Equilibration of electrons and conductance of quantum wires

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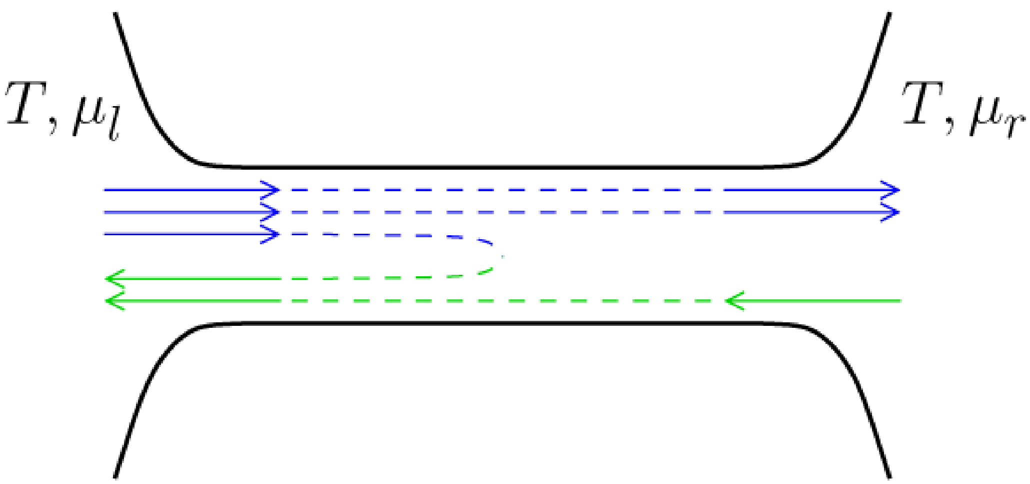
Nordita, September 19, 2012



$$\mu_l \neq \mu_r$$

↓

no equilibrium

Collaborations and Publications

Collaborators: A. Andreev, T. Micklitz, M. Pustilnik, J. Rech

Publications:

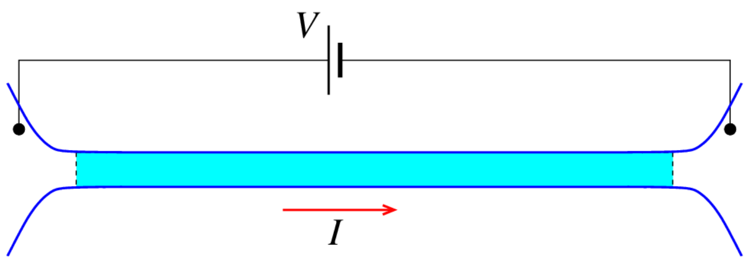
PRL **102**, 116402 (2009)
PRB **81**, 115313 (2010)
PRL **105**, 046401 (2010)
PRL **107**, 056405 (2011)
PRB **85**, 041102 (2012)
PRB 86, 045136 (2012)

Outline

Motivation: Experiments with quantum point contacts

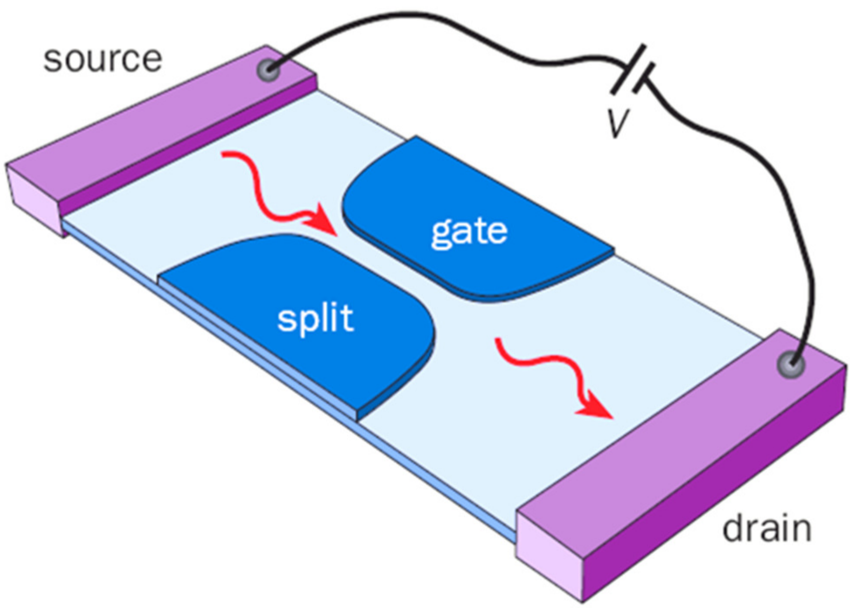
- Quantized conductance;
- Temperature dependent corrections & 0.7 structure.

Theory: Transport in long uniform quantum wires

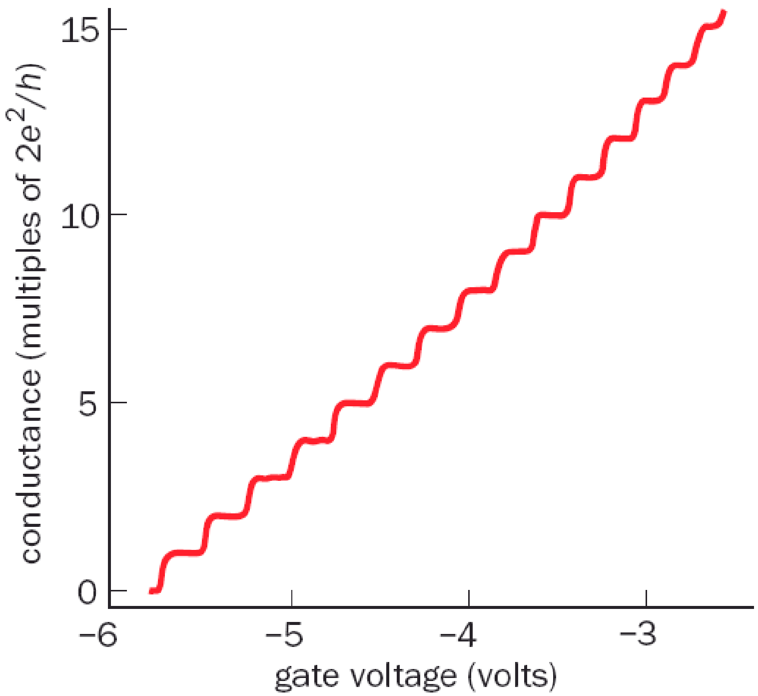


- Corrections to conductance are caused by backscattering of electrons;
- Backscattering can be quantified in terms of rate of equilibration of electrons in the wire;
- Equilibration is controlled by spin excitations.

