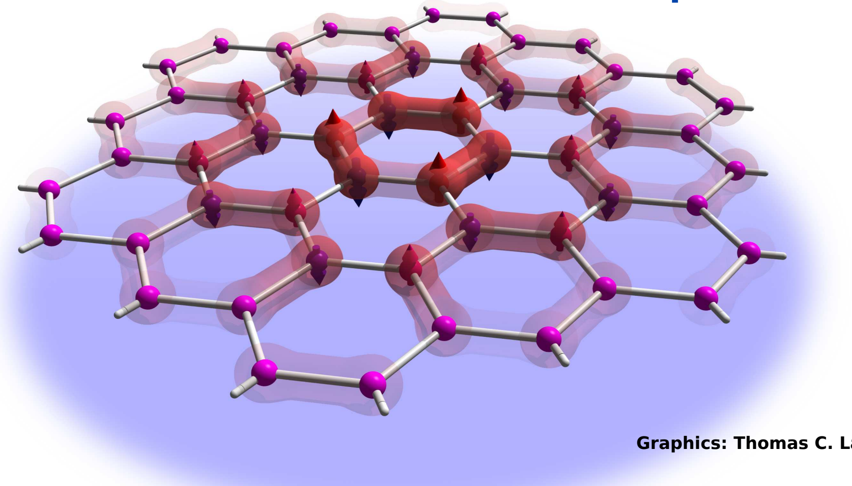
Stability of spin liquid phases of alkaline earth atoms at finite temperature



Graphics: Thomas C. Lang

P. Sinkovicz, A. Zamora, E. Szirmai, G. Szirmai, M. Lewenstein

Wigner Research Centre of the Hungarian Academy of Sciences

ICFO - The Institute of Photonic Sciences, Barcelona

The Institute of Photonic Sciences

Motivation: magnetic systems without long range order

$$H = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j$$

$J > 0$	antiferromagnetic coupling

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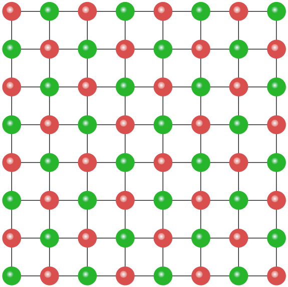

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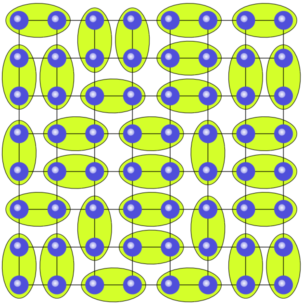

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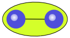
Singlet pairs

 $= \frac{1}{\sqrt{2}} \left(|{\uparrow\downarrow}\rangle - |{\downarrow\uparrow}\rangle \right)$

rotational symmetry preserved

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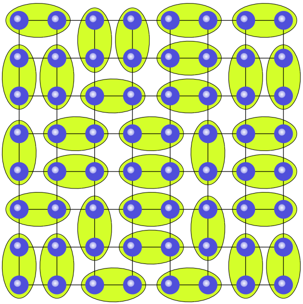

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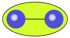 $= \frac{1}{\sqrt{2}}\left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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1 particle / site

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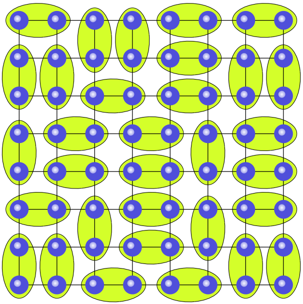

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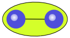

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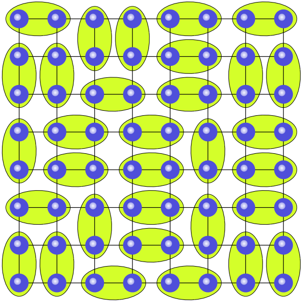

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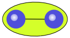 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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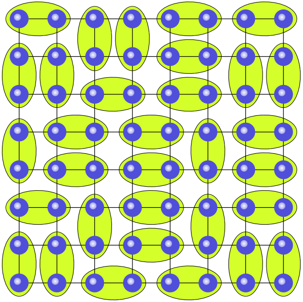

