Indirect Excitons

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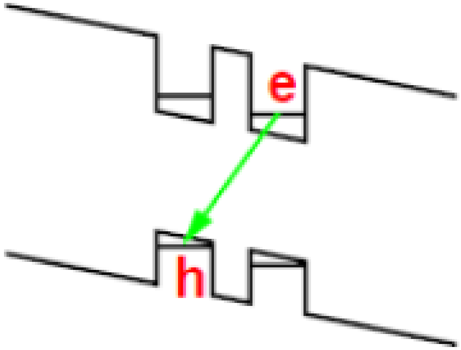
- Cold exciton gases in coupled quantum wells
- Inner and outer rings; localised bright spots; MOES
- Spontaneous coherence and polarisation build-up
- Spin dynamics around sources of cold excitons
- Magnetic field effect: polarisation currents
- Topology of 4-fold degenerate condensates

Spatially indirect excitons in coupled quantum wells

Recent review: D.W. Snoke, "Coherence and Optical Emission from Bilayer Exciton Condensates," Advances in Condensed Matter Physics **2011**, 938609 (2011).

L.V. Butov, San Diego



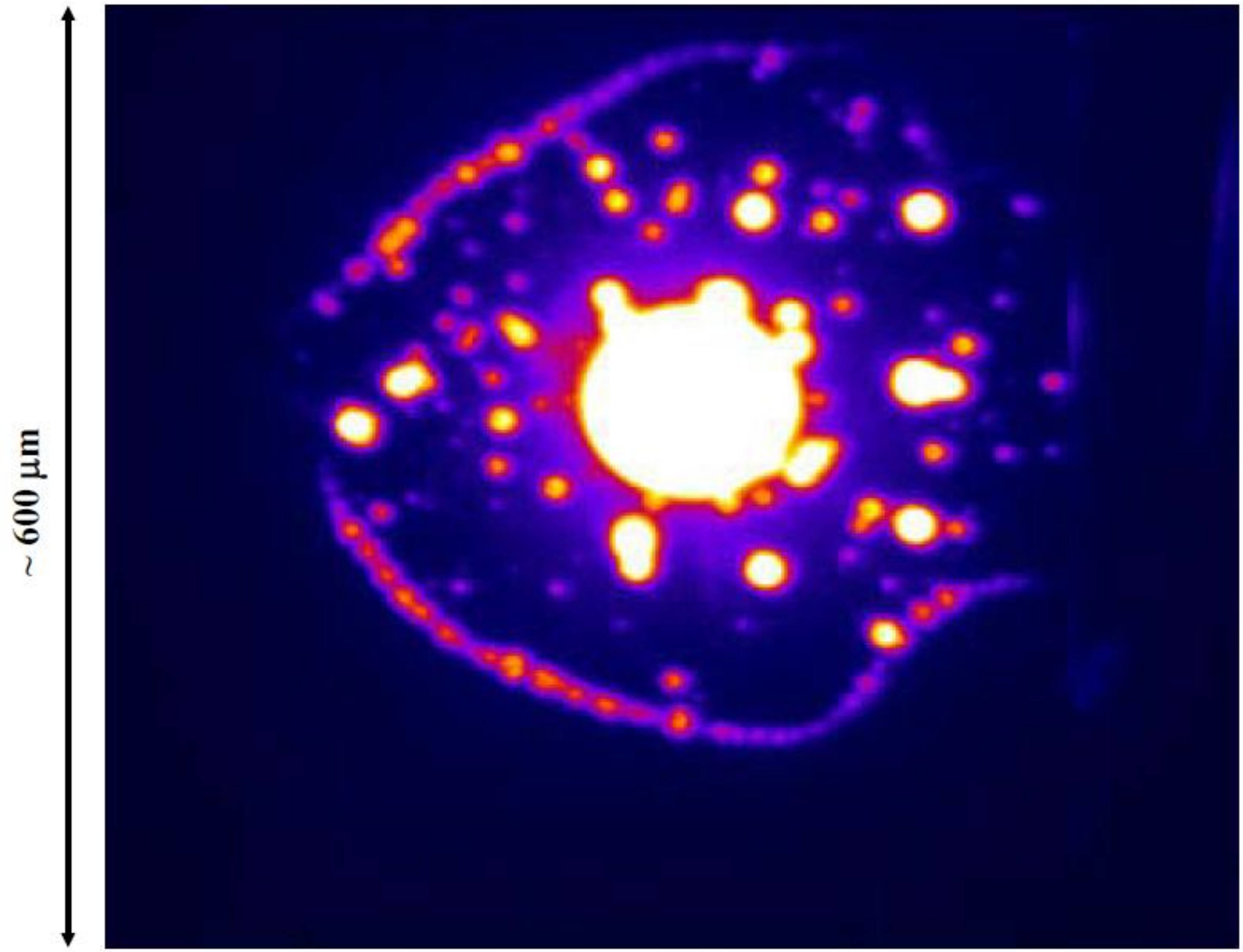
D. Snoke, Pittsburgh

- Controllable exciton life-time
- Controllable exciton interaction strength
- Controllable exchange splitting of dark-bright excitons

$$k_B T_{BKT} = \frac{\pi \hbar^2 n}{2m} \approx (1-2)K$$

D.R. Nelson and J.M. Kosterlitz, Phys. Rev. Lett., 39, 1977

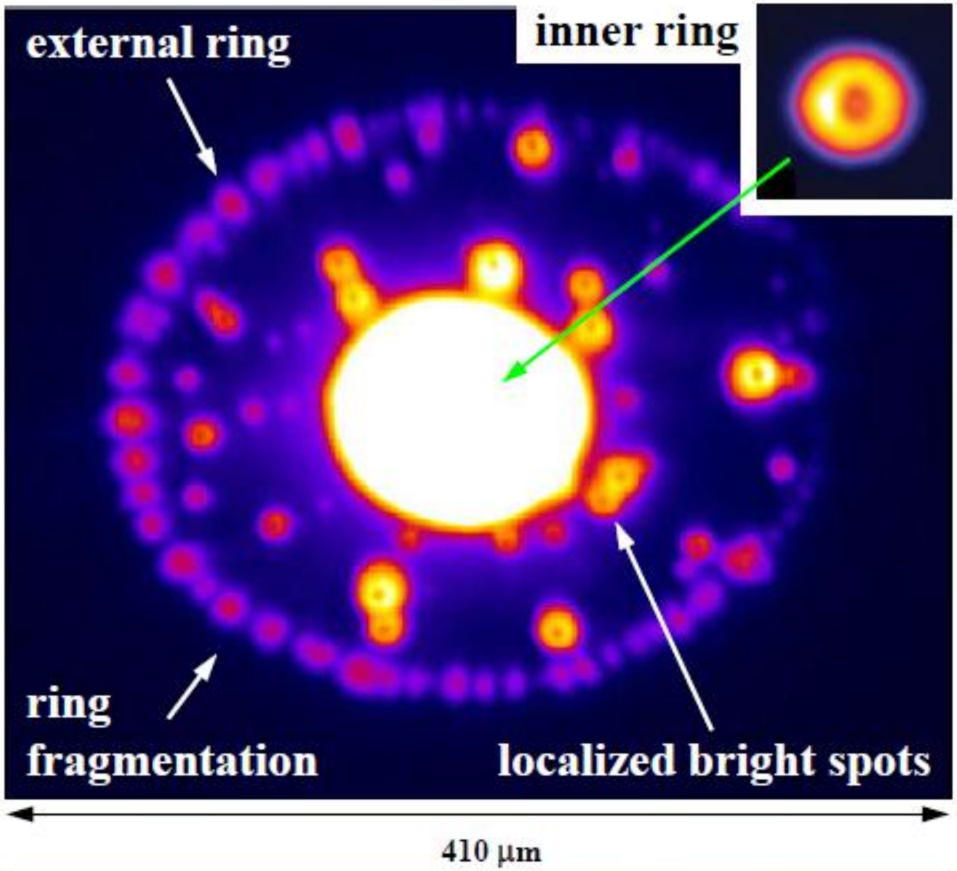
2D image of indirect exciton PL vs P_{ex}



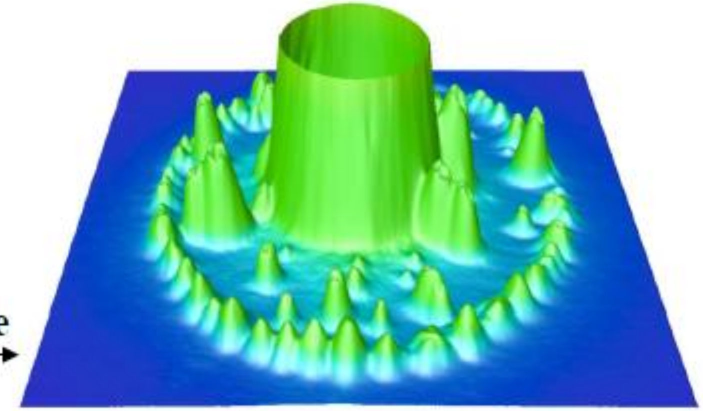
L.V. Butov, A.C. Gossard, D.S. Chemla, cond-mat/0204482 [Nature 418, 751 (2002)]

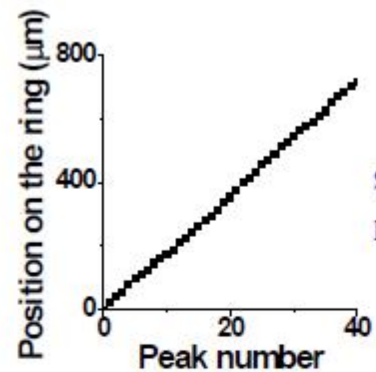
See also: D. Snoke, S. Denev, Y. Liu, L. Pfeiffer, and K. West, Nature, 418, 754 (2002).

