Efimov Physics beyond Universality

Parity-Order in Mott-Insulators, Duality and Gauge Fields

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S. P. Rath, W. Simeth, M. Endres, W. Zw. arXiv:1302.0693

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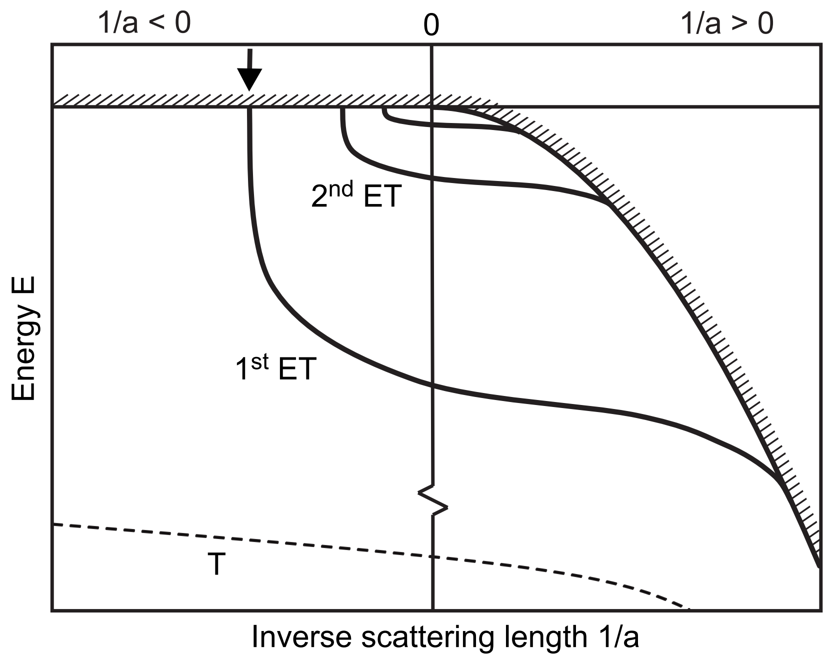
Efimov '70 three bosons form trimer states even at $a < 0$

sequence of scattering lengths at threshold $a_-^{(n+1)}/a_-^{(n)} \to 22.6942..$

exp. observed via maxima in 3−body loss rate $L_3(a)$ Grimm.. '06



universal Efimov number

$22.69.. = \exp \pi/s_0 \quad s_0 = 1.00624$

also shows up in energies at $a = \infty$

$E^{(n)}/E^{(n+1)} \to \exp 2\pi/s_0 \simeq 515.03$

position of **first** Efimov trimer

at $a_- = a_-^{(0)}$ is usually fixed by

an adjustable **three-body parameter** (i.e. is **not** universal)

Efimov spectrum and **broken scale invariance**

eff. one-body potential for hyperradius $R = \sqrt{\left(r_{12}^2 + r_{13}^2 + r_{23}^2\right)/3}$

$$V_{eff}(R) = \frac{\hbar^2}{2m}\frac{\nu^2(R) - 1/4}{R^2} \quad \text{with} \quad \nu(R, a=\infty) = is_0$$

prevent **Thomas collapse** by boundary condition as $R \to 0$

$\Psi(R_0) \equiv 0 \qquad R_0 \simeq 1/\Lambda^\star$ depends on short distance details

but experiments indicate a **universal (?) three-body parameter**

	^{7}Li	^{85}Rb	^{133}Cs
$-a_-/l_{vdW}$	9.17, 8.13, 8.25	9.24	8.63, 10.19, 9.48, 9.46

van der Waals potentials $V(r) = \begin{cases} -C_6/r^6 & \text{if} \quad r > r_c \\ \infty & \text{if} \quad r \leq r_c \end{cases}$

characteristic length $2\, l_{\mathrm{vdw}} = \left(mC_6/\hbar^2\right)^{1/4} \simeq 10$ nm

