Role of Spin in Quantum Wire Transport

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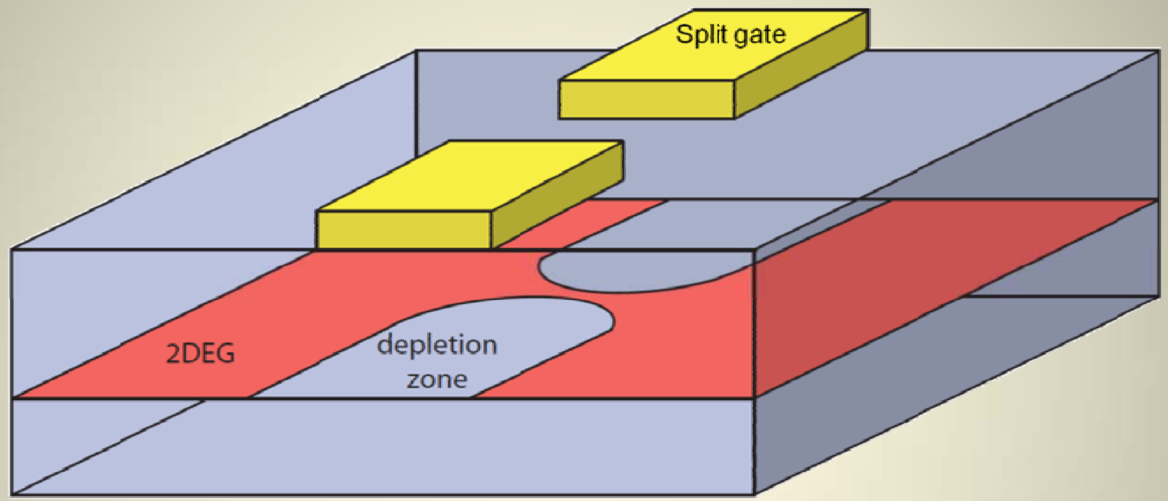
Spin-Related Phenomena in Mesoscopic Transport, Nordita, September 19, 2012

Outline

- Introduction
- ballistic transport
- 0.7 structure

- Low density quantum wires
- spin incoherent transport
- zig-zag Wigner crystal

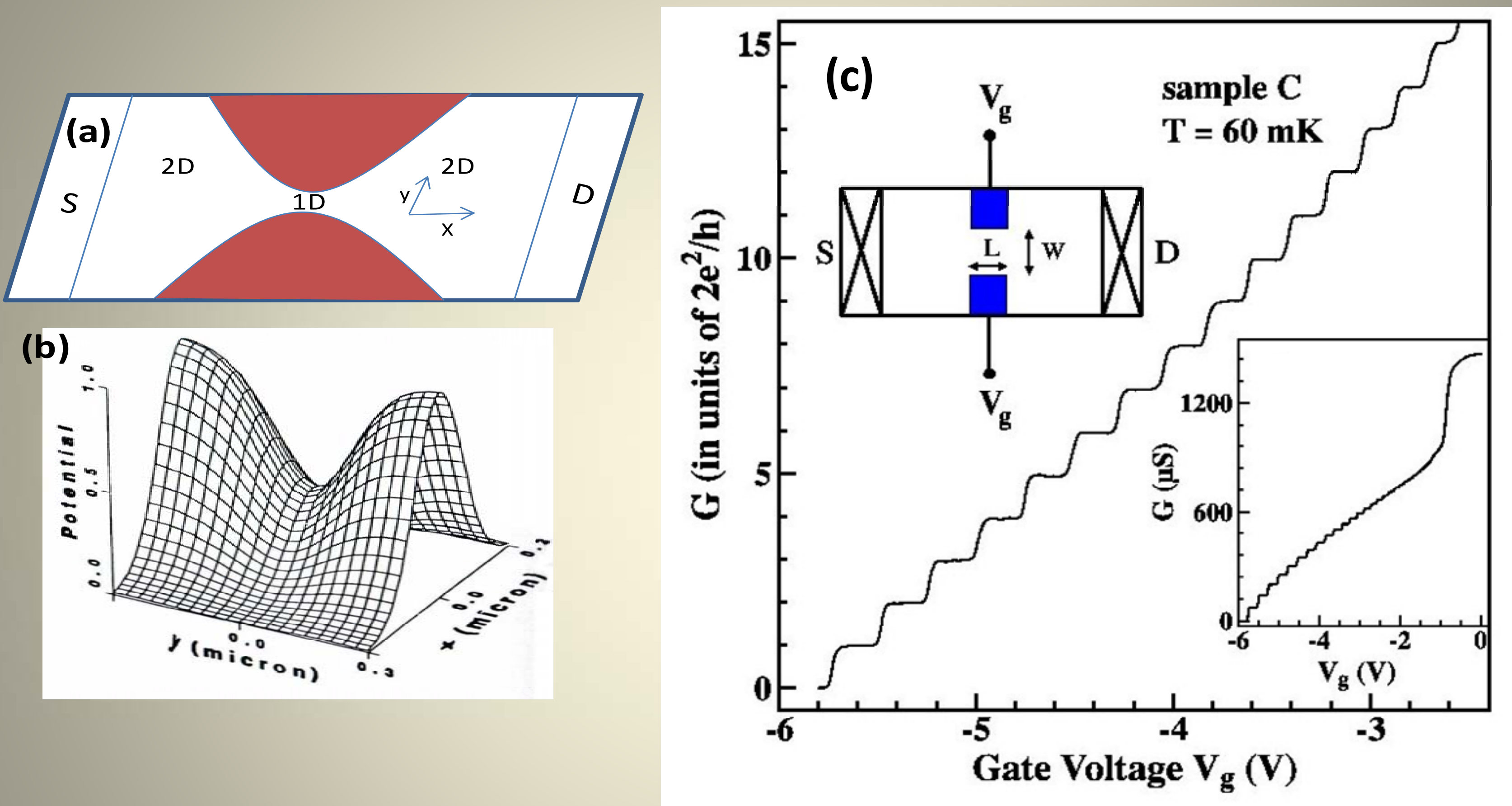
Split-gate devices



- Schematic diagram of a split-gate device

- Gates defined using electron-beam lithography.
- Negative voltage on gates depletes electrons from the 2DEG below.
- First split-gate devices on GaAs: T. J. Thornton *et al.*, Phys. Rev. Lett. **56**, 1198 (1986).
- Confinement in transverse direction sets up 1D subbands.

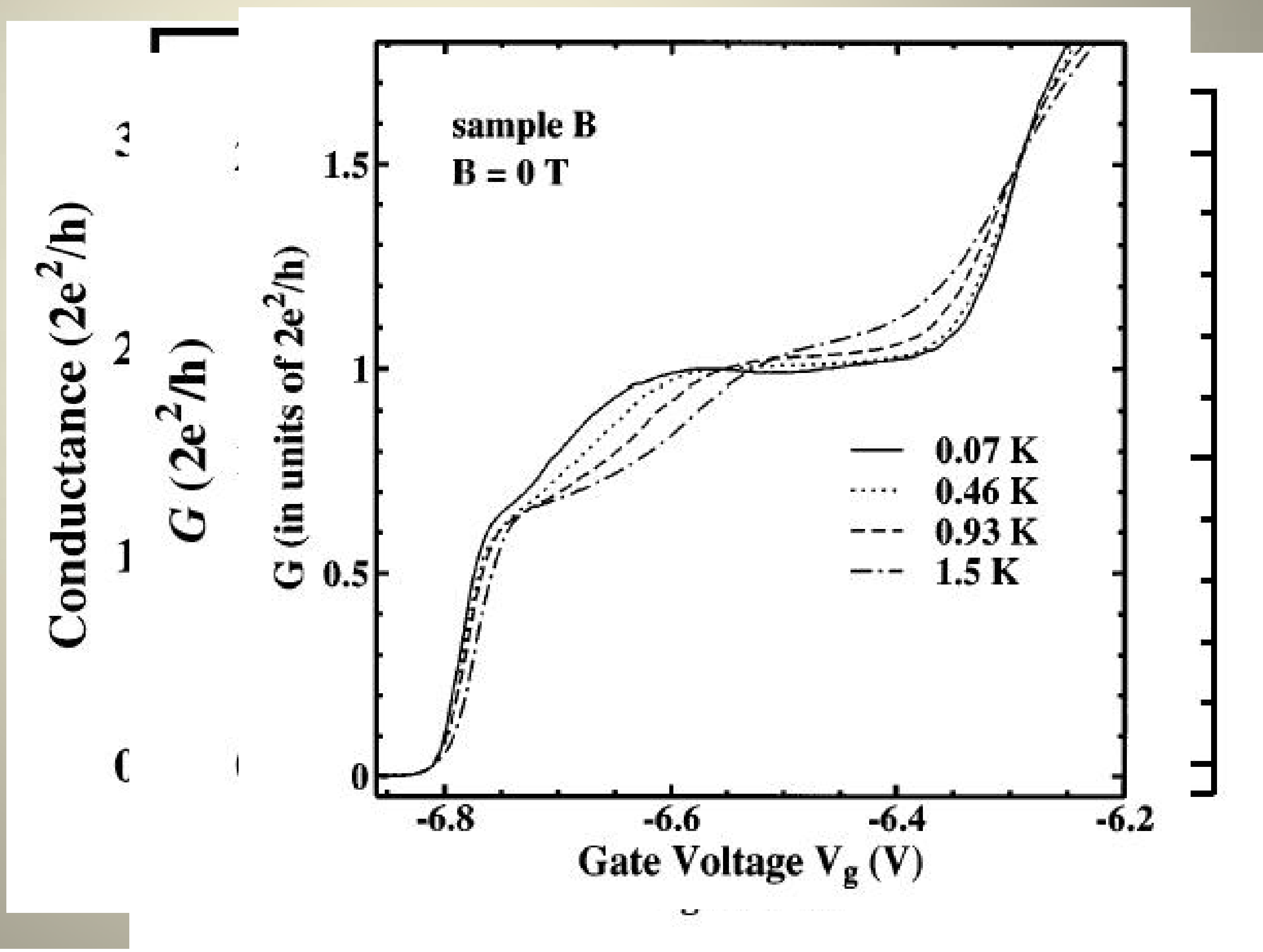
Quantized Conductance



Discovered in 1988, understood within the non-interacting electron picture :
Landauer – Buttiker Formalism – Conduction is transmission.

The 0.7 structure

K. J. Thomas *et al.*, Phys. Rev. Lett. **77**, 135 (1996)



The Lieb-Mattis Theorem

Lieb & Mattis (1962):

- *The ground state of a system of electrons in one dimension subject to an arbitrary symmetric potential is unmagnetised.*
- Based on very general mathematical properties of the Schrödinger equation describing such interacting electronic systems.

However….

It is now generally accepted that for a realistic quantum wire structure such as a split-gate device, with finite dimensions, Lieb & Mattis theorem may not be applicable.

Models to explain 0.7 Structure include..

•Spin polarisation:
C. K. Wang and K.-F. Berggren, Phys. Rev. B **57,** 4552 1998.

•Kondo effect:
Cronenwett (2001), Meir

•Ferromagnetic correlations in a Wigner crystal
Spivak (2000)

PHYSICAL REVIEW B VOLUME 61, NUMBER 24 15 JUNE 2000-II

Ferromagnetic correlations in quasi-one-dimensional conducting channels

Boris Spivak
Physics Department, University of Washington, Seattle, Washington 98195

Fei Zhou
Physics Department, Princeton University, Princeton, New Jersey 08544

(Received 26 January 2000)

We propose a model that explains the experimental observation of spontaneous spin polarization of conducting electrons in quasi-one-dimensional $Al_xGa_{1-x}As/GaAs$ channels [K. J. Thomas, J. T. Nicholls, M. Y. Simmons, M. Pepper, D. R. Mace, and D. A. Ritchie, Phys. Rev. Lett. 77, 135 (1996); K. J. Thomas, J. T. Nicholls, M. J. Appeyard, M. Y. Simmons, M. Pepper, D. R. Mace, W. R. Tribe, and D. A. Ritchie, Phys. Rev. B 58, 4846 (1998)]. We show that a ferromagnetic order is a generic property of a quasi-one-dimensional conducting channel embedded in a Wigner crystal. We also discuss gate voltage, magnetic field, and temperature dependences of the channel's conductance.

Interacting 1D System

Luttinger Liquid

Spin charge separation

Temperature << typical bandwidths of spin and charge modes, both transport without scattering: *power law behaviour*.