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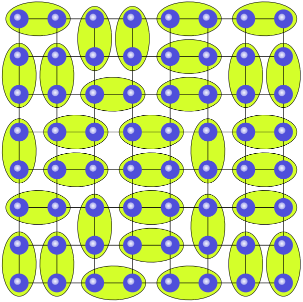

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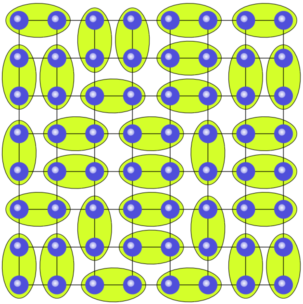

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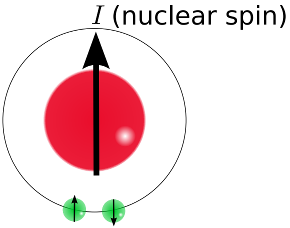
$$H_{\mathrm{mf}} = -J \sum_{\langle i,j \rangle} \left(\chi_{i,j} c^\dagger_{j,\alpha} c_{i,\alpha} + \chi_{j,i} c^\dagger_{i,\alpha} c_{j,\alpha} - |\chi_{i,j}|^2 \right) + \sum_i \varphi_i \left(c^\dagger_{i,\alpha} c_{i,\alpha} - 1 \right)$$

Works well for SU(N) spins when $\mathrm{N} \to \infty$

J. B. Marston, I. Affleck, Phys. Rev. B **39**, 11538 (1989)

SU(N) systems are realized with alkaline earth atoms



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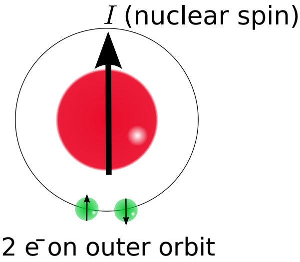

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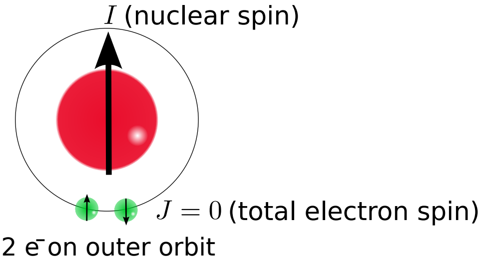

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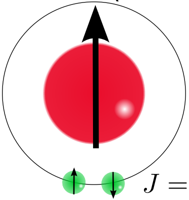
I (nuclear spin)



$J = 0$ (total electron spin)

2 e^- on outer orbit

$F = I$ (total hyperfine spin)

Collisions are spin independent

A. Gorshkov et al, Nat. Phys. **6**, 289 (2010)

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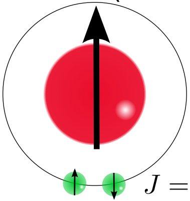

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$\mathrm{SU}(N = 2I + 1)$ symmetric models

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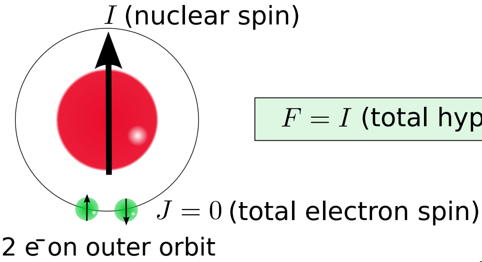

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A. Gorshkov et al, Nat. Phys. **6**, 289 (2010)

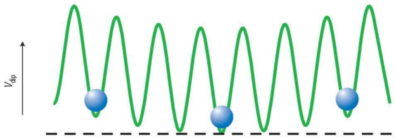
$\mathrm{SU}(N = 2I + 1)$ symmetric models

Example $^{173}\mathrm{Yb}$: $I = \frac{5}{2} \Rightarrow 2I + 1 = 6$ spin components

