Spin dynamics of cold exciton condensates

Oleksandr Kyriienko*, E. B. Magnusson, and Ivan A. Shelykh

Science Institute, University of Iceland

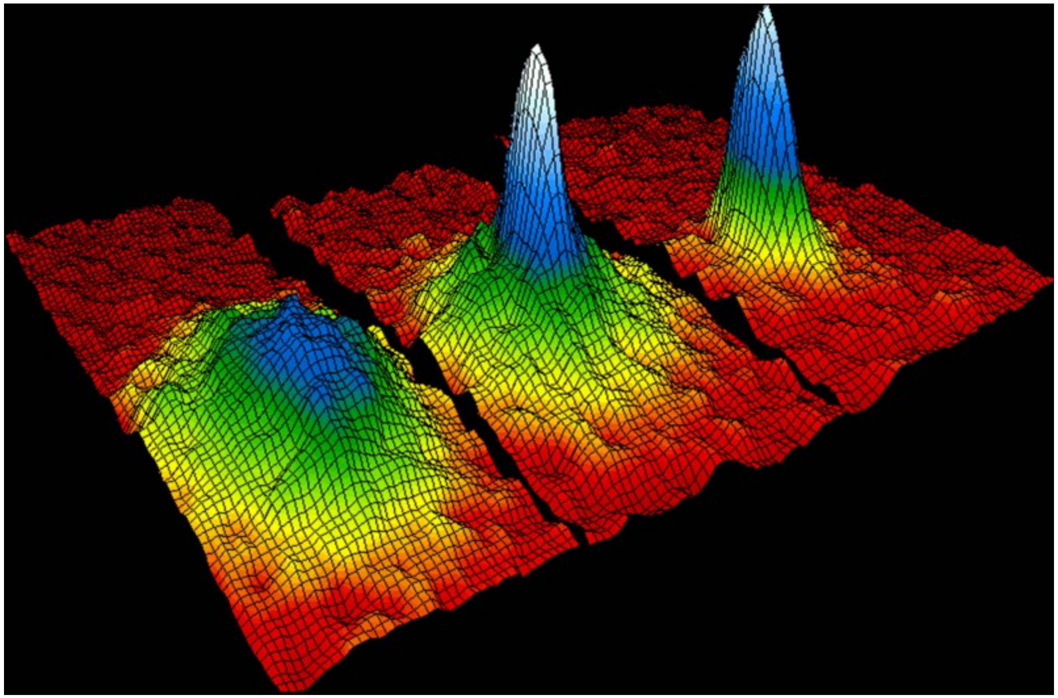
Division of Physics and Applied Physics, Nanyang Technological University, Singapore

HÁSKÓLI ÍSLANDS

Outline

- Introduction
- Model
- Elementary excitations
- Dynamics
- Conclusions

Introduction



$$T_c \approx \frac{3.3\hbar}{mk_B} n$$

Bose-Einstein condensate

[M. H. Anderson et al., Science 269, 198 (1995)]

Introduction

- helium-4
- alkali metal atoms (Li, Na, K, Rb, Cs)
- alkaline earth metal atoms (Ca, Be, Mn, Sr etc)

- direct excitons
- exciton-polaritons
- magnons
- quantum Hall bilayer excitons
- indirect excitons

Introduction

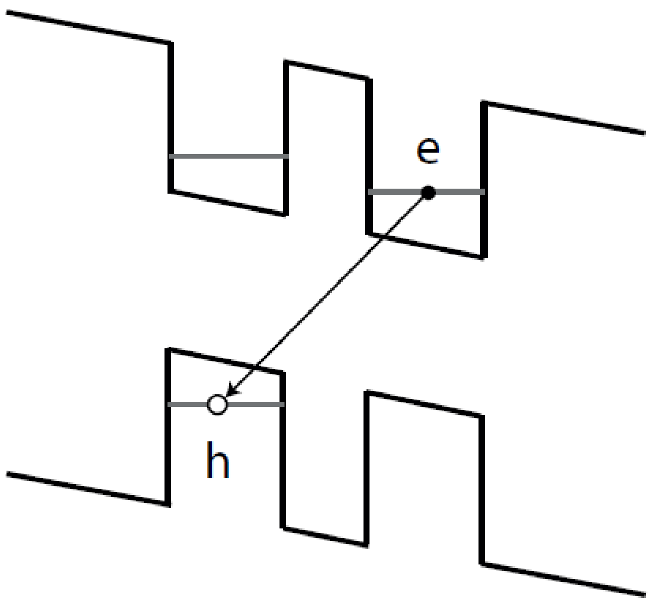
What is indirect exciton?

"Spatially indirect exciton is a bound state of an electron and a hole localized in coupled parallel 2D layers".

Theoretical prediction: **[L. V. Keldysh and Yu. V. Kopaev, Sov. Phys. Solid State 6, 2219 (1965)]**

Experimental obervation of spontaneous coherence:
[L. V. Butov et al., Phys. Rev. Lett. 86, 5608 (2001)]

Introduction



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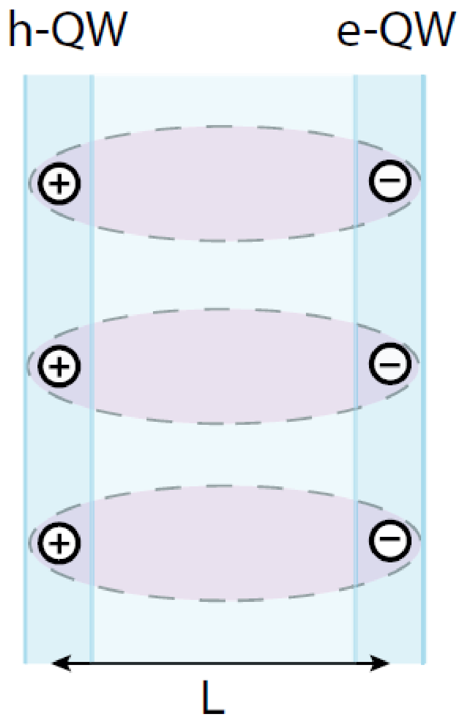
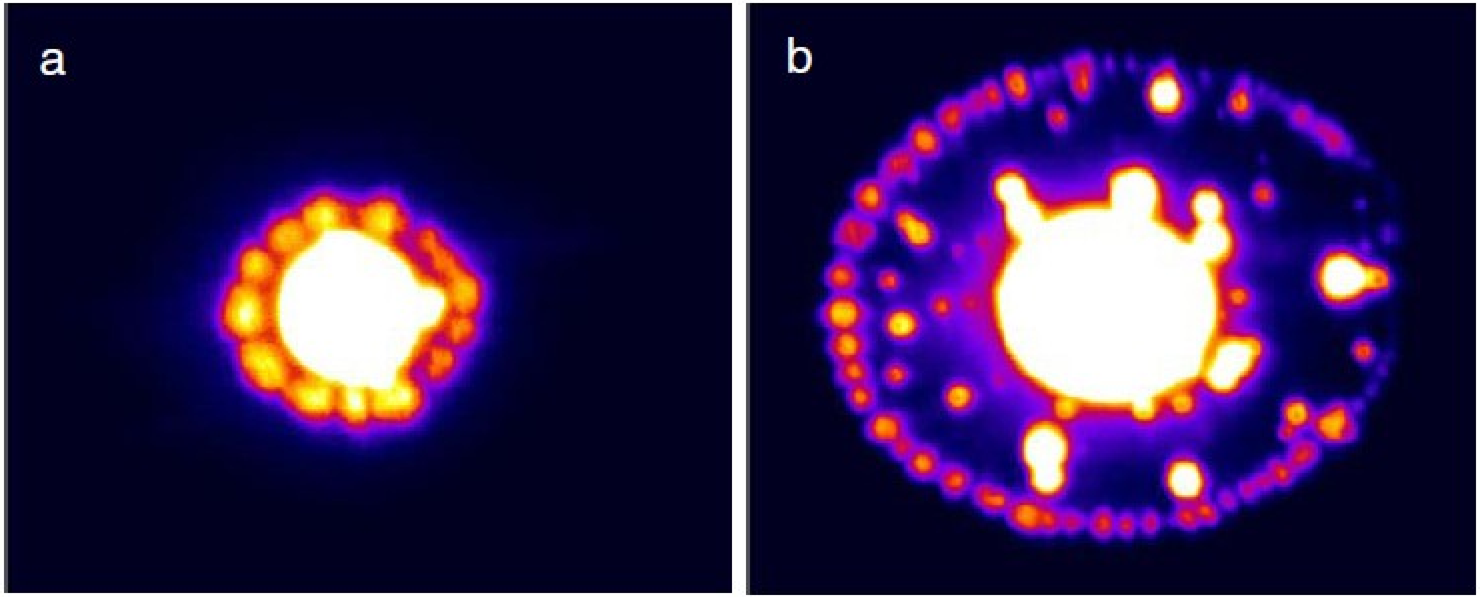


(b)

