Interacting gauge theories with Bose gases: continuum and lattice effects

M. J. Edmonds[1], M. Valiente[1], G. Juzeliūnas[2], L. Santos[3] & P. Öhberg[1].

[1] *SUPA, Heriot-Watt University, Edinburgh*

[2] *Institute of Theoretical Physics and Astronomy, Vilnius University, Lithuania*

[3] *Institute für Theoretische Physik, Leibniz Universität, Hannover, Germany*

14th February 2013

Contents of talk

Gauge theories with cold atoms

- Theoretical framework: optically induced gauge potentials
- Single particle theory: geometric phases
- Many particles: density dependent vector potential
- Many body equation of motion: currents
- One dimensional example: Chiral solitons.
- Lattice formulation
- Pendulum model+phase-space analysis
- Outlook and Conclusions.

The premise: giving cold atoms a charge

Charged particles

- Want to simulate Hamiltonian of charged particles in a magnetic field:

$$\hat{\mathcal{H}} = \sum_i \frac{1}{2m}(\hat{p}_i - q_i \vec{A}(\mathbf{r}, t))^2 \tag{1}$$

- Consider atomic cloud of Ultracold Bosons/Fermions: $q_i^{tot} = 0 \rightarrow$ No natural coupling to gauge potential, $\vec{A}(\mathbf{r}, t)$.
- Methodologies to address this: stir condensate with laser, Rabi/Raman optical couplings.

The story so far...

Many theoretical and experimental papers on continuum gauge potentials with cold atoms, progress so far:

Gauge potentials with cold atoms

$\vec{A}(\mathbf{r}, t) \propto$	Theory	Experiment	Symmetry
$f(\mathbf{r})$	✓	✓	$U(1)$
$\hat{\sigma}_x \hat{e}_x$	✓	✓	U(1) (spin-orbit)
$\hat{\sigma}_x \hat{e}_x + \hat{\sigma}_y \hat{e}_y$	✓	×	SU(2) (non-abelian)
λ_i	✓	×	SU(3)
$f[\psi(\mathbf{r})]$	?	×	...

So far, all vector potentials have been *static*: they do not depend on $\psi(\mathbf{r})$.
Open question: how do we generate an interacting gauge potential?

J. Ruseckas et al, *Phys. Rev. Lett.* **95** 010404 (2005),

G. Juzeliūnas et al, *Phys. Rev. A.* **81** 053403 (2010),

M. Edmonds et al, *New J. Phys.* **14** 073056 (2012).

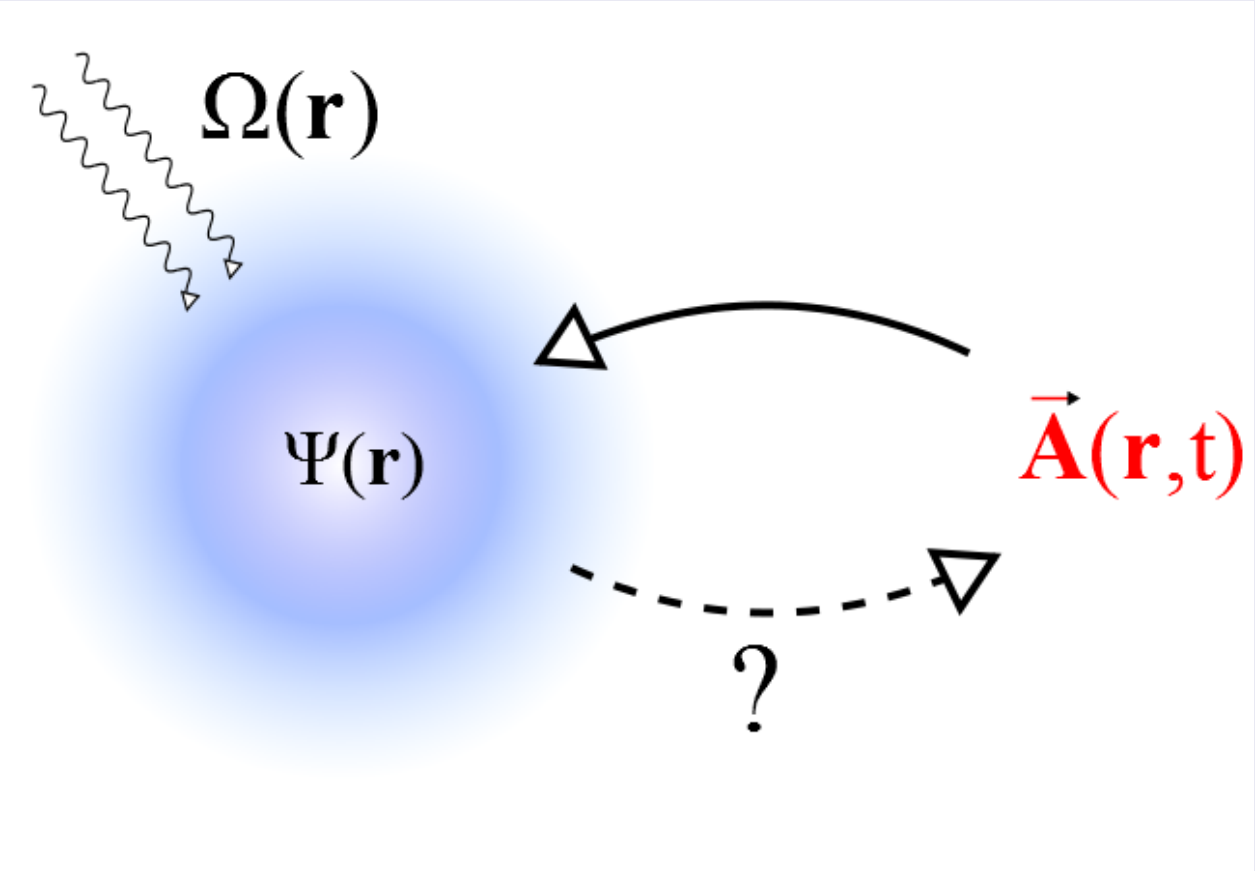
Y.-J. Lin et al, *Nature* **471** 83-86 (2011),