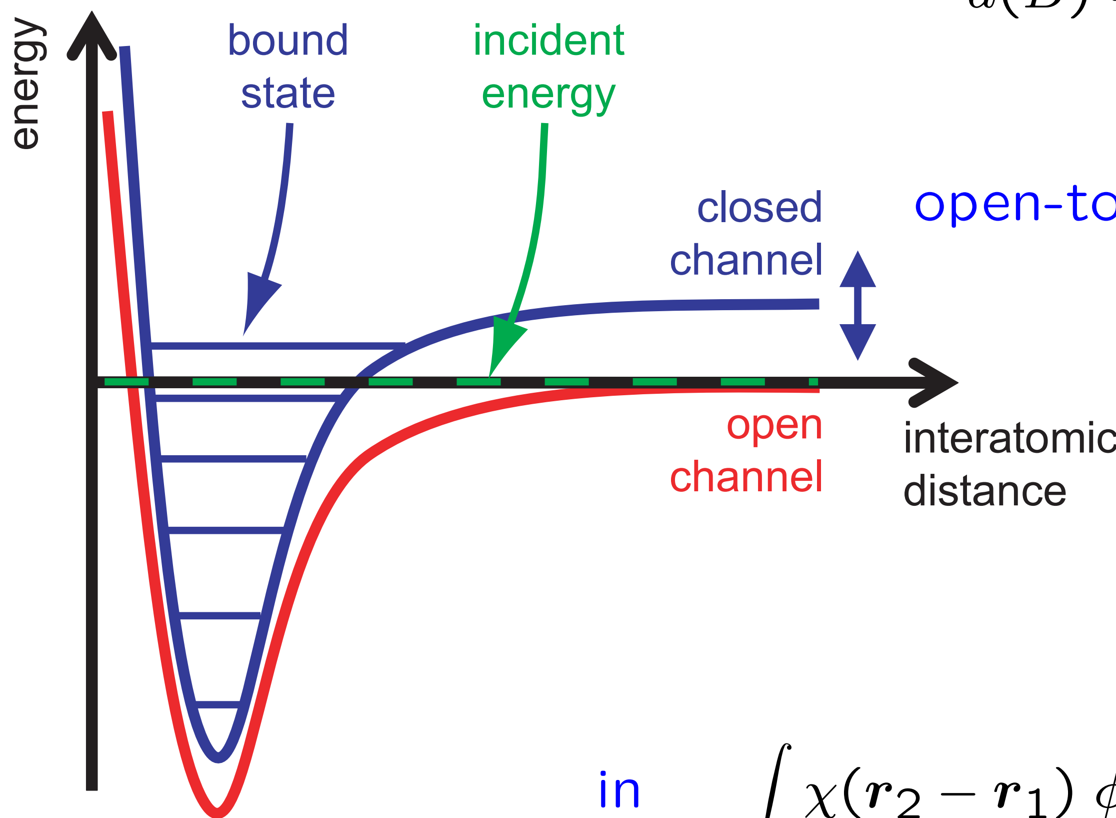
scatt. length $a = \bar{a}\,[1 - \tan(\Phi - 3\pi/8)]$ with $\bar{a} = 0.956\, l_{\mathrm{vdw}}$

WKB-phase $\Phi = \int dr \sqrt{m|V(r)|}/\hbar = 2\, l_{\mathrm{vdw}}^2/r_c^2 \simeq \pi \cdot N_b \gg 1$

effective range $r_e = 2.92\, \bar{a} \left(1 - 2\frac{\bar{a}}{a} + 2\left(\frac{\bar{a}}{a}\right)^2\right)$ Flambaum et al. '99

Chin, Greene, Ueda: l_{vdw} sets scale **also** in the three-body problem

Feshbach resonance within a two-channel model



$$a(B) = a_{bg} - \frac{\hbar^2}{m\, r^\star \mu(B - B_0)}$$

open-to-closed channel transfer

range function

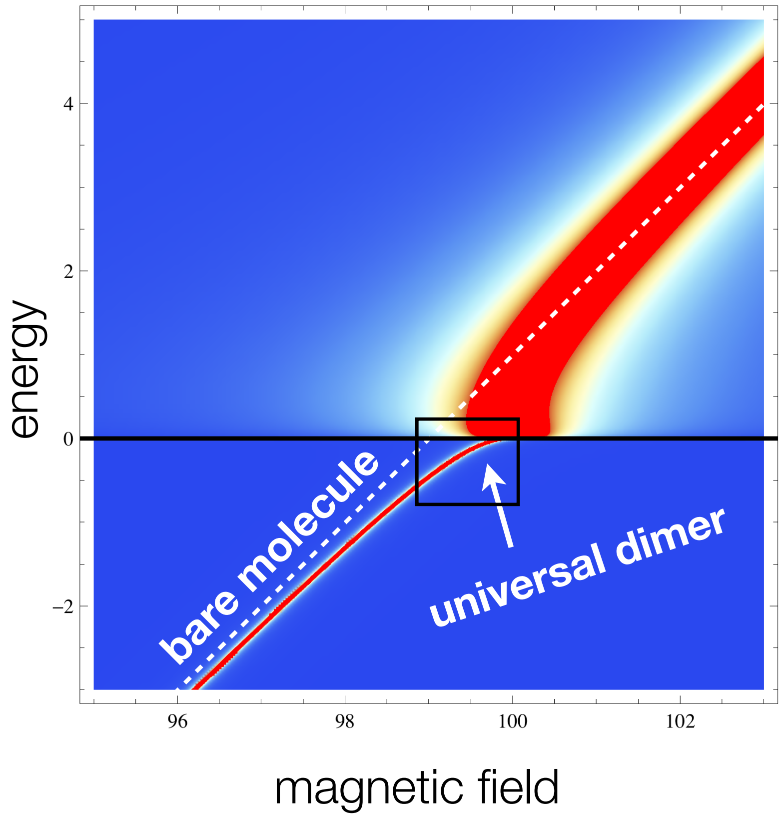
$$\chi(r) \sim h \cdot \exp -r/\sigma$$

in $$\int \chi(\boldsymbol{r}_2 - \boldsymbol{r}_1)\, \phi(\frac{\boldsymbol{r}_1 + \boldsymbol{r}_2}{2})\, \psi^*(\boldsymbol{r}_1)\psi^*(\boldsymbol{r}_2)$$

