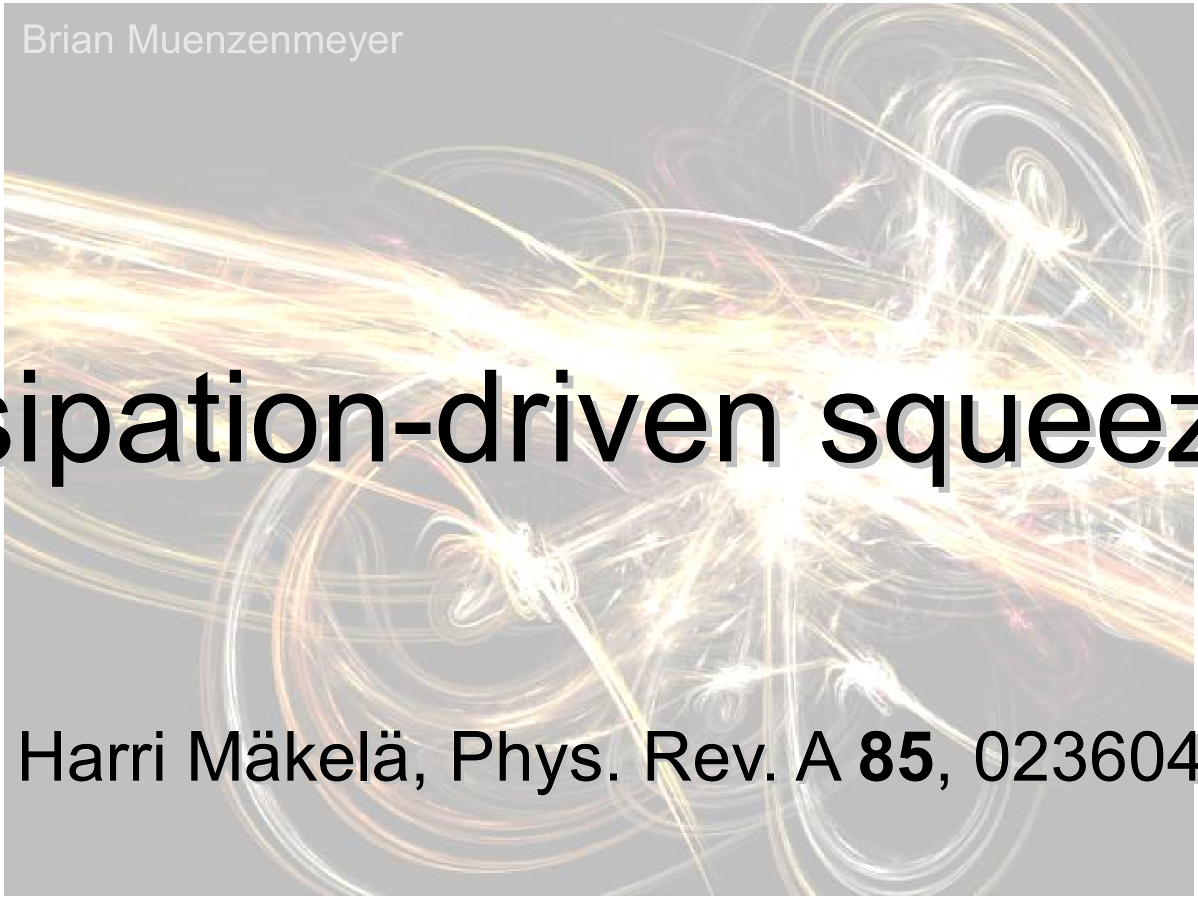


Dissipation-driven squeezing

GW & Harri Mäkelä, Phys. Rev. A **85**, 023604 (2012).

Gentaro Watanabe

(APCTP, RIKEN)

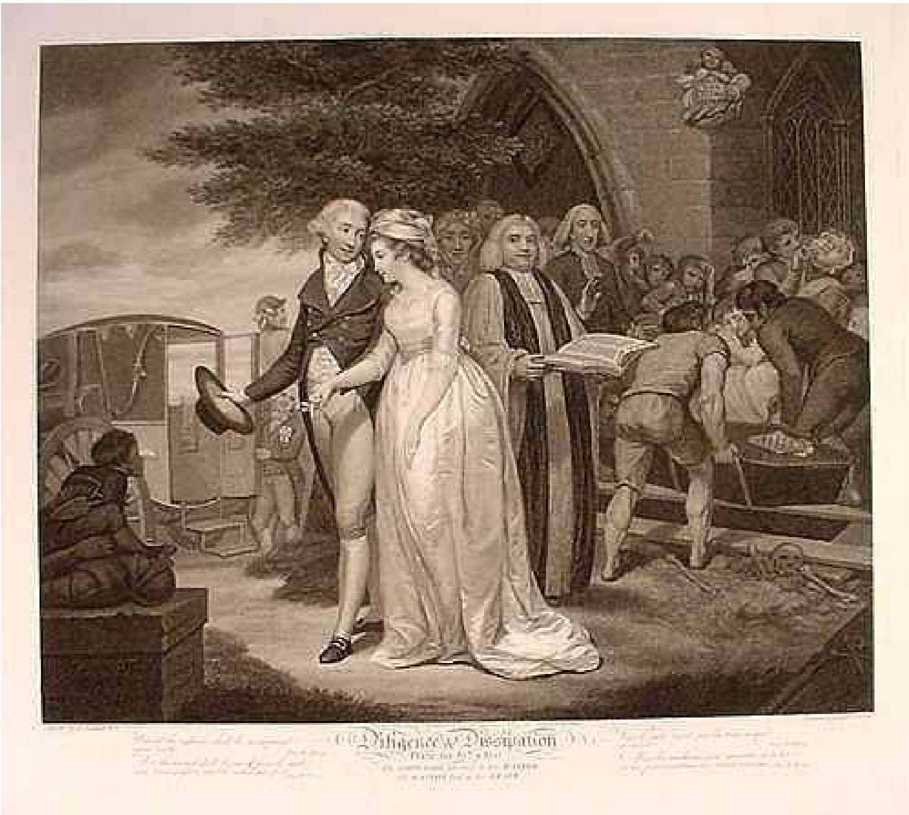
Contents

Quantum state engineering using dissipation

What is dissipation?

Dissipation (*Longman Advanced American Dictionary*)

1. the process of making something disappear or scatter
2. the act of wasting money, time, energy etc.
3. the enjoyment of physical pleasures that are harmful to your health



"Diligence and Dissipation" by Northcote

Dissipation: caused by coupling with environment

Dissipation in cold atom gases

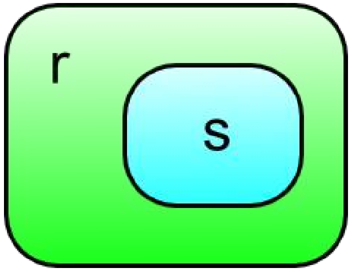
Dissipation: caused by coupling with environment

- Particle losses (1-, 2-, & 3-body losses)
- Incoherent scattering of trap lasers etc.

Usually, "dissipation" $\approx$ "decoherence"

But, "dissipation" $\neq$ "decoherence"

Quantum sys. coupled to a reservoir.



(total) = (system) + (reservoir)

ρ_{sr} : total density operator (system + reservoir)

$\rho \equiv \rho_s = \mathrm{Tr}_r[\rho_{sr}]$: system density operator

What we need is the system density operator ρ.

Master equation: EOM for the system density op. within Born-Markov approx.

$$\frac{d\rho}{dt} = -i[H,\rho] + \underbrace{\boxed{\frac{\gamma}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)}}_{\text{Dissipation}}$$

c : jump op.

γ : dissipation rate

Born-Markov approx.

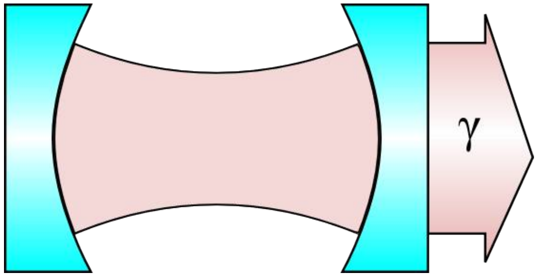
- Weak system-bath coupling (Born)
  coupling term << *H* or reservoir
- Short correlation time of the reservoir (Markov)
  << dynamical timescale of sys
  Reservoir has no memory.

