

# *Transport of electrons through a photon cavity*

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# Cooperation

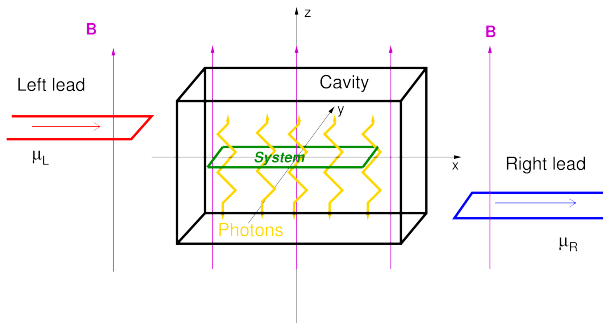
- Andrei Manolescu
  - Valeriu Moldoveanu
  - Chi-Shung Tang
  - Hsi-Sheng Goan
  - Ólafur Jónasson
  - Thorsten Arnold
  - Nzar Rauf Abdullah
  - Kristinn Torfason
- 
- New Journal of Physics 11, 073019 (2009)
  - New Journal of Physics 11, 113007 (2009)
  - Phys. Rev. B81, 155442 (2010)
  - Phys. Rev. B81, 205319 (2010)
  - New Journal of Physics, 14, 013036 (2012)
  - Phys. Rev. B85, 075306 (2012)
  - (arXiv:1203.3048), (arXiv:1203.5980), (arXiv:1207.6797)

# Approach

$$H(t) = \underbrace{H_e + H_{\text{Coul}}}_{\text{diagonalize+cut}} + H_{\text{EM}} + H_{e\text{-EM}} + H_{\text{LR}} + H_{\text{T}}(t)$$

diagonalize+cut  $\rightarrow H_S$

project on system( $H_S$ )  $\rightarrow$  GME



## e-EM coupling

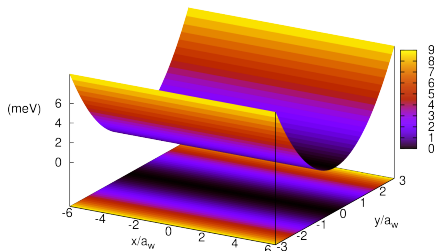
### Full electron-photon coupling

$$\begin{aligned} & \int d\mathbf{r} \psi^\dagger \left\{ \frac{1}{2m^*} \left( -i\hbar\nabla + \frac{e}{c} [\mathbf{A} + \mathbf{A}_{\text{ext}}] \right)^2 \right\} \psi \\ &= \int d\mathbf{r} \psi^\dagger \left\{ \frac{1}{2m^*} \left( -i\hbar\nabla + \frac{e}{c} \mathbf{A} \right)^2 \right\} \psi \\ & - \frac{1}{c} \int d\mathbf{r} \mathbf{j} \cdot \mathbf{A}_{\text{ext}} - \frac{e^2}{2m^*c} \int d\mathbf{r} \rho A_{\text{ext}}^2 \\ &= H_e + H_{e\text{-EM}} \end{aligned}$$

$$\begin{aligned} \mathbf{j} &= -\frac{e}{2m^*} \{ \psi^\dagger (\boldsymbol{\pi}\psi) + (\boldsymbol{\pi}^*\psi^\dagger) \psi \} \\ \rho &= -e\psi^\dagger\psi, \quad \boldsymbol{\pi} = \left( \mathbf{p} + \frac{e}{c} \mathbf{A}_{\text{ext}} \right) \end{aligned}$$

# Central system

- Finite parabolic quantum wire  $L_x = 300$  nm
- GaAs parameters



- External perpendicular magnetic field,  $\mathbf{B} = B\hat{z}$   
Confinement energy in  $y$ -direction  $\hbar\Omega_0 = 1.0$  meV
- Semi-infinite leads in magnetic field, parabolic  $y$ -confinement