Quantum point contacts

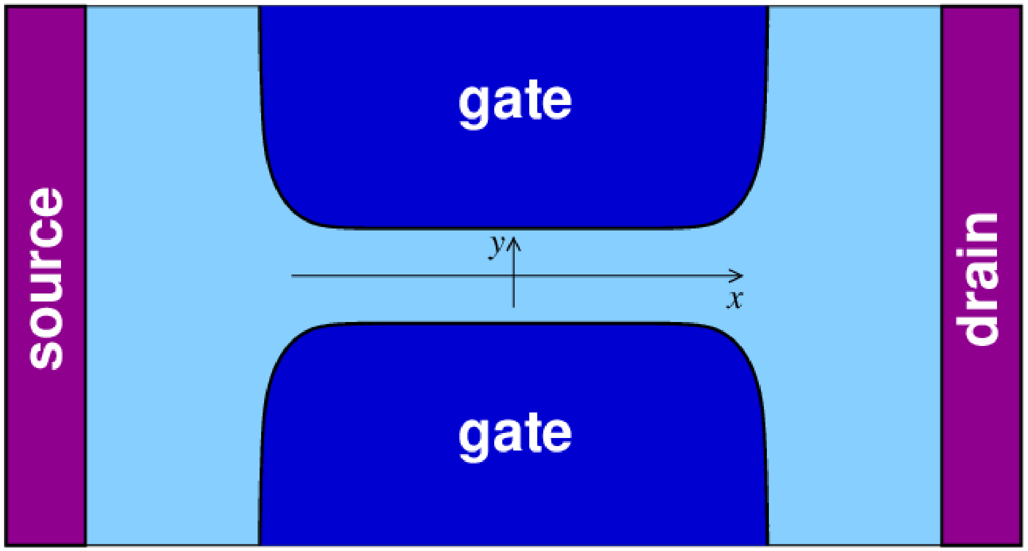


From Berggren & Pepper, 2002

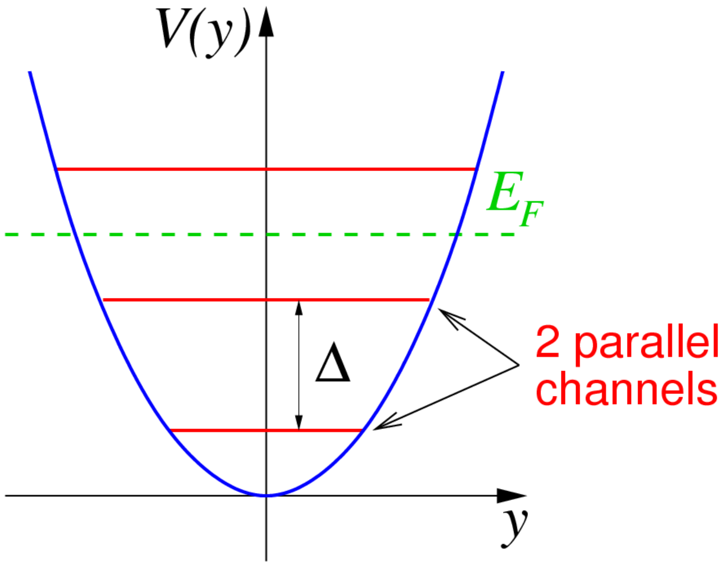


As a function of gate voltage conductance shows multiple steps of height $G_0 = \frac{2e^2}{h}$

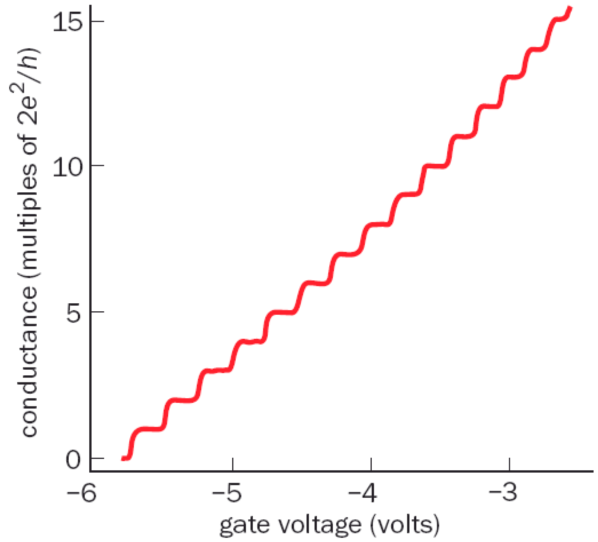
Why steps?



Top view

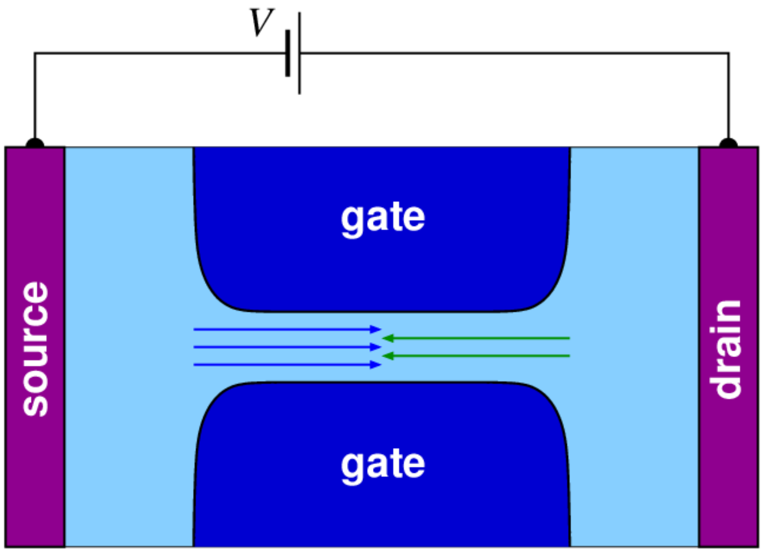


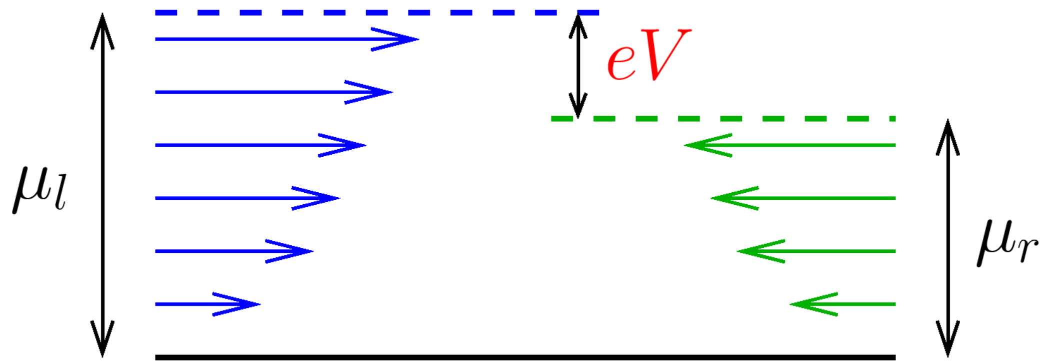
Electron motion across the channel



- Gate voltage changes the number of parallel channels in the wire
- Each channel has conductance $G_0 = 2e^2/h$

Conductance of a single channel





Current:

$$I = 2e\int_0^\infty \frac{dp}{h} v_p[n_F(\epsilon_p-\mu_l)-n_F(\epsilon_p-\mu_r)]$$
$$= \frac{2e}{h}(\mu_l-\mu_r)\int_0^\infty d\epsilon\left(-\frac{\partial n_F}{\partial \epsilon}\right)$$
$$= \frac{2e^2}{h} V n_F(0)$$

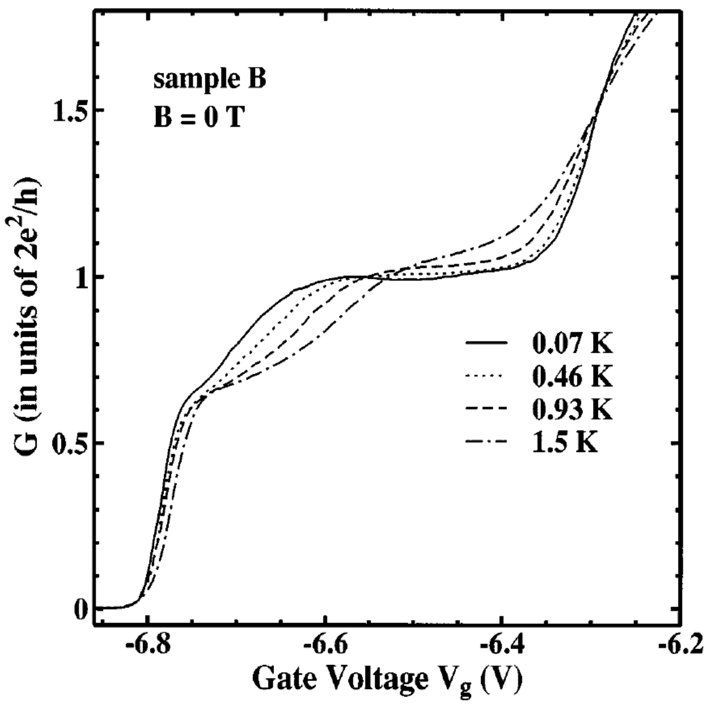
Conductance:

$$G = \frac{2e^2}{h}\frac{1}{1+e^{-\mu/T}}$$

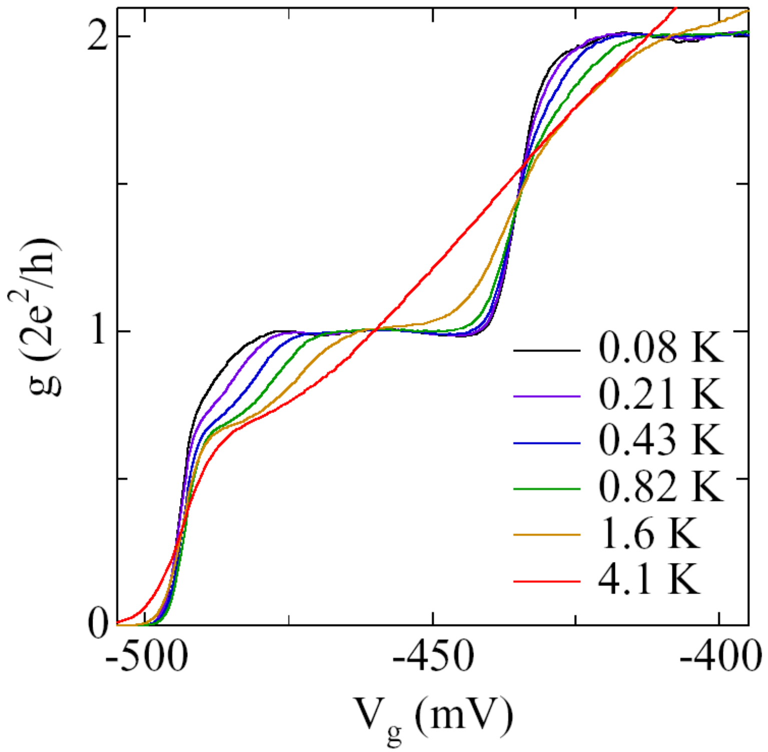
$$G = \frac{2e^2}{h} \text{ at } \mu \gg T$$

0.7 structure

Conductance vs. gate voltage at different temperatures:



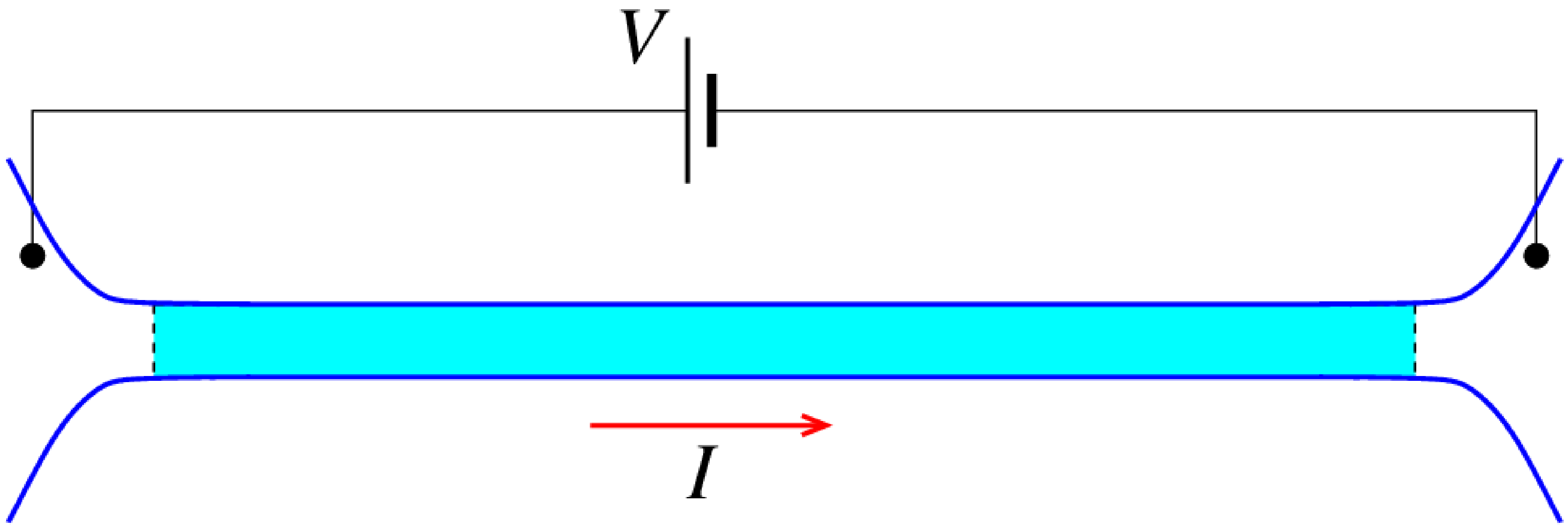
Thomas *et al.*, 1996



Cronenwett *et al.*, 2001

As the temperature grows, a negative correction to the conductance appears, often developing into a shoulder near $0.7(2e^2/h)$

The problem



Long uniform quantum wire:

- Interactions inside the wire
- No interactions in the leads
- No disorder

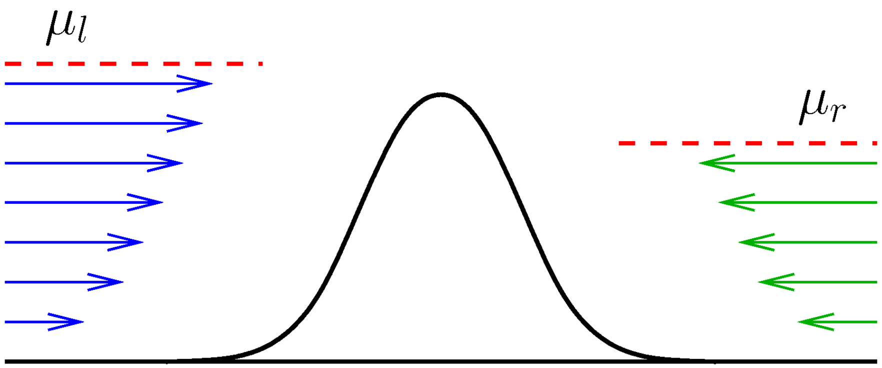
What is the conductance?

Origin of the corrections to conductance: Backscattering

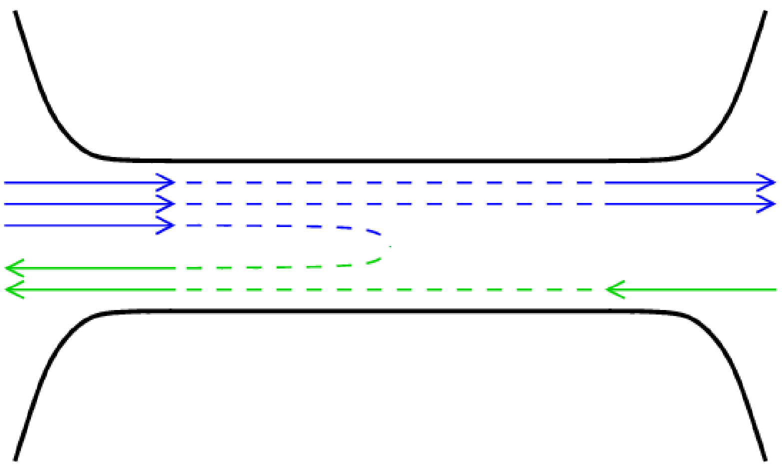
Special case: Potential barrier

Landauer formula $G = \frac{e^2}{h} T(E_F)$

Only the electrons that pass through the barrier contribute to conductance



General case: Any backscattering in the wire



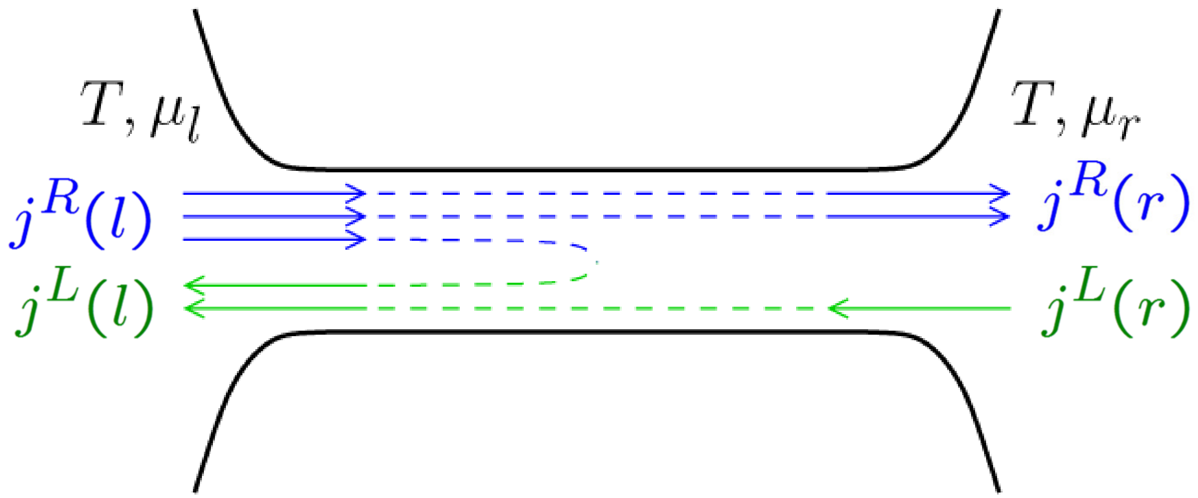
$$I = \frac{e^2}{h} V + e\dot{N}^R$$

Even if there is no barrier, backscattering can be caused by interactions between electrons