Poor man's derivation of master eq. (1)

ex. Photons in a lossy cavity

γ: loss rate

a: annihilation op. of photons



Prob. of a photon escaping from cavity in δt.

$$\Delta P = \gamma \langle \Psi | a^\dagger a | \Psi \rangle \delta t$$

$$|\Psi\rangle \xrightarrow{\delta t} \begin{cases} |\Psi_{\mathrm{emit}}\rangle & \Delta P \\ |\Psi_{\mathrm{no\ emit}}\rangle & 1 - \Delta P \end{cases}$$

$$|\Psi_{\mathrm{emit}}\rangle = \frac{a|\Psi\rangle}{\langle \Psi | a^\dagger a | \Psi \rangle^{1/2}} = (\gamma \delta t / \Delta P)^{1/2} a |\Psi\rangle$$

$$|\Psi_{\mathrm{no\ emit}}\rangle = \frac{e^{-iH_{\mathrm{eff}}\delta t}|\Psi\rangle}{\langle \Psi | e^{iH_{\mathrm{eff}}^\dagger \delta t} e^{-iH_{\mathrm{eff}}\delta t} | \Psi \rangle^{1/2}} \simeq \frac{(1 - iH\delta t - \frac{\gamma}{2}\delta t a^\dagger a)|\Psi\rangle}{(1-\Delta P)^{1/2}}$$

$$H_{\mathrm{eff}} = H - i\frac{\gamma}{2} a^\dagger a$$

Measurement of "no emission". ➡ non-Hermitian

Poor man's derivation of master eq. (2)

Density operator at $t + \delta t$

$$\rho(t+\delta t) = \Delta P |\Psi_{\rm emit}\rangle\langle\Psi_{\rm emit}| + (1-\Delta P)|\Psi_{\rm no\ emit}\rangle\langle\Psi_{\rm no\ emit}|$$

$$\simeq |\Psi\rangle\langle\Psi| - i\delta t\ [H, |\Psi\rangle\langle\Psi|]$$

$$+\frac{\gamma}{2}\delta t\left(2a|\Psi\rangle\langle\Psi|a^\dagger - a^\dagger a|\Psi\rangle\langle\Psi| - |\Psi\rangle\langle\Psi|a^\dagger a\right)$$

Since $|\Psi\rangle\langle\Psi| = \rho(t)$

$$\Rightarrow \quad \frac{d\rho}{dt} = -i[H,\rho] + \frac{\gamma}{2}\left(\boxed{2a\rho a^\dagger}\boxed{- a^\dagger a\rho - \rho a^\dagger a}\right)$$

Quantum jump by emission

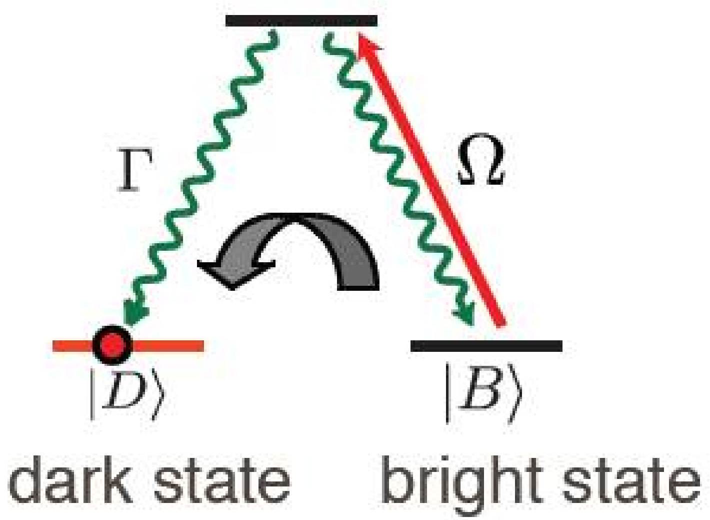
Damping by non-unitary evolution.

Jump operator = a

State preparation using dissipation (1)

Usually, "dissipation" ≈ "decoherence"

But, "dissipation" ≠ "decoherence"



(presentation by P. Zoller)

Analogy to optical pumping

$$\rho(t) \xrightarrow[t \to \infty]{} |D\rangle\langle D|$$

Source of Γ: Dissipation in cold atom gases

- By spontaneous emission of photons
  ⇐ coupling with vac. st.
- By emission of Bogoliubov excitations
  ⇐ coupling with background BEC

Dissipation can be used for preparation of pure states.

[Agarwal (1988), Aspect *et al*. (1988), Kasevich & Chu (1992), Diehl *et al*. (2008), Kraus *et al*. (2008)]

State preparation using dissipation (2)

Kraus *et al*. PRA **78**, 042307 (2008).

Master eq.

$$\frac{d\rho}{dt} = \mathcal{L}(\rho) = -i[H,\rho] + \sum_i \frac{\gamma_i}{2}(2c_i\rho c_i^\dagger - c_i^\dagger c_i \rho - \rho c_i^\dagger c_i)$$

1. $H|\Psi\rangle = E|\Psi\rangle$
2. $\forall i \quad c_i|\Psi\rangle = 0$: dark st.
3. Uniqueness
   $|\Psi\rangle$ is the only st.;
   $\mathcal{L}(|\Psi\rangle\langle\Psi|) = 0$

target st.

$\Rightarrow \quad \rho \xrightarrow[t\to\infty]{} |\Psi\rangle\langle\Psi|$

Task: Construct a parent Liouvillian

Condition 3. $\Rightarrow$ c_i must be non-Hermitian.

$\because$ If c_i is Hermitian $\Rightarrow$ $\dot\rho = -i[H,\rho] + \sum_i \frac{\gamma_i}{2}\left[[c_i,\rho],c_i\right]$

$\therefore \quad \rho \propto I$ is also a steady st.; $\mathcal{L}(I) = 0$

State preparation using dissipation (3)

State preparation using dissipation!

Advantages

Works for any initial state.

No need of dynamical manipulations.

"Just wait."

Dissipation-induced coherence

Example: Pumped single-mode field with loss

[G. S. Agarwal, J. Opt. Soc. Am. B, 5, 1940 (1988)]

Pumping Hamiltonian:

$$H = \frac{g}{2}(a + a^\dagger)$$

Master eq.:

$$\frac{d\rho}{dt} = -i[H,\rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a\rho - \rho a^\dagger a)$$

$$= \frac{\gamma}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c) \equiv \mathcal{D}[c]\rho$$

with $c \equiv a + ig/\gamma$

$\frac{d\rho}{dt} = \mathcal{D}[c]\rho = 0$ if & only if $(a + ig/\gamma)\rho = 0 = \rho(a + ig/\gamma)^\dagger$

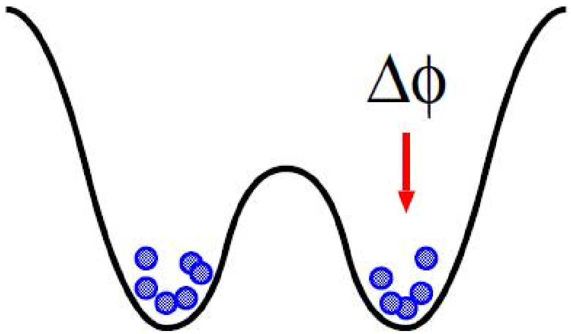
$\Rightarrow$ $\rho = |-ig/\gamma\rangle\langle -ig/\gamma|$: coherent st.

"Dissipation-induced coherence"

Matter-wave interf. and 2-mode sys.

Matter-wave interferometer : interferometer with cold atoms

Ex. cold Bose gases in a double-well pot.



Release from the trap.

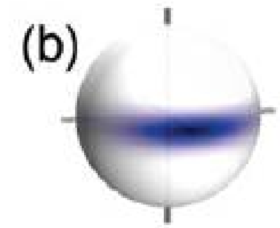
Expand & interfere

Measure the force field, etc.
from the interference pattern.

Squeezed st. in matter-wave interf.

Uncertainty relation $\Delta X_1 \Delta X_2 \gtrsim \hbar$

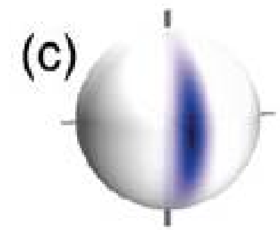
Squeezing: decreasing uncertainty of one variable
(in expense of increasing the other's)

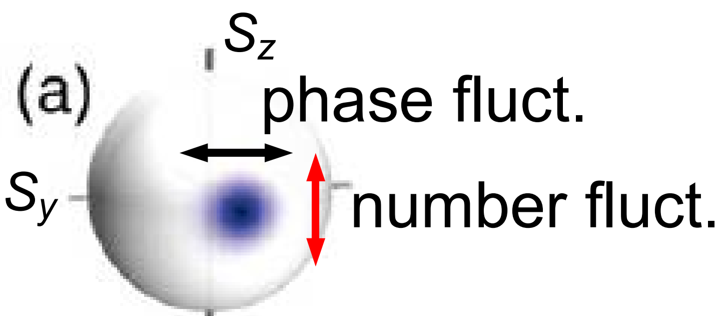


Two-mode case

Number squeezing $\xi_N \equiv \langle \Delta S_z^2 \rangle^{1/2} / \sqrt{N/2} < 1$

Phase squeezing $\xi_{\mathrm{phase}} \equiv \langle \Delta S_y^2 \rangle^{1/2} / \sqrt{N/2} < 1$





Bloch sphere

$$S_x = (a_1^\dagger a_2 + a_2^\dagger a_1)/2$$
$$S_y = (a_1^\dagger a_2 - a_2^\dagger a_1)/2i$$
$$S_z = (a_1^\dagger a_1 - a_2^\dagger a_2)/2$$

Phase sq. st.: High visibility of interf. pattern.
Good for readout.

(Number sq. st.: Small phase diffusion.
Longer phase accum. time.)

GW & Mäkelä, PRA **85**, 023604 (2012).

A method to create number & phase sq. st. in any 2-mode Bose sys. using dissipation.

Proposal of the physical setup

Create number/phase squeezing/anti-squeezing in a controllable manner.

