Indirect Excitons

A.A. High, A.T. Hammack, J.R. Leonard, Sen Yang, Leonid Butov
Department of Physics, University of California at San Diego, La Jolla, CA, USA

M.R. Vladimirova and A.V. Kavokin
Laboratoire Charles Coulomb, CNRS/Universite de Montpellier II, Montpellier, France, Spin Optics Laboratory, St-Petersburg, Physics and Astronomy School, University of Southampton, Southampton, UK

T. Ostatnicky
Faculty of Mathematics and Physics, Charles University in Prague
A.C. Gossard
Materials Department, University of California at Santa Barbara

- Cold exciton gases in coupled quantum wells
- Inner and outer rings; localised bright spots; MOES
- Spontaneous coherence and polarisation build-up
- Spin dynamics around sources of cold excitons
- Magnetic field effect: polarisation currents
- Topology of 4-fold degenerate condensates
Spatially indirect excitons in coupled quantum wells


- Controllable exciton life-time
- Controllable exciton interaction strength
- Controllable exchange splitting of dark-bright excitons

\[ k_B T_{BKT} = \frac{\pi \hbar^2 n}{2m} \approx (1 - 2)K \]


Pattern Formation: Exciton Rings and Macroscopically Ordered Exciton State

- external ring
- inner ring
- ring fragmentation
- localized bright spots

410 μm

- Position on the ring (μm)
- Peak number
- Amplitude of the Fourier Transform

Spatial order on macroscopic lengths appears abruptly at low T

laser excitation creates excitons in CQW

inner ring forms due to exciton transport and cooling

Inner ring

flow of excitons out of excitation spot due to exciton drift, diffusion, etc.

exciton transport

excitation spot
high $T_X$
lower occupation of radiative zone

inner ring
lower $T_X$
higher occupation of radiative zone

above barrier laser excitation creates additional number of holes in CQW

heavier holes have higher collection efficiency to CQW

external ring forms at interface between electron-rich and hole-rich regions

Theoretical model for MOES consistent with the experimental observations

instability requires positive feedback to density variations

instability can result from quantum degeneracy in a cold exciton system due to stimulated kinetics of exciton formation

\[
\frac{\partial n_e}{\partial t} = D_e \nabla^2 n_e - wn_e n_h + J_e \\
\frac{\partial n_h}{\partial t} = D_h \nabla^2 n_h - wn_e n_h + J_h \\
\frac{\partial n_x}{\partial t} = D_x \nabla^2 n_x + wn_e n_h - n_x / \tau_{opt}
\]

\[
w \sim 1 + N_{E=0} = e^{T_{db} / T} = e^{2\pi \hbar^2 / mg k_B T n_x}
\]


Problem: stimulation of classical particles in real space (need to develop a quantum theory).
Polarisation textures appear near the sources of cold excitons (T=0.1 K)
The textures disappear at $T=7K$
Appearance of polarisation textures is correlated with the build up of spatial coherence

Magnetic field effect (unpublished)
Coupled quantum wells: Four component exciton condensates:
-2, -1, +1, +2 states are degenerate
Exciton spin density matrix

Exciton spinor wave-function (no coordinate-dependent part here)

$$\Psi = (\Psi_{+1}, \Psi_{-1}, \Psi_{+2}, \Psi_{-2},) = (\Psi_{e,-1/2}\Psi_{h,+3/2}, \Psi_{e,+1/2}\Psi_{h,-3/2}, \Psi_{e,+1/2}\Psi_{h,+3/2}, \Psi_{e,-1/2}\Psi_{h,-3/2})$$

Normalisation condition

$$\Psi_{+1}\Psi_{+1}^* + \Psi_{-1}\Psi_{-1}^* + \Psi_{+2}\Psi_{+2}^* + \Psi_{-2}\Psi_{-2}^* = \Psi_{e,+1/2}\Psi_{e,+1/2}^* + \Psi_{e,-1/2}\Psi_{e,-1/2}^* = \Psi_{h,+3/2}\Psi_{h,+3/2}^* + \Psi_{h,-3/2}\Psi_{h,-3/2}^* = 1$$