parameters h, σ, B_{res} determine **strength, range** and **bare position**

scatt. amplitude $f(k) = h^2 \chi^2(k)\, \mathcal{G}_\phi(2k^2, 0) \to \dfrac{1}{-1/a + r_e k^2/2 - ik}$

$$\frac{1}{a} = \frac{1}{2\sigma} - \frac{16\pi}{h^2}\tilde{\nu}_{\text{res}}(B)$$ with bare detuning $\tilde{\nu}_{\text{res}}(B) = \mu(B - B_{\text{res}})$



$h^2 = 32\pi/r^\star$ gives $a(B) = -\dfrac{1}{\tilde{\nu}(B) r^\star}$

resonance shift $\mu(B_0 - B_{\text{res}}) =$

$= 1/(r^\star \bar{a})$ Julienne et al. '04

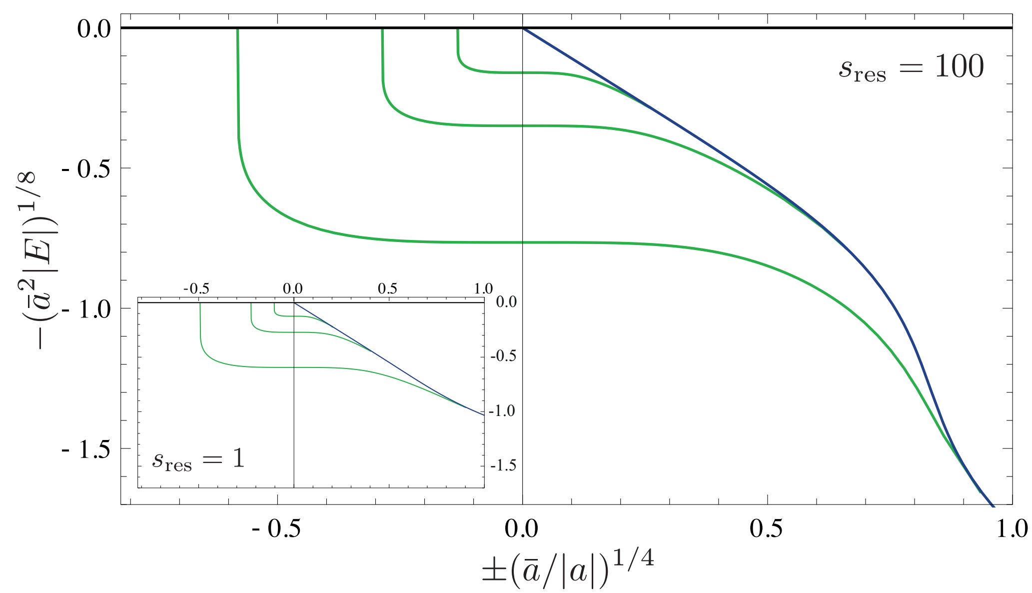
fixes range $\sigma = \bar{a} \simeq l_{\text{vdw}}$

effective range $r_e = -2r^* + 3\,\sigma\left(1 - \dfrac{4\sigma}{3a}\right) \to \begin{cases} \simeq 3\,\bar{a} \gg r^\star & \textbf{open} \\ -2\,r^\star & \textbf{closed} \end{cases}$

Efimov spectrum from Functional RG atom-dimer interaction

$$\int_{Q_1,Q_2,Q_3} \lambda_3^{(k)}(Q_1,Q_2,Q_3)\,\phi^*(Q_1)\psi^*(Q_2)\phi(Q_3)\psi(Q_1+Q_2-Q_3)$$

at momentum scale k, solve flow equ. with initial condition $\lambda_3^{(\Lambda)} \equiv 0$