Extension to optical lattices

- New phase characterized by non-zero cond. fraction and thermal-like particle statistics.

Two-mode Bose systems

Squeezing jump operator

Coherent st.: $|\phi\rangle \propto (e^{i\phi/2}a_1^\dagger + e^{-i\phi/2}a_2^\dagger)^N|0\rangle$

Squeezing jump op.

$$c = \boxed{(a_1^\dagger + a_2^\dagger)(a_1 - a_2)} + \epsilon\boxed{(a_1^\dagger - a_2^\dagger)(a_1 + a_2)} \quad (-1 < \epsilon < 1)$$

$|\phi = 0\rangle \propto (a_1^\dagger + a_2^\dagger)^N|0\rangle$
is a dark st.
$\because \ [a_1 - a_2, a_1^\dagger + a_2^\dagger] = 0$

$|\phi = \pi\rangle \propto (a_1^\dagger - a_2^\dagger)^N|0\rangle$
is a dark st.
$\because \ [a_1 + a_2, a_1^\dagger - a_2^\dagger] = 0$

$$c = 2(1+\epsilon)S_z - 2i(1-\epsilon)S_y$$

when ε =1, $c = 4S_z \propto \Delta N$

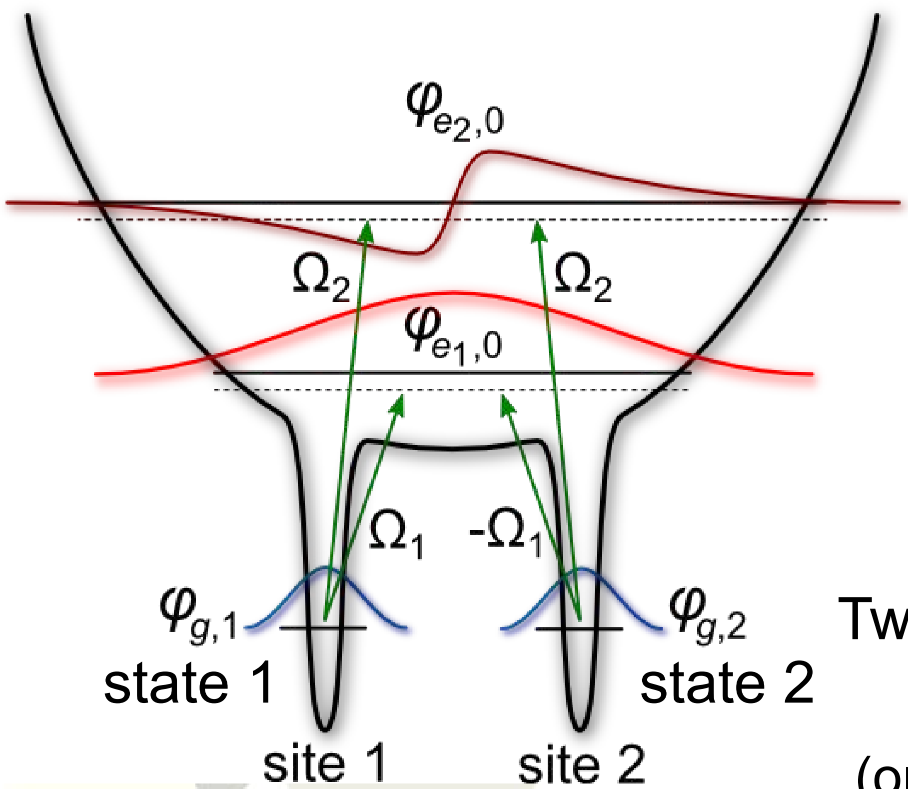
Dark st. is a Fock st. with ΔN=0.

Physical realization (preliminary)

Setup: trapped atoms + background BEC

system — reservoir of s.f. phonons

- States 1& 2 are Raman coupled to 2 excited states with even & odd parity.
- Atoms in the excited states decay into states 1 & 2 through s.f. phonon emission.



$$c = \underbrace{\boxed{(a_1^\dagger + a_2^\dagger)(a_1 - a_2)}}_{c_1} + \epsilon \underbrace{\boxed{(a_1^\dagger - a_2^\dagger)(a_1 + a_2)}}_{c_2}$$

→ c_2

→ c_1

$$\epsilon = -\frac{\Omega_2}{\Omega_1}\frac{\Delta_1}{\Delta_2}\frac{\phi_{e_2,0}(x_0)}{\phi_{e_1,0}(x_0)}$$

Ω_i : effective Rabi freq.

Δ_i : detuning

Two excited states are adiabatically eliminated.

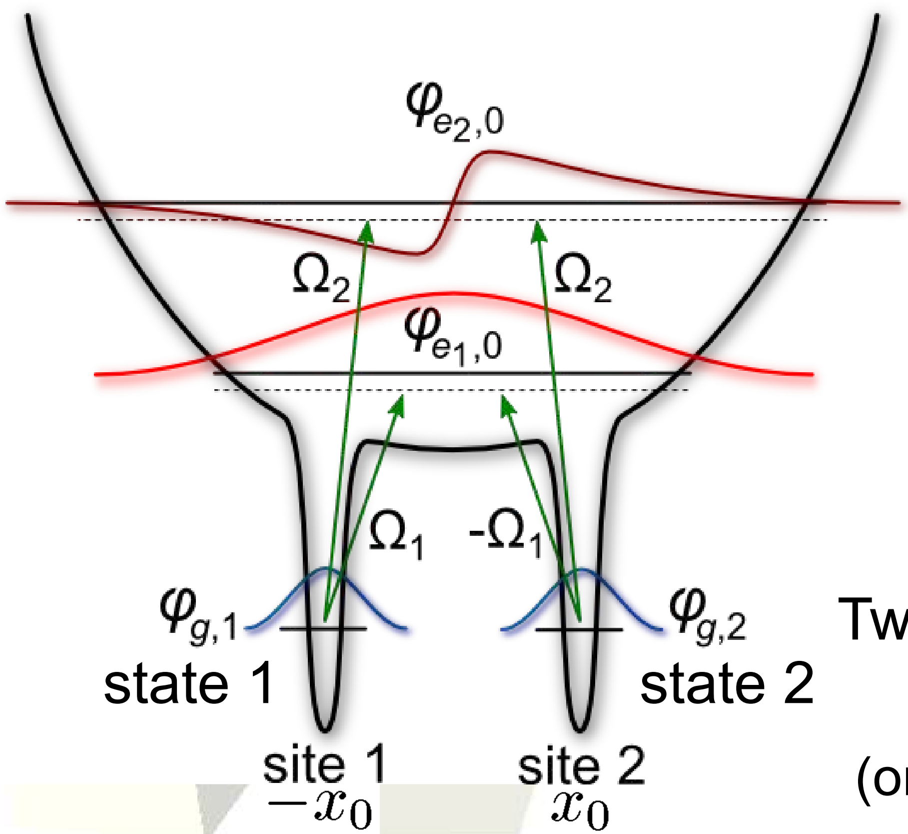
(on-going project with Caballar, Mäkelä, and Diehl)

Physical realization (preliminary)

Conditions

- For the coherence btwn site 1 & 2: $k_n x_0 \ll 1$
- For the coherence btwn C_1 & C_2: $|k_2 - k_1| \ll k_1, k_2$

$$\Rightarrow \quad x_0 \ll 1/k_n \ll a_{\text{trap}}$$



$$c = \underbrace{\boxed{(a_1^\dagger + a_2^\dagger)(a_1 - a_2)}}_{c_1} + \epsilon \underbrace{\boxed{(a_1^\dagger - a_2^\dagger)(a_1 + a_2)}}_{c_2}$$

$\Rightarrow c_2$

$\Rightarrow c_1$

$$\epsilon = -\frac{\Omega_2}{\Omega_1}\frac{\Delta_1}{\Delta_2}\frac{\phi_{e_2,0}(x_0)}{\phi_{e_1,0}(x_0)}$$

Ω_i : effective Rabi freq.
Δ_i : detuning

Two excited states are adiabatically eliminated.

(on-going project with Caballar, Mäkelä, and Diehl)

Coherent st. analysis (valid for N>>1)

$$\langle S_x^2\rangle \simeq \langle S_x\rangle^2 \quad \text{with} \quad \langle S_x\rangle \simeq N/2 + O(N^0)$$

$$\frac{d}{dt}\langle S_{y,z}^2\rangle = \mathrm{Tr}\left[\dot\rho S_{y,z}^2\right]$$

master eq. ➡ $$\frac{d}{dt}\langle S_{y,z}^2\rangle \simeq -4N\gamma(1-\epsilon^2)\langle S_{y,z}^2\rangle + N^2\gamma(1\pm\epsilon)^2$$

In the steady state

Number squeezing param.: $$\xi_N = \frac{\langle S_z^2\rangle^{1/2}}{\sqrt{N/2}} \simeq \sqrt{\frac{1-\epsilon}{1+\epsilon}}$$

Phase squeezing param.: $$\xi_{\mathrm{phase}} = \frac{\langle S_y^2\rangle^{1/2}}{\sqrt{N/2}} \simeq \sqrt{\frac{1+\epsilon}{1-\epsilon}}$$

ε > 0 ⇒ number sq. & phase anti-sq.