Optical lattice

periodic potential created by standing wave laser light

$$V_{1d}(r,z) = V_0 e^{-2r^2/w^2(z)} \sin^2(k_L z)$$

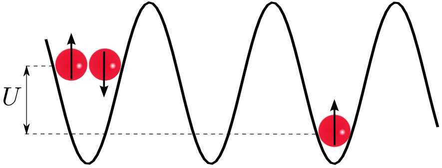


Tight binding Hamiltonian / Hubbard model

$$H = -t \sum_{\langle i,j \rangle} \left(c^\dagger_{i\alpha} c_{j\alpha} + \mathrm{H.c.} \right) + \frac{U}{2} \sum_i c^\dagger_{i\alpha} c^\dagger_{i\beta} c_{i\beta} c_{i\alpha},$$

... and in the strongly interacting limit the SU(N) Heisenberg model

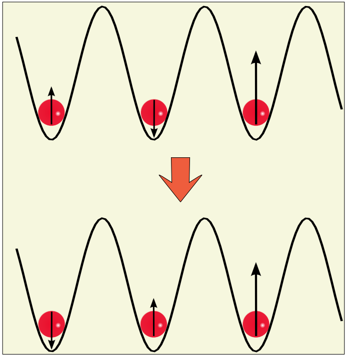
1 particle per site



$U \gg t$

particle transport is forbidden, **but**

spins can exchange without current



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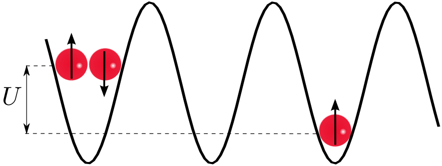

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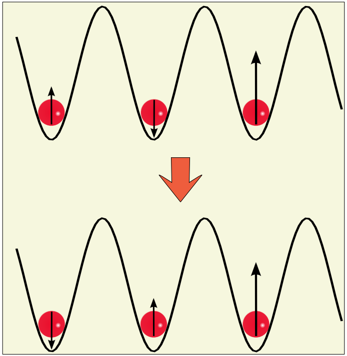

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Low energy effective Hamiltonian (2nd order)

$$H = -J \sum_{\langle i,j \rangle} c^\dagger_{i,\alpha} c_{j,\alpha} c^\dagger_{j,\beta} c_{i,\beta} + \sum_i \varphi_i \left(c^\dagger_{i,\alpha} c_{i,\alpha} - 1 \right), \qquad J = \frac{4t^2}{U}$$

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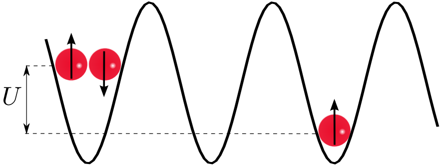

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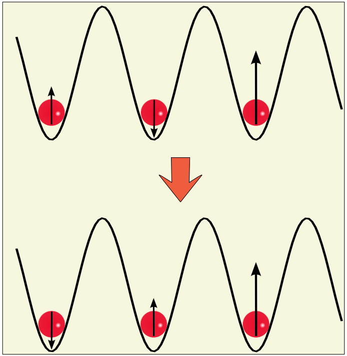

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for square lattice: M. Hermele et al, Phys. Rev. Lett. **103**, 135301 (2009)

Partition function and free energy

$$Z = \int D[c,\bar{c}]e^{-S[c,\bar{c}]}$$

$$S[c,\bar{c}] = \int_0^\beta \mathrm{d}\tau \Big[\sum_i \bar{c}_{i,\alpha}(\partial_\tau + \varphi_i)c_{i,\alpha} \\ - J\sum_{\langle i,j\rangle} \bar{c}_{i,\alpha}c_{j,\alpha}\bar{c}_{j,\beta}c_{i,\beta} + \sum_i \varphi_i(\bar{c}_{i,\alpha}c_{i,\alpha} - 1)$$

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the action is quartic, we need to introduce the mean-fields with a Hubbard-Stratonovich transformation.