If J<<T << E_F - Spin modes are mostly reflected, characteristics different from a normal Luttinger liquid- ***Spin-incoherent Luttinger liquid*** results. - *Fiete 2008, Matveev 2007*

J becomes extremely small at low densities where strong Coloumb repulsion push electrons equidistant along a linear chain: **Wigner Crystal.**

Conductance of a 1D Wigner Crystal

VOLUME 92, NUMBER 10 PHYSICAL REVIEW LETTERS week ending 12 MARCH 2004

Conductance of a Quantum Wire in the Wigner-Crystal Regime

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(Received 3 July 2003; published 9 March 2004)

We study the effect of Coulomb interactions on the conductance of a single-mode quantum wire connecting two bulk leads. When the density of electrons in the wire is very low, they arrange in a finite-length Wigner crystal. In this regime the electron spins form an antiferromagnetic Heisenberg chain with an exponentially small coupling J. An electric current in the wire perturbs the spin chain and gives rise to a temperature-dependent contribution of the spin subsystem to the resistance. At low temperature $T \ll J$ this effect is small, and the conductance of the wire remains close to $2e^2/h$. At $T \gg J$ the spin effect reduces the conductance to e^2/h.

Rev. Mod. Phys. **71** 801 (2007)

Colloquium: **The spin-incoherent Luttinger liquid**

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(Dated: March 15, 2007)

In contrast to the well known Fermi liquid theory of three dimensions, interacting one-dimensional and quasi one-dimensional systems of fermions are described at low energy by an effective theory known as Luttinger liquid theory. This theory is expressed in terms of collective many-body excitations that show exotic behavior such as spin-charge separation. Luttinger liquid theory is commonly applied on the premise that "low energy" describes both the spin and charge sectors. However, when the interactions in the system are very strong, as they typically are at low particle densities, the ratio of spin to charge energy may become exponentially small. It is then possible at very low temperatures for the energy to be low compared to the characteristic charge energy, but still high compared to the characteristic spin energy. This energy window of near ground-state charge degrees of freedom, but highly thermally excited spin degrees of freedom is called a spin-incoherent Luttinger liquid. The spin-incoherent Luttinger liquid exhibits a higher degree universality than the Luttinger liquid and its properties are qualitatively distinct. In this colloquium I detail some of the recent theoretical developments in the field and describe experimental indications of such a regime in gated semiconductor quantum wires.

Wigner Crystallisation

The possibility that an electron gas might freeze out into a crystalline state was suggested by E Wigner in 1934:

- occurs in the low-density limit, where Coulomb energy dominate kinetic energy
- electrons occupy equidistant positions to minimise Coulomb repulsion

Quasi-1D Wigner Crystallisation

PHYSICAL REVIEW B **69**, 045324 (2004)

Generic properties of a quasi-one-dimensional classical Wigner crystal

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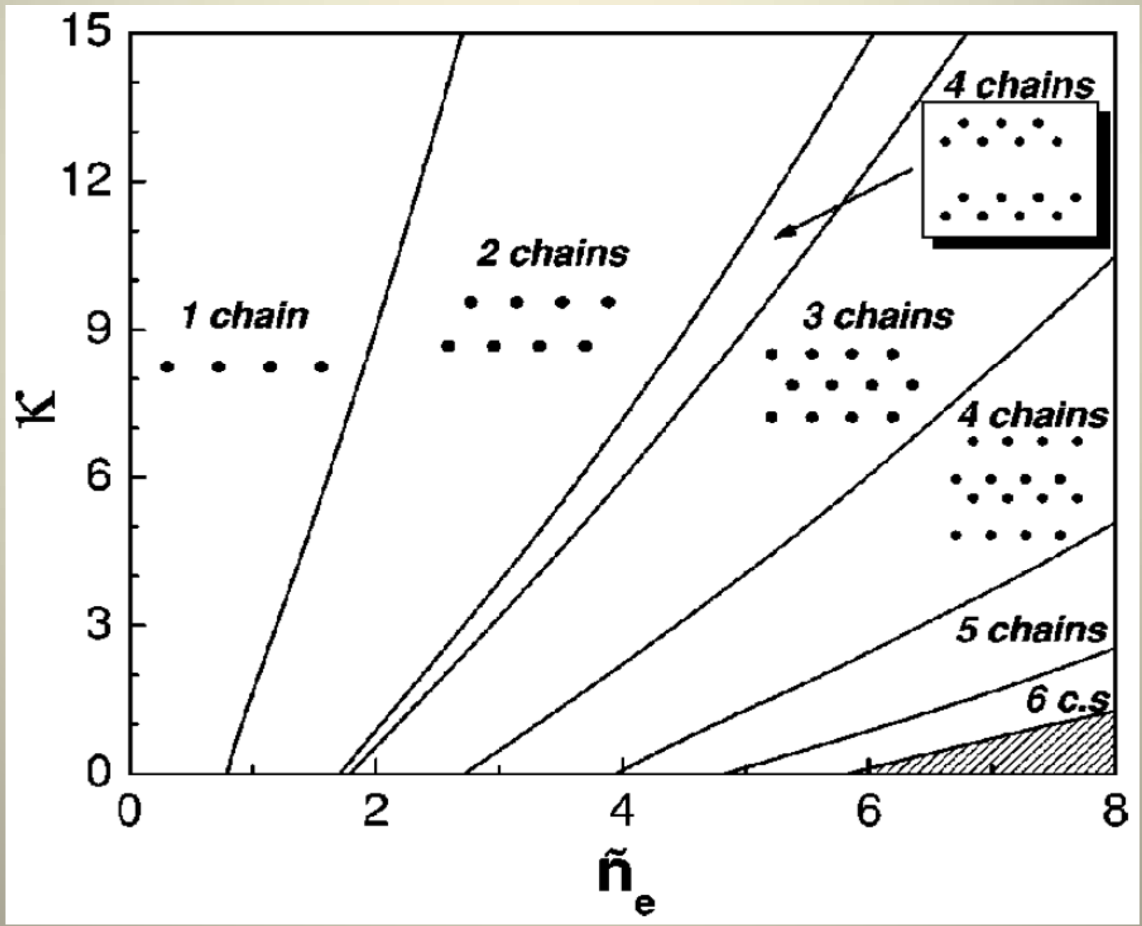
Department of Physics, University of Antwerp (Campus Drie Eiken), B-2610 Antwerpen, Belgium
(Received 24 June 2003; published 30 January 2004)

We studied the structural, dynamical properties and melting of a quasi-one-dimensional system of charged particles, interacting through a screened Coulomb potential. The ground-state energy was calculated and, depending on the density and the screening length, the system crystallizes in a number of chains. As a function of the density (or the confining potential), the ground state configurations and the structural transitions between them were analyzed both by analytical and Monte Carlo calculations. The system exhibits a rich phase diagram at zero temperature with continuous and discontinuous structural transitions. We calculated the normal modes of the Wigner crystal and the magnetophonons when an external constant magnetic field B is applied. At finite temperature the melting of the system was studied via Monte Carlo simulations using the *modified Lindemann criterion* (MLC). The melting temperature as a function of the density was obtained for different screening parameters. Reentrant melting as a function of the density was found as well as evidence of directional dependent melting. The single-chain regime exhibits anomalous melting temperatures according to the MLC and as a check we study the pair-correlation function at different densities and different temperatures, which allowed us to formulate a different melting criterion. Possible connection with recent theoretical and experimental results are discussed and experiments are proposed.

DOI: 10.1103/PhysRevB.69.045324 PACS number(s): 64.60.Cn, 64.70.Dv, 61.46.+w

Structural Phase Diagram of a Quasi-1D Wigner Crystal

Physical Review B 69, 045324 (2004)



A Zigzag Wigner Crystal

PHYSICAL REVIEW B **76**, 075302 (2007)

Spin coupling in zigzag Wigner crystals

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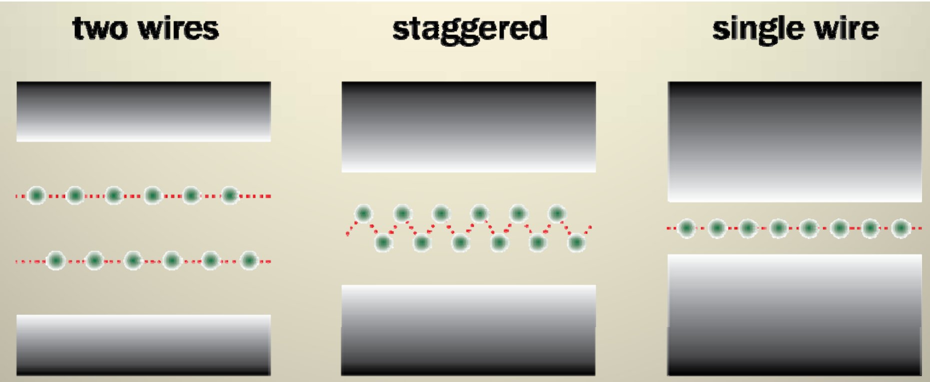
[2]*Department of Physics, The Ohio State University, Columbus, Ohio 43210, USA*
[3]*Department of Physics, Hokkaido University, Sapporo 060-0810, Japan*
(Received 5 April 2007; published 2 August 2007)

We consider interacting electrons in a quantum wire in the case of a shallow confining potential and low electron density. In a certain range of densities, the electrons form a two-row (zigzag) Wigner crystal whose spin properties are determined by nearest and next-nearest neighbor exchange as well as by three- and four-particle ring exchange processes. The phase diagram of the resulting zigzag spin chain has regions of complete spin polarization and partial spin polarization in addition to a number of unpolarized phases, including antiferromagnetism and dimer order as well as a novel phase generated by the four-particle ring exchange.

DOI: 10.1103/PhysRevB.76.075302 PACS number(s): 73.21.Hb, 71.10.Pm



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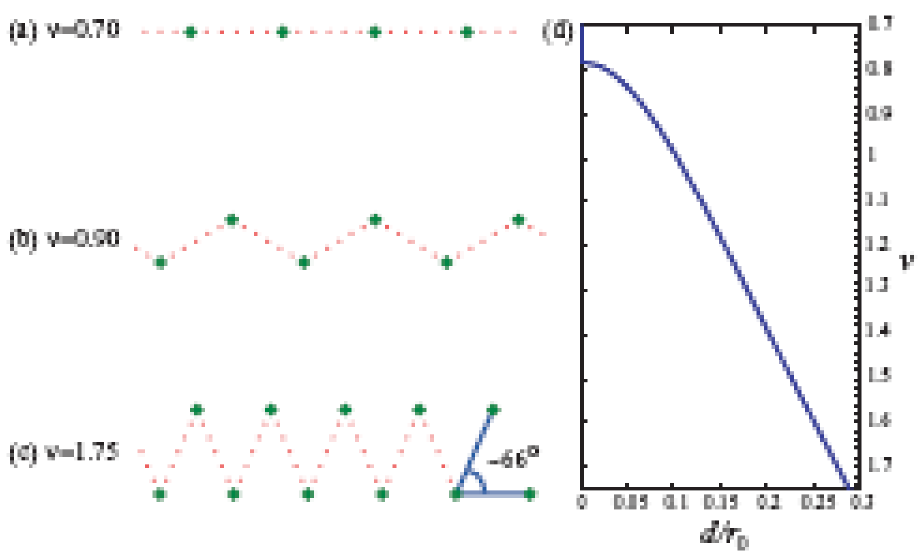

FIG. 1. (Color online) Wigner crystal of electrons in a quantum wire. The structure as determined by the dimensionless distance between rows d/r_0 depends on the parameter ν proportional to electron density (see text). As density grows, (a) the one-dimensional crystal gives way to [(b) and (c)] a zigzag chain.

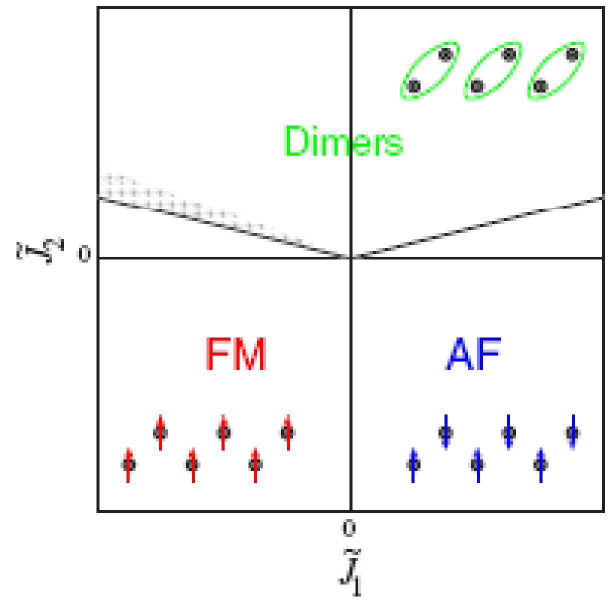

FIG. 2. (Color online) The phase diagram including nearest neighbor, next-nearest neighbor, and three-particle ring exchanges. The effective couplings $\tilde{J}_1$ and $\tilde{J}_2$ are defined in the text. The shaded region between the dimer and ferromagnetic phases corresponds to the exotic phase predicted in Ref. 48.