ε < 0 ⇒ phase sq. & number anti-sq.

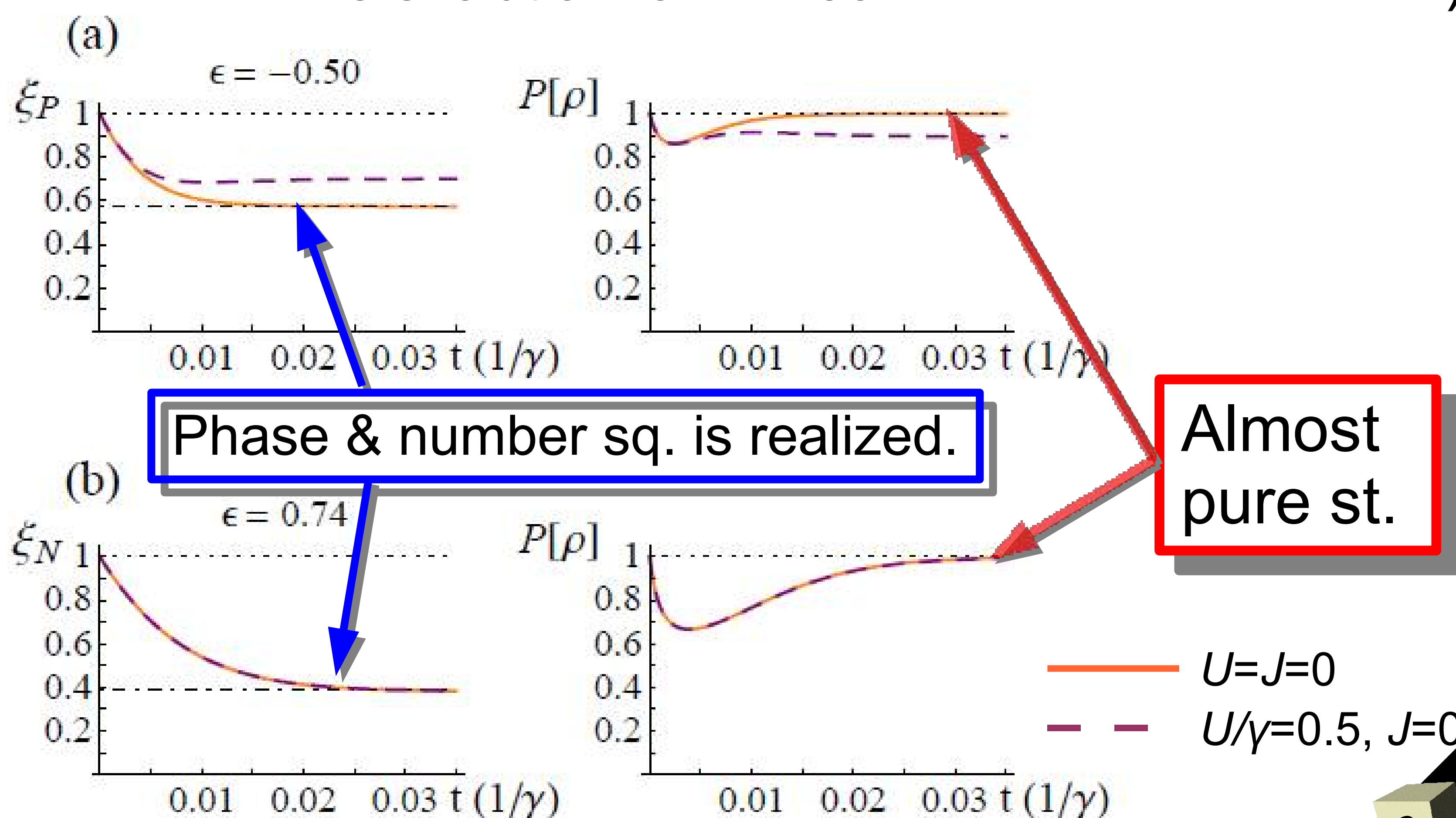
Squeezing parameter and purity $P[\rho]$

purity $P[\rho] \equiv \{(N+1)\mathrm{Tr}[\rho^2] - 1\}/N = \begin{cases} 1 & \text{(pure st.)} \\ 0 & \text{(completely mixed st.)} \end{cases}$

Time evolution for N=100

$\varepsilon < 0$ Phase squeezing

$\varepsilon > 0$ Number squeezing



Phase & number sq. is realized.

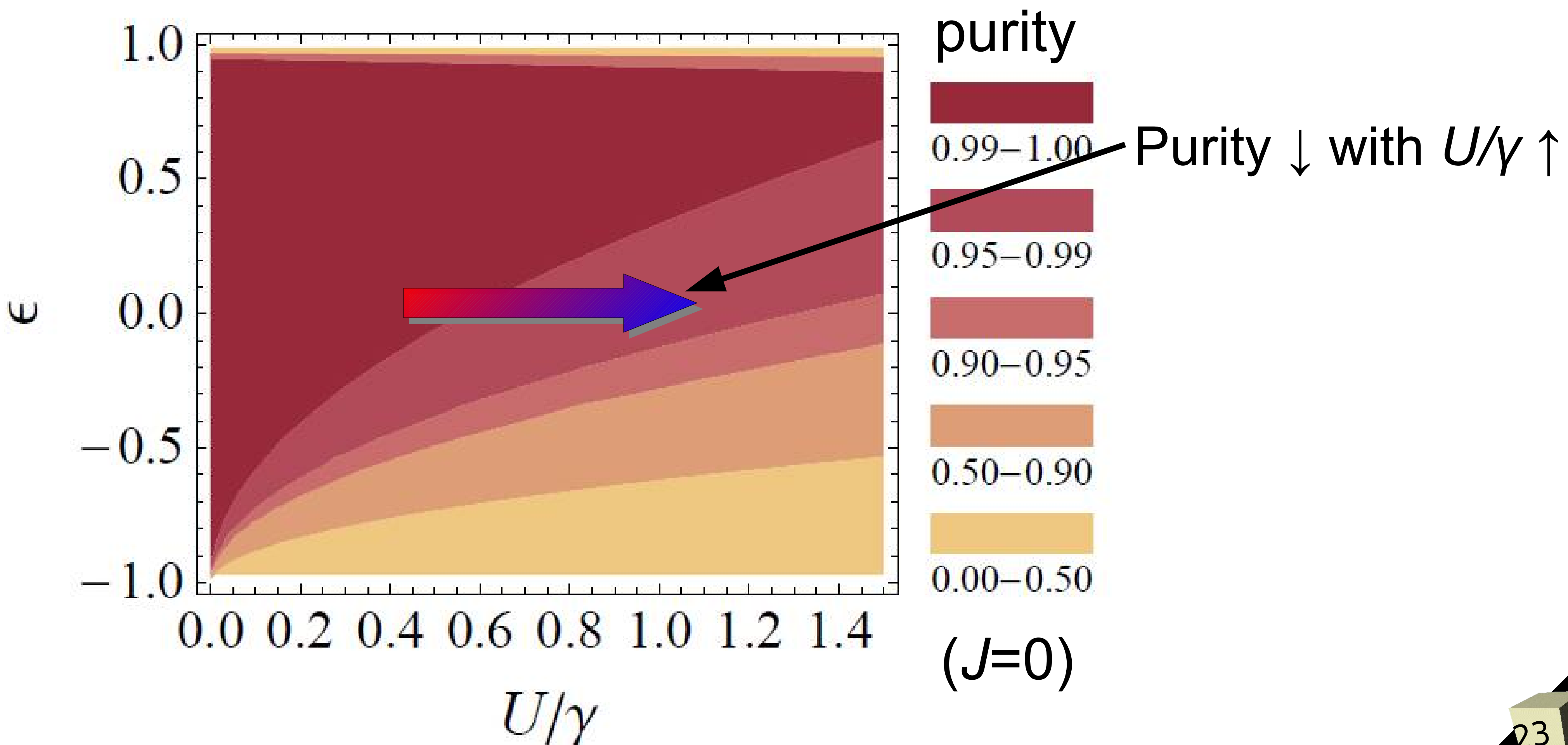
Almost pure st.

Purity of steady st. with BH Hamiltonian

2-site BH Hamiltonian: $H = -2JS_x + US_z^2$

S_x : eigenst. are coherent st.
S_z : eigenst. are Fock st.

Nonzero-U reduces purity.



