau}$$

Kinetic equation

$$\rho_{\boldsymbol{k}} = f_{\boldsymbol{k}} I + \boldsymbol{S}_{\boldsymbol{k}} \cdot \boldsymbol{\sigma} \qquad \frac{\partial \boldsymbol{S}_{\boldsymbol{k}}}{\partial t} + \boldsymbol{S}_{\boldsymbol{k}} \times \boldsymbol{\Omega}_{\boldsymbol{k}} = \mathrm{St}\{\boldsymbol{S}\}.$$

$$\mathrm{St}\{\boldsymbol{S}\} = \sum_{\boldsymbol{k}'} [W_{\boldsymbol{k},k'} \boldsymbol{S}_{\boldsymbol{k}'} - W_{\boldsymbol{k}',\boldsymbol{k}} \boldsymbol{S}_{\boldsymbol{k}}] = -\frac{\delta \boldsymbol{S}_{\boldsymbol{k}}}{\tau}.$$

$$\frac{\partial \bar{\boldsymbol{S}}_{\boldsymbol{k}}}{\partial t} + \hat{\Gamma} \bar{\boldsymbol{S}}_{\boldsymbol{k}} = 0, \qquad \Gamma_{ij} = \left(\overline{\Omega_k^2} \delta_{ij} - \overline{\Omega_{\boldsymbol{k},i} \Omega_{\boldsymbol{k},j}} \right) \tau.$$

Example: asymmetric quantum well

$$\boldsymbol{\Omega}_{\boldsymbol{k}} = \alpha_R (k_y, -k_x, 0).$$

$$\Gamma_{zz} = \alpha_R^2 k^2 \tau, \quad \Gamma_{xx} = \alpha_R^2 \bar{k_x^2} \tau = \frac{\alpha_R^2 k^2 \tau}{2}, \quad \Gamma_{yy} = \alpha_R^2 \bar{k_y^2} \tau = \frac{\alpha_R^2 k^2 \tau}{2}.$$

$\tau \sim 1$ ps, $\hbar\Omega \sim 0.1$ meV, => $\Omega \sim 0.1\ \mathrm{ps}^{-1}$. $\quad \frac{1}{\tau_s} \sim 10^{-2}\ \mathrm{ps}^{-1}$.

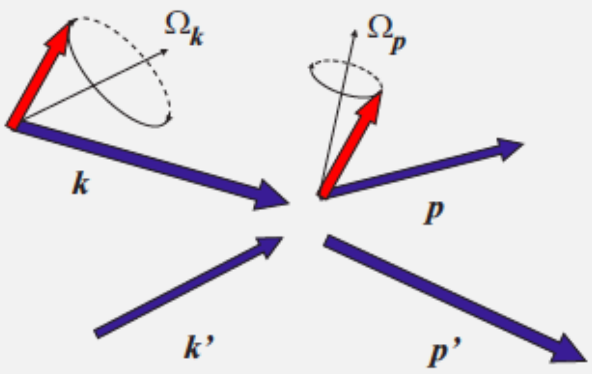
Relevant scattering mechanisms

$$\frac{1}{\tau_s} \sim \Omega^2 \tau$$

the stronger scattering $(\tau \downarrow)$ the longer spin relaxation time $(\tau_s \uparrow)$

What are the relevant scattering processes?

- elastic momentum scattering (impurities, interface roughnesses, phonons. . .)
- electron-electron collisions (although the total momentum is conserved, the **spins** of electrons are **intermixed**)



MMG, Ivchenko (2002)

M.-W. Wu (2003)

Inclusion of e-e scattering

Average spin in the $\boldsymbol{k}$ state $\boldsymbol{s_k}$, average occupation $f_{\boldsymbol{k}}$

$$\frac{d\boldsymbol{s_k}}{dt} + \boldsymbol{s_k} \times (\boldsymbol{\Omega_k} + \boldsymbol{\Omega}_{C,\boldsymbol{k}}) + \boldsymbol{Q_k}\{\boldsymbol{s}\} = 0,$$

Collision integral (Boltzmann statistics, no exchange)

$$\boldsymbol{Q_k}\{\boldsymbol{s}, f\} = \sum_{\boldsymbol{k}'\boldsymbol{p}\boldsymbol{p}'} W(\boldsymbol{k}, \boldsymbol{k}' \to \boldsymbol{p}, \boldsymbol{p}')(\boldsymbol{s_k} f_{\boldsymbol{k}'} - \boldsymbol{s_p} f_{\boldsymbol{p}'})$$

See lecture of Prof. Wu for more details

Its solution allows to obtain the electron-electron scattering rates which govern Dyakonov-Perel' spin relaxation mechanism

$$\frac{1}{\tau_{ee}} \approx 3.4 \frac{E_F}{\hbar} \left(\frac{k_B T}{E_F}\right)^2, \quad k_B T \ll E_F$$

$$\tau_s \propto \frac{1}{\langle \Omega^2 \rangle \tau_{ee}}$$

$$\frac{1}{\tau_{ee}} \approx 35.7 \frac{e^4 N_S}{\hbar \varkappa^2 k_B T}, \quad k_B T \gg E_F$$