poles in $\lambda_3^{(k=0)}(q_1,q_2,E) =$

$$\mathcal{B}(q_1,q_2)/\left(E + E^{(n)} + i\Gamma^{(n)}\right)$$

trimer energies $E^{(n)}$ fixed by

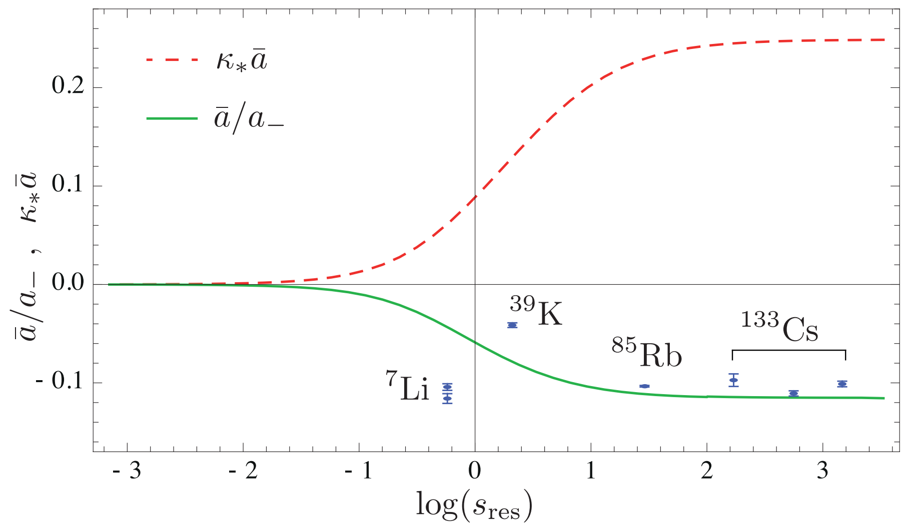
$\bar{a} \simeq l_{\mathrm{vdw}}$ and the dim.less

resonance strength $s_{\rm res} = \bar{a}/r^\star$

Results: we obtain $a_- = -8.3\, l_{\mathrm{vdw}}, \;\; \kappa_\star l_{\mathrm{vdw}} = 0.26$ for $r^\star \ll \bar{a} \quad \rightarrow$

a) explains the 'universality' of a_- for resonances with $s_{\mathrm{res}} \gg 1$

b) predicts a smooth crossover to $a_- = -10.3\, r^\star$ for $s_{\mathrm{res}} \ll 1$

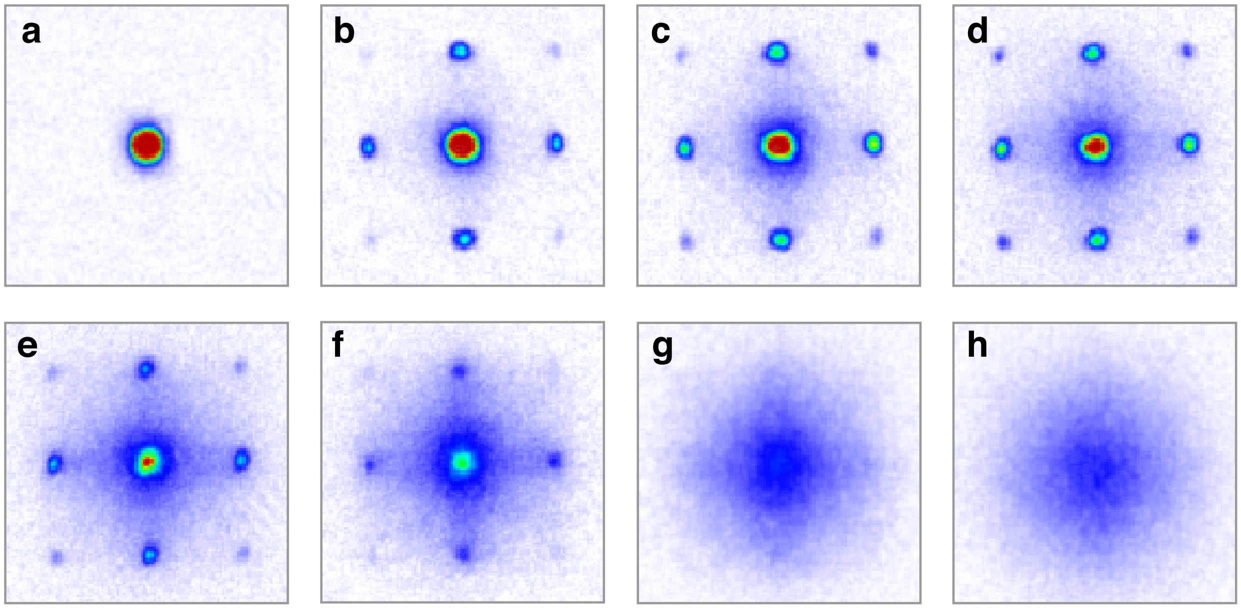


crossover from open-channel dominated limit $s_{\mathrm{res}} \gg 1$ to $s_{\mathrm{res}} \ll 1$ where $a_- = -10.3\, r^\star$