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PHYSICAL REVIEW B **82**, 073307 (2010)

Localization of electrons and formation of two-dimensional Wigner spin lattices in a special cylindrical semiconductor stripe

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(Received 24 March 2010; revised manuscript received 9 June 2010; published 27 August 2010)

We consider a two-dimensional (2D)electron gas residing on the surface of a cylinder of a given radius R in the presence of a parabolic confinement along the axis of the cylinder. In this way the system of electrons forms a closed cylindrical stripe (wire). Using the local spin-density technique we first consider localization of electrons within of a potential barrier embedded in the wire. Barriers with sharp retangularlike features are populated in steps because of Coulomb blockade. The nature of a single bound state in a short soft barrier (quantum point contacts) at pinch-off is discussed in terms Coulomb blockade. For a shallow barrier-free wire we retrace the structural transitions at low electron densities from a single chain of localized states to double and triple chains (Wigner spin lattices). The present system is related to the model of a inhomogeneous quantum wire introduced recently by Güçlü *et al.* [Phys. Rev. B **80**, 201302(R) (2009)]. An important aspect is, however, the present extension into higher electron densities as well as to the low-density regime and the formation of 2D Wigner microlattices.

DOI: 10.1103/PhysRevB.82.073307 PACS number(s): 73.21.−b, 73.23.−b, 74.78.Na

Is Double-Row Transport Realisable?

The transition from a 1D Wigner crystal to a zig-zag chain occurs when the Coulomb interaction energy:

$$V_{\text{int}}(r) = \frac{e^2}{\varepsilon r}$$

becomes comparable to the confining potential (which, for a quantum point contact, is assumed to be parabolic):

$$V_{\text{conf}}(y) = \frac{m\Omega^2 y^2}{2}$$

where **Ω** is the harmonic frequency.

$$V_{\text{int}}(r_0) = V_{\text{conf}}(r_0) \longrightarrow r_0 = \sqrt[3]{\frac{2e^2}{\varepsilon m \Omega^2}}$$

Can We Observe This?

To be in the Wigner-crystal régime, we need

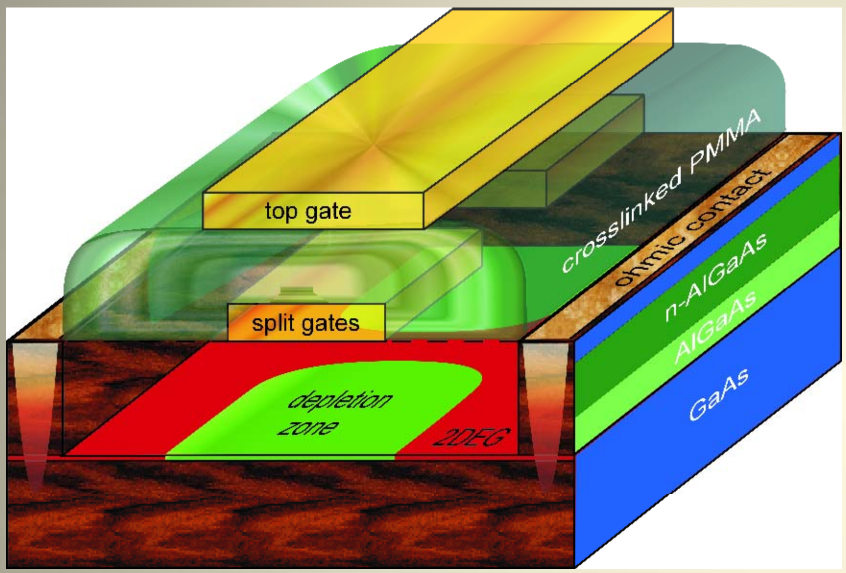
$$r_0 >> a_{\mathrm{B}} = \frac{\hbar^2 \varepsilon}{me^2}$$

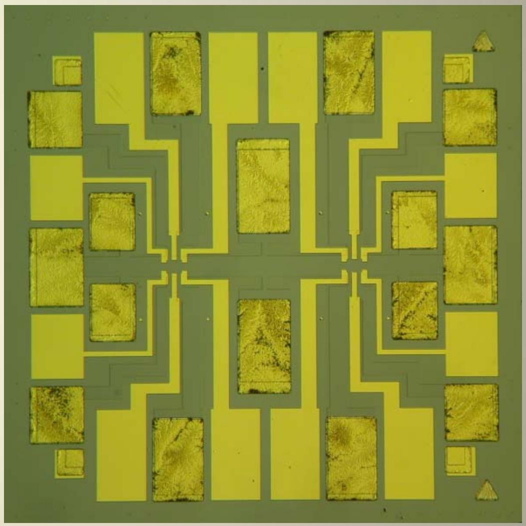
or, substituting for r_0,

$$\left(\frac{me^4}{\varepsilon^2 \Omega}\right)^{2/3} >> \hbar\sqrt{2}$$

Ω, which describes the steepness of the confinement, is the only parameter in this relation that is amenable to tuning for any given realisation of a quantum wire.

Top-Gated Split-Gate Device





GaAs/AlGaAs heterostructure

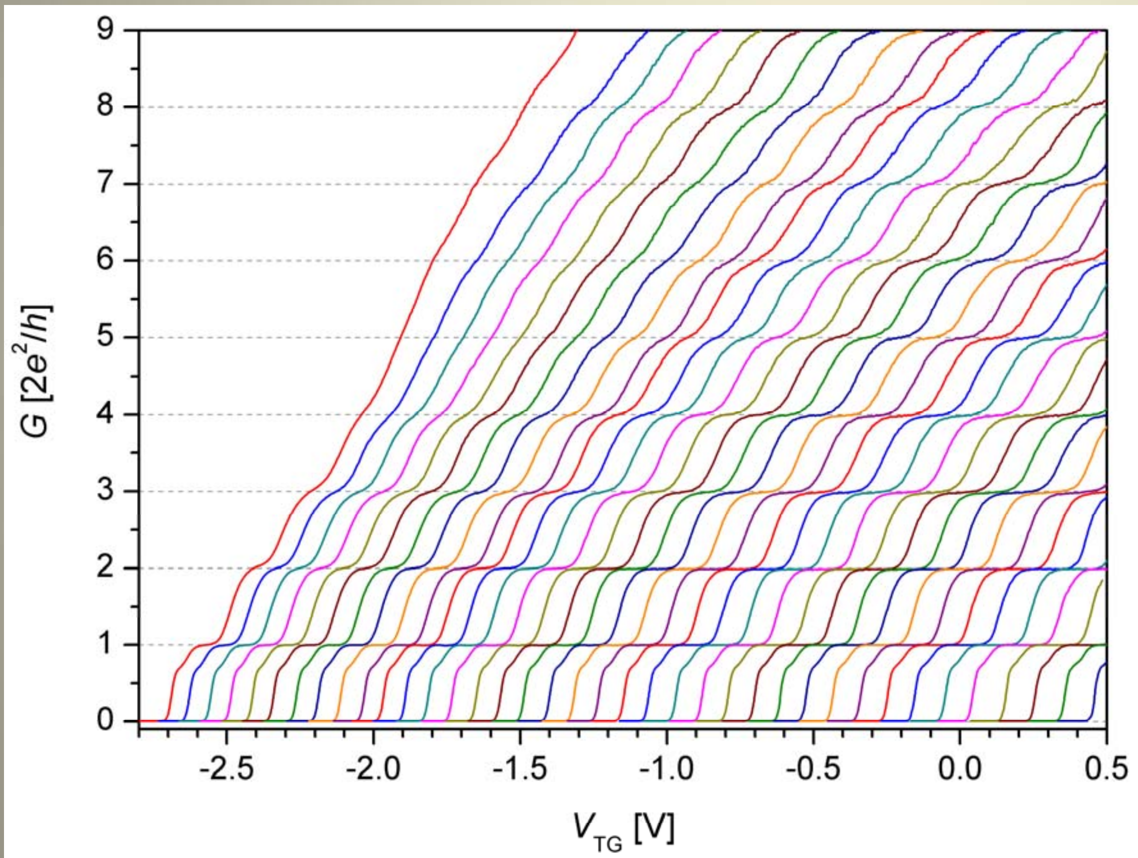
- $n_{2D} \sim 2 \times 10^{11}/\text{cm}^2$
- $\mu \sim 2 \times 10^6\ \text{cm}^2/\text{Vs}$

split gates

- length 0.4 µm
- separation 0.7 µm

Typical Device Characteristics

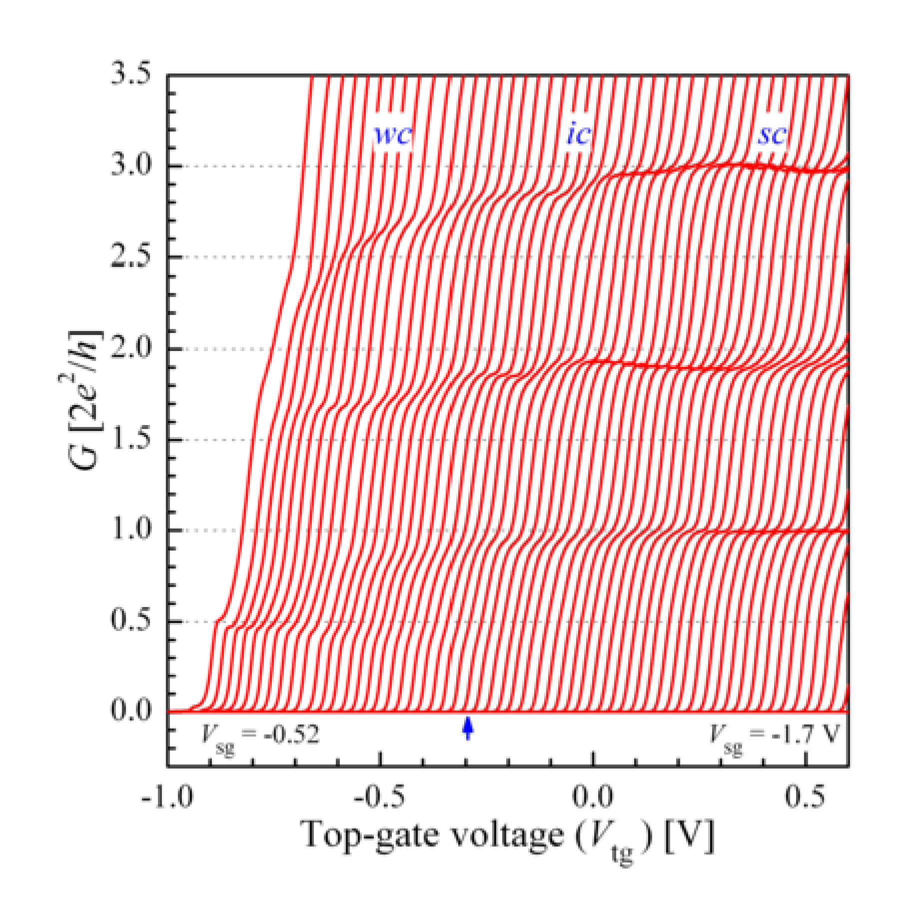
increasing confinement strength → <u>higher electron densities</u>