Y.-J. Lin et al, *Phys. Rev. Lett.* **102** 130401 (2009),

Y.-J. Lin et al, *Nature* **462** 628-632 (2009).

Interacting gauge potentials for a BEC?

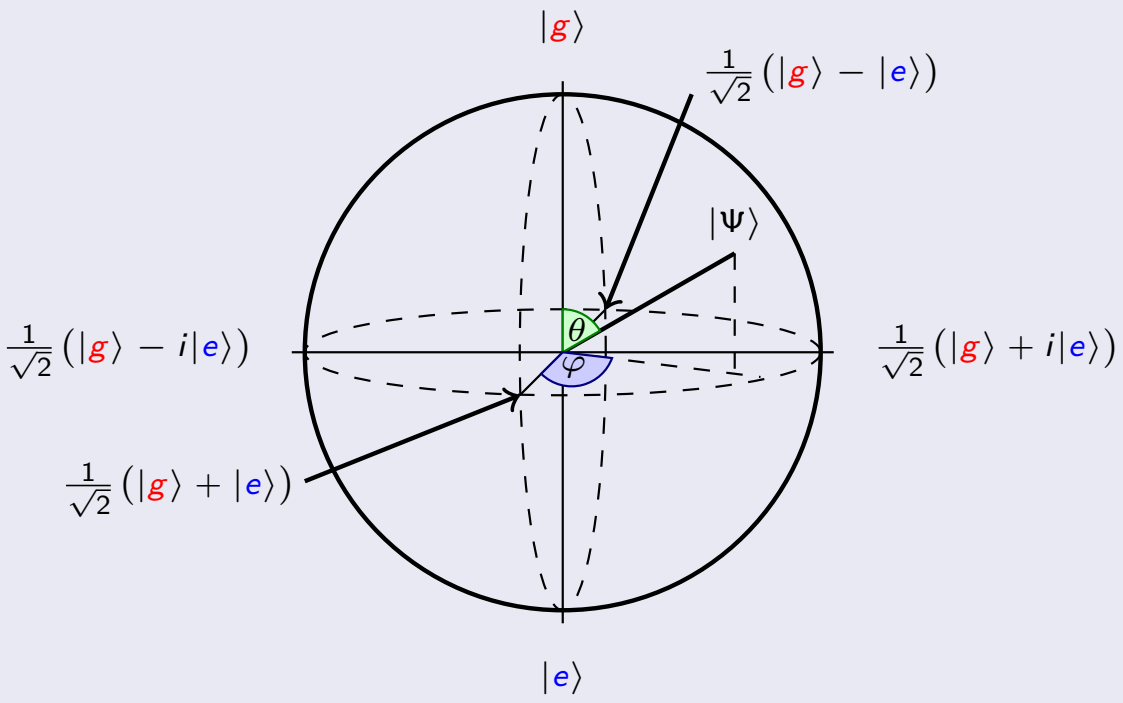
The goal: An effective back-action



- Can we make the gauge potential depend on the motion of the atoms?