Pattern Formation: Exciton Rings and Macroscopically Ordered Exciton State

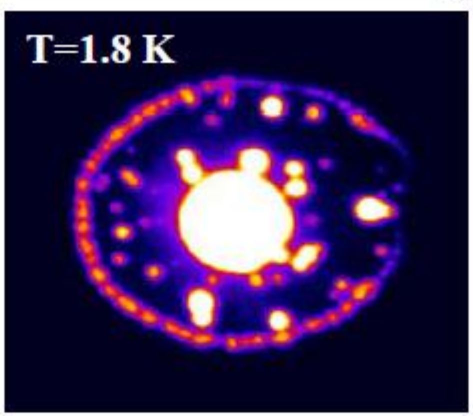


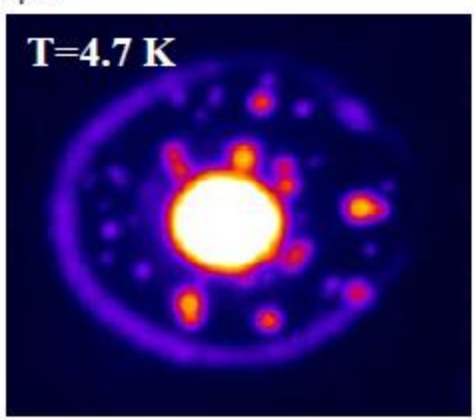
same

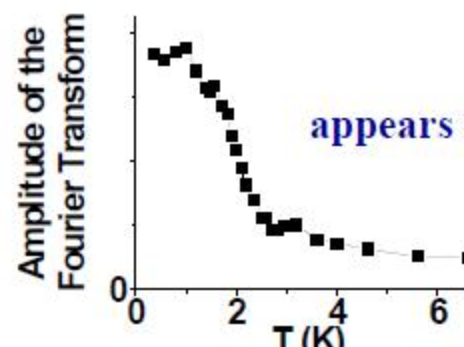




spatial order on macroscopic lengths





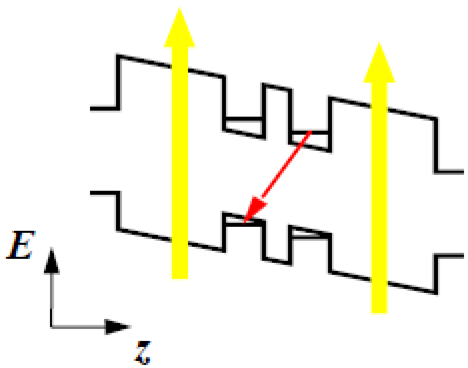


appears abruptly at low *T*

L.V. Butov, A.C. Gossard, D.S. Chemla, Nature 418, 751 (2002)

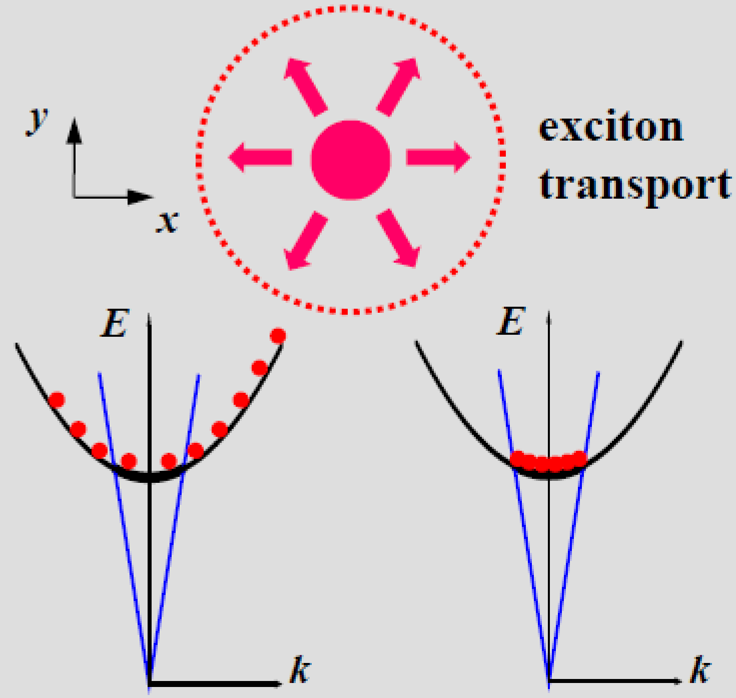
Inner ring

laser excitation creates excitons in CQW



inner ring forms due to exciton transport and cooling

flow of excitons out of excitation spot due to exciton drift, diffusion, etc.



<u>excitation spot</u>
high T_X
lower occupation of radiative zone

<u>inner ring</u>
lower T_X
higher occupation of radiative zone

L.V. Butov, A.C. Gossard, D.S. Chemla, Nature 418, 751 (2002)

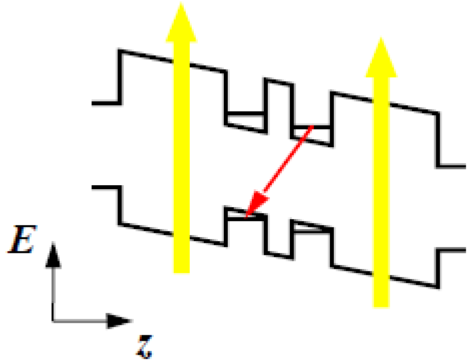
A.L. Ivanov, L. Smallwood, A. Hammack, Sen Yang, L.V. Butov, A.C. Gossard, EPL 73, 920 (2006)

External ring

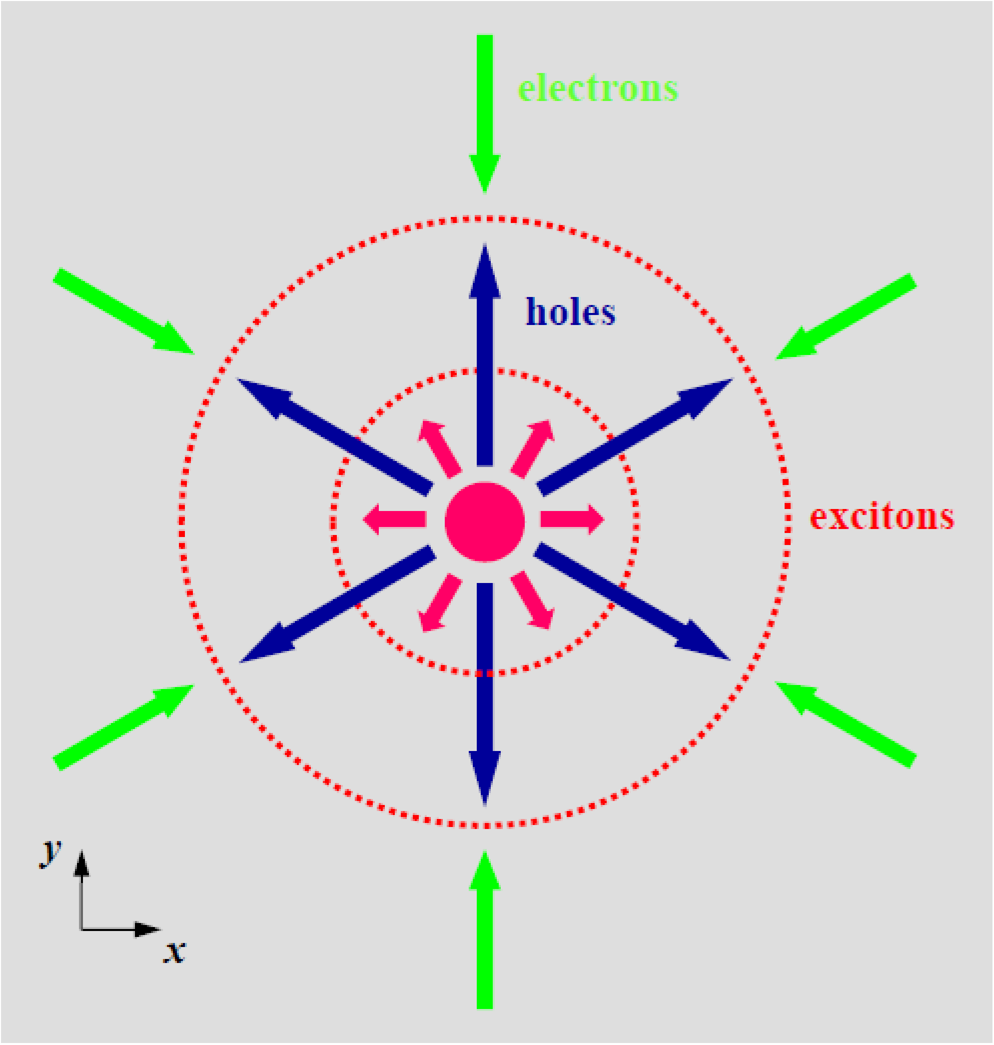
above barrier laser excitation creates
additional number of holes in CQW

⬆

heavier holes have higher
collection efficiency to CQW



external ring forms at interface between
electron-rich and hole-rich regions



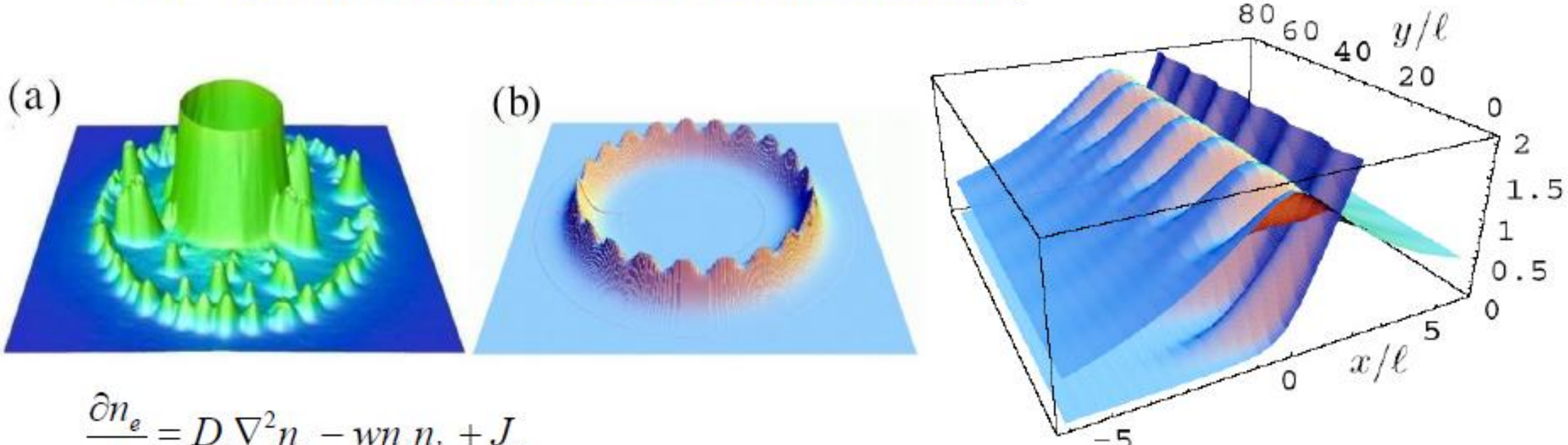
L.V. Butov, L.S. Levitov, B.D. Simons, A.V. Mintsev, A.C. Gossard, D.S. Chemla, PRL 92, 117404 (2004)
R. Rapaport, G. Chen, D. Snoke, S.H. Simon, L. Pfeiffer, K.West, Y.Liu, S.Denev, PRL 92, 117405 (2004)

