BCS-BEC crossover in a quasi-2D Fermi gas

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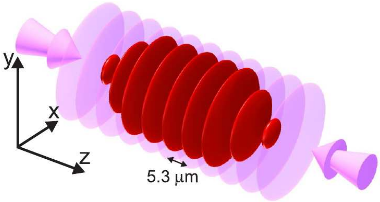
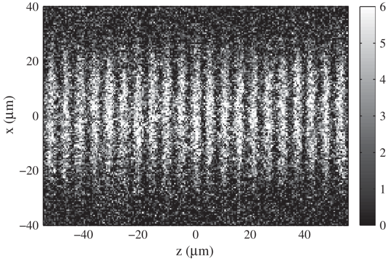
Making a quasi-2D Fermi gas

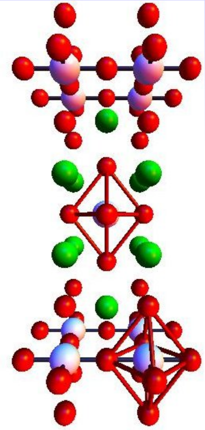
- Strong harmonic confinement in one direction: $\omega_z \gg \omega_x = \omega_y$
- Stacks of pancakes created using 1D optical lattice
- Tune between 2D and 3D via optical lattice depth
- Dimensionality depends on $\frac{\varepsilon_F}{\hbar\omega_z}$, $\frac{\varepsilon_B}{\hbar\omega_z}$, $\frac{k_B T}{\hbar\omega_z}$
- *s*-wave interactions controlled via 3D Feshbach resonance: BCS-BEC





K. Martiyanov, V. Makhalov, A. Turlapov, PRL, 2010

Motivation



La_2CuO_4

Martin Long, Andy Schofield

- 2 is critical number of dimensions for long range order (Mermin-Wagner theorem)
- High T_c superconductors: big mystery of condensed matter physics
- How relevant is coupling between planes to superconducting mechanism?
- Use quasi-2D Fermi gas to model complicated system: **quantum simulation**

Talk Outline

- Overview of previous work
- Explanation of method:
  Non-interacting system, interactions, inclusion of multiple (infinite!) bands
- Analytic approach for two band case
- Relationship between Δ, μ, ε_B, ε_F: departure from 2D behaviour, hints of 3D dimers confined to 2D in extreme BEC limit
- Comparing with experiment:
  Momentum distribution function, quasiparticle dispersions, radio-frequency spectroscopy

Summary of previous work

Experiment

- Massimo Inguscio, *PRA 2003*
- Andrey Turlapov, *PRL 2010*
- Martin Zwierlein, *PRL 2012*
- Chris Vale, *PRL 2011*
- Michael Köhl, *PRL 2011, Nature 2012*
- John Thomas, *PRL 2012*

Theory

- Sascha Zöllner, Georg Bruun, Chris Pethick *PRA 2011*
- Meera Parish, Jesper Levinsen, Stefan Baur, Wave Ngampruetikorn
- Jani-Petri Martikainen, Päivi Törmä, *PRL 2005*
- Dmitry Petrov, Gora Shlyapnikov, *PRA 2001, 2003*

And many more ...

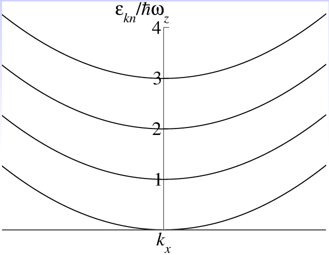
Theoretical system

- $T = 0$
- Model single pancake by oscillator potential, $\frac{1}{2} m\omega_z^2 z^2$
- Population balanced two-component gas
- Attractive contact interaction controlled by broad Feshbach resonance

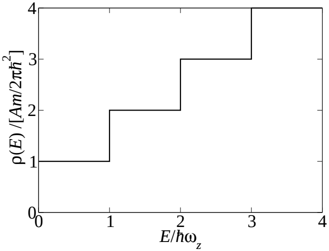
Quantum numbers

2D momentum $\mathbf{k}$
harmonic oscillator number n
hyperfine state $\sigma = \uparrow, \downarrow$

- Single particle energies are $\varepsilon_{\mathbf{k}n} = \frac{\hbar^2 k^2}{2m} + n\hbar\omega_z$