Optically induced gauge potentials (i)

Illustrative model: two level system



Consider two states $\{|g\rangle, |e\rangle\}$, driven by classical field $\Omega(\mathbf{r})$, light-matter coupling described operator $\hat{H}_{lm}$:

$$\hat{H}_{lm} = \frac{\hbar\Omega_{eg}}{2}\left\{\cos\theta(|g\rangle\langle g| - |e\rangle\langle e|) + \sin\theta(e^{-i\varphi}|g\rangle\langle e| + h.c.)\right\}. \tag{2}$$

J. Dalibard et al, Rev. Mod. Phys. **83** *1523 (2012)*

Optically induced gauge potentials (ii)

- Eigenstates of $\hat{H}_{lm}$ (dressed states) given by $\{|\chi_+\rangle, |\chi_-\rangle\}$, use these as basis for molecular gauge theory:

$$|\Psi(\mathbf{r}, t)\rangle = \sum_{j \in \{+,-\}} \psi_j(\mathbf{r}, t) \otimes |\chi_j(\mathbf{r})\rangle \tag{3}$$

- We can use this ansatz to derive an equation of motion for ψ_j:

$$i\hbar\dot{\psi}_\pm = \left[\frac{1}{2m}(\hat{\mathbf{p}} - \mathbf{A}_{jl}(\theta, \varphi))^2 \pm \frac{\hbar\Omega}{2} + W\right]\psi_\pm. \tag{4}$$

- The vector potential $\mathbf{A}_{jl} = i\hbar\langle\chi_j|\nabla\chi_l\rangle$ and the scalar potential $W = \frac{\hbar^2}{2m}|\langle\chi_+|\nabla\chi_-\rangle|^2$ are geometric phases: they depend on the spatial variation of lasers through the "angles" θ & φ.

M. Berry, Proc. R. Soc. Lond. A **392** 45-47 (1984).

Many body description (i)

To go beyond the simple "static" single particle vector potentials, we can take N two level atoms, whose effective mean-field Hamiltonian is:

$$\hat{H}_{GP} = \frac{\hat{\mathbf{p}}^2}{2m} \otimes \hat{1} + \hat{H}_{lm} + \frac{1}{2}\begin{pmatrix} g_{11}\rho_1 + g_{12}\rho_2 & 0 \\ 0 & g_{22}\rho_2 + g_{12}\rho_1 \end{pmatrix}. \quad (5)$$

where $\rho_i = |\Psi_i|^2$, $\langle \hat{H} \rangle_{GP} = \langle \Psi | \hat{H}_{GP} | \Psi \rangle$ and $|\Psi\rangle = \otimes_{M=1}^{N} |\Psi_M\rangle$.
$\Rightarrow$ Introduce perturbed dressed states:

$$|\chi_\pm\rangle = |\chi_\pm^{(0)}\rangle \pm \frac{g_{11} - g_{22}}{8\hbar\Omega} |\Psi_\pm|^2 |\chi_\mp^{(0)}\rangle. \quad (6)$$

In the $\{|\chi_+\rangle, |\chi_-\rangle\}$ basis, the effective Hamiltonian then becomes:

$$\boxed{\hat{H}_\pm = \frac{1}{2m}(\hat{\mathbf{p}} - \mathbf{A}_\pm[\mathbf{r}; |\Psi_\pm|^2])^2 + W \pm \frac{\hbar\Omega}{2} + \frac{1}{2} g |\Psi_\pm|^2.} \quad (7)$$

Where $\mathbf{A}_\pm = \mathbf{A}_\pm^{(0)} \pm \mathbf{a}_1 |\Psi_\pm|^2$.

Many body description (ii)

The final step is to minimize $\langle \hat{H}_{GP} \rangle$ in order to get a Gross-Pitaevskii like equation for the condensate $i\hbar\dot{\Psi} = \delta\langle \hat{H}_{GP} \rangle / \delta\Psi^*$:

$$i\hbar\partial_t\Psi = \left[\frac{\mathbf{\Pi}_\pm^2}{2m} \pm \mathbf{a}_1 \cdot \mathbf{j} + W + g|\Psi_\pm|^2\right]\Psi. \tag{8}$$

where $\mathbf{\Pi}_\pm = \mathbf{p} - \mathbf{A}_\pm$, and the current: $\mathbf{j} = \frac{h}{m}\mathrm{Im}\{\Psi^*(\nabla - \frac{i}{\hbar}\mathbf{A}_\pm)\Psi\}$.

- Mean-field description of an Interacting gauge potential.
- Strength of gauge potential is controlled via $\mathbf{a}_1$ term.
- Scalar potential is given by $W \approx \frac{1}{2m}(-\frac{\hbar}{2}\nabla\phi)^2$.
- The constant $g = \frac{1}{4}(g_{11} + g_{22} + 2g_{12})$ parameterizes the scattering.
- Current $\mathbf{j}$ gives rise to novel effects...