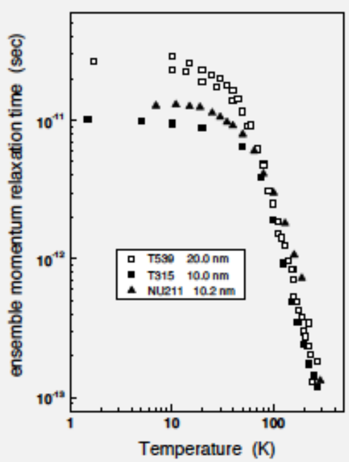
$$\tau_s \sim \begin{cases} T^2 N_S^{-2}, & T \ll E_F \\ T^{-2} N_S, & T \gg E_F \end{cases}$$

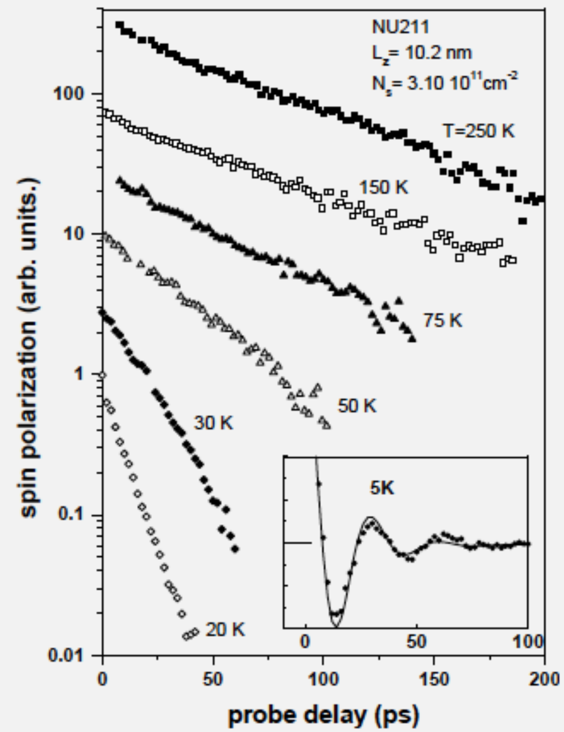
MMG, Ivchenko (2004)

Experimental proof - I

Spin polarization decay in
GaAs/AlGaAs n-type quantum wells

- Electron density
  $N \sim 3 \times 10^{11}$ cm^{-2}
- Well width $a \sim 100$ Å
- Momentum scattering time
  $\tau_p \sim 10$ ps

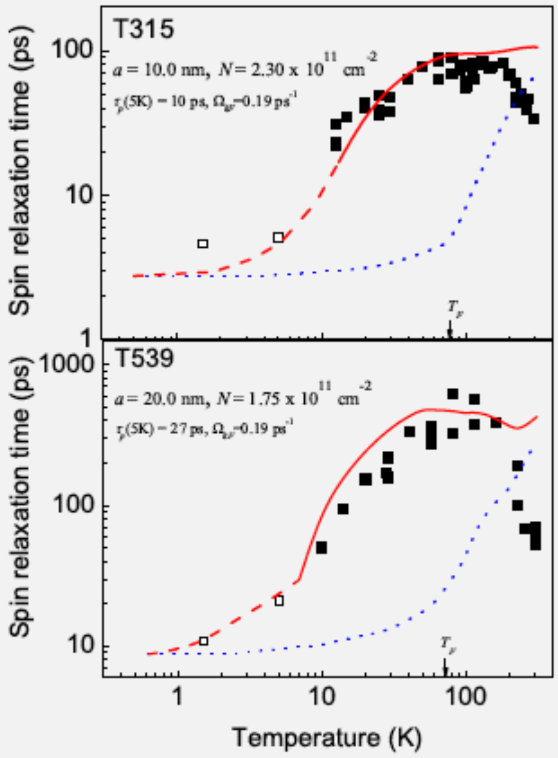




Leyland, John, Harley, MMG, Ivchenko, Ritchie, Farrer, Shields, Henini (2007)

Experimental proof - II

GaAs/AlGaAs quantum wells



Leyland, John, Harley, MMG, Ivchenko, Ritchie, Farrer, Shields, Henini (2007)