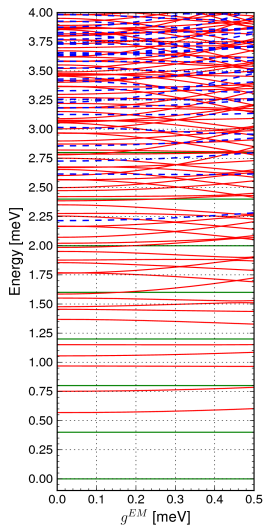
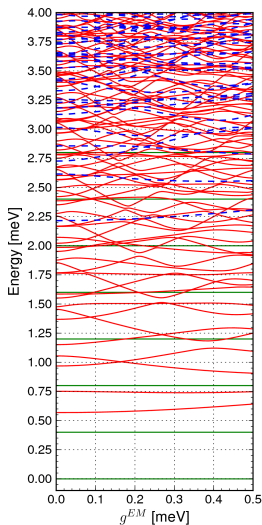
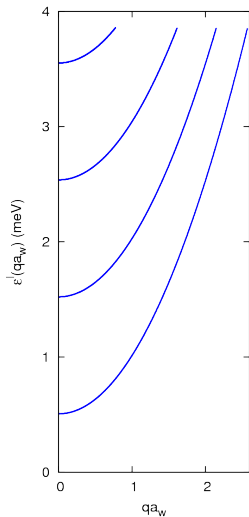
# Single cavity mode

$$H_S = \sum_a E_a d_a^\dagger d_a + \hbar \nu a^\dagger a + \frac{1}{2} \sum_{abcd} \langle ab | V_{\text{Coul}} | cd \rangle d_a^\dagger d_b^\dagger d_d d_c \\ + \sum_{ab} d_a^\dagger d_b \{ g_{ab} a + g_{ab}^* a^\dagger \}$$

$$H_S = \sum_\mu |\mu\rangle \tilde{E}_\mu(\mu) + \hbar \omega a^\dagger a + g^{\text{EM}} \sum_{\mu\nu ij} |\mu\rangle \langle \mu | \mathcal{V}^+ d_i^\dagger d_j \mathcal{V} | \nu \rangle \langle \nu | g_{ij} \{ a + a^\dagger \} \\ + g^{\text{EM}} \left( \frac{g^{\text{EM}}}{\hbar \Omega_w} \right) \sum_{\mu\nu i} |\mu\rangle \langle \mu | \mathcal{V}^+ d_i^\dagger d_i \mathcal{V} | \nu \rangle \langle \nu | \left\{ \left( a^\dagger a + \frac{1}{2} \right) + \frac{1}{2} (aa + a^\dagger a^\dagger) \right\}$$

$$\underbrace{|\mu\rangle \otimes |N_{\text{ph}}\rangle}_{\text{diagonalization}} \longrightarrow |\mu\rangle_{\text{e-EM}}$$

# Energy spectra, leads, e-EM central system x- and y-pol., $\hbar\nu = 0.4$ meV, $B = 0.1$ T



# Opening the system to the leads $\rightarrow$ GME

- Weak coupling to leads
- Non-Markovian
- Memory effects
- $\mathcal{P} = \rho_L \rho_R \text{Tr}_{LR}$
- $H_T^l(t) = \chi^l(t) \sum_{q,a} \{ T_{qa}^l d_{ql}^\dagger c_a + (T_{qa}^l)^* c_a^\dagger d_{ql} \}$
- Reduced statistical operator  
 $\rho_S(t) = \mathcal{P}\{W(t)\}$

Liouville-von Neumann equation

$$\dot{W}(t) = -\frac{i}{\hbar} [H(t), W(t)] = -i\mathcal{L}W(t)$$

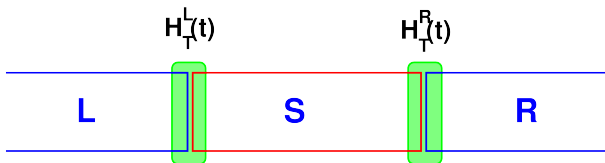
$$\langle A(t) \rangle = \text{Tr}\{W(t)A\} = \text{Tr}_S\{\rho_S(t)A\}$$

$$i\hbar\dot{\rho}_S(t) \approx \underbrace{\mathcal{L}_S \rho_S(t)}_{\text{closed system}} + \underbrace{\frac{1}{i\hbar} \text{Tr}_{LR} \left\{ \mathcal{L}_T(t) \int_0^t ds e^{-i(t-s)\mathcal{L}_0} \mathcal{L}_T(s) \rho_L \rho_R \rho_S(s) \right\}}_{\text{dissipation, memory}}$$



# Coupling Hamiltonian

Contact area



Coupling tensor with sensitivity to geometry

$$T_{aq}^l = \int_{\Omega_S^l \times \Omega_I} d\mathbf{r} d\mathbf{r}' (\psi_q^l(\mathbf{r}'))^* \psi_a^S(\mathbf{r}) g_{aq}^l(\mathbf{r}, \mathbf{r}')$$