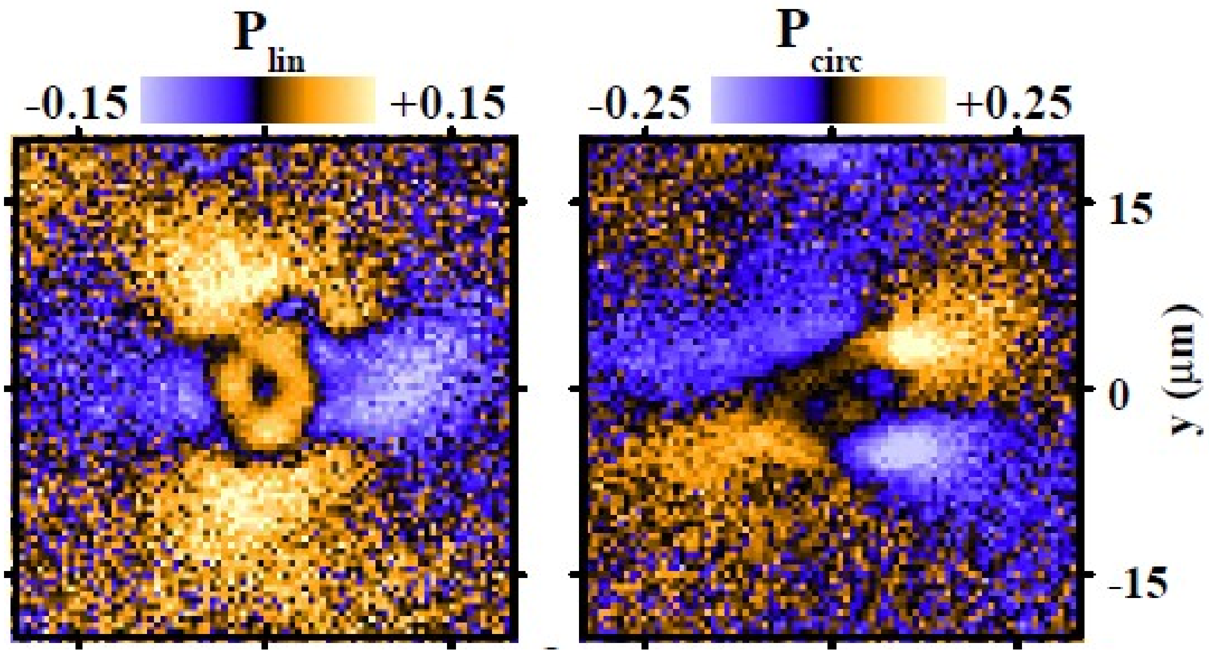
Introduction



pattern formation in cold exciton condensate

[L. V. Butov, J. Phys.: Condens. Matter 16 (2004) R1577]

Introduction



polarization patterns

[A. A. High et al., Nature 483, 584 (2012); arXiv:1103.0321]

Model

0. $\Psi(\mathbf{r},t) = (\Psi_{+2}(\mathbf{r},t), \Psi_{+1}(\mathbf{r},t), \Psi_{-1}(\mathbf{r},t), \Psi_{-2}(\mathbf{r},t))$

account for both bright and dark states

$H = H_0 + H_{int}$ Hamiltonian density

1. H_0 – single particle part including SOI

2. H_{int} – interaction part

3. $i\hbar\partial_t\Psi_\sigma = \frac{\delta H}{\delta \Psi_\sigma^*}$ mean field Gross-Pitaevskii equation

Model

single particle Hamiltonian: $H_0 = \Psi^\dagger(\mathbf{r},t)\hat{\mathbf{T}}\Psi(\mathbf{r},t)$

where $$\hat{\mathbf{T}} = \begin{pmatrix} \hat{\mathbf{T}}_{12} & 0 \\ 0 & \hat{\mathbf{T}}_{12} \end{pmatrix}$$

and $$\hat{\mathbf{T}}^K_{12} = \begin{pmatrix} \hbar^2K^2/2M & \hat{S}_K \\ \hat{S}^*_K & \hbar^2K^2/2M \end{pmatrix}$$

SOI

SOI

Model

spin-orbit interaction (SOI):

Rashba SOI

$$H_R = \alpha \left(\sigma_x \hat{k}_y - \sigma_y \hat{k}_x \right)$$

Dresselhaus SOI

$$H_D = \beta \left(\sigma_x \hat{k}_x - \sigma_y \hat{k}_y \right)$$

Model

Derivation of single particle of Hamiltonian for indirect exciton:

$$\hat{\mathbf{T}} = \begin{pmatrix} \hat{\mathbf{T}}_0 & 0 \\ 0 & \hat{\mathbf{T}}_0 \end{pmatrix}$$ acts on two particle wave function

$$\Psi = (\Psi_{++}, \Psi_{-+}, \Psi_{+-}, \Psi_{--})$$ and

$$\hat{\mathbf{T}}_0 = \begin{pmatrix} -\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 & \alpha(-\partial_x^e + i\partial_y^e) \\ \alpha(\partial_x^e + i\partial_y^e) & -\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 \end{pmatrix}$$

Model

going to CM frame, one obtains

$$\hat{\mathbf{T}}_0 = \begin{pmatrix} -\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{2\mu}\nabla^2_{\mathbf{r}} & \alpha(-\chi\partial_X - \partial_x + i\chi\partial_Y + i\partial_y) \\ \alpha(\chi\partial_X + \partial_x + i\chi\partial_Y + i\partial_y) & -\frac{\hbar^2}{2M}\nabla^2_{\mathbf{R}} - \frac{\hbar^2}{2\mu}\nabla^2_{\mathbf{r}} \end{pmatrix}$$

We are interested in CM motion and neglect electron-hole relative motion.

Model

$$\hat{S}_K = \chi \left[\beta(\hat{K}_X + i\hat{K}_Y) + \alpha(\hat{K}_Y + i\hat{K}_X) \right]$$

off-diagonal spin-flip operator with $\hat{K}_X = -i\partial_X, \hat{K}_Y = -i\partial_Y$

These terms lead to effective bright-to-dark exciton conversion.

$$\pm 1 \rightarrow \pm 2$$

Model

0. $\Psi(\mathbf{r},t) = (\Psi_{+2}(\mathbf{r},t), \Psi_{+1}(\mathbf{r},t), \Psi_{-1}(\mathbf{r},t), \Psi_{-2}(\mathbf{r},t))$

account for both bright and dark states

$H = H_0 + H_{int}$ Hamiltonian density

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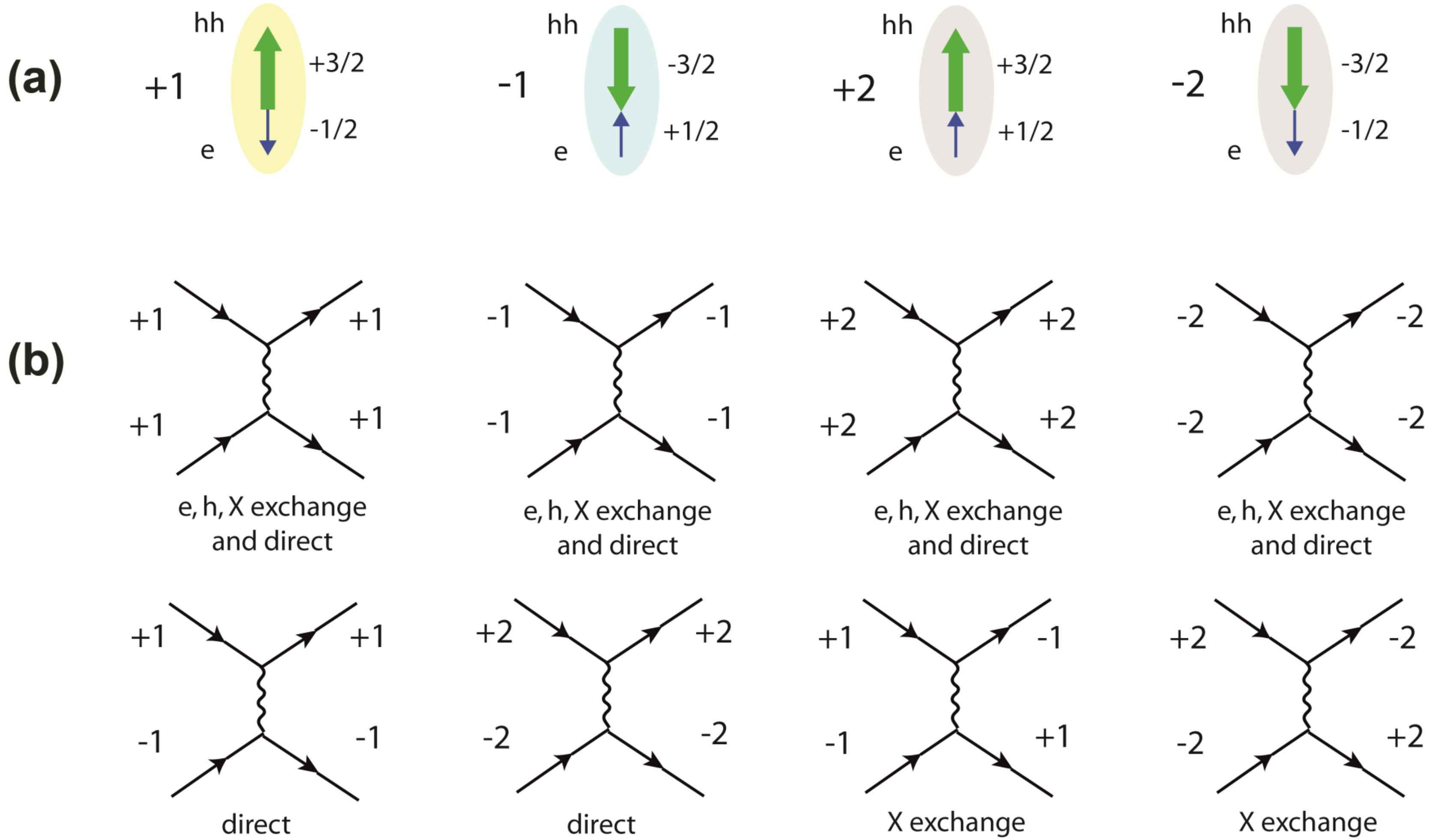
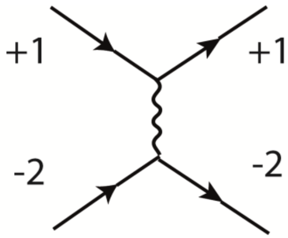
Model