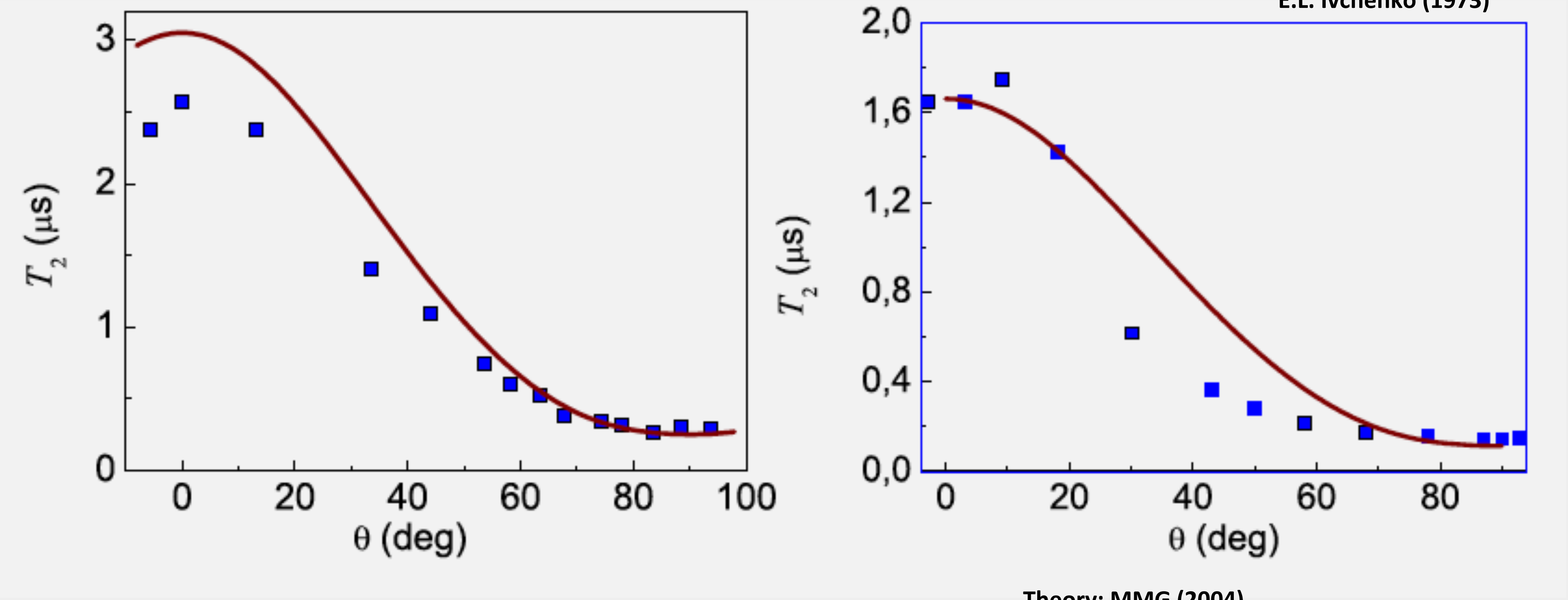
$$\frac{1}{\tau_s} \sim \Omega^2 \tau$$

- Very low T's: $\tau = \tau_p$ (remote impurities, ...)
- Degenerate gas (ee-scattering)
  $\tau = \tau_{ee} \propto \frac{\hbar}{E_F} \left(\frac{E_F}{k_B T}\right)^2$
- Slightly non-degenerate gas (ee-scattering) $\tau \propto k_B T$
- High T's $\tau = \tau_p$ (optical phonons)

Features of DP mechanism

- Spin relaxation is slowed down by scattering
- Spin relaxation is quenched by magnetic field

E.L. Ivchenko (1973)



Theory: MMG (2004)
Experiment: Wilamowski, Jantch (2004)

Spin dephasing in (ultra) high mobility structures

Spin precesses around $\boldsymbol{\Omega_k}$: spin makes $\geqslant 1$ revolutions between scattering

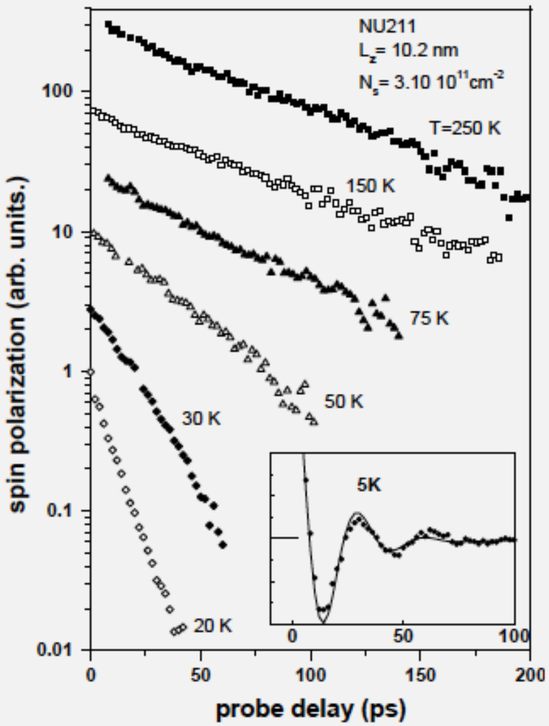
$$s_z(t) \propto \left[\cosh\left(\frac{qt}{2\tau}\right) + \frac{1}{q}\sinh\left(\frac{qt}{2\tau}\right)\right] \mathrm{e}^{-\frac{t}{2\tau}}$$

