Repulsive polarons
&
Itinerant Ferromagnetism

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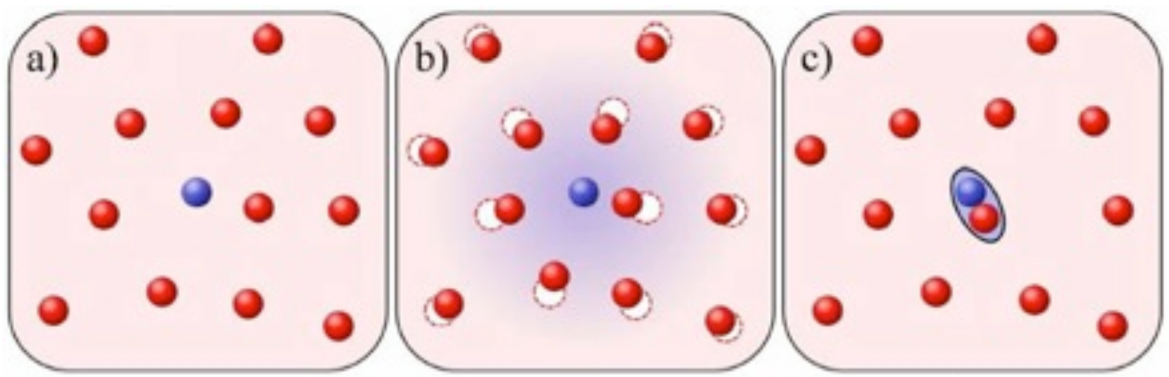
-C. Kohstall, M. Zaccanti, M. Jag, A. Trenkwalder, P. Massignan, GMB, F. Schreck, and R. Grimm, Nature **485**, 615 (2012)
-GMB and P. Massignan, PRL **105** 020401 (2010)
-P. Massignan and GMB, EPJD **65**, 83 (2011)
-P. Massignan, Z. Yu, and GMB, arXiv:1301.3163

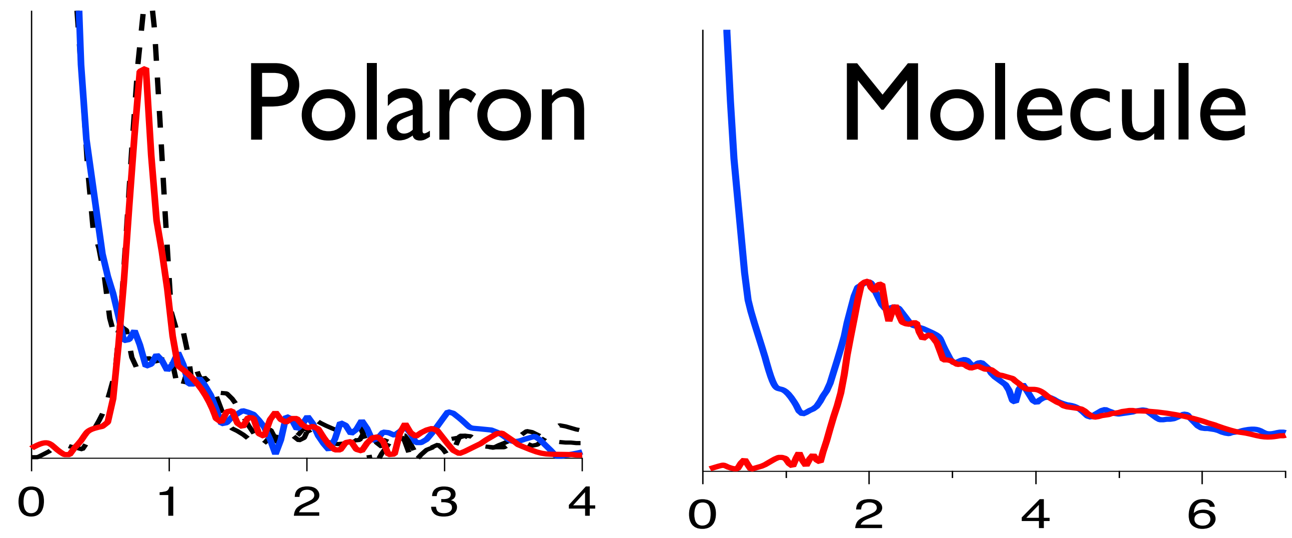
Outline

- Polarons & molecules: Main concepts & results
- 2-body physics: broad vs. narrow resonances
- Many-body theory & comparison with experiments
- Itinerant Ferromagnetism

Polarons and molecules

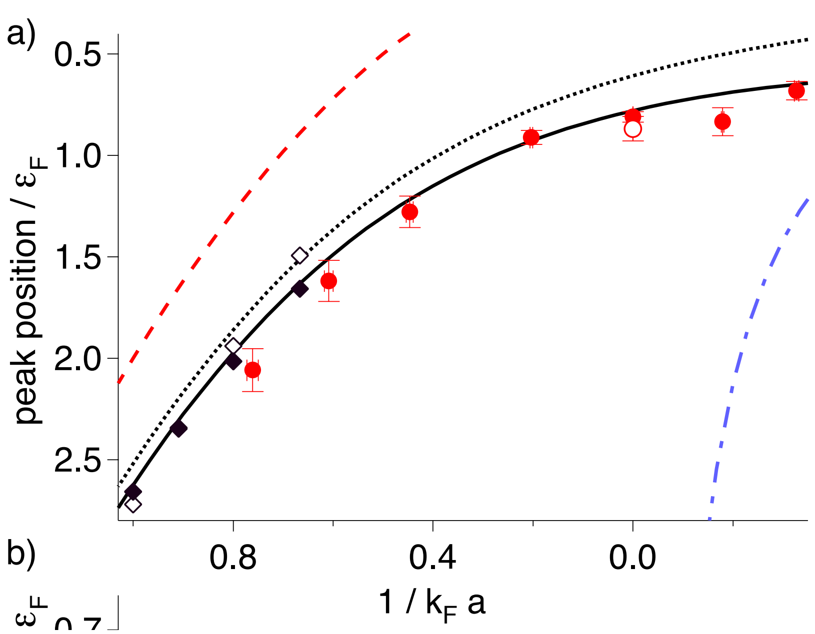
One ↓ in a Fermi sea of ↑'s

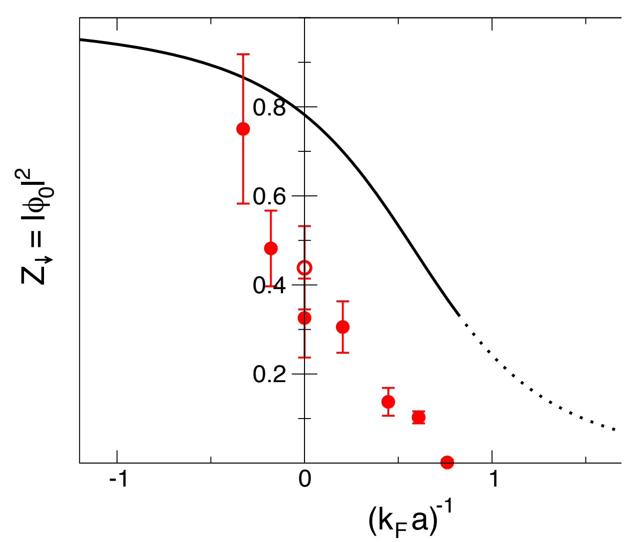




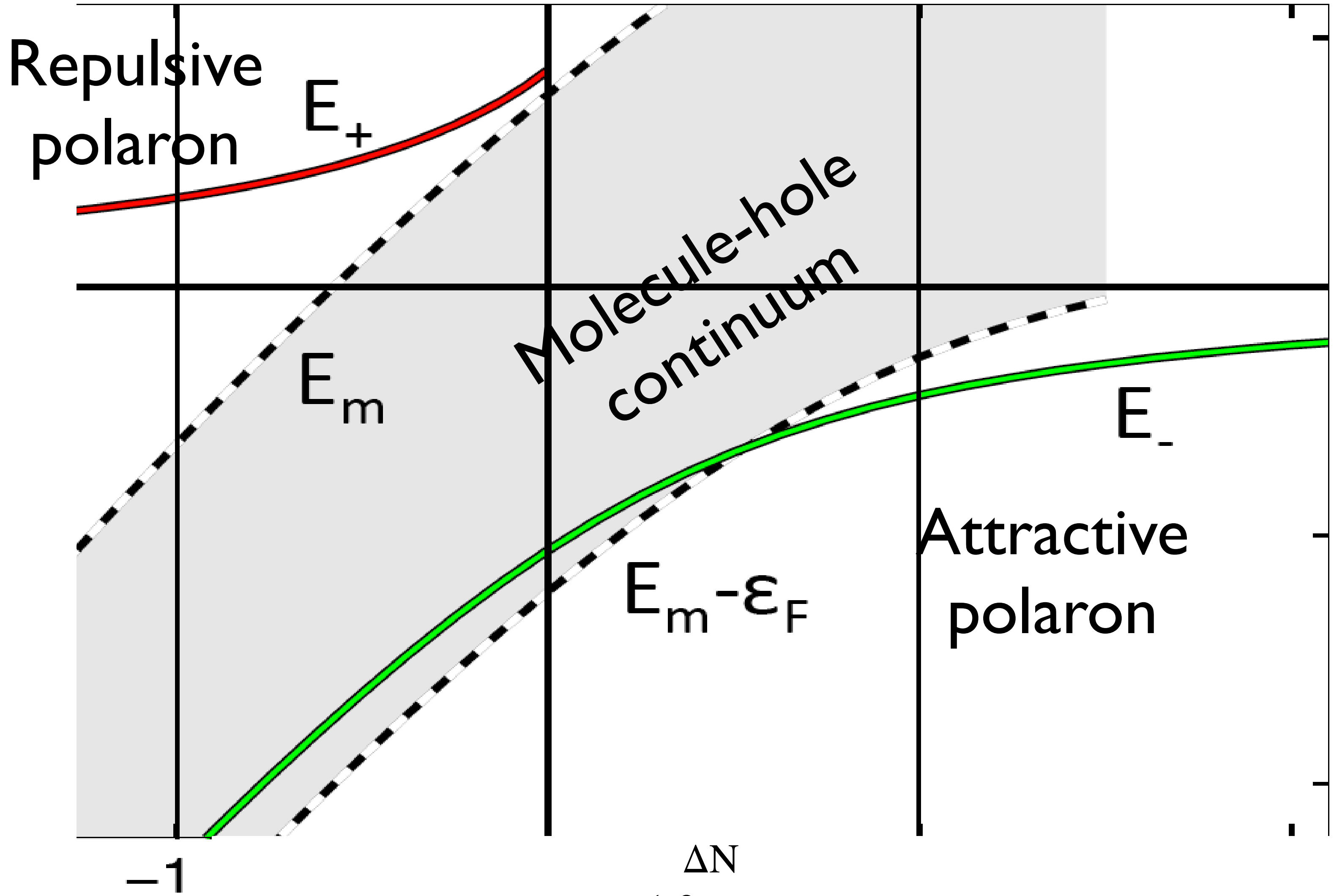
Increasing interaction

Zwierlein group





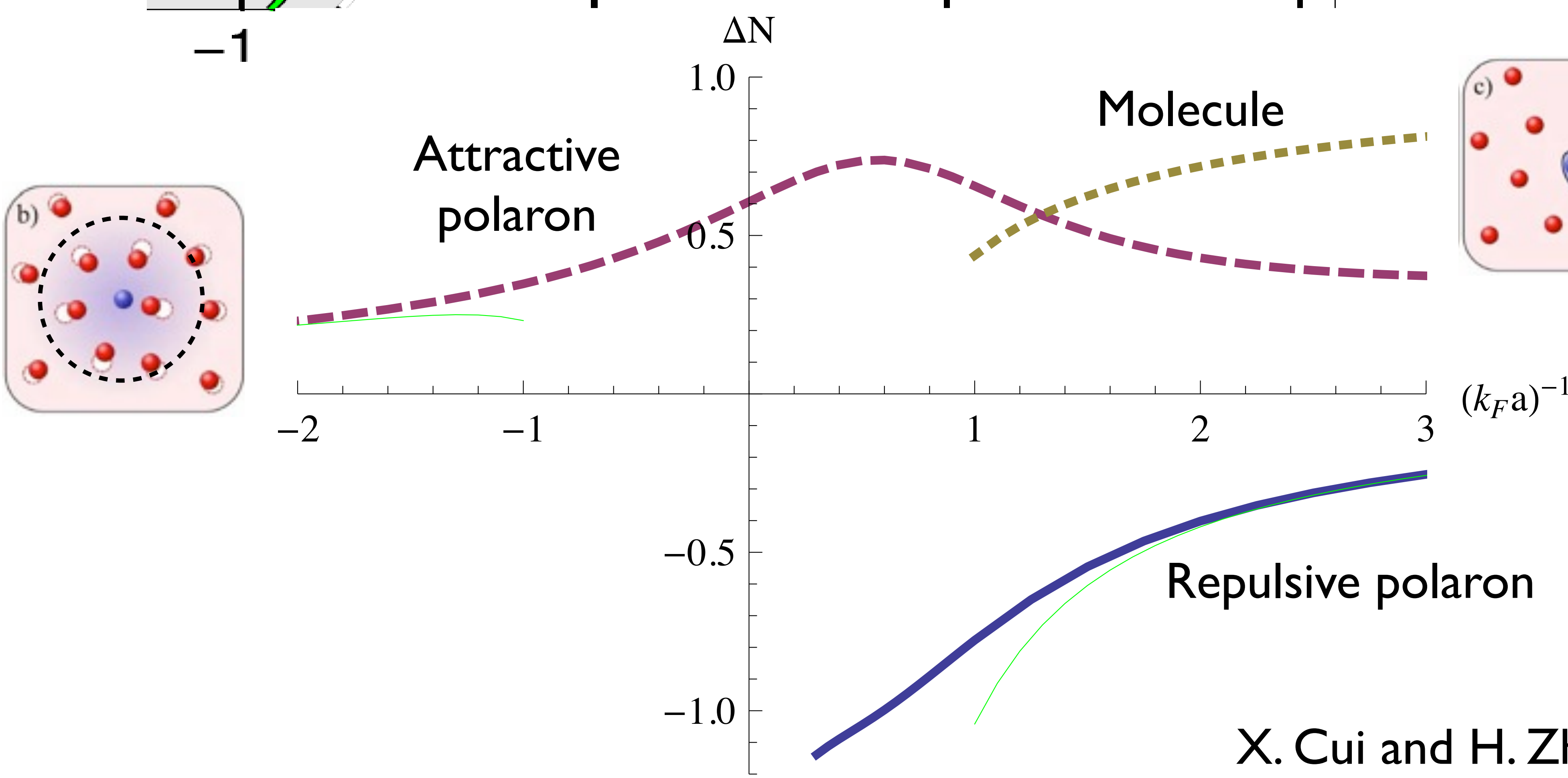
Chevy, Mora, Zwerger, Punk, Combescot, Leyronas, Recati, Lobo, Prokof'ev, Svistunov, ...