Nonlocal overlap

$$g_{aq}^l(\mathbf{r}, \mathbf{r}') = g_0^l \exp[-\delta_1^l(x-x')^2 - \delta_2^l(y-y')^2] \exp\left(\frac{-|E_a - \epsilon^l(q)|}{\Delta_E^l}\right)$$

# Basis transformations

Transform the coupling tensor into the Coulomb interacting many-electron basis  $\{|\mu\rangle\}$  and the electron-photon basis  $\{|\check{\mu}\rangle\}$

$$\tilde{\mathcal{T}}^l(q) = \mathcal{W}^+ \mathcal{V}^+ \mathcal{T}^l(q) \mathcal{V} \mathcal{W}, \quad \left(\tilde{\mathcal{T}}^l(q)\right)^* = \mathcal{W}^+ \mathcal{V}^+ \left(\mathcal{T}^l(q)\right)^* \mathcal{V} \mathcal{W}$$

and the rest of the Hamiltonian after each diagonalization

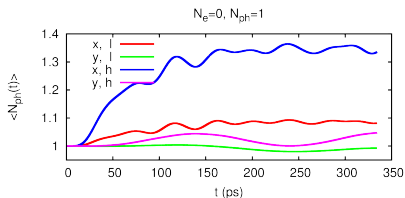
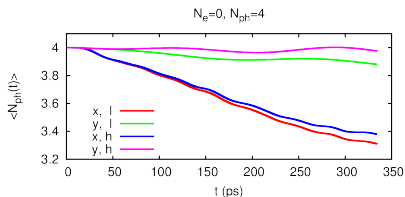
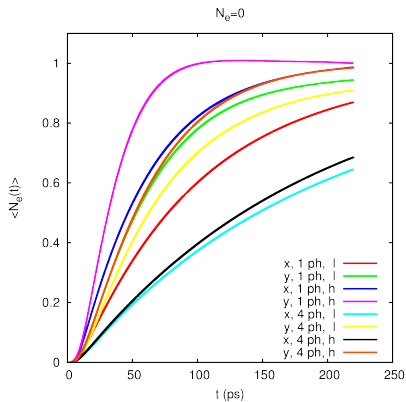
- Fock space construction and truncation schemes

## Step by step guide

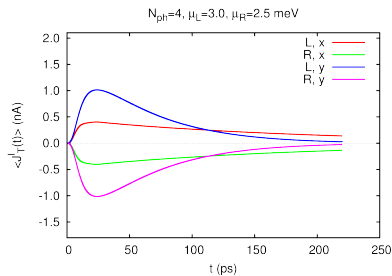
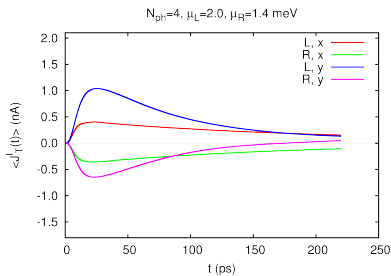
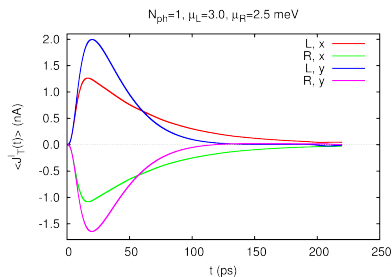
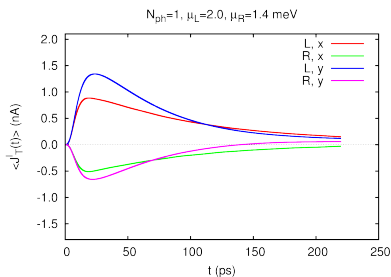
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- <http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-2.pdf>
- <http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-3.pdf>
- <http://hartree.raunvis.hi.is/~vidar/Nam/TE/GME-4.pdf>

# Empty system charged

$\hbar\Omega_0 = 1.0$  meV,  $\hbar\omega = 0.4$  meV,  $g^{\text{EM}} = 0.1$  meV,  $B = 0.1$  T,  
18 SES, 64 MES + 27 photons  $\rightarrow$  64 MBS, no spin