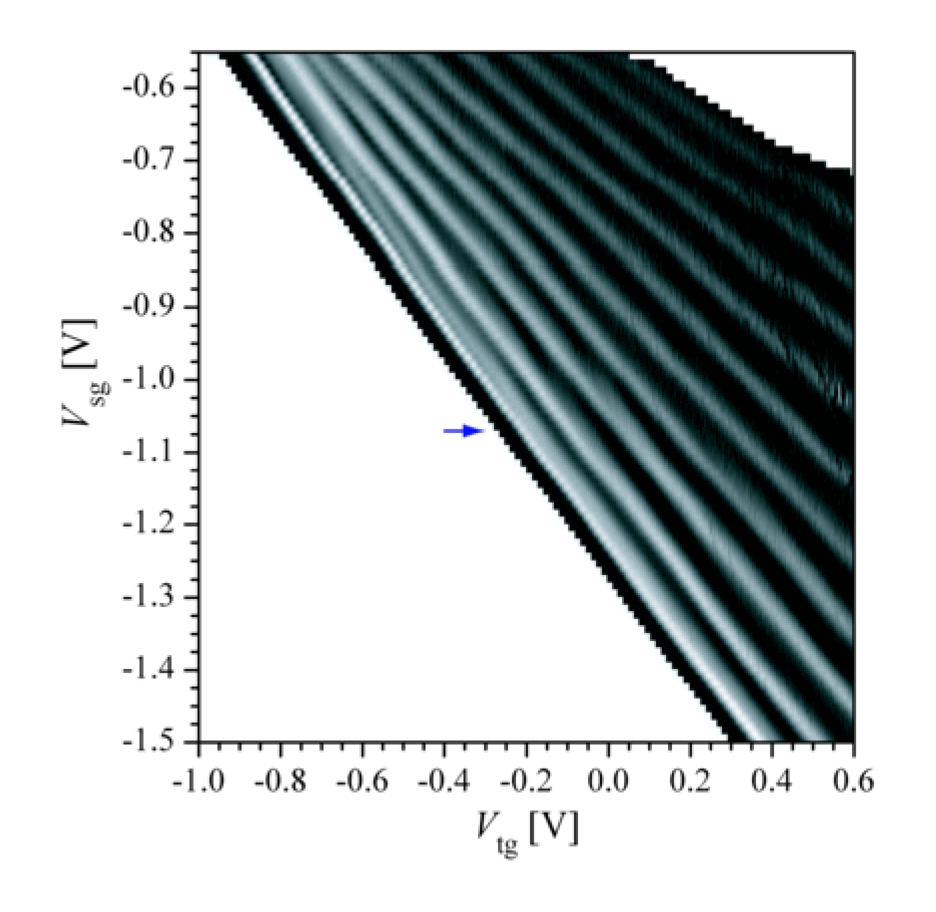


- n_{2D} ~ $1.8 \times 10^{11}/\text{cm}^2$
- There are no significant deviations from expected quantisation of conductance plateaux.
- The 0.7 structure is present throughout.
- The increasing spacing between successive traces with strengthening confinement corresponds to an increase in the sub-band spacing.

Spin-incoherent Regime

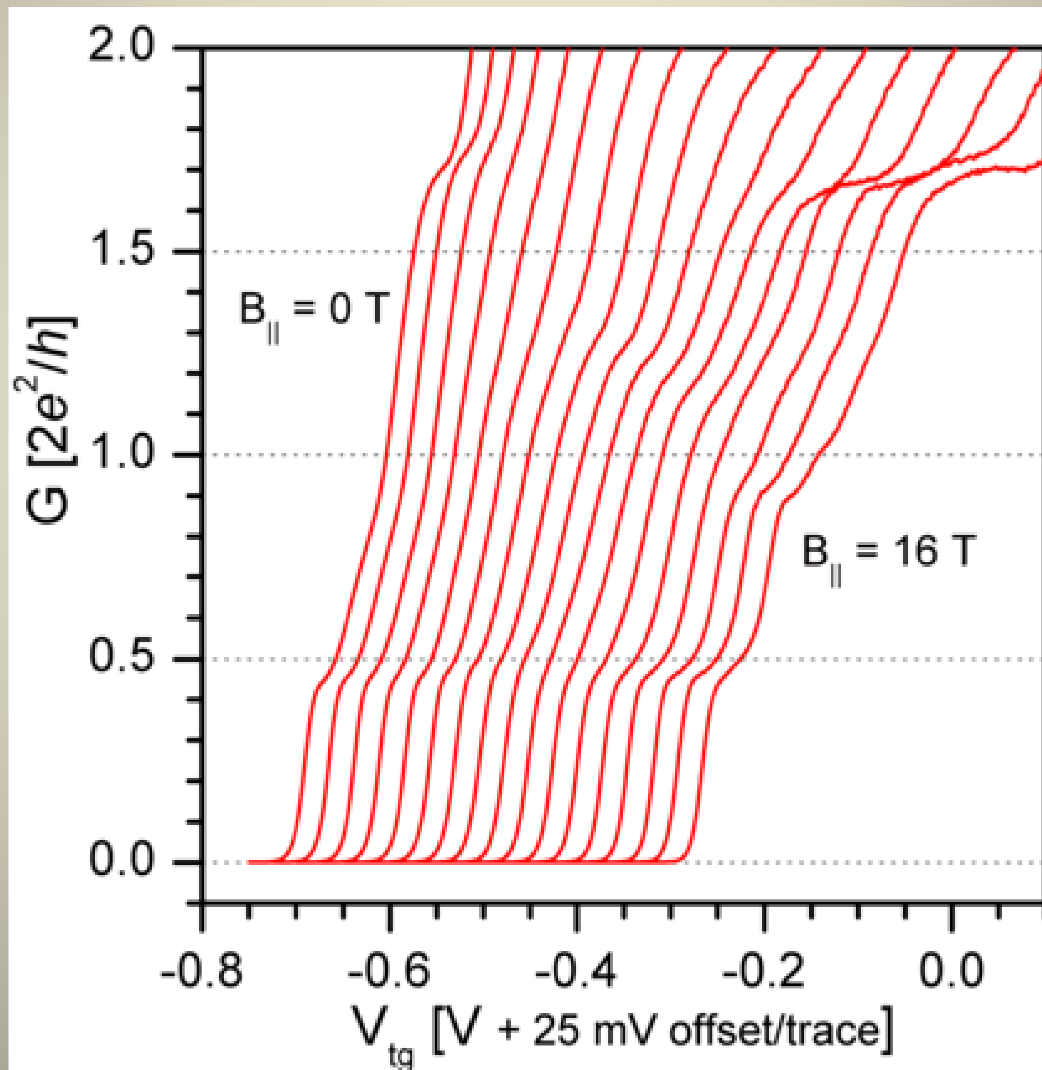
Structure at e^2/h at weak confinement strengths



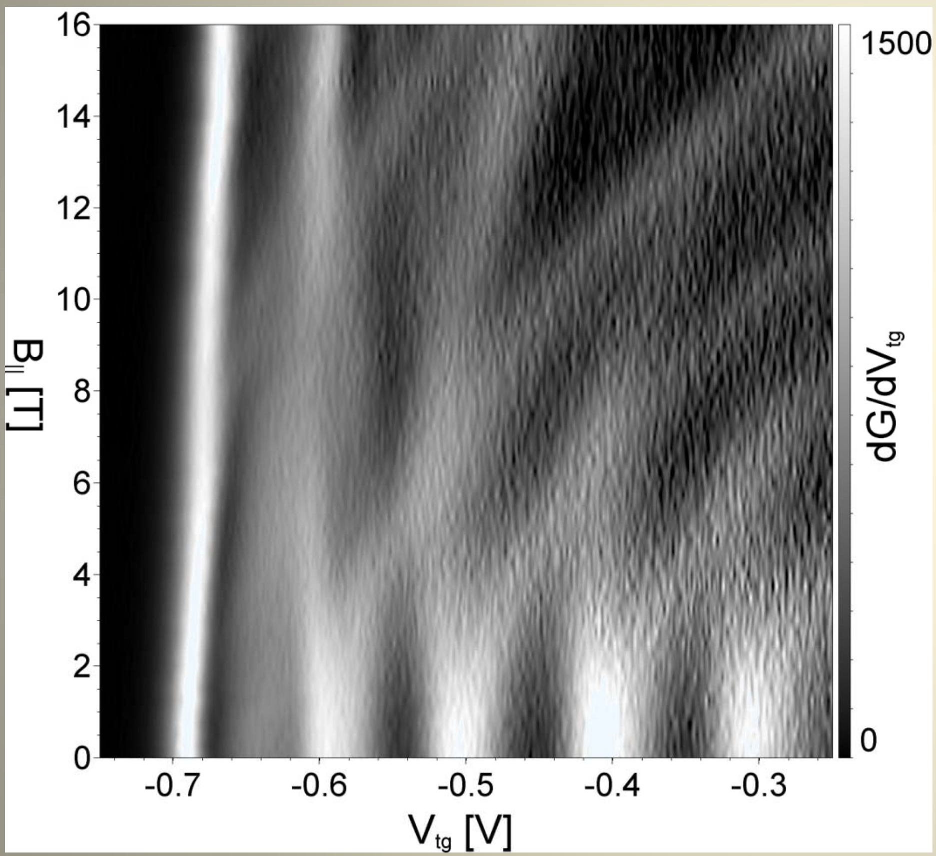


W. K. Hew, K. J. Thomas, M. Pepper et al., *Phys. Rev. Lett.*, **101**, 036801 (2008).

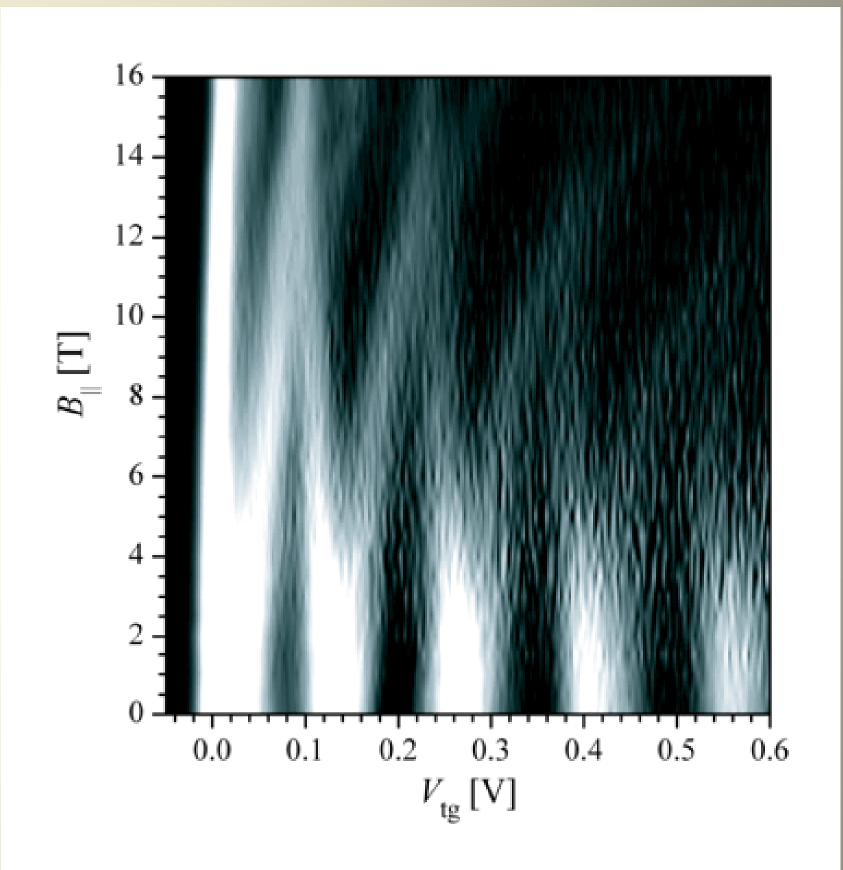
Parallel magnetic field dependence of e^2/h