Number of atoms $\Delta N_\uparrow$ in dressing cloud:

$$\delta\mu_\uparrow = \frac{\partial^2\epsilon}{\partial n_\uparrow \partial n_\downarrow} + \frac{\partial^2\epsilon}{\partial n_\uparrow \partial n_\uparrow}\Delta N_\uparrow = 0$$

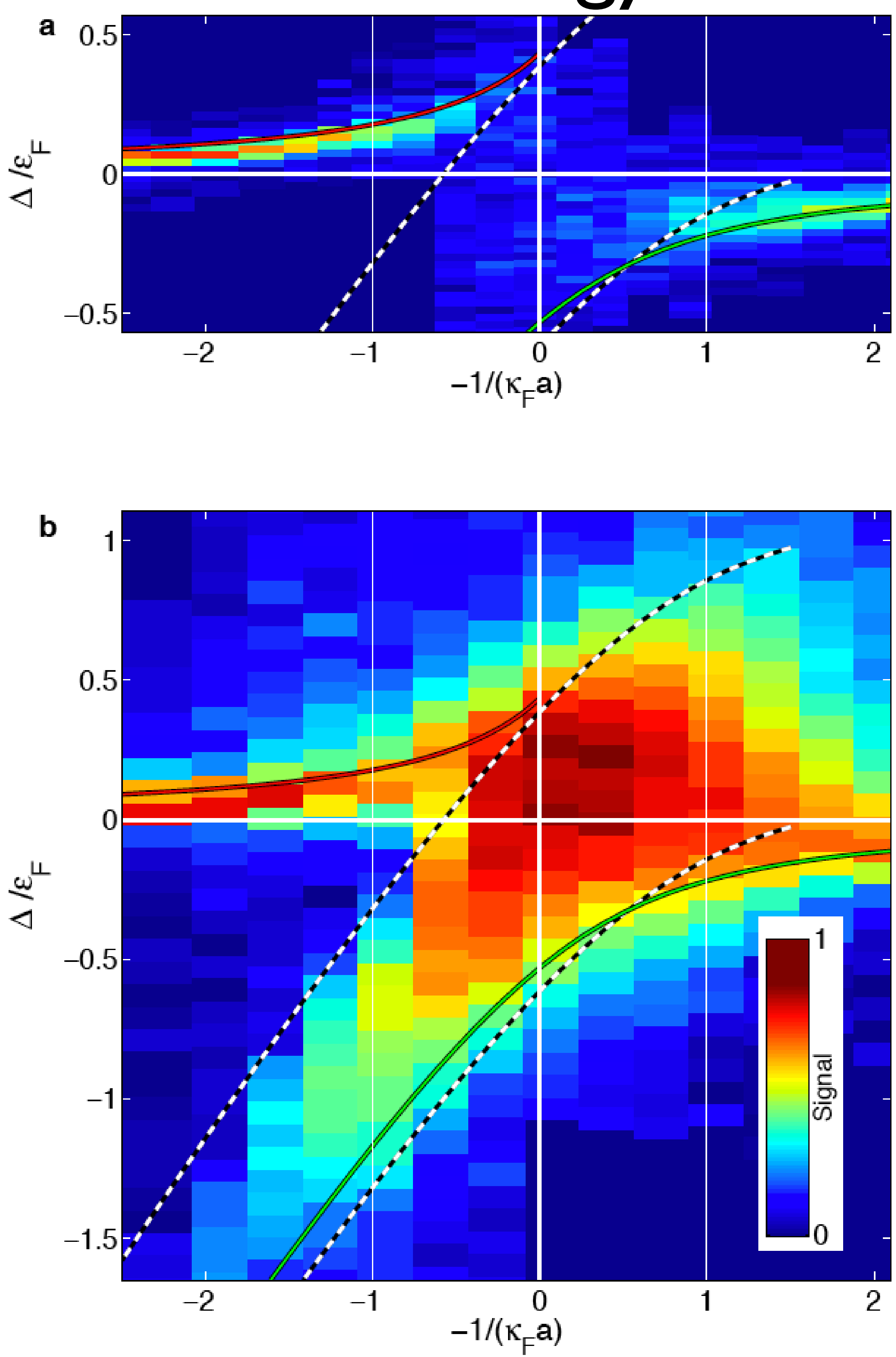
$$\Delta N_\uparrow = -\left(\frac{\partial \mu_\downarrow}{\partial \epsilon_F}\right)_{n_\downarrow}$$



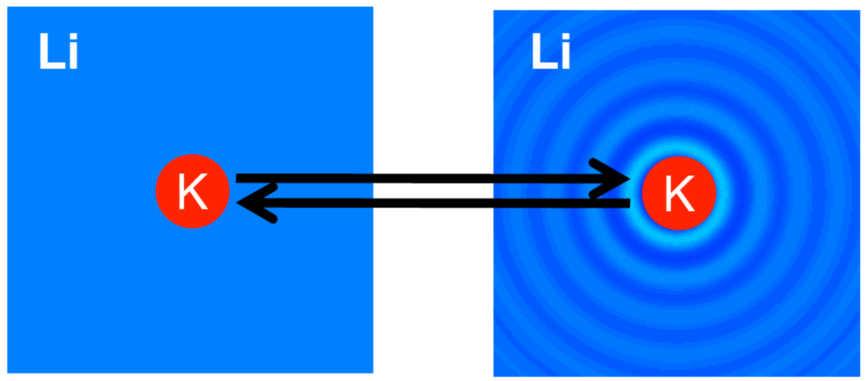
X. Cui and H. Zhai, PRA **81**, 041602 (2010)

^{40}K-^{6}Li experiments by Grimm group

Energy



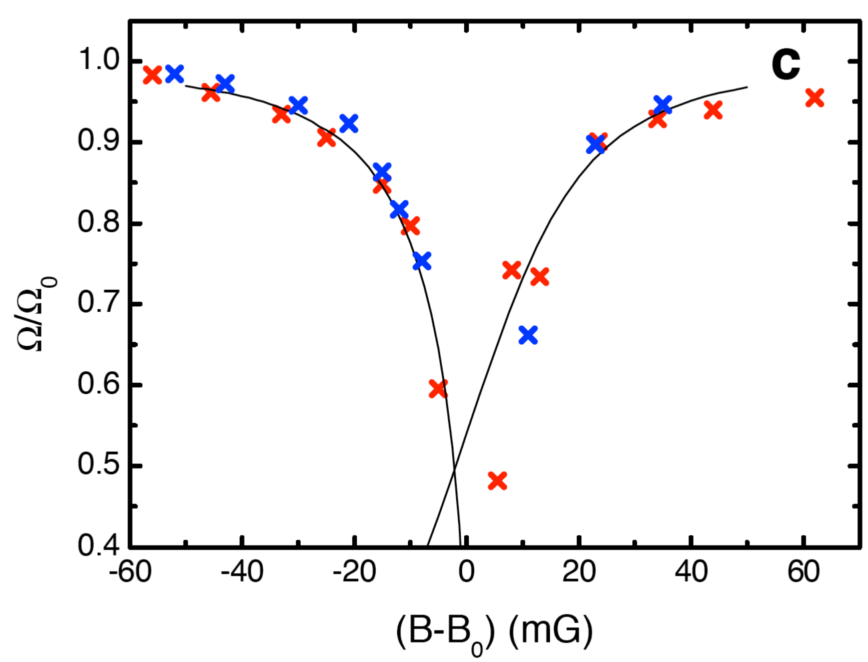
RF flip



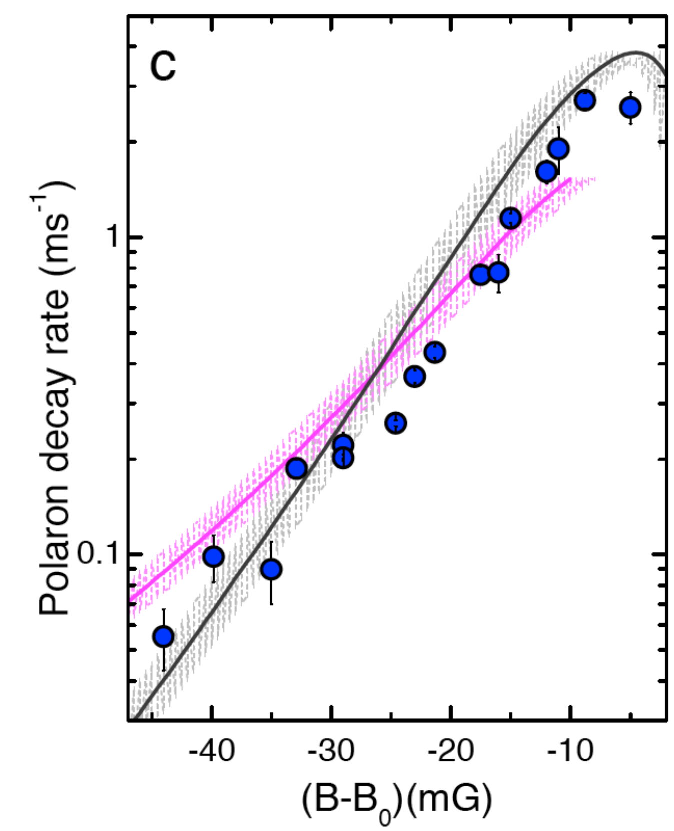
Non-interacting

Strongly interacting

QP residue

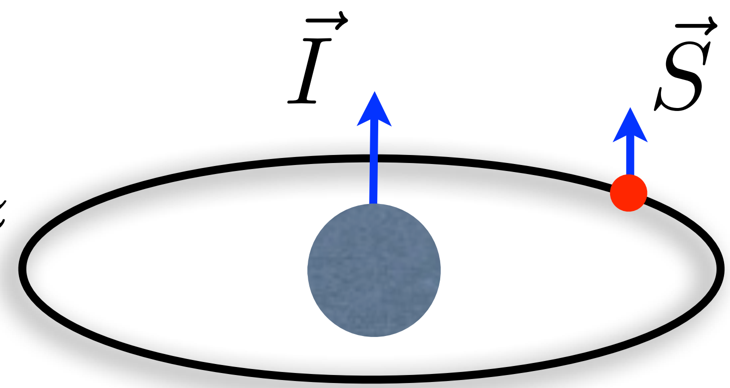


Decay rate

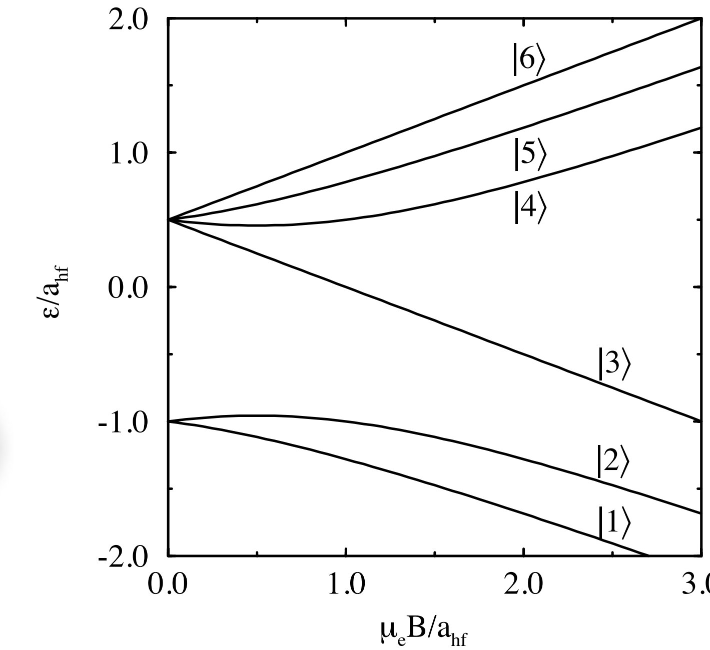


2-body physics

Hyperfine Hamiltonian:



^{6}Li

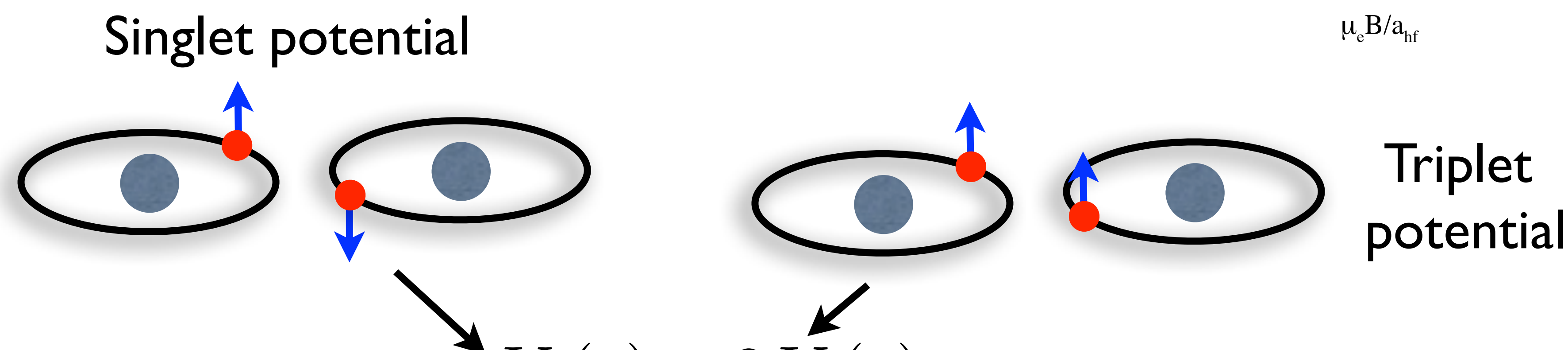


$$\hat{H}_{\rm spin} = A\,\vec{I}\cdot\vec{S} + C\,S_z + D\,I_z$$

$$\hat{H}_{\rm spin}|\alpha\rangle = \epsilon_\alpha|\alpha\rangle$$

$$|\alpha\rangle \equiv |F, m_F\rangle$$

$$\vec{F} = \vec{S} + \vec{I}$$



Atom-atom interaction: $V(r) = \dfrac{V_s(r) + 3\,V_t(r)}{4} + [V_t(r) - V_s(r)]\,\vec{S}_1\cdot\vec{S}_2$

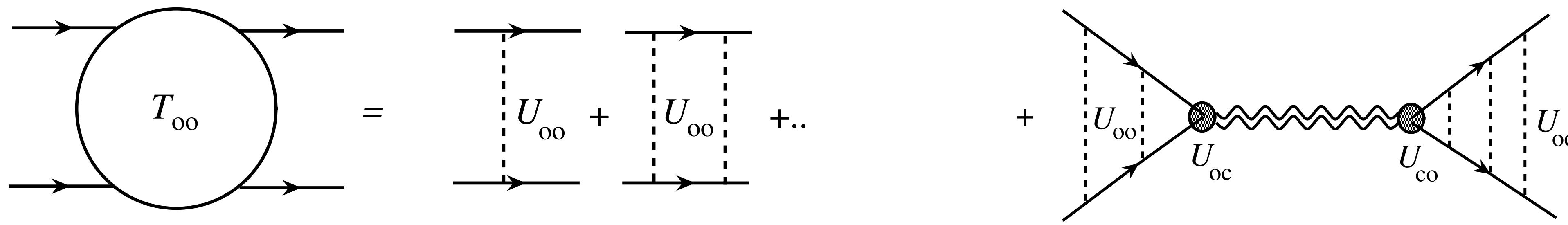
$[\hat{H}_{\rm spin}, \hat{V}] \neq 0$ Mixes hyperfine states $\Rightarrow$ Scattering channels

Low-energy interaction:

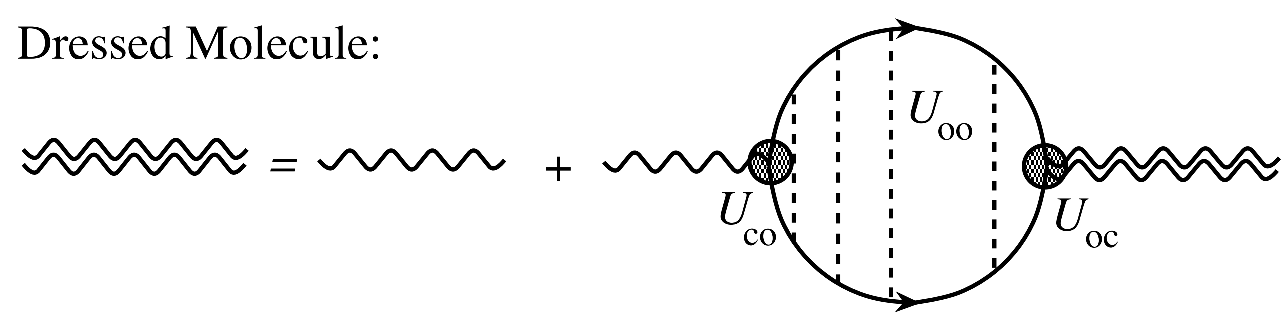
$$U = \frac{2\pi}{m_r}\left[\frac{a_s + 3a_t}{4} + (a_t - a_s)\vec{S}_1\cdot\vec{S}_2\right]$$

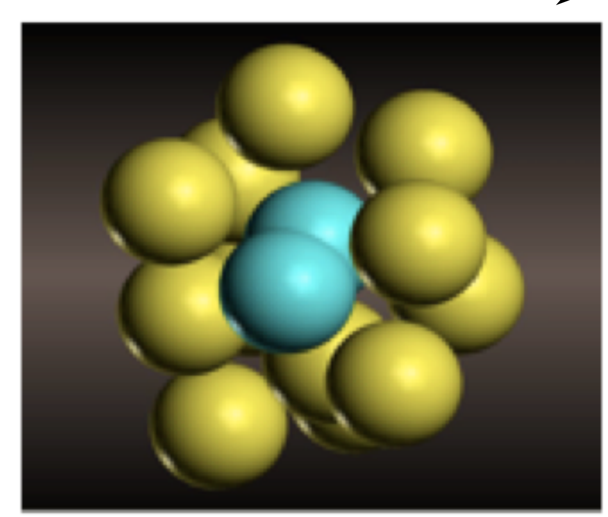
Scattering matrix:

$$T = \frac{T_{\rm bg}}{1 - T_{\rm bg}\Pi} + \frac{g^2}{\omega - K^2/2M - \Delta\mu(B - B_0) + g^2\Pi}$$



“Dressed” molecule





“<u>Landau Theory</u>”

Interaction expressed in terms of observable 2-body parameters

$$\boxed{r_{\rm eff}\, a_{\rm bg} = -\frac{1}{\Delta\mu \Delta B m_r} \propto \frac{1}{g^2}}$$

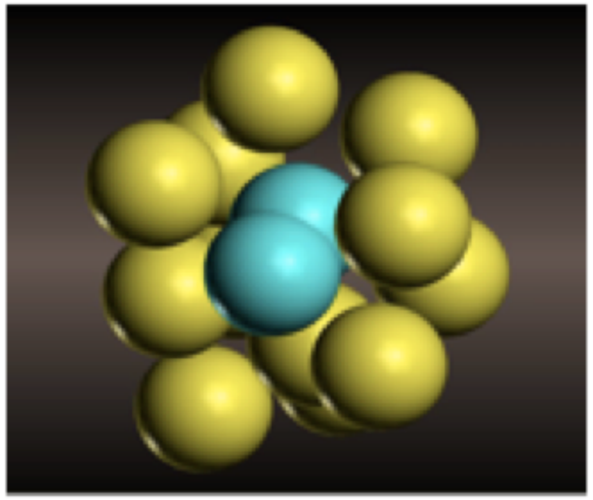
$$\boxed{g^2 = T_{\rm bg} \Delta\mu \Delta B}$$

GMB, E. Kolomeitsev, and A. D. Jackson, PRA **71** 052713 (2005)

“Broad” resonance

$$k_F r_{\text{eff}} \ll 1 \qquad \frac{g^2}{\epsilon_F} \gg \frac{1}{m_r k_F}$$

Single channel

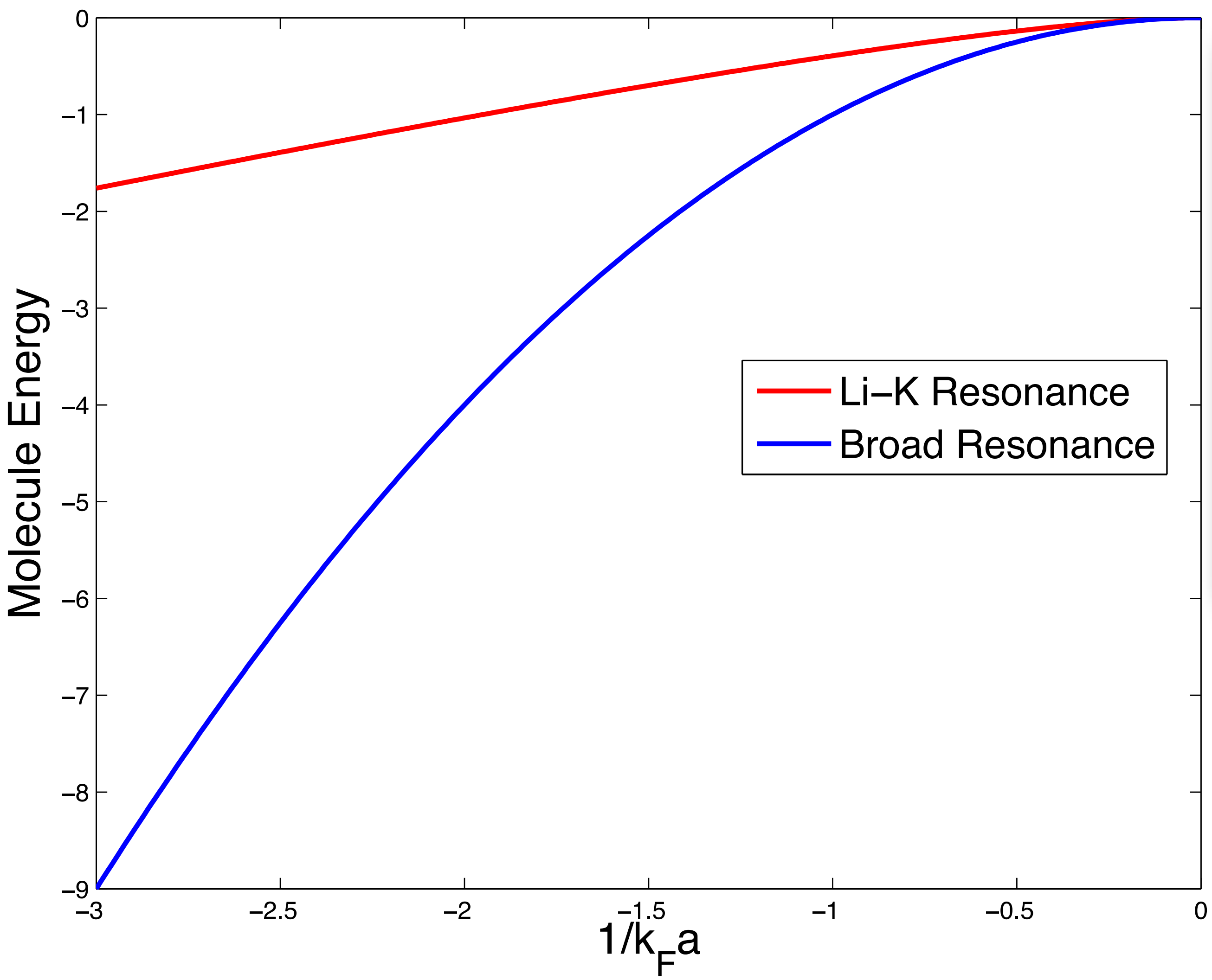


“Narrow” resonance

$$k_F r_{\text{eff}} \gtrsim 1 \qquad \frac{g^2}{\epsilon_F} \ll \frac{1}{m_r k_F}$$

Multi-channel

Molecule energy



^{40}K - ^{6}Li resonance:

$$B_0 = 154,72G \quad \Delta B = 880mG$$

$$a_{bg} = 63,0a_0 \quad \Delta\mu = 1,64\mu_B$$

$$|k_F r_{\text{eff}}| \simeq 1,9$$

$$E_B = \frac{\hbar^2}{2m_r a^{*2}} \qquad a^* = \frac{r_{\text{eff}}}{1-\sqrt{1-2r_{\text{eff}}/a}}$$

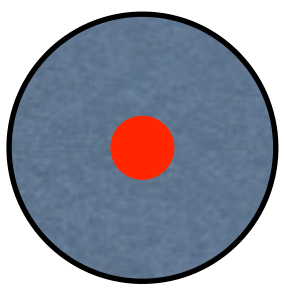
$$\to \frac{\hbar^2}{2m_r a^2}$$

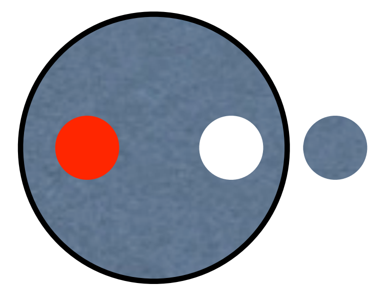
for broad resonance

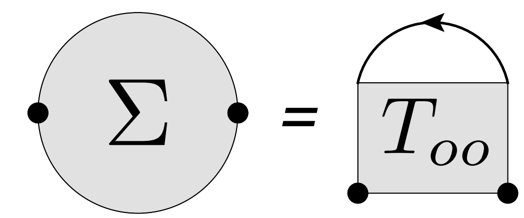
Many-body theory

Polaron:

$$|\psi_P\rangle = \sqrt{Z}a^\dagger_{0\downarrow}|\mathrm{FS}\rangle + \sum_{q<k_F<k} \phi_{\mathbf{k},\mathbf{q}} a^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} a^\dagger_{\mathbf{k}\uparrow} a_{\mathbf{q}\uparrow}|\mathrm{FS}\rangle + \ldots$$

Zero holes: 

One hole: 

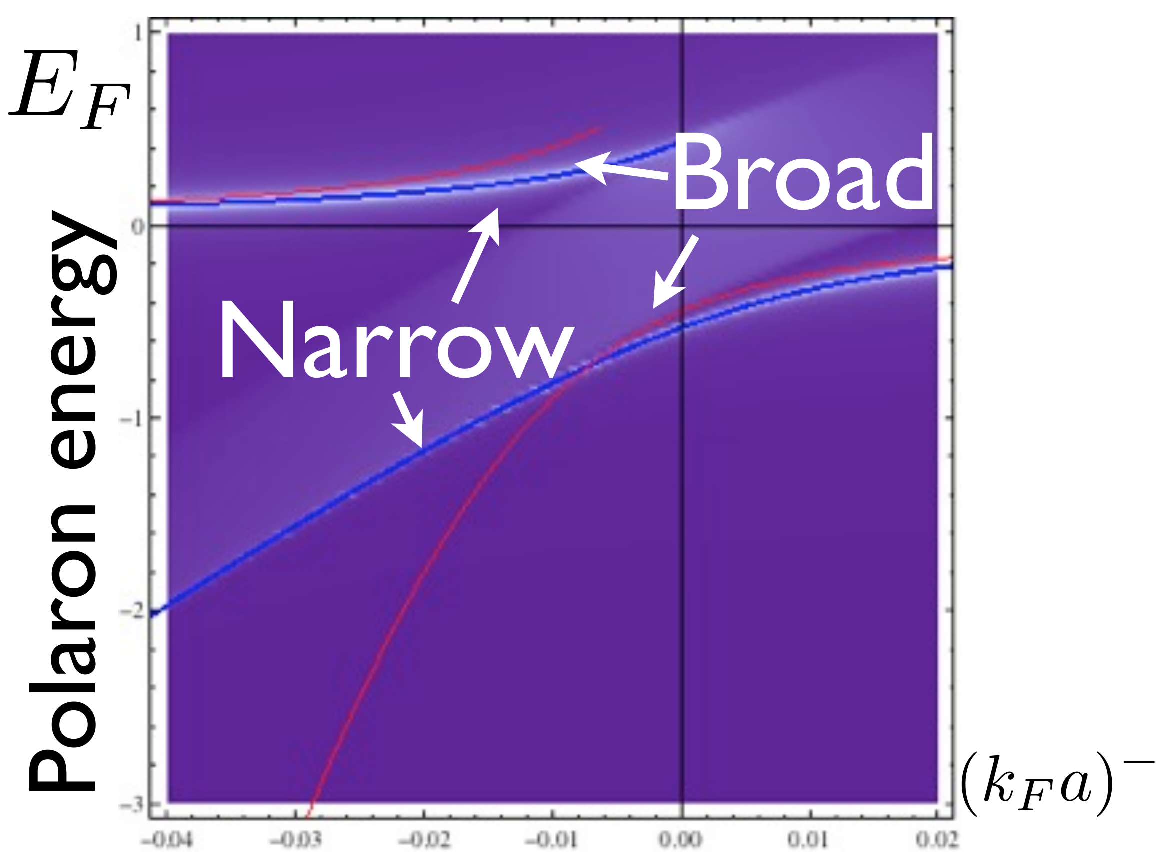


F. Chevy PRA **74** 063628 (2006)

Molecule:

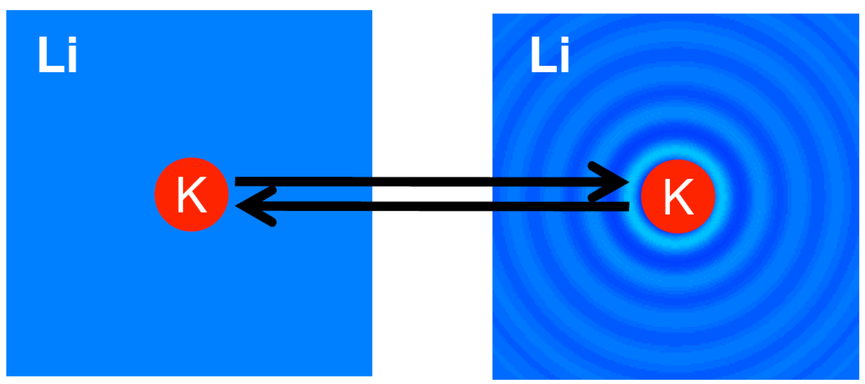
$$|\psi_M\rangle = \sum_{\mathbf{k}} \psi_k a^\dagger_{-\mathbf{k}\downarrow} a^\dagger_{\mathbf{k}\uparrow}|\mathrm{FS}\rangle + \ldots$$

$$\psi_{\mathbf{k}} \propto \frac{1}{1+k^2a^2} \Leftrightarrow \psi_M(r) \propto \frac{e^{-r/a}}{r}$$