(Petrov '04, Gogolin .... '08)

need: more exp. data on resonances with $s_{\mathrm{res}} < 1$ Modugno **LENS**

II) Nonlocal Order in Mott-Insulators



SF-MI transition

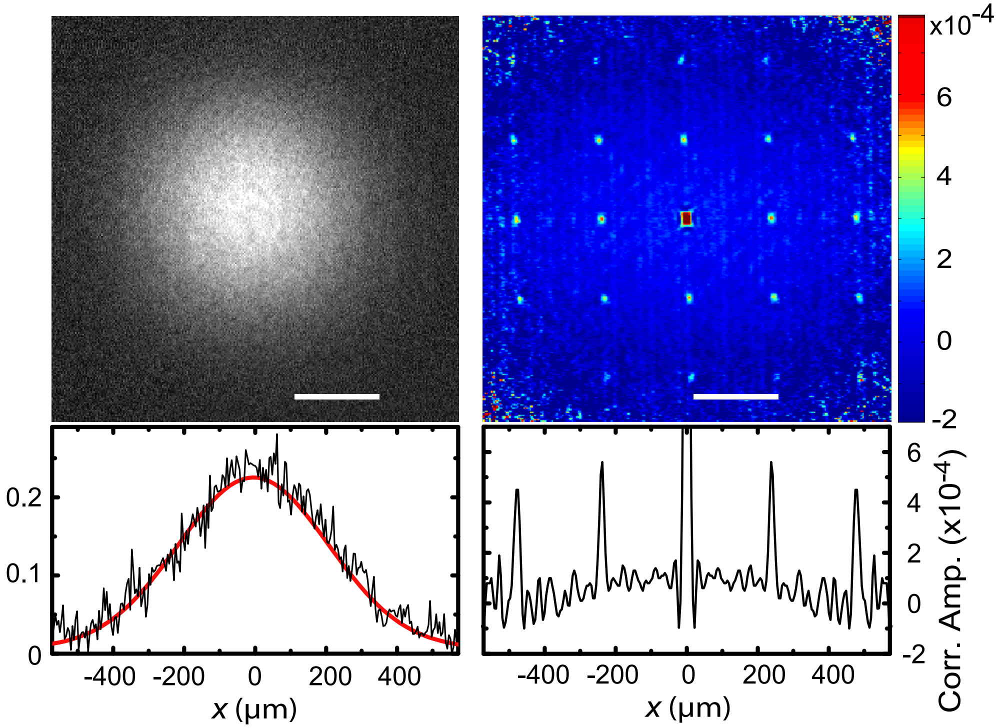
Greiner et al. '02

Bragg peaks in $\quad n(\boldsymbol{k}) = n|w(\boldsymbol{k})|^2 \sum_R e^{i\boldsymbol{k}\cdot\boldsymbol{R}} \rho_1(\boldsymbol{R})$ at $\quad \boldsymbol{k} = \boldsymbol{G}$

show LRO of amplitudes in the SF $\quad \rho_1(\boldsymbol{R}) = \langle \hat{a}^\dagger(\boldsymbol{R})\hat{a}(0)\rangle \to n_0$

pair correlation in the Mott-phase from noise in the

absorption image (Fölling et al. '05) **bunching** at $\boldsymbol{k}-\boldsymbol{k}'=\boldsymbol{G}$



reflects large Fourier-comp.

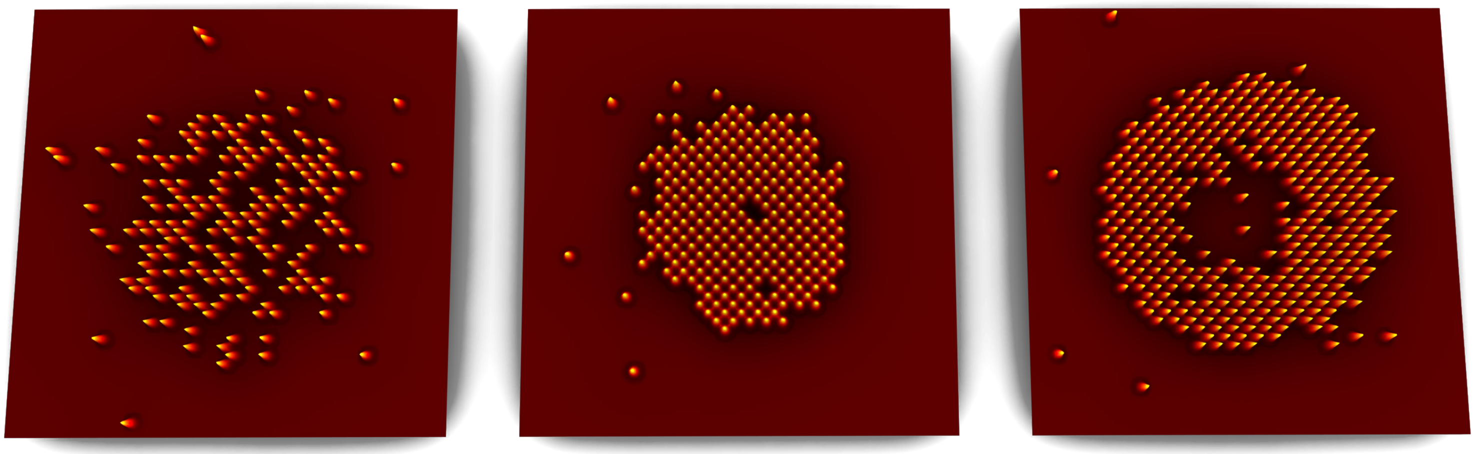
$$\sum_{\boldsymbol{R}} e^{i(\boldsymbol{k}-\boldsymbol{k}')\boldsymbol{R}} n_{\boldsymbol{R}} = \mathcal{O}(N)$$