Chiral solitons

We can solve the Schrödinger equation exactly to give *chiral* solitons:

$$\hat{H} = \frac{1}{2m}(-i\hbar\partial_x - a_1|\Psi|^2)^2 - a_1\, j(x) + g|\Psi|^2. \tag{9}$$

Make gauge transformation: $\Psi = \exp\left(ia_1 \int_{-\infty}^{x} dx'|\Psi(x',t)|^2\right)\Phi$:

$$\hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 - 2a_1\, j(x) + g|\Phi|^2. \tag{10}$$

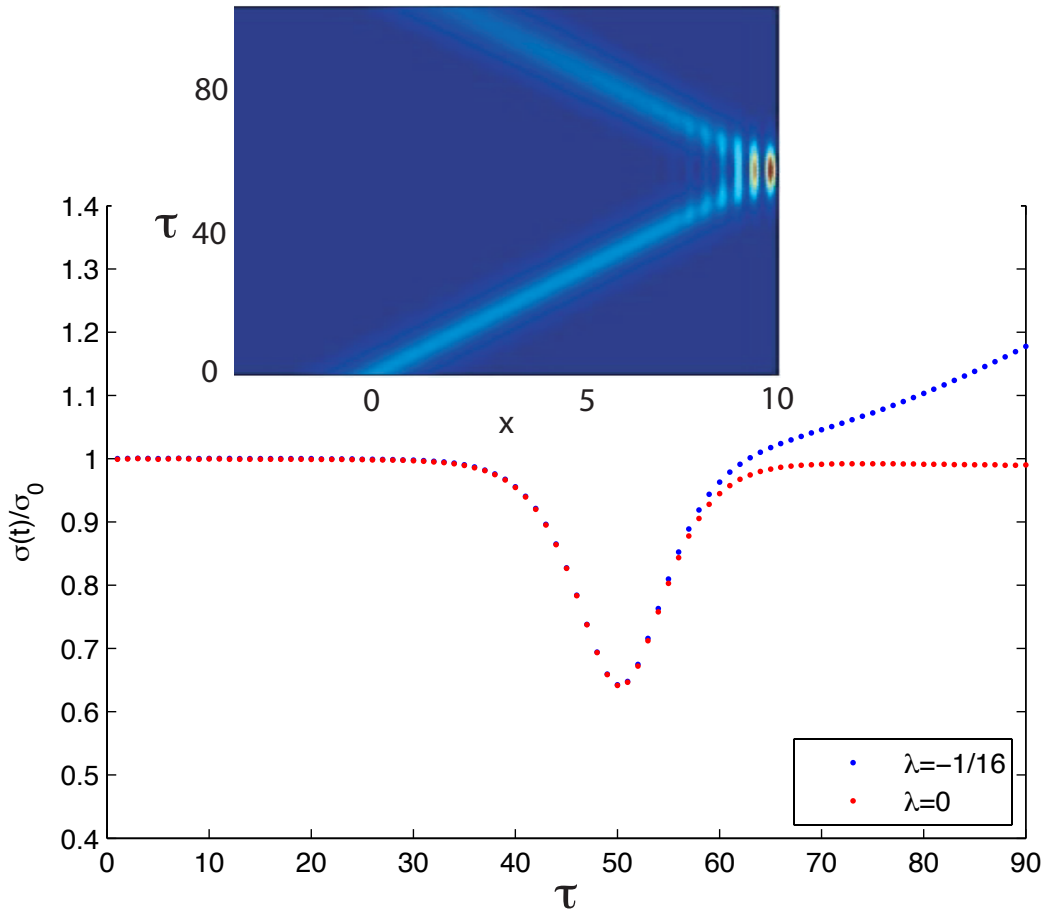
The current $j(x)$ can itself be transformed into a density $|\Psi|^2$ using $\Psi = \sqrt{\rho}e^{ikx}$:

$$\hat{H} = -\frac{\hbar^2}{2m}\partial_x^2 + (g - v)|\Phi|^2. \tag{11}$$

Where $v = \hbar k/m$ gives the velocity of the soliton solutions: the type of soliton depends on sign of v!