Theoretical model for MOES consistent with the experimental observations

instability requires <u>positive feedback</u> to density variations

↕

instability can result from quantum degeneracy in a cold exciton system due to <u>stimulated kinetics of exciton formation</u>



$$\frac{\partial n_e}{\partial t} = D_e \nabla^2 n_e - w n_e n_h + J_e$$

$$\frac{\partial n_h}{\partial t} = D_h \nabla^2 n_h - w n_e n_h + J_h$$

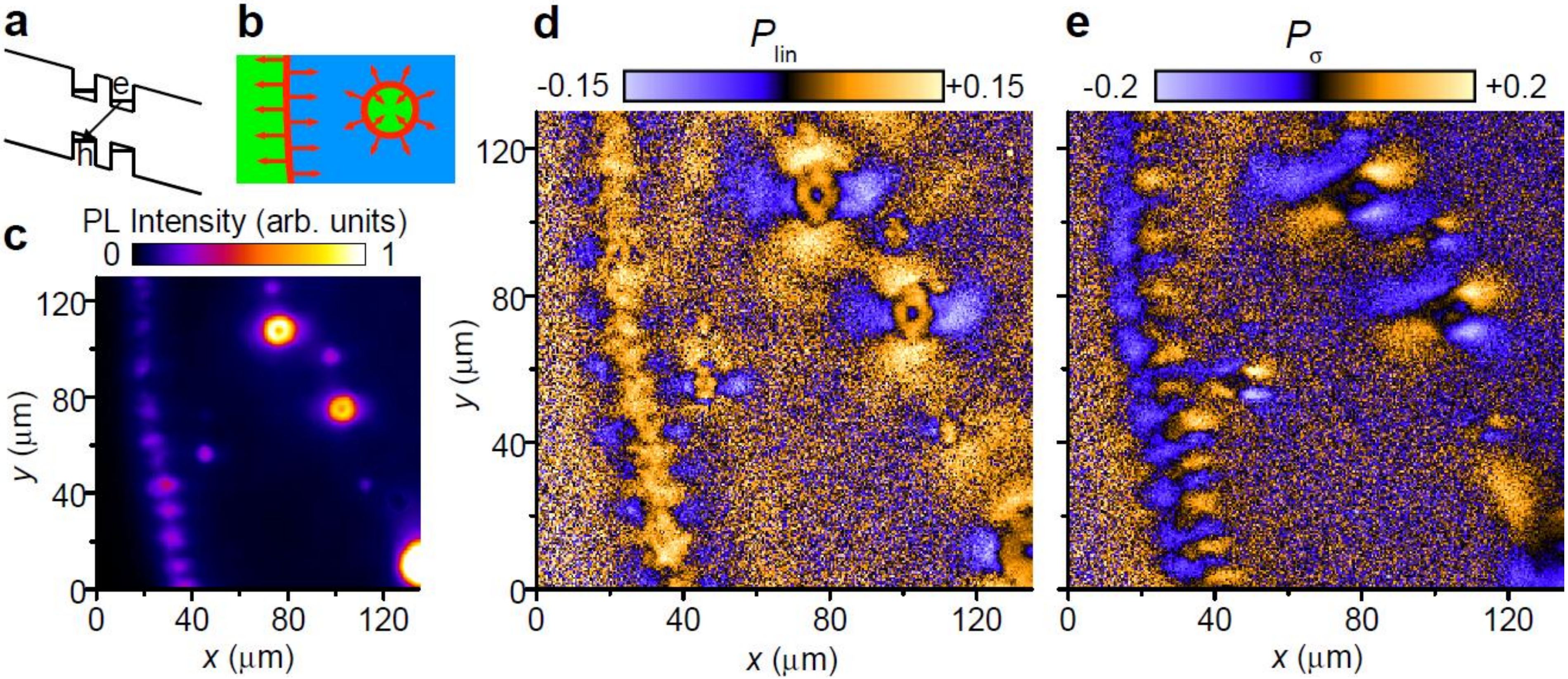
$$\frac{\partial n_X}{\partial t} = D_X \nabla^2 n_X + \underline{w n_e n_h} - n_X / \tau_{opt}$$

$$\underline{w \sim 1 + N_{E=0} = e^{\frac{T_{dB}}{T}} = e^{\frac{2\pi\hbar^2}{m g k_B T} n_x}}$$

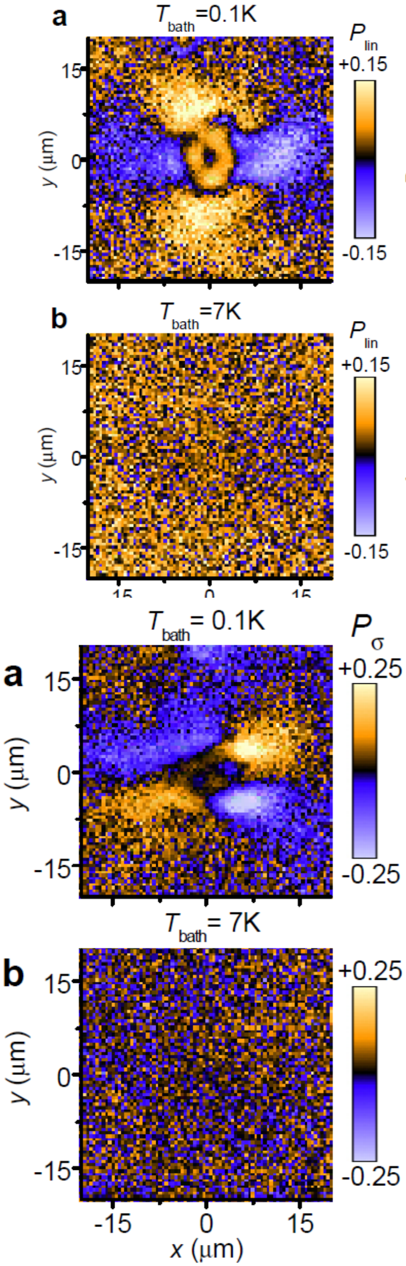
L.S. Levitov, B.D. Simons, L.V. Butov, PRL 94, 176404 (2005)
R. Rapaport et al, Physical Review Letters **92**, 117405 (2004).

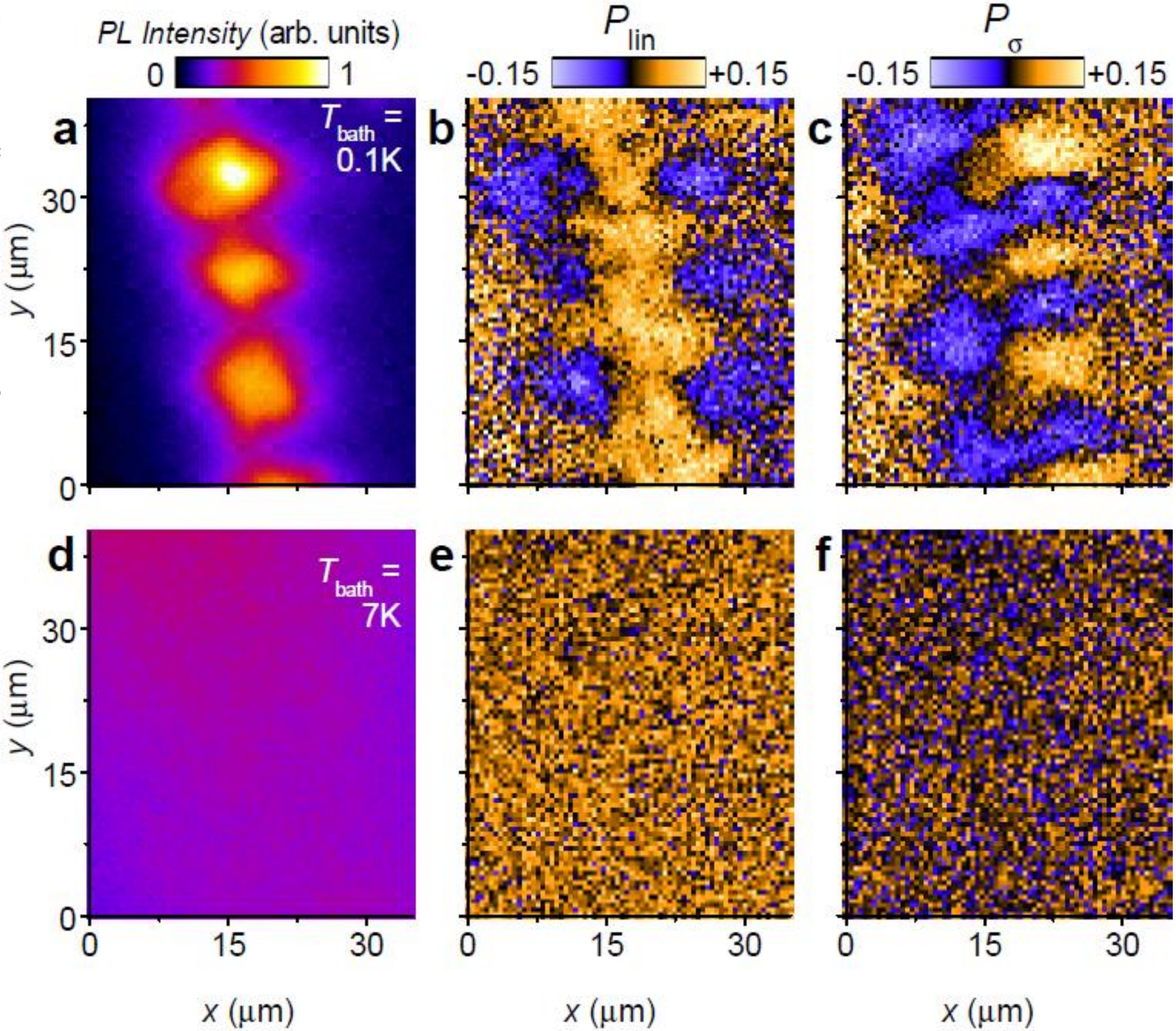
Problem: stimulation of classical particles in real space (need to develop a quantum theory).