Correction to conductance due to backscattering of electrons

- Consider the current of right-moving electrons

$$j^R(r) = j^R(l) + \dot{N}^R$$



- The total current

$$j = j^R(r) + j^L(r)$$

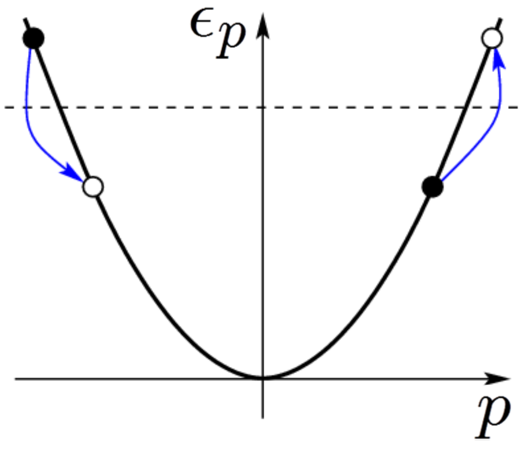
- Exclude the outgoing current $j^R(r)$

$$j = j^R(l) + j^L(r) + \dot{N}^R$$

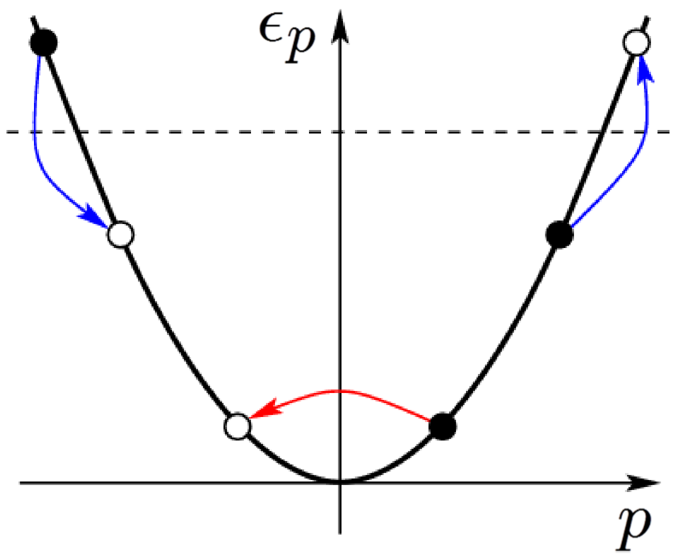
- Apply Landauer formula to incoming currents

$$I = \frac{e^2}{h} V + e\dot{N}^R$$

Interactions give rise to backscattering



Two-particle collisions do not change the momenta of the electrons because of the conservation laws



Three particle collisions can change the number of right-moving electrons:

Correction to the conductance of a quantum wire:

$$\delta G = -\frac{2e^2}{h}\frac{L}{l_{eee}}e^{-\mu/T}$$

$$\frac{1}{l_{eee}} \propto \left(\frac{T}{\mu}\right)^7$$

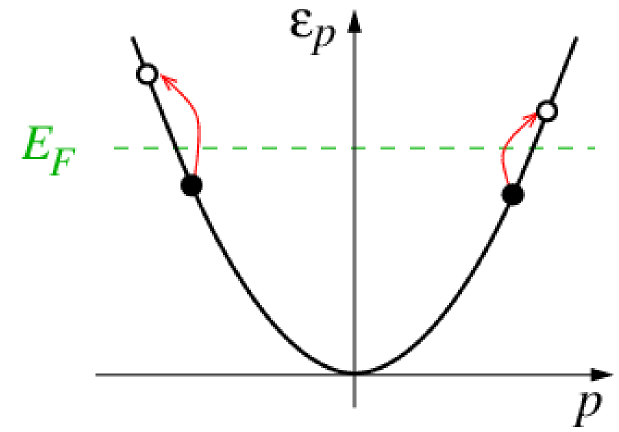
[Lunde, Flensberg & Glazman, 2007]

We can do more

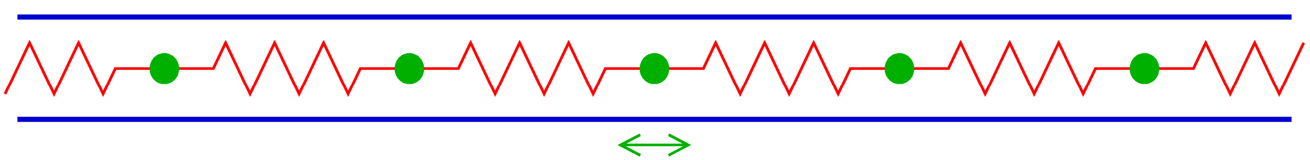
- Study the whole dependence of the length of the wire,
- Account for interactions of arbitrary strength,
- Explore the thermoelectric coefficients,
- Include the effects of smooth disorder,
- …

Interacting electrons in one dimension: Luttinger liquid

Particle-hole excitations become acoustic bosons



Example: Wigner crystal



- Strong repulsion between electrons make them form a one-dimensional crystal
- Excitations of a crystal, the "phonons", are bosons with acoustic spectrum, cf. Luttinger liquid

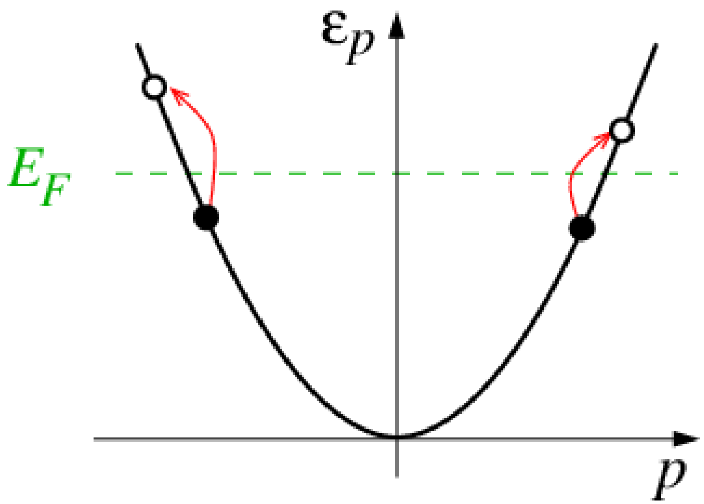
Luttinger liquid theory describes the low-energy properties of one-dimensional electron systems at any interaction strength.