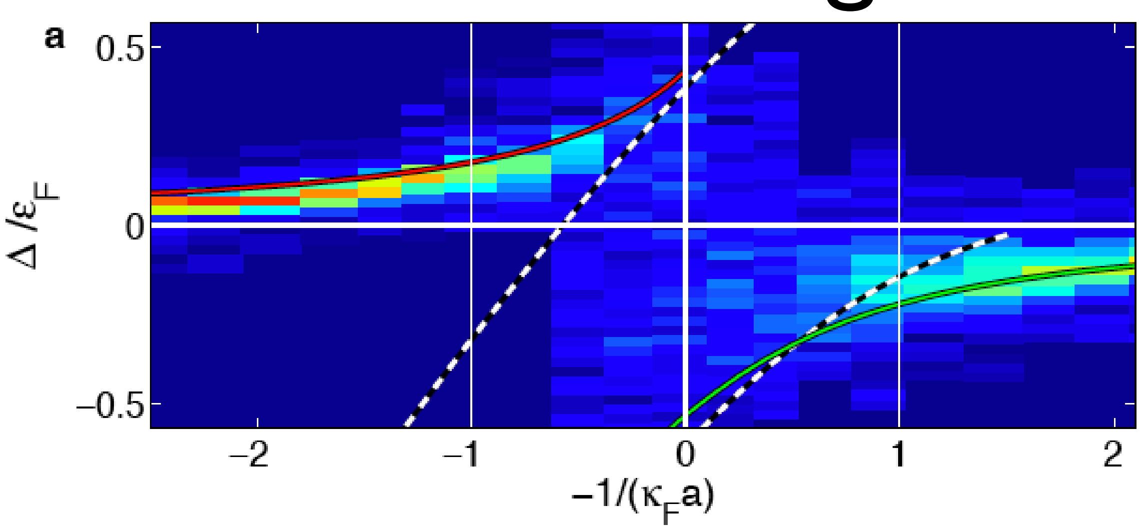
Results & experiments

RF flip

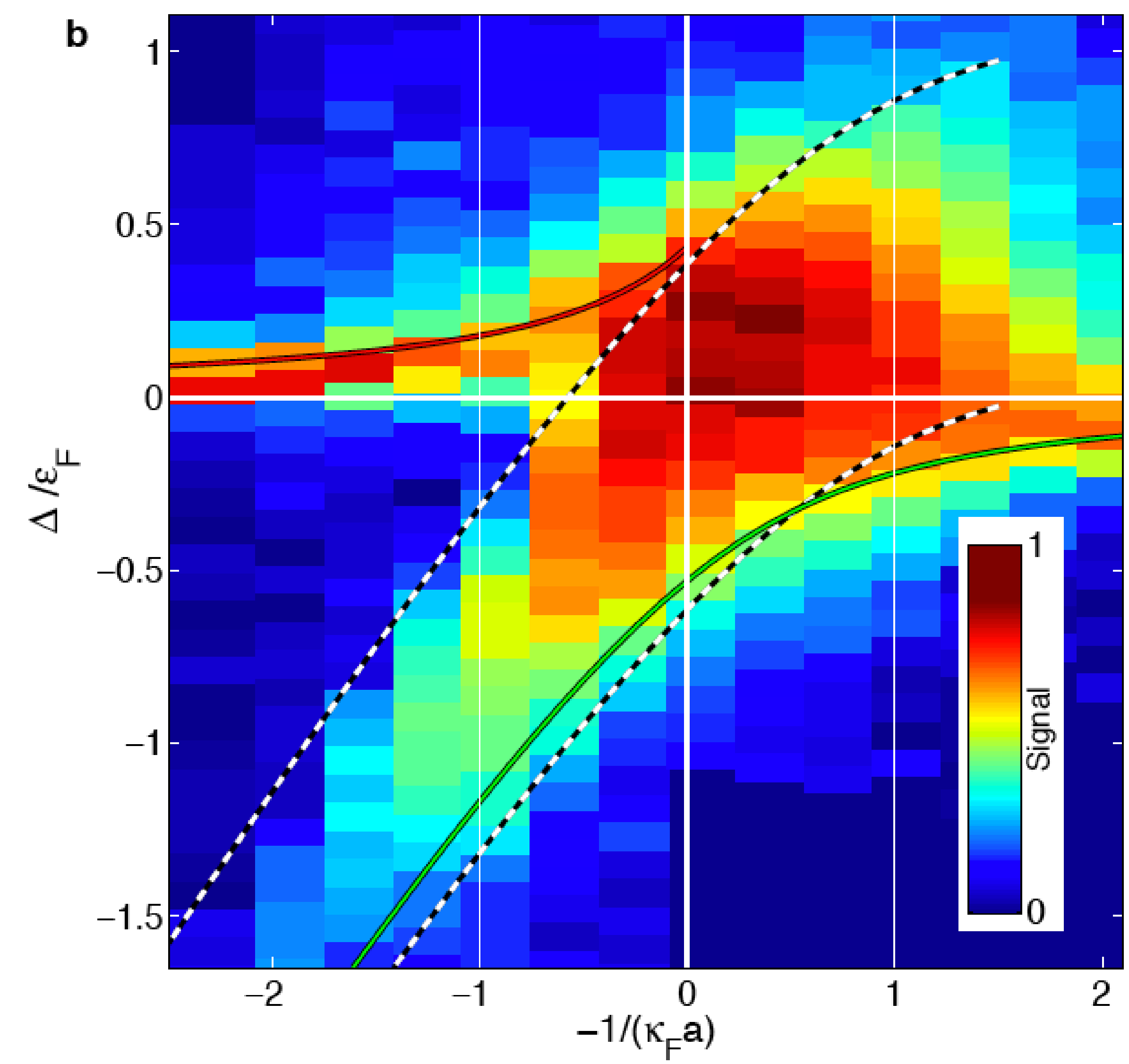


Non-interacting Strongly interacting

Polaron energies



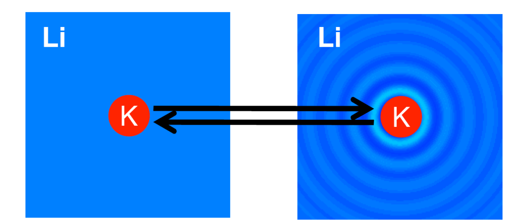
Molecule-hole continuum



Polaron quasiparticle residue

$$|\psi_P\rangle = \sqrt{Z} a^\dagger_{0\downarrow}|\mathrm{FS}\rangle + \sum_{q<k_F<k} \phi_{\mathbf{k},\mathbf{q}} a^\dagger_{\mathbf{q}-\mathbf{k}\downarrow} a^\dagger_{\mathbf{k}\uparrow} a_{\mathbf{q}\uparrow}|\mathrm{FS}\rangle + \ldots$$

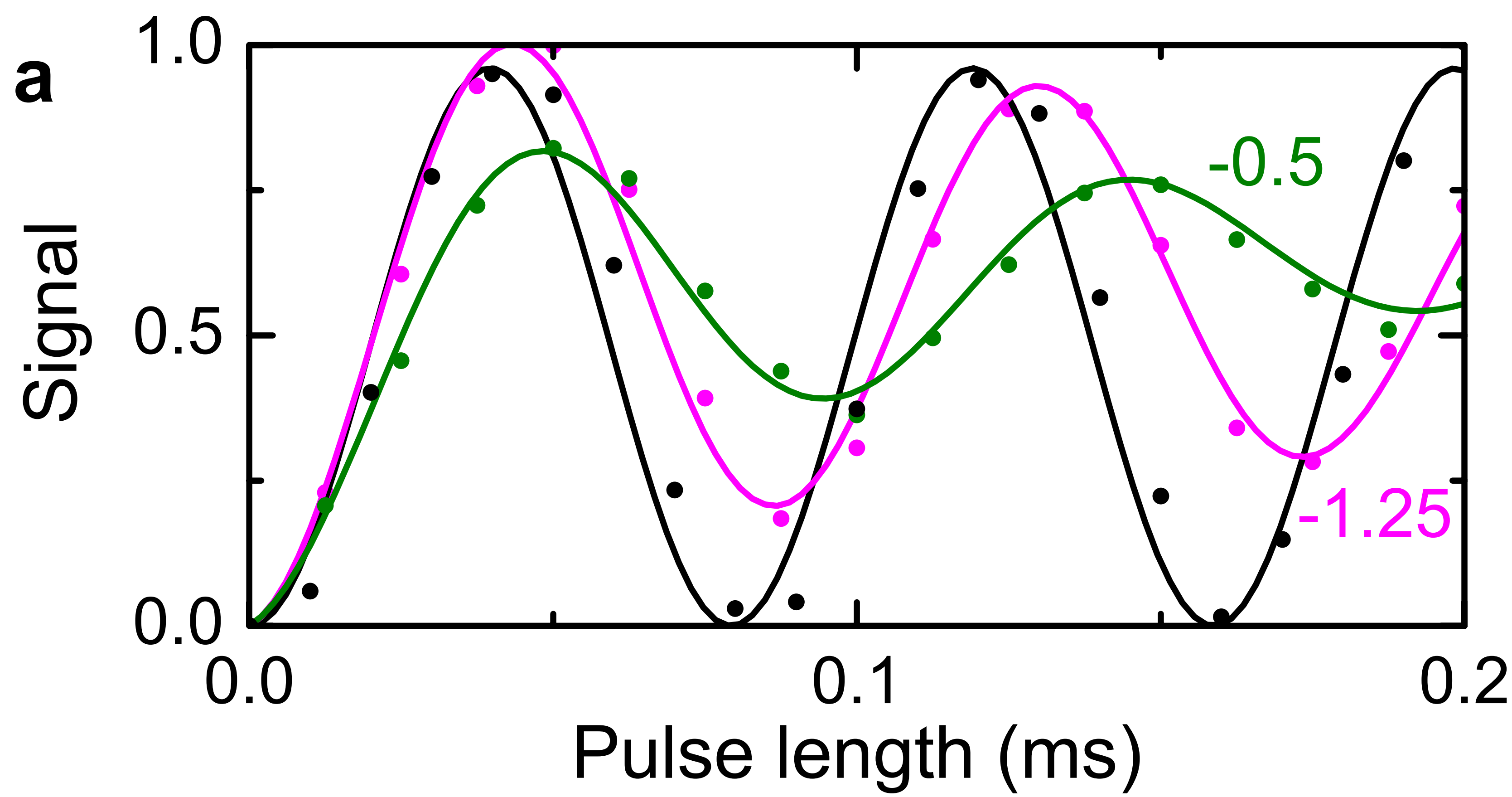
RF-probe momentum conserving $R \propto \Omega_0 \sum_{\mathbf{k}} (b^\dagger_{\downarrow\mathbf{k}} a_{\downarrow\mathbf{k}} + h.c.)$