$$q = \sqrt{1 - 4\Omega_{k_F}^2 \tau^2}$$

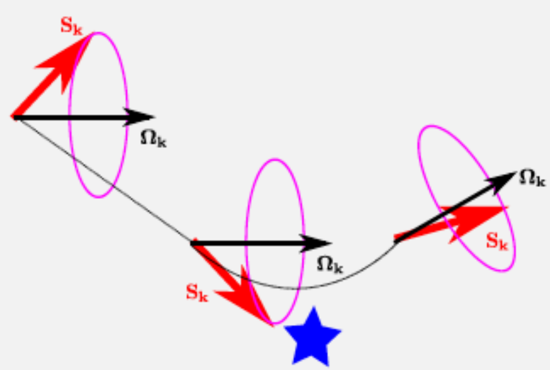
MMG (2007)

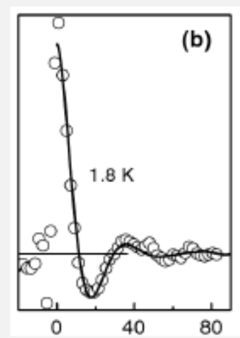
$$\propto e^{-t/2\tau} \cos \Omega_{k_F} t$$

Gridnev (2001)



W. Leyland et. al. (2007)





M. A. Brand et. al. (2002)

Thank you for attention

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   – Free electrons
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3. Dyakonov-Perel spin relaxation
   – Spin splittings in nanostructures
   – Spin density matrix & kinetic equation
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   – Spin relaxation anisotropy
   – Features of high-mobility structures

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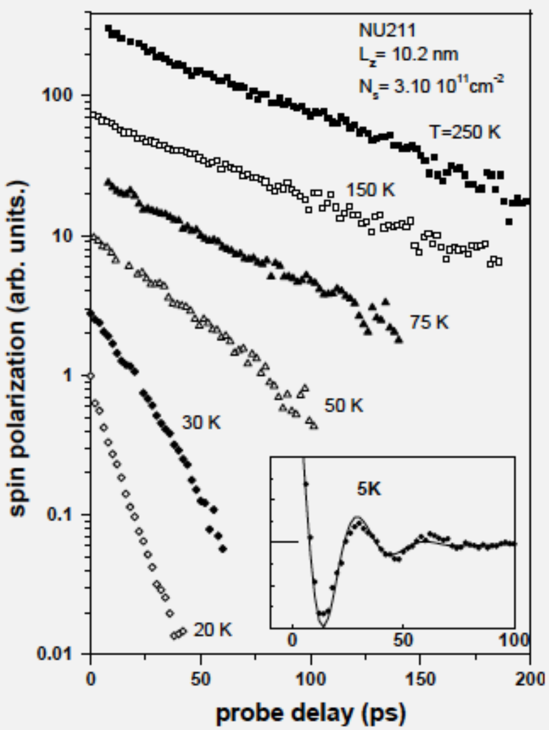

W. Leyland et. al. (2007)

$$s_z(t) \propto \left[\cosh\left(\frac{qt}{2\tau}\right) + \frac{1}{q}\sinh\left(\frac{qt}{2\tau}\right)\right] e^{-\frac{t}{2\tau}}$$

$$q = \sqrt{1 - 4\Omega_{k_F}^2 \tau^2}$$

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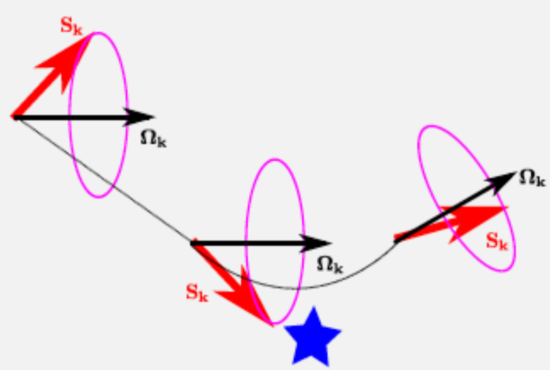