Partition function after a Hubbard-Stratonovich transformation

$$Z = \int D[c, \bar{c}, \chi, \chi^*] e^{-S_{\mathrm{HS}}[c,\bar{c},\chi,\chi^*]}$$

$$S_{\mathrm{HS}}[c, \bar{c}] = \int_0^\beta \mathrm{d}\tau \Bigg[\sum_i \bar{c}_{i,\alpha}(\partial_\tau + \varphi_i) c_{i,\alpha} - \sum_{\langle i,j \rangle} \left(\chi_{i,j} \bar{c}_{j,\alpha} c_{i,\alpha} + \chi^*_{i,j} \bar{c}_{i,\alpha} c_{j,\alpha} - \frac{1}{J} \left| \chi_{i,j} \right|^2 \right) \Bigg]$$

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Integrating out the fermions

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Introducing the mean fields

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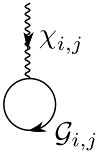
finite temperature Green's function

The saddle point provides the mean field equations



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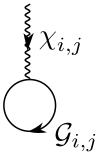

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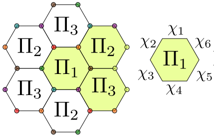
Gauge invariance:

$$\chi_{i,j} \to \chi_{i,j} e^{i(\theta_j - \theta_i)}$$

infinitely many solutions

Solutions on a honeycomb lattice @ T=0



$$\Pi_1 = \chi_1 \cdot \chi_2 \cdot \chi_3 \cdot \chi_4 \cdot \chi_5 \cdot \chi_6 = |\Pi| \, e^i \Phi$$

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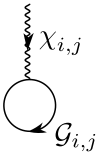

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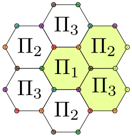
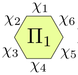
E	Π_1	Π_2	Π_3





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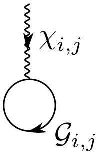

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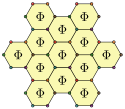
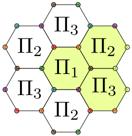
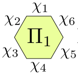
E	Π_1	Π_2	Π_3
-6.148	$-0.159 - 0.276i$	$-0.159 - 0.276i$	$-0.159 - 0.276i$
-6.148	$-0.159 + 0.276i$	$-0.159 + 0.276i$	$-0.159 + 0.276i$







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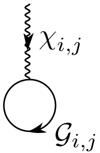

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Solutions on a honeycomb lattice @ T=0

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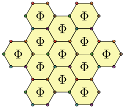

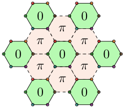

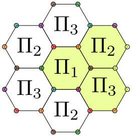

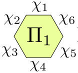

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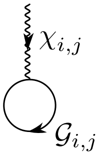

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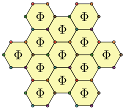
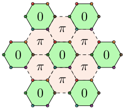
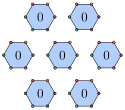
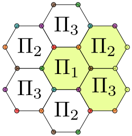
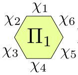
-6	0	1	0
-6	0	0	1











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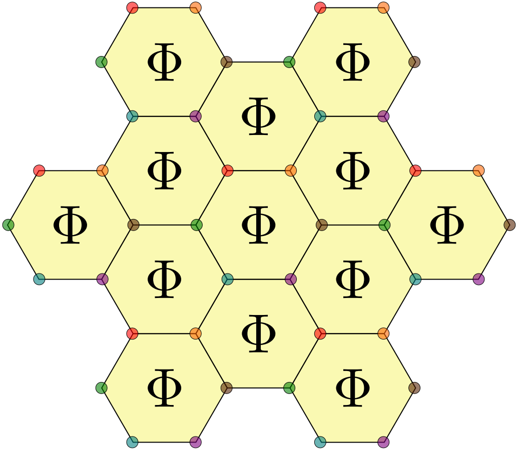

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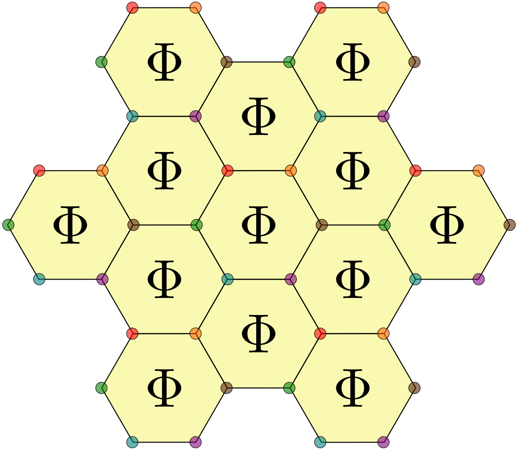