Single particle energies



Single particle density of states

Introducing interactions

- Interaction matrix elements calculated by switching to relative and centre of mass oscillator quantum numbers, ν, N:

$$\langle n_1 n_2 | \hat{g} | n_3 n_4 \rangle = g \sum_N f_\nu \langle n_1 n_2 | N\nu \rangle f_{\nu'} \langle N\nu' | n_3 n_4 \rangle \equiv g \sum_N V_N^{n_1 n_2} V_N^{n_3 n_4},$$

where f_ν is the sum over the Fourier transform of the oscillator wave function, $f_\nu = \sum_{k_z} \tilde{\phi}_\nu(k_z)$, so $f_{2\nu+1} = 0$.

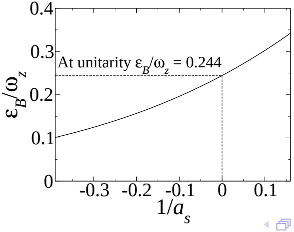
- Coupling constant expressed in terms of binding energy, ε_B

$$-\frac{1}{g} = \sum_{\mathbf{k}, n_1, n_2} \frac{f_{n_1+n_2}^2 |\langle 0 \quad n_1 + n_2 | n_1 n_2 \rangle|^2}{\varepsilon_{\mathbf{k}n_1} + \varepsilon_{\mathbf{k}n_2} + \varepsilon_B}$$

Also have g in terms of 3D scattering length a_s and UV cut-off Λ:

$$\frac{1}{g} = \frac{m}{4\pi}\left(\frac{1}{a_s} - \frac{2\Lambda}{\pi}\right).$$

D.S. Petrov and G. Shlyapnikov, PRA 2001 and I. Bloch, J. Dalibard and W. Zweger, RMP 2008



Mean field approach

- Many body Hamiltonian is

$$\hat{H} = \sum_{\mathbf{k},n,\sigma} (\varepsilon_{\mathbf{k}n} - \mu) c^\dagger_{\mathbf{k}n\sigma} c_{\mathbf{k}n\sigma} + \sum_{\substack{\mathbf{k},n_1,n_2 \\ \mathbf{k}',n_3,n_4 \\ \mathbf{q}}} \langle n_1 n_2 | \hat{g} | n_3 n_4 \rangle c^\dagger_{\mathbf{k}n_1\uparrow} c^\dagger_{\mathbf{q}-\mathbf{k}n_2\downarrow} c_{\mathbf{q}-\mathbf{k}'n_3\downarrow} c_{\mathbf{k}'n_4\uparrow}$$

- Define superfluid order parameter as

$$\Delta_{\mathbf{q}N} = g \sum_{\mathbf{k},n_1,n_2} V_N^{n_1 n_2} \left\langle c_{\mathbf{q}-\mathbf{k}n_2\downarrow} c_{\mathbf{k}n_1\uparrow} \right\rangle$$

- Assume fluctuations are small, so we can use mean field Hamiltonian

$$\begin{aligned} \hat{H}_{\mathrm{MF}} &= \sum_{\mathbf{k},n,\sigma} (\epsilon_{\mathbf{k}n} - \mu) c^\dagger_{\mathbf{k}n\sigma} c_{\mathbf{k}n\sigma} \\ &+ \sum_{\mathbf{q},N} \Bigg(\Delta_{\mathbf{q}N} \sum_{\mathbf{k},n_1,n_2} V_N^{n_1 n_2} c^\dagger_{\mathbf{k}n_1\uparrow} c^\dagger_{\mathbf{q}-\mathbf{k}n_2\downarrow} \\ &+ \Delta^*_{\mathbf{q}N} \sum_{\mathbf{k}',n_3,n_4} V_N^{n_3 n_4} c_{\mathbf{q}-\mathbf{k}'n_3\downarrow} c_{\mathbf{k}'n_4\uparrow} - \frac{|\Delta_{\mathbf{q}N}|^2}{g} \Bigg) \end{aligned}$$