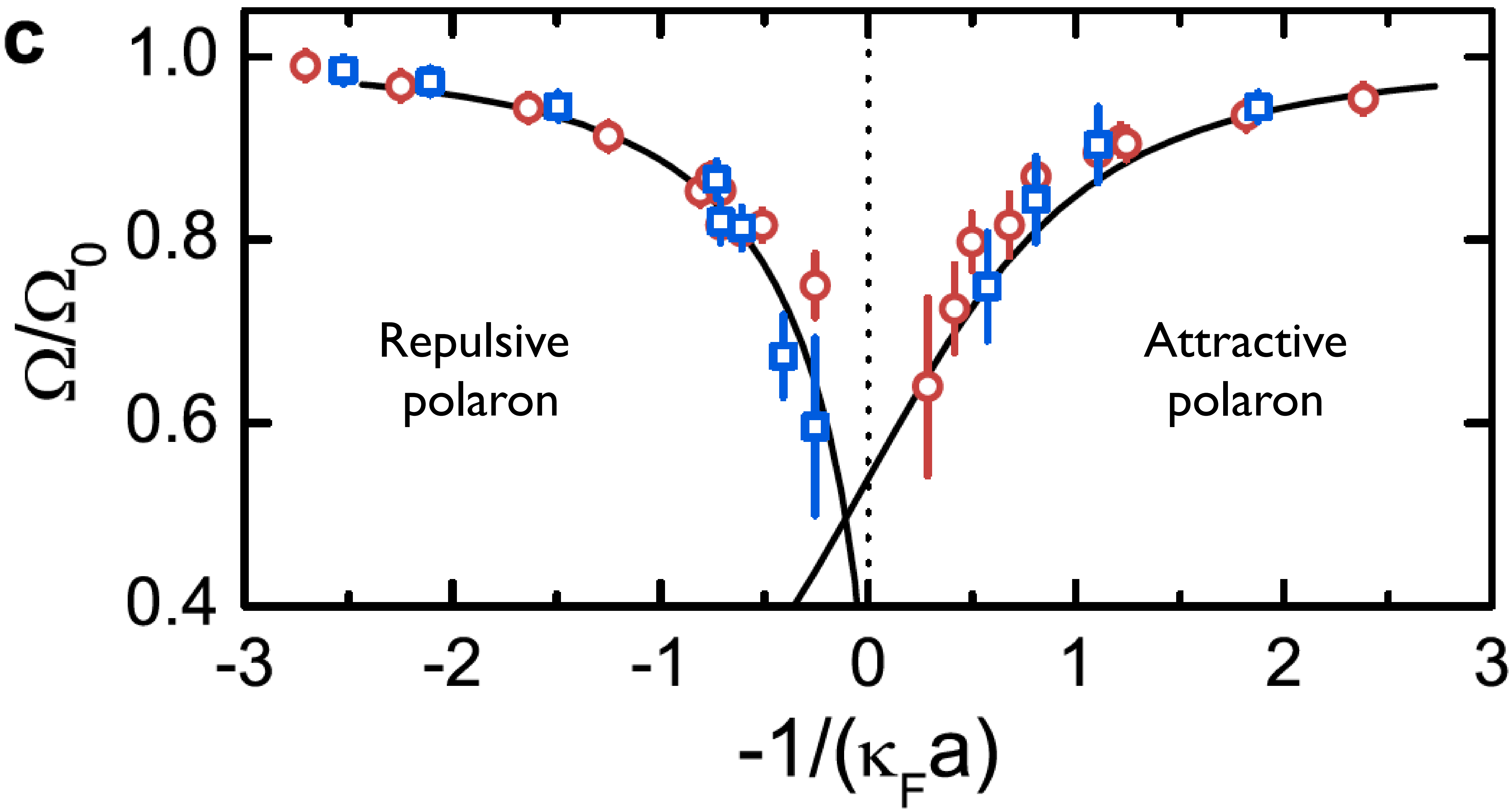


Initial state: $|I\rangle = b^\dagger_{\downarrow 0}|\mathrm{FS}\rangle$

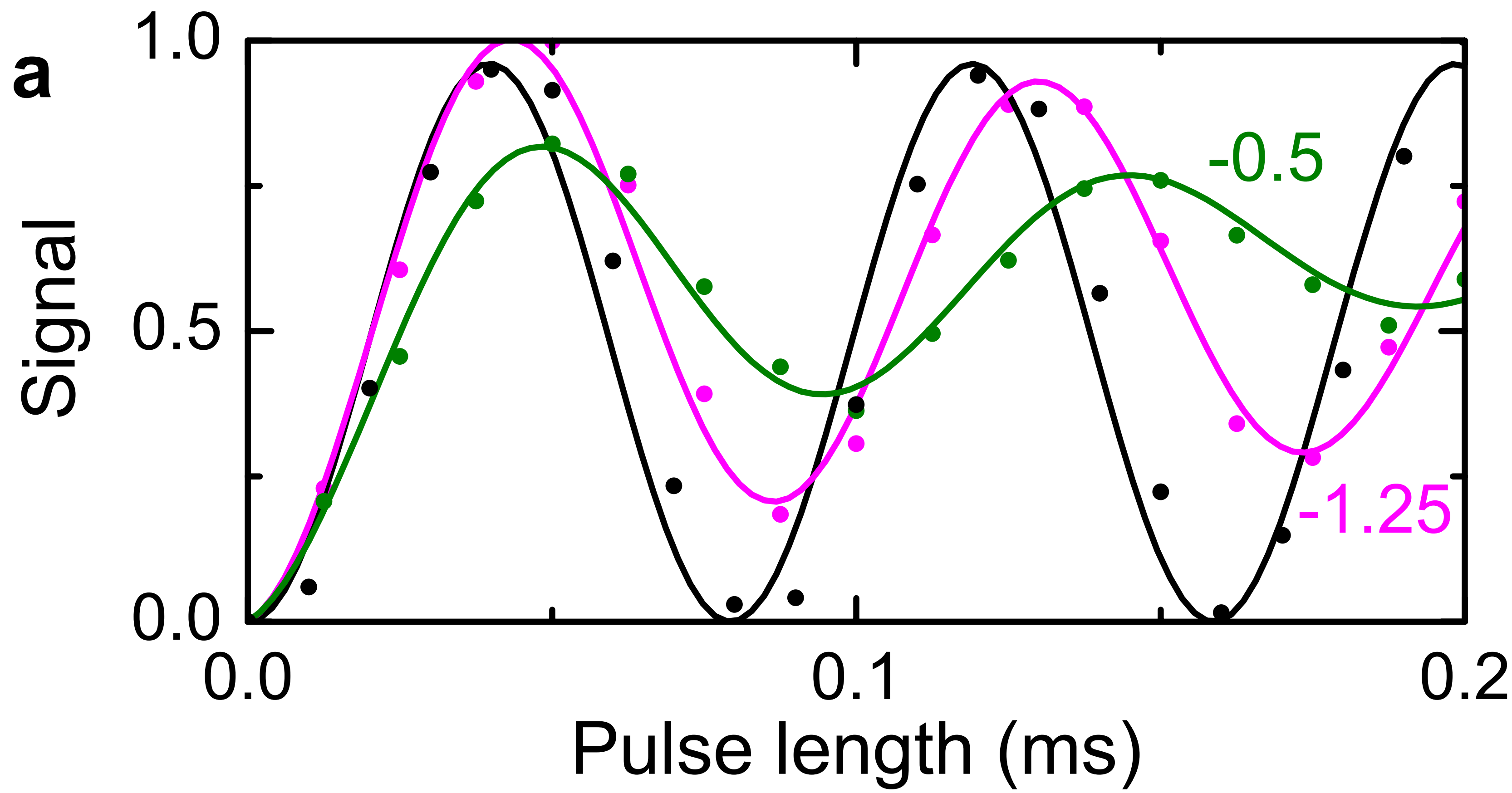
Rabi flipping frequency:

$$\begin{aligned} \Omega &= \langle\psi_P|R|I\rangle \\ &= \sqrt{Z}\Omega_0 \end{aligned}$$

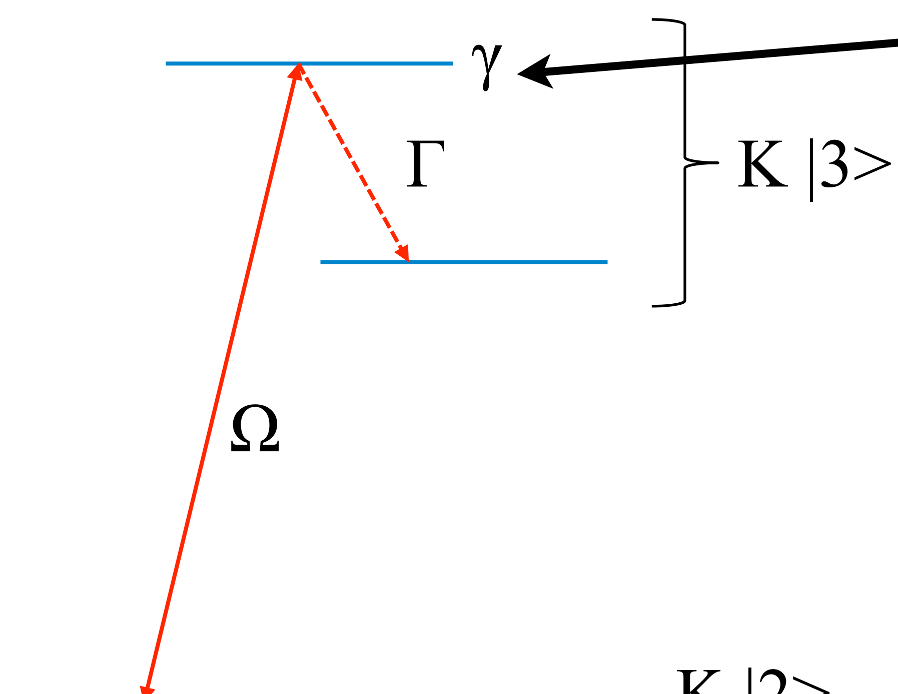



c
1.0
0.8
0.6
0.4
-3
-2
-1
0
1
2
3
Ω/Ω0
-1/(κFa)
Repulsive polaron
Attractive polaron

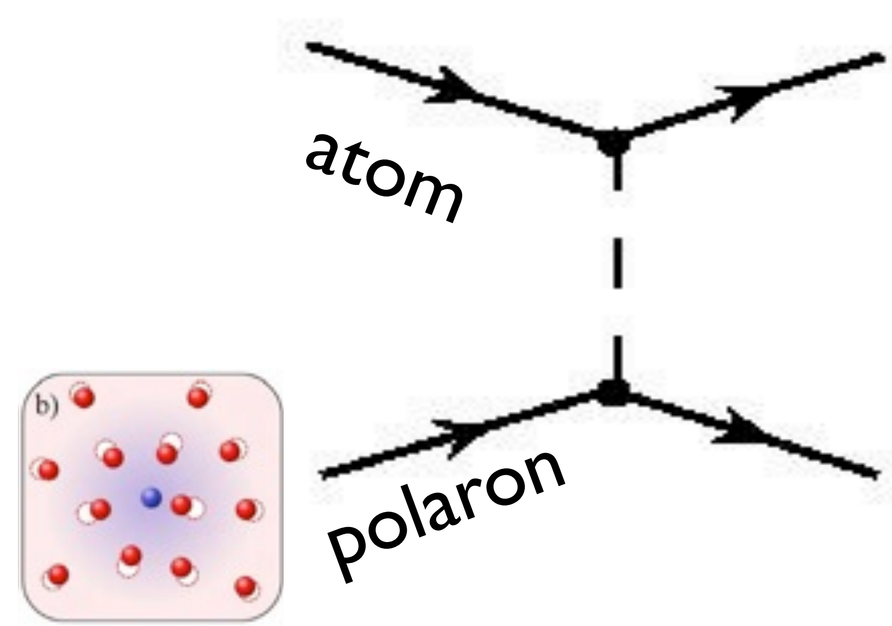
Damping of oscillations:



3-state model

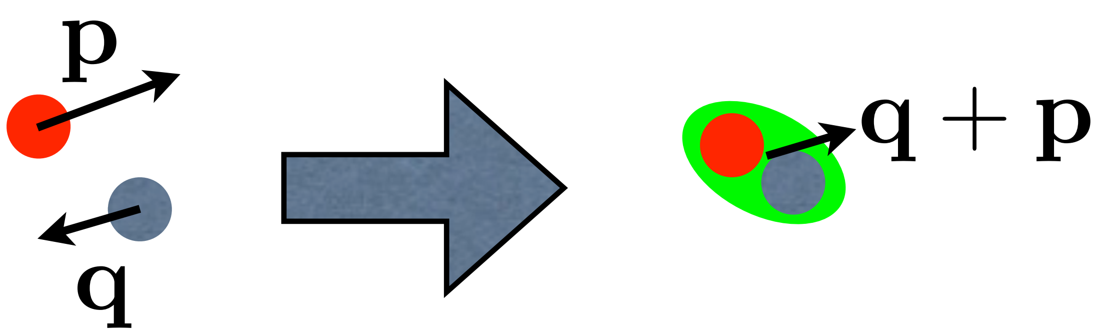


Collisional broadening:



Molecule wave function

RF-signal to molecule-hole continuum (BEC limit):



$$\Gamma_{2\mathrm{B}}(\omega_{\mathrm{rf}}) \quad \propto \quad \iint \frac{d^3p}{(2\pi)^3}\frac{d^3q}{(2\pi)^3} f(\xi_{p\downarrow}) f(\xi_{q\uparrow})$$

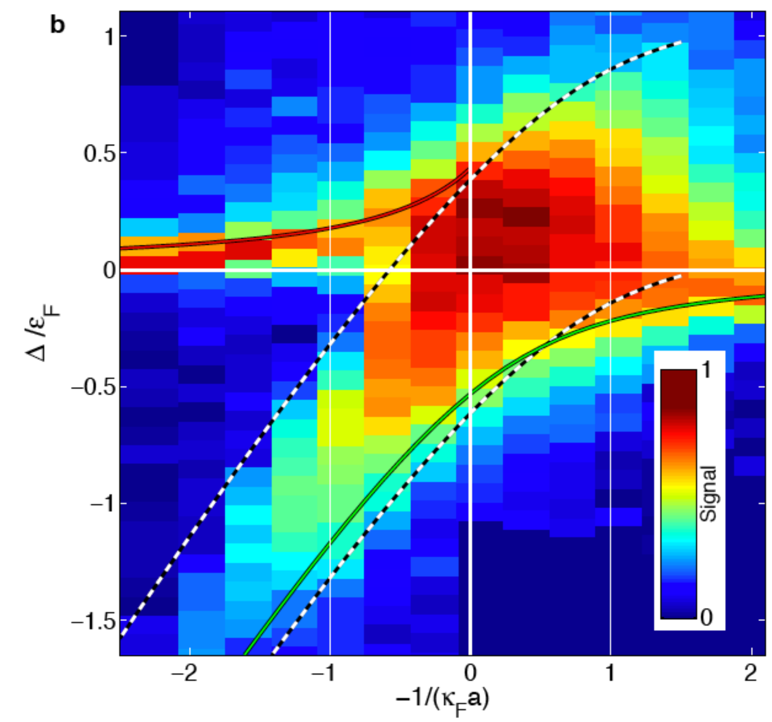
Overlap between molecule and plane wave

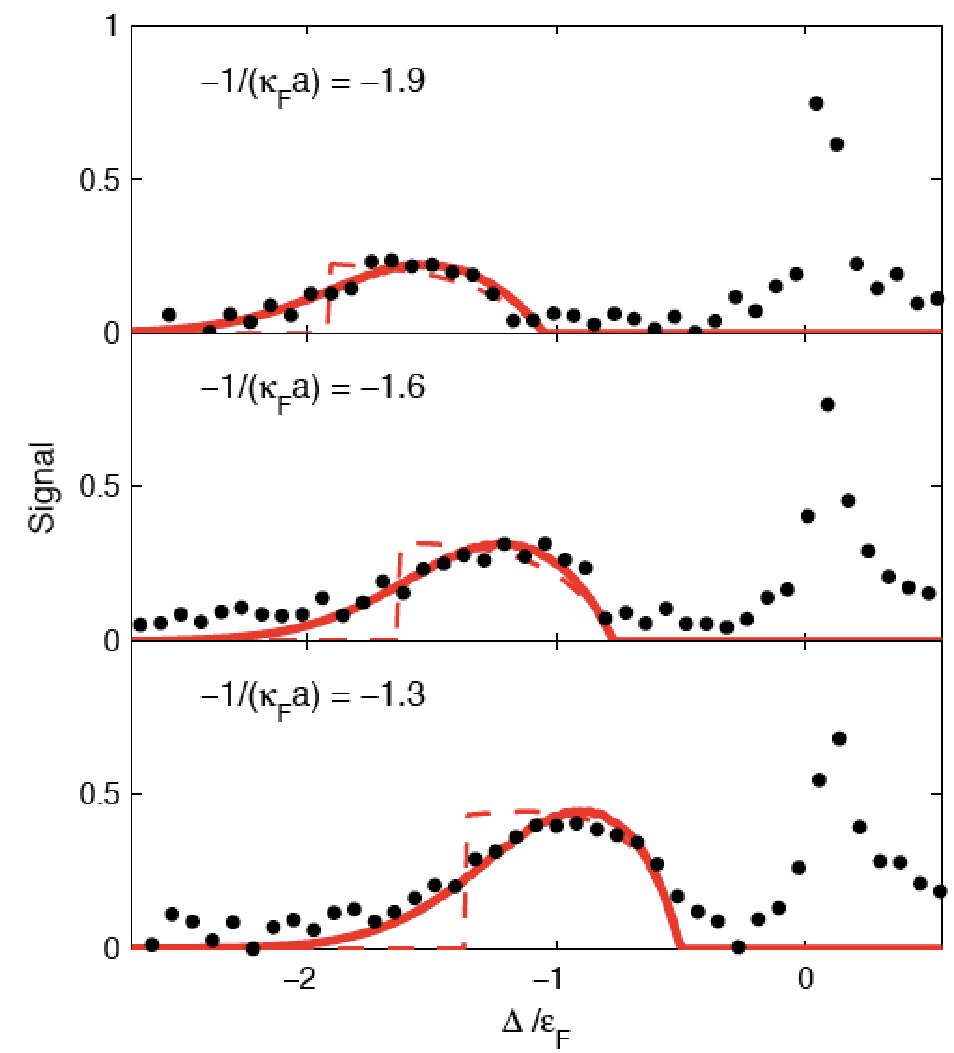
$$\frac{1}{\sqrt{1+4R^*/a}}\frac{8\pi {a^*}^3}{(1+k'^2{a^*}^2)^2}$$

$$\delta(\omega_{\mathrm{rf}} + |\omega_M| + \frac{k'^2}{2m_r})$$

$$\rightarrow \frac{8\pi a^3}{(1+k'^2a^2)^2}$$

for $|a/r_{\mathrm{eff}}| \gg 1$

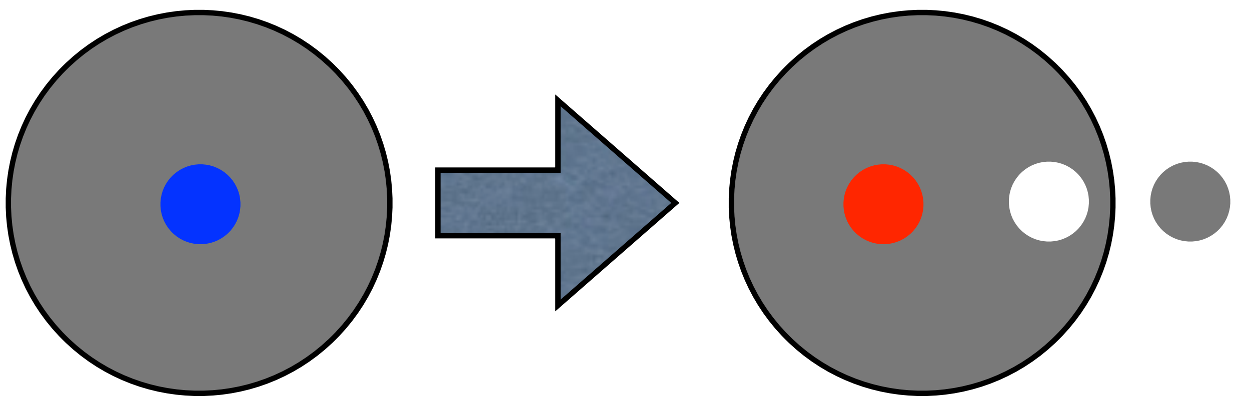




Repulsive Polaron Decay

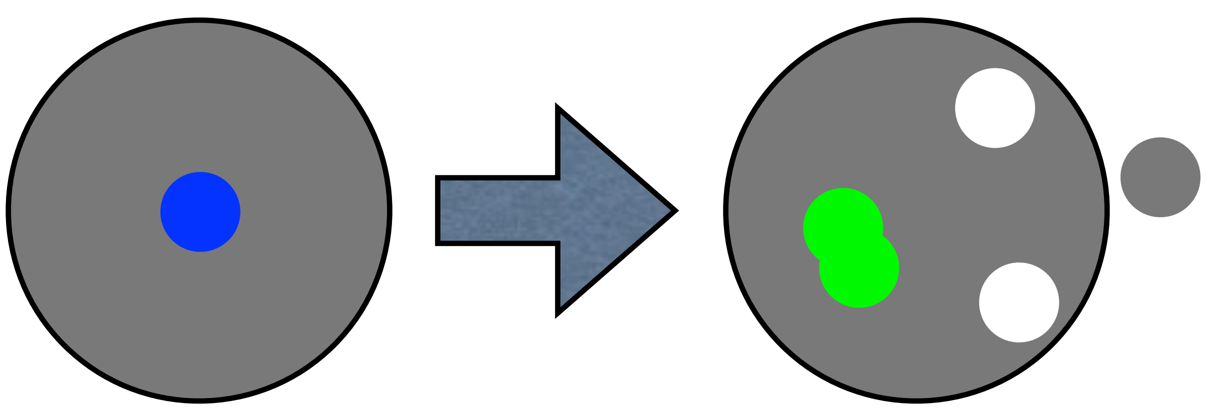
❶ Decay to attractive polaron:

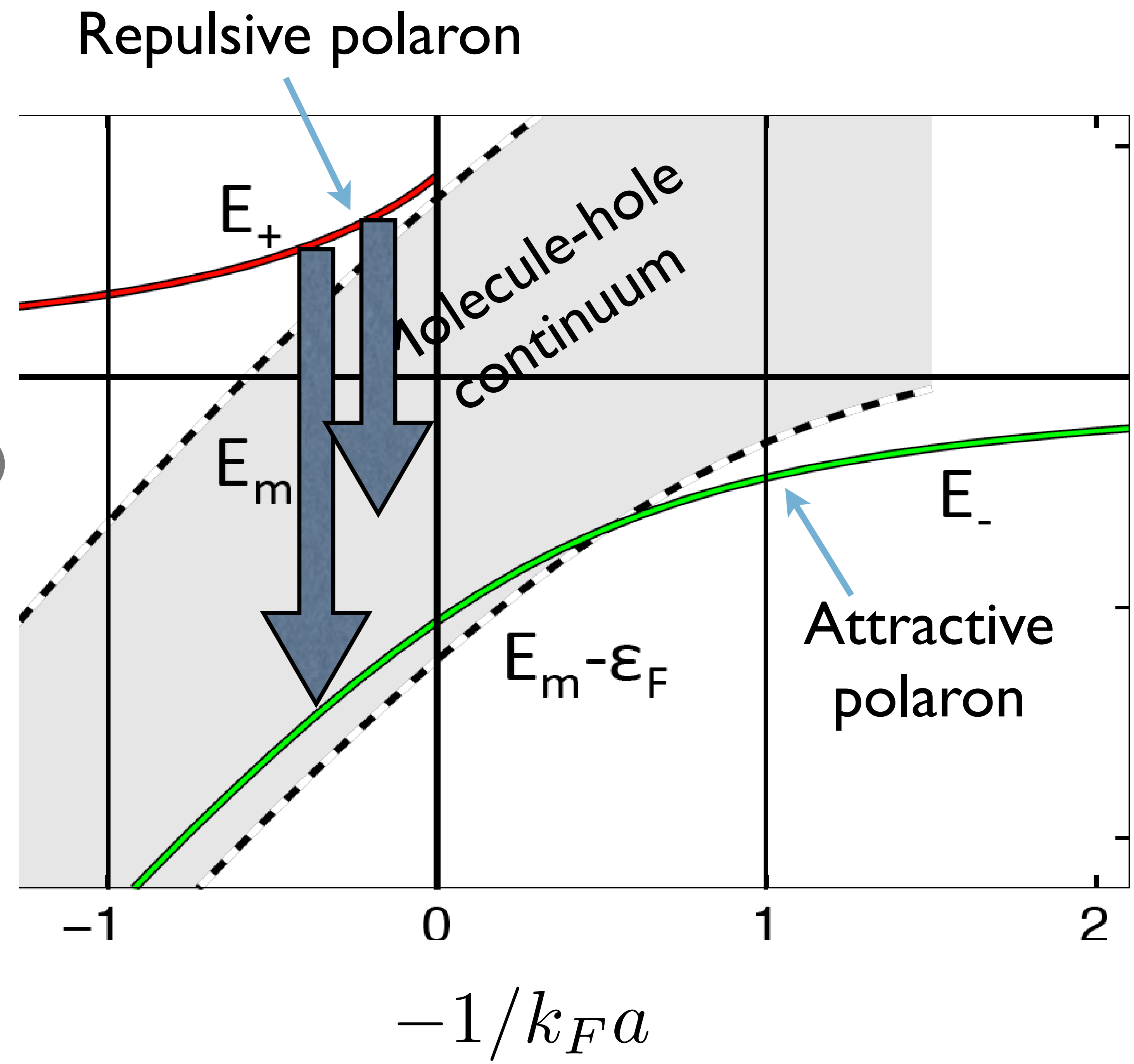
2-body process



❷ Decay to molecule:

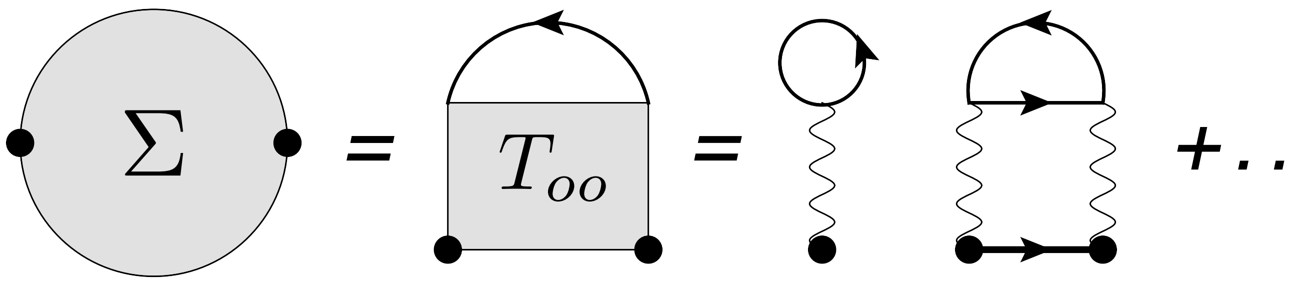
3-body process

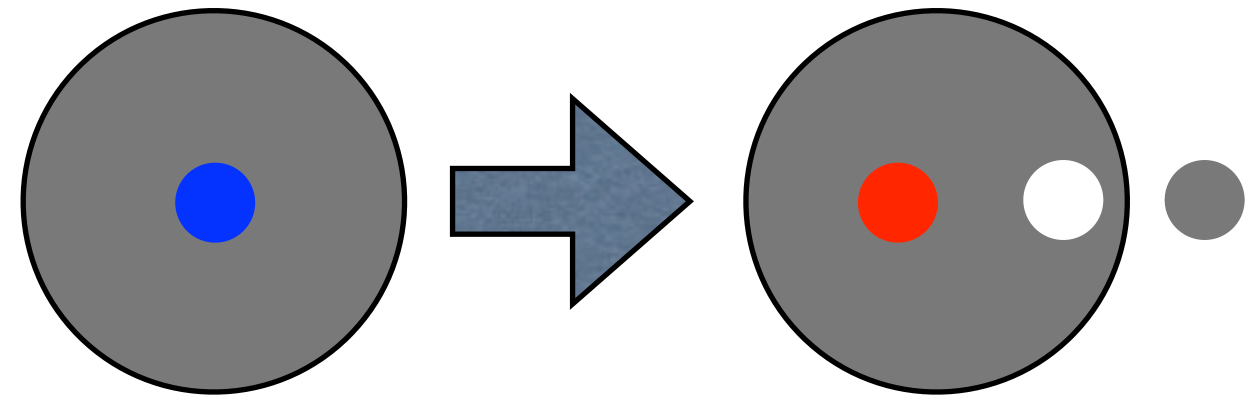




R. Schmidt and T. Enns, PRA **83** 063620 (2011)

2-body decay to attractive polaron:





$$G_\downarrow = \frac{Z_+}{\hbar\omega - E_+} + \frac{Z_-}{\hbar\omega - E_-}$$

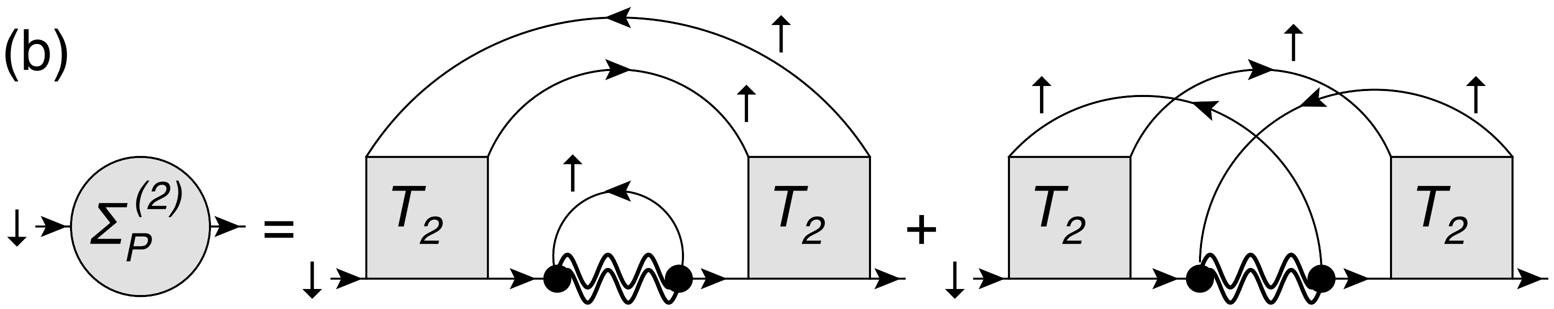
BEC-limit

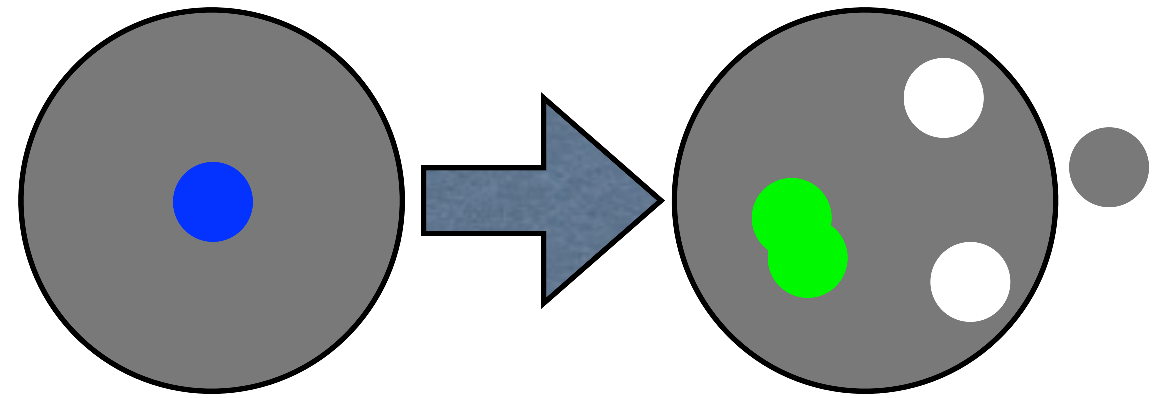
$$\begin{aligned}
\Gamma_{PP} &= \pi T_0^2 Z_- \int_{q<k_F<k} d^3\check{q}\, d^3\check{k}\, \delta(\Delta E + \epsilon_{\uparrow q} - \epsilon_{\uparrow k} - \epsilon^*_{\downarrow \mathbf{q}-\mathbf{k}}) \\
&= Z_- \frac{2}{3\pi} \sqrt{\frac{m_\uparrow (m_r^*)^3}{m_r^4}} \sqrt{\frac{\Delta E_{PP}}{\epsilon_F}} (k_F a)^2 \epsilon_F \propto k_F a
\end{aligned}$$

P. Massignan and GMB, EPJD **65**, 83 (2011)

3-body decay to molecule + hole:

(b)





$$F(\mathbf{q},\mathbf{k},\omega) = T_2(\mathbf{q},\omega+\xi_{q\uparrow})G^0_\downarrow(\mathbf{q}-\mathbf{k},\omega+\xi_{q\uparrow}-\xi_{k\uparrow})$$

$$D(\mathbf{p},\omega) \simeq \frac{Z_M}{\omega-\omega_M-p^2/2m^*_M}.$$

$$\Gamma_P = \frac{g^2 Z_M}{2}\int d^3\check{q}d^3\check{k}d^3\check{q}'\left[F(\mathbf{q},\mathbf{k},\omega_P)-F(\mathbf{q}',\mathbf{k},\omega_P)\right]^2$$
$$\times\delta\left(\Delta\omega+\xi_{q\uparrow}+\xi_{q'\uparrow}-\xi_{k\uparrow}-(\mathbf{q}+\mathbf{q}'-\mathbf{k})^2/2m^*_M\right)$$