G($B_{||}$): Spin-incoherent to coherent transition

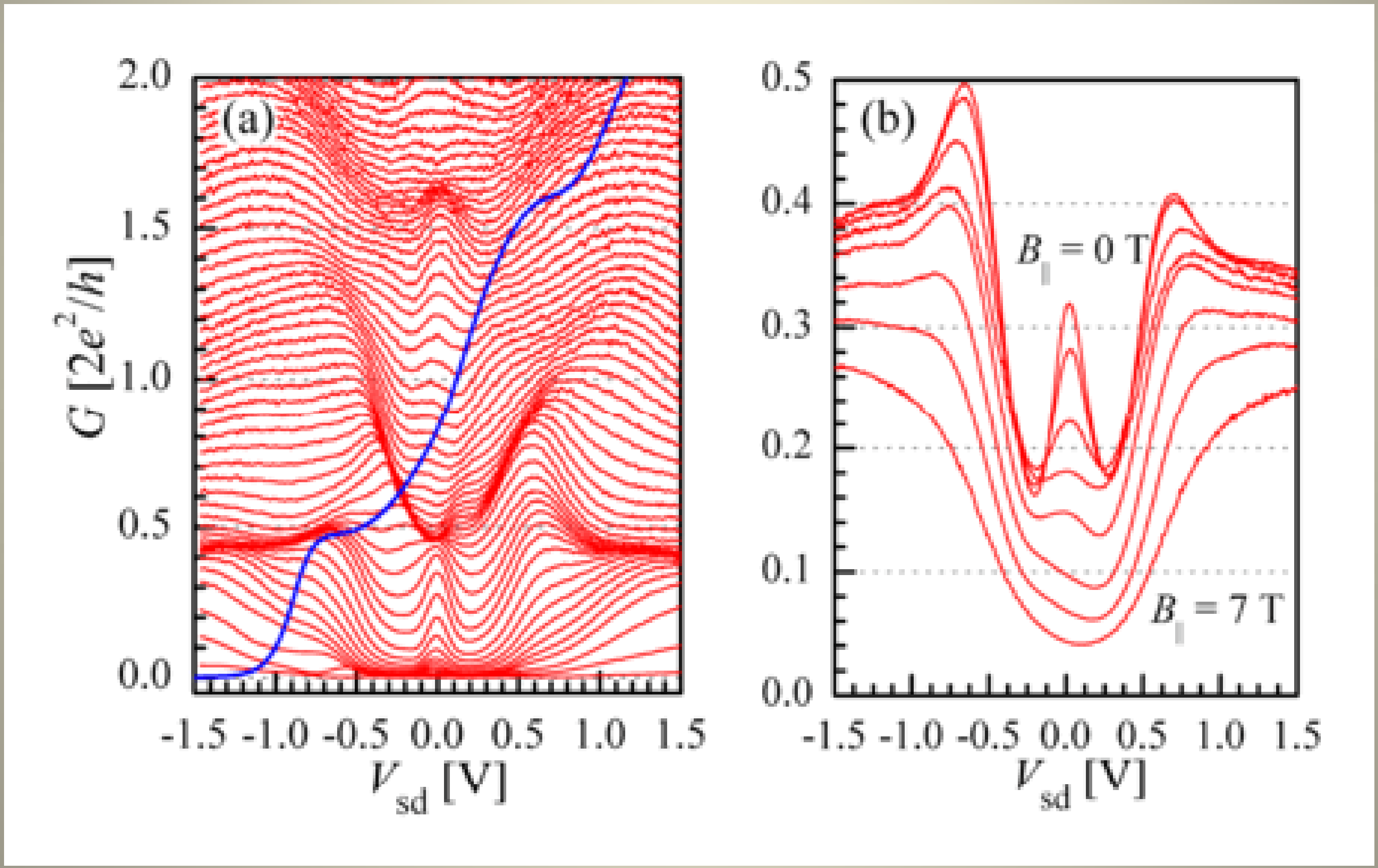


Spin-incoherent regime

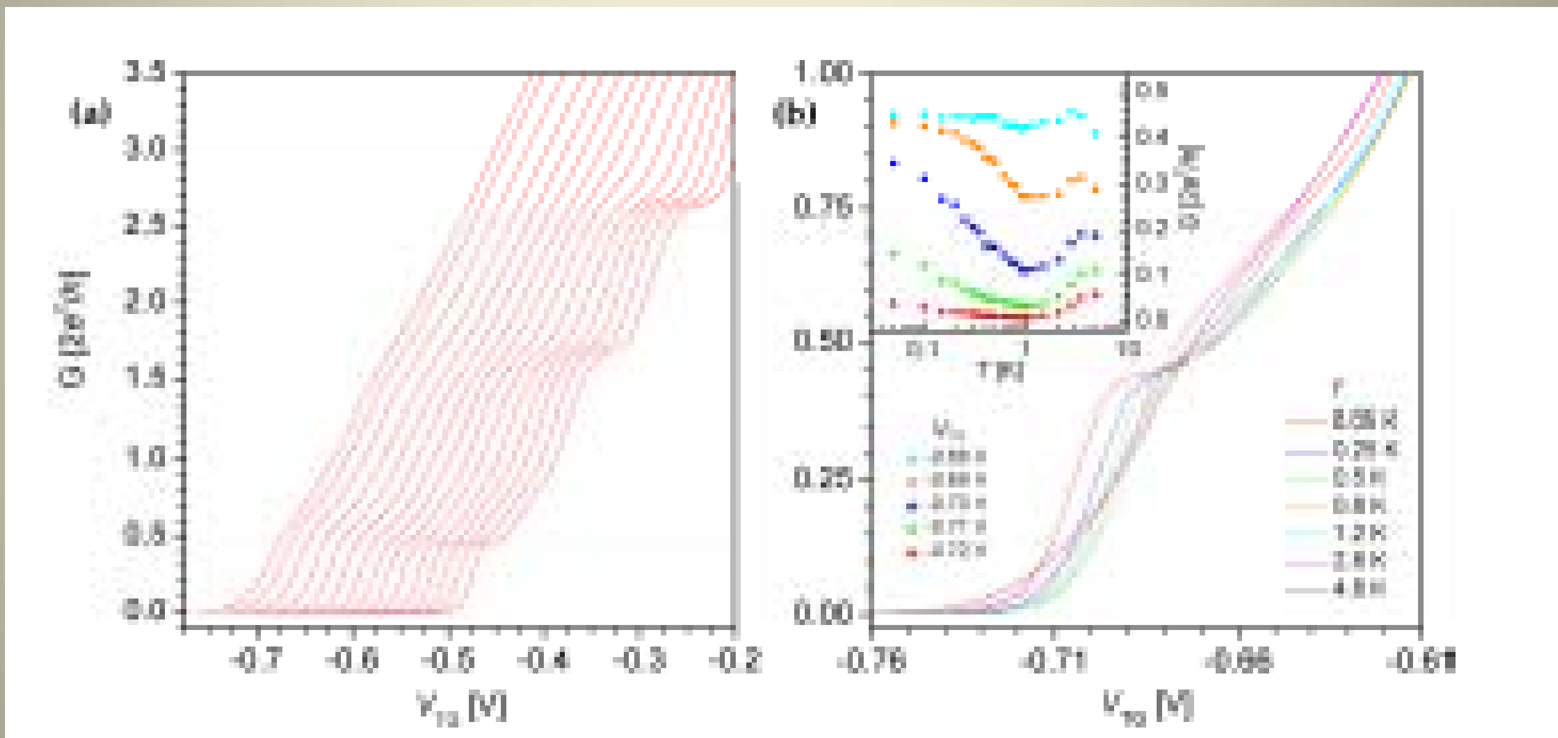


Spin-coherent (usual) transport regime

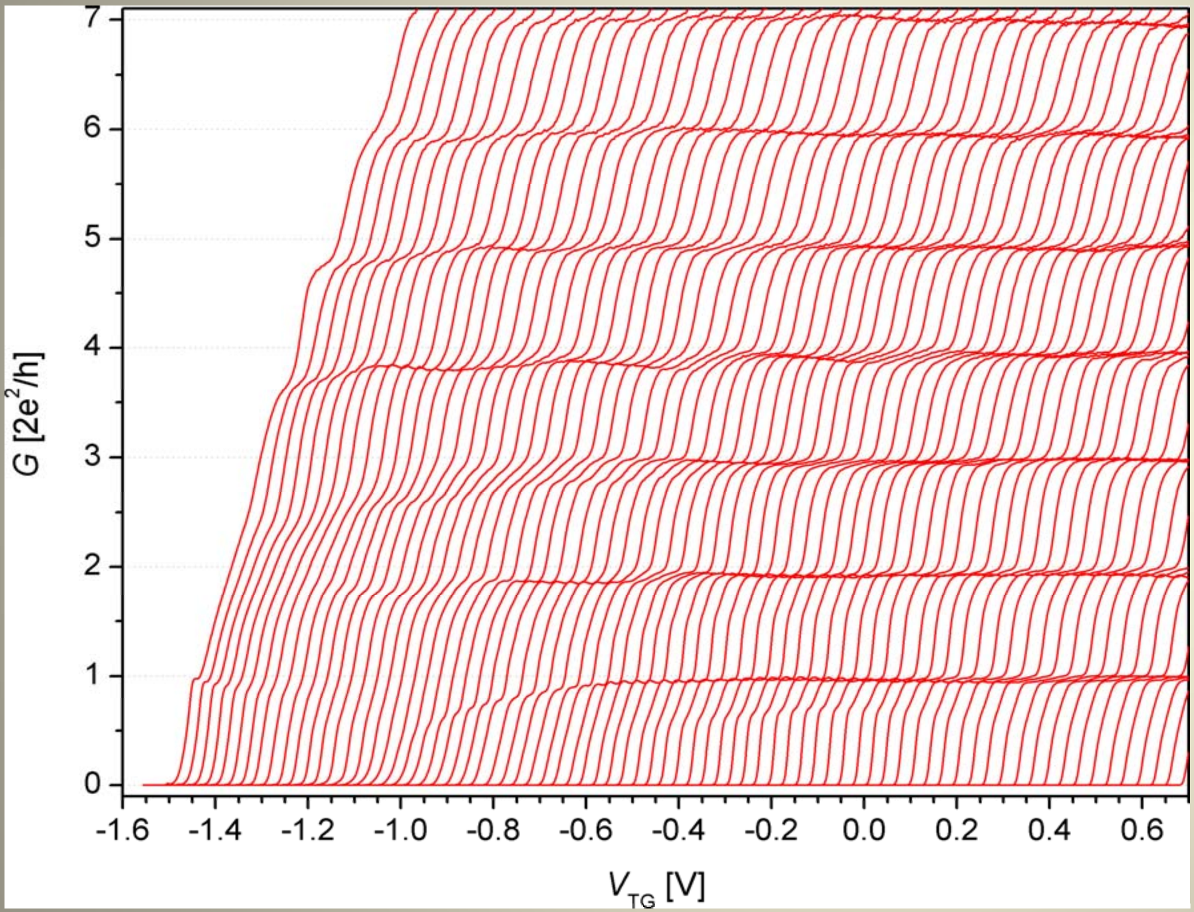
Bias dependence below and above e^2/h



Temperature dependence of spin-incoherent e^2/h plateau



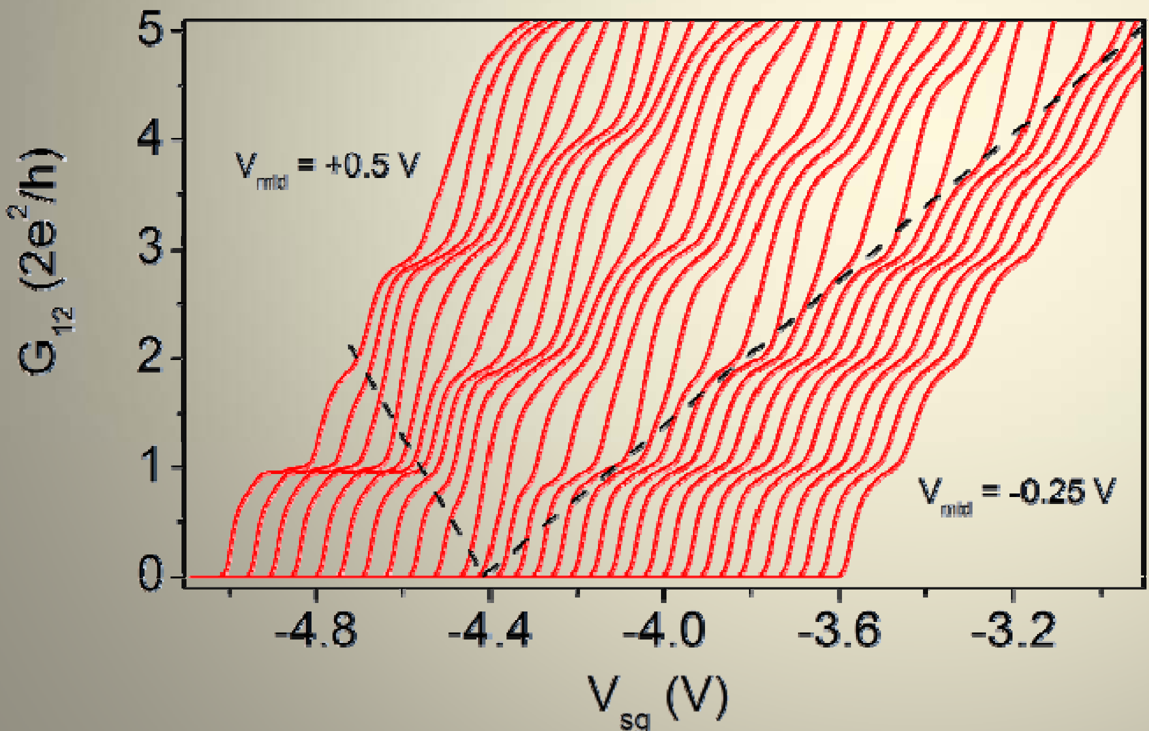
Double row Formation

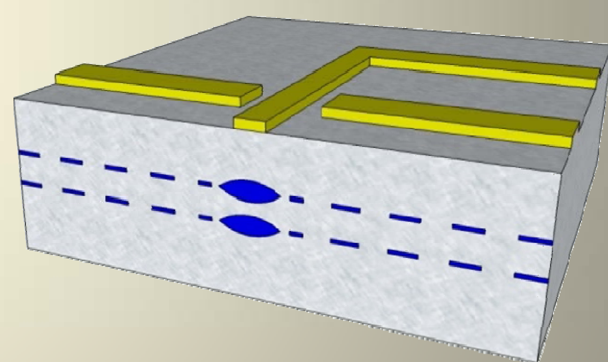


The double-row régime of transport manifests at lower electron densities when confinement is weakened.