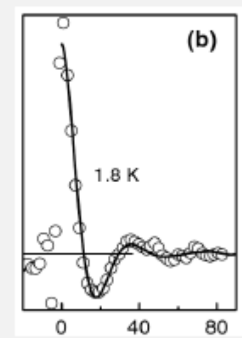

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Spin relaxation - II

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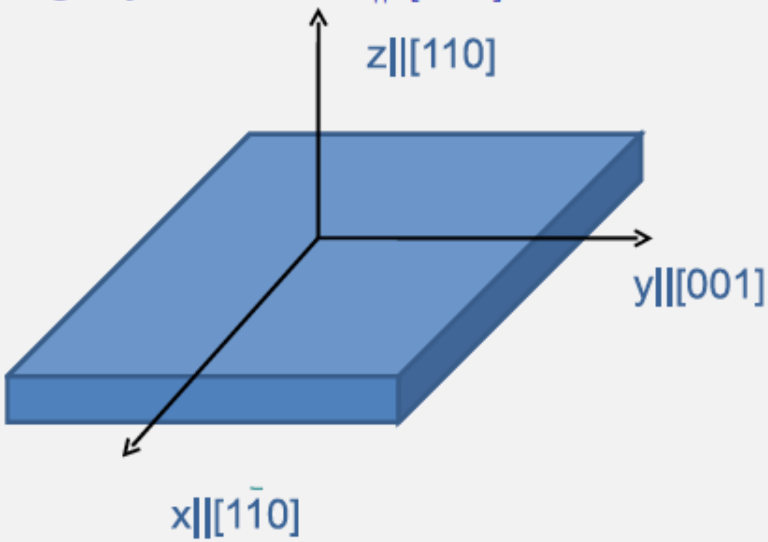
   – Classical model of spin relaxation
   – Effects of magnetic field
   – Spin relaxation in quantum wires
2. Spin decoherence in quantum dots (localized electrons)
   – Spread of electron *g*-factors
   – Nuclei-induced spin dephasing

Spin-orbit coupling fluctuations

E.g. symmetric $z \parallel [110]$ QW



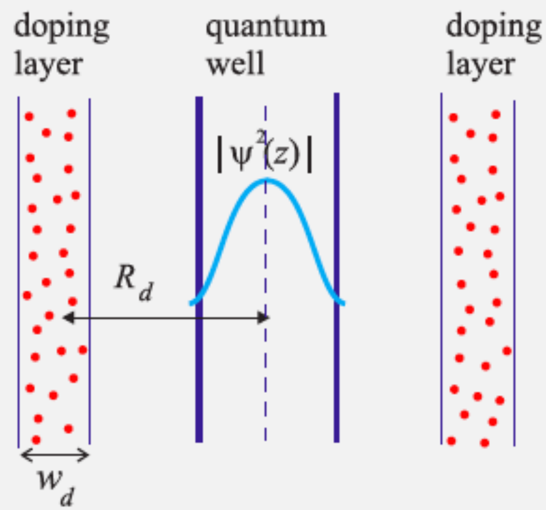
Dresselhaus term

$$\boldsymbol{\Omega}_{\mathrm{D}} \propto (0, 0, k_x) \parallel z$$

does not affect spin z component

What about Rashba term?

Nominally symmetric structure



Symmetry is locally broken:

$$\boldsymbol{\Omega}_{\mathrm{R}} = \frac{2\alpha_{\mathrm{R}}(\boldsymbol{\rho})}{\hbar}(k_y, -k_x, 0)$$

$$\alpha_{\mathrm{R}}(\boldsymbol{\rho}) = \xi E_z(\boldsymbol{\rho}), E_z(\rho) = \int \frac{z n(\boldsymbol{r}) \mathrm{d}\boldsymbol{r}}{|\boldsymbol{\rho} - \boldsymbol{r}|^3}$$

Spin relaxation for ballistic electrons

Random spin-orbit coupling

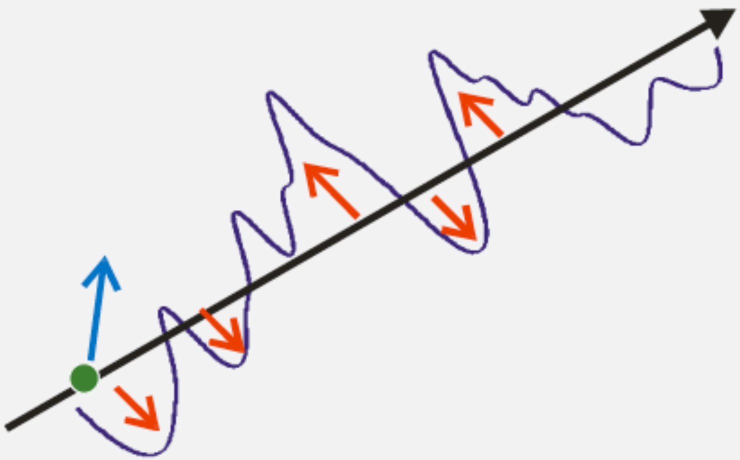
$$\mathbf{\Omega}_{\rm R} = \frac{2\alpha_{\rm R}(\boldsymbol{\rho})}{\hbar}(k_y, -k_x, 0)$$