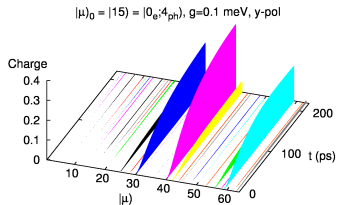
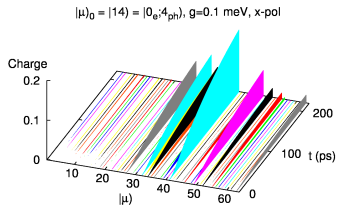
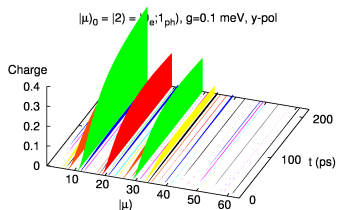
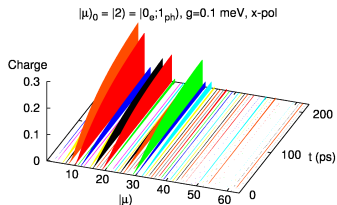


# Charging current



# Charging of many-body states

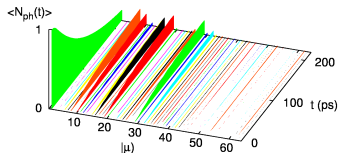
$$\mu_L = 2.0 \text{ meV}, \mu_R = 1.4 \text{ meV} \leftrightarrow |\check{1}\check{1}\rangle \text{ and } |\check{2}\check{1}\rangle$$



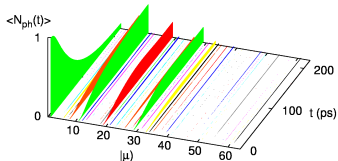
# Photon content of many-body states

$$\mu_L = 2.0 \text{ meV}, \mu_R = 1.4 \text{ meV} \leftrightarrow |\check{1}\check{1}\rangle \text{ and } |\check{2}\check{1}\rangle$$

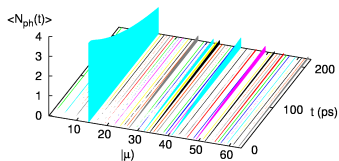
$$|\mu\rangle_0 = |2\rangle = |0_e; 1_{ph}\rangle, \text{ x-pol}$$



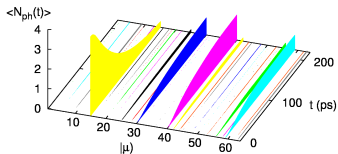
$$|\mu\rangle_0 = |2\rangle = |0_e; 1_{ph}\rangle, \text{ y-pol}$$