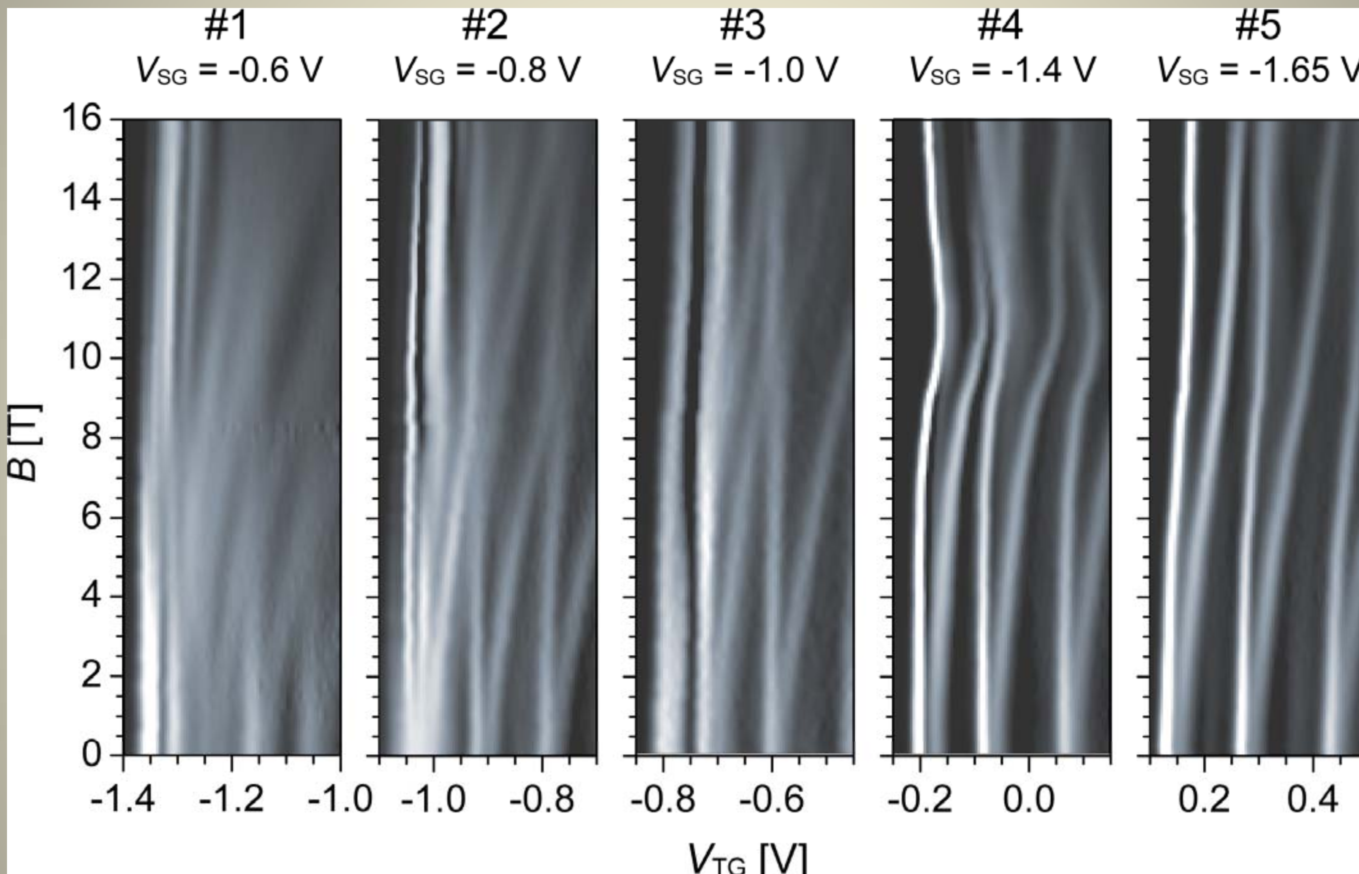
- n_{2D} ~ 1.6 × 10^{11}/cm^2
- The quantised plateaux are depressed as the 1D confinement weakens.
- The first plateau (at $2e^2/h$) falls and disappears in the weak-confinement régime, leading to a jump in conductance directly to $4e^2/h$ as the wire is populated.
- The $2e^2/h$ plateau reappears at the weakest confinements.

Two Quantum Wires Defined in a Double Quantum Well



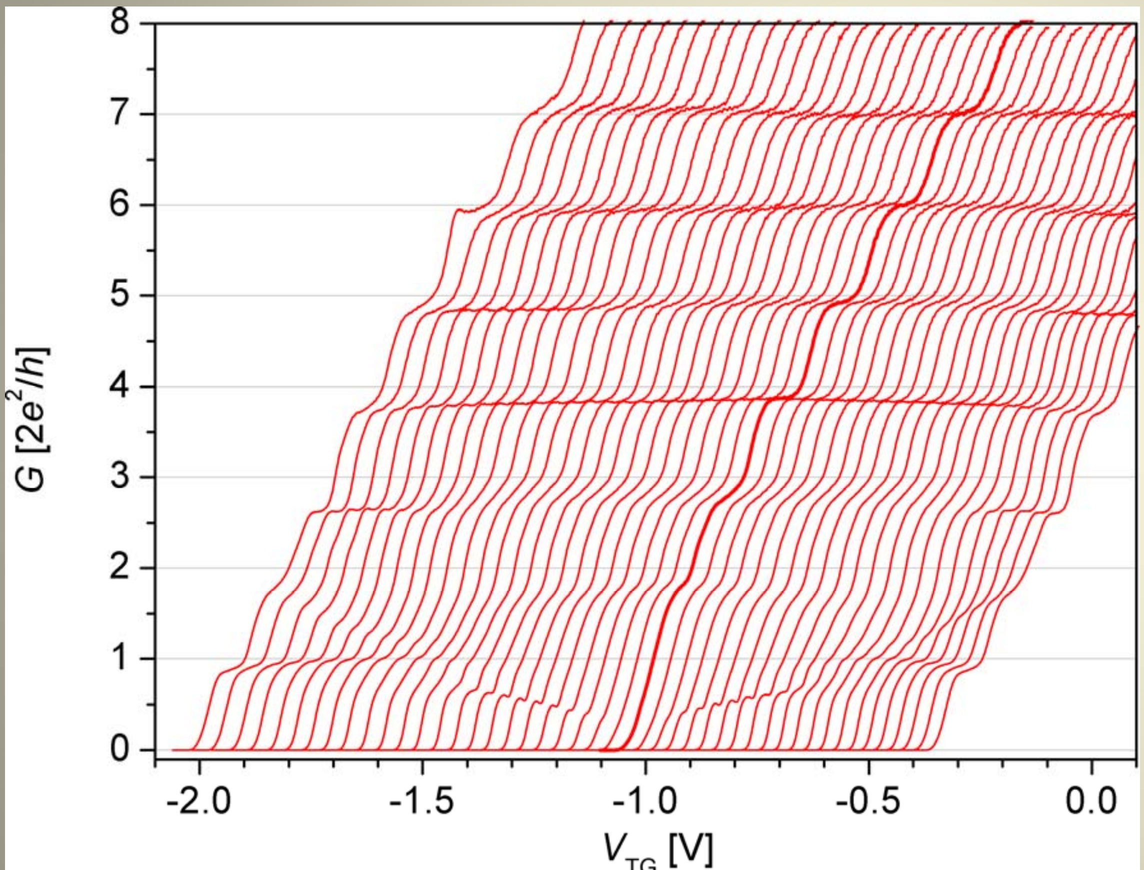


Magnetic-Field Dependence



grey-scale plot of the transconductance dG/dV_{TG} as a function of an in-plane magnetic field at five different confinement potentials, weakening from right to left

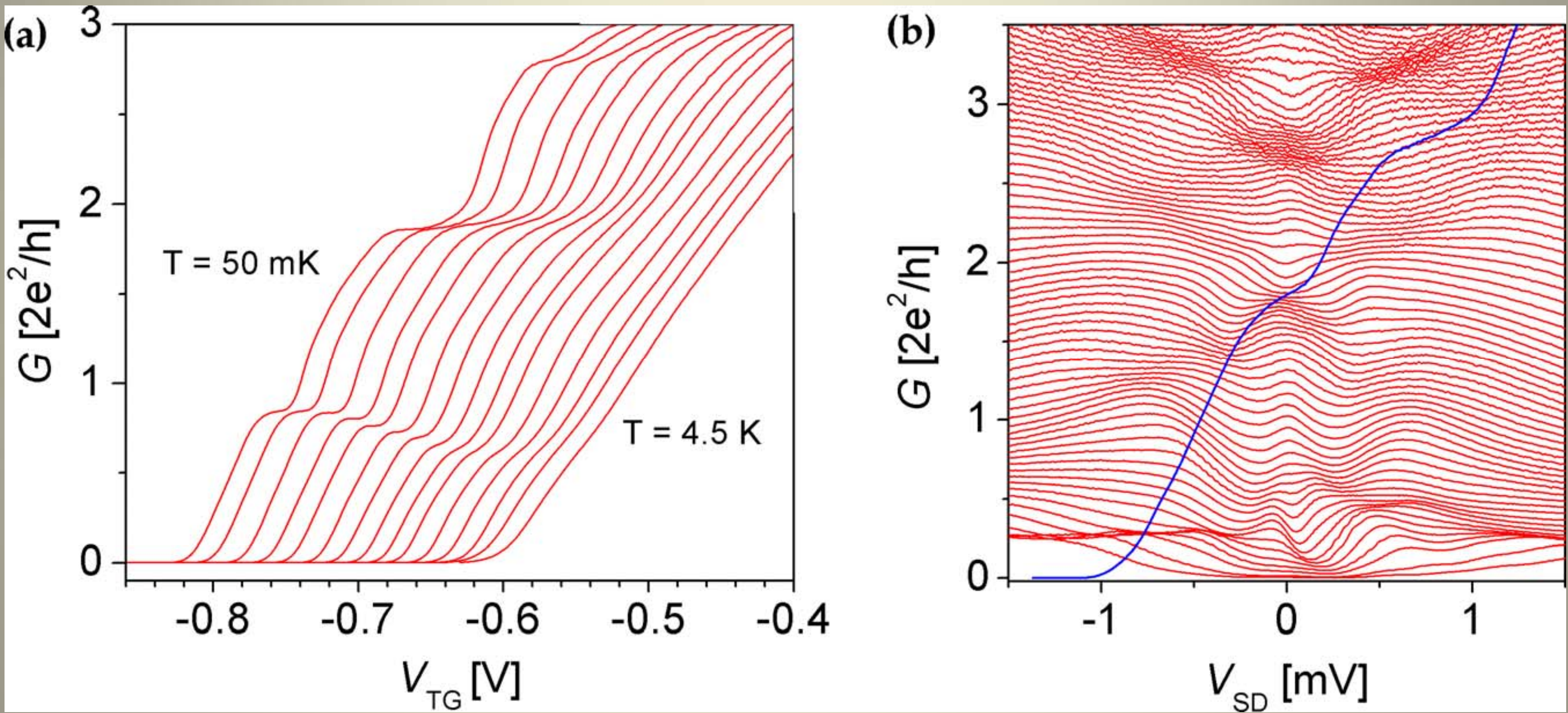
Lateral Displacement of the Channel



The quantum wire can be laterally displaced by applying a differential bias to the split gates.