$\langle \alpha \rangle = 0$: no spin-orbit coupling on average

Fluctuations: $\langle \alpha^2 \rangle = \xi^2 \langle E_z^2 \rangle$

Spin relaxation rate

$$\frac{1}{\tau_s} \sim \langle \Omega_{\rm R}^2 \rangle \tau_d$$



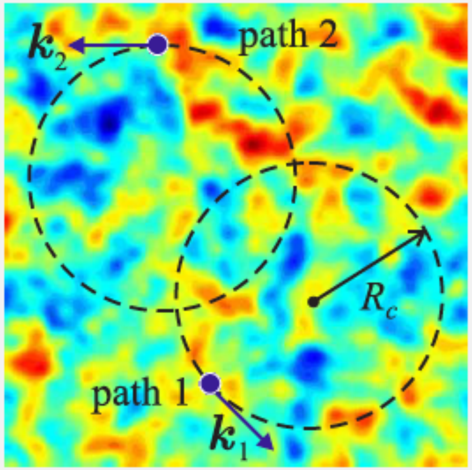
$\tau_d = l_d/v$ is the domain passage time

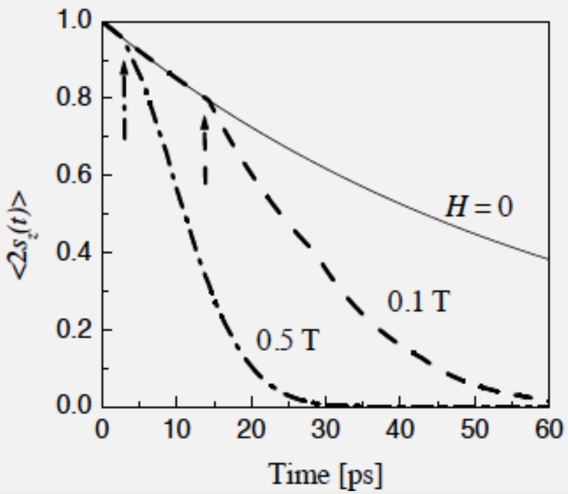
Momentum scattering is not needed!

$$\frac{s_z(t)}{s_z(0)} = \exp\left[-\int_0^t dt_1 \int_0^{t_1} dt_2 C_{\Omega\Omega}(t_1 - t_2) \right]$$

MMG, Sherman (2005)

Magnetic field effect





Electron passes the same configuration of the disorder:

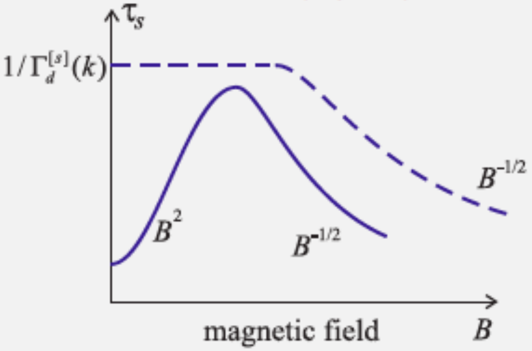
spin relaxation speeds up

$$\frac{s_z(t)}{s_z(0)} = \exp\left[-\frac{4\langle\alpha_R^2\rangle k^2}{\hbar^2}\frac{\tau_d t^2}{T_c}\right]$$

cyclotron period $T_c = 2\pi/\omega_c$

Classically strong magnetic fields:

$E_F \gg \omega_c\tau \gg (\tau/\tau_d)^{2/3}$

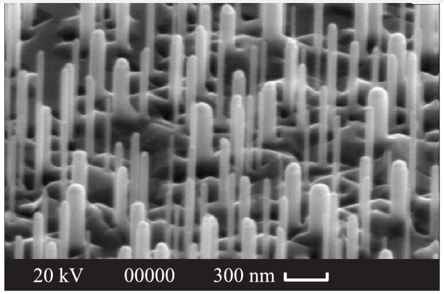


MMG, Sherman (2005)

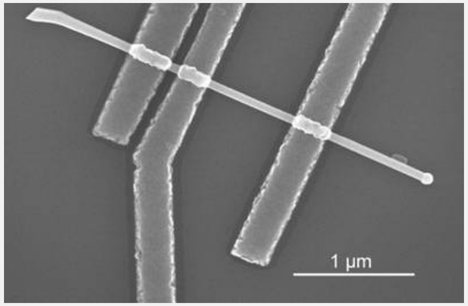
Spin dephasing in nanowires

– demonstrate 1D physics, between localized (0D) and free (2D, 3D)

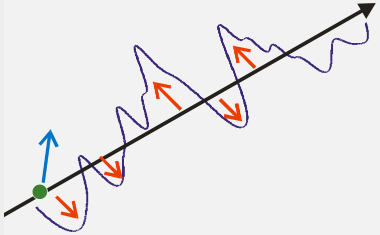
$$\mathcal{H}_{\text{so}} = \frac{1}{2}[\alpha(x)k_x + k_x\alpha(x)]\sigma_\lambda \quad \Rightarrow \quad \boldsymbol{\Omega}_{k_x} \parallel \boldsymbol{\lambda}$$