of average density

on sites of a regular lattice

Full Counting Statistics of Atomic Gases

experimentally accessible by **single site resolution images**



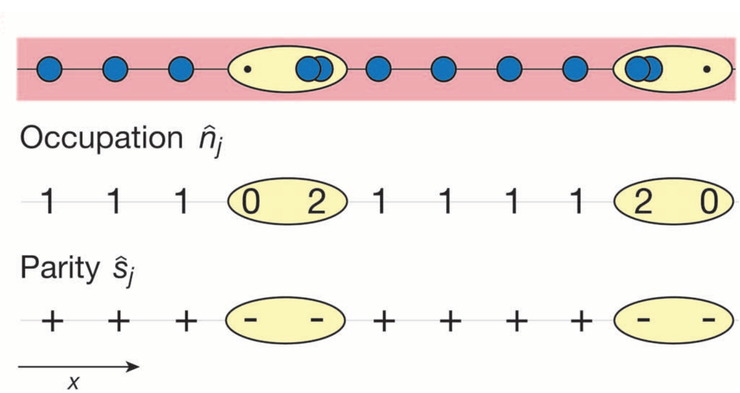
Bakr et al.'10, Sherson et al.'10

'Hidden' Order in Mott-Insulators (Berg et al '08)

MI has no local order parameter: it is defined by **incompressibility**

questions: a) is there some long range order in a MI ?

b) deconfinement of particle-hole pairs at transition ?

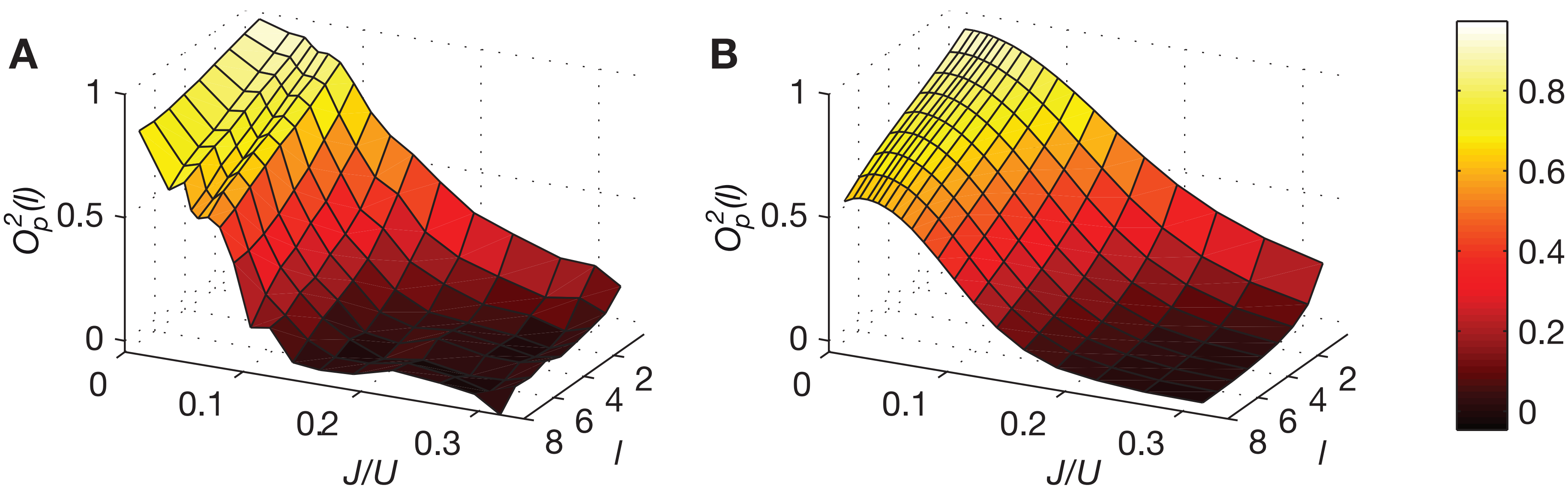


hidden order in a MI

due to confined ph-pairs

Endres et al **Science** '11

parity order parameter exp. versus numerics Mazza ...