(J=0)

Purity of steady st. with BH Hamiltonian

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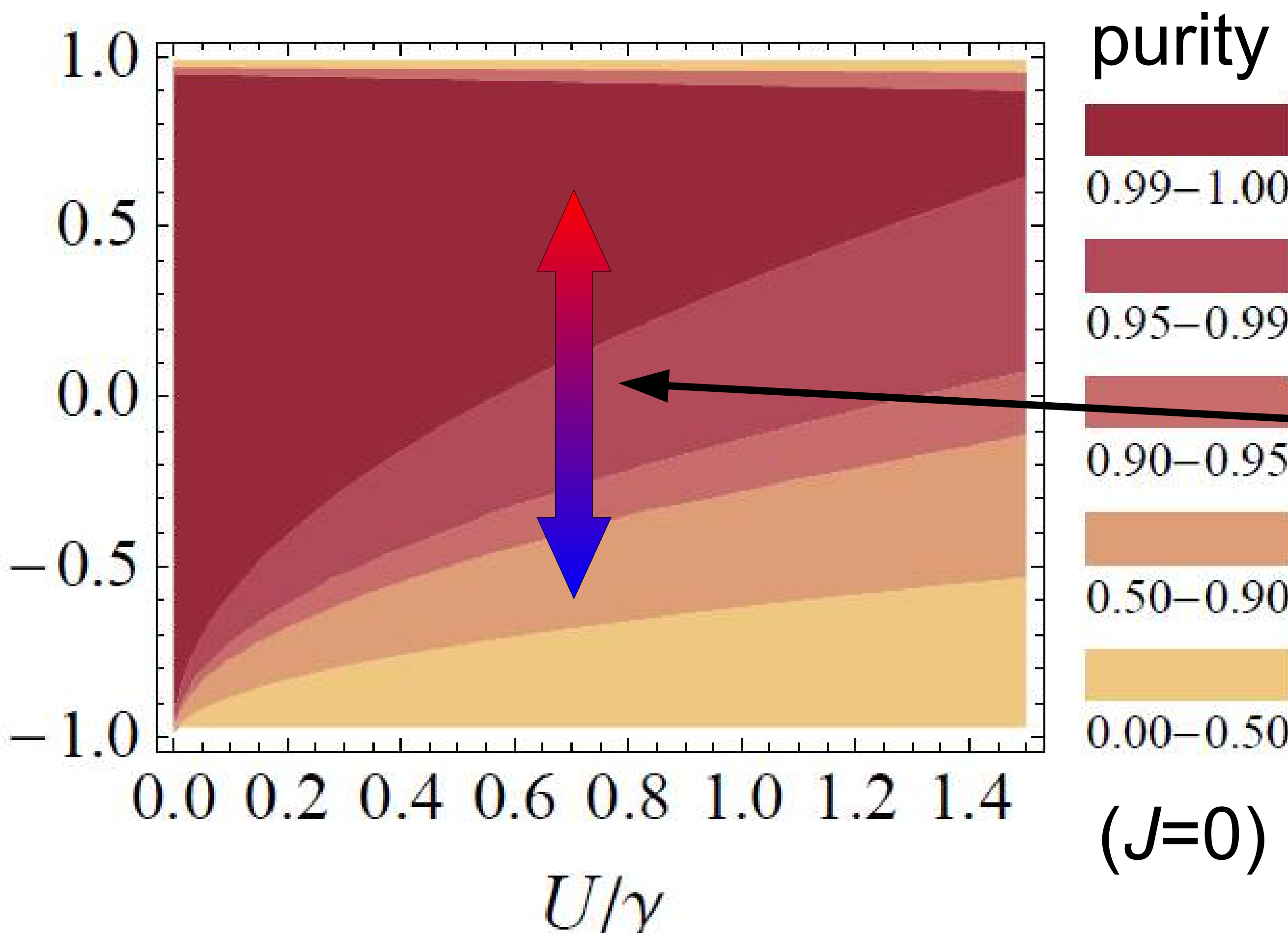
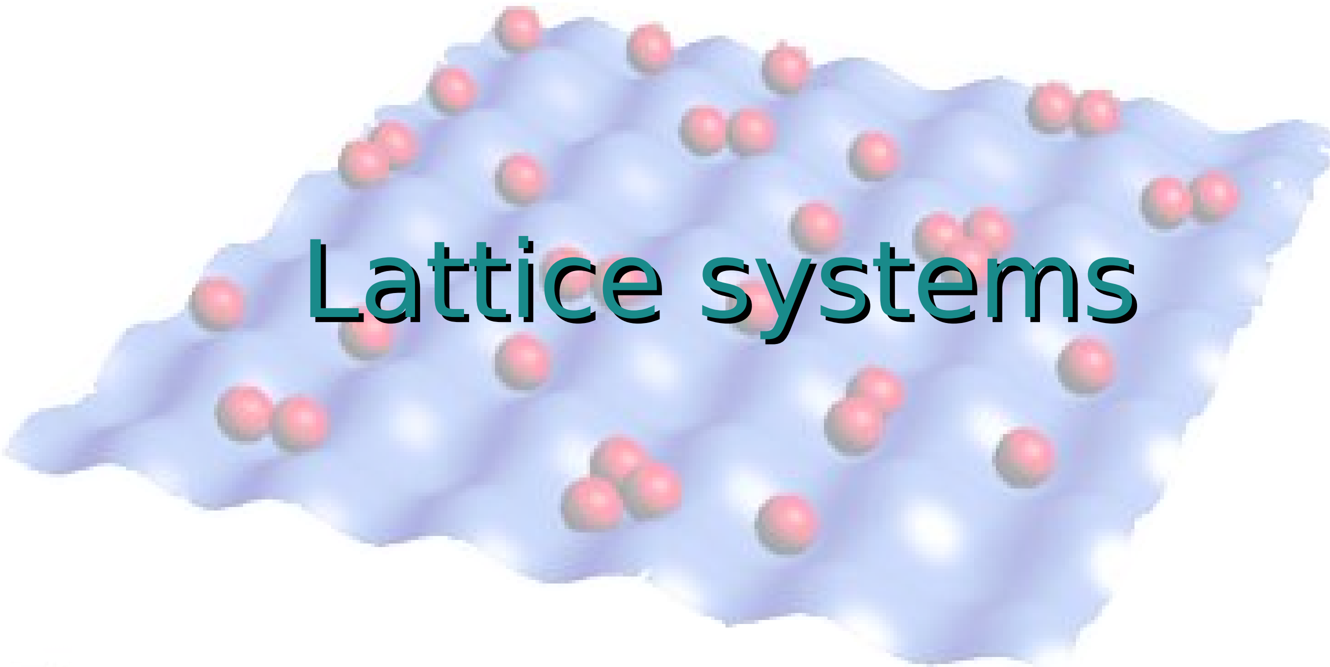


Purity ↓ with U/γ ↑

Purity is higher for $\varepsilon > 0$ (number sq.)

(J=0)

Lattice systems



Squeezing jump operator on lattices

Jump op. acting on site *i* and *j*.

$$c_{ij} = (a_i^\dagger + a_j^\dagger)(a_i - a_j) + \epsilon(a_i^\dagger - a_j^\dagger)(a_i + a_j)$$

Master eq.

$$\partial_t \rho = -i[H,\rho] + \boxed{\frac{\gamma}{2}\sum_{\langle i,j\rangle}(2c_{ij}\rho c_{ij}^\dagger - c_{ij}^\dagger c_{ij}\rho - \rho c_{ij}^\dagger c_{ij})}$$

Drive the system into a squeezed st.

BH Hamiltonian

$$H = \boxed{-J\sum_{\langle i,j\rangle}(a_i^\dagger a_j + a_j^\dagger a_i)} + \boxed{\frac{U}{2}\sum_i n_i(n_i-1)} - \mu\sum_i n_i$$

Delocalizes atoms.

Builds coherence.

Localizes atoms.

Destroys coherence.

Competition btw. **dissipative** term and **interaction** term.

Generalized MF Gutzwiller approach

Solve the master eq. within
generalized MF Gutzwiller approx.

[Diehl et al., PRL **105**, 015702 (2010)]

Product ansatz

$$\rho = \bigotimes_i \rho_i \quad \text{with} \quad \rho_i \equiv \mathrm{Tr}_{\neq i}[\rho]$$

(reduced density op. for site *i*)

Site-decoupling MF approx.

$$H = \sum_i h_i$$

with $h_i = -J \sum_{\langle i' | i \rangle} (\langle a_{i'} \rangle a_i^\dagger + \langle a_{i'}^\dagger \rangle a_i) + \frac{U}{2} n_i (n_i - 1) - \mu n_i$