Dubrovskii, Cirlin, Ustinov



http://phys.org/news5043.html



Spin rotation angle is determined by the electron displacement

$$\mathcal{C}_{ss}(t) = \int_{-\infty}^{\infty} \mathrm{d}x \; p(x,t) \cos\left(\frac{2m\alpha_0}{\hbar^2}x\right) \exp\left[-\langle\theta_{\mathrm{r}}^2(x)\rangle/2\right].$$

Spin relaxation - II

1. Spin orbit coupling fluctuations (crossover between DP and EY mechanisms)
   - Classical model of spin relaxation
   - Effects of magnetic field
   - Spin relaxation in quantum wires
2. Spin decoherence in quantum dots (localized electrons)
   - Spread of electron *g*-factors
   - Nuclei-induced spin dephasing

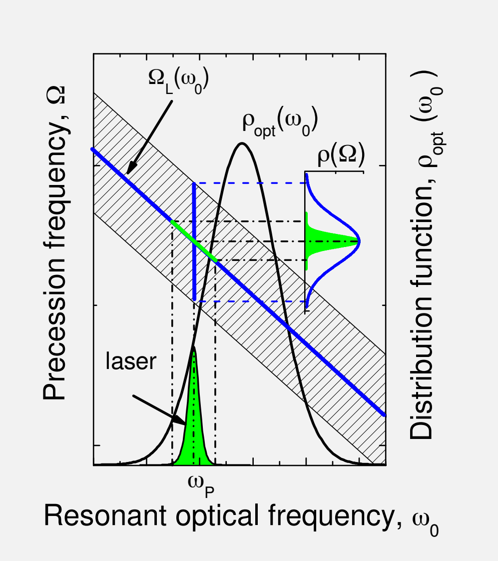
Spin relaxation mechanisms

Localized electrons:

- Spin-orbit mediated electron-phonon interaction
- Hyperfine interaction (electron-nuclei coupling)
- Inhomogeneous broadening (in magnetic field: *g*-factor spread)

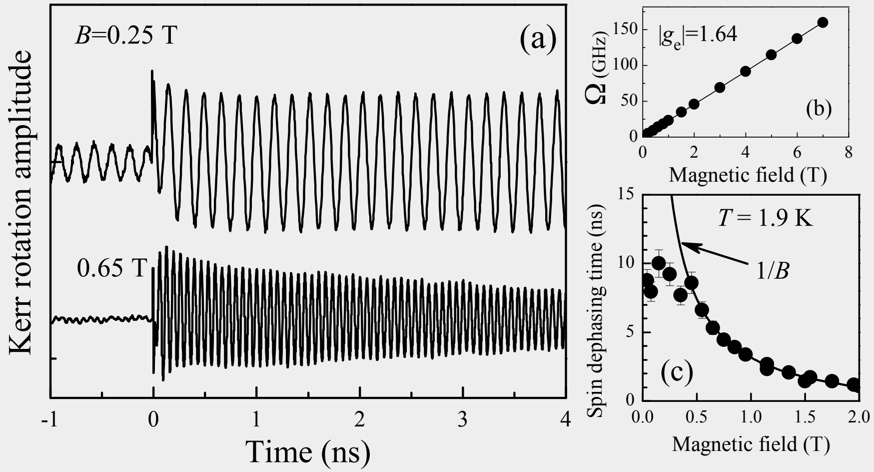
Orbital motion is quenched, hence spin-orbit coupling plays minor role

Spread of Larmor frequencies



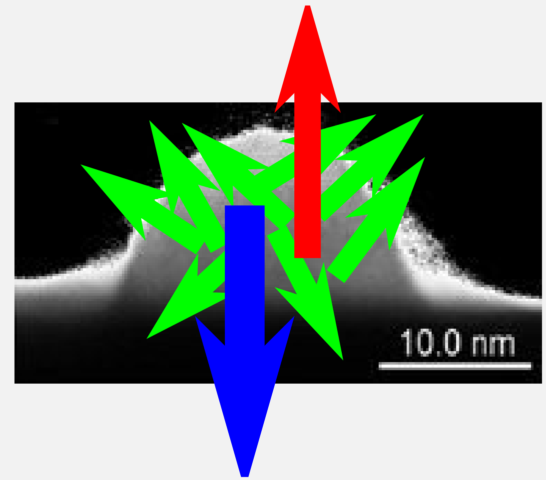
$$g \approx 2 - \frac{4|p_{cv}|^2}{3m_0} \frac{\Delta}{E_g^{QD}(E_g^{QD} + \Delta)}$$