Quasiparticles

- Assume for ground state $\Delta_{\mathbf{q}N} = \delta_{\mathbf{q}0}\delta_{N0}\Delta_0$
- Require $\hat{H}_{\rm MF} = \sum_{\mathbf{k},n,\sigma} E_{\mathbf{k}n}\gamma^\dagger_{\mathbf{k}n\sigma}\gamma_{\mathbf{k}n\sigma} + E_g$, where $\gamma^\dagger_{\mathbf{k}n\sigma}(\gamma_{\mathbf{k}n\sigma})$ are quasiparticle creation (annihilation) operators:

$$\gamma^\dagger_{\mathbf{k}n\uparrow} = \sum_{n'}(u_{\mathbf{k}n'n}c^\dagger_{\mathbf{k}n'\uparrow} + v_{\mathbf{k}n'n}c_{-\mathbf{k}n'\downarrow}), \quad \gamma_{-\mathbf{k}n\downarrow} = \sum_{n'}(u_{\mathbf{k}n'n}c_{-\mathbf{k}n'\downarrow} - v_{\mathbf{k}n'n}c^\dagger_{\mathbf{k}n'\uparrow})$$

- Can show $E_g = \sum_{\mathbf{k},n}(\epsilon_{\mathbf{k}n} - \mu - E_{\mathbf{k}n}) - \frac{{\Delta_0}^2}{g}$ and derive BdG equations

$$\begin{pmatrix} \epsilon-\mu & \Delta \\ -\Delta^* & -(\epsilon-\mu) \end{pmatrix}\begin{pmatrix} U \\ V \end{pmatrix} = \begin{pmatrix} U \\ V \end{pmatrix} E$$

- For a fixed μ, find Δ_0 by minimising E_g. μ is chosen to fix the particle density, $\rho = 2\sum_{\mathbf{k},n',n}|v_{\mathbf{k}n'n}|^2$, at the value for an ideal gas.
- Much faster than diagonalising BdG equations. Can do calculations for up to 100 bands and then **extrapolate to infinitely many bands**, since Δ_0, μ scale linearly with the inverse number of bands.

Analytic results

- For the pure 2D system, Mohit Randeria *et al.* (PRL 1989 and PRB 1990), solved the mean field equations analytically

$$\Delta \equiv \Delta V_0^{00} = \sqrt{2\varepsilon_B \varepsilon_F}, \qquad \mu = \varepsilon_F - \frac{\varepsilon_B}{2}$$

- For two band case, relevant for very small ε_B/ω_z, ε_F/ω_z, there is **no inter-band pairing**, so equations simplified:

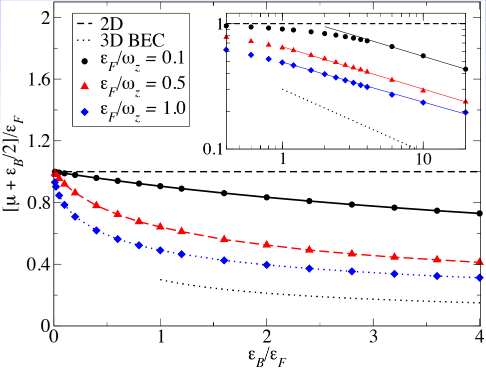
$$E_{\mathbf{k}n} = \sqrt{(\varepsilon_{\mathbf{k}n} - \mu)^2 + (V_0^{nn}\Delta_0)^2}, \; -\frac{1}{g} = \sum_{\mathbf{k},n} \frac{(V_0^{nn})^2}{2\sqrt{(\varepsilon_{\mathbf{k}n}-\mu)^2+(V_0^{nn}\Delta_0)^2}}$$

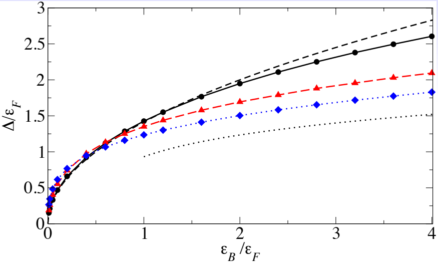
- Can thus derive simultaneous equations in μ, Δ

$$\left(\frac{\varepsilon_B}{-\mu+\sqrt{\Delta^2+\mu^2}}\right)^4 \left(\frac{\varepsilon_B+2\omega_z}{\omega_z-\mu+\sqrt{\Delta^2/4+(\omega_z-\mu)^2}}\right) = 1$$

$$2\varepsilon_F = \sqrt{\Delta^2 + \mu^2} + \sqrt{\Delta^2/4 + (\omega_z - \mu)^2} + 2\mu - \omega_z$$