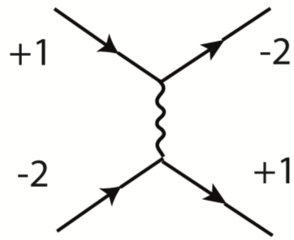


interactions

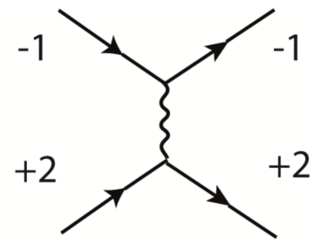
Model



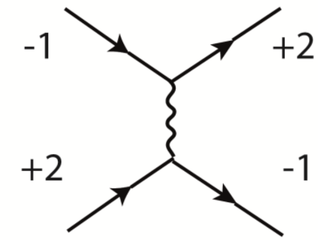
e exchange and direct



h exchange and X exchange



e exchange and direct



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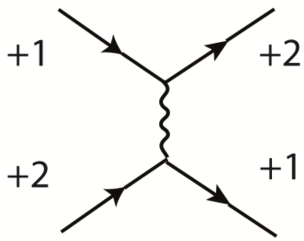
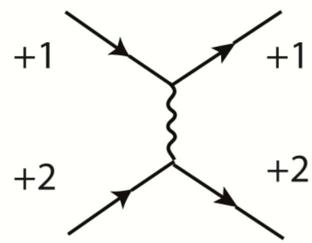
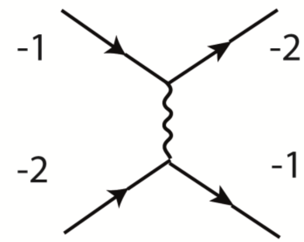


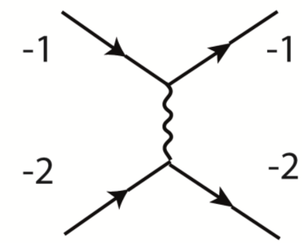
e exchange and X exchange



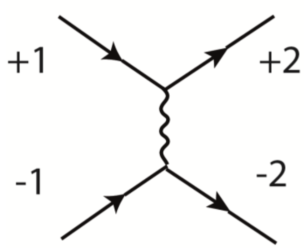
h exchange and direct



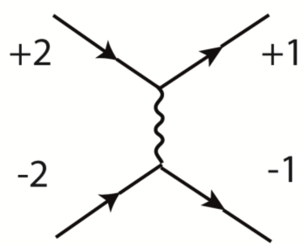
e exchange and X exchange



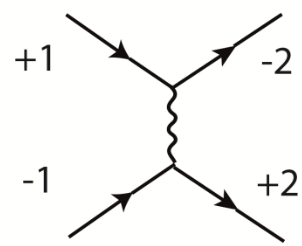
h exchange and direct



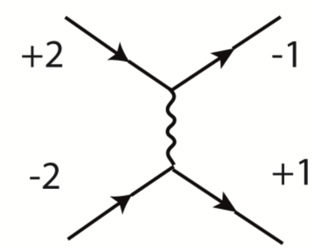
e exchange



e exchange



h exchange



h exchange

Model

interaction Hamiltonian:

$$H_{int} = \frac{V_{dir} + V_X + V_e + V_h}{2} \sum_{\sigma=\pm1\pm2} |\Psi_\sigma|^4 + (V_{dir} + V_X)(|\Psi_{+1}|^2|\Psi_{-1}|^2 + |\Psi_{+2}|^2|\Psi_{-2}|^2)+$$

$$+ (V_{dir} + V_X + V_e + V_h)(|\Psi_{+1}|^2|\Psi_{+2}|^2 + |\Psi_{-1}|^2|\Psi_{-2}|^2 + |\Psi_{+1}|^2|\Psi_{-2}|^2 + |\Psi_{-1}|^2|\Psi_{+2}|^2)+$$

$$+ \boxed{(V_e + V_h)(\Psi^*_{+1}\Psi^*_{-1}\Psi_{+2}\Psi_{-2} + \Psi^*_{+2}\Psi^*_{-2}\Psi_{+1}\Psi_{-1})}$$

spin-conversion

Model

matrix elements of exciton-exciton interaction

$$V_{dir}(\mathbf{Q},\mathbf{Q}',\mathbf{q}) = \int d^2\mathbf{r}_\mathrm{e} d^2\mathbf{r}_\mathrm{h} d^2\mathbf{r}_{\mathrm{e}'} d^2\mathbf{r}_{\mathrm{h}'} \Psi^*_\mathrm{Q}(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h})\Psi^*_{\mathrm{Q}'}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'}) V_I(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h},\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'})\Psi_{\mathrm{Q+q}}(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h})\Psi_{\mathrm{Q}'-\mathrm{q}}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'})$$

$$V_X^{exch}(\mathbf{Q},\mathbf{Q}',\mathbf{q}) = \int d^2\mathbf{r}_\mathrm{e} d^2\mathbf{r}_\mathrm{h} d^2\mathbf{r}_{\mathrm{e}'} d^2\mathbf{r}_{\mathrm{h}'} \Psi^*_\mathrm{Q}(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h})\Psi^*_{\mathrm{Q}'}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'}) V_I(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h},\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'})\Psi_{\mathrm{Q+q}}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'})\Psi_{\mathrm{Q}'-\mathrm{q}}(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h})$$

$$V_e^{exch}(\mathbf{Q},\mathbf{Q}',\mathbf{q}) = -\int d^2\mathbf{r}_\mathrm{e} d^2\mathbf{r}_\mathrm{h} d^2\mathbf{r}_{\mathrm{e}'} d^2\mathbf{r}_{\mathrm{h}'} \Psi^*_\mathrm{Q}(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h})\Psi^*_{\mathrm{Q}'}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'}) V_I(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h},\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'})\Psi_{\mathrm{Q+q}}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_\mathrm{h})\Psi_{\mathrm{Q}'-\mathrm{q}}(\mathbf{r}_\mathrm{e},\mathbf{r}_{\mathrm{h}'})$$

$$V_h^{exch}(\mathbf{Q},\mathbf{Q}',\mathbf{q}) = -\int d^2\mathbf{r}_\mathrm{e} d^2\mathbf{r}_\mathrm{h} d^2\mathbf{r}_{\mathrm{e}'} d^2\mathbf{r}_{\mathrm{h}'} \Psi^*_\mathrm{Q}(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h})\Psi^*_{\mathrm{Q}'}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'}) V_I(\mathbf{r}_\mathrm{e},\mathbf{r}_\mathrm{h},\mathbf{r}_{\mathrm{e}'},\mathbf{r}_{\mathrm{h}'})\Psi_{\mathrm{Q+q}}(\mathbf{r}_\mathrm{e},\mathbf{r}_{\mathrm{h}'})\Psi_{\mathrm{Q}'-\mathrm{q}}(\mathbf{r}_{\mathrm{e}'},\mathbf{r}_\mathrm{h})$$

Model

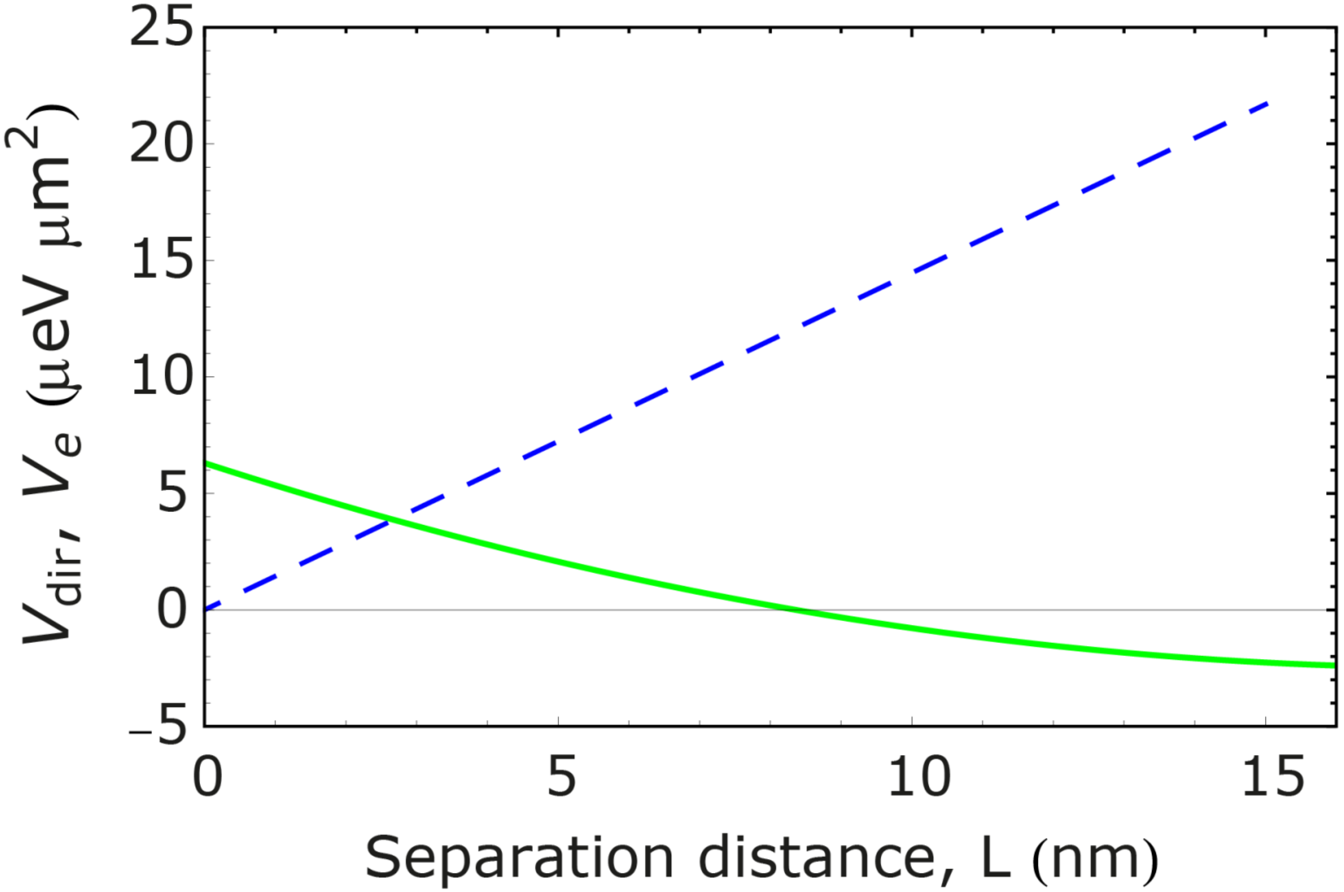
$$V_{dir} = \frac{e^2 L}{\epsilon\epsilon_0}$$