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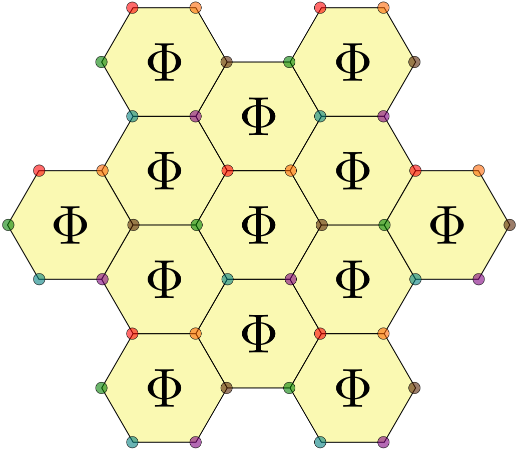

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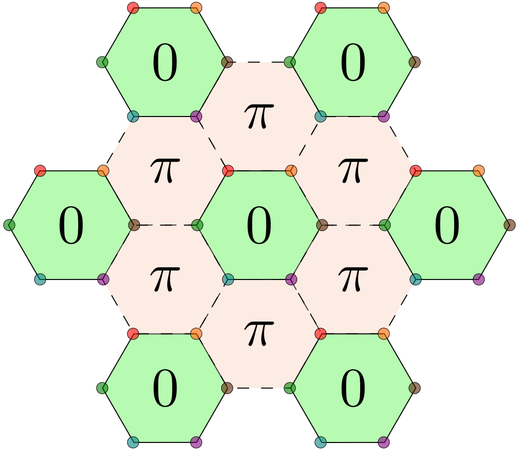

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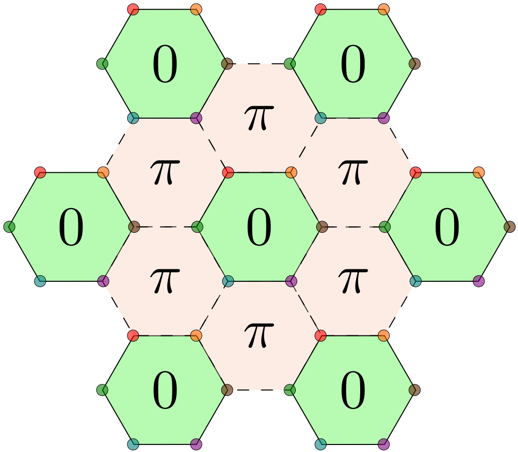

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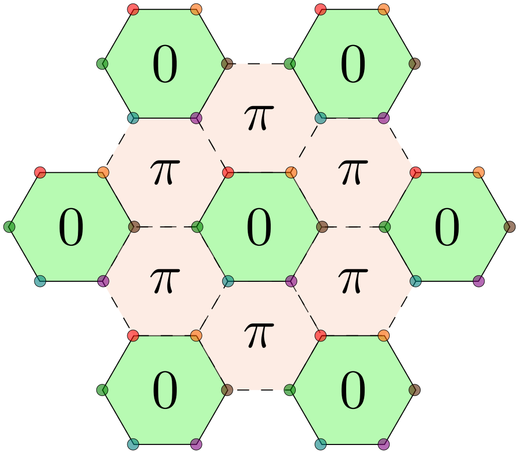

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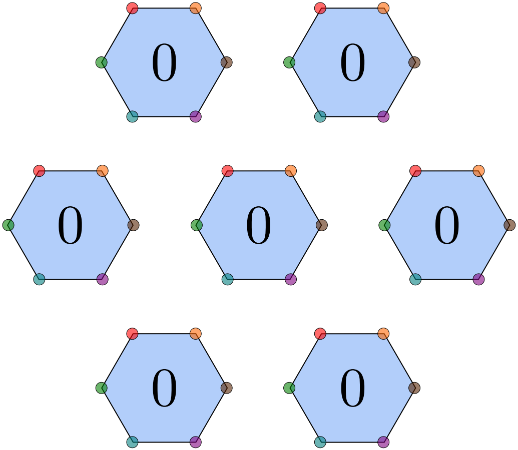