U. Aglietti et al, *Phys. Rev. Lett.* **77**, 4406 (1996).

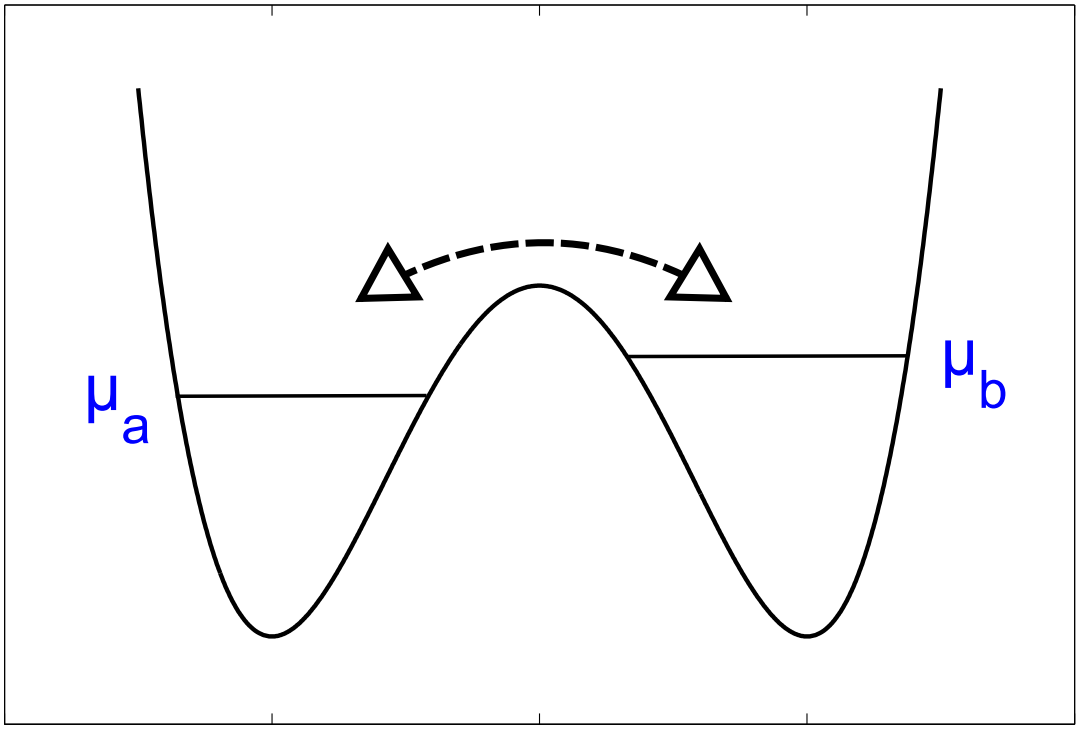
Chiral solitons (ii)



M. J. Edmonds et al, arXiv 1212.0445 PRL (in press)

Lattice formulation (i)

Double well potential



The double well is described by $V(x) = \alpha(x^2 - x_{\text{min}}^2)^2$.

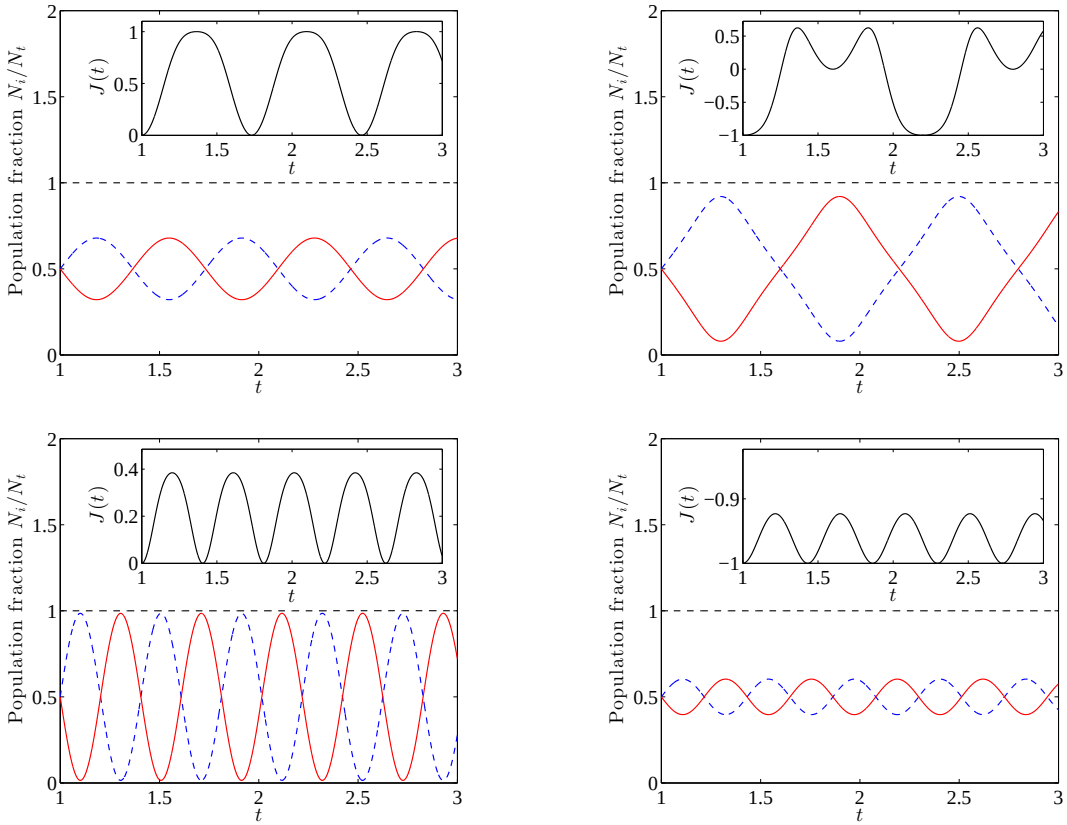
Lattice formulation (ii)