Polarisation textures appear near the sources of cold excitons (T=0.1 K)

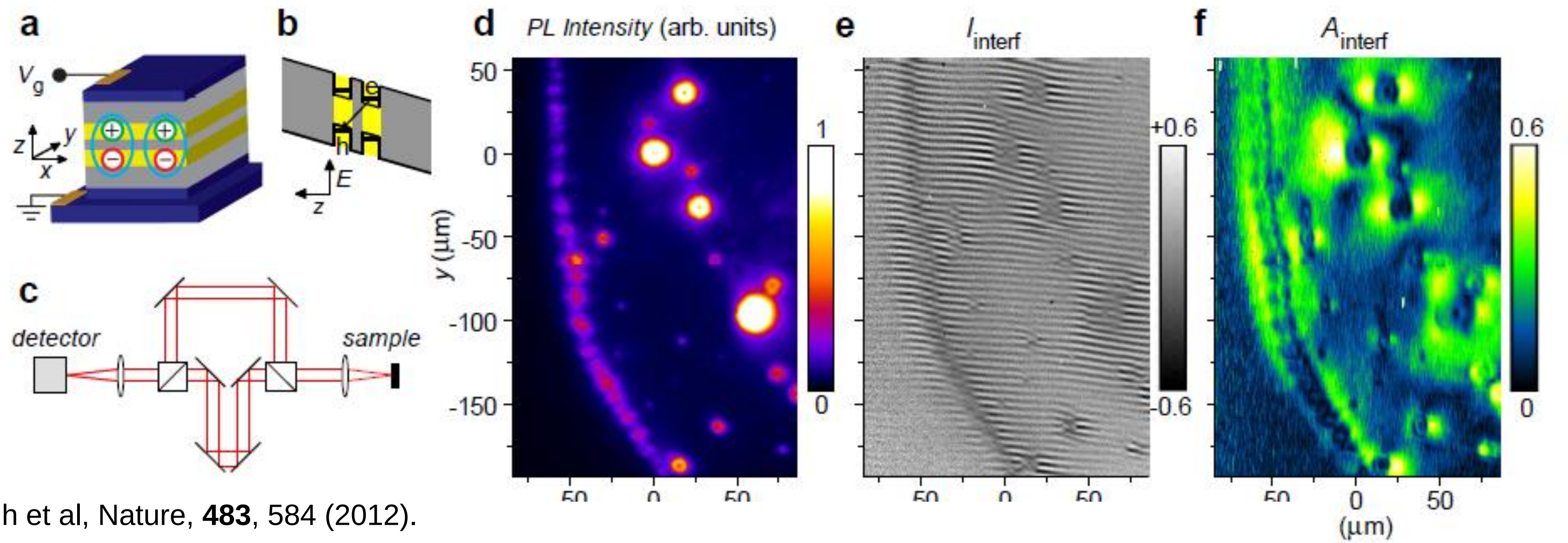


The textures disappear at T=7K

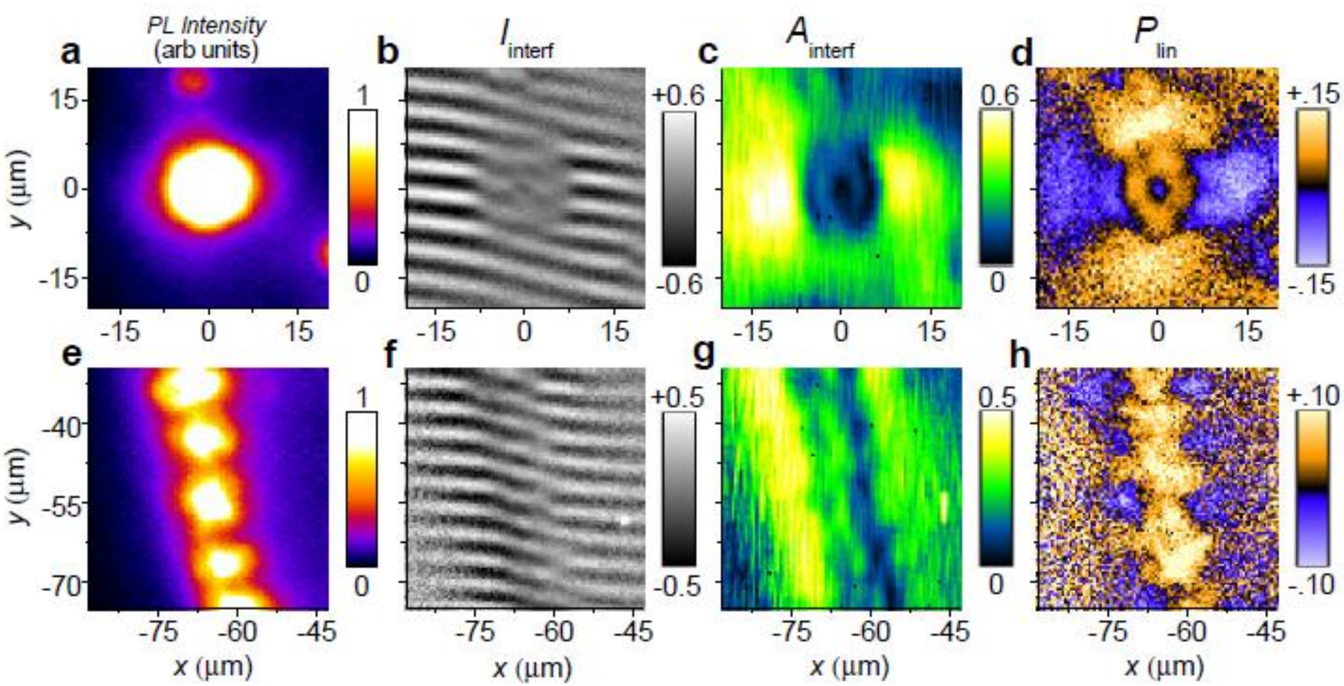




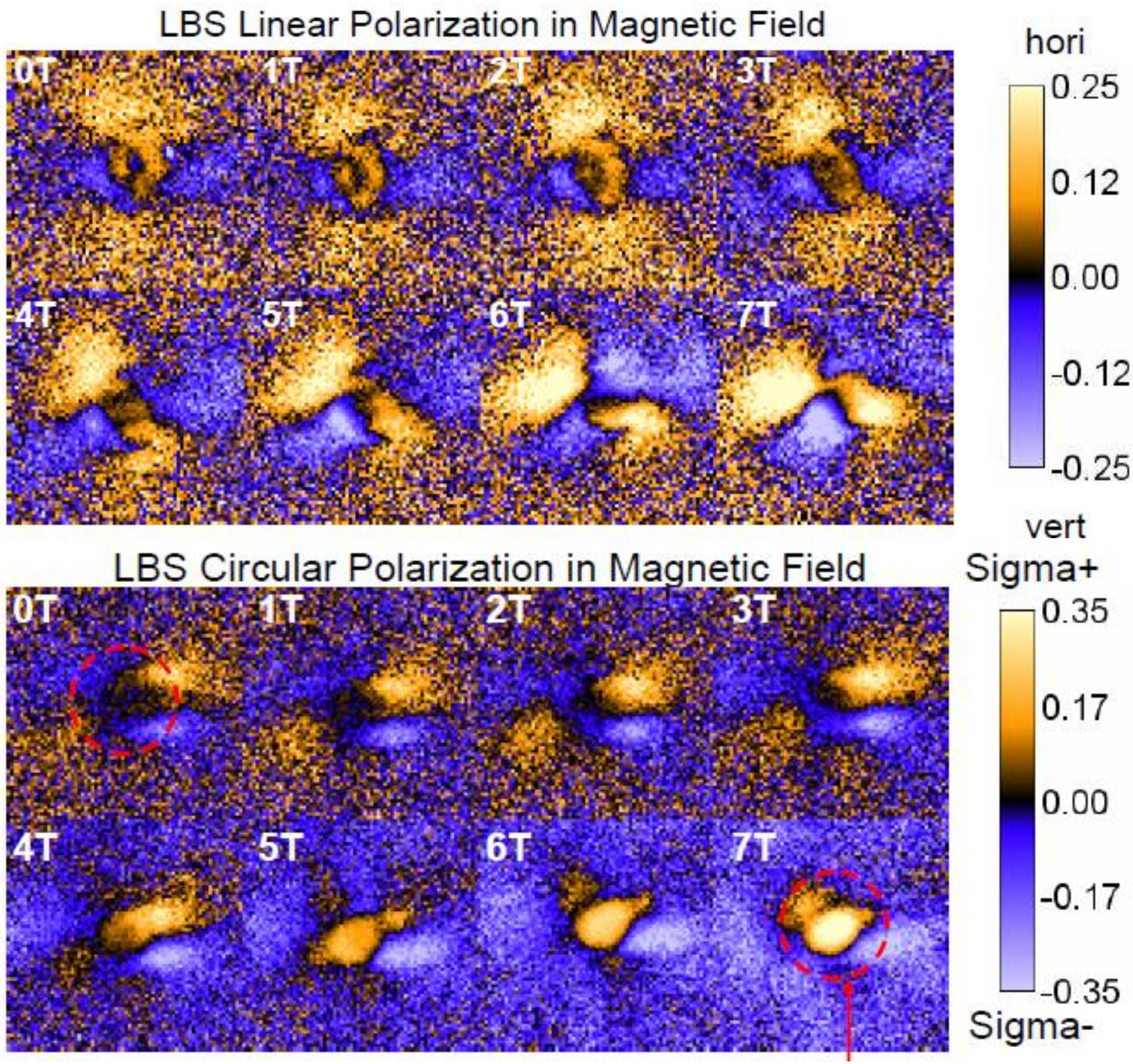
Appearance of polarisation textures is correlated with the build up of spatial coherence



A. High et al, Nature, **483**, 584 (2012).

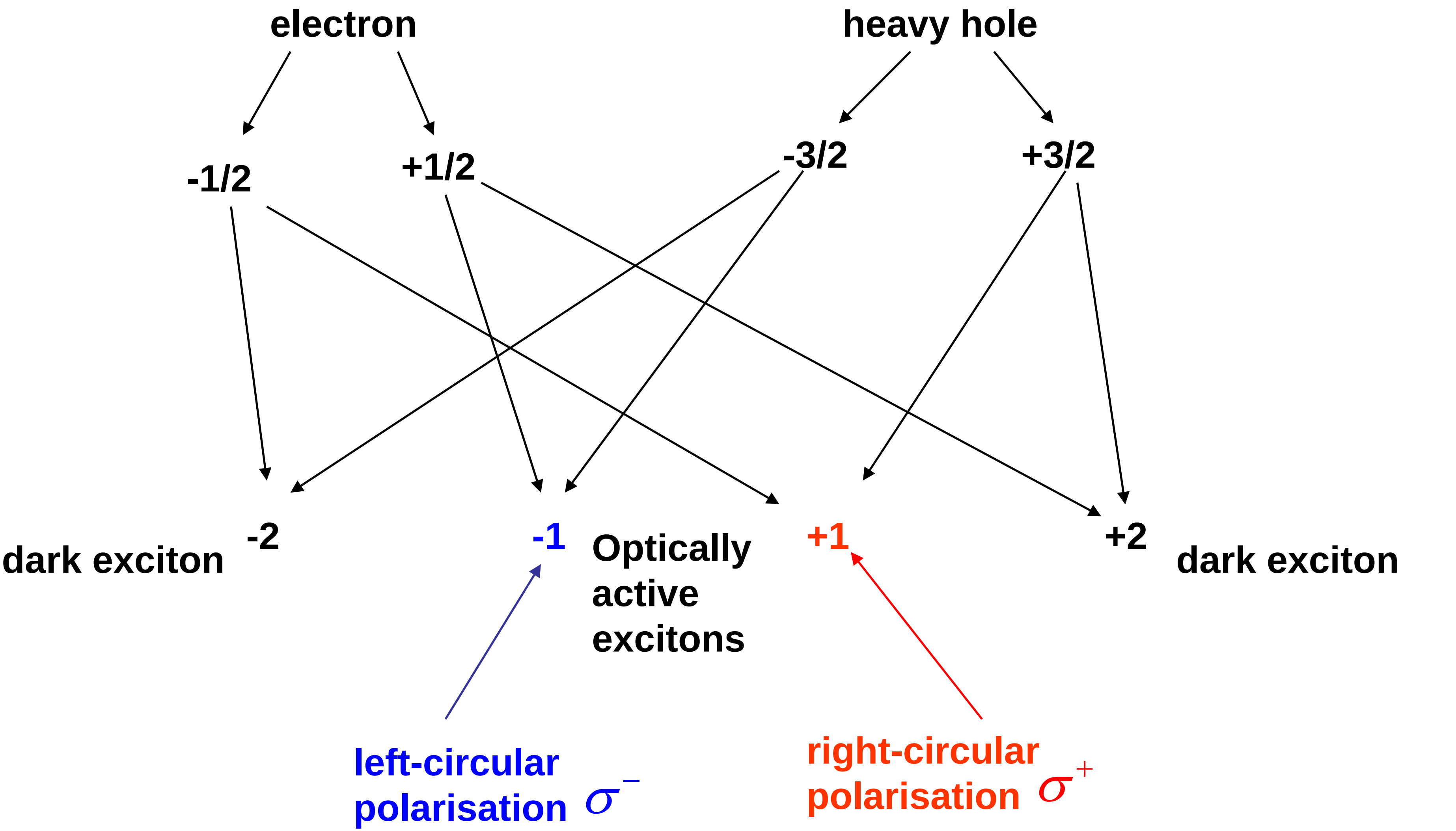


Magnetic field effect (unpublished)