– direct and exciton exchange in q=0 limit

$$V_e^{exch} = -\frac{e^2}{4\pi\epsilon\epsilon_0 A}\left(\frac{2}{\pi}\right)^2 a_B \cdot \mathcal{I}_e^{exch}(\Delta Q, q, \Theta, \beta_e)$$

– electron and hole exchange

Model



V_{dir}, V_e (μeV μm^2)
25
20
15
10
5
0
−5
0
5
10
15
Separation distance, L (nm)

Model

0. $\Psi(\mathbf{r},t) = (\Psi_{+2}(\mathbf{r},t), \Psi_{+1}(\mathbf{r},t), \Psi_{-1}(\mathbf{r},t), \Psi_{-2}(\mathbf{r},t))$

account for both bright and dark states

$H = H_0 + H_{int}$ Hamiltonian density

1. H_0 – single particle part including SOI

2. H_{int} – interaction part

3. $i\hbar\partial_t\Psi_\sigma = \frac{\delta H}{\delta \Psi_\sigma^*}$ mean field Gross-Pitaevskii equation

Model

set of Gross-Pitaevskii equations

$$
\begin{cases}
i\hbar\frac{\partial\Psi_{+1}}{\partial t} = \hat{E}\Psi_{+1} - \hat{S}^*_{12}\Psi_{+2} + V_0\Psi_{+1}|\Psi_{+1}|^2 + (V_0 - W)\Psi_{+1}|\Psi_{-1}|^2 + V_0\Psi_{+1}(|\Psi_{-2}|^2 + |\Psi_{+2}|^2) + W\Psi^*_{-1}\Psi_{+2}\Psi_{-2} \\
i\hbar\frac{\partial\Psi_{-1}}{\partial t} = \hat{E}\Psi_{-1} + \hat{S}_{12}\Psi_{-2} + V_0\Psi_{-1}|\Psi_{-1}|^2 + (V_0 - W)\Psi_{-1}|\Psi_{+1}|^2 + V_0\Psi_{-1}(|\Psi_{+2}|^2 + |\Psi_{-2}|^2) + W\Psi^*_{+1}\Psi_{+2}\Psi_{-2} \\
i\hbar\frac{\partial\Psi_{+2}}{\partial t} = \hat{E}\Psi_{+2} + \hat{S}_{12}\Psi_{+1} + V_0\Psi_{+2}|\Psi_{+2}|^2 + (V_0 - W)\Psi_{+2}|\Psi_{-2}|^2 + V_0\Psi_{+2}(|\Psi_{-1}|^2 + |\Psi_{+1}|^2) + W\Psi^*_{-2}\Psi_{+1}\Psi_{-1} \\
i\hbar\frac{\partial\Psi_{-2}}{\partial t} = \hat{E}\Psi_{-2} - \hat{S}^*_{12}\Psi_{-1} + V_0\Psi_{-2}|\Psi_{-2}|^2 + (V_0 - W)\Psi_{-2}|\Psi_{+2}|^2 + V_0\Psi_{-2}(|\Psi_{+1}|^2 + |\Psi_{-1}|^2) + W\Psi^*_{+2}\Psi_{+1}\Psi_{-1}
\end{cases}
$$

where $V_0 = V_{dir} + V_X + V_e + V_h$ and $W = V_e + V_h$

Elementary excitations

1. Minimize free energy to find a ground state

$$F(\Psi_{+1}, \Psi_{-1}, \Psi_{+2}, \Psi_{-2}, \mu) = H - \mu f(\Psi_{+1}, \Psi_{-1}, \Psi_{+2}, \Psi_{-2}),$$

$$f(\Psi_{+1}, \Psi_{-1}, \Psi_{+2}, \Psi_{-2}) = |\Psi_{+1}|^2 + |\Psi_{-1}|^2 + |\Psi_{+2}|^2 + |\Psi_{-2}|^2 = n$$

μ denotes chemical potential

Firstly, we disregard SOI terms and define ground state from interaction Hamiltonian.

Elementary excitations

4 different configurations:

(i) $|\Psi^0{}_{+1}| = \sqrt{n},\ \Psi^0{}_{-1,\pm 2} = 0$ or $|\Psi^0{}_{-1}| = \sqrt{n},\ \Psi^0{}_{+1,\pm 2} = 0$ or

$|\Psi^0{}_{+2}| = \sqrt{n},\ \Psi^0{}_{-2,\pm 1} = 0$ or $|\Psi^0{}_{-2}| = \sqrt{n},\ \Psi^0{}_{+2,\pm 1} = 0$

– one-component condensate

(ii) $|\Psi^0{}_{+1,-1}| = \sqrt{n/2},\ \Psi^0{}_{\pm 2} = 0$ or $|\Psi^0{}_{+2,-2}| = \sqrt{n/2},\ \Psi^0{}_{\pm 1} = 0$

– "ii" two-component condensate

Elementary excitations

(iii) $|\Psi^0_{+1,-2}| = \sqrt{n/2},\ \Psi^0_{-1,+2} = 0$ or $|\Psi^0_{-1,+2}| = \sqrt{n/2},\ \Psi^0_{+1,-2} = 0$

– "ij" two-component condensate

(iv) $|\Psi^0_{\pm 1,\pm 2}| = \sqrt{n/4}$ **– four-component condensate**

Elementary excitations

First, consider negative exchange interaction (large QW separations)

$$W = V_e + V_h < 0$$

$$\begin{cases} H^{(1)} = (V_{dir} + V_X + V_e + V_h)\dfrac{n^2}{2} = \dfrac{V_0 n^2}{2} \\ H^{(2)}_{ii} = (V_{dir} + V_X + \dfrac{V_e + V_h}{2})\dfrac{n^2}{2} = (V_0 - \dfrac{W}{2})\dfrac{n^2}{2} \\ H^{(2)}_{ij} = (V_{dir} + V_X + V_e + V_h)\dfrac{n^2}{2} = \dfrac{V_0 n^2}{2} \\ H^{(4)} = (V_{dir} + V_X + V_e + V_h)\dfrac{n^2}{2} = \dfrac{V_0 n^2}{2} \end{cases}$$

$$\mu_< = (V_{dir} + V_X + V_e + V_h)n = V_0 n$$

7 times degenerate ground state

Model

interaction Hamiltonian:

$$H_{int} = \frac{V_{dir} + V_X + V_e + V_h}{2} \sum_{\sigma=\pm1\pm2} |\Psi_\sigma|^4 + (V_{dir} + V_X)(|\Psi_{+1}|^2|\Psi_{-1}|^2 + |\Psi_{+2}|^2|\Psi_{-2}|^2)+$$

$$+ (V_{dir} + V_X + V_e + V_h)(|\Psi_{+1}|^2|\Psi_{+2}|^2 + |\Psi_{-1}|^2|\Psi_{-2}|^2 + |\Psi_{+1}|^2|\Psi_{-2}|^2 + |\Psi_{-1}|^2|\Psi_{+2}|^2)+$$

$$+ (V_e + V_h)(\Psi^*_{+1}\Psi^*_{-1}\Psi_{+2}\Psi_{-2} + \Psi^*_{+2}\Psi^*_{-2}\Psi_{+1}\Psi_{-1})$$

spin-conversion

Elementary excitations

Second, consider positive exchange interaction $W = V_e + V_h > 0$
(small QW separations)

$$\begin{cases} H^{(1)} = (V_{dir} + V_X + V_e + V_h)\frac{n^2}{2} = \frac{V_0 n^2}{2} \\ H^{(2)}_{ii} = (V_{dir} + V_X + \frac{V_e + V_h}{2})\frac{n^2}{2} = (V_0 - \frac{W}{2})\frac{n^2}{2} \\ H^{(2)}_{ij} = (V_{dir} + V_X + V_e + V_h)\frac{n^2}{2} = \frac{V_0 n^2}{2} \\ H^{(4)} = (V_{dir} + V_X + \frac{V_e + V_h}{2})\frac{n^2}{2} = (V_0 - \frac{W}{2})\frac{n^2}{2} \end{cases}$$

$$\mu_> = (V_0 - W/2)n$$

3 times degenerate ground state

Elementary excitations

2. Linearize Gross-Pitaevskii equation with respect to the small perturbation

$$\Psi_i^0 = \sqrt{n/4} + A_i e^{i(\mathbf{kr}-\omega t)} + B_i^* e^{-i(\mathbf{kr}-\omega t)}$$