$$\langle \hat{O}_P^2(L) \rangle = \langle e^{i\pi \sum_j \delta \hat{n}_j} \rangle \;\rightarrow\; \begin{cases} O_P^2(\infty) \neq 0 & \text{Mott-Insulator} \\ 0 & \text{Superfluid} \end{cases}$$

Area Law for Number Fluctuations

Gaussian approximation $\langle \hat{O}_P^2(L) \rangle \approx \exp -\pi^2 \langle \delta \hat{N}^2 \rangle (L)/2$

measures number fluctuations in a domain of size L

$$\langle \delta \hat{N}^2 \rangle (L) = \int d1 \int d2 \; n\nu(|\boldsymbol{x}_1 - \boldsymbol{x}_2|) \;\rightarrow\; \begin{cases} \sim \; L^{\,d-1} & \text{MI} \\ \sim \alpha \, L^{\,d-1} \ln(L) & \text{SF} \end{cases}$$

static structure factor $S(\boldsymbol{q}) \;\rightarrow\; \begin{cases} \sim \boldsymbol{q}^2 & \textbf{in}\text{-compress.} \\ \alpha \cdot |\boldsymbol{q}| + \ldots & \text{compressible} \end{cases}$

compare **entanglement entropy** $S(L) = b\, L^{d-1} + \ln\left(\rho_s L^{d-1}/c_s\right)/2$

Duality mapping to a classical interface problem Zw. '89

consider Bosons with integer average occupation $\bar{n} \gg 1$

$$\hat{H}_{\text{eff}} = \frac{U}{2} \sum_{\ell} \hat{n}_{\ell}^2 + J_{\text{eff}} \sum_{\ell} \left[1 - \cos\left(\hat{\varphi}_{\ell+1} - \hat{\varphi}_{\ell} \right) \right] \qquad \hat{n}_{\ell} = \hat{b}_{\ell}^{\dagger} \hat{b}_{\ell} - \bar{n}$$

map 1D quantum problem to a 2D classical interface

$$S_{DG} = \frac{1}{2} \sqrt{\frac{U}{J_{\text{eff}}}} \sum_{\ell,j} \left[\left(\nabla_x h_{\ell,j} \right)^2 + \left(\nabla_\tau h_{\ell,j} \right)^2 \right] \qquad \hat{n}_{\ell} = \nabla_x \hat{h}_{\ell}$$

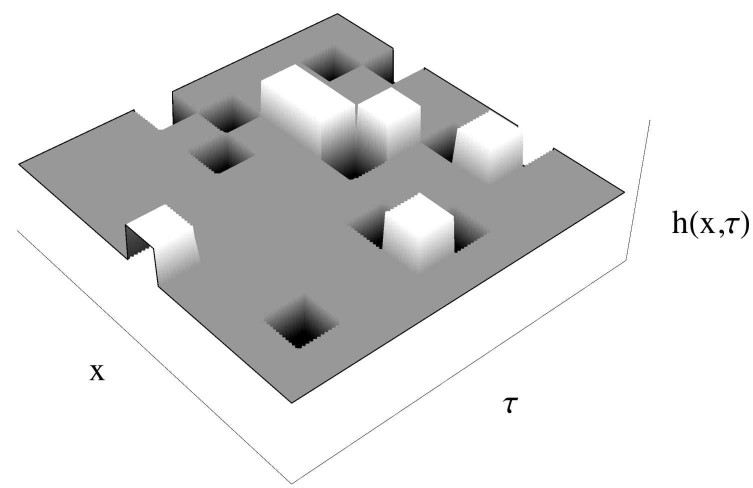
discrete Gaussian model with integer valued height variables $h_{\ell,j}$

roughening at $\quad \tilde{T} = \sqrt{J_{\text{eff}}/U} \geq \tilde{T}_R \simeq 1 \quad$ Fröhlich/Sp. '81

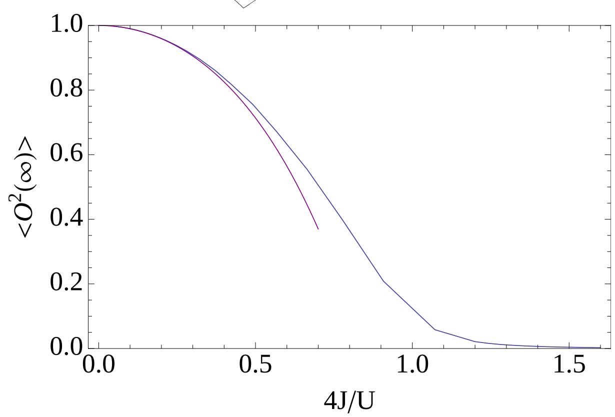
MI with gap $\Delta\mu \neq 0$ corresponds to a **smooth interface** with