Hamiltonian of a Luttinger liquid

$$H = \sum_q \hbar v |q| b_q^\dagger b_q + \frac{\pi\hbar}{2L}[v_N(N - N_0)^2 + v_J J^2]$$

N is the total number of particles, J is the current quantum number:

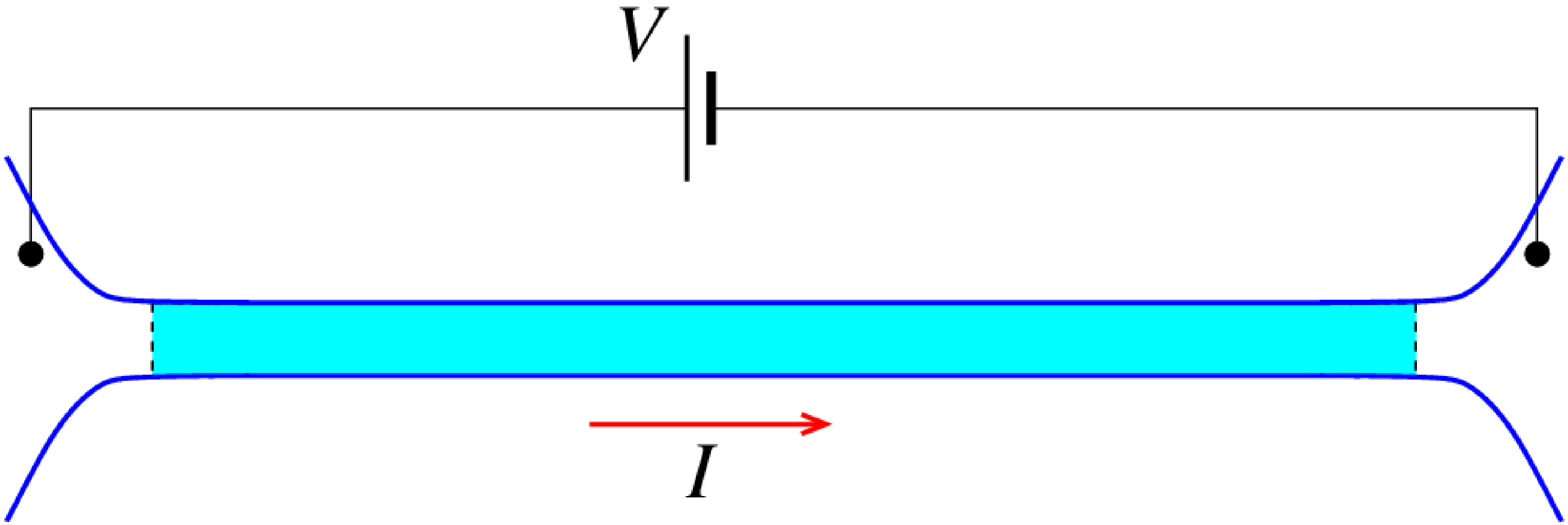
$$N = N^R + N^L, \quad J = N^R - N^L$$



In a uniform Luttinger liquid the numbers of right- and left-movers (N^R, N^L) are conserved: No correction to conductance!

[cf. Maslov & Stone 1995; Ponomarenko 1995; Safi and Schulz, 1995]

Equilibrium state of electrons in a long wire



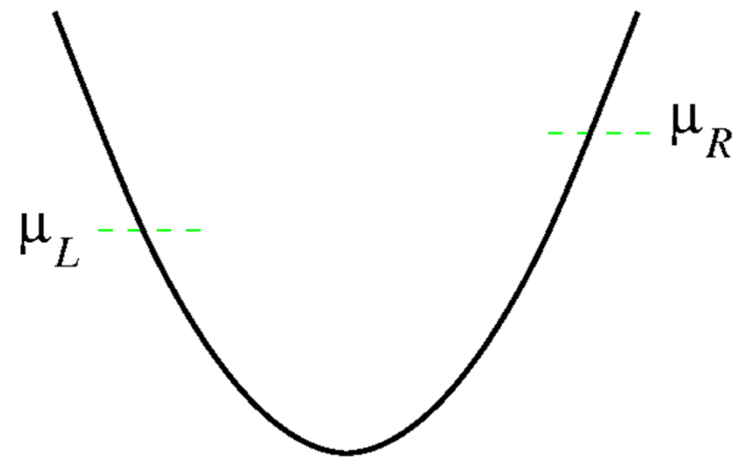
In a long wire collisions between electrons will bring the system to an equilibrium state. Since the collisions conserve momentum, the distribution has the form

$$w_i = \exp\left(-\frac{E_i - uP_i}{T}\right)$$

Momentum of a Luttinger liquid

$$P = \sum_q \hbar q\, b_q^\dagger b_q + p_F J$$

$$J = N^R - N^L$$



Distribution of the bosons in equilibrium

$$N_q = \frac{1}{e^{\hbar(v|q|-uq)/T} - 1}$$

Distribution of *J:* $w_J \propto \exp\left\{-\frac{1}{T}\left(\frac{\pi\hbar v_J}{2L}J^2 - up_F J\right)\right\}$

In equilibrium, u becomes drift velocity: $u = v_d$

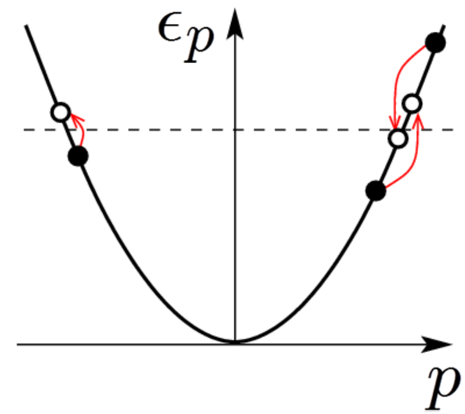
$v_d = \frac{I}{en}$ (the velocity at which electron liquid moves as a whole)

Two relaxation times

Quadratic Hamiltonian:
No relaxation

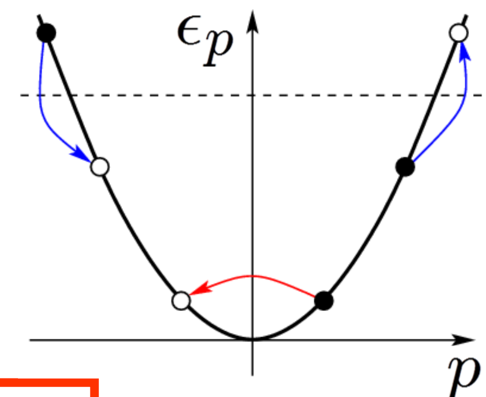
$$H = \sum_q \hbar v |q| b_q^\dagger b_q + \frac{\pi\hbar}{2L}[v_N(N-N_0)^2 + v_J J^2]$$

Relaxation of bosons (particle-hole excitations)



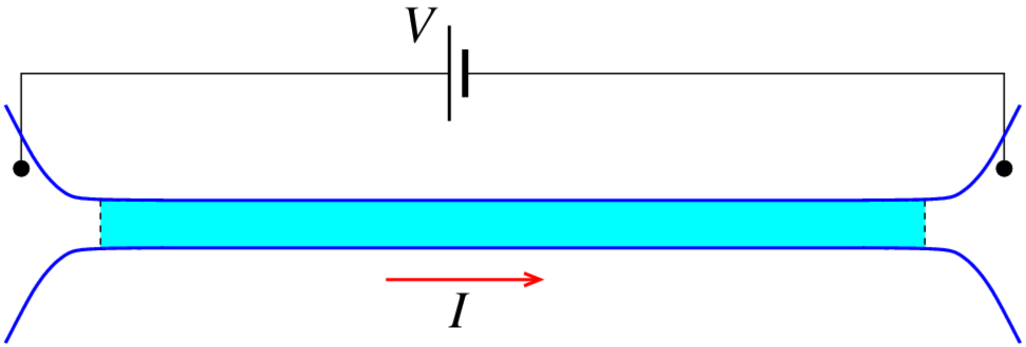
- Relaxation rate: $\tau_0^{-1} \propto T^\alpha$