and solve algebraic equation of amplitudes A_i and B_i

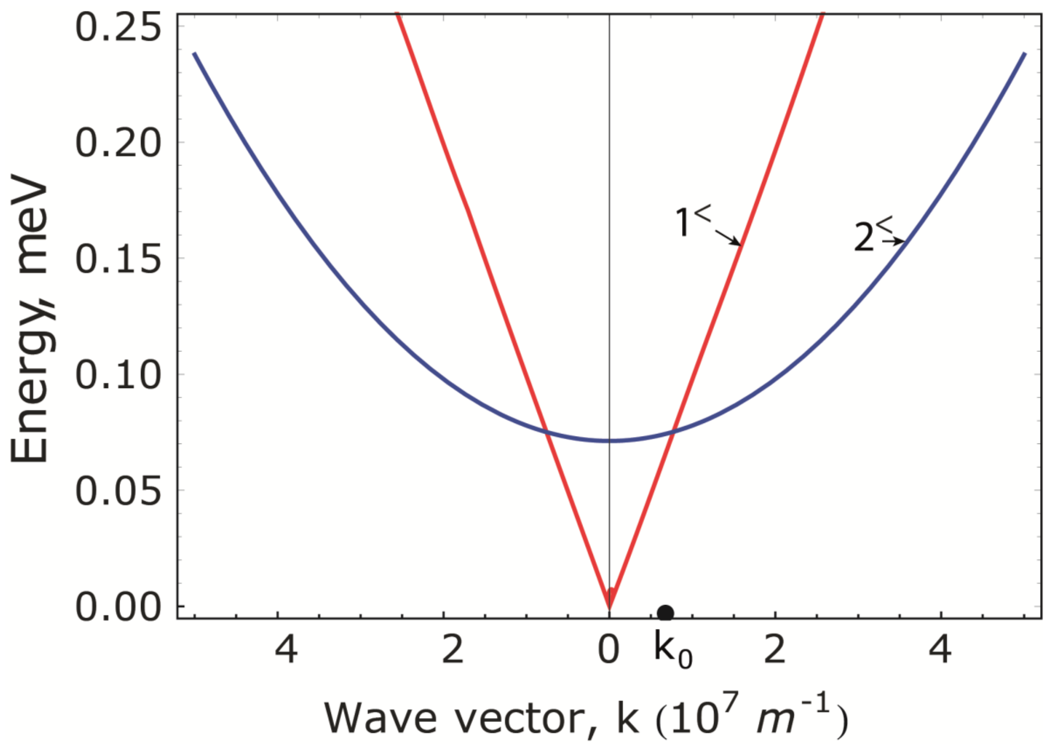
⇩

dispersions of elementary excitations

Elementary excitations

$$W = V_e + V_h < 0$$

$$\hbar\omega_1^< = \sqrt{E_k(E_k + 2\mu_<)}$$
$$\hbar\omega_2^< = E_k + n|W|$$
$$\hbar\omega_{3,4}^< = E_k$$



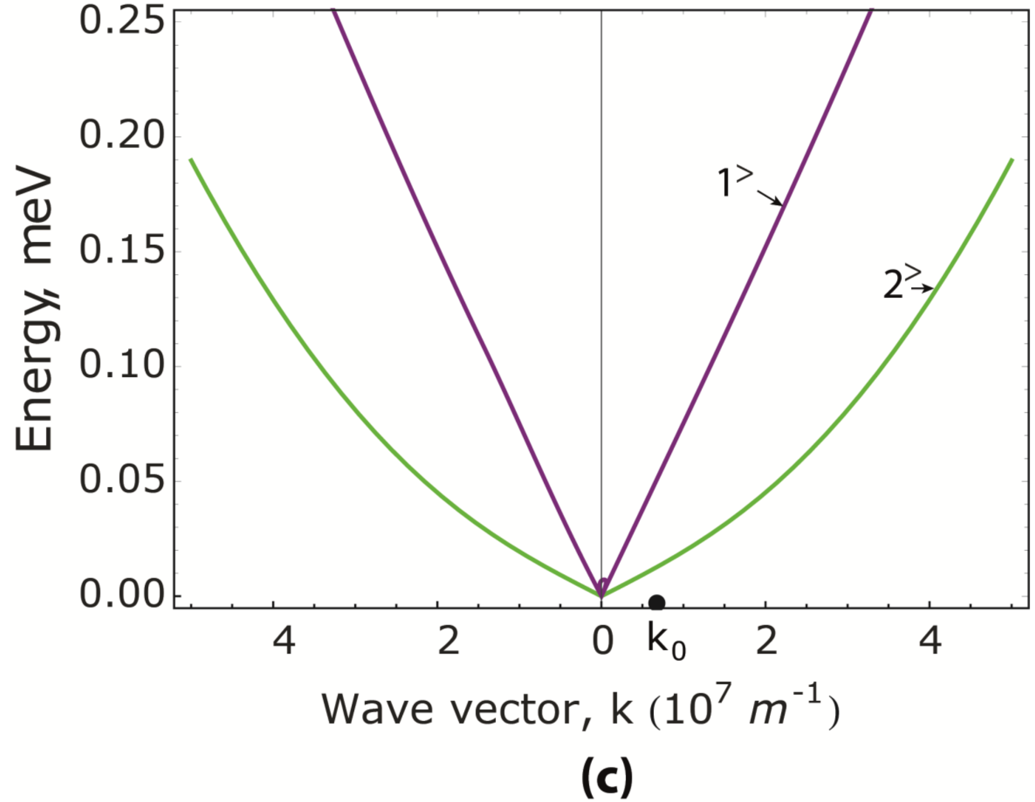
(a)

[Yu. G. Rubo, A.V. Kavokin, PRB 84, 045309]

Elementary excitations

$$W = V_e + V_h > 0$$

$$\hbar\omega^{>}_{1,3,4} = \sqrt{E_k[E_k + 2\mu_{>}]}$$
$$\hbar\omega^{>}_{2} = \sqrt{E_k(E_k + nW)}$$