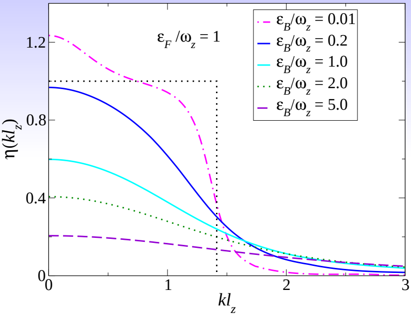
Numerical results





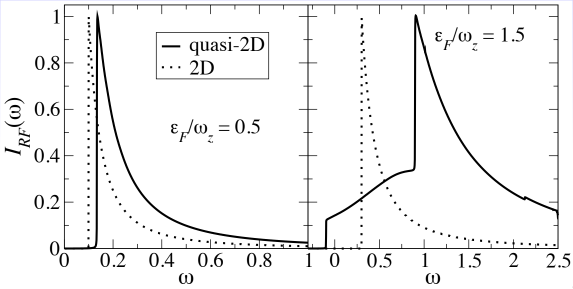
- As ε_B/ω_z, ε_F/ω_z increase, departure from 2D curves also generally increases
- Can obtain first order correction to 2D results in BCS regime ($\varepsilon_B/\varepsilon_F \ll 1$) from 2 band equations:
$\frac{\mu}{\varepsilon_F} \simeq 1 - \frac{\varepsilon_B}{2\varepsilon_F}\left(1 + \frac{\varepsilon_F}{4\omega_z}\right)$ $\qquad$ $\frac{\Delta}{\varepsilon_F} \simeq \sqrt{\frac{2\varepsilon_B}{\varepsilon_F}}\left(1 + \frac{\varepsilon_F}{8\omega_z}\right)$,

Momentum distribution function



- Momentum distribution function can be observed in time of flight experiments
- Calculated (per spin) according to $\eta(\mathbf{k}) = \sum_{n,n'} |v_{\mathbf{k}nn'}|^2$
- In BCS regime get a kink at low momentum: presence of higher bands felt because of interactions

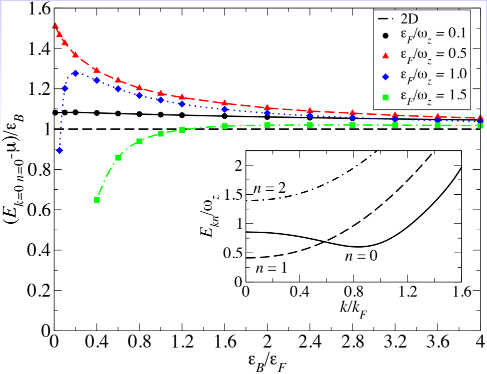
Radio frequency spectra

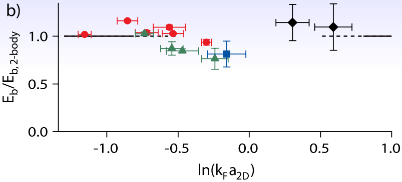


$\varepsilon_B/\varepsilon_F = 0.2$

- RF pulse takes atoms from initial occupied state to final unoccupied state (Groups of Michael Köhl, John Thomas, Martin Zwierlein)
- $I_{RF}(\omega) \propto \sum_{k,n',n} |v_{kn'n}|^2 \delta(\epsilon_{kn'} - \mu + E_{kn} - \omega)$
- For perfect 2D, $I_{RF}(\omega) \propto \frac{\Delta^2}{\omega^2}\Theta(\omega - \varepsilon_B)$
- Get more complicated structure in the BCS regime for higher ε_F/ω_z values, ill-defined pairing threshold, possible relevance to John Thomas experiment

Excitation gap





A.T. Sommer *et al.*, PRL 2012

- Lowest measurable energy gap corresponds to $E_{k=0\ n=0} - \mu$
- This is exactly equal to ε_B in 2D
- For smaller ε_F/ω_z see an enhancement in $(E_{k=0\ n=0} - \mu)/\varepsilon_B$, but for larger values there is a reduction due to level repulsion

Conclusions and Outlook

- Have studied pairing of quasi-2D Fermi gas throughout BCS-BEC crossover
- All harmonic bands of confining potential taken into account
- Proper renormalisation of contact interaction
- Radio frequency spectroscopy drastically deviating from 2D case for high enough ε_F/ω_z - possible explanation for John Thomas experiment?
- Next: include finite temperature

Thank you for listening!

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