$$T_{\rm inh} \sim \frac{1}{\Delta\Omega} \sim \frac{1}{B}$$



Hyperfine interaction

Element	^{27}Al	$^{69(71)}$Ga	^{75}As	^{115}In
Z	13	31	33	49
I	$5/2$	$3/2$	$3/2$	$9/2$



$$\boldsymbol{A} = \frac{\boldsymbol{\mu} \times \boldsymbol{r}}{r^3} = \mathrm{rot}\,\frac{\boldsymbol{\mu}}{r}, \qquad \boldsymbol{\mu} = \frac{\mu_I \boldsymbol{I}}{I}$$

$$\mathcal{H} = \frac{1}{2m_0}\left(\boldsymbol{p} - \frac{e}{c}\boldsymbol{A}\right)^2 + \frac{1}{2} g_0 \mu_B \boldsymbol{\sigma} \cdot \mathrm{rot}\,\boldsymbol{A}$$

$$V = \frac{2\mu_B \mu_I}{I}\boldsymbol{I}\left[\frac{\boldsymbol{L}}{r^3} - \frac{\boldsymbol{s}}{r^3} + 3\frac{\boldsymbol{r}(\boldsymbol{r}\cdot\boldsymbol{s})}{r^5} + \frac{8}{3}\boldsymbol{s}\pi\delta(\boldsymbol{r})\right]$$

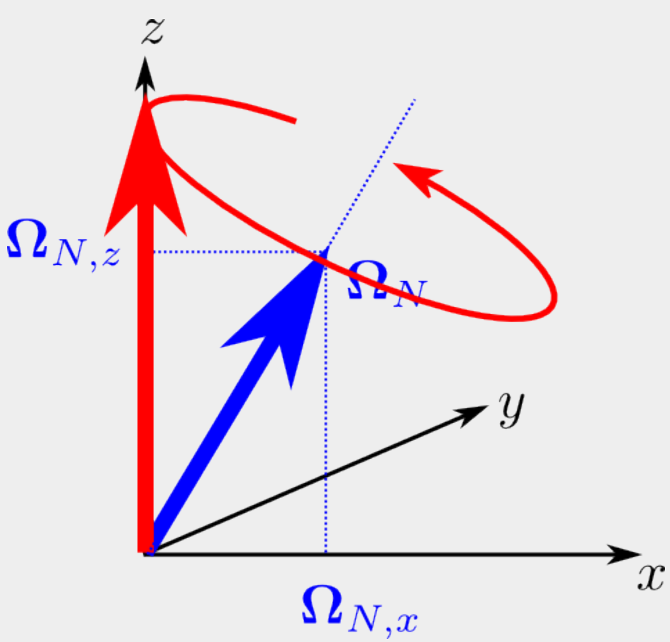
$$\tilde{V} = A v_0 (\boldsymbol{I}\cdot\boldsymbol{s})|\Psi(\boldsymbol{R}_i)|^2$$

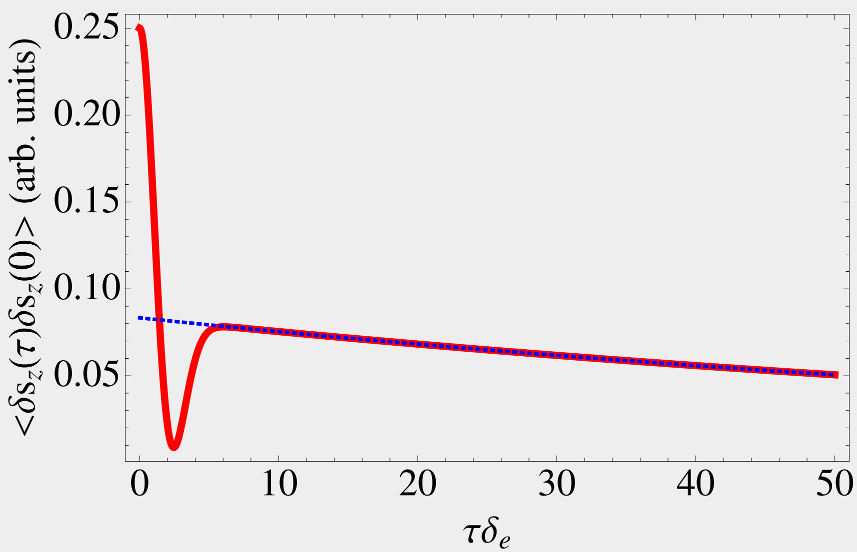
$$A \sim 10^2 \;\; \mu\mathrm{eV}$$

Nuclear fluctuations in quantum dots

$$\mathcal{F}(\mathbf{\Omega}_N) = \frac{1}{(\sqrt{\pi}\delta_e)^3} \exp\left(-\frac{\Omega_N^2}{\delta_e^2}\right)$$

$$S_z(t) = \frac{S_z(0)}{3}\left[1 + (2 - \delta_e^2 t^2)\exp\left(-\frac{t^2\delta_e^2}{4}\right)\right]$$





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