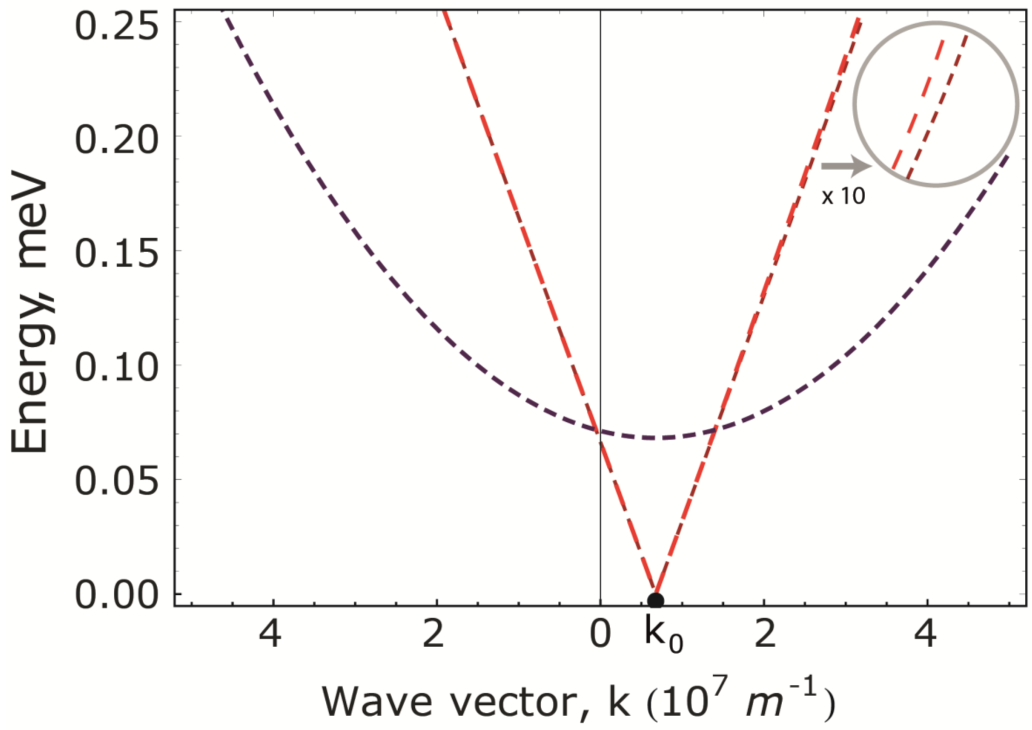


Elementary excitations

$$W = V_e + V_h < 0$$

$$k_0 = \pm \frac{M\alpha}{\hbar^2}$$

$$\varepsilon_\pm = E_k \pm \alpha k$$



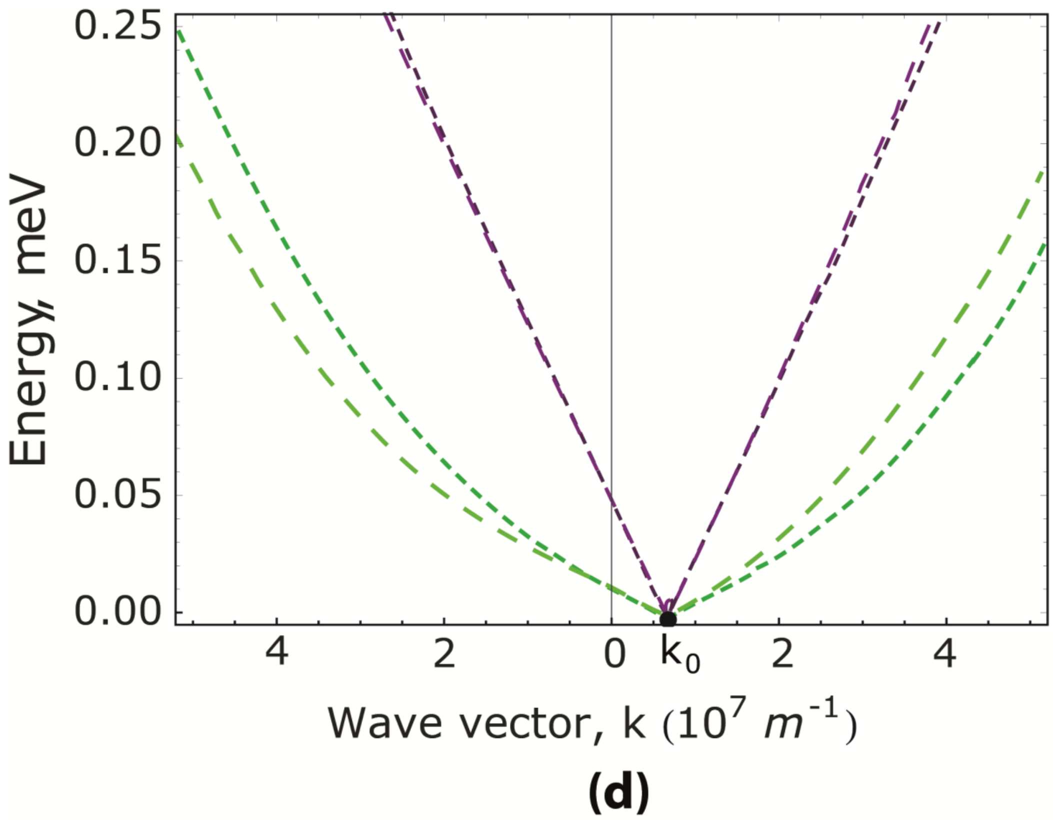
(b)

Elementary excitations

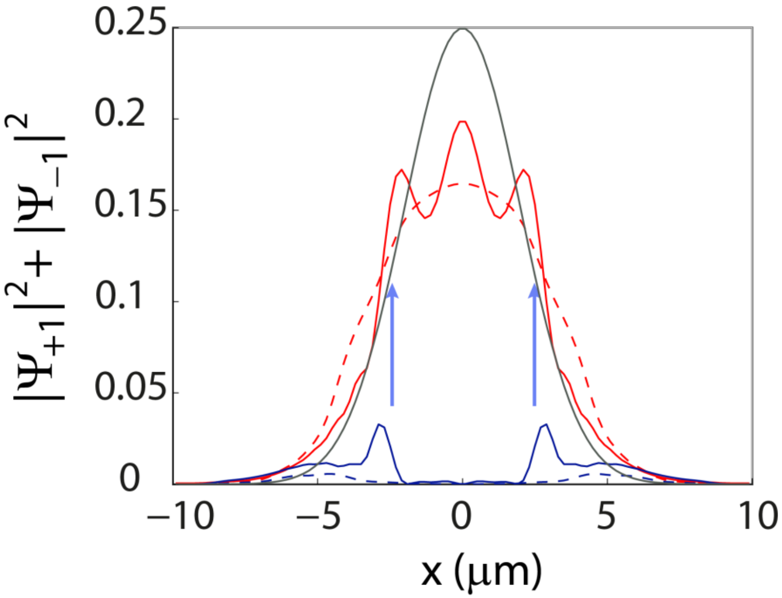
$$W = V_e + V_h > 0$$

$$k_0 = \pm \frac{M\alpha}{\hbar^2}$$

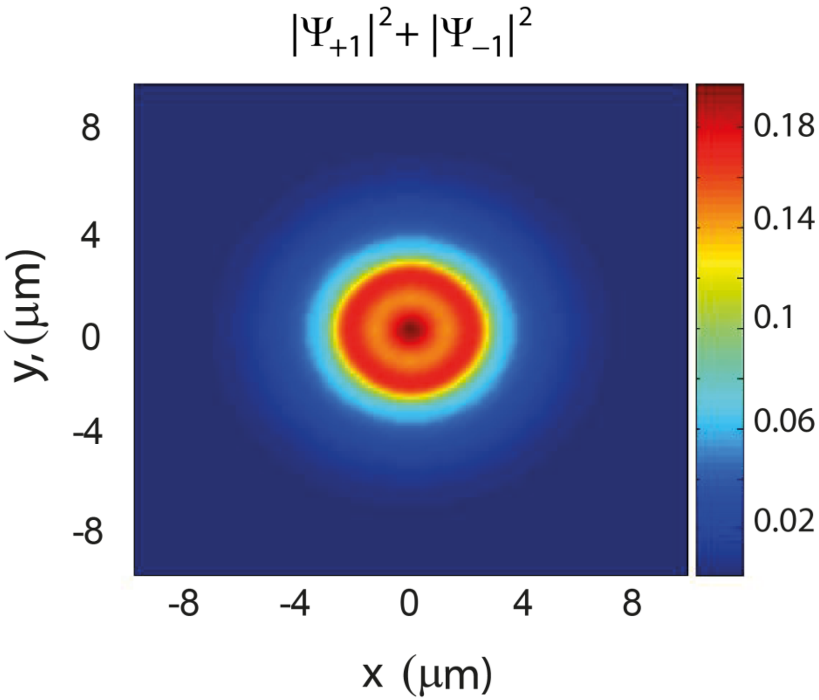
$$\varepsilon_{\pm} = E_k \pm \alpha k$$



Dynamics



(a)



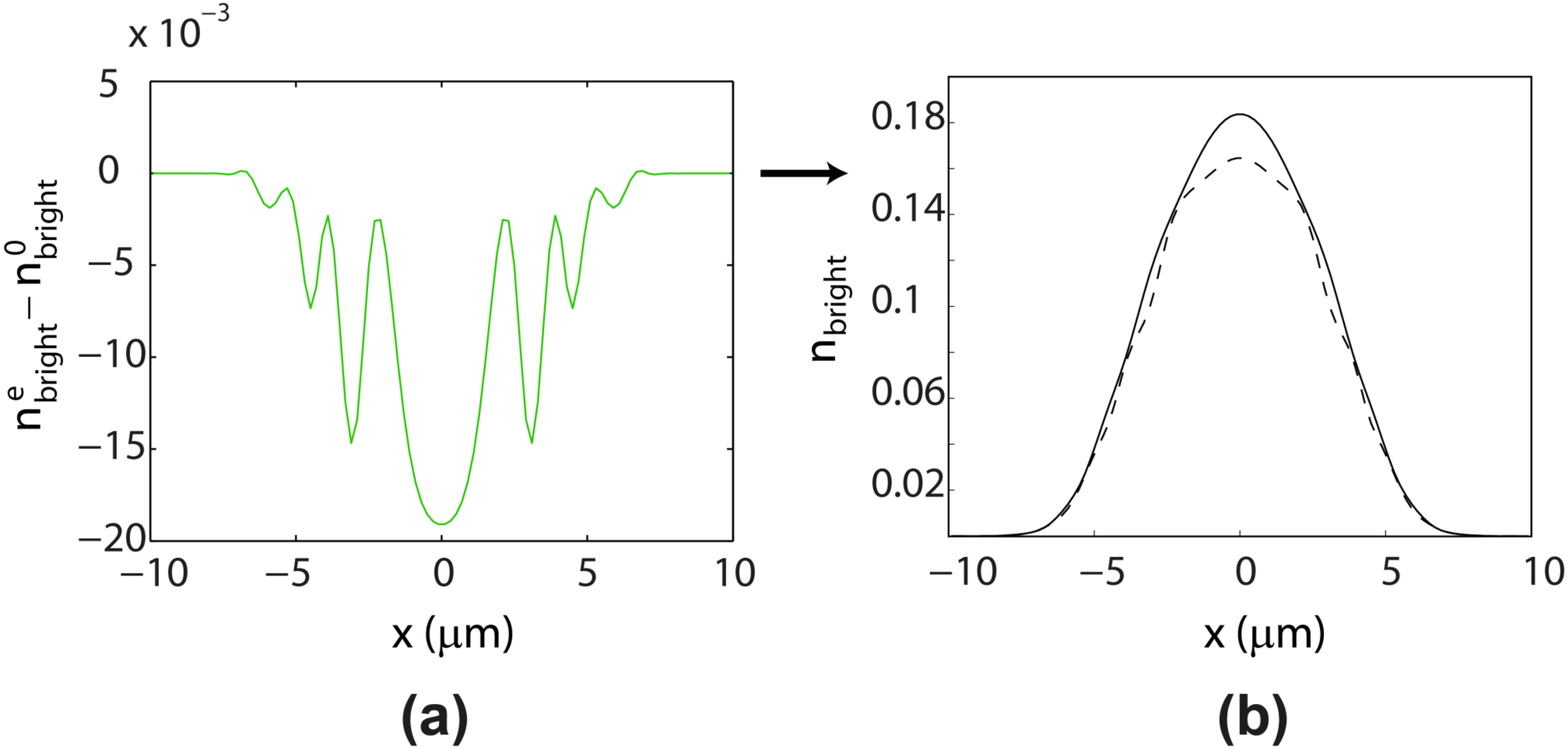
(b)