- Relaxation of J (backscattering) $\tau^{-1} \propto e^{-D/T}$



Bosons equilibrate much faster than J

Partially equilibrated wire



Wire of intermediate length

$$v\tau_0 \ll L \sim v\tau$$

The bosons are in equilibrium with each other:

$$N_q = \frac{1}{e^{\hbar(v|q|-uq)/T} - 1}$$

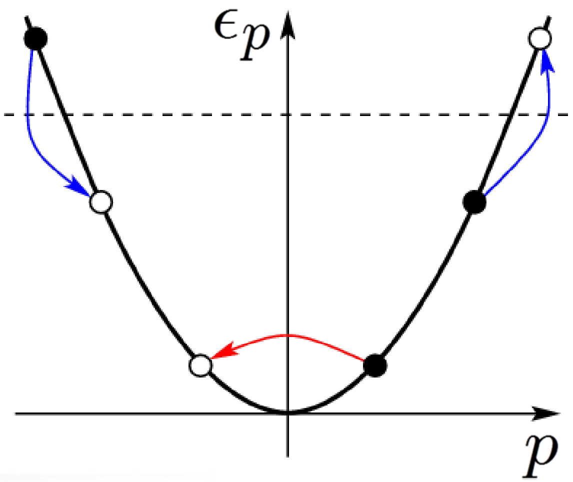
but not with the system as a whole:

$$u \neq v_d$$

Slow relaxation to full equilibrium

$$\dot{u} = -\frac{u - v_d}{\tau}$$

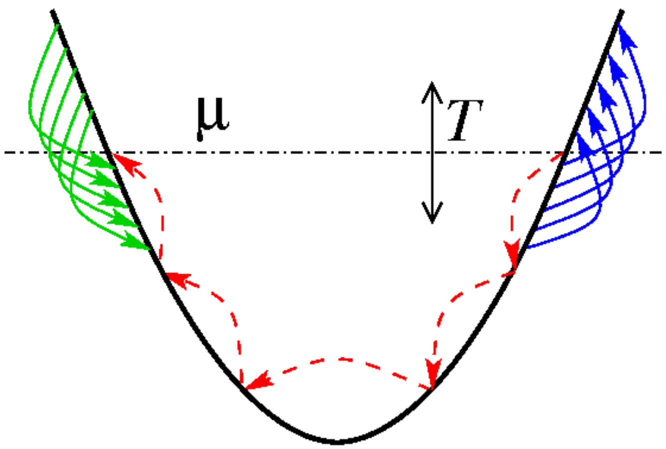
$$\frac{1}{\tau} \propto e^{-D/T}$$



Momentum conservation

$$P = \sum_q \hbar q \, b_q^\dagger b_q + p_F J$$

$$J = N^R - N^L$$



Single backscattering event:

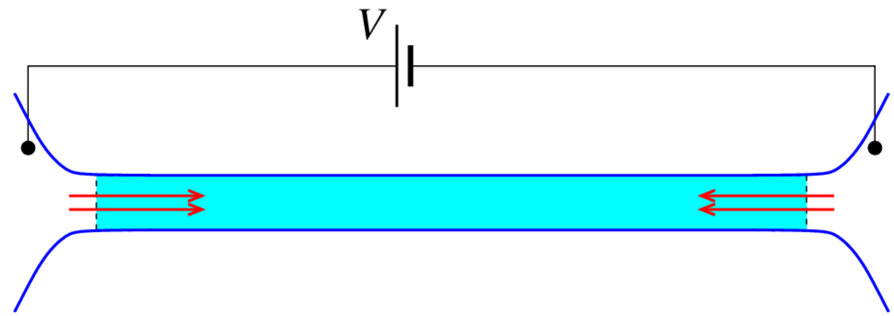
$$\Delta N^R = -1, \ \Delta J = -2, \ \Delta P_b = 2p_F \quad \Rightarrow \quad \dot{N}^R = -\frac{\dot{P}_b}{2p_F}$$

Momentum carried by the bosons: $P_b \propto u, \ \dot{P}_b \propto \dot{u} = -\frac{u - v_d}{\tau}$

Electron backscattering rate:

$$\dot{N}^R = L \frac{\pi}{6\hbar} \frac{T^2}{v^3 p_F} \frac{u - v_d}{\tau}$$

Energy conservation



Cf. particle number conservation

$$I = \frac{e^2}{h}V + e\dot{N}^R$$

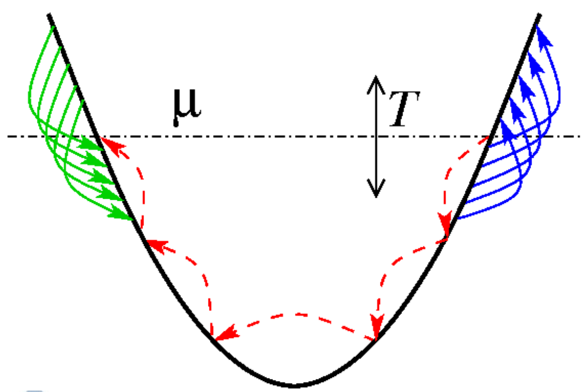
$$\boxed{j_E = \dot{E}^R}$$

(leads supply no energy current)

Boson distribution

$$N_q = \frac{1}{e^{\hbar(v|q|-uq)/T} - 1}$$

$$\boxed{j_E = \frac{\pi}{3}\frac{T^2}{\hbar v}u}$$



Momentum change: $\Delta p^L + \Delta p^R = 2p_F$

Energy change: $-v\Delta p^L + v\Delta p^R = 0$

$$\Delta p^L = \Delta p^R = p_F$$

$$\boxed{\dot{E}^R = -vp_F\dot{N}^R}$$

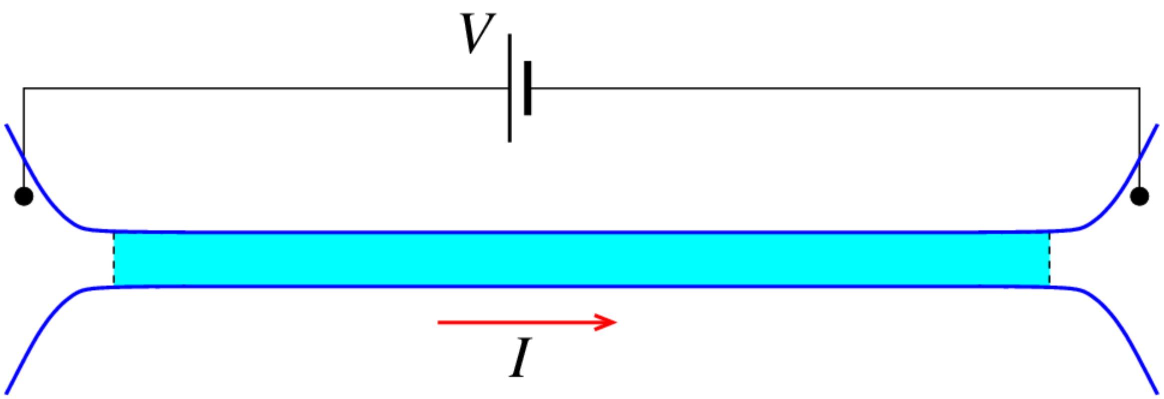
Backscattering rate

Conservation laws:

$$\dot{N}^R = -\frac{\pi}{3}\frac{T^2}{\hbar v^2 p_F}u$$

Kinetics:

$$\dot{N}^R = L\frac{\pi}{6\hbar}\frac{T^2}{v^3 p_F}\frac{u - v_d}{\tau}$$



$$I = \frac{e^2}{h}V + e\dot{N}^R$$

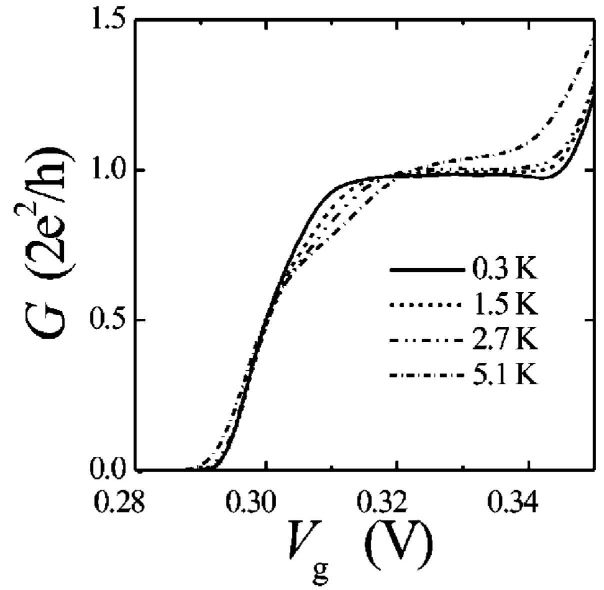
$$G = \frac{e^2}{h}\left[1 - \frac{\pi^2}{3}\left(\frac{T}{vp_F}\right)^2 \frac{L}{L + 2v\tau}\right]$$