- Place continuum theory on two-site lattice, and study the coherent dynamics of the resulting theory.
- Use two-mode approximation, $\hat{\Phi} = \eta_l(x)\hat{c}_l + \eta_r(x)\hat{c}_r$:

$$\begin{aligned}\hat{H} = &- J(\hat{c}_l^\dagger \hat{c}_r + \hat{c}_r^\dagger \hat{c}_l) + U[\hat{n}_l(\hat{n}_l - 1) + \hat{n}_r(\hat{n}_r - 1)] \\ &+ \Gamma_1[\hat{c}_l^\dagger \hat{j} \hat{c}_l + \hat{c}_r^\dagger \hat{j} \hat{c}_r] + \Gamma_2[\hat{c}_l^\dagger \hat{j} \hat{c}_r + \hat{c}_r^\dagger \hat{j} \hat{c}_l]. \end{aligned} \tag{12}$$

- $\hat{H}$ contains discrete current $\hat{j} = -i(\hat{c}_r^\dagger \hat{c}_l - \hat{c}_l^\dagger \hat{c}_r)$ as well as number operator $\hat{n}_i = \hat{c}_i^\dagger \hat{c}_i$.
- $\Rightarrow$ Study population dynamics in Heisenberg picture $i\hbar\hat{c}_i = [\hat{c}_i, \hat{H}]$...

Lattice formulation (iii)

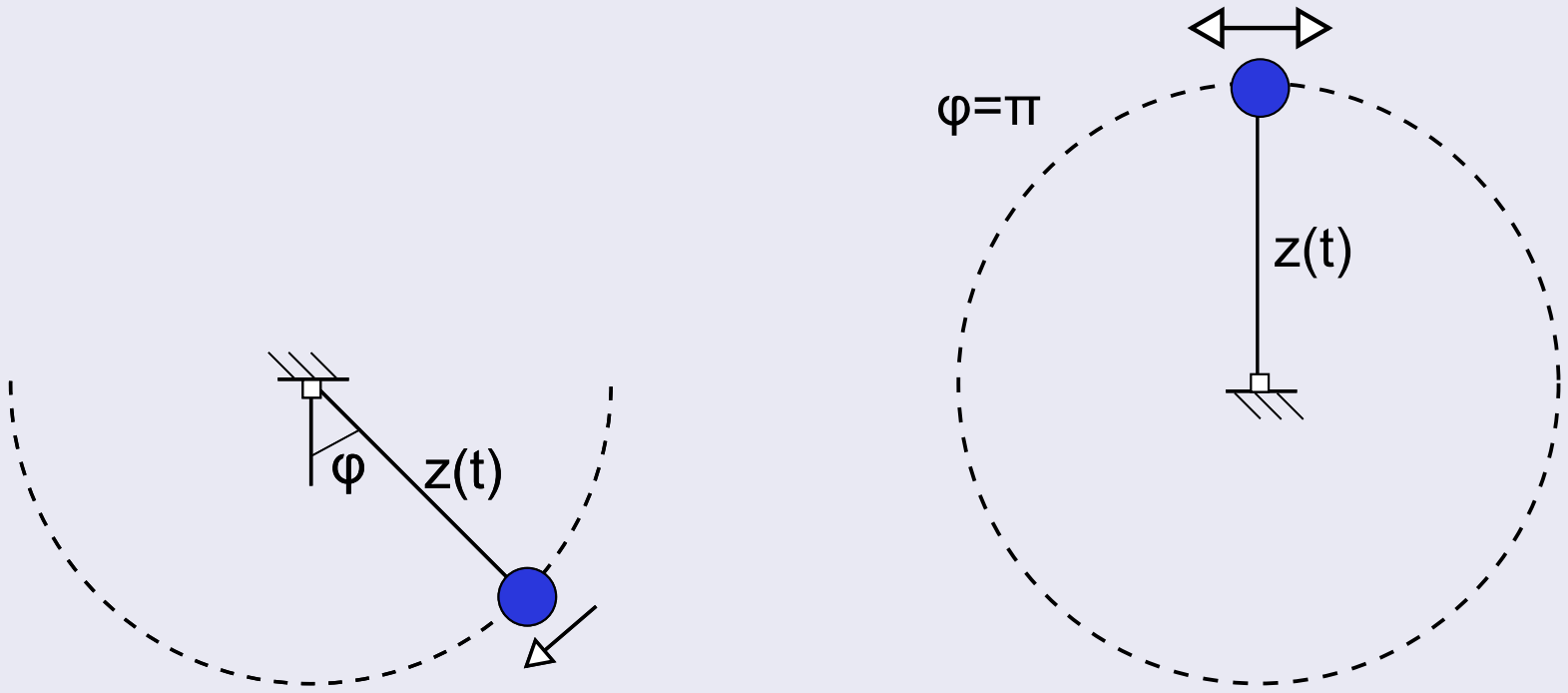


- Legend: $|c_l|^2$, $|c_r|^2$ and $\sum_i |c_i|^2$ with current J(t) inset.