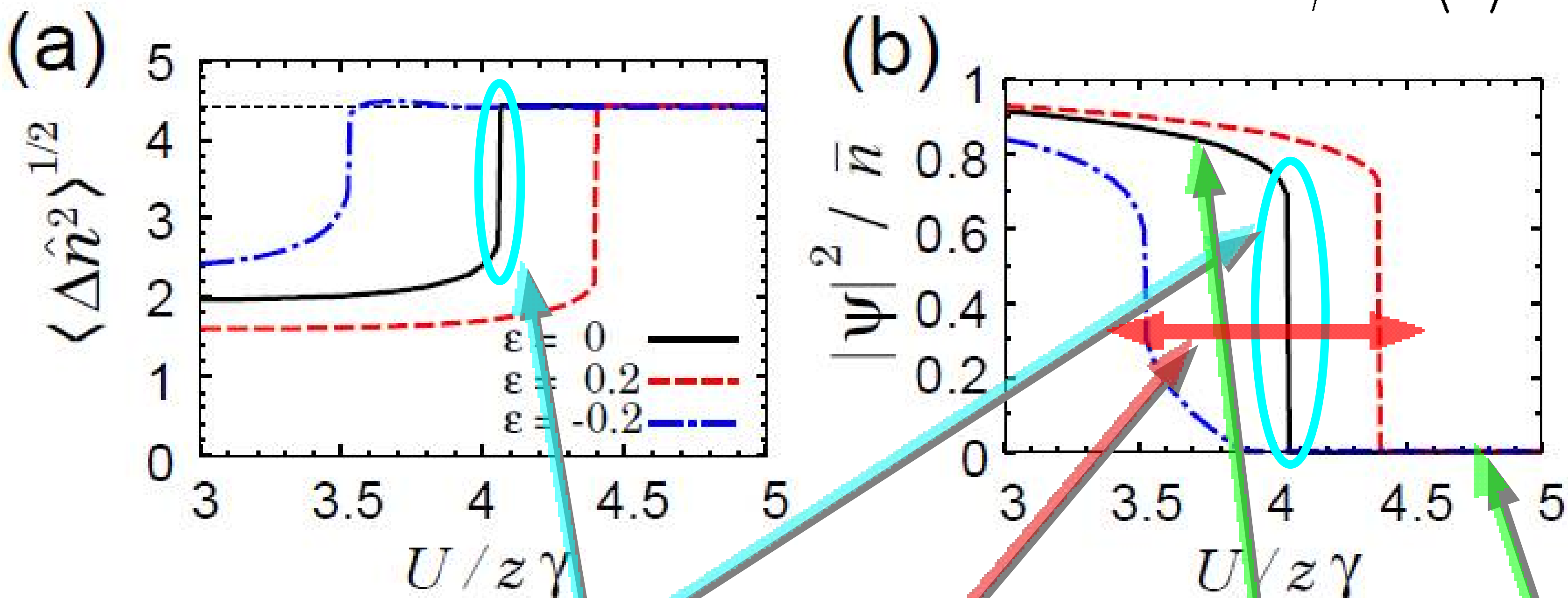
Good approx. for local properties
in higher dimensions & at larger filling factors.

Results: number fluct. & cond. frac.

<u>Number fluctuation & condensate fraction of steady st.</u>

filling factor $n = 4$ $J/\gamma = 1$ z: coordination #

$\psi \equiv \langle a \rangle$



First order-like transition

cond. phase

non-cond. phase

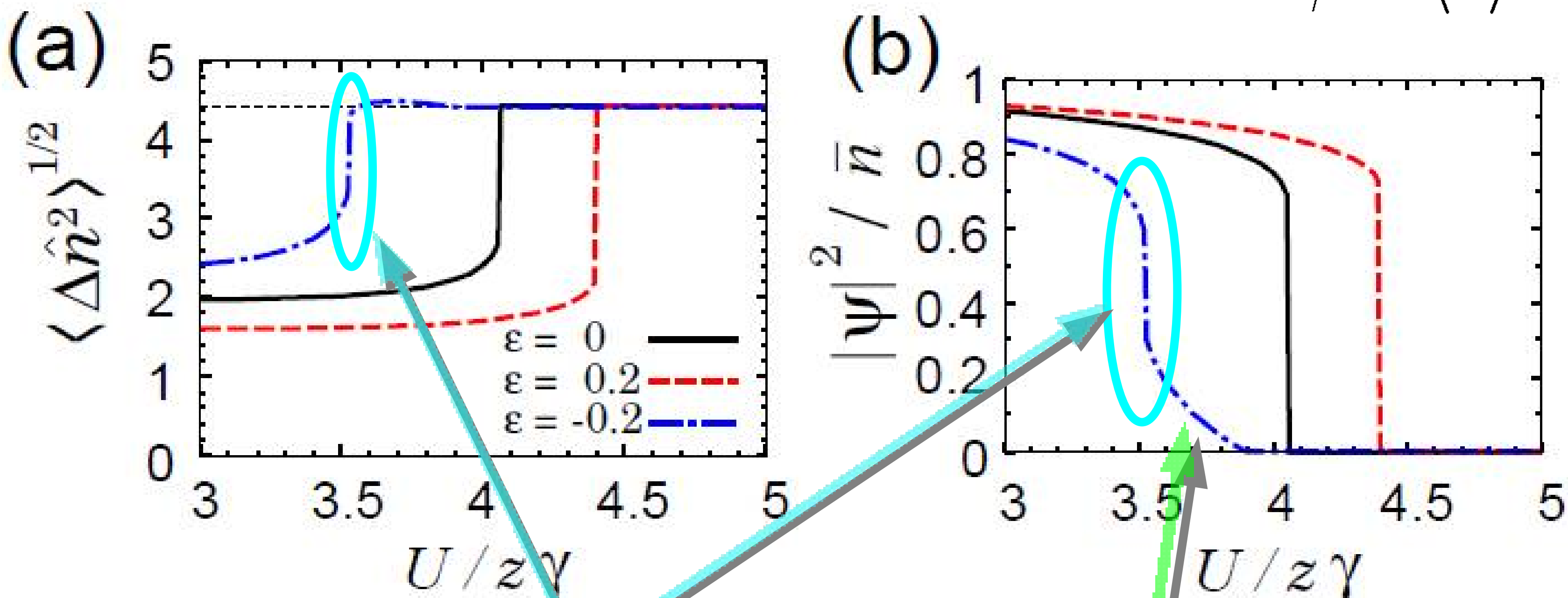
Parameter ε control the phase boundary.

$\varepsilon > 0$: expand the cond. phase

Number fluctuation & condensate fraction of steady st.

filling factor $n = 4$ $J/\gamma = 1$ z: coordination #

$\psi \equiv \langle a \rangle$



First order-like transition

Non-zero cond. fraction

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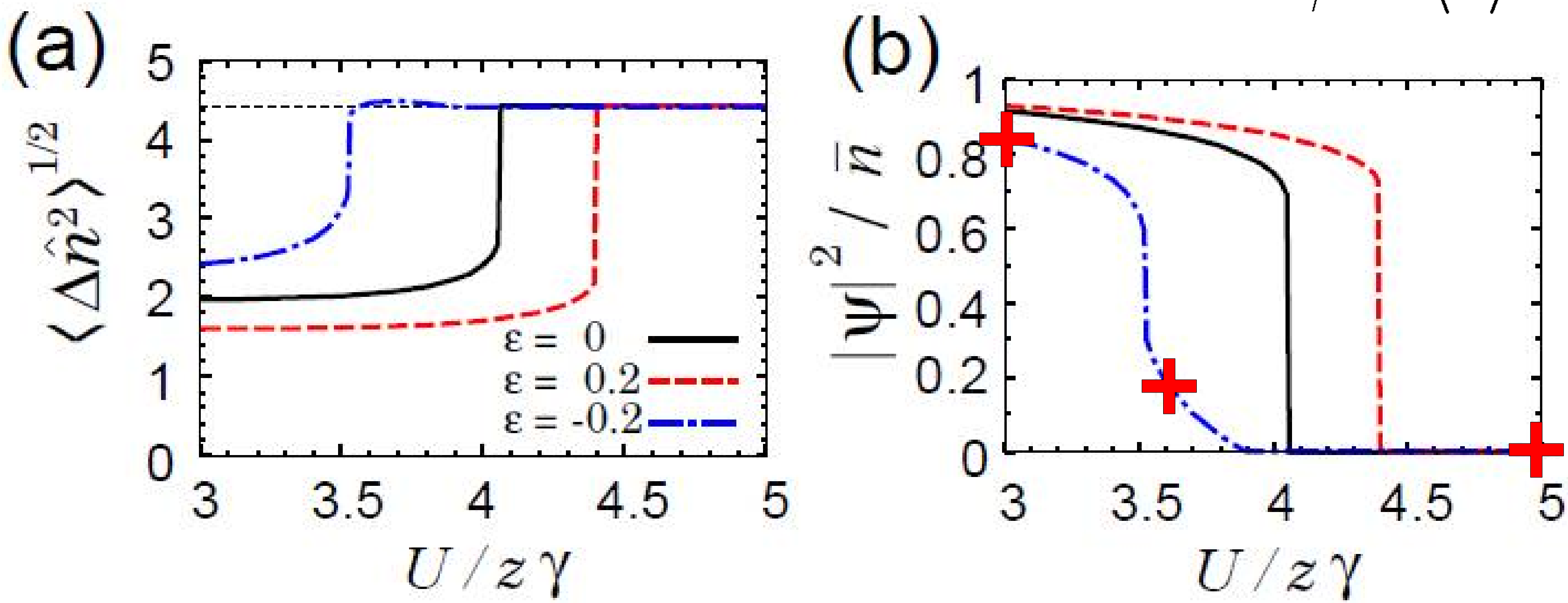
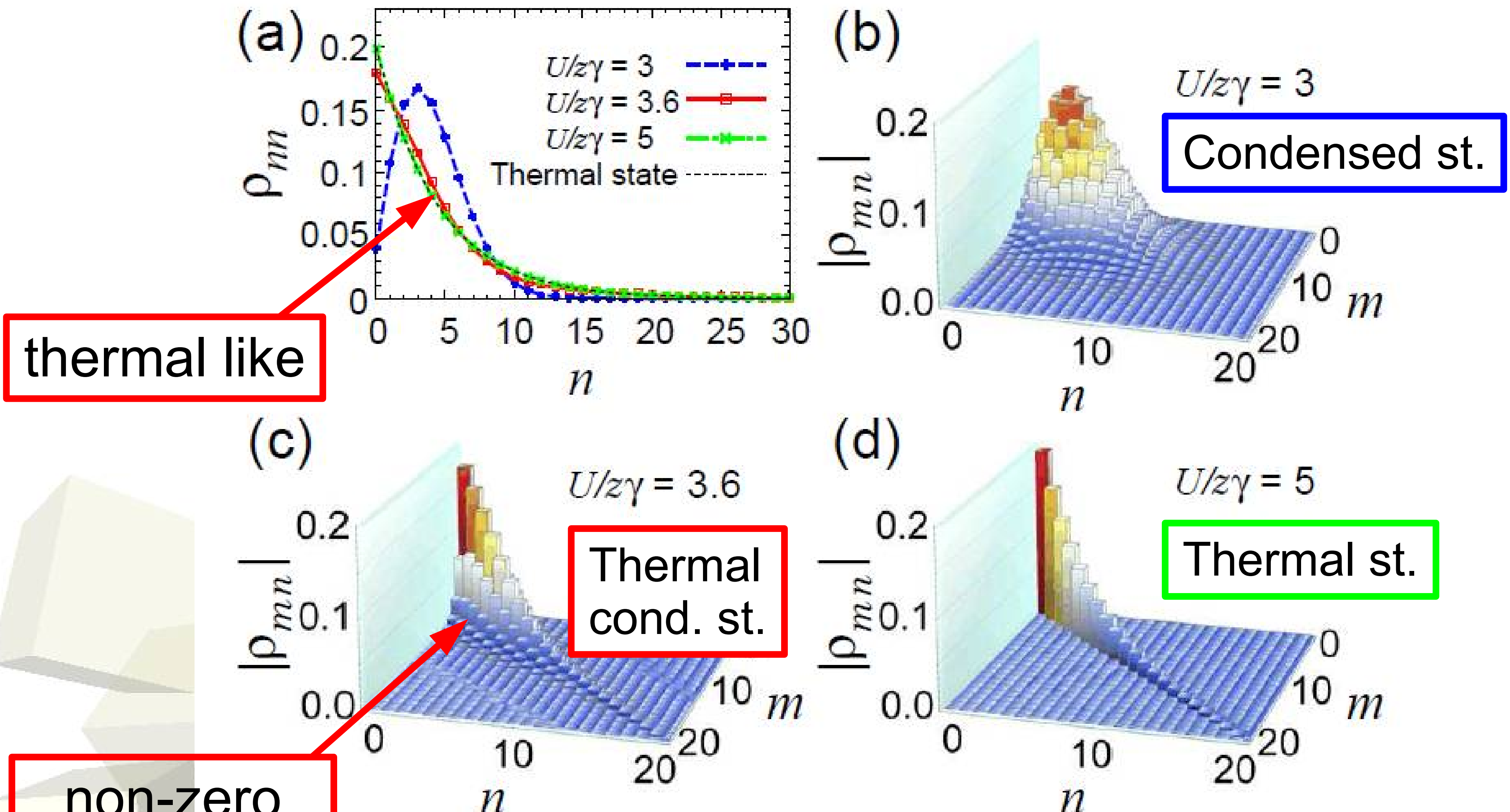