sample: GaAs/AlGasAs/GaAs, 8nm/4nm/8nm; linear pump

$$V_{dir} = 19.9 \ \mu eV \ \ \mu m^2 \qquad V_e = -1.78 \ \mu eV \ \ \mu m^2$$

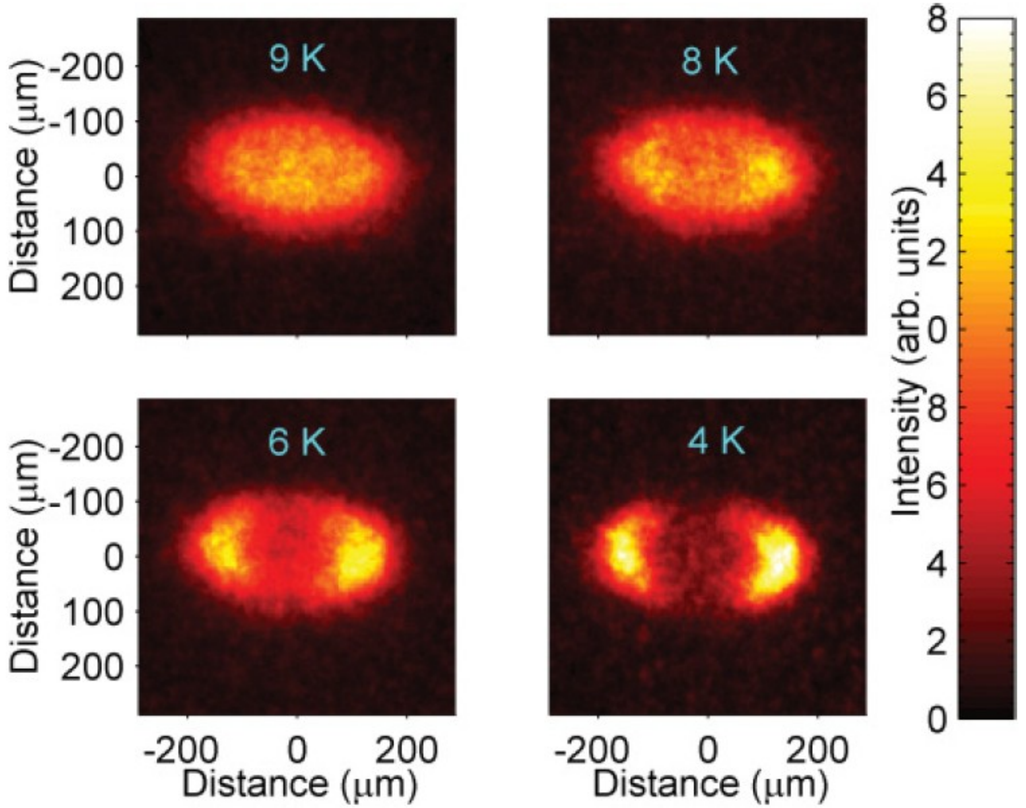
Rashba <u>or</u> Dresselhaus

Dynamics



bright-to-dark conversion due to e and h exchange

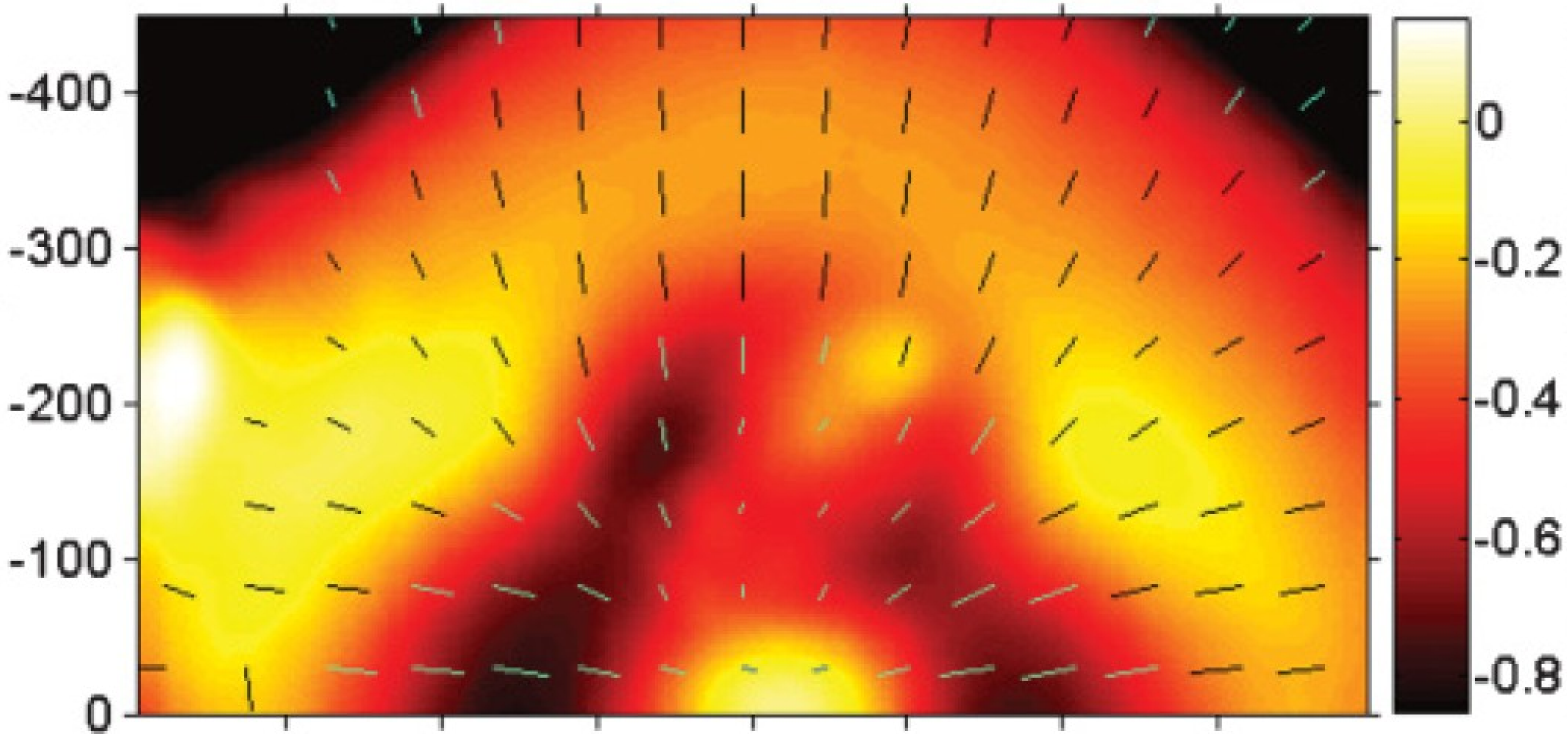
Dynamics



condensate in a trap

[N. W. Sinclair et al., PRB 83, 245304 (2011)]

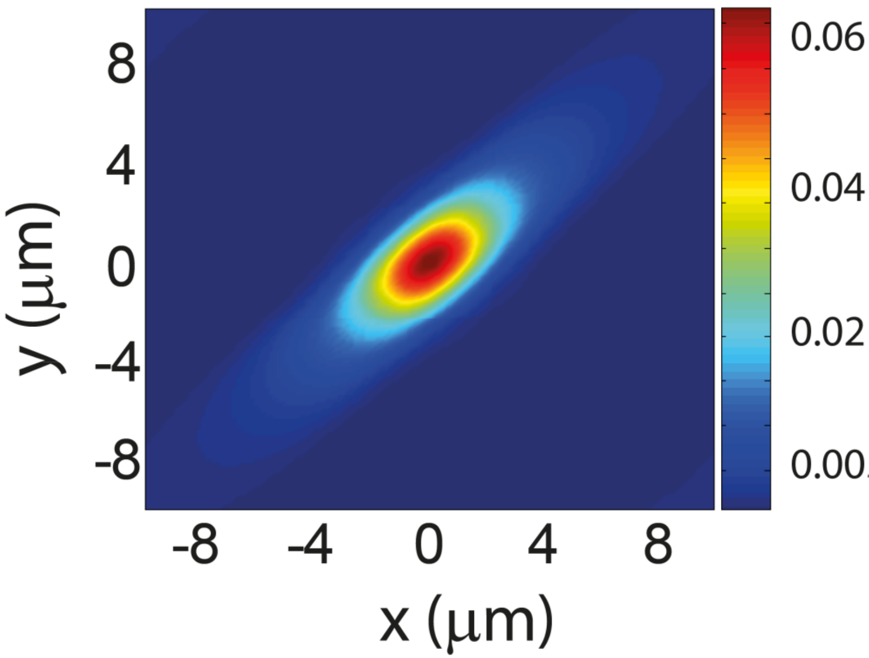
Dynamics



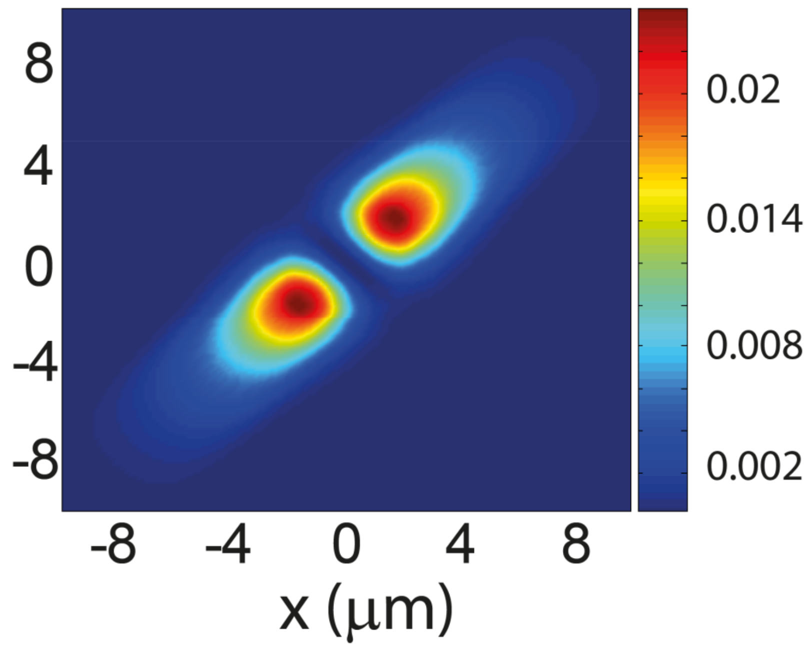
“mexican hat”-like profile for high strain trap

