Inertial waves

A Laboratory Experiment with an astrophysical perspective

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A Laboratory Experiment: that other, other side of the screen

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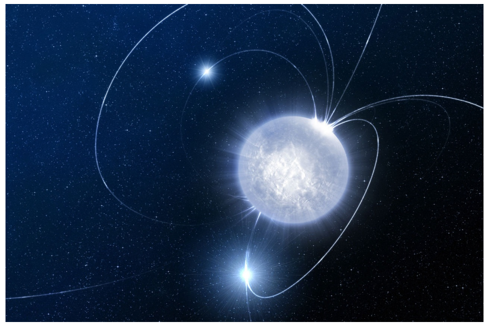
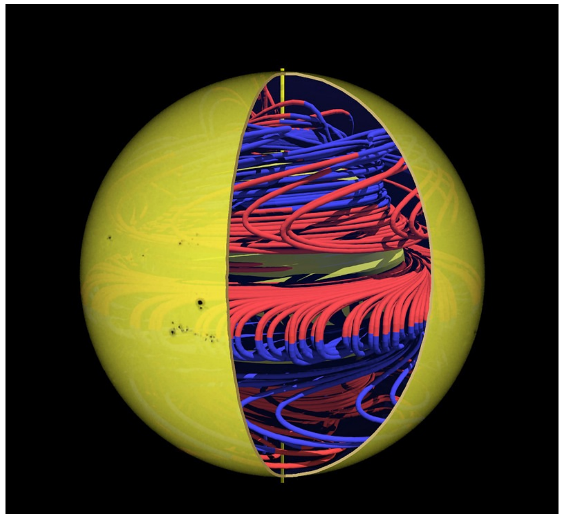
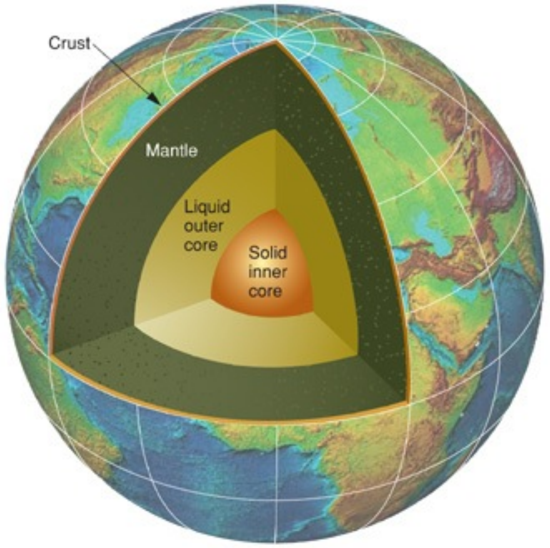
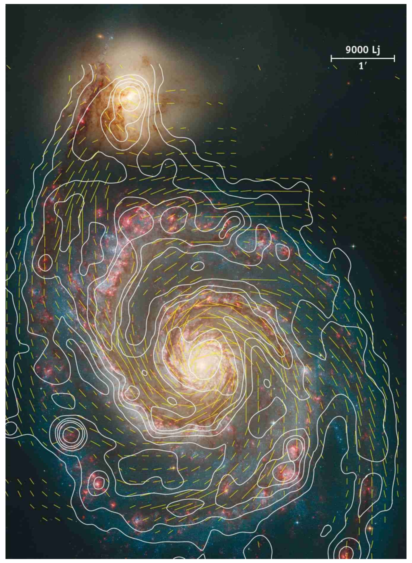
Dynamo action:

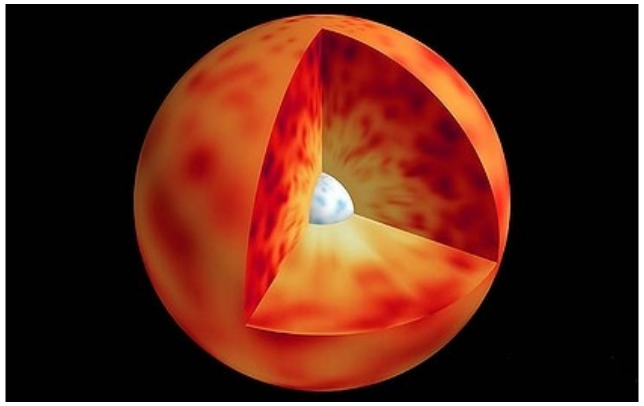




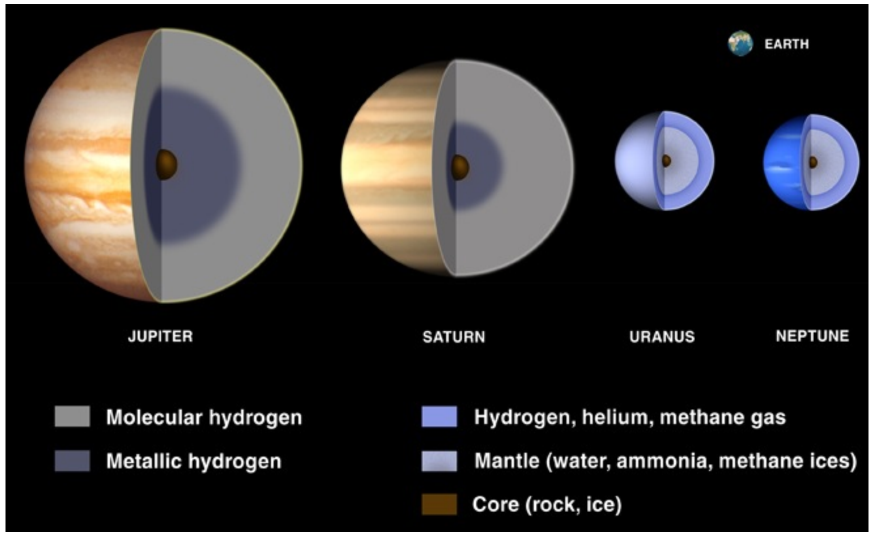
Arlt





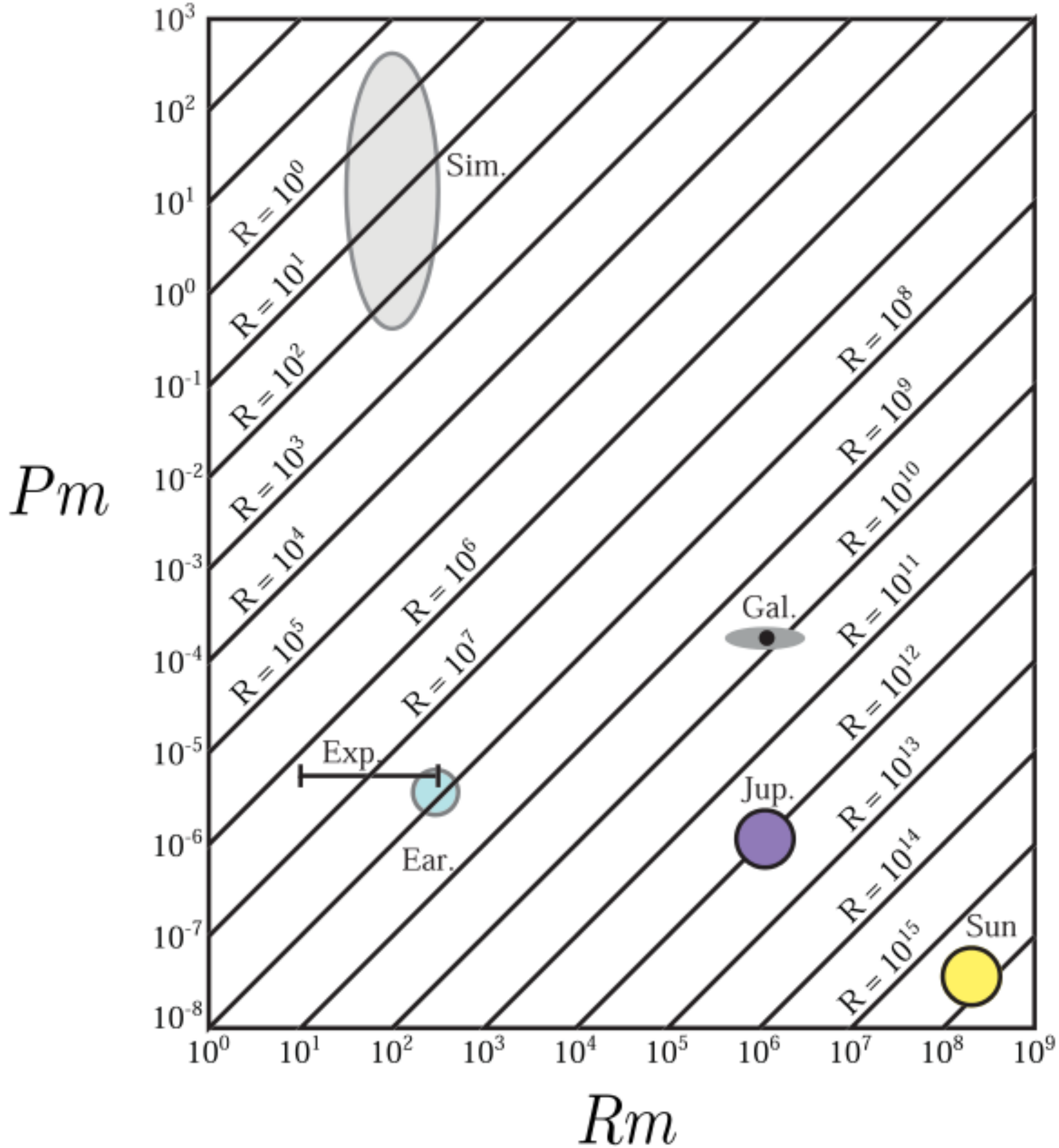


Beck



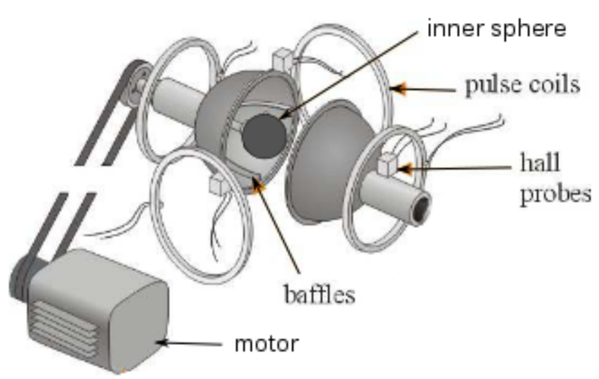
Almost always present whenever there is an electrically conducting fluid in some state of rotation

If it is so ubiquitous,
let's see if we can
make it in the lab!

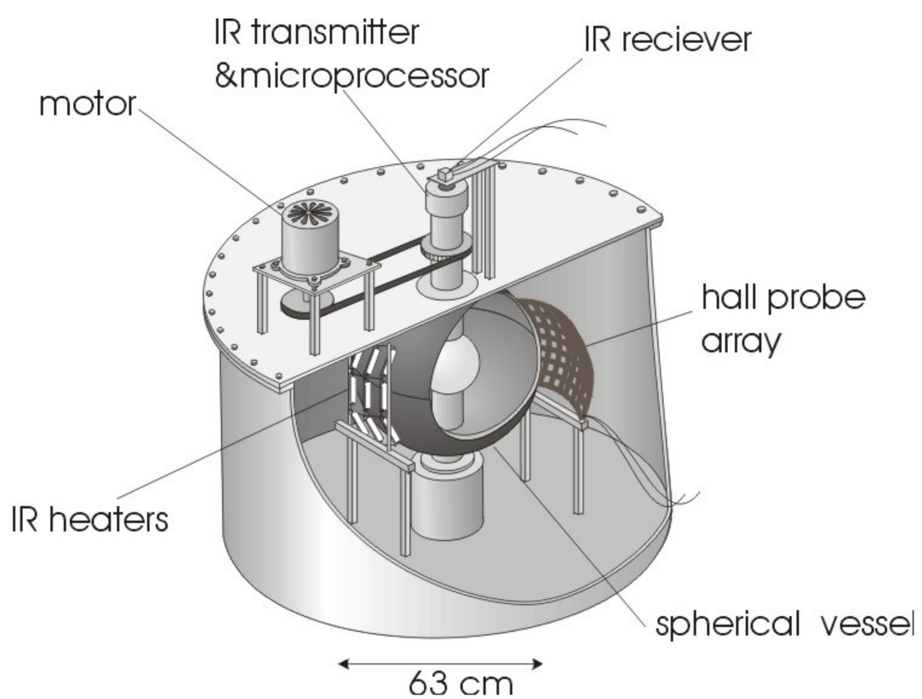


Pm
Rm
Sim.
Gal.
Exp.
Ear.
Jup.
Sun
R = 10^0
R = 10^1
R = 10^2
R = 10^3
R = 10^4
R = 10^5
R = 10^6
R = 10^7
R = 10^8
R = 10^9
R = 10^10
R = 10^11
R = 10^12
R = 10^13
R = 10^14
R = 10^15

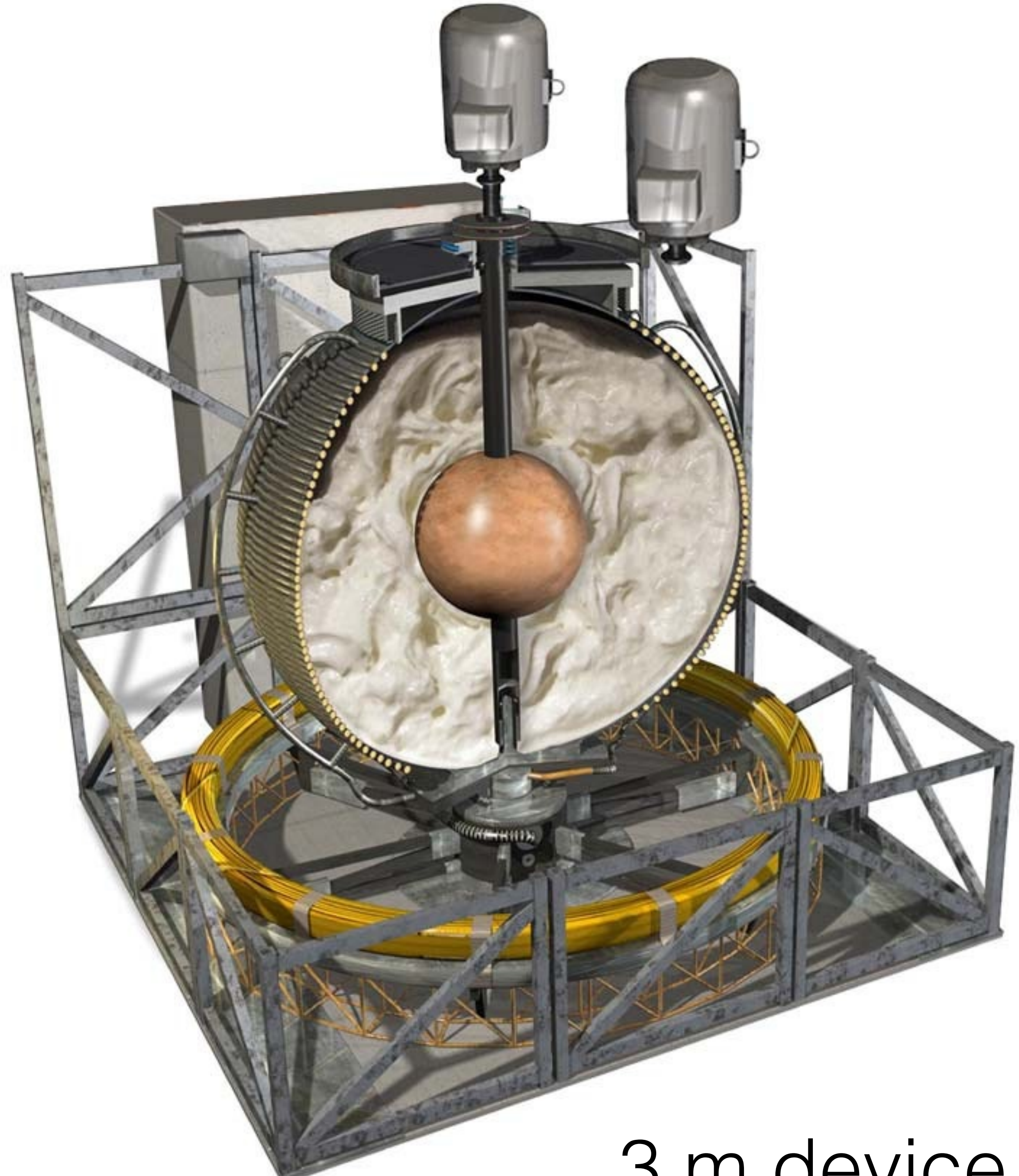
The Liquid Sodium Experiments at Maryland



30 cm device



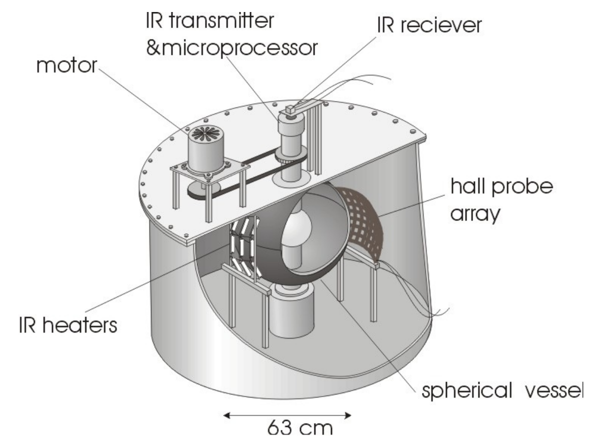
60 cm device



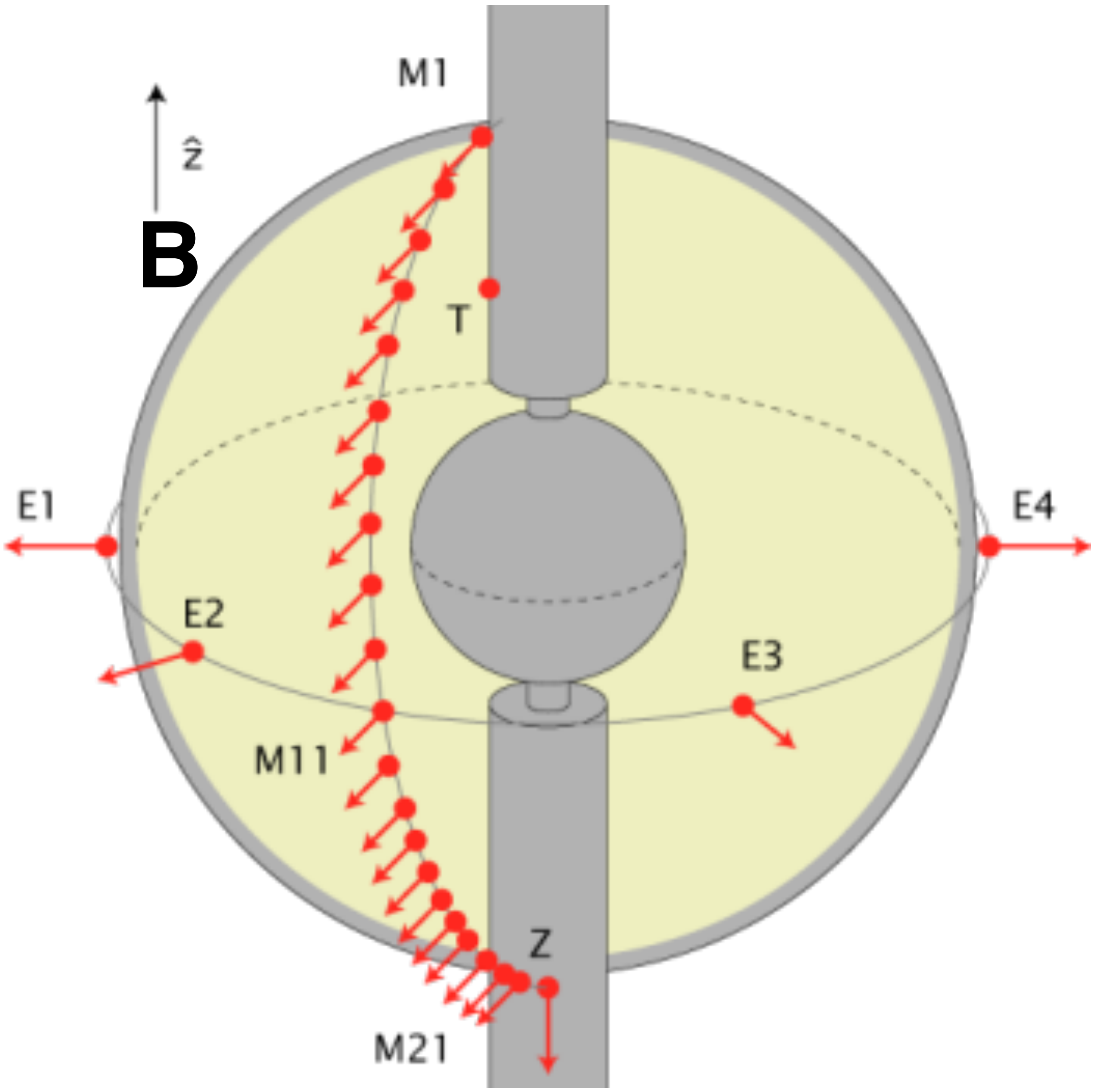
3 m device

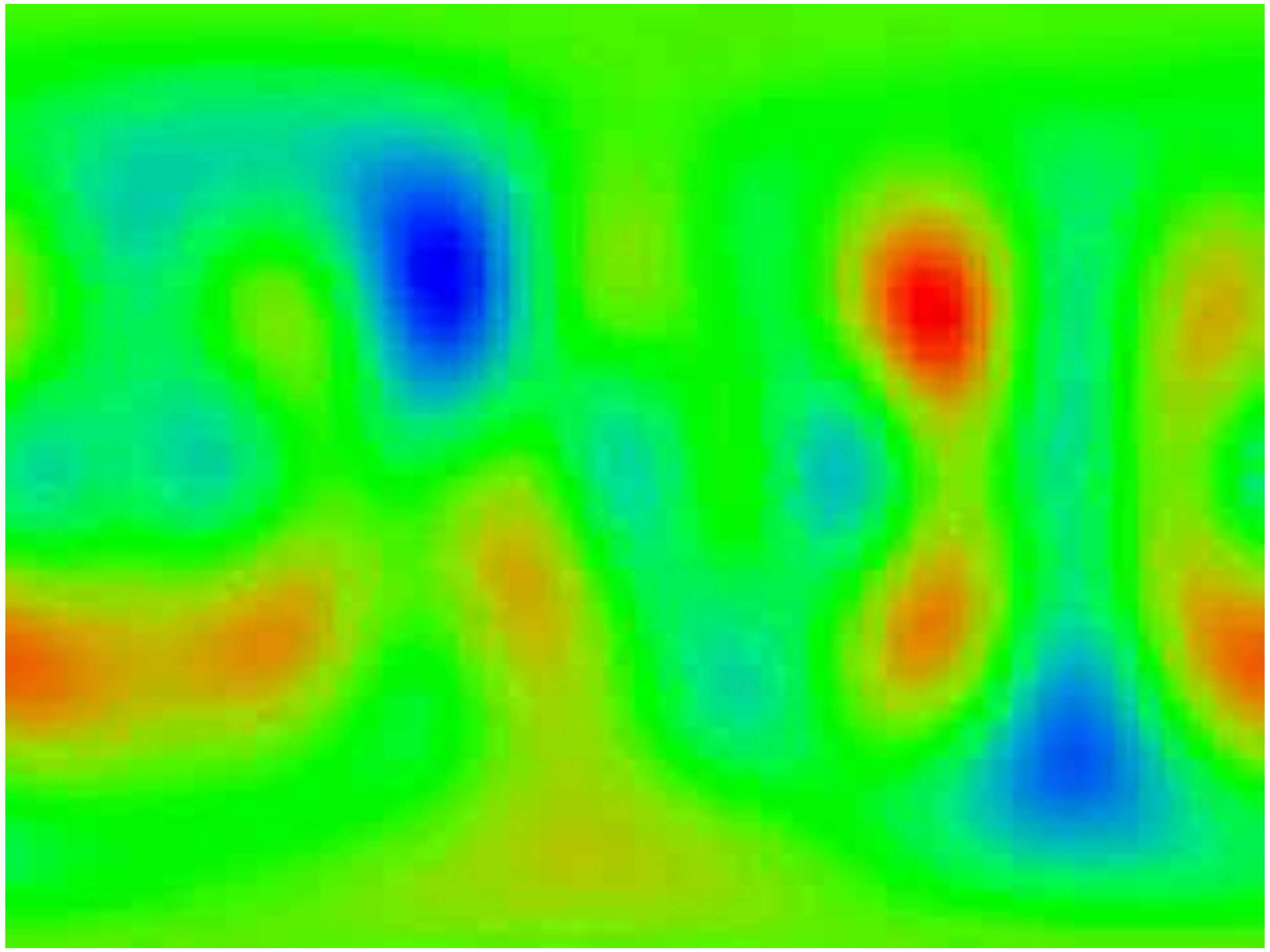
13 Tonnes of liquid sodium

Results from the 60 cm device: Inertial waves



Independently rotating spheres



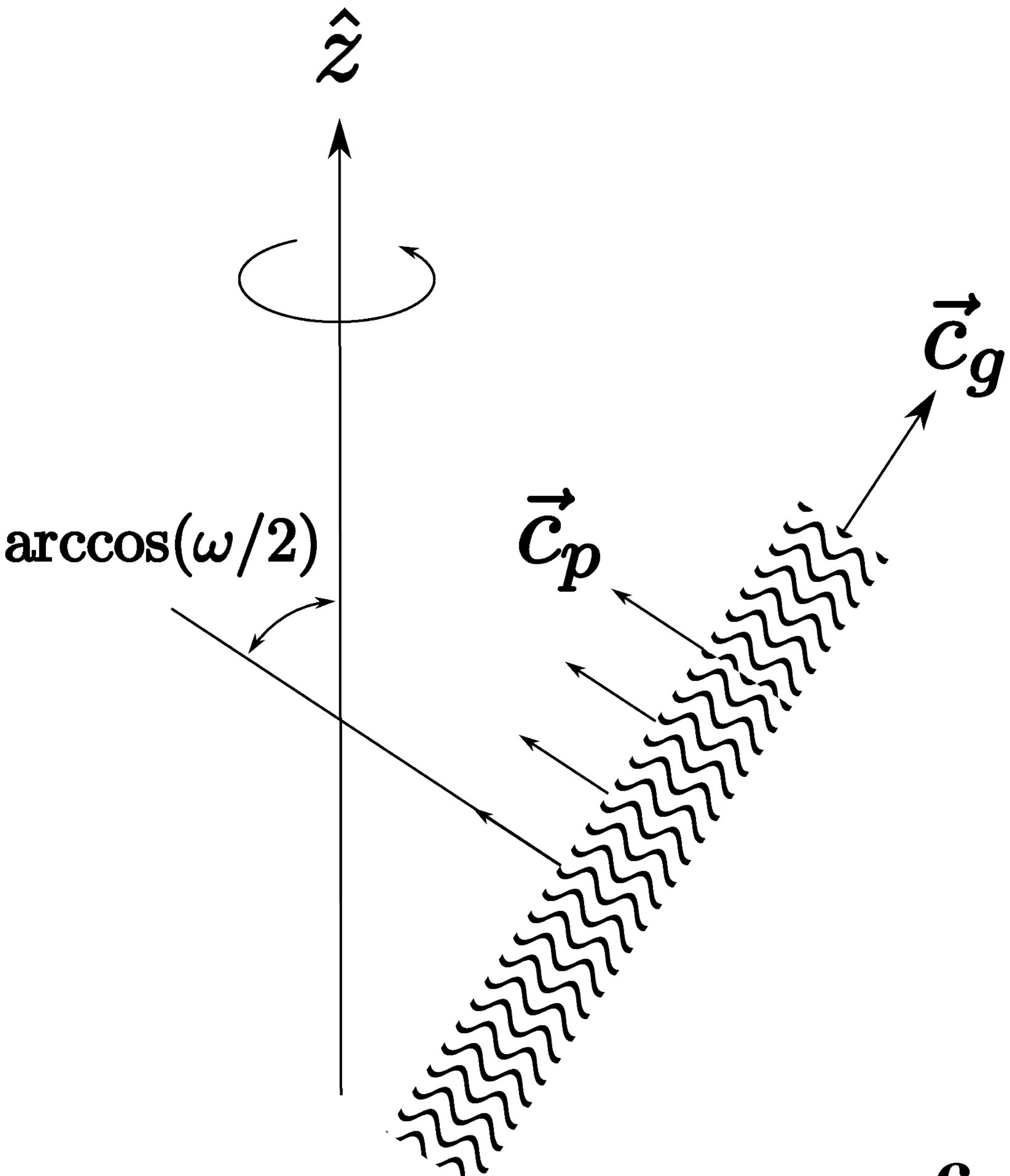


Counter-rotation ramp

The magnetic field here is passive, it only aids the imaging of the mode

Kelley, Triana, et al., GAFD 101, Nos. 5-6, 2007

Inertial waves, very peculiar!



$$\mathbf{u} \propto e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$$

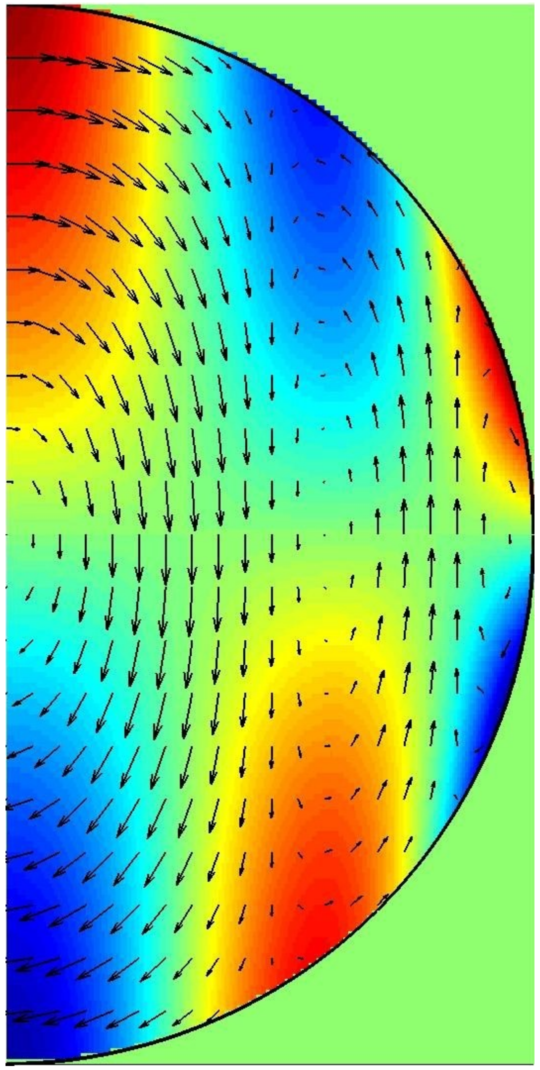
Plane wave solution

$$\omega = \pm 2\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}$$

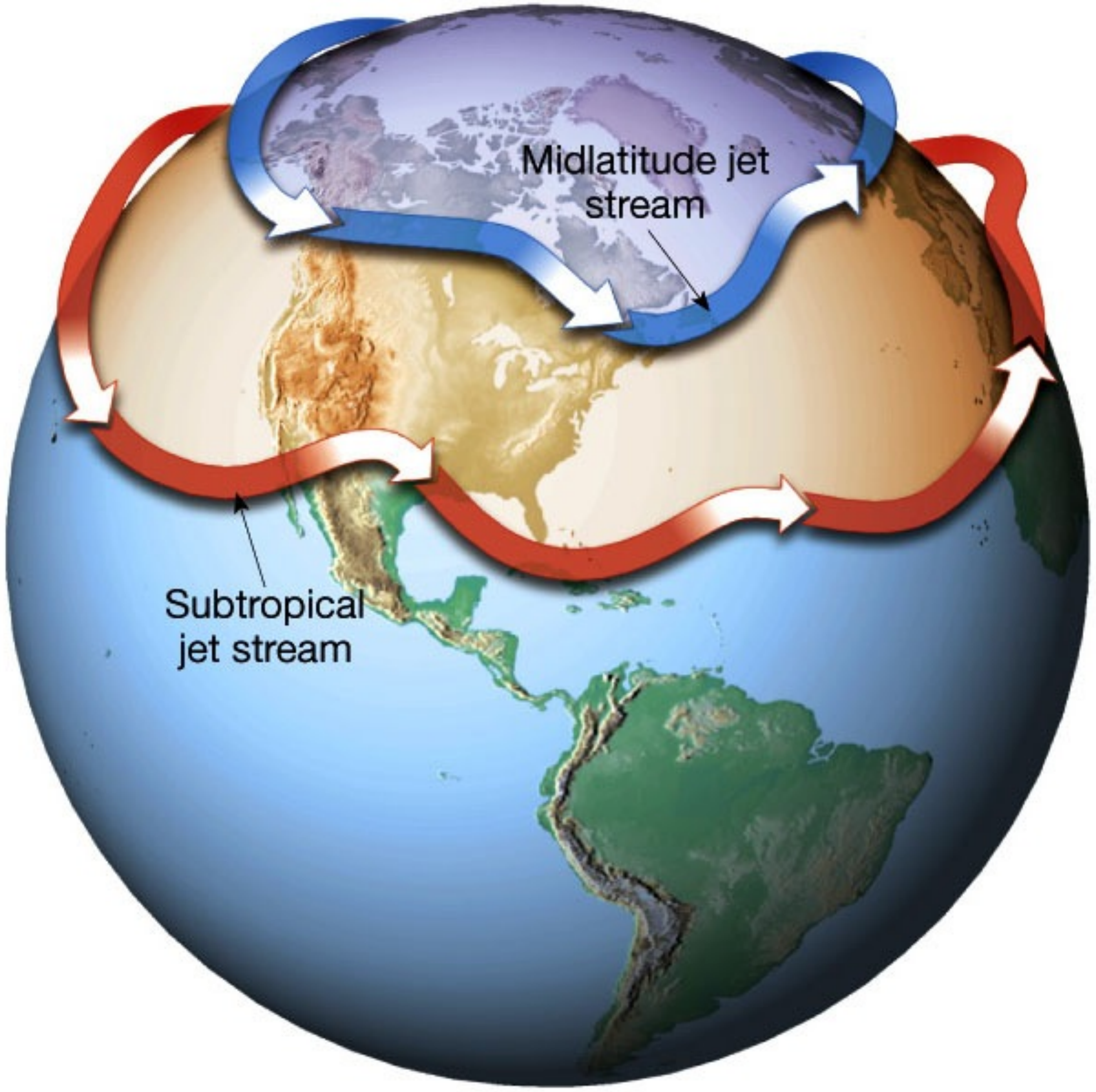
Dispersion relation

$$\mathbf{c_p} = \pm 2\,\frac{\hat{\mathbf{k}}\cdot\hat{\mathbf{z}}}{k}\,\hat{\mathbf{k}} \qquad \mathbf{c_g} = \pm\,\frac{2}{k}\,\hat{\mathbf{k}}\times(\hat{\mathbf{z}}\times\hat{\mathbf{k}})$$

Inertial modes: Coriolis-restored oscillations

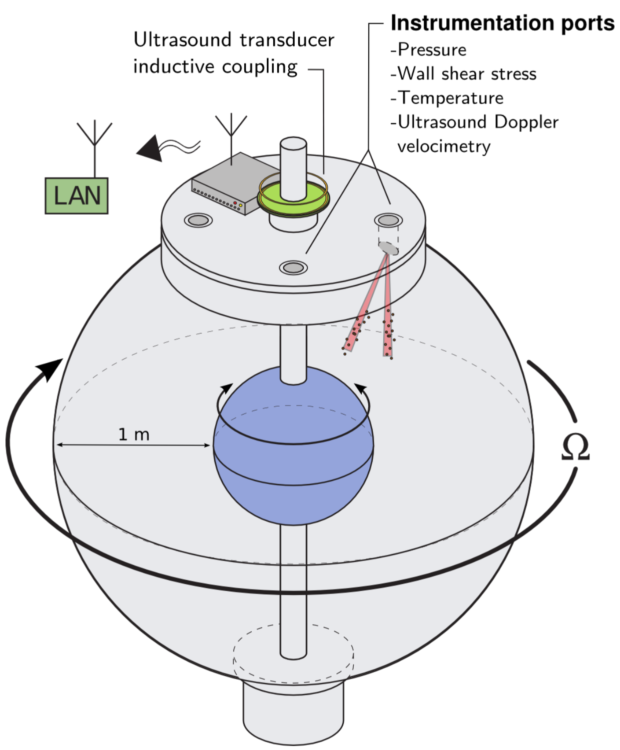


In a full sphere



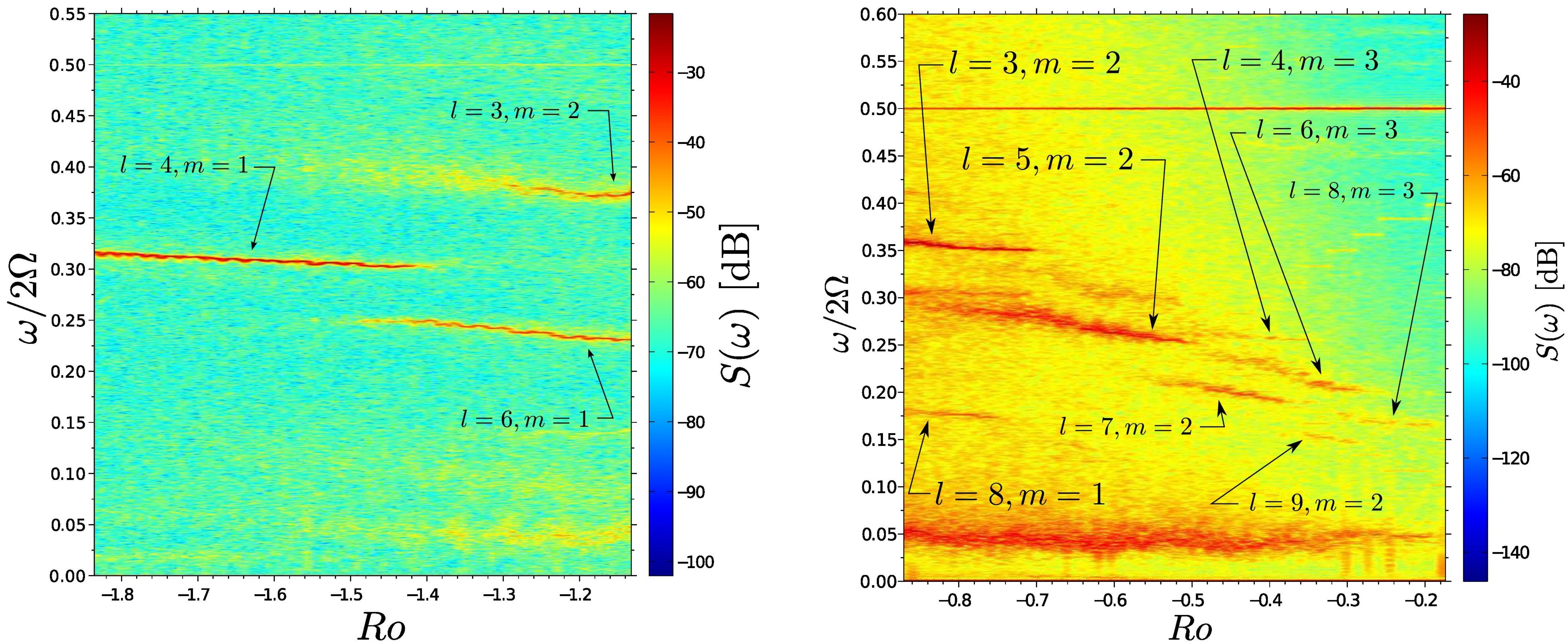
In the Earth's atmosphere

The 3-meter Experiment



20 Tonnes,
2x 350 Hp motors,
135 Km/h at the equator

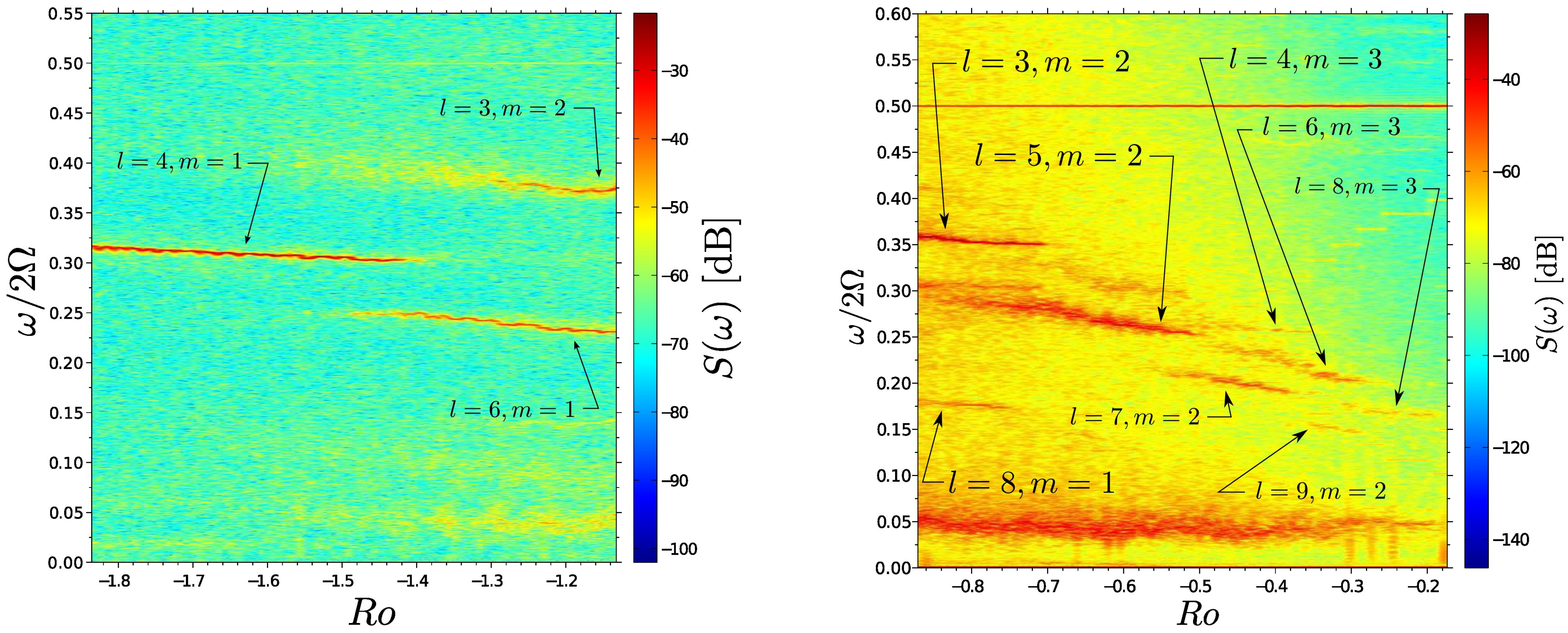
Inertial waves in the 3 meter system



Pressure spectrograms

$$Ro \equiv \frac{\Delta\Omega}{\Omega}$$

Inertial waves revisited



How are these modes excited?

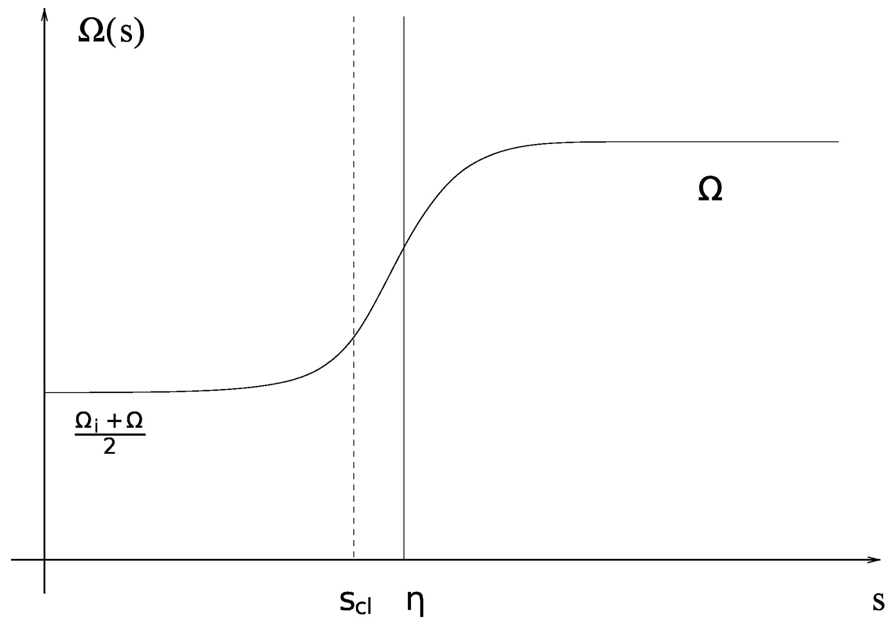
Why are they retrograde?

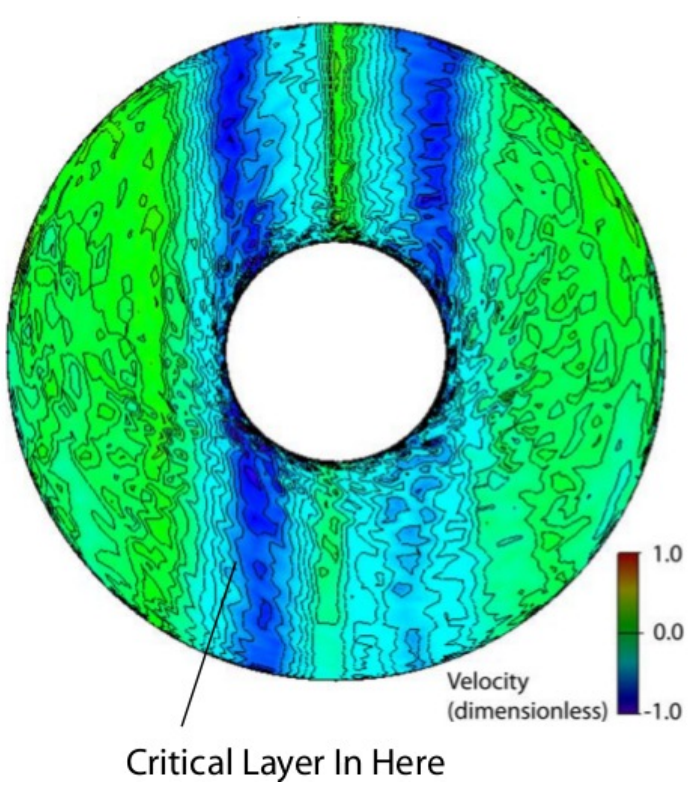
Why are they equatorially antisymmetric?

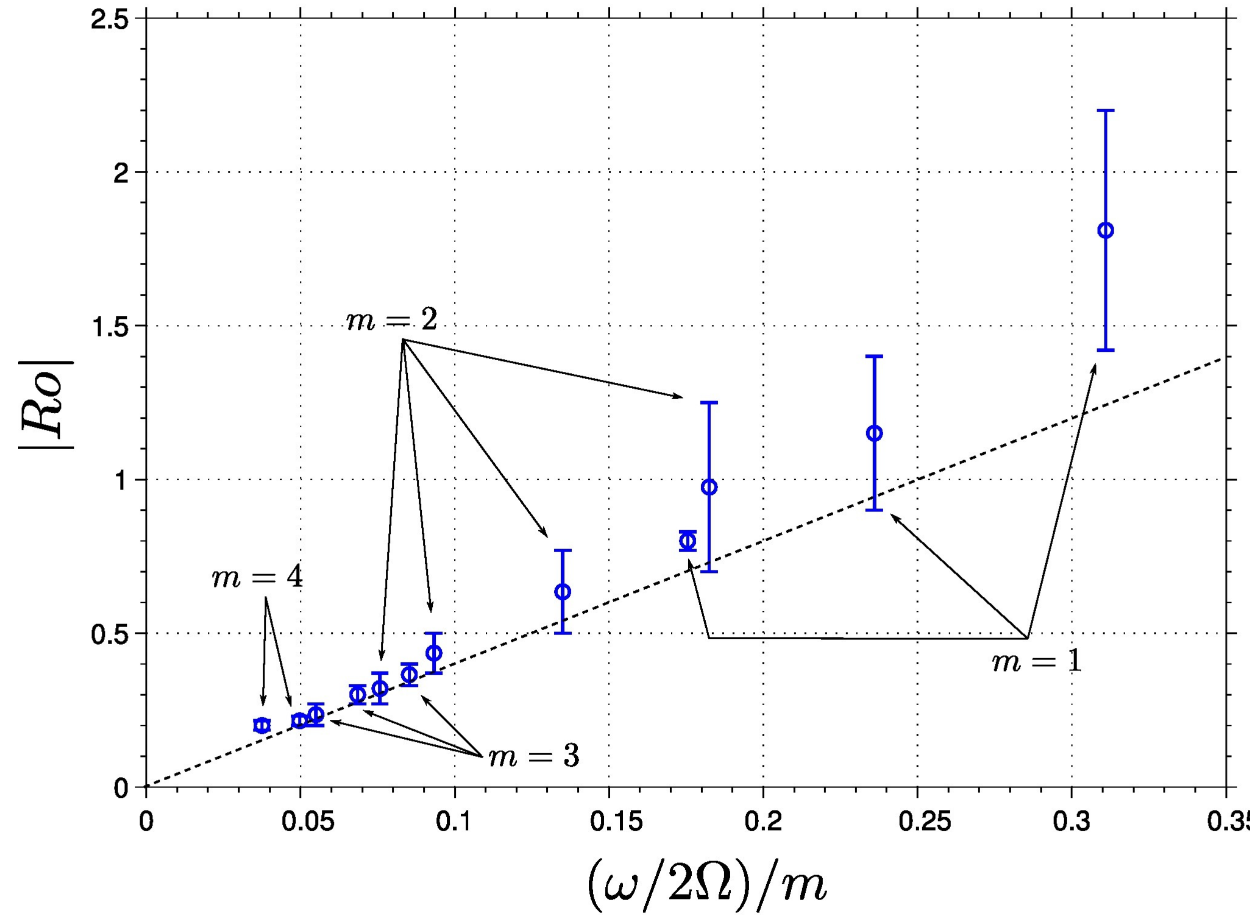
Why do we see only the ones we see?

Why do they appear in that order?

Why in that order?



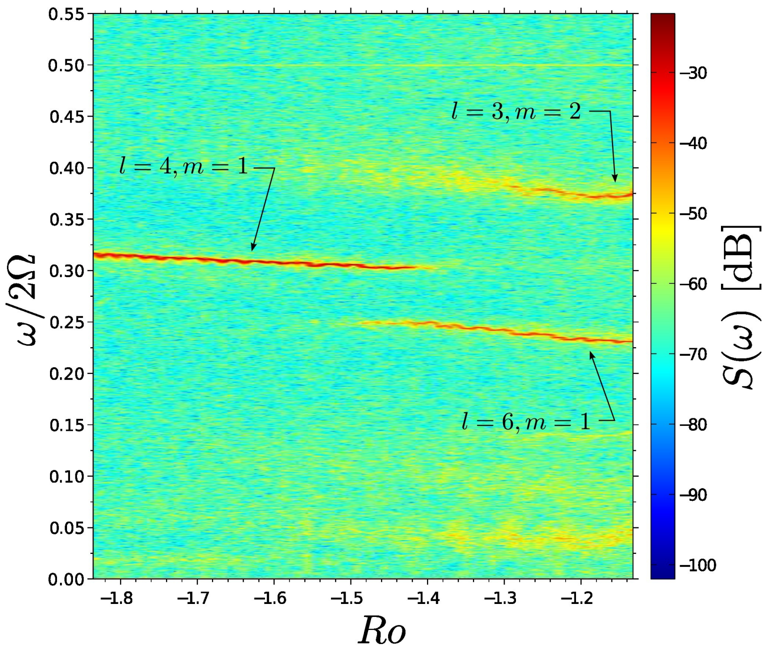


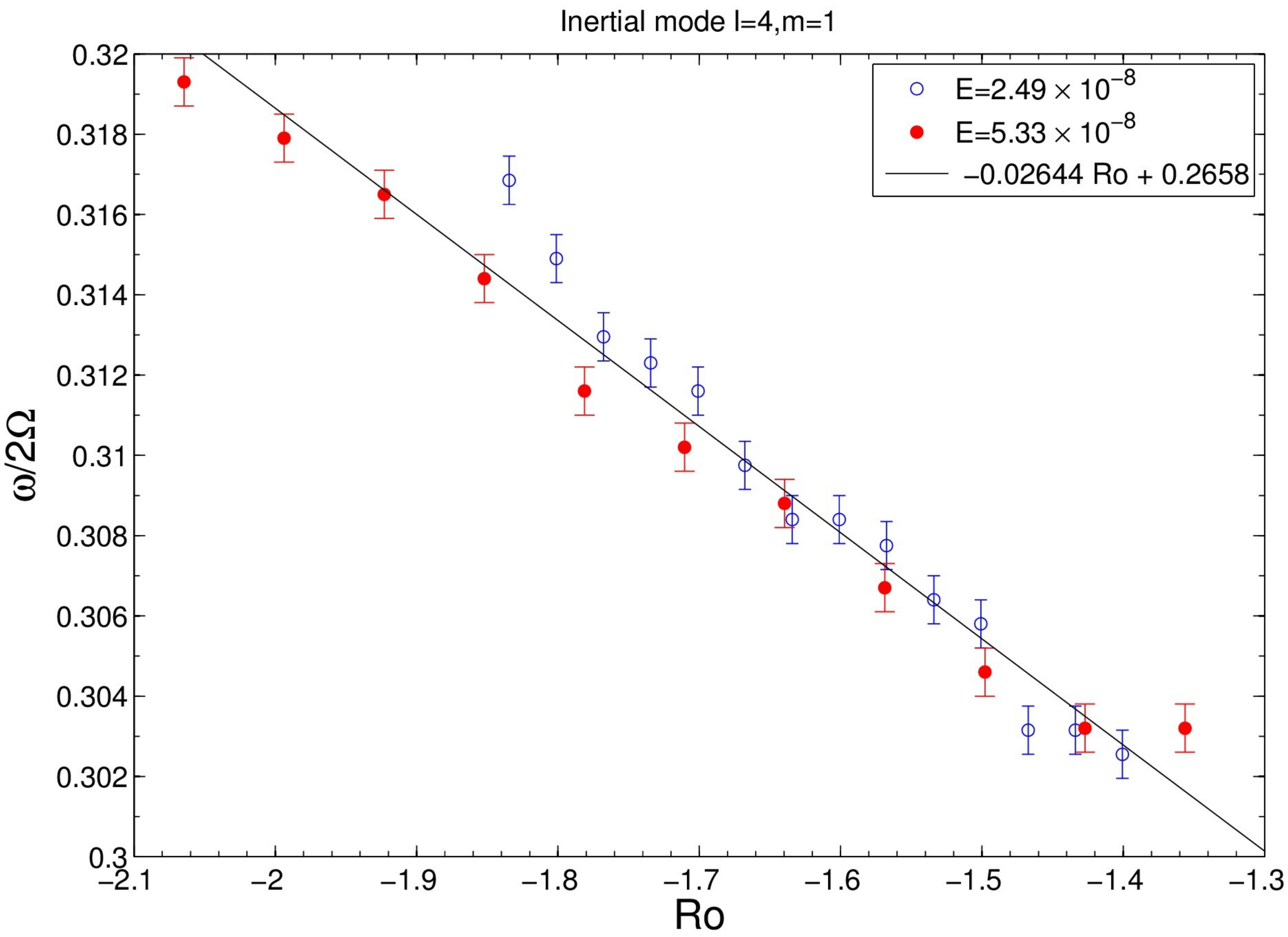


The modes are excited (perhaps by over-reflection?) at a critical layer in the neighborhood of the tangent cylinder

Matsui et al., PEPI 188, Issues 3-4, Pages 194-202, 2011

Mode frequency varies slightly with the amount of differential rotation



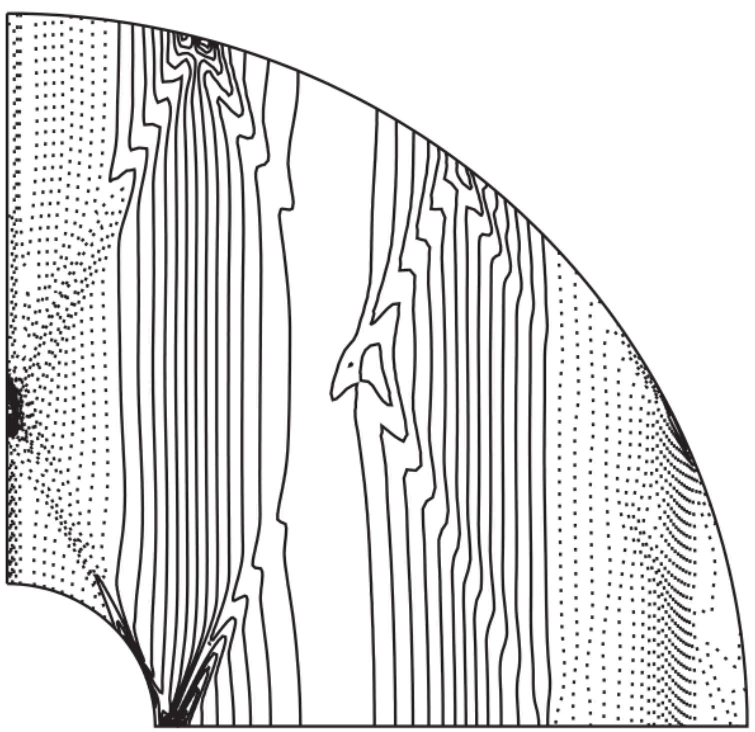


Inverse problem: differential rotation from frequency shifts

Rieutord, Triana, et al., PRE 86, 026304, 2012

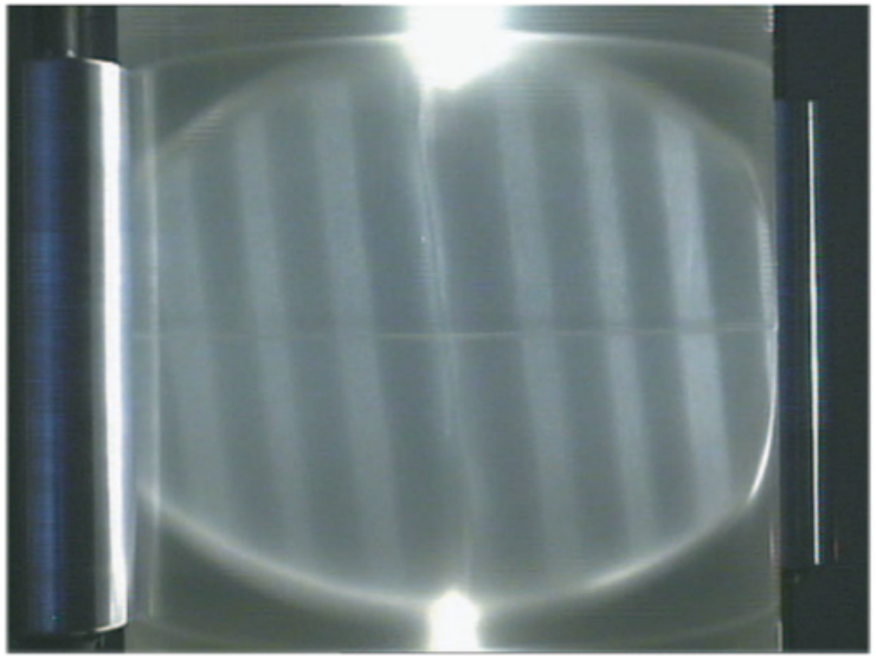
Nonlinear self-interaction of an inertial modes: Zonal flow generation

A. Tilgner, PRL 99, 194501 (2007)



Zonal wind contour plot

C. Morize et al., PRL 104, 214501 (2010)

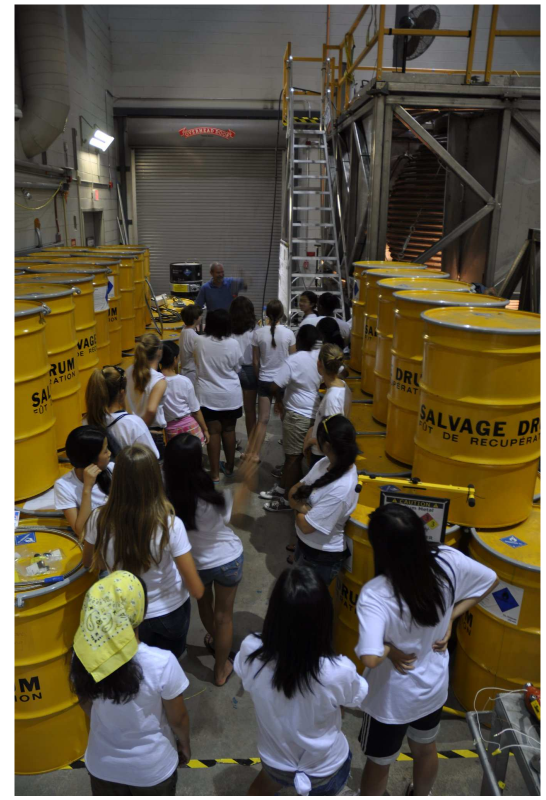


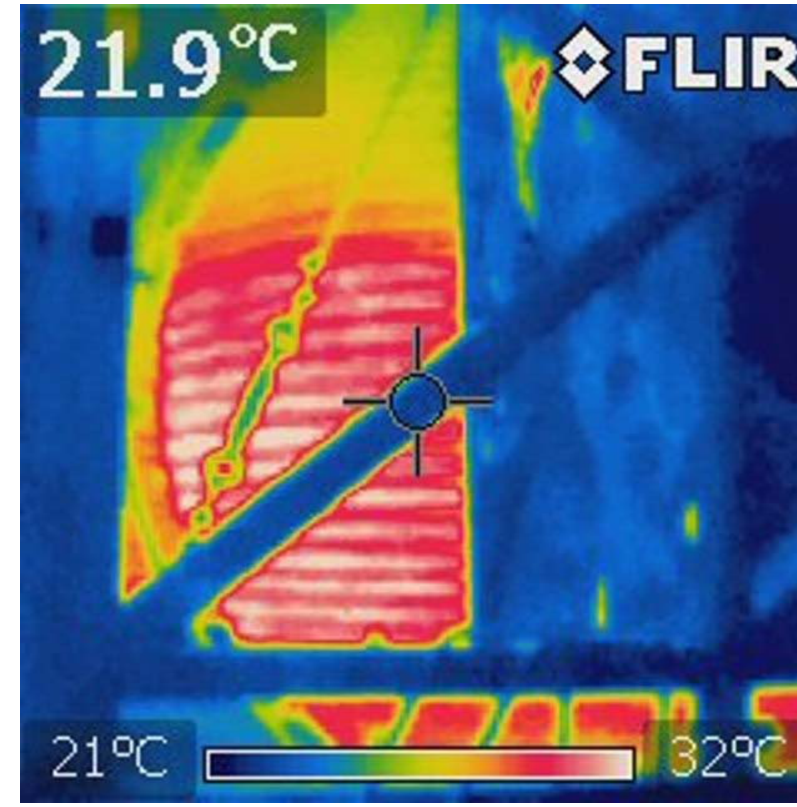
Shear cylinders, resonant tidal forcing

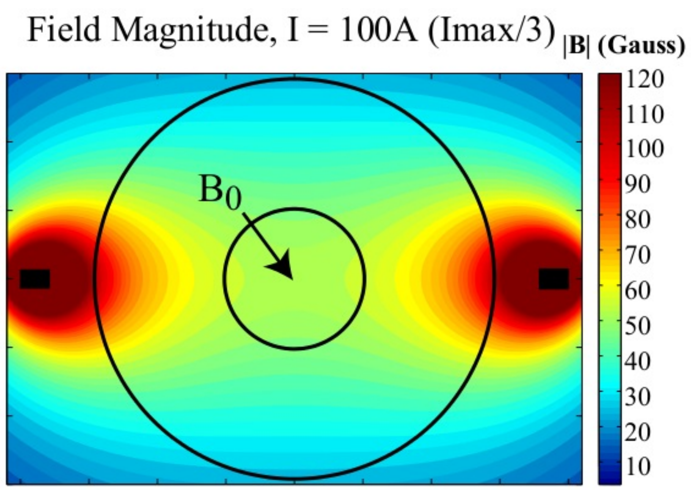
Geostrophic velocity $\sim E^{-3/10}$

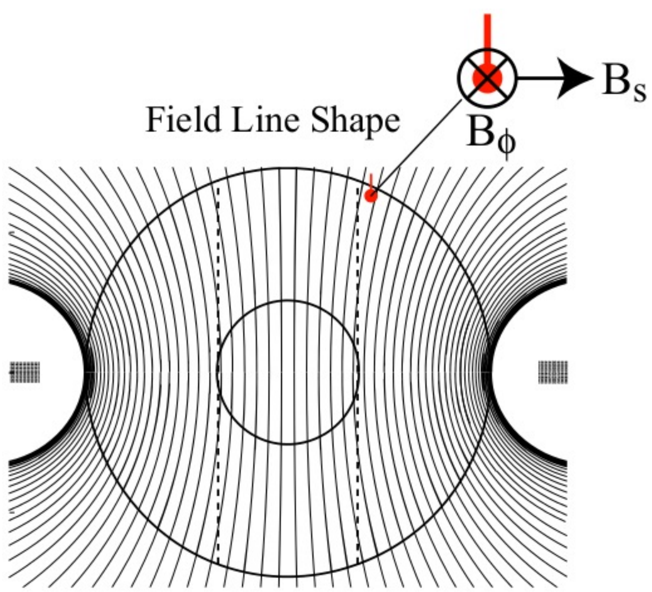
Sodium experiments!

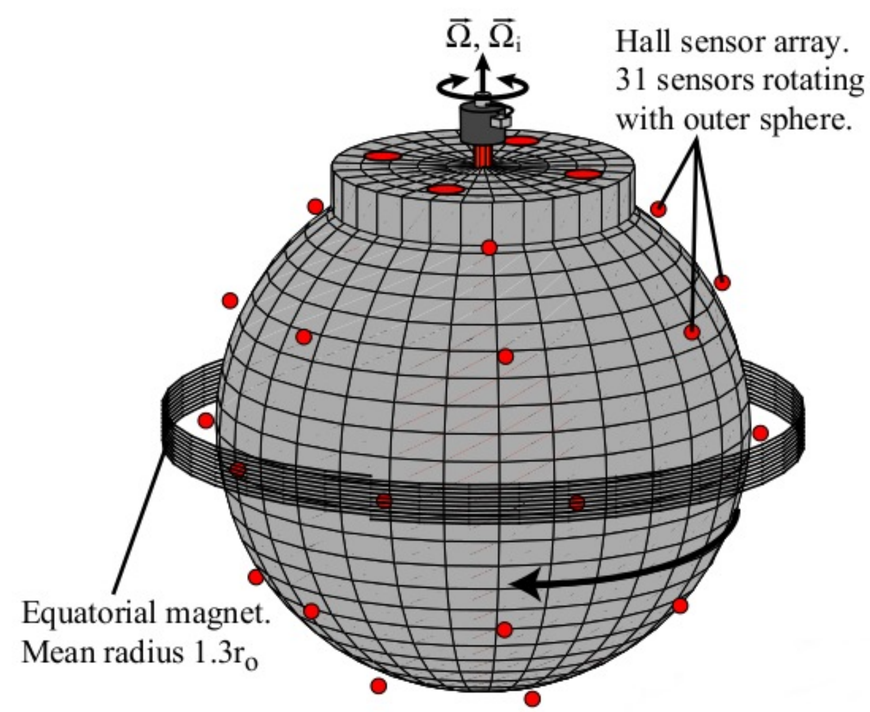
The fill







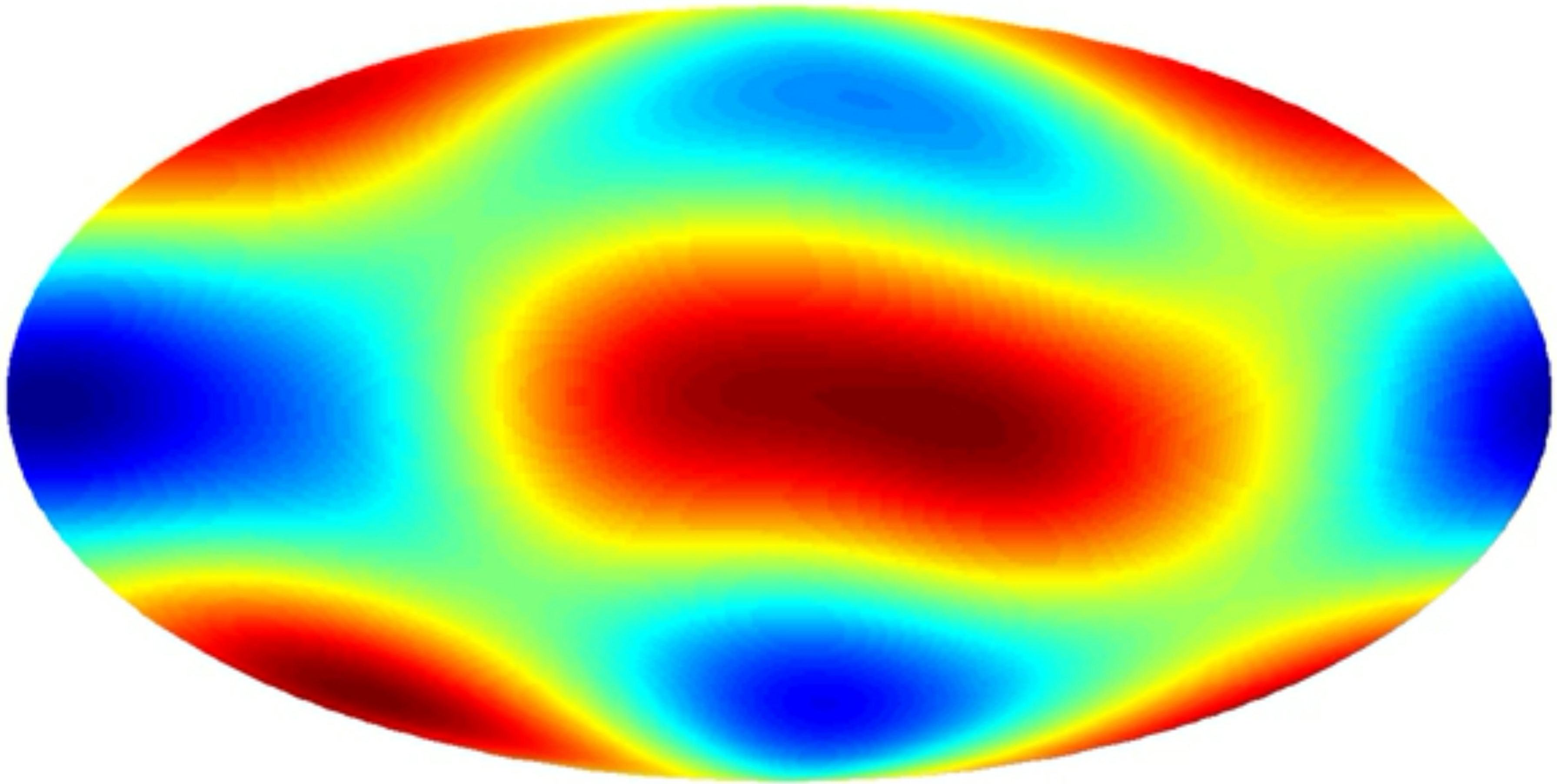




The external coils

Inertial mode imaged magnetically

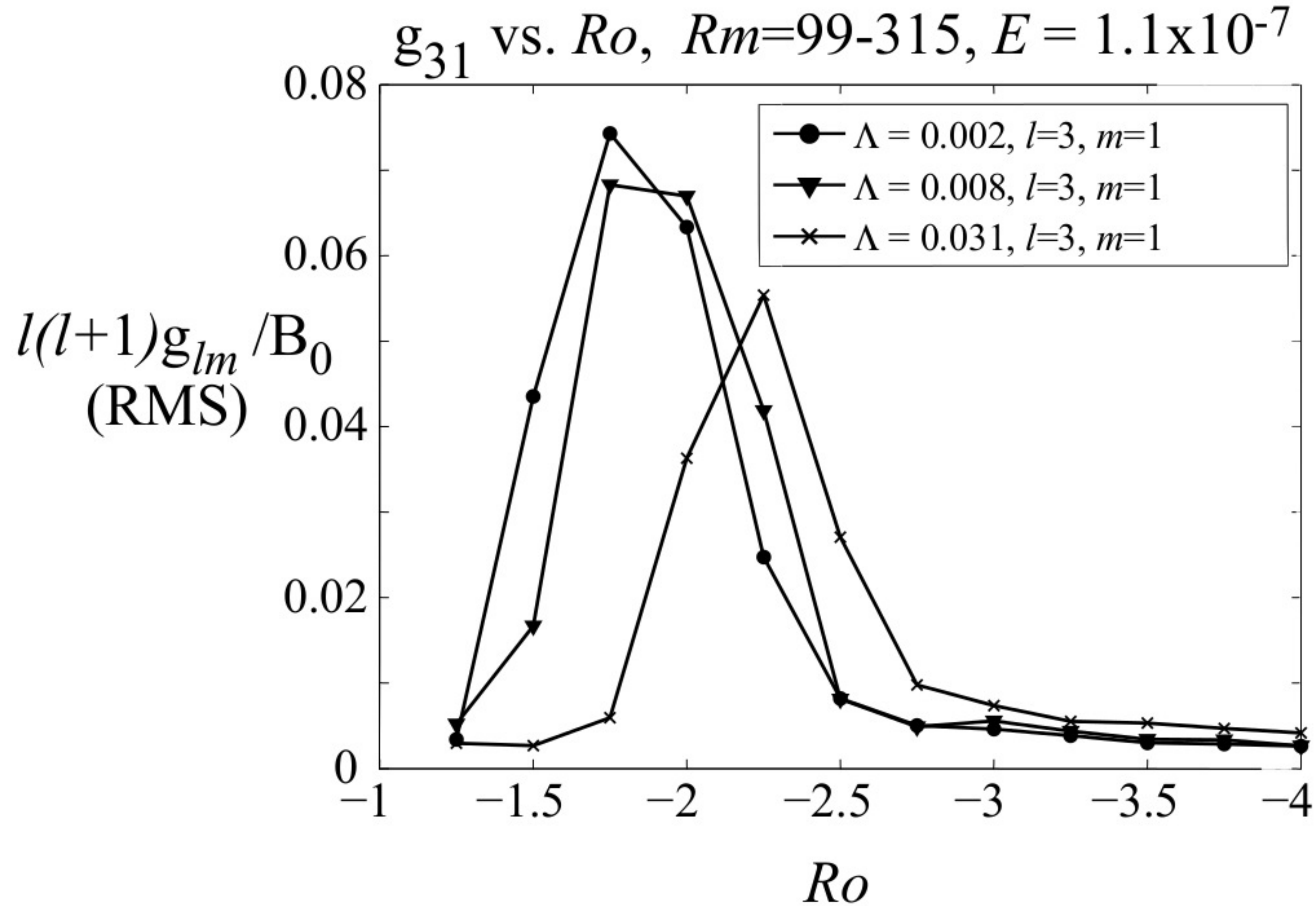
(Just as in the 60cm experiment)



$$l = 4, m = 1$$

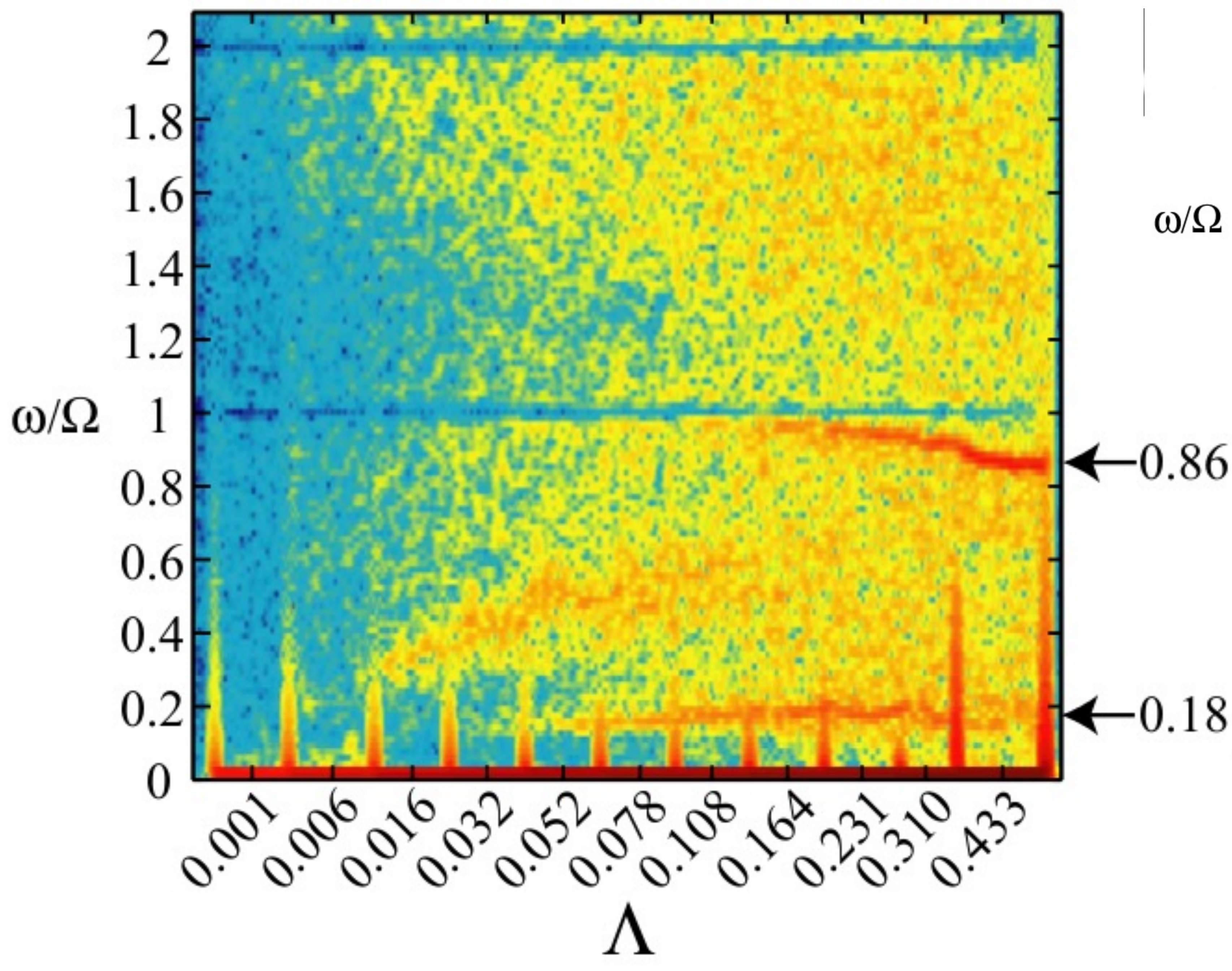
$$\Lambda = 0.001, Rm = 268, E = 5.6 \times 10^{-8}$$

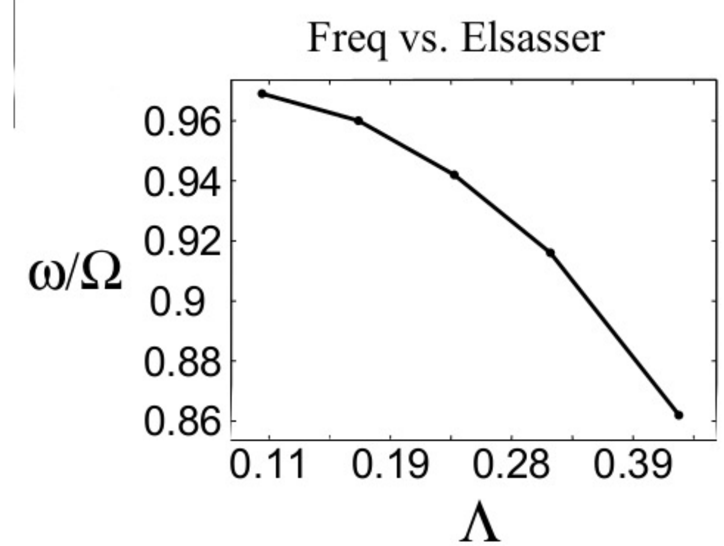
Damping/detuning with applied field



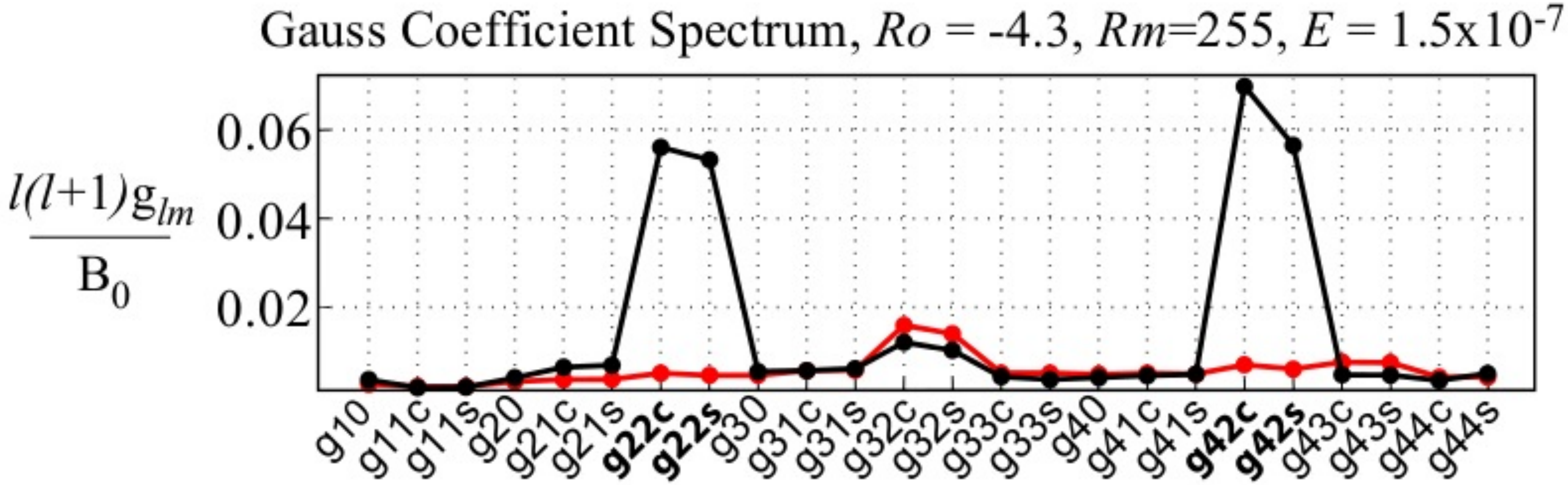
Strong field

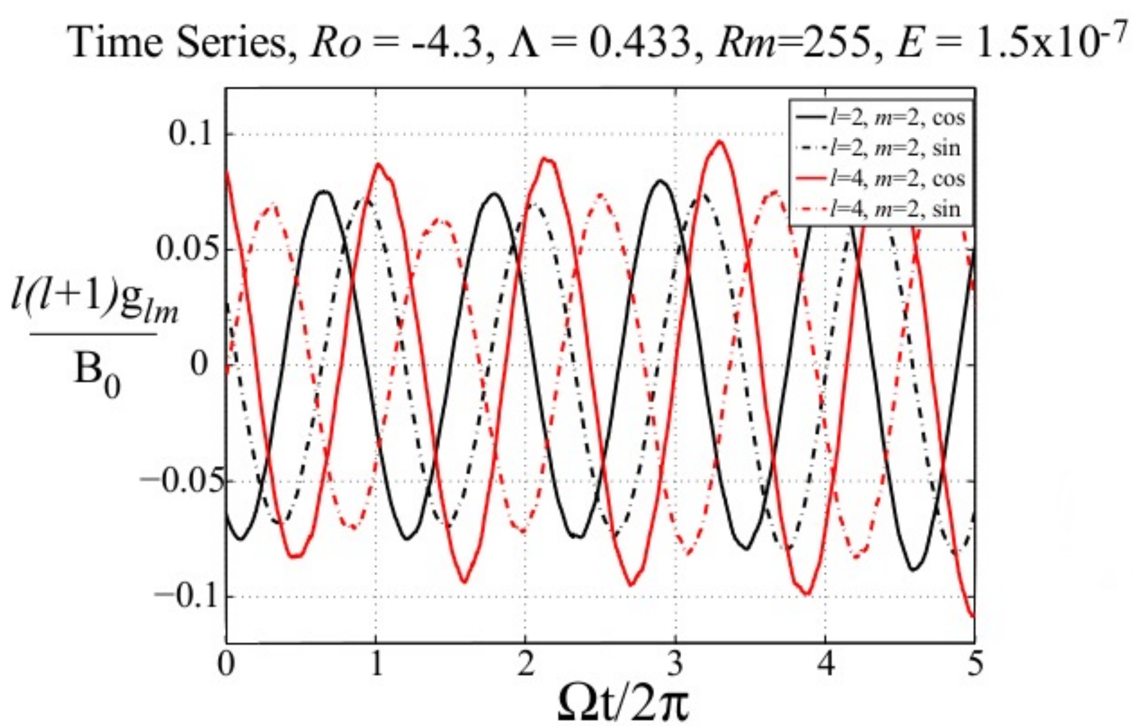
Midlat Spectrogram, Ro = -4.3, Rm=255, $E = 1.5 \times 10^{-7}$

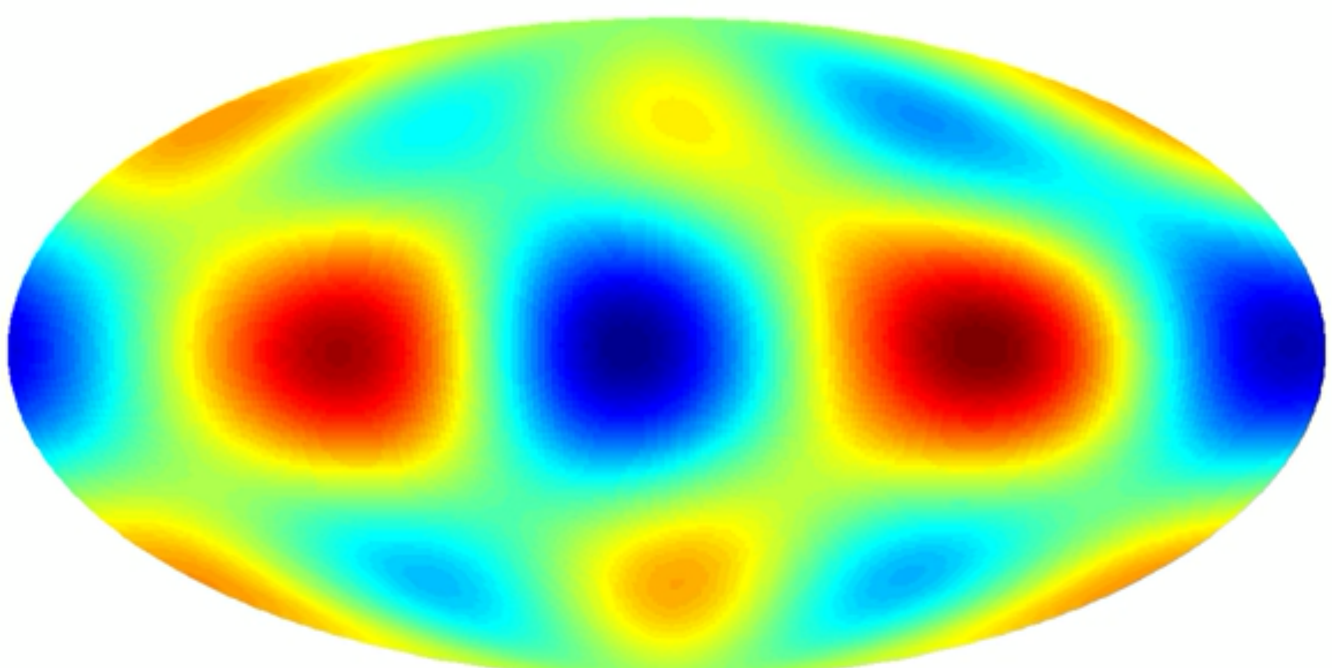


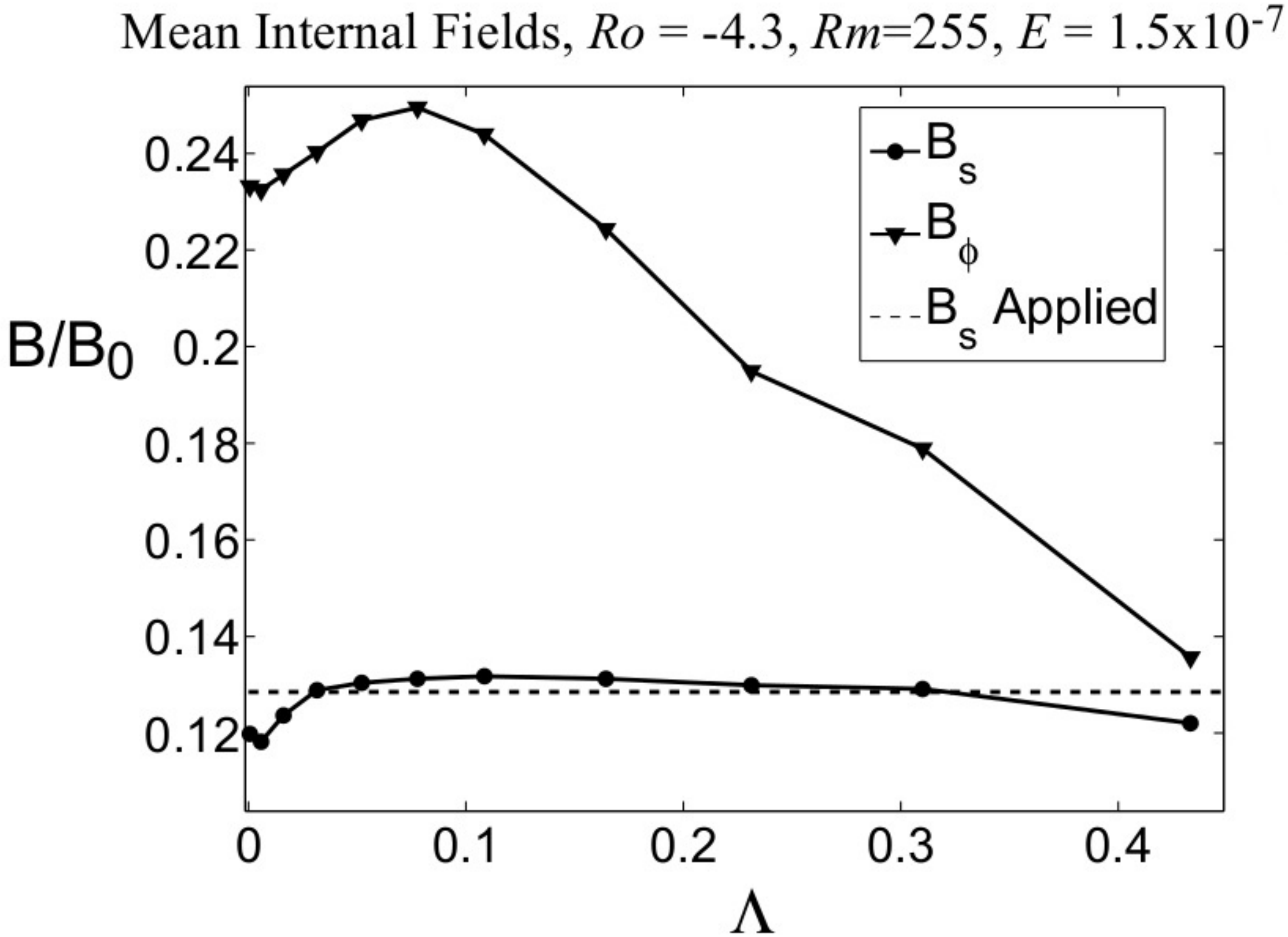