Results: density matrices

Particle number statistics & density matrix

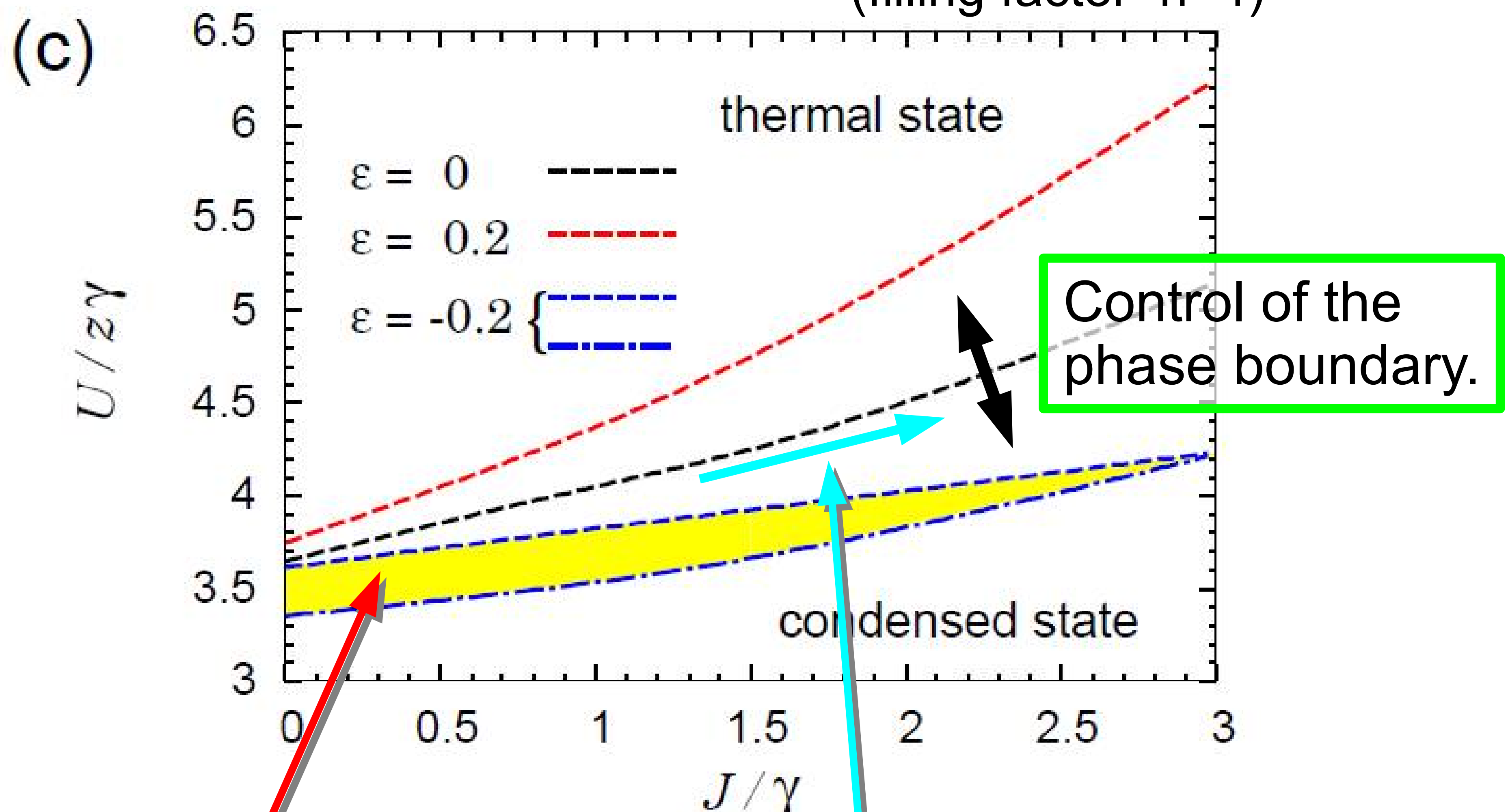


- Thermal-like particle number statistics.
- Non-zero off-diagonal matrix elements.

Results: steady state phase diagram

Non-equilibrium steady st. phase diagram

(filling factor n=4)



Thermal condensed phase

Bigger cond. st. region for large *J*.
(*J*-term builds the coherence.)

Summary & conclusion

GW & Mäkelä, PRA 85, 023604 (2012).