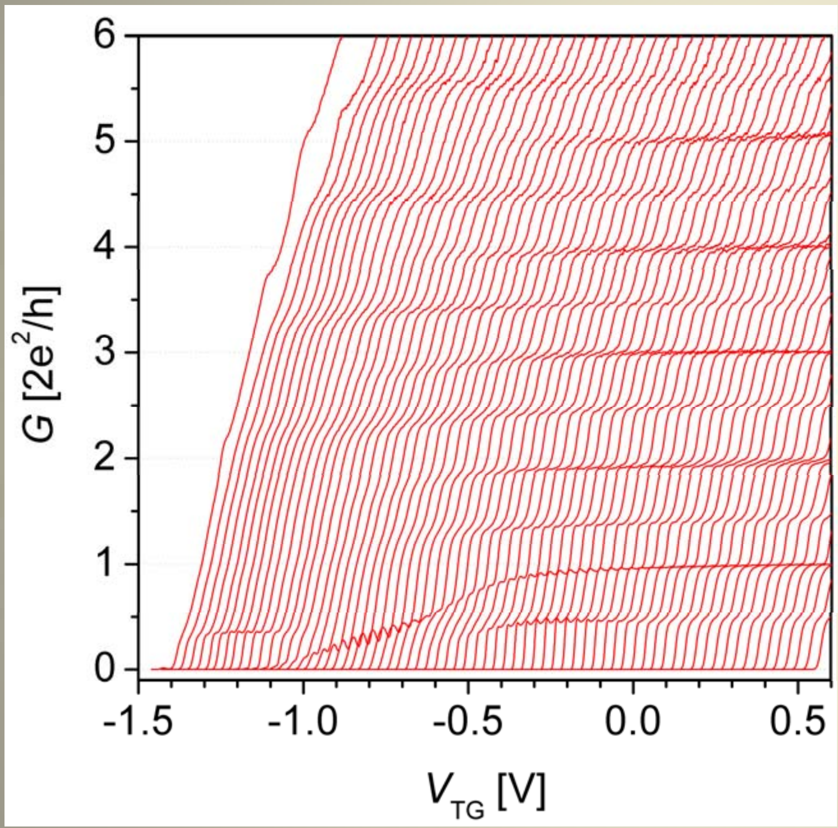
- total displacement of some 350 nm across the 2DEG
- The double-row régime is sensitive to lateral channel symmetry, for a small differential bias is sufficient to restore the plateau at $2e^2/h$.
- The higher plateaux are remarkably invariant.
- The high degree of symmetry of the traces with respect to the bias direction attests to the cleanliness of the 2DEG.