Exciton spin-density matrix:

$$\hat{\rho} = |\Psi\rangle \langle \Psi| = \begin{bmatrix}
\Psi_{+1}\Psi_{+1}^* & \Psi_{+1}\Psi_{-1}^* & \Psi_{+1}\Psi_{+2}^* & \Psi_{+1}\Psi_{-2}^* \\
\Psi_{-1}\Psi_{+1}^* & \Psi_{-1}\Psi_{-1}^* & \Psi_{-1}\Psi_{+2}^* & \Psi_{-1}\Psi_{-2}^* \\
\Psi_{+2}\Psi_{+1}^* & \Psi_{+2}\Psi_{-1}^* & \Psi_{+2}\Psi_{+2}^* & \Psi_{+2}\Psi_{-2}^* \\
\Psi_{-2}\Psi_{+1}^* & \Psi_{-2}\Psi_{-1}^* & \Psi_{-2}\Psi_{+2}^* & \Psi_{-2}\Psi_{-2}^*
\end{bmatrix}$$
Relation between electron, hole and exciton spin density matrices

\[ \hat{\rho} = |\Psi\rangle \langle \Psi| = \begin{bmatrix} \psi_{+1} \psi_{+1}^* & \psi_{+1} \psi_{-1}^* & \psi_{+1} \psi_{+2}^* & \psi_{+1} \psi_{-2}^* \\ \psi_{-1} \psi_{+1}^* & \psi_{-1} \psi_{-1}^* & \psi_{-1} \psi_{+2}^* & \psi_{-1} \psi_{-2}^* \\ \psi_{+2} \psi_{+1}^* & \psi_{+2} \psi_{-1}^* & \psi_{+2} \psi_{+2}^* & \psi_{+2} \psi_{-2}^* \\ \psi_{-2} \psi_{+1}^* & \psi_{-2} \psi_{-1}^* & \psi_{-2} \psi_{+2}^* & \psi_{-2} \psi_{-2}^* \end{bmatrix} \]

Useful relations:

\[ \hat{\rho}_e = |\Psi_e\rangle \langle \Psi_e| = \begin{bmatrix} \psi_{e,+ \frac{1}{2}} \psi_{e,+ \frac{1}{2}}^* & \psi_{e,+ \frac{1}{2}} \psi_{e,- \frac{1}{2}}^* \\ \psi_{e,- \frac{1}{2}} \psi_{e,+ \frac{1}{2}}^* & \psi_{e,- \frac{1}{2}} \psi_{e,- \frac{1}{2}}^* \end{bmatrix} = \begin{bmatrix} \rho_{22} + \rho_{33} & \rho_{24} + \rho_{31} \\ \rho_{13} + \rho_{42} & \rho_{11} + \rho_{44} \end{bmatrix} \]

\[ \hat{\rho}_h = |\Psi_h\rangle \langle \Psi_h| = \begin{bmatrix} \psi_{h,+ \frac{3}{2}} \psi_{h,+ \frac{3}{2}}^* & \psi_{h,+ \frac{3}{2}} \psi_{h,- \frac{3}{2}}^* \\ \psi_{h,- \frac{3}{2}} \psi_{h,+ \frac{3}{2}}^* & \psi_{h,- \frac{3}{2}} \psi_{h,- \frac{3}{2}}^* \end{bmatrix} = \begin{bmatrix} \rho_{11} + \rho_{33} & \rho_{14} + \rho_{32} \\ \rho_{23} + \rho_{41} & \rho_{22} + \rho_{44} \end{bmatrix} \]

Here we used the normalisation condition!
Electron and hole spins are linked with the exciton density matrix elements!

\[
\hat{\rho}_e = \begin{bmatrix}
\frac{1}{2} + S_{e,z} & S_{e,x} - i S_{e,y} \\
S_{e,x} + i S_{e,y} & \frac{1}{2} - S_{e,z}
\end{bmatrix},
\hat{\rho}_h = \begin{bmatrix}
\frac{1}{2} + S_{h,z} & S_{h,x} - i S_{h,y} \\
S_{h,x} + i S_{h,y} & \frac{1}{2} - S_{h,z}
\end{bmatrix}
\]

\[
S_{e,z} = (\rho_{22} + \rho_{33} - \rho_{11} - \rho_{44})/2,
\]

\[
S_{h,z} = (\rho_{11} + \rho_{33} - \rho_{22} - \rho_{44})/2.
\]

\[
S_{e,x} = (\rho_{13} + \rho_{31} + \rho_{24} + \rho_{42})/2,
\]

\[
S_{h,x} = (\rho_{14} + \rho_{23} + \rho_{32} + \rho_{41})/2,
\]

\[
S_{e,y} = i(-\rho_{13} + \rho_{31} + \rho_{24} - \rho_{42})/2.
\]

\[
S_{h,y} = i(\rho_{14} - \rho_{23} + \rho_{32} - \rho_{41})/2.
\]