Coupled quantum wells: Four component exciton condensates:

-2, -1, +1, +2 states are degenerate



Exciton spin density matrix

Exciton spinor wave-function (no coordinate-dependent part here)

$$\Psi = (\Psi_{+1}, \Psi_{-1}, \Psi_{+2}, \Psi_{-2},) = \left(\Psi_{e,-1/2}\Psi_{h,+3/2}, \Psi_{e,+1/2}\Psi_{h,-3/2}, \Psi_{e,+1/2}\Psi_{h,+3/2}, \Psi_{e,-1/2}\Psi_{h,-3/2}\right)$$

Normalisation condition

$$\Psi_{+1}\Psi^*_{+1} + \Psi_{-1}\Psi^*_{-1} + \Psi_{+2}\Psi^*_{+2} + \Psi_{-2}\Psi^*_{-2} = \Psi_{e,+\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}} + \Psi_{e,-\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}} = \Psi_{h,+\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} + \Psi_{h,-\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} = 1$$

Exciton spin-density matrix:

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \begin{bmatrix} \Psi_{+1}\Psi^*_{+1} & \Psi_{+1}\Psi^*_{-1} & \Psi_{+1}\Psi^*_{+2} & \Psi_{+1}\Psi^*_{-2} \\ \Psi_{-1}\Psi^*_{+1} & \Psi_{-1}\Psi^*_{-1} & \Psi_{-1}\Psi^*_{+2} & \Psi_{-1}\Psi^*_{-2} \\ \Psi_{+2}\Psi^*_{+1} & \Psi_{+2}\Psi^*_{-1} & \Psi_{+2}\Psi^*_{+2} & \Psi_{+2}\Psi^*_{-2} \\ \Psi_{-2}\Psi^*_{+1} & \Psi_{-2}\Psi^*_{-1} & \Psi_{-2}\Psi^*_{+2} & \Psi_{-2}\Psi^*_{-2} \end{bmatrix}$$

Relation between electron, hole and exciton spin density matrices

$$\hat{\rho} = |\Psi\rangle\langle\Psi| = \begin{bmatrix} \Psi_{+1}\Psi^*_{+1} & \Psi_{+1}\Psi^*_{-1} & \Psi_{+1}\Psi^*_{+2} & \Psi_{+1}\Psi^*_{-2} \\ \Psi_{-1}\Psi^*_{+1} & \Psi_{-1}\Psi^*_{-1} & \Psi_{-1}\Psi^*_{+2} & \Psi_{-1}\Psi^*_{-2} \\ \Psi_{+2}\Psi^*_{+1} & \Psi_{+2}\Psi^*_{-1} & \Psi_{+2}\Psi^*_{+2} & \Psi_{+2}\Psi^*_{-2} \\ \Psi_{-2}\Psi^*_{+1} & \Psi_{-2}\Psi^*_{-1} & \Psi_{-2}\Psi^*_{+2} & \Psi_{-2}\Psi^*_{-2} \end{bmatrix} =$$

$$= \begin{bmatrix} \Psi_{e,-\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} \\ \Psi_{e,+\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} \\ \Psi_{e,+\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,+\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} \\ \Psi_{e,-\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}}\Psi_{h,-\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} \end{bmatrix}$$

Useful relations:

$$\hat{\rho}_e = |\Psi_e\rangle\langle\Psi_e| = \begin{bmatrix} \Psi_{e,+\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}} & \Psi_{e,+\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}} \\ \Psi_{e,-\frac{1}{2}}\Psi^*_{e,+\frac{1}{2}} & \Psi_{e,-\frac{1}{2}}\Psi^*_{e,-\frac{1}{2}} \end{bmatrix} = \begin{bmatrix} \rho_{22}+\rho_{33} & \rho_{24}+\rho_{31} \\ \rho_{13}+\rho_{42} & \rho_{11}+\rho_{44} \end{bmatrix}$$

$$\hat{\rho}_h = |\Psi_h\rangle\langle\Psi_h| = \begin{bmatrix} \Psi_{h,+\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{h,+\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} \\ \Psi_{h,-\frac{3}{2}}\Psi^*_{h,+\frac{3}{2}} & \Psi_{h,-\frac{3}{2}}\Psi^*_{h,-\frac{3}{2}} \end{bmatrix} = \begin{bmatrix} \rho_{11}+\rho_{33} & \rho_{14}+\rho_{32} \\ \rho_{23}+\rho_{41} & \rho_{22}+\rho_{44} \end{bmatrix}$$

Here we used the normalisation condition!

Electron and hole spins are linked with the exciton density matrix elements!

$$\hat{\rho}_e = \begin{bmatrix} \frac{1}{2} + S_{e,z} & S_{e,x} - iS_{e,y} \\ S_{e,x} + iS_{e,y} & \frac{1}{2} - S_{e,z} \end{bmatrix}, \hat{\rho}_h = \begin{bmatrix} \frac{1}{2} + S_{h,z} & S_{h,x} - iS_{h,y} \\ S_{h,x} + iS_{h,y} & \frac{1}{2} - S_{h,z} \end{bmatrix}$$

$$S_{e,z} = (\rho_{22} + \rho_{33} - \rho_{11} - \rho_{44})/2,$$

$$S_{h,z} = (\rho_{11} + \rho_{33} - \rho_{22} - \rho_{44})/2.$$

$$S_{e,x} = (\rho_{13} + \rho_{31} + \rho_{24} + \rho_{42})/2,$$

$$S_{h,x} = (\rho_{14} + \rho_{23} + \rho_{32} + \rho_{41})/2,$$

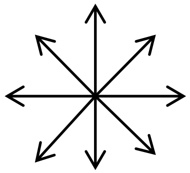
$$S_{e,y} = i(-\rho_{13} + \rho_{31} + \rho_{24} - \rho_{42})/2,$$

$$S_{h,y} = i(\rho_{14} - \rho_{23} + \rho_{32} - \rho_{41})/2.$$

All we know from traditional spintronics can be applied to excitons once we remember that they are composed by electrons and holes!

Our model:

- Excitons propagate ballistically in radial directions; *r=vt*
- Dark and bright excitons are mixed by spin-orbit interaction.
- Electron and hole spins are rotated by the Dresselhaus field;
- Supplementary beats appear due to linear polarisation splittings;



We solve the Liouville equation for a 4x4 spin density matrix:

$$i\hbar \frac{d\hat{p}}{dt} = [\widehat{H}, \hat{p}] \qquad \hat{\rho} = |\Psi><\Psi| \qquad \Psi = (\Psi_{+1}, \Psi_{-1}, \Psi_{+2}, \Psi_{-2})$$

The polarisation degrees are given by:

$$\rho_l = \frac{2S_x}{I} = (\rho_{12} + \rho_{21})/(\rho_{11} + \rho_{22})$$

$$\rho_c = \frac{2S_z}{I} = (\rho_{11} - \rho_{22})/(\rho_{11} + \rho_{22})$$

What contributes to the exciton Hamiltonian?

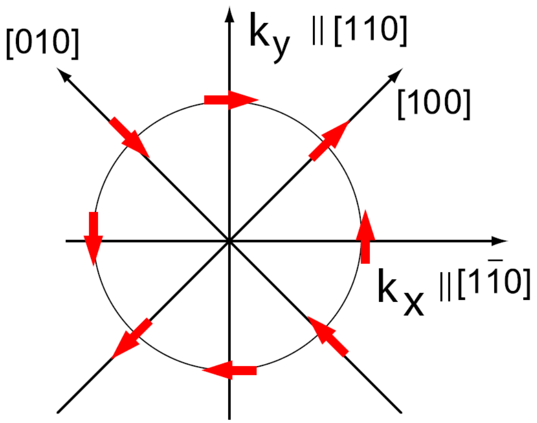
Dresselhaus and Zeeman effects on electrons:

Dresselhaus spin-orbit Hamiltonian

$$H_{eff(e,h)} = -\frac{1}{2} g_{e,h} \mu_B \left(\mathbf{B}_{eff} \hat{\sigma} \right)$$

$$-\frac{1}{2} g_e \mu_B \mathbf{B}_{eff} = \beta_e \left(k_{e,x}, -k_{e,y} \right)$$

$$H_e = \beta_e \left(k_{e,x} \sigma_x - k_{e,y} \sigma_y \right) - \frac{1}{2} g_e \mu_B B \sigma_z$$



Dresselhaus and Zeeman effects on electrons: exciton Hamiltonian

$$H_e = \beta_e\left(k_{e,x}\sigma_x - k_{e,y}\sigma_y\right) - \frac{1}{2}g_e\mu_B B\sigma_z$$

Hence:

$$H_e = \begin{bmatrix} -\frac{1}{2}g_e\mu_B B & \beta_e\left(k_{e,x} + ik_{e,y}\right) \\ \beta_e\left(k_{e,x} - ik_{e,y}\right) & \frac{1}{2}g_e\mu_B B \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}g_e\mu_B B & \beta_e k_e e^{i\varphi} \\ \beta_e k_e e^{-i\varphi} & \frac{1}{2}g_e\mu_B B \end{bmatrix}.$$

In the exciton basis:

(-1/2, +1/2, +1/2, -1/2) $\longrightarrow$ (+1,-1,+2,-2)

$$\widehat{H_e} = \begin{bmatrix} g_e\mu_B B/2 & 0 & k_e\beta_e e^{-i\varphi} & 0 \\ 0 & -g_e\mu_B B/2 & 0 & k_e\beta_e e^{i\varphi} \\ k_e\beta_e e^{i\varphi} & 0 & -g_e\mu_B B/2 & 0 \\ 0 & k_e\beta_e e^{-i\varphi} & 0 & g_e\mu_B B/2 \end{bmatrix}.$$

Dresselhaus and Zeeman effects for holes: $-\frac{1}{2}g_h\mu_B\mathbf{B}_{eff} = \beta_h\left(k_{e,x}, k_{e,y}\right)$

In the (+3/2,-3/2) basis:

$$H_h = \beta_h\left(k_{h,x}\sigma_x + k_{h,y}\sigma_y\right) - \frac{1}{2}g_h\mu_B B\sigma_z.$$

$$H_h = \begin{bmatrix} -\frac{1}{2}g_h\mu_B B & \beta_h\left(k_{h,x} - ik_{h,y}\right) \\ \beta_h\left(k_{h,x} + ik_{h,y}\right) & \frac{1}{2}g_h\mu_B B \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}g_h\mu_B B & \beta_h k_h e^{-i\varphi} \\ \beta_h k_h e^{i\varphi} & \frac{1}{2}g_h\mu_B B \end{bmatrix}$$

In the exciton basis:

$$(+3/2,\ -3/2,\ +3/2,\ -3/2) \longrightarrow (+1,-1,+2,-2)$$

$$\widehat{H_h} = \begin{bmatrix} -g_h\mu_B B/2 & 0 & 0 & k_h\beta_h e^{-i\varphi} \\ 0 & g_h\mu_B B/2 & k_h\beta_h e^{i\varphi} & 0 \\ 0 & k_h\beta_h e^{-i\varphi} & -g_h\mu_B B/2 & 0 \\ k_h\beta_h e^{i\varphi} & 0 & 0 & g_h\mu_B B/2 \end{bmatrix}.$$

Long- and short-range exchange splittings of 4 exciton states:

Splitting of bright excitons:

$$H_b = E_b I - \delta_b \sigma_x = \begin{bmatrix} E_b & -\delta_b \\ -\delta_b & E_b \end{bmatrix},$$

Splitting of dark excitons:

$$H_d = E_d I - \delta_d \sigma_x = \begin{bmatrix} E_d & -\delta_d \\ -\delta_d & E_d \end{bmatrix}$$

In the exciton basis (+1,-1,+2,-2)

$$H_0 = \begin{bmatrix} E_b & -\delta_b & 0 & 0 \\ -\delta_b & E_b & 0 & 0 \\ 0 & 0 & E_d & -\delta_d \\ 0 & 0 & -\delta_d & E_d \end{bmatrix}.$$

All terms together, we obtain the exciton Hamiltonian:

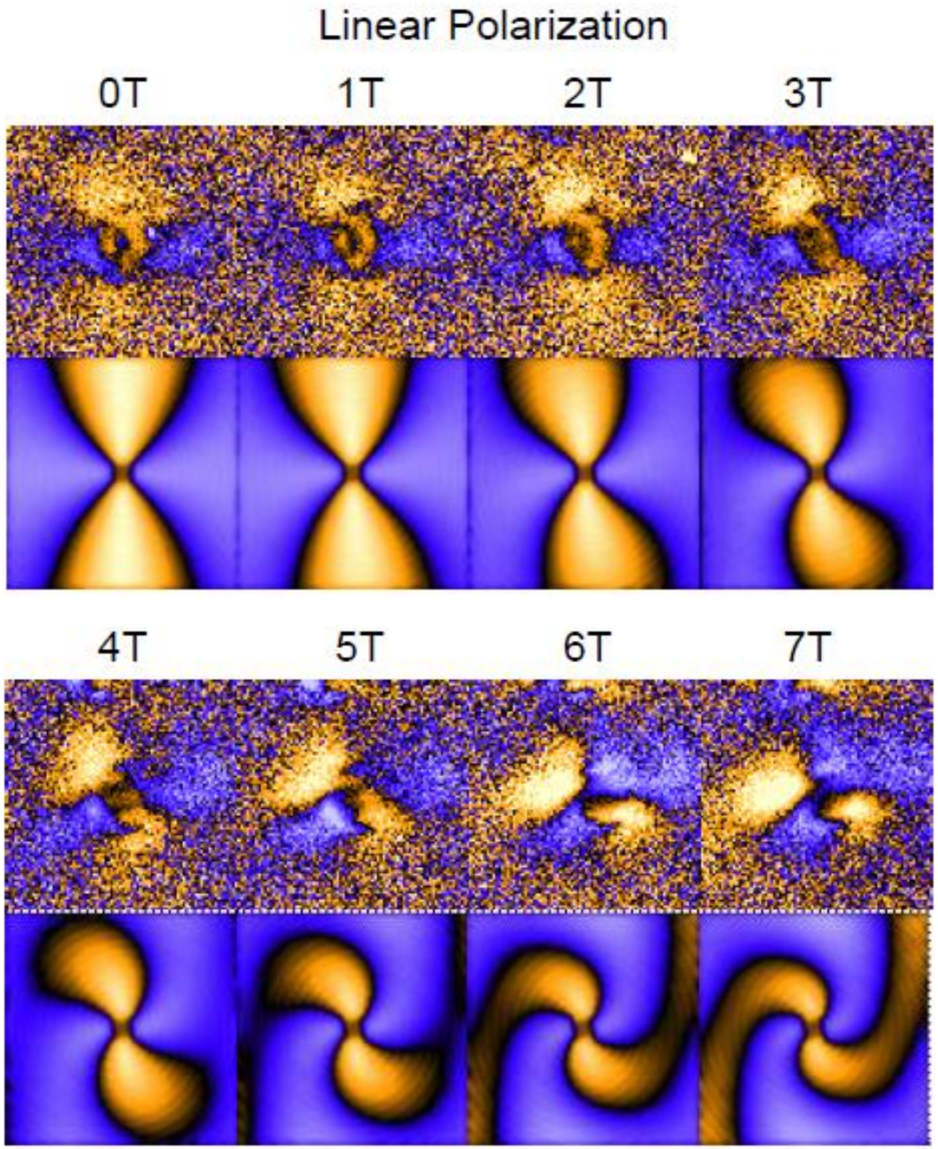
$$\hat{H} = \begin{bmatrix} E_b - (g_h - g_e)\mu_B B/2 & -\delta_b & k_e\beta_e e^{-i\varphi} & k_h\beta_h e^{-i\varphi} \\ -\delta_b & E_b + (g_h - g_e)\mu_B B/2 & k_h\beta_h e^{i\varphi} & k_e\beta_e e^{i\varphi} \\ k_e\beta_e e^{i\varphi} & k_h\beta_h e^{-i\varphi} & E_d - (g_h + g_e)\mu_B B/2 & -\delta_d \\ k_h\beta_h e^{i\varphi} & k_e\beta_e e^{-i\varphi} & -\delta_d & E_d + (g_h + g_e)\mu_B B/2 \end{bmatrix}.$$

$$k_{ex} = k_h + k_e$$

$$k_e = \frac{m_e}{m_e + m_{hh}} k_{ex}, \qquad k_h = \frac{m_{hh}}{m_e + m_{hh}} k_{ex}.$$

Hidden asymmetry between bright and dark excitons!

Theory vs experiment:



$$g_e = 0.010$$

$$g_h = -0.0085$$

$$E_b - E_d = -1\mu eV$$

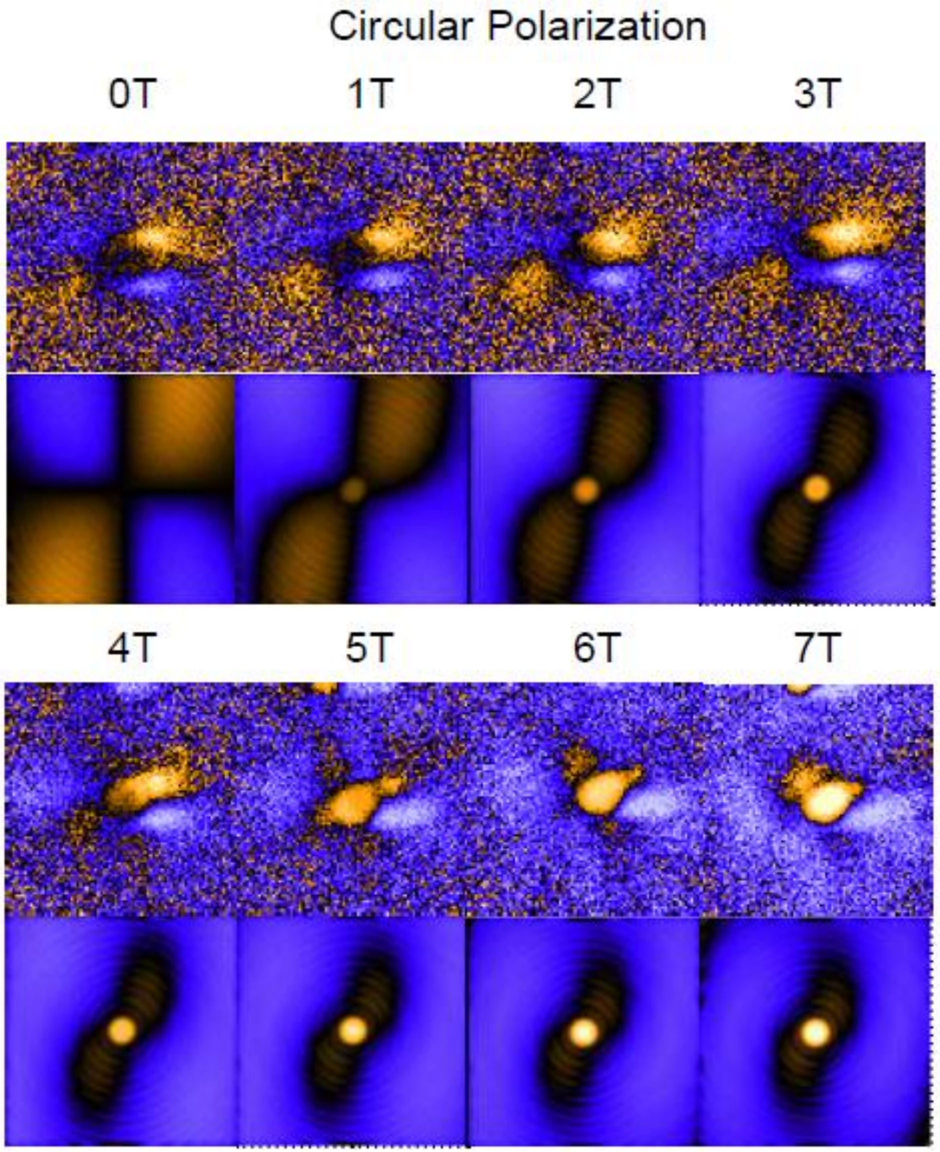
$$\delta_b = 0.55\mu eV$$

$$\delta_d = -13\mu eV$$

$$k = 15.4\mu m^{-1}$$

T=0.1 K

Theory vs experiment:



$g_e = 0.010$

$g_h = -0.0085$

$E_b - E_d = -1\mu eV$

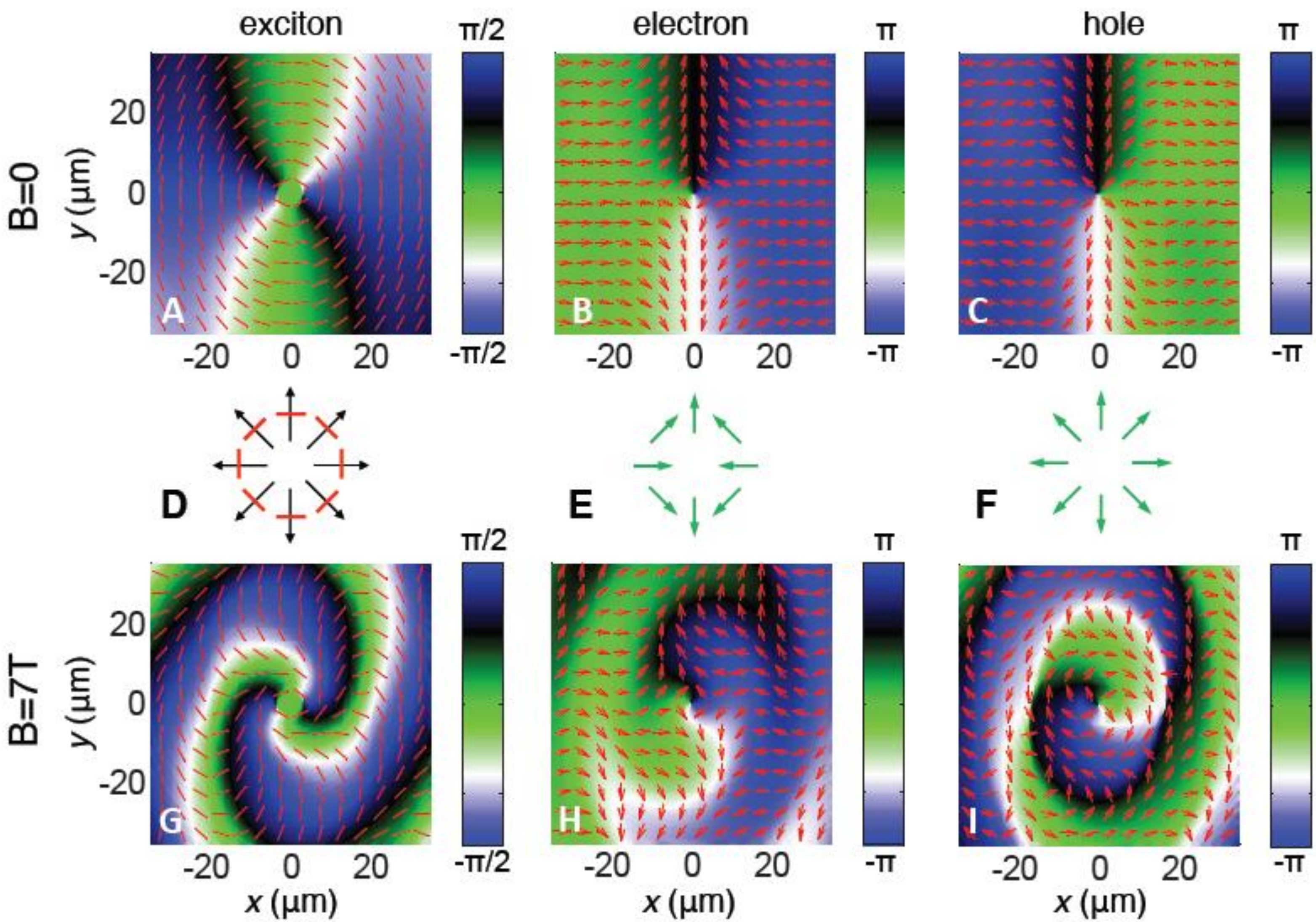
$\delta_b = 0.55\mu eV$

$\delta_d = -13\mu eV$

$k = 15.4\mu m^{-1}$

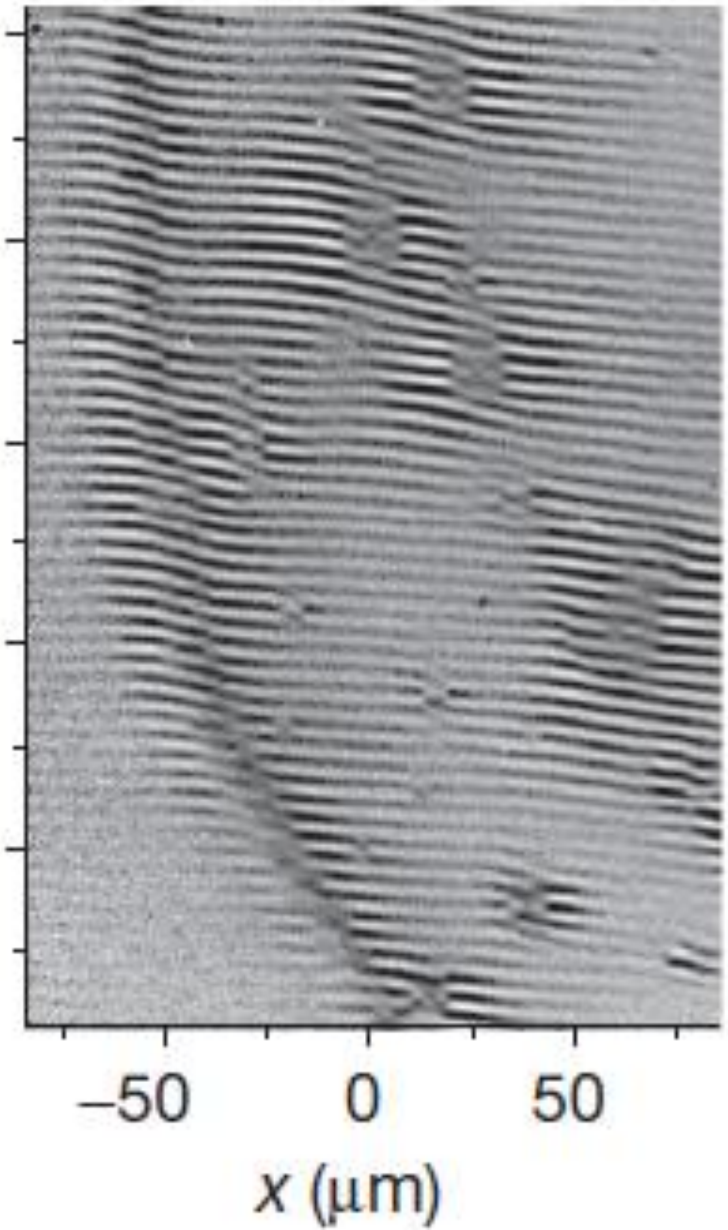
T=0.1 K

Excitons carry electron and hole spin currents!

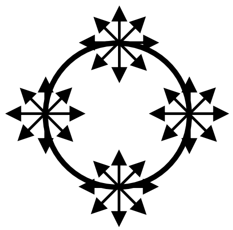


Simulations of the interferograms: forks are not vortices but sources of excitons!

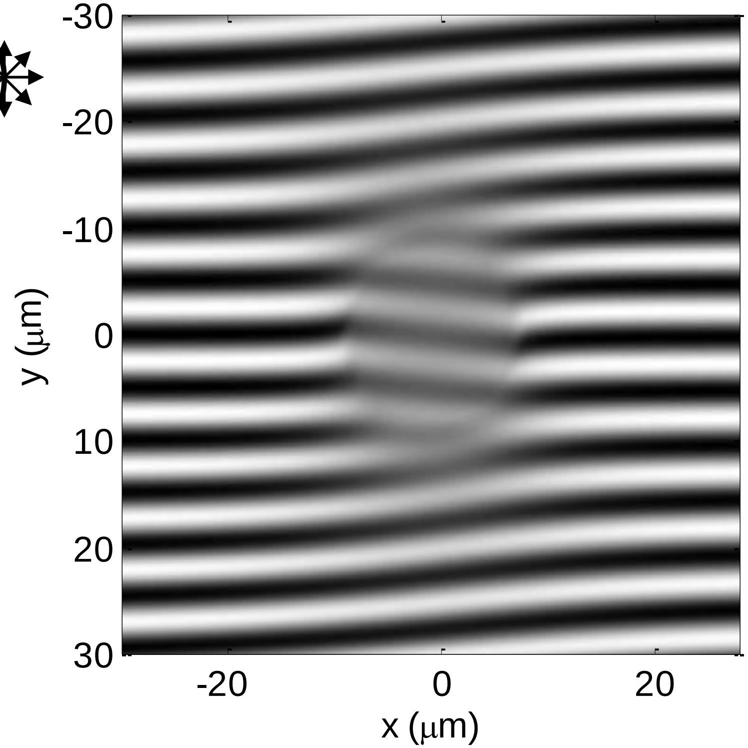
experiment



A small “inner ring” emits excitons in all directions



theory



$$I_{12} \doteq \sum_{\vec{R}_0} \left| e^{i\phi\left(\left|\vec{R}-\vec{R}_0\right|\right)} + e^{ik_y y} e^{i\phi\left(\left|\vec{R}-\vec{R}_0+\delta\vec{r}\right|\right)} \right|^2$$