Temperature dependence and DC bias measurement in the double row regime

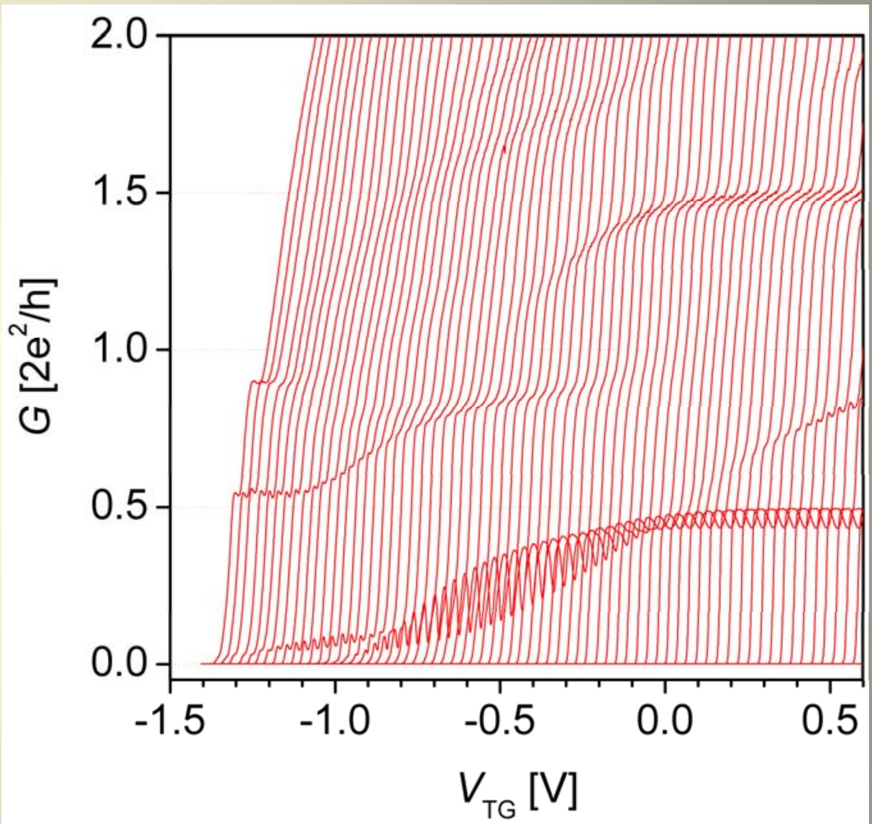


In-Plane Magnetic Field

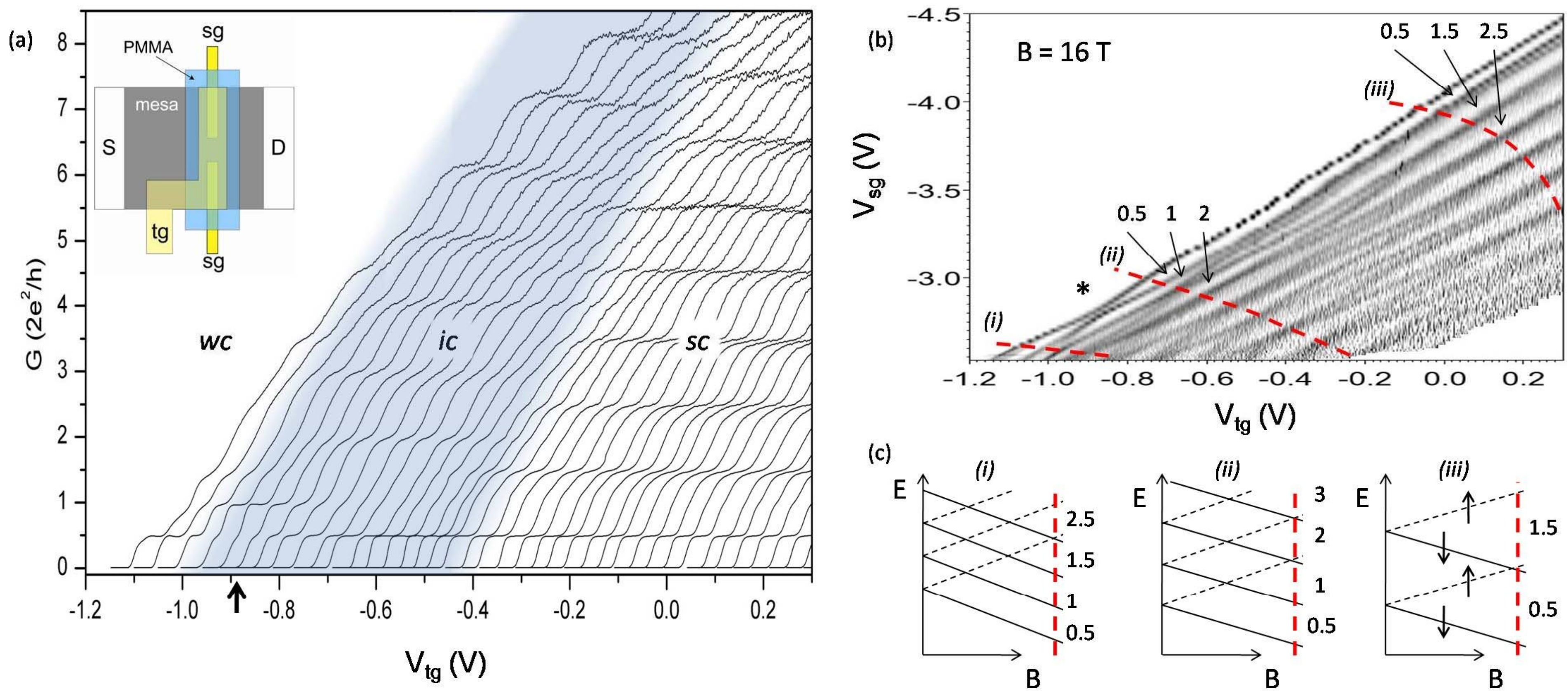
B = 7 T



B = 16 T



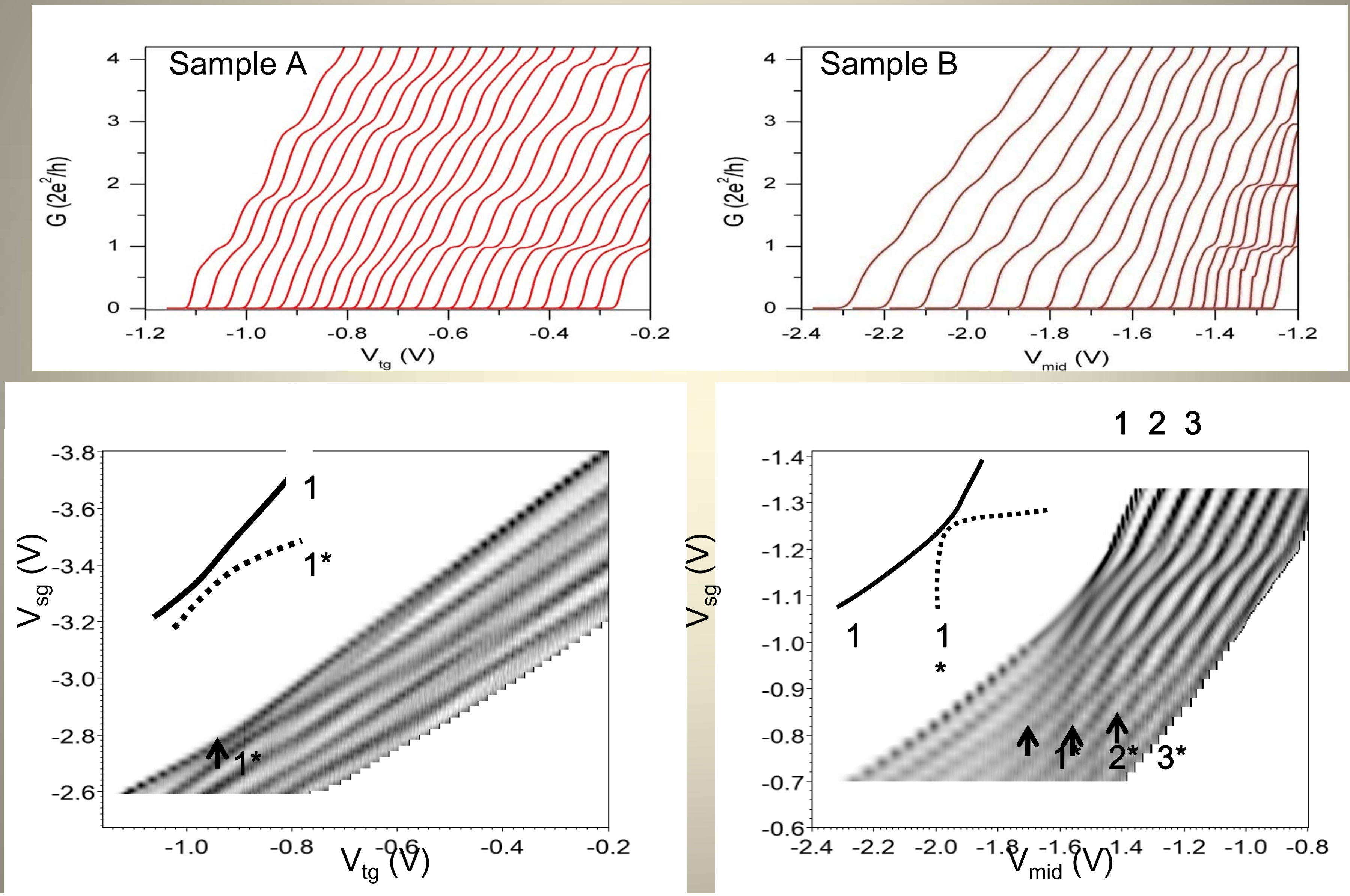
Coupling between rows



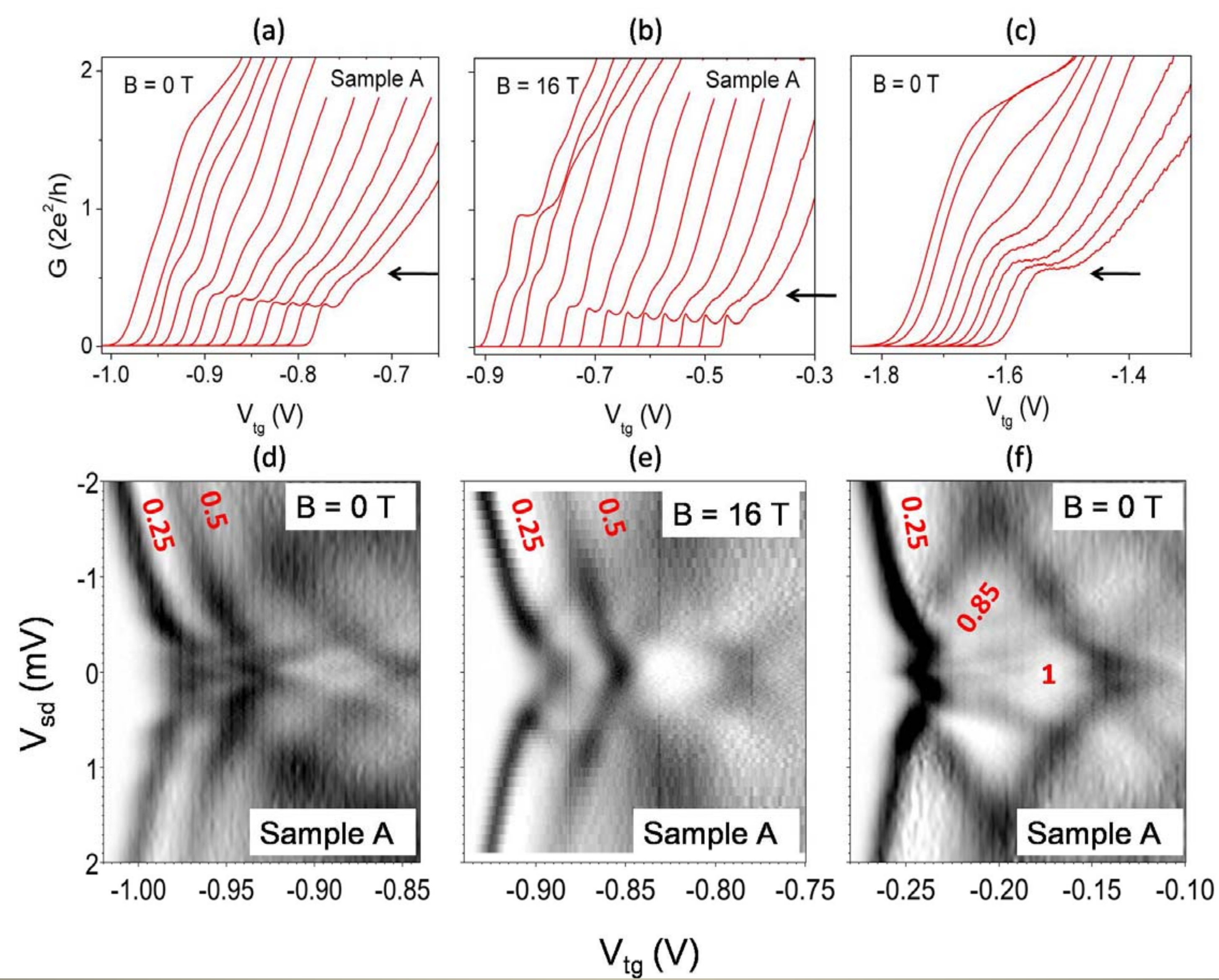
Split-gate width = 1 μm

L. W. Smith *et al.*, *Phys. Rev. B.*, **80**, 041306® (2009).

Coupling of Rows in Zero Magnetic Field

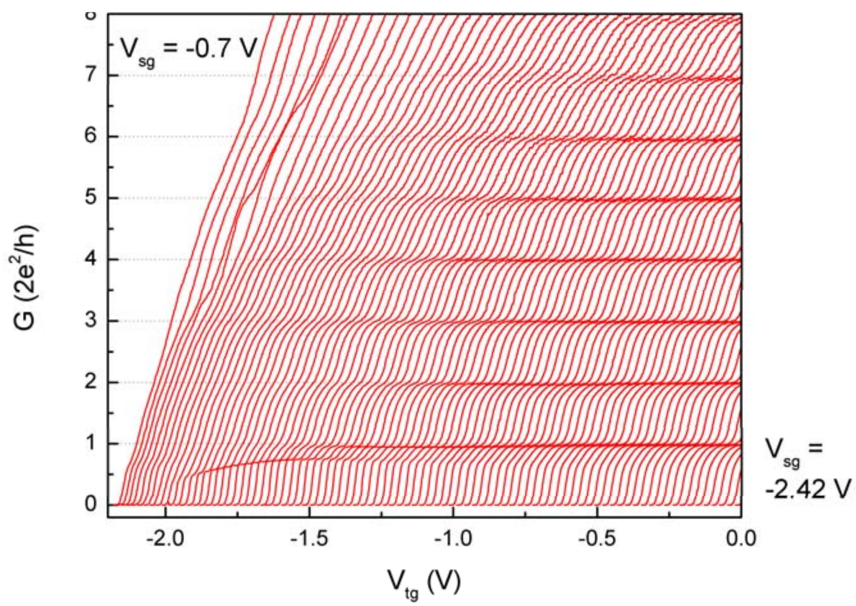


Source-Drain Bias Measurements

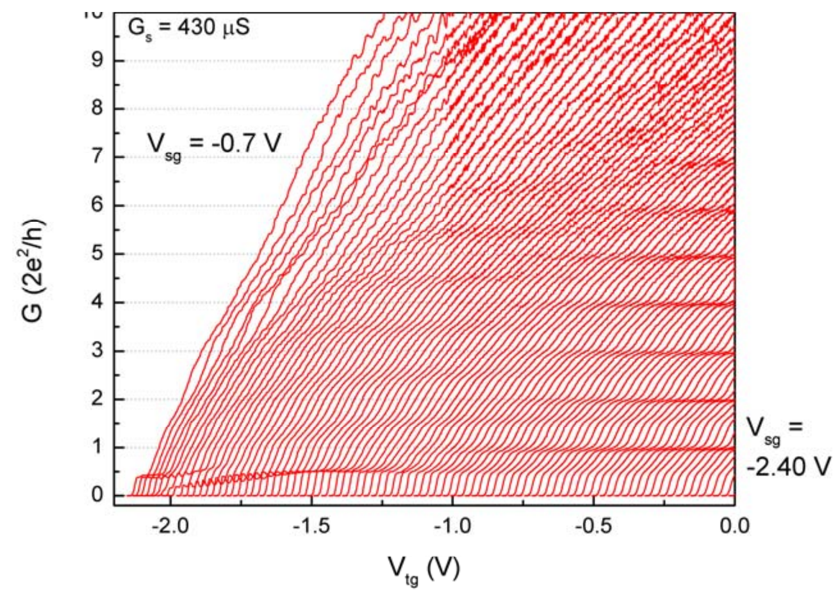


0.7 Structure at Weak Confinement

B = 0

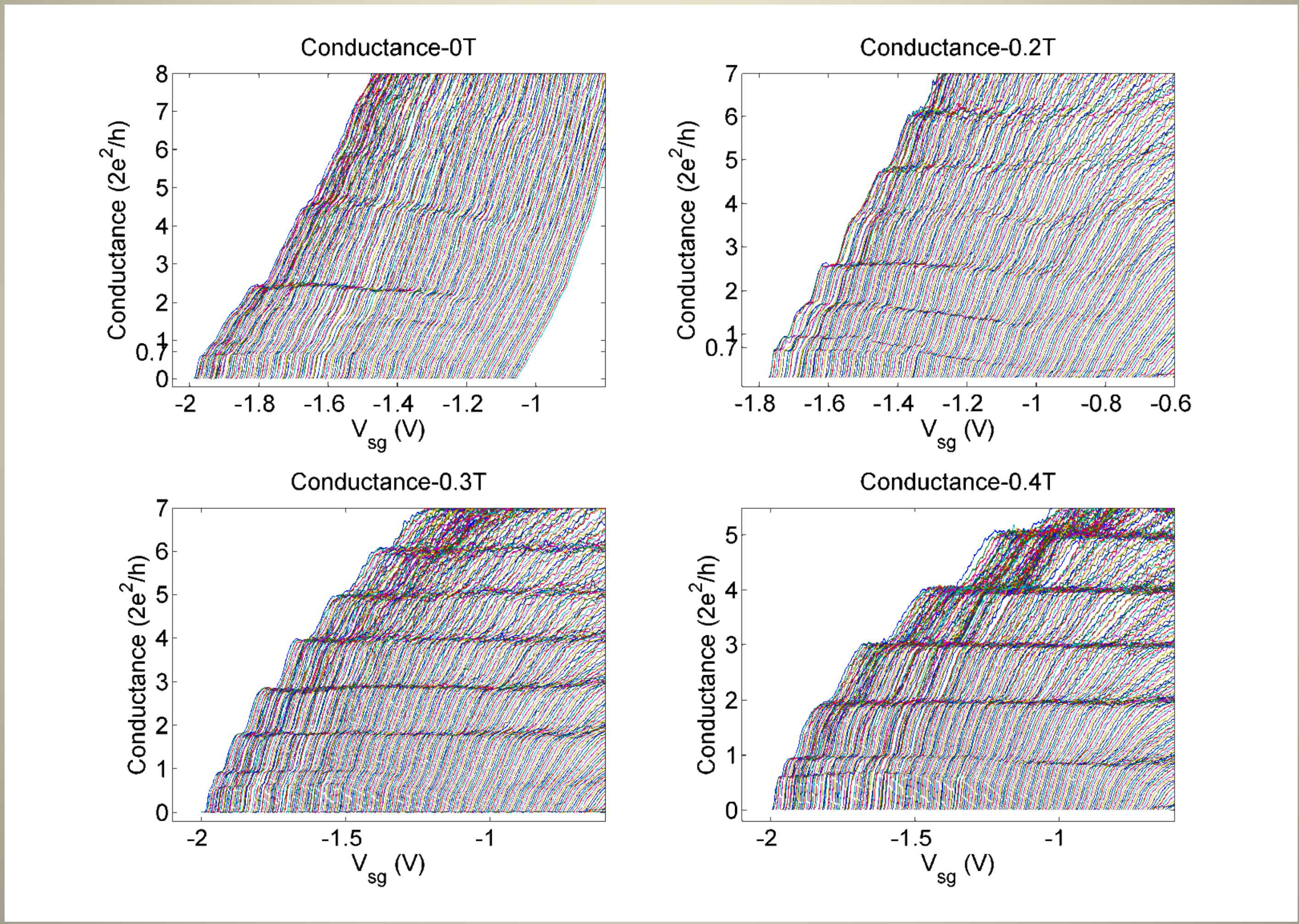


B = 8 T



Thomas *et al.*, (Unpublished 2012)

Tuning double row transport – Effect of transverse magnetic field



Sanjeev Kumar, KJ Thomas, M Pepper *et al.*, (Unpublished 2012)

Conclusions

- Theoretical literature, both classical and quantum, suggest that the electrons in a quasi-1D quantum wire will relax laterally as the transverse confinement potential weakens, staggering into a zigzag before bifurcating into two rows.

- At extremely low densities, it is possible to access the spin-incoherent Luttinger liquid transport regime. [W. K. Hew, K. J. Thomas et al., *Phys. Rev. Lett.* 101, 036801(2008)]

- The addition of a top gate to the standard split-gate device has permitted greater flexibility to explore the régime of weak confinement in quantum wires.

- Evidence has been obtained for the transition into double-row transport in the form of a jump in conductance to $4e^2/h$ at weak confinements. [W. K. Hew, K. J. Thomas *et al., Phys. Rev. Lett.* **102**, 056804 (2009)]

- Finite coupling between rows could be important in the context of a zig-zag Wigner crystal. [L. W. Smith, W. K. Hew, K J Thomas *et al., Phys. Rev. B* **80**, 041306R (2009)]

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Sanjeev Kumar
Mike Pepper, Ian Farrer, Dave Ritchie,
Dave Anderson, Geb Jones

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