All we know from traditional spintronics can be applied to excitons once we remember that they are composed by electrons and holes!
Our model:

- Excitons propagate ballistically in radial directions; \( r = vt \)
- Dark and bright excitons are mixed by spin-orbit interaction.
- Electron and hole spins are rotated by the Dresselhaus field;
- Supplementary beats appear due to linear polarisation splittings;

We solve the Liouville equation for a 4x4 spin density matrix:

\[
\frac{i\hbar}{\hbar} \frac{d\hat{\rho}}{dt} = [\hat{H}, \hat{\rho}]
\]

\[\hat{\rho} = |\Psi><\Psi| \quad \Psi = (\Psi_+, \Psi_-, \Psi_+, \Psi_-)\]

The polarisation degrees are given by:

\[\rho_l = \frac{2S_x}{I} = \frac{\rho_{12} + \rho_{21}}{\rho_{11} + \rho_{22}}\]

\[\rho_c = \frac{2S_z}{I} = \frac{\rho_{11} - \rho_{22}}{\rho_{11} + \rho_{22}}\]
What contributes to the exciton Hamiltonian?

Dresselhaus and Zeeman effects on electrons:

Dresselhaus spin-orbit Hamiltonian

\[ H_{eff(e,h)} = -\frac{1}{2}g_e\hbar \mu_B (B_{eff} \hat{\sigma}) \]

\[ -\frac{1}{2} g_e \mu_B B_{eff} = \beta_e (k_{e,x}, -k_{e,y}) \]

\[ H_e = \beta_e (k_{e,x} \sigma_x - k_{e,y} \sigma_y) - \frac{1}{2} g_e \mu_B B \sigma_z \]
Dresselhaus and Zeeman effects on electrons: exciton Hamiltonian

\[ H_e = \beta_e \left( k_{e,x} \sigma_x - k_{e,y} \sigma_y \right) - \frac{1}{2} g_e \mu_B B \sigma_z \]

Hence:

\[ H_e = \begin{bmatrix} -\frac{1}{2} g_e \mu_B B & \beta_e (k_{e,x} + i k_{e,y}) \\ \beta_e (k_{e,x} - i k_{e,y}) & \frac{1}{2} g_e \mu_B B \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} g_e \mu_B B & \beta_e k_e e^{i\varphi} \\ \beta_e k_e e^{-i\varphi} & \frac{1}{2} g_e \mu_B B \end{bmatrix}. \]

In the exciton basis:

\((-1/2, +1/2, +1/2, -1/2) \rightarrow (+1, -1, +2, -2)\)

\[ \tilde{H}_e = \begin{bmatrix} g_e \mu_B B/2 & 0 & k_e \beta_e e^{-i\varphi} & 0 \\ 0 & -g_e \mu_B B/2 & 0 & k_e \beta_e e^{i\varphi} \\ k_e \beta_e e^{i\varphi} & 0 & -g_e \mu_B B/2 & 0 \\ 0 & k_e \beta_e e^{-i\varphi} & 0 & g_e \mu_B B/2 \end{bmatrix}. \]
Dresselhaus and Zeeman effects for holes: $-\frac{1}{2} g_h \mu_B B_{\text{eff}} = \beta_h (k_{e,x}, k_{e,y})$

In the $(+3/2,-3/2)$ basis:

$$H_h = \beta_h (k_{h,x} \sigma_x + k_{h,y} \sigma_y) - \frac{1}{2} g_h \mu_B B \sigma_z.$$ 

$$H_h = \begin{bmatrix} -\frac{1}{2} g_h \mu_B B & \beta_h (k_{h,x} - i k_{h,y}) \\ \beta_h (k_{h,x} + i k_{h,y}) & \frac{1}{2} g_h \mu_B B \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} g_h \mu_B B & \beta_h k_h e^{-i\phi} \\ \beta_h k_h e^{i\phi} & \frac{1}{2} g_h \mu_B B \end{bmatrix}$$

In the exciton basis:

$(+3/2, -3/2, +3/2, -3/2) \quad \rightarrow \quad (+1, -1, +2, -2)$

$$\tilde{H}_h = \begin{bmatrix} -g_h \mu_B B/2 & 0 & 0 & k_h \beta_h e^{-i\phi} \\ 0 & g_h \mu_B B/2 & k_h \beta_h e^{i\phi} & 0 \\ 0 & k_h \beta_h e^{-i\phi} & -g_h \mu_B B/2 & 0 \\ k_h \beta_h e^{i\phi} & 0 & 0 & g_h \mu_B B/2 \end{bmatrix}.$$
Long- and short-range exchange splittings of 4 exciton states:

Splitting of bright excitons:

\[
H_b = E_b I - \delta_b \sigma_x = \begin{bmatrix}
E_b & -\delta_b \\
-\delta_b & E_b \\
\end{bmatrix},
\]

Splitting of dark excitons:

\[
H_d = E_d I - \delta_d \sigma_x = \begin{bmatrix}
E_d & -\delta_d \\
-\delta_d & E_d \\
\end{bmatrix}
\]

In the exciton basis \((+1,-1,+2,-2)\)

\[
H_0 = \begin{bmatrix}
E_b & -\delta_b & 0 & 0 \\
-\delta_b & E_b & 0 & 0 \\
0 & 0 & E_d & -\delta_d \\
0 & 0 & -\delta_d & E_d \\
\end{bmatrix}.
\]
All terms together, we obtain the exciton Hamiltonian:

\[
\hat{H} = \begin{pmatrix}
    E_b - (g_h - g_e)\mu_B B/2 & -\delta_b & k_e \beta e^{-i\phi} & k_h \beta h e^{-i\phi} \\
    -\delta_b & E_b + (g_h - g_e)\mu_B B/2 & k_h \beta h e^{i\phi} & k_e \beta e^{i\phi} \\
    k_e \beta e^{i\phi} & k_h \beta h e^{-i\phi} & E_d - (g_h + g_e)\mu_B B/2 & -\delta_d \\
    k_h \beta h e^{i\phi} & k_e \beta e^{-i\phi} & -\delta_d & E_d + (g_h + g_e)\mu_B B/2
\end{pmatrix}.
\]

\[k_{ex} = k_h + k_e\]

\[k_e = \frac{m_e}{m_e + m_{hh}} k_{ex}, \quad k_h = \frac{m_{hh}}{m_e + m_{hh}} k_{ex}.
\]

Hidden asymmetry between bright and dark excitons!
Theory vs experiment:

\[ g_e = 0.010 \]
\[ g_h = -0.0085 \]
\[ E_b - E_d = -1\mu eV \]
\[ \delta_b = 0.55\mu eV \]
\[ \delta_d = -13\mu eV \]
\[ k = 15.4\mu m^{-1} \]
\[ T = 0.1 K \]
Theory vs experiment:

Circular Polarization

$g_e = 0.010$

$g_h = -0.0085$

$E_b - E_d = -1 \mu eV$

$\delta_b = 0.55 \mu eV$

$\delta_d = -13 \mu eV$

$k = 15.4 \mu m^{-1}$

$T=0.1 K$
Excitons carry electron and hole spin currents!
Simulations of the interferograms: forks are not vortices but sources of excitons!

A small “inner ring” emits excitons in all directions

\[
I_{12} = \sum_{\tilde{R}_0} \left| e^{i\phi(\tilde{R}-\tilde{R}_0)} + e^{ik_{xy} \phi(\tilde{R}-\tilde{R}_0+\delta r)} \right|^2
\]
Spin-orbital coupling and topology of spin-degenerate cold exciton gases

M. Matuszewski,¹ T. C. H. Liew,² ³ ⁴ Y. G. Rubo,² and A. V. Kavokin⁵ ⁶

¹Instytut Fizyki Polskiej Akademii Nauk, Aleja Lotników 32/46, PL-02-668 Warsaw, Poland
²Centro de Investigación en Energía, Universidad Nacional Autónoma de México, Temixco, Morelos, 62580, Mexico
³Mediterranean Institute of Fundamental Physics, 31, via Appia Nuova, 00040, Rome, Italy
⁴School of Physical and Mathematical Sciences, Nanyang Technological University, 637371, Singapore
⁵School of Physics and Astronomy, University of Southampton, Southampton, SO17 1BJ, United Kingdom
⁶Spin Optics Laboratory, St-Petersburg State University, 1, Ulianovskaya, St-Petersburg, Russia