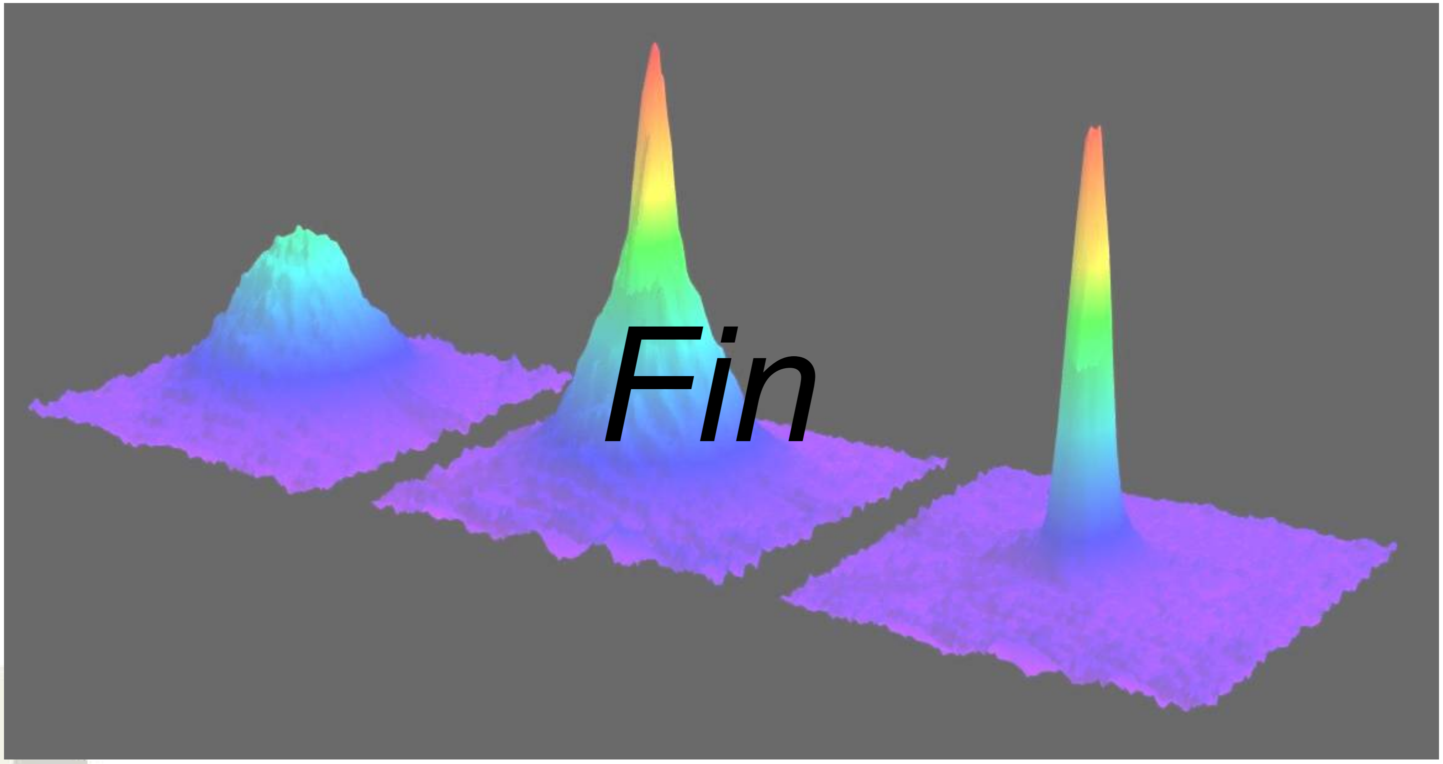
We found a method to create number & phase sq. st. in any 2-mode Bose sys. using dissipation.

- Physical setup to realize the squeezing jump op.

- Create number/phase squeezing/anti-squeezing st. in a controllable manner.

- Extension to optical lattices gives control of the phase boundaries in steady-st. phase diagram.

- "Thermal condensed phase":
  A new phase characterized by non-zero cond. fraction
  & thermal-like particle statistics.



Fin

Thank you for your attention.

Master eq. based on the measurement theory (1)

A measurement is done in the time interval (t, $t+T$).

Measurement op. $M_\alpha(T)$ — state after measurement $\propto M_\alpha(T)|\psi\rangle$

α: result of the measurement related to the measurement basis $|\alpha\rangle$.

Completeness: $$\sum_\alpha M_\alpha^\dagger(T) M_\alpha(T) = 1$$

Prob. of result α

$$P_\alpha = \mathrm{Tr}[M_\alpha(T)\rho(t)M_\alpha^\dagger(T)] \equiv \mathrm{Tr}[\tilde{\rho}_\alpha(t+T)]$$

State after the measurement conditioned by α

$$\rho_\alpha(t+T) = \tilde{\rho}_\alpha(t+T)/P_\alpha$$

Non-selective evolution

$$\rho(t+T) = \sum_\alpha P_\alpha \rho_\alpha(t+T) = \sum_\alpha M_\alpha(T)\rho(t)M_\alpha^\dagger(T)$$

Master eq. based on the measurement theory (2)

Continuous measurement: $T \rightarrow dt$

Set of measurement op. (photons in a lossy cavity)

Emission: $M_1(dt) = \sqrt{\gamma dt}\, a$ Prob. of escape.

$$\Delta P = \gamma \langle \Psi | a^\dagger a | \Psi \rangle \delta t$$

No emission: $M_0(dt) = 1 - \left(iH + \boxed{\frac{\gamma}{2} a^\dagger a} \right) dt$

Completeness

Non-selective evolution:

$$\rho(t+dt) = M_0(dt)\rho(t)M_0^\dagger(dt) + M_1(dt)\rho(t)M_1^\dagger(dt)$$

$$\frac{d}{dt}\rho(t) = -i[H,\rho] + \frac{\gamma}{2}(2a\rho a^\dagger - a^\dagger a \rho - \rho a^\dagger a)$$

System-reservoir coupling

System-reservoir coupling: Atoms & Bogoliubov excit.

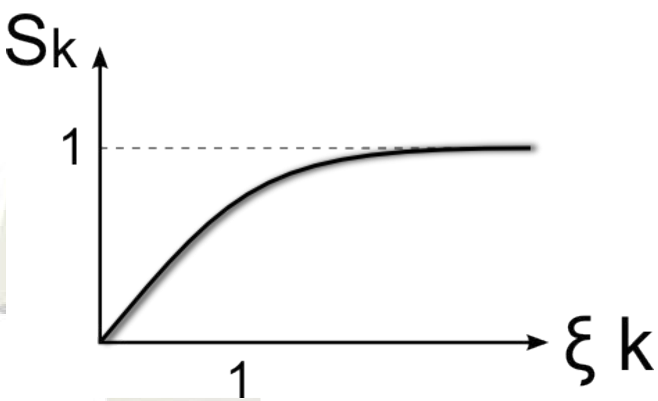
$$\hat{H}_{ab} = \frac{1}{2}\frac{4\pi a_{ab}}{\mu}\int d^3r\ \hat{\psi}_a^\dagger\hat{\psi}_a\hat{\psi}_b^\dagger\hat{\psi}_b \simeq \sum_{\mathbf{k}\neq 0} g_k(\hat{A}_{\mathbf{k}}^\dagger\hat{b}_{\mathbf{k}} + \mathrm{h.c})$$

$\hat{\psi}_a$: trapped atoms

$\hat{\psi}_b = \sqrt{\rho_b} + \delta\hat{\psi}_b(\mathbf{r})$: background s.f. atoms

$$\delta\hat{\psi}_b(\mathbf{r}) = \frac{1}{\sqrt{V}}\sum_{\mathbf{k}}(u_{\mathbf{k}}\hat{b}_{\mathbf{k}}e^{i\mathbf{k}\cdot\mathbf{r}} + v_{\mathbf{k}}\hat{b}_{\mathbf{k}}^\dagger e^{-i\mathbf{k}\cdot\mathbf{r}})$$

$\hat{b}_{\mathbf{k}}$: Bogoliubov excitations



$$g_k \propto S_k^{1/2} = \sqrt{\frac{k^2}{2m_b E_{\mathbf{k}}}}$$: static structure factor of BEC

Exact form of the steady state

$$c = 4\sqrt{\epsilon}\, e^{\chi S_x} S_z e^{-\chi S_x}$$

$$\chi \equiv \operatorname{arctanh}\left[\frac{1-\epsilon}{1+\epsilon}\right]$$

For even *N*

Eigenst. of *c* with zero eigenvalue

$$\phi_{\rm sq} \propto e^{\chi S_x} |N/2\rangle$$

$$= \sum_{n=-N/2}^{N/2} \alpha_n |N/2 - n\rangle$$

$$\alpha_n = \binom{N}{N/2}^{1/2} \binom{N}{N/2+n}^{-1/2} \left(\frac{1+\sqrt{\epsilon}}{2\epsilon^{1/4}}\right)^N \sum_{s=|n|}^{N/2} \binom{N/2}{s} \binom{N/2}{s+n} \left(\frac{1-\sqrt{\epsilon}}{1+\sqrt{\epsilon}}\right)^{2s+n}$$

For odd *N*

Non-zero elements of $c\phi_{\rm sq} \sim \epsilon^{(N+1)/2}$

$\phi_{\rm sq}$ is a dark st. for $N \to \infty$.

Density operator

$$\rho_{\text{th}} = \frac{1}{1+\bar{n}} \sum_{n=0}^{\infty} \left(\frac{\bar{n}}{1+\bar{n}} \right)^n |n\rangle\langle n|$$

$$\bar{n} \equiv \langle n \rangle = \text{Tr}[\rho_{\text{th}} n]$$

$$\text{Tr}[\rho_{\text{th}}^2] = \frac{1}{1+2\bar{n}}$$

$$\langle n^2 \rangle = \bar{n}(1+2\bar{n})$$

von Neumann entropy

$$S(\rho_{\text{th}}) = (1+\bar{n}) \ln (1+\bar{n}) - \bar{n} \ln \bar{n}$$

Coincides with the thermodynamic entropy.

Dissipative term within generalized Gutzwiller approx.

$$\mathcal{L}_\ell[\rho_\ell] = \gamma \sum_{\langle \ell' | \ell \rangle} \sum_{r,s=1}^{4} \Gamma_{\ell'}^{r,s} [2A_\ell^r \rho_\ell A_\ell^{s\,\dagger} - A_\ell^{s\,\dagger} A_\ell^r \rho_\ell - \rho_\ell A_\ell^{s\,\dagger} A_\ell^r]$$

$$\boldsymbol{A}_\ell = (1, b_\ell^\dagger, b_\ell, n_\ell)$$

$$\Gamma_\ell^{r,s} = \sigma^r \sigma^s \mathrm{Tr}_\ell A_\ell^{(5-s)\,\dagger} A_\ell^{(5-r)} \rho_\ell$$

$$\sigma = (-1-\epsilon, -1+\epsilon, 1-\epsilon, 1+\epsilon)$$