$$|\mu\rangle_0 = |14\rangle = |0_e; 4_{ph}\rangle, \text{ x-pol}$$

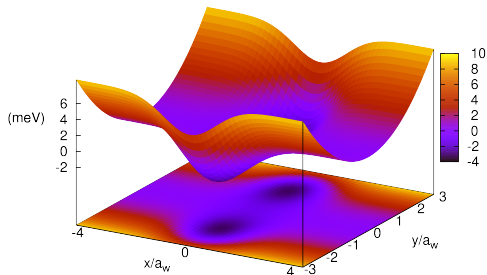


$$|\mu\rangle_0 = |15\rangle = |0_e; 4_{ph}\rangle, \text{ y-pol}$$

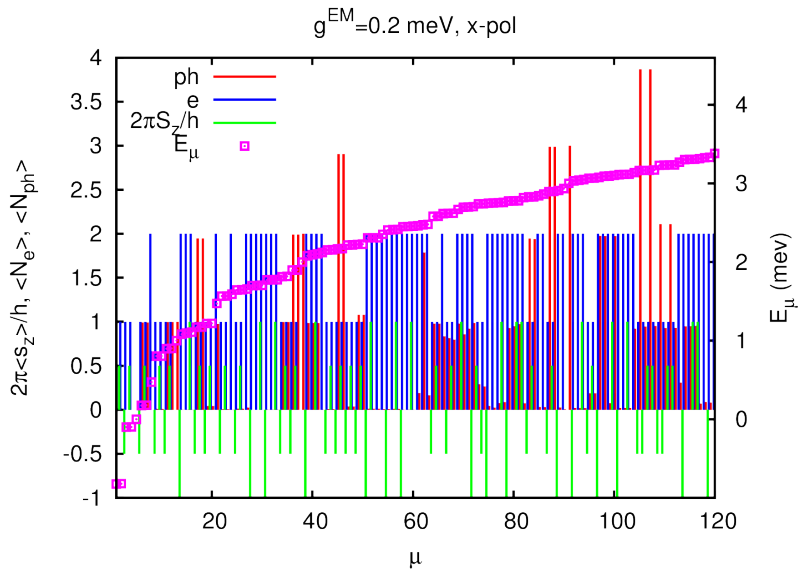


# Spin and details

- Spin
- Parallel dots
- $\hbar\Omega_0 = 2.0$  meV  
 $\hbar\omega = 1.0$  meV  
 $g^{\text{EM}} = 0.2$  meV  
 $g = 0.44$
- 36 SES, 120 MES
- 120 MBS

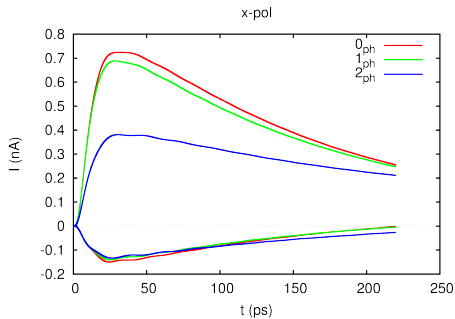
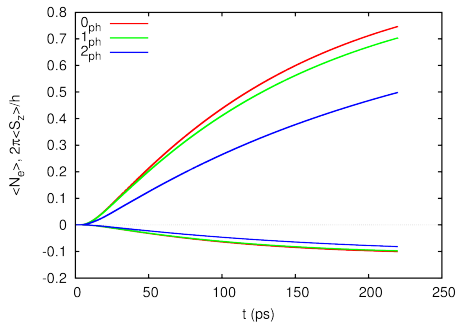


# Closed system





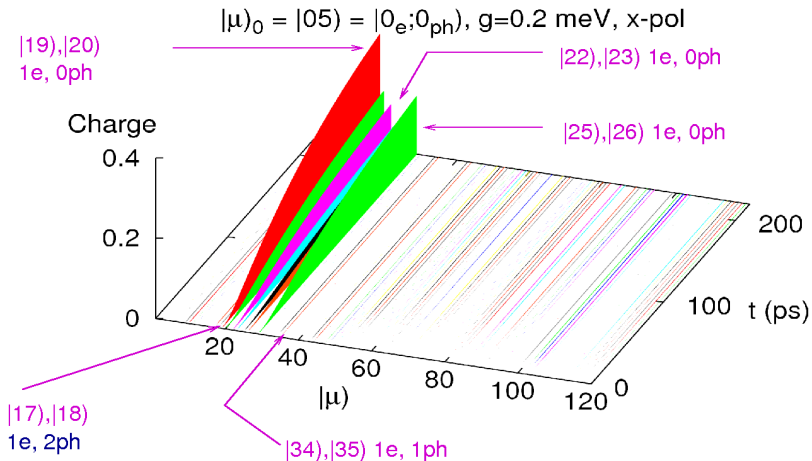
# Electron transport



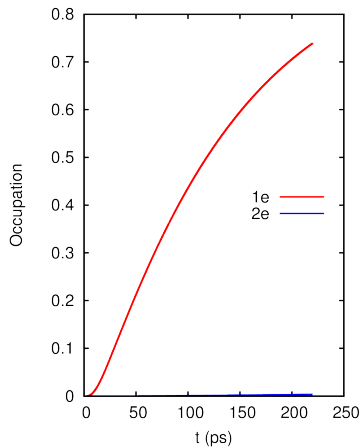
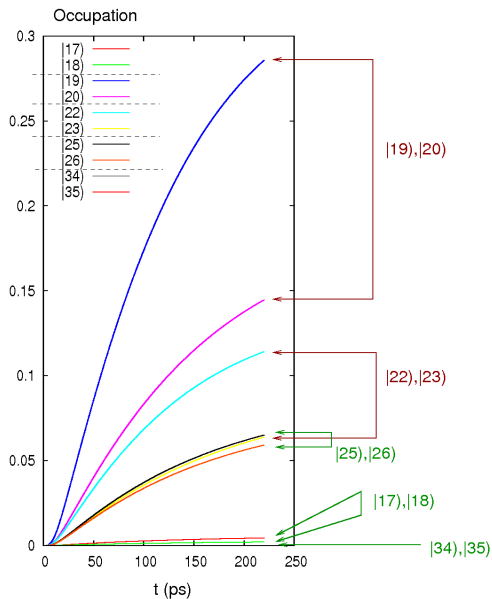
# Multi-photon processes