Spin-orbital coupling and topology of spin-degenerate cold exciton gases

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Gross-Pitaevskii equations:

$$i\hbar\frac{\partial\Psi_{+1}}{\partial t}=-\frac{\hbar^2\nabla^2}{2m_{ex}}\Psi_{+1}+\beta_e\left(\hat{k}_x-i\hat{k}_y\right)\Psi_{+2}+\beta_h\left(\hat{k}_x-i\hat{k}_y\right)\Psi_{-2}$$
$$+\alpha_1|\Psi_{+1}|^2\Psi_{+1}+\alpha_2|\Psi_{-1}|^2\Psi_{+1}+\alpha_3|\Psi_{+2}|^2\Psi_{+1}+\alpha_4|\Psi_{-2}|^2\Psi_{+1}+\underline{W\Psi^*_{-1}\Psi_{+2}\Psi_{-2}}$$

$$i\hbar\frac{\partial\Psi_{-1}}{\partial t}=-\frac{\hbar^2\nabla^2}{2m_{ex}}\Psi_{-1}+\beta_e\left(\hat{k}_x+i\hat{k}_y\right)\Psi_{-2}+\beta_h\left(\hat{k}_x+i\hat{k}_y\right)\Psi_{+2}$$
$$+\alpha_1|\Psi_{-1}|^2\Psi_{-1}+\alpha_2|\Psi_{+1}|^2\Psi_{-1}+\alpha_3|\Psi_{-2}|^2\Psi_{-1}+\alpha_4|\Psi_{+2}|^2\Psi_{-1}+\underline{W\Psi^*_{+1}\Psi_{+2}\Psi_{-2}}$$

$$i\hbar\frac{\partial\Psi_{+2}}{\partial t}=-\frac{\hbar^2\nabla^2}{2m_{ex}}\Psi_{+2}+\beta_e\left(\hat{k}_x+i\hat{k}_y\right)\Psi_{+1}+\beta_h\left(\hat{k}_x-i\hat{k}_y\right)\Psi_{-1}$$
$$+\alpha_1|\Psi_{+2}|^2\Psi_{+2}+\alpha_2|\Psi_{-2}|^2\Psi_{+2}+\alpha_3|\Psi_{+1}|^2\Psi_{+2}+\alpha_4|\Psi_{-1}|^2\Psi_{+2}+\underline{W\Psi^*_{-2}\Psi_{+1}\Psi_{-1}}$$

$$i\hbar\frac{\partial\Psi_{-2}}{\partial t}=-\frac{\hbar^2\nabla^2}{2m_{ex}}\Psi_{-2}+\beta_e\left(\hat{k}_x-i\hat{k}_y\right)\Psi_{-1}+\beta_h\left(\hat{k}_x+i\hat{k}_y\right)\Psi_{+1}$$
$$+\alpha_1|\Psi_{-2}|^2\Psi_{-2}+\alpha_2|\Psi_{+2}|^2\Psi_{-2}+\alpha_3|\Psi_{-1}|^2\Psi_{-2}+\alpha_4|\Psi_{+1}|^2\Psi_{-2}+\underline{W\Psi^*_{+2}\Psi_{+1}\Psi_{-1}}$$

$$k_{x,y}=-i\nabla_{x,y}$$

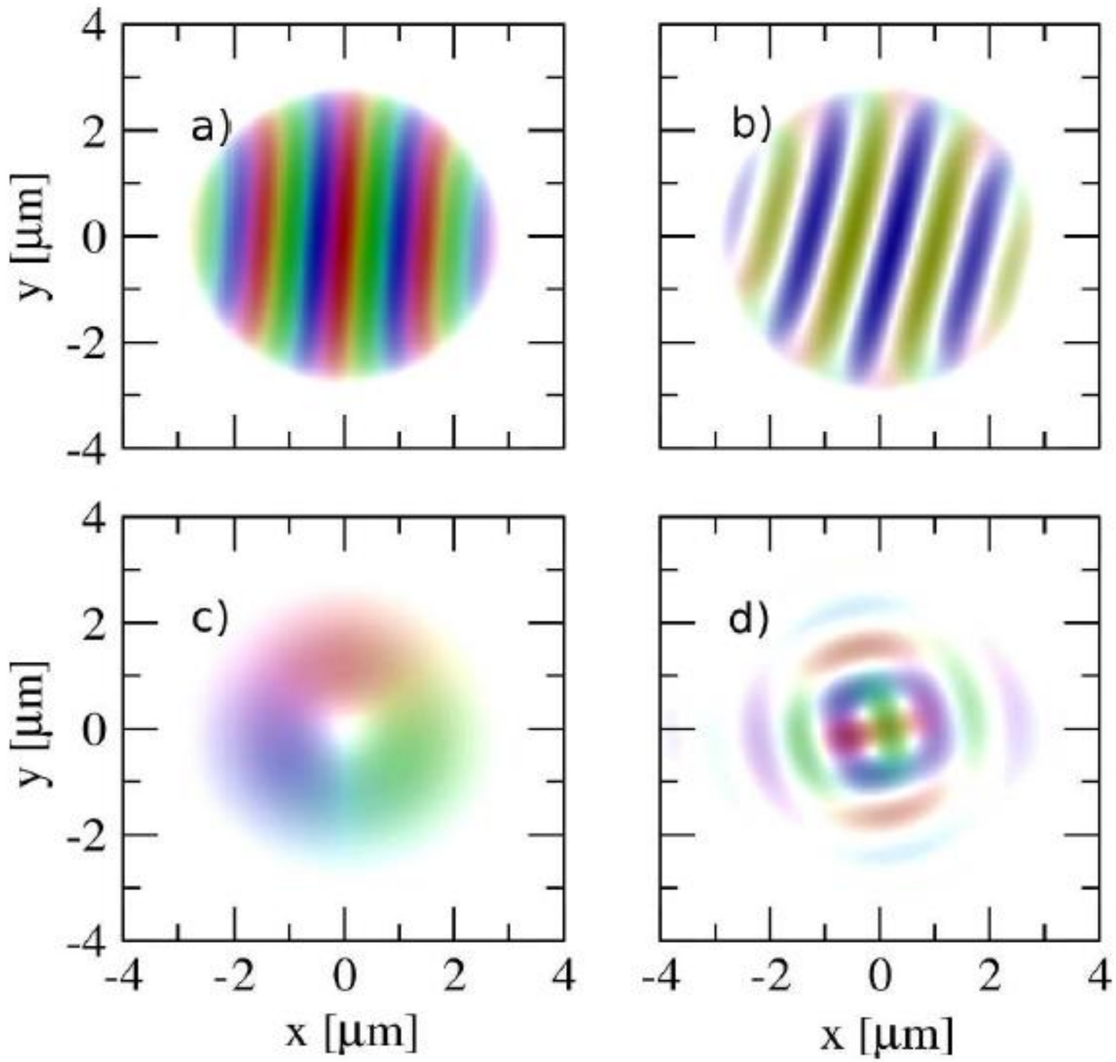
with external harmonic potential $U(\mathbf{r}_\perp)=M\omega^2\mathbf{r}_\perp^2/2$

Examples of spin structures of the condensates in the traps

Color=phase,
Darkness=
intensity
Weak trap

plane wave
(W<0)

Polarisation
domains
(W>0)



Vortex (weak spin-orbit coupling)

multiple vortices (strong spin-orbit coupling)

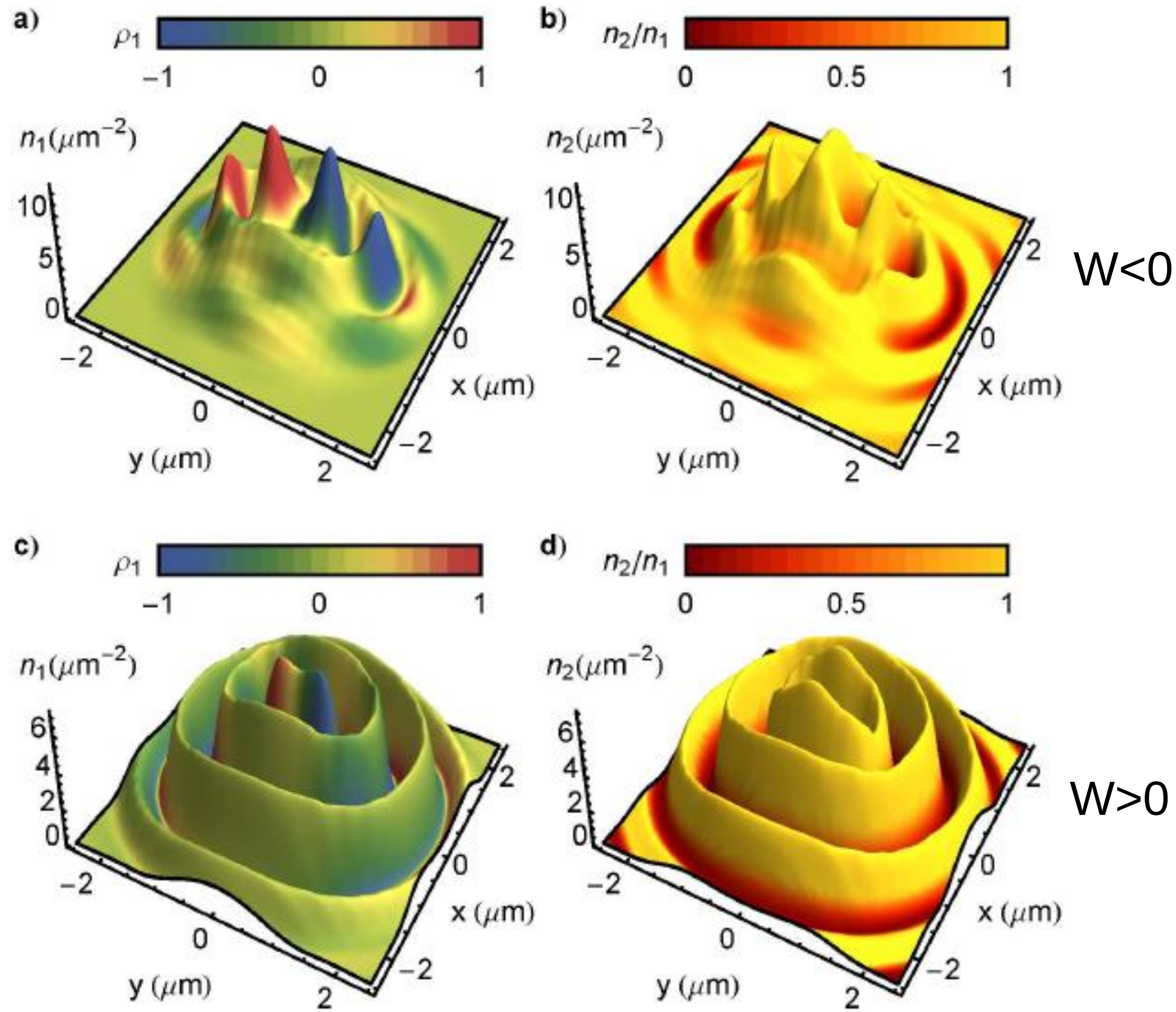
Conditions on winding numbers:

$$m_{+2} + m_{-2} = m_{+1} + m_{-1}$$

$$m_{+2} = m_{+1} + 1 = m_{-1} - 1 = m_{-2}$$

Exciton condensates in a “strong trap”

- Phase separation of bright and dark excitons
- Spin polarisation of bright excitons



Bright excitons Dark excitons

Strong similarity with experimental observations: N. W. Sinclair, et al, Strain-induced darkening of trapped excitons in coupled quantum wells at low temperature, Phys. Rev. B **83**, 245304 (2011)

Conclusions:

- At 100 mK the gas of indirect excitons forms coherent polarisation textures
- The experiment is interpreted in terms of ballistic motion of excitons and Dresselhaus effect on electrons and holes
- Magnetic field creates polarisation currents non colinear to the mass currents
- Magnetic field induced classical polarisation vortices are observed
- Excitons carry electron and hole spin currents
- Interesting non-trivial topology of condensates in traps is expected