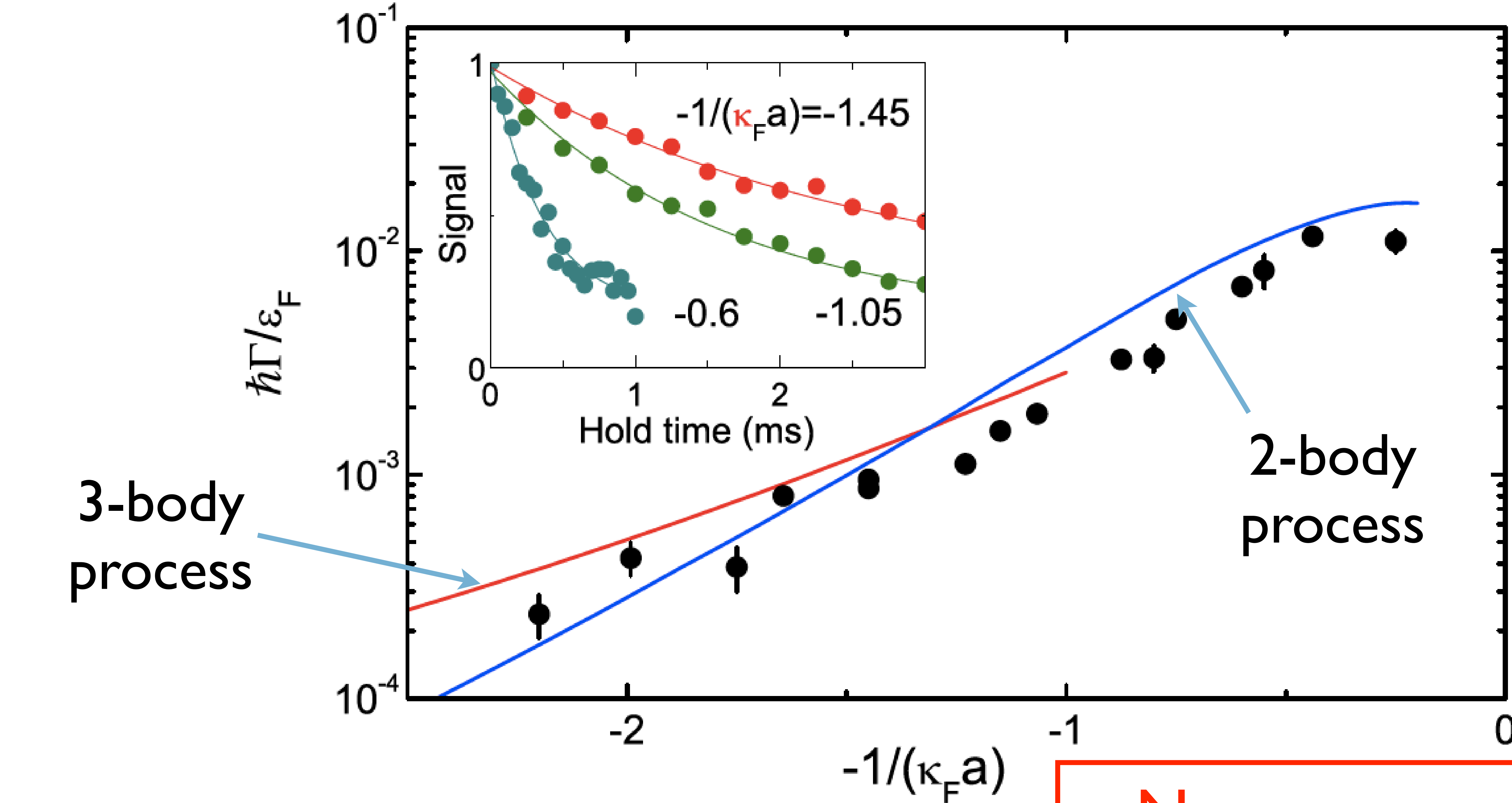
Broad resonance $\Gamma_P \propto (k_F a)^6\epsilon_F \propto n_\uparrow^2\epsilon_F$

Due to Fermi exclusion principle

GMB and P. Massignan, PRL **105** 020401 (2010)

D. S. Petrov, PRA **67** 010703 (2003)

Experiment



$(k_F a)^{-1}$=-0.25:

$$\hbar\Gamma/\epsilon_F = 0.01$$

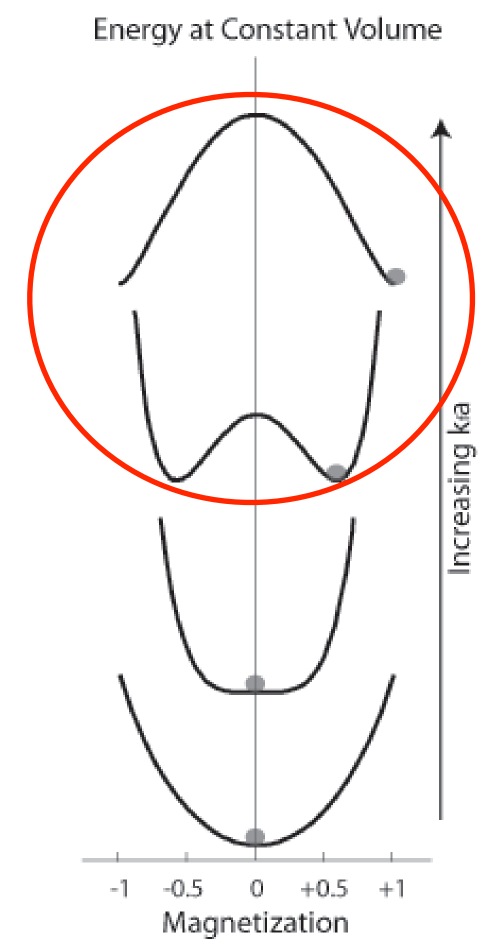
$$\hbar\Gamma/E_+ = 0.03$$

Non-zero range gives ≈ 10 times longer life time.
1/e life time ≈400μs

Itinerant ferromagnetism

Fermi gas with short range repulsive interactions

$$\hat{H} = -\int d^3r \hat{\psi}^\dagger_\sigma(\mathbf{r}) \frac{\nabla^2}{2m} \hat{\psi}_\sigma(\mathbf{r}) + g \int d^3r \hat{\psi}^\dagger_\uparrow(\mathbf{r}) \hat{\psi}^\dagger_\downarrow(\mathbf{r}) \hat{\psi}_\downarrow(\mathbf{r}) \psi_\downarrow(\mathbf{r})$$



Stoner (1933):

$$E = \frac{3}{5} n \epsilon_F [(1+\eta)^{5/3} + (1-\eta)^{5/3} + A(1+\eta)(1-\eta)]$$

$$\eta = \frac{n_\uparrow - n_\downarrow}{n_\uparrow + n_\downarrow} \qquad A \propto g \propto k_F a$$

Strong coupling phenomenon. Complicated.
Never realized in condensed matter systems

Realized with cold atoms?

Yes : Gyo-Boon Jo *et al*., Science **325**, 1521(2009)

No: C. Sanner *et al*., PRL **108**, 240404 (2012)

D. Pekker *et al*., PRL **106**, 050402 (2011)

Not observable due to pairing instability

Used broad resonance
Balanced system

Still hope?

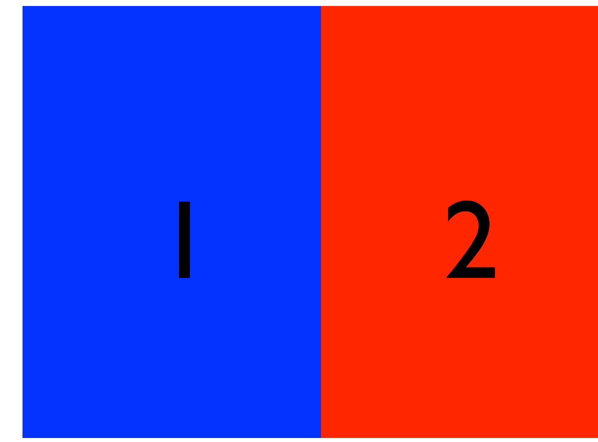
① Non-zero range gives longer lifetime

② Have reliable theory for $N_{\downarrow} \ll N_{\uparrow}$

Thermodynamic analysis

Energy per particle of phase separated state:

$$\varepsilon_{\rm sep} = (1-y)\varepsilon_1(N_1/V_1, T) + y\varepsilon_2(N_2/V_2, T)$$

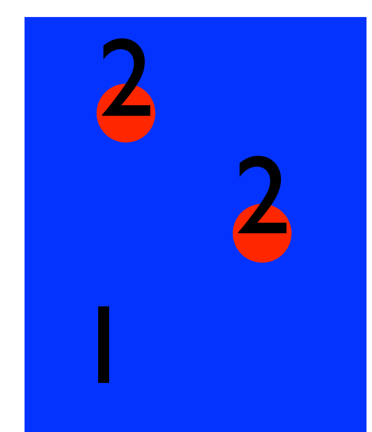


$$y = N_2/N \ll 1$$

Energy per particle of mixed state:

$$\varepsilon_{\rm mix} = (1-y)\varepsilon_1(N_1/V, T) + y\varepsilon_2(N_2/V, T) + y(1-y)^{2/3}E_+$$

E_+ the polaron energy



S. Pilati *et al.*, PRL **105**, 030405 (2010)

Look at the difference in free energies

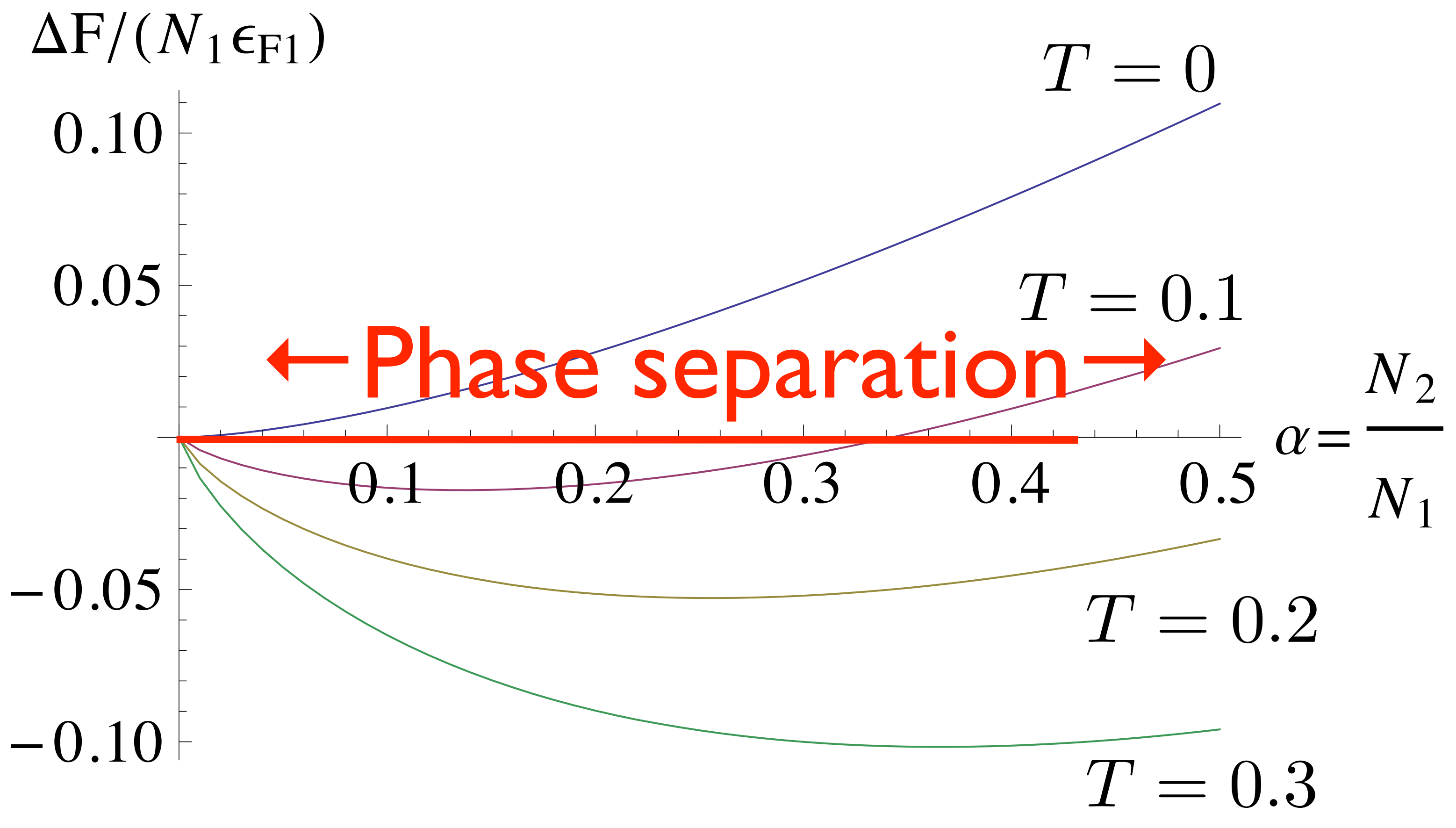
$$\Delta f = (1-y)\varepsilon(n_1) + y\varepsilon(n_2) + y(1-y)^{2/3}E_+ - \varepsilon - T\Delta s$$

Ideal mixture entropy of mixing (purely combinatorial):

$$\Delta s = -k_B[y \ln y + (1-y)\ln(1-y)]$$

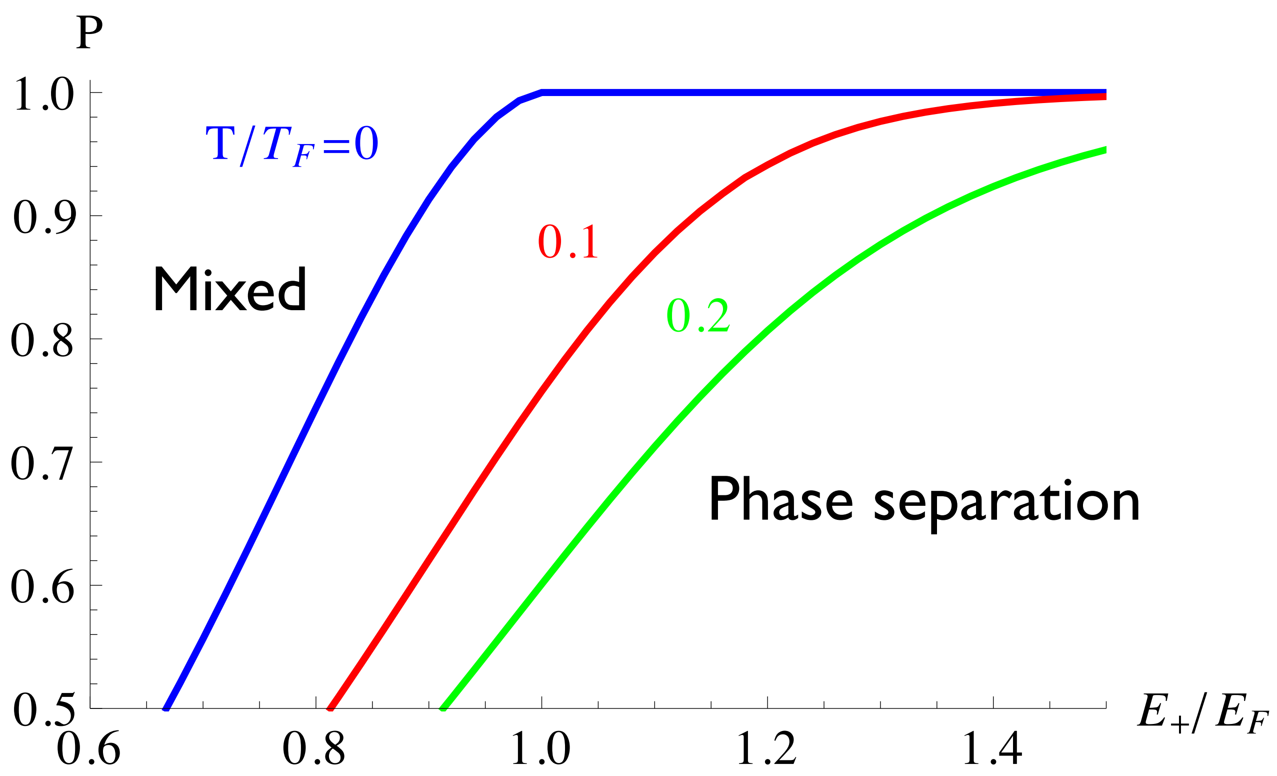
Maxwell construction:

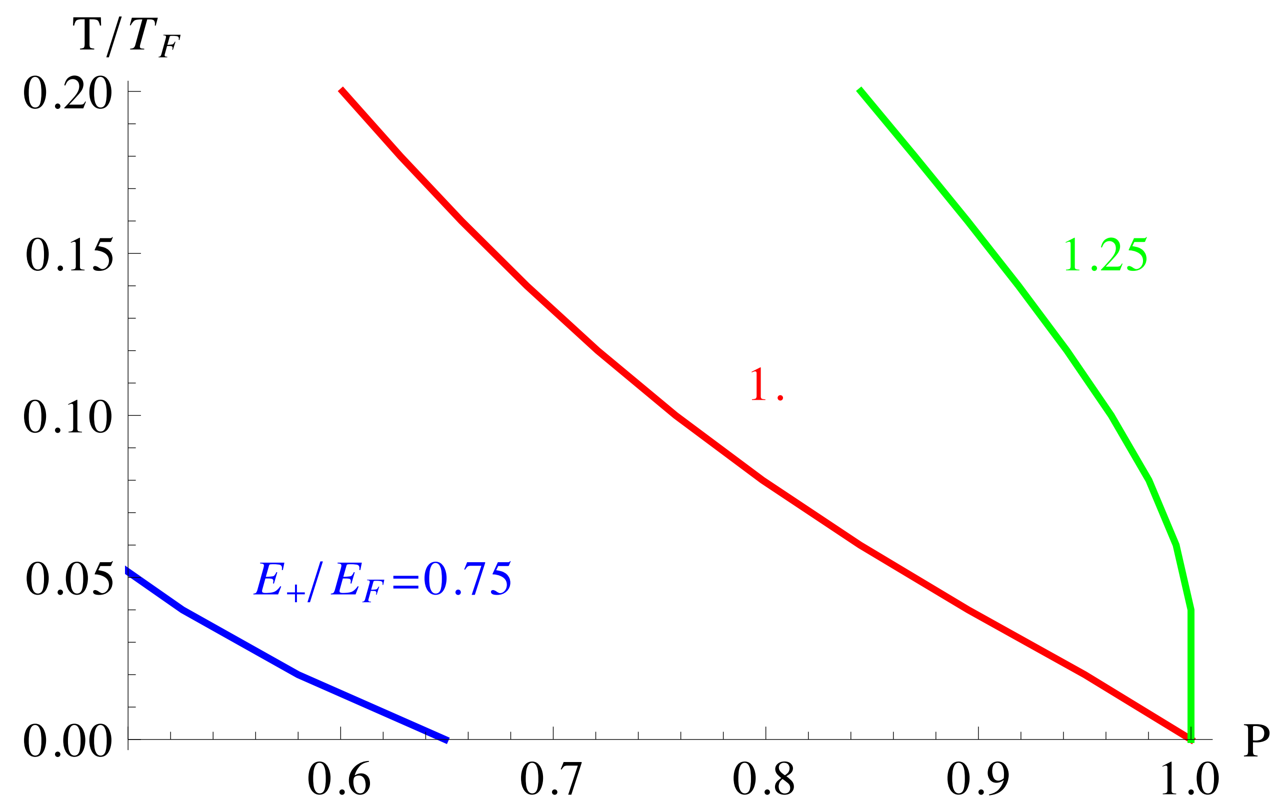
$$E_+/\epsilon_F = 1$$



Phase diagrams

$$P = \frac{N_1 - N_2}{N_1 + N_2}$$



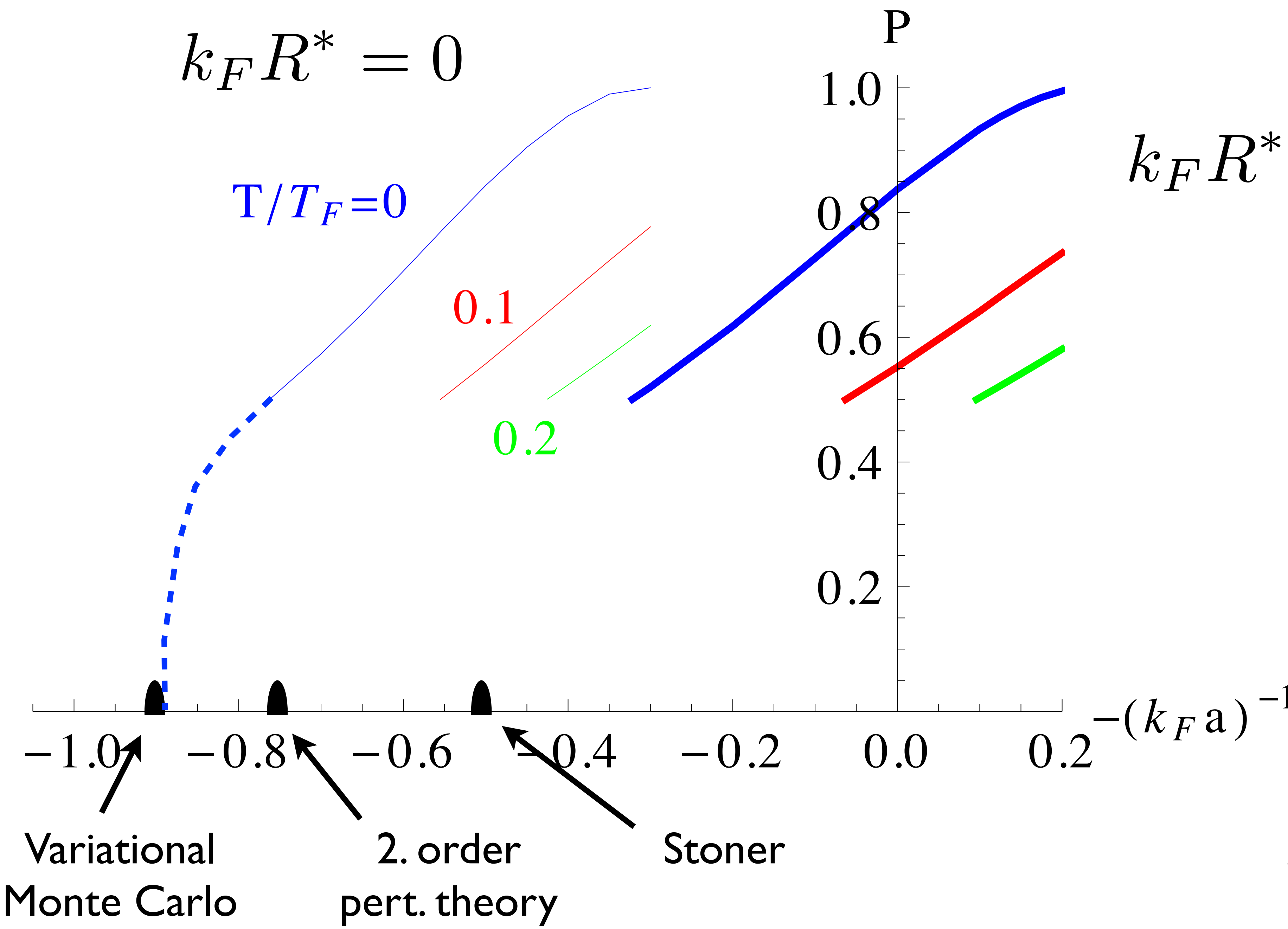


Generic, depends on:

① Polaron ansatz

② Ideal mixture

$E_+(k_F a, k_F R^*)$ gives phase diagram in terms of physical parameters



Transition moves to BEC side with increasing R^*

Results consistent with Monte-Carlo

S. Pilati *et al.*, PRL **105**, 030405 (2010)
G. J. Conduit *et al.*, PRL **103**, 207201 (2009)
S.-Y. Chang *et al.*, Proc. Nat. Acad. Sci. **108**, 51 (2011)
R. A. Duine and A. H. MacDonald, PRL **95**, 230403 (2005)
E. Stoner, Philos. Mag. **15**, 1018 (1933)

Different masses $m_1 \neq m_2$

Phase separation for T=0 and P→1 for

$$E_+ > \left(\frac{m_1}{m_2}\right)^{3/5} E_{F1}(n) = \frac{(6\pi n)^{2/3}}{2m_2^{3/5} m_1^{2/5}}$$

Phase separation favored by making the masses larges, since reduced kinetic energy cost

C. W. von Keyserlink and G. J. Conduit, PRA **83**, 053625 (2011)
X. Cuio and T.-L. Ho, arXiv:1208.2211

Problem of decay

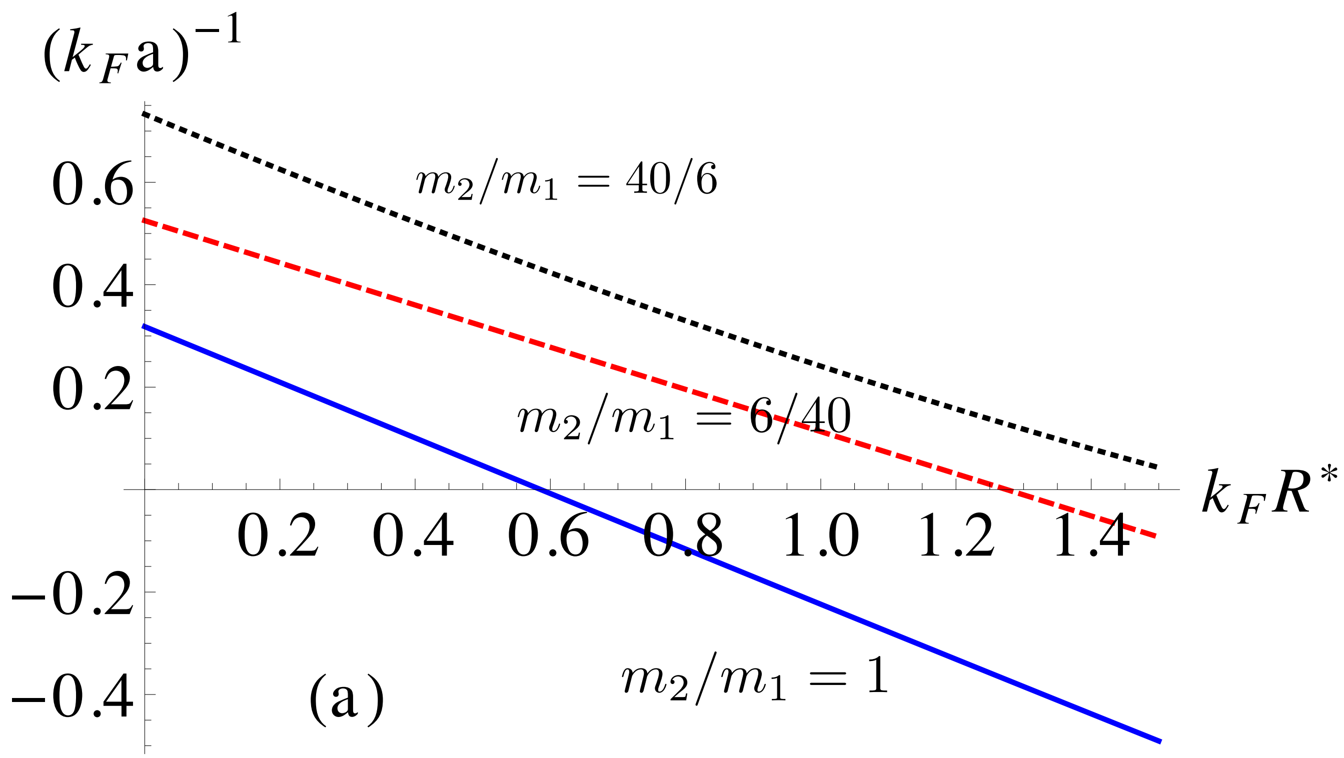
Ferromagnetism was not observed in MIT experiment due to fast decay to lower branch

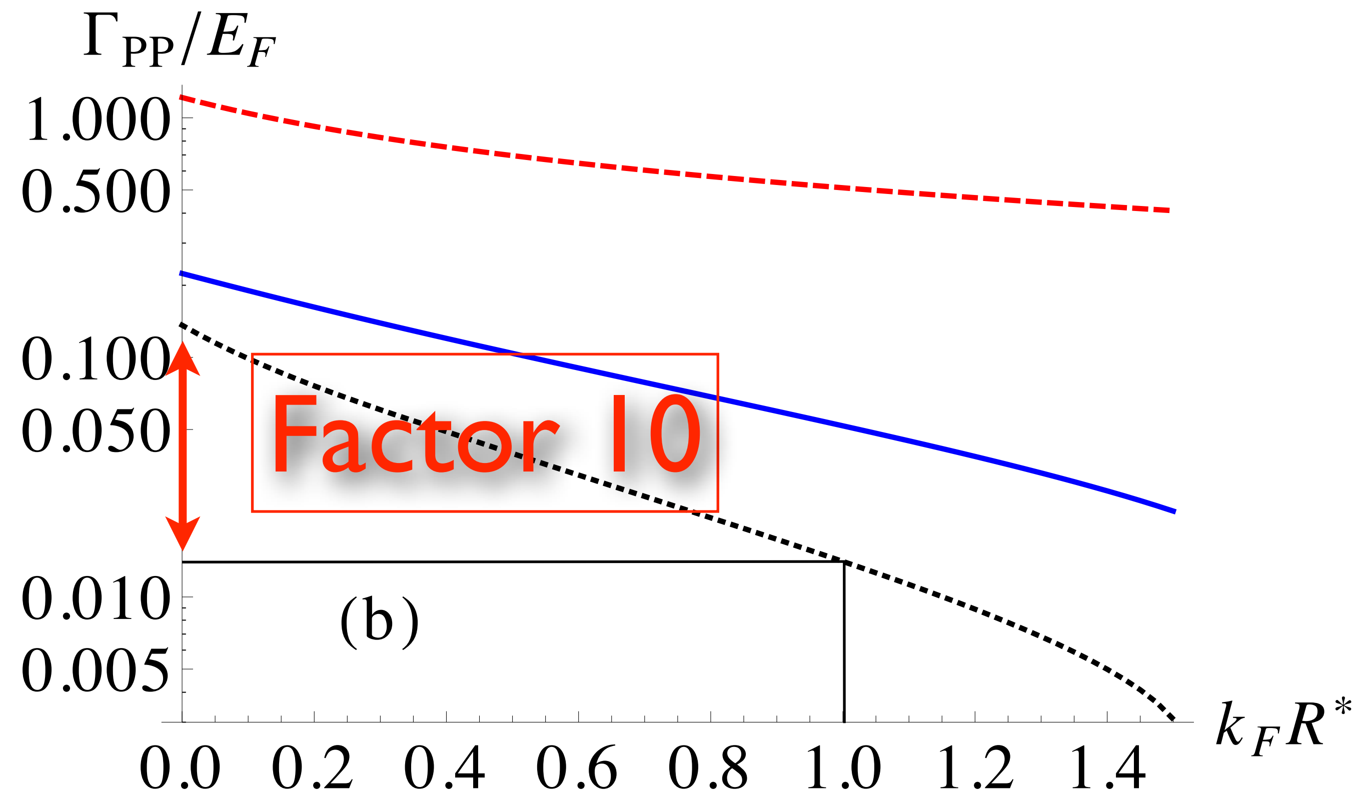
Used ^{6}Li atoms interacting via a broad resonance

^{6}Li-^{40}K mixture has much longer lifetime due to $k_F R^{*} \sim 1$

Critical coupling strength for T=0 and P→1

Decay rate at critical coupling strength





Ferromagnetism with narrow Feschbach resonance?

Conclusions

- Long lived repulsive polaron
- Excellent agreement between theory & experiment
- Narrow resonance increases stability of repulsive polaron
- Reliable phase diagrams for itinerant ferromagnetism
- Ferromagnetism for narrow resonance?