Bias window:  $|11\rangle$  ---  $|21\rangle$

Spin pairs

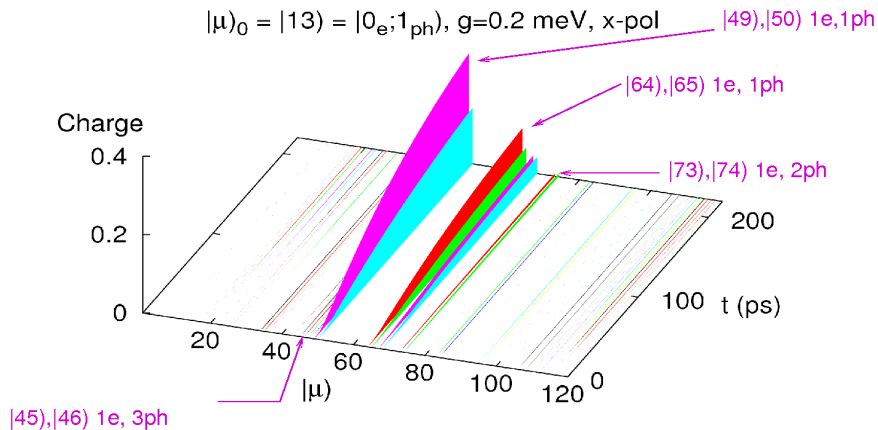


# Spin

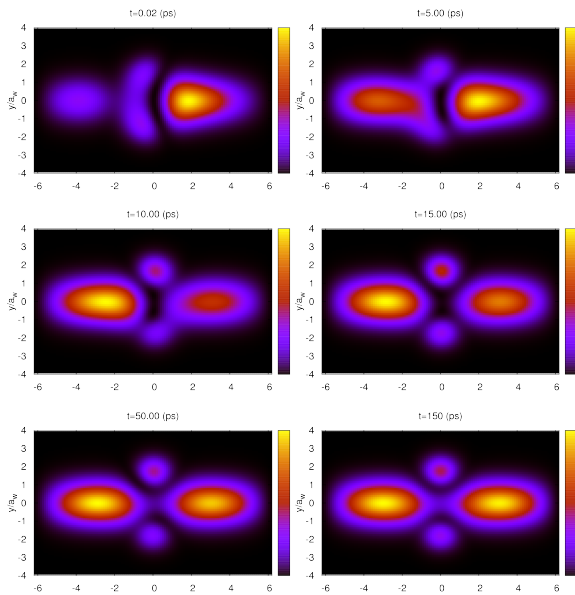


# Multi-photon processes

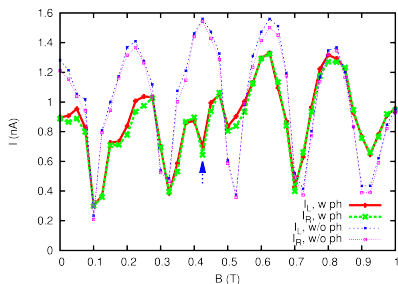
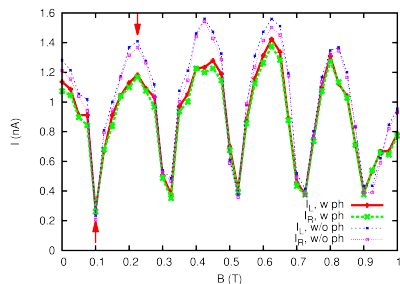
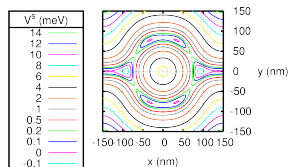
## Spin pairs



# Time-dependent charge



# Photon attenuated and assisted transport



# Summary

- Non markovian time-dependent electron transport
- Weak lead-system coupling – strong coupling to a cavity photon mode
- Numerically exact Coulomb and one-photon-mode interaction
- Finite bias, beyond linear response
- Geometrical effects, external homogeneous magnetic field
- Many-body correlations of photons and electrons with spin
- Freedom in choosing initial states
- Parallelization for CPU's and GPU's