Gross-Pitaevskii equations:

\[ i\hbar \frac{\partial \Psi_{+1}}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m_{\text{ex}}} \Psi_{+1} + \beta_e \left( \hat{k}_x - i\hat{k}_y \right) \Psi_{+2} + \beta_h \left( \hat{k}_x - i\hat{k}_y \right) \Psi_{-2} \]
\[ + \alpha_1 |\Psi_{+1}|^2 \Psi_{+1} + \alpha_2 |\Psi_{-1}|^2 \Psi_{+1} + \alpha_3 |\Psi_{+2}|^2 \Psi_{+1} + \alpha_4 |\Psi_{-2}|^2 \Psi_{+1} + W\Psi^*_{-1} \Psi_{+2} \Psi_{-2} \]

\[ i\hbar \frac{\partial \Psi_{-1}}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m_{\text{ex}}} \Psi_{-1} + \beta_e \left( \hat{k}_x + i\hat{k}_y \right) \Psi_{-2} + \beta_h \left( \hat{k}_x + i\hat{k}_y \right) \Psi_{+2} \]
\[ + \alpha_1 |\Psi_{-1}|^2 \Psi_{-1} + \alpha_2 |\Psi_{+1}|^2 \Psi_{-1} + \alpha_3 |\Psi_{-2}|^2 \Psi_{-1} + \alpha_4 |\Psi_{+2}|^2 \Psi_{-1} + W\Psi^*_{+1} \Psi_{+2} \Psi_{-2} \]

\[ i\hbar \frac{\partial \Psi_{+2}}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m_{\text{ex}}} \Psi_{+2} + \beta_e \left( \hat{k}_x + i\hat{k}_y \right) \Psi_{+1} + \beta_h \left( \hat{k}_x - i\hat{k}_y \right) \Psi_{-1} \]
\[ + \alpha_1 |\Psi_{+2}|^2 \Psi_{+2} + \alpha_2 |\Psi_{-2}|^2 \Psi_{+2} + \alpha_3 |\Psi_{+1}|^2 \Psi_{+2} + \alpha_4 |\Psi_{-1}|^2 \Psi_{+2} + W\Psi^*_{-2} \Psi_{+1} \Psi_{-1} \]

\[ i\hbar \frac{\partial \Psi_{-2}}{\partial t} = -\frac{\hbar^2 \nabla^2}{2m_{\text{ex}}} \Psi_{-2} + \beta_e \left( \hat{k}_x - i\hat{k}_y \right) \Psi_{-1} + \beta_h \left( \hat{k}_x + i\hat{k}_y \right) \Psi_{+1} \]
\[ + \alpha_1 |\Psi_{-2}|^2 \Psi_{-2} + \alpha_2 |\Psi_{+2}|^2 \Psi_{-2} + \alpha_3 |\Psi_{-1}|^2 \Psi_{-2} + \alpha_4 |\Psi_{+1}|^2 \Psi_{-2} + W\Psi^*_{+2} \Psi_{+1} \Psi_{-1} \]

\[ k_{x,y} = -i\nabla_{x,y} \]

with external harmonic potential \[ U(r_{\perp}) = M\omega^2 r_{\perp}^2 / 2 \]
Examples of spin structures of the condensates in the traps

Color=phase, Darkness=intensity
Weak trap plane wave (W<0)

Vortex (weak spin-orbit coupling)

Polarisation domains (W>0)
multiple vortices (strong spin-orbit coupling)

Conditions on winding numbers:

\[ m_{+2} + m_{-2} = m_{+1} + m_{-1} \]
\[ m_{+2} = m_{+1} + 1 = m_{-1} - 1 = m_{-2} \]
Exciton condensates in a “strong trap”

- Phase separation of bright and dark excitons
- Spin polarisation of bright excitons

Conclusions:

• At 100 mK the gas of indirect excitons forms coherent polarisation textures
• The experiment is interpreted in terms of ballistic motion of excitons and Dresselhaus effect on electrons and holes
• Magnetic field creates polarisation currents non colinear to the mass currents
• Magnetic field induced classical polarisation vortices are observed
• Excitons carry electron and hole spin currents
• Interesting non-trivial topology of condensates in traps is expected