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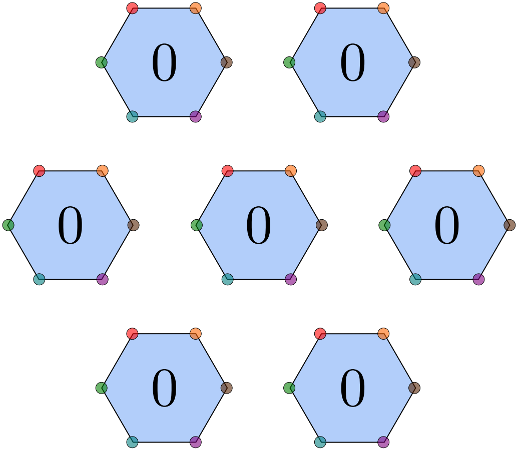

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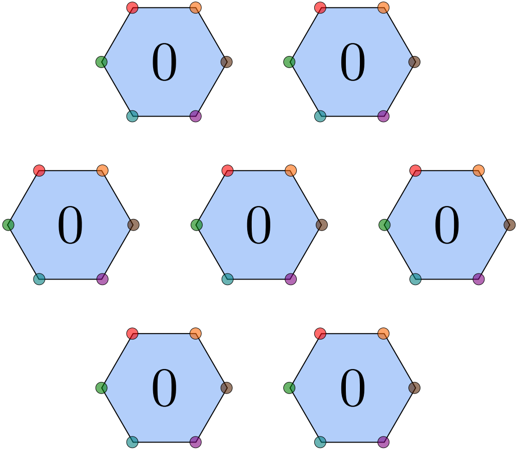

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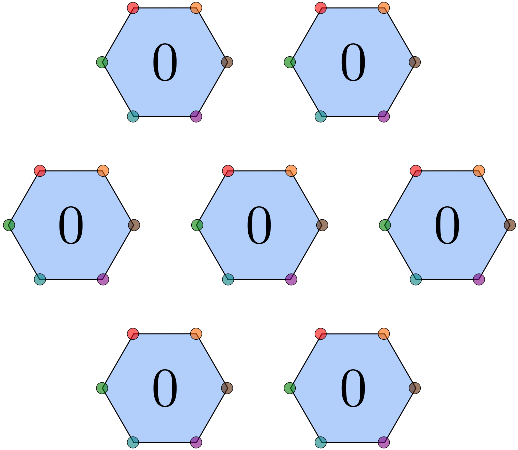

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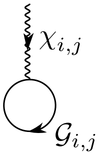

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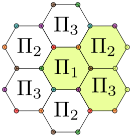

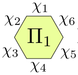

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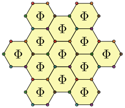

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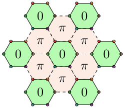

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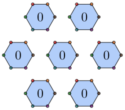

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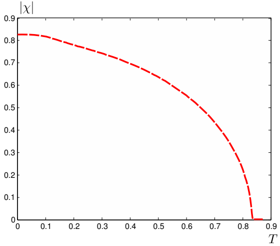
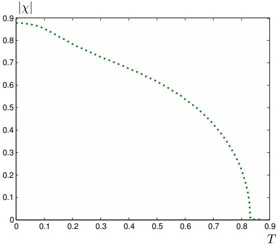
The spin liquid phases at finite temperature

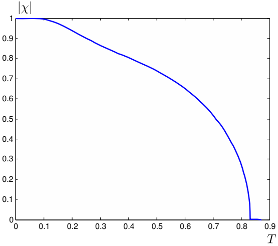
chiral phase



π-flux phase



plaquette phase



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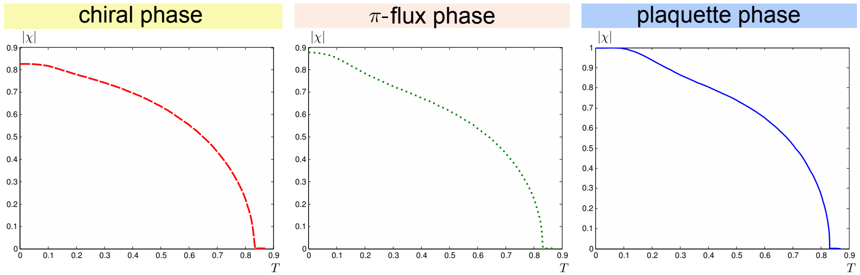
The spin liquid phases at finite temperature