Conductance of long and short wires

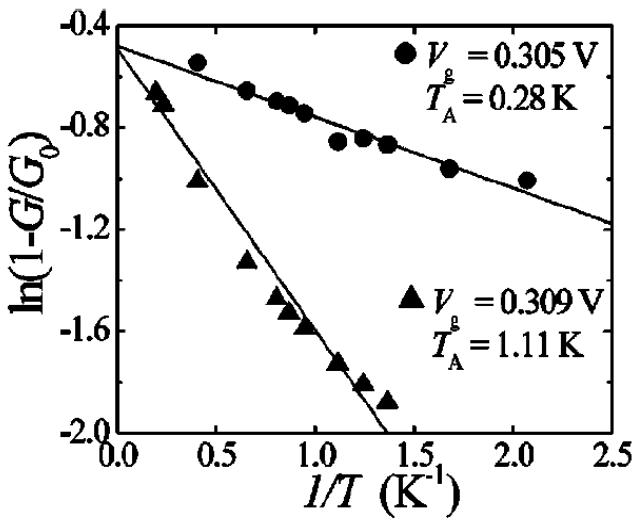
$$G \simeq \frac{e^2}{h}\left[1 - \frac{\pi^2}{3}\left(\frac{T}{v p_F}\right)^2 \frac{L}{L + 2v\tau}\right]$$

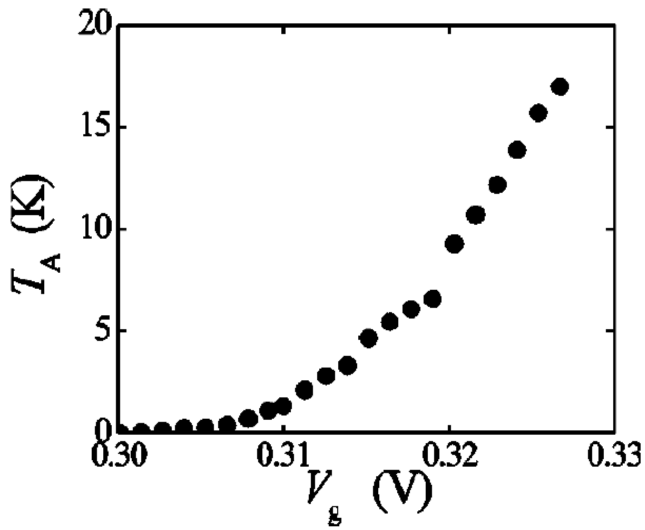
$$\tau \propto e^{T_A/T}$$

- Short wire: $\delta G \propto L e^{-T_A/T}$
- Long wire: $\delta G \propto T^2$



Kristensen *et al.*, 2000

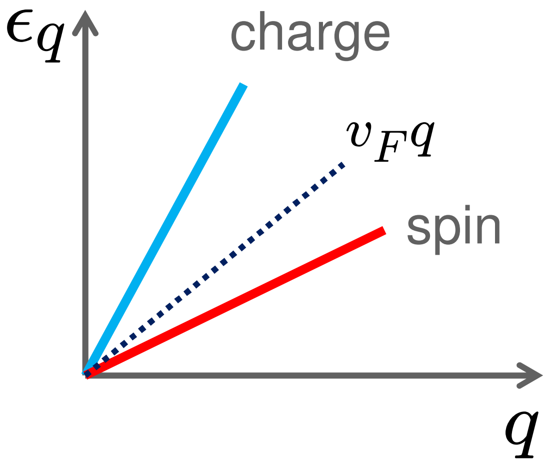




$T_A \sim 1\text{K}$

Spins!

What about spins?



Luttinger liquid with two types of bosonic excitations: the charge and spin ones

For repulsive interactions:

velocity of charge excitations $v_c > v_F$
velocity of spin excitations $v_s < v_F$

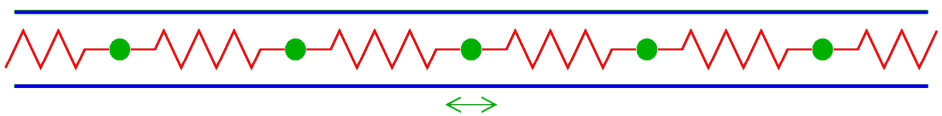
In a long spinless wire the conductance saturates at

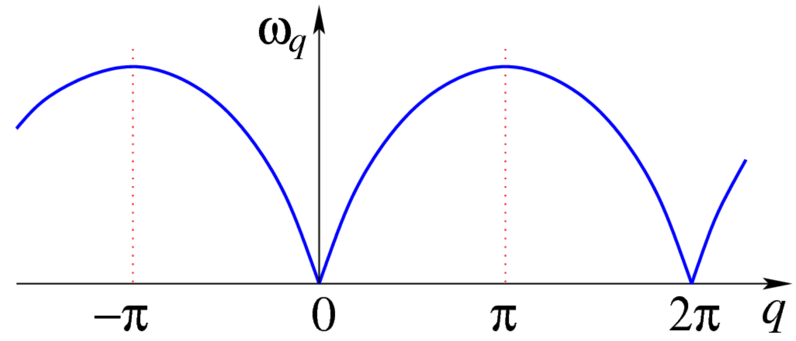
$$G = \frac{e^2}{h}\left[1 - \frac{\pi^2}{3}\left(\frac{T}{v p_F}\right)^2\right]$$

Slower spin excitations are more important than the faster charge excitations!

1D electrons at low density: Wigner crystal

$na_B \ll 1$



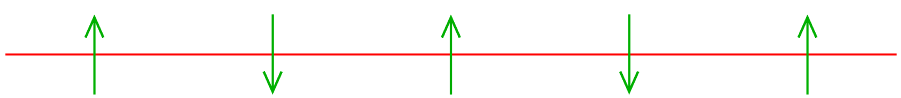


Charge excitations: phonons in the Wigner crystal

The maximum phonon energy is larger than the Fermi energy for free electrons:

$$\hbar\omega_D \approx 6.8 \frac{E_F}{\sqrt{na_B}}$$

Spin excitations: spinons of the Heisenberg model with weak exchange



$$H_s = \sum J \boldsymbol{S}_l \cdot \boldsymbol{S}_{l+1}$$

Spinon spectrum: $\epsilon_q = \frac{\pi J}{2} \sin q$

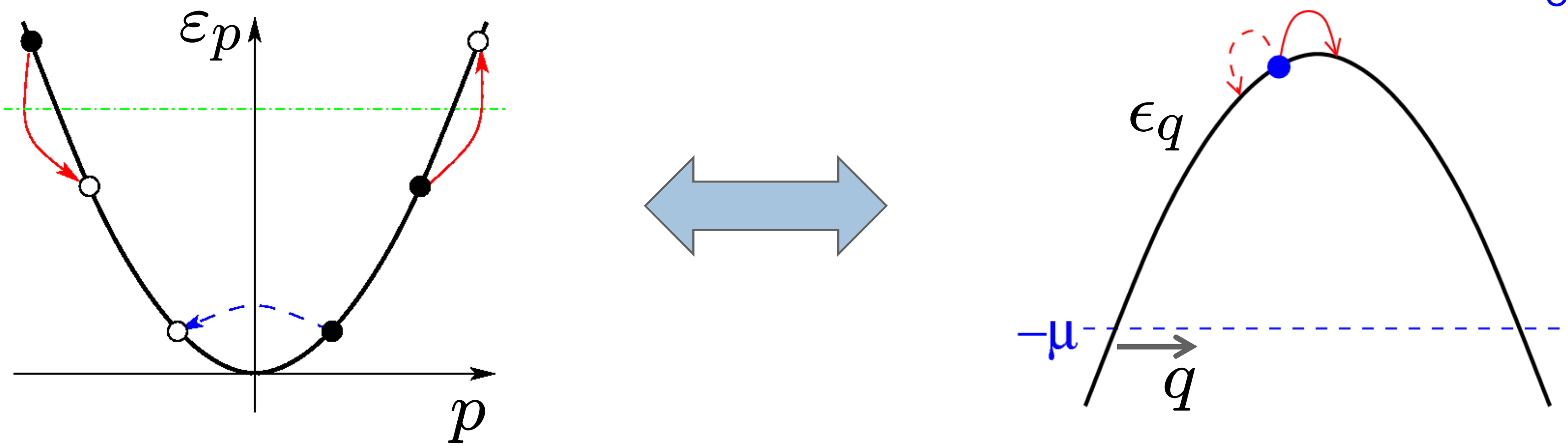
The maximum spinon energy is exponentially small: $\epsilon_\pi \sim \frac{E_F}{(na_B)^{3/4}} \exp\left(-\frac{2.79}{\sqrt{na_B}}\right)$