M. J. Edmonds et al, arXiv 1302.0198

Classical Pendulum model

Nonrigid Pendulum



- Number/phase variables $c_i = \sqrt{N_i} e^{i\theta_i}$ and define population/phase differences: $z(t) = (N_l - N_r)/N_t$, $\varphi(t) = \theta_r - \theta_l$.

Classical Pendulum model (ii)

- Gives Hamiltonian for nonrigid pendulum of length $z(t)$ and angle $\varphi(t)$:

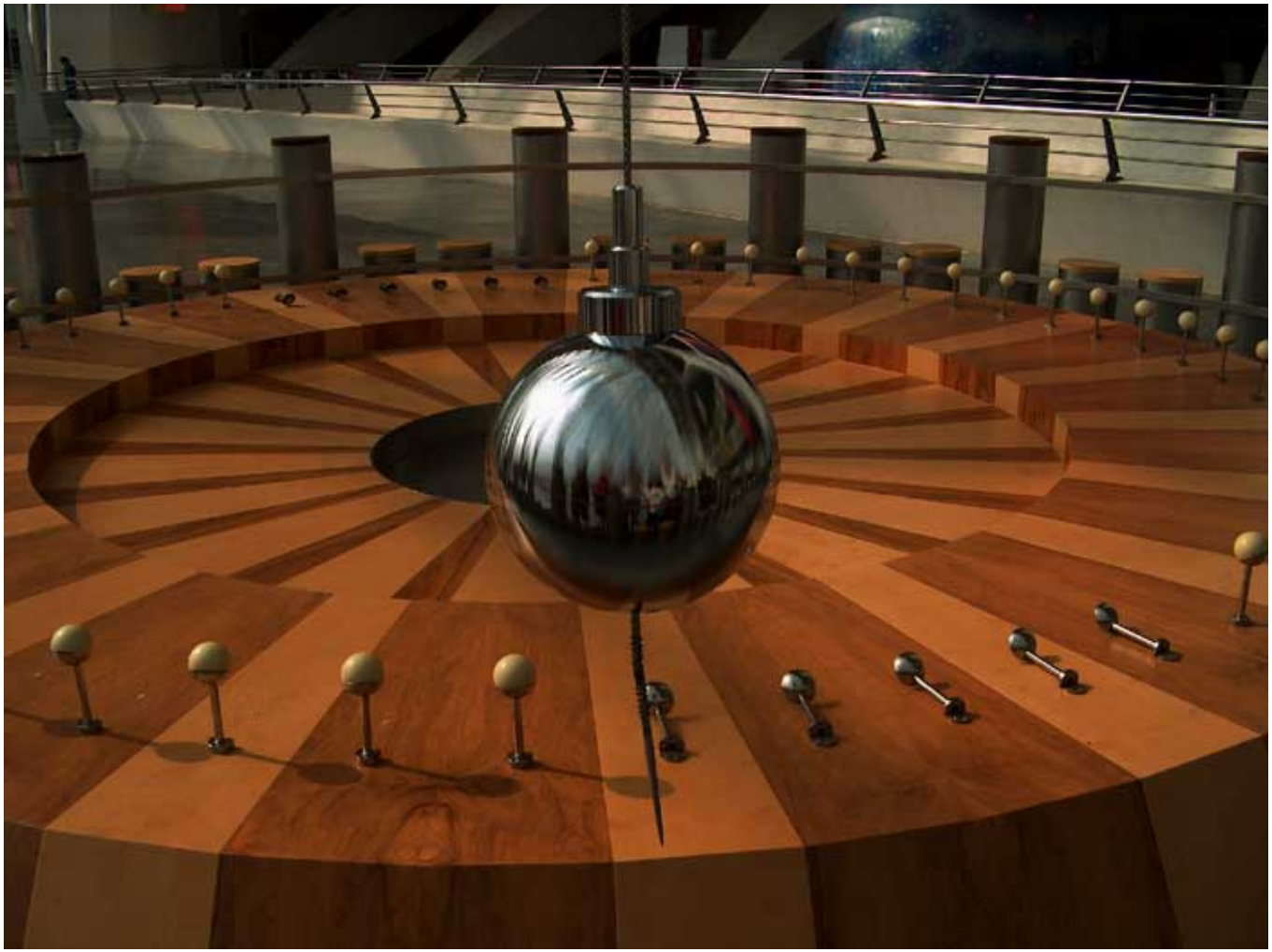
$$H = \frac{\Lambda z^2}{2} - \sqrt{1-z^2}\cos(\varphi) - \gamma_1\sqrt{1-z^2}\sin(\varphi) - \gamma_2(1-z^2)\sin(2\varphi) \tag{13}$$