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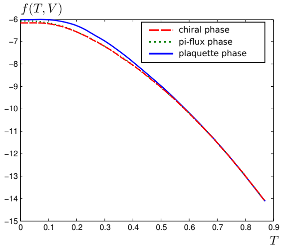
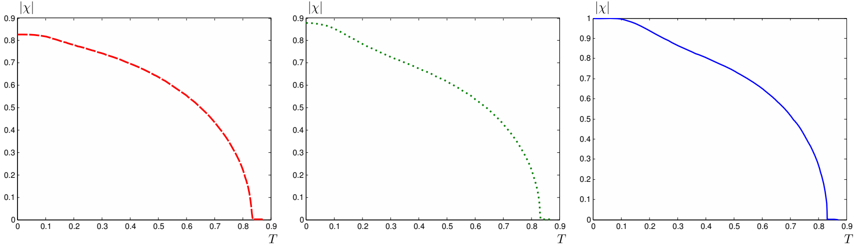
the free energy per plaquette:

$$f(T,V) = -\frac{k_B T}{V}\ \ln Z(\beta)$$



The spin liquid phases at finite temperature

chiral phase | π-flux phase | plaquette phase



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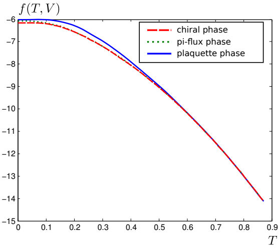
the free energies at mean-field level approach each other without crossing

the chiral spin liquid phase has the lowest free energy also for T>0.



Gaussian approximation of the effective action around the saddle point

$$S_{\mathrm{eff}} = S_0 + \sum_{\langle i,j\rangle,\langle k,l\rangle,n} \begin{bmatrix} \delta\chi^*_{k,l}(i\omega_n) \\ \delta\chi_{k,l}(-i\omega_n) \end{bmatrix} \begin{bmatrix} D_{i,j;k,l}(i\omega_n) & A_{i,j;k,l}(i\omega_n) \\ A^*_{i,j;k,l}(-i\omega_n) & D_{k,l;i,j}(-i\omega_n) \end{bmatrix} \begin{bmatrix} \delta\chi_{i,j}(i\omega_n) \\ \delta\chi^*_{i,j}(-i\omega_n) \end{bmatrix}$$

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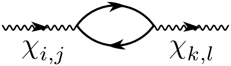
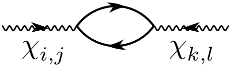


$$D_{i,j;k,l} = \mathrm{tr}\left(\mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\chi_{i,j}}\mathcal{G}\frac{\delta\mathcal{G}^{-1}}{\delta\chi^*_{k,l}}\right)$$



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Beyond the saddle point: stability analysis / fluctuations

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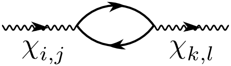

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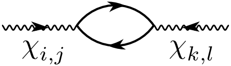

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Beyond the saddle point: stability analysis / fluctuations

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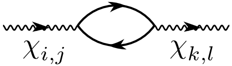

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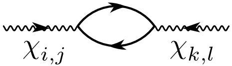

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the excitations also give corrections to the free energy.

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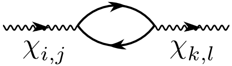

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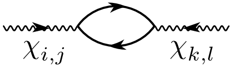

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the excitations also give corrections to the free energy.

difficulties arise due to the constraints and also due to gauge invariance.

Summary

We considered thermodynamic properties of spin-5/2 alkaline-earth-metal fermions in a honeycomb lattice.

At low temperatures the charge degrees of freedom are frozen, and the spin dynamics realizes a chiral spin liquid state with a dynamically generated flux that violates time reversal invariance.

The low energy excitations in an infinite system are gauge bosons of U(1) Chern-Simons field theory.

The higher energy spin liquid states are also interesting generalizations of their square lattice counterparts.

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