[N. W. Sinclair et al., PRB 83, 245304 (2011)]

Dynamics



bright exciton density

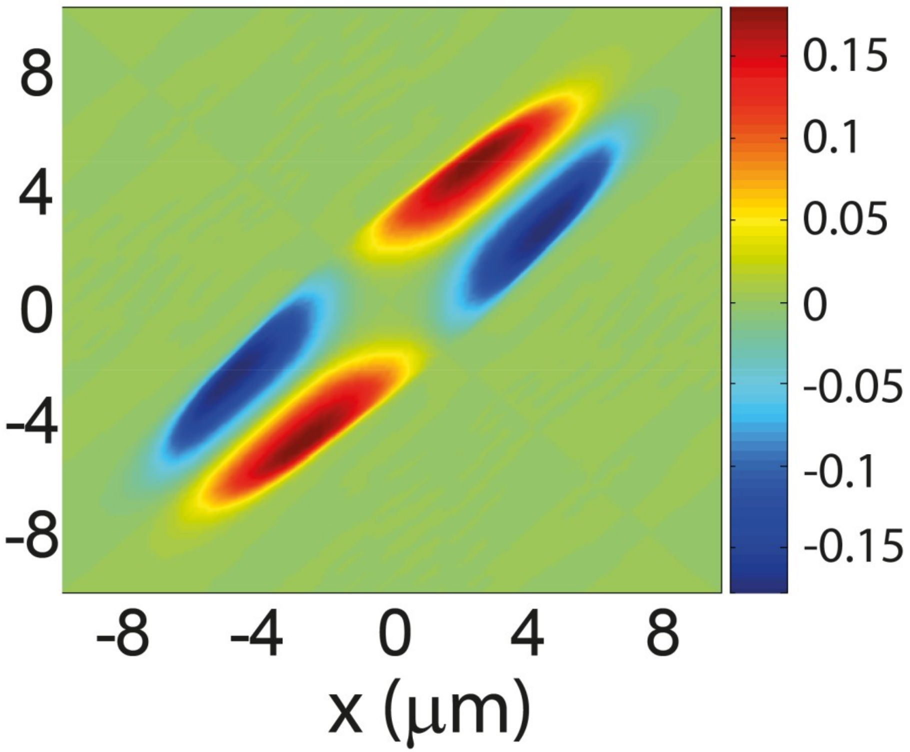


dark exciton density

$$\varepsilon_{\pm}(\mathbf{k}) = \frac{\hbar^2 k^2}{2m} \pm k\sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \sin(2\theta_{\mathbf{k}})}$$

Rashba <u>and</u> Dresselhaus

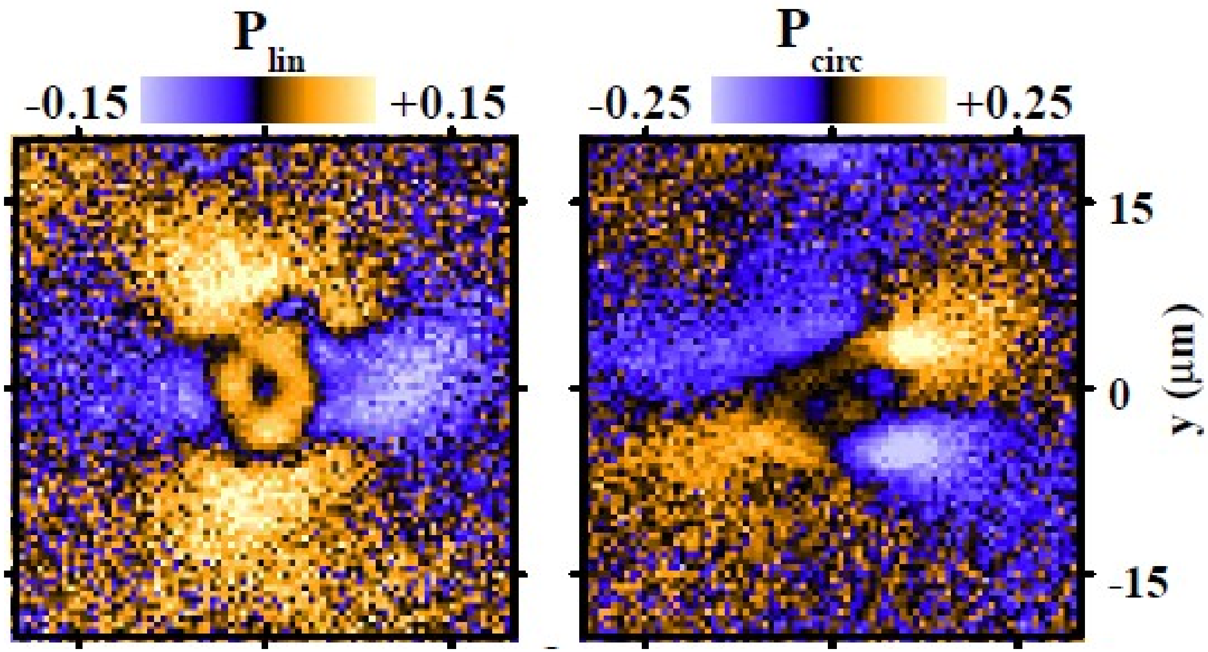
Dynamics



circular polarization $\wp = (|\Psi_{+1}|^2 - |\Psi_{-1}|^2)/(|\Psi_{+1}|^2 + |\Psi_{-1}|^2)$

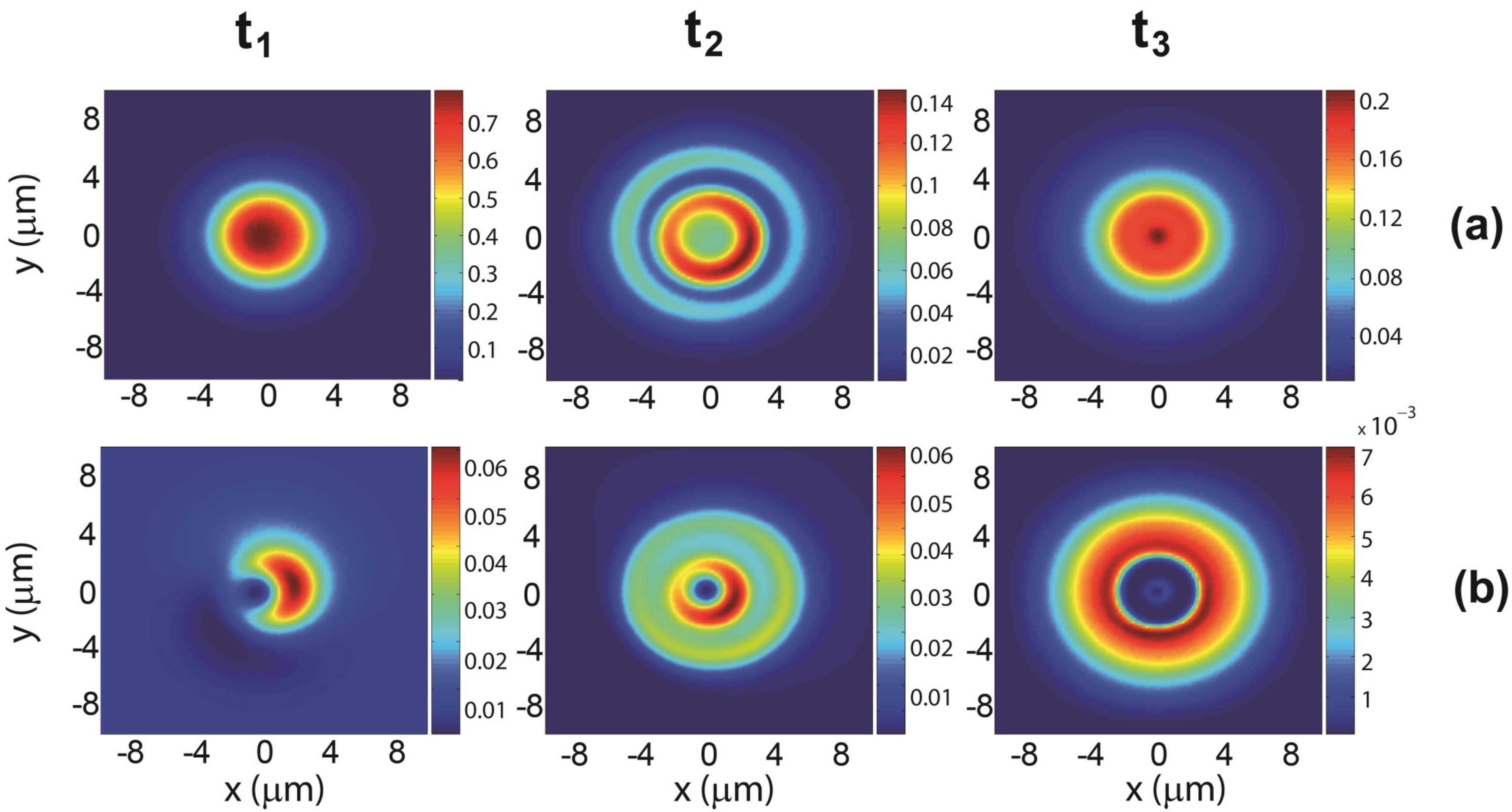
Rashba and Dresselhaus

Dynamics



polarization patterns

[A. A. High et al., Nature 483, 584 (2012); arXiv:1103.0321]



bright (a) and dark (b) exciton density; circular pump

Rashba and Dresselhaus

Conclusions

- included SOI and calculated interactions
- found elementary excitation spectrum
- real-time dynamics: “mexican hat”, dip in the center, four-leaf polarization pattern, artificial magnetic field

This work: **[arXiv:1204.2721], to appear in PRB**

Thank you for attention!