finite **step free energy** $\quad f_s = \Delta\mu/(2\sqrt{U\, J_{\text{eff}}}) \neq 0$ below $\tilde{T}_R$

parity order $\quad \langle \hat{O}_P^2(L) \rangle = \langle e^{i\pi(\hat{h}_L - \hat{h}_0)} \rangle \;\rightarrow$



$$\begin{cases} 1 - \left(\frac{4J}{U}\right)^2 - \frac{19}{12}\left(\frac{4J}{U}\right)^4 & \text{Mott-Insulator} \\ f_s \sim \exp -\frac{C}{\sqrt{J_c - J}} & \text{near transition} \\ \sim L^{-\tilde{\eta}} \rightarrow 0 & \text{SF} \quad (\tilde{\eta}_c = 1) \end{cases}$$



analytical results consistent with DMRG

Duality to a Lattice Gauge Theory in 2D   Peskin '78

part. function $$Z = \sum_{[\boldsymbol{A}]} \exp\left(-\frac{1}{2\tilde{T}}\sum_{\boldsymbol{x},\tau}(\nabla\wedge\boldsymbol{A})^2\right) \qquad \tilde{T} = \sqrt{J_{\text{eff}}/U}$$

discrete lattice electrodynamics: $\boldsymbol{A}(x,y,\tau)$ takes only integer values

MI-SF transition $\rightarrow$ **deconfinement of magnetic monopoles**

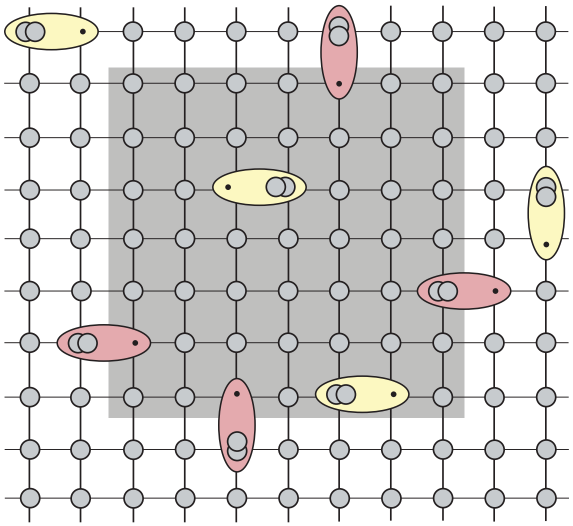
$$\langle \hat{a}^\dagger(\boldsymbol{x})\,\hat{a}\rangle = \bar{n}\,\langle e^{i\hat{\phi}(\boldsymbol{x})}e^{-i\hat{\phi}(0)}\rangle = Z[\boldsymbol{x},0]/Z = \exp(-\Delta F[\boldsymbol{x},0])$$

eff. interaction $$\Delta F[\boldsymbol{x},0] = \begin{cases} |\boldsymbol{x}|/\xi & \text{MI} \\ \text{const.} - c_s/(4\pi\rho_s|\boldsymbol{x}|) & \text{SF} \end{cases}$$

massive **'photons'** $\omega = c_s\sqrt{m^2c_s^2+|\boldsymbol{q}|^2}$ become massless $\omega = c_s|\boldsymbol{q}|$

parity order maps to an effective **Wilson loop** at $\tau = \text{const}$

$$\langle \hat{O}_P^2(L) \rangle = \left\langle \exp\left[i\pi \int_{\mathcal{D}} \mathrm{d}^2 x \, (\nabla \wedge \boldsymbol{A})_\tau \right] \right\rangle = \langle \exp\left[i\pi \int_{\partial\mathcal{D}} \boldsymbol{A} \cdot \mathrm{d}\boldsymbol{s} \right] \rangle \quad \rightarrow$$



$$\begin{cases} \sim \exp -\tilde{T} \cdot L/m^2 & \text{MI} \quad \textbf{perimeter-law} \\ \sim \exp -\tilde{T} \cdot L \ln L & \text{Superfluid} \quad (\tilde{T} > \tilde{T}_c) \end{cases}$$

decay of parity order in the MI determines **Mott gap** $\Delta \sim 1/m$

- The first **Efimov trimer** appears at a scattering length

  which crosses over from $a_{-} \approx -8.3\, l_{\mathrm{vdw}}$ to $a_{-} \approx -10.3\, r^{\star}$

- Single site resolution measures **non-local parity order.**

  Duality transf. show that $-\ln \langle \widehat{O}_P^2(L) \rangle$ exhibits an **area law**

  and is related to a Wilson-loop in a $2+1$-dim. gauge theory