[Klironomos, Ramazashvili & KM 2005; Fogler & Pivovarov 2005]

Evaluation of the equilibration rate at weak interactions

Scattering of a hole near the bottom of the band

$$\tau^{-1} = \tau_0^{-1} e^{-E_F/T}$$



The hole moves in small random steps in momentum space, $\delta q \sim \dfrac{T}{v_F}$

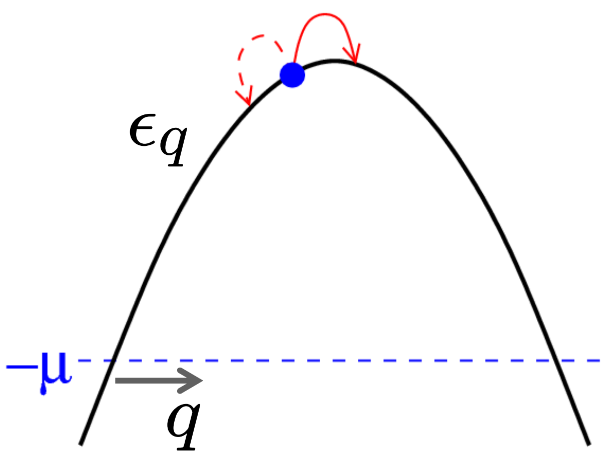
Such diffusion is described by the Fokker-Planck equation, where B is the diffusion constant

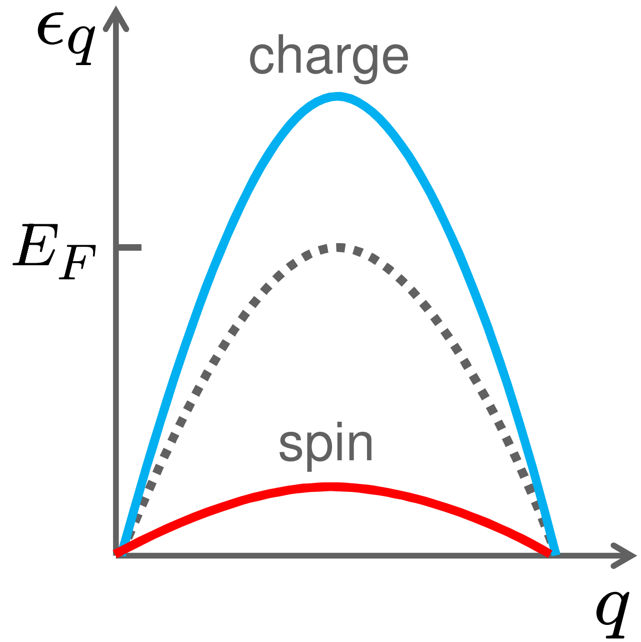
$$\partial_t P = -\frac{B}{2} \partial_q \left(\frac{\partial_q \epsilon_q}{T} + \partial_q \right) P$$

Its solution enables one to find the prefactor $\tau_0^{-1} \propto T^{3/2}$

Equilibration rate for strong interactions

$$\tau^{-1} = \tau_0^{-1} e^{-E_F/T}$$





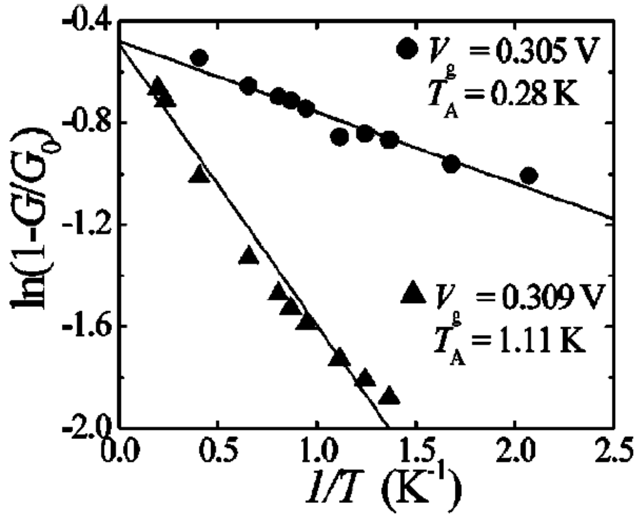
Equilibration of the electron system via backscattering of spinons is controlled by an exponentially small activation energy

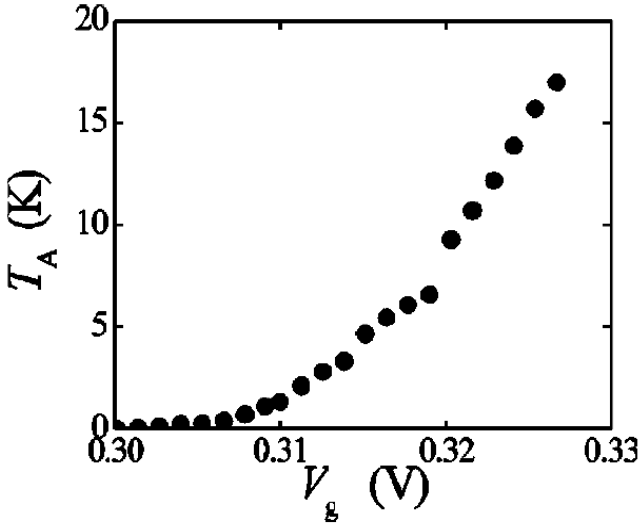
$$\tau^{-1} \propto \exp\left(-\frac{T_A}{T}\right) \qquad T_A \sim \frac{E_F}{(na_B)^{3/4}} \exp\left(-\frac{2.79}{\sqrt{na_B}}\right)$$

Correction to the conductance

Our result for spinless Luttinger liquid is easily generalized to the case of two boson modes (charge and spin). At low electron density (strong interactions) it simplifies to the form

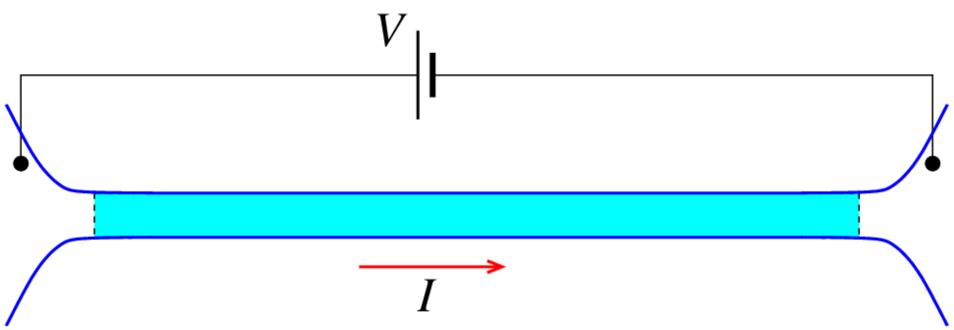
$$G = \frac{2e^2}{h}\left[1 - \frac{2}{3}\left(\frac{T}{T_A}\right)^2 \frac{L}{L + l_{\text{eq}}}\right] \qquad l_{\text{eq}} \propto e^{T_A/T}, \quad T_A \ll E_F$$





Summary

For a long quantum wire we have expressed the conductance in terms of the equilibration rate and wire length



$$G = \frac{2e^2}{h}\left[1 - \frac{2}{3}\left(\frac{T}{T_A}\right)^2 \frac{L}{L + l_{\text{eq}}}\right]$$

The characteristic length scale grows exponentially in the zero temperature limit

$$l_{\text{eq}} \propto e^{T_A/T}$$

However, at low electron density the activation temperature is exponentially small

$$T_A \sim \frac{E_F}{(na_B)^{3/4}} \exp\left(-\frac{2.79}{\sqrt{na_B}}\right)$$

At temperatures near T_A the correction is of order e^2/h even in relatively short wires