- If we drop the γ_2 term then we can write this as:

$$H = \frac{\Lambda z^2}{2} - R\sqrt{1-z^2}\cos(\varphi - \varphi_0) \tag{14}$$

- With $R = \sqrt{1+\gamma_1^2}$ and $\varphi_0 = \arctan(\gamma_1)$

Classical Pendulum model (iii)



Motion of pendulum in rotating frame of reference affects dynamics...

Phase-space analysis

- Can calculate equations of motion for z and φ using Hamilton's equations: $\dot{z} = -\frac{\partial H}{\partial \varphi}$ and $\dot{\varphi} = \frac{\partial H}{\partial z}$.

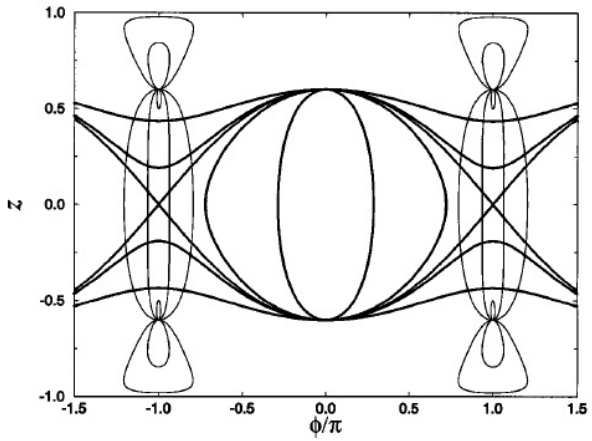
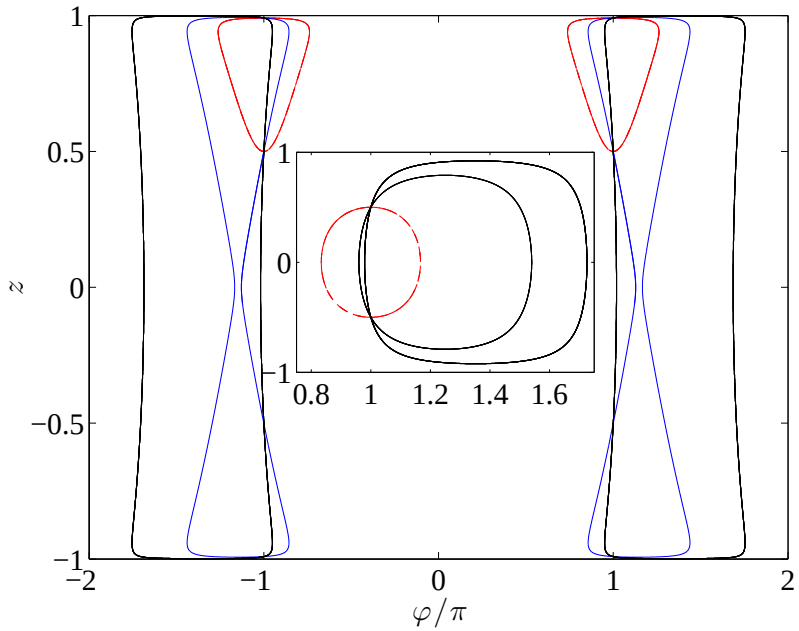


FIG. 3. Constant energy lines in a phase-space plot of population imbalance z versus phase difference ϕ. Bold solid line: $z(0) = 0.6$, $\phi(0) = 0$, $\Lambda = 1$, 8, 10, 11, and 20. Solid line: $z(0) = 0.6$, $\phi(0) = \pi$, $\Lambda = 0$, 1, 1.2, 1.5, and 2.



- Left: PS trajectories of system with $\Gamma_i = 0$.
- Right: PS trajectories of system with $\Lambda = 2$ and $\Lambda = 0$. (inset)

A. Smerzi et al Phys. Rev. Lett. **79** 4950 (1997).

Conclusions & Outlook

- Possible to go beyond single particle description $\Rightarrow$ Interacting gauge potentials.
- Appearance of current nonlinearity at mean-field level.
- Novel 1D effects including chirality.
- Lattice formulation includes discrete currents
- Phase-space properties strongly affected by gauge potential.
- Interacting gauge potential leads to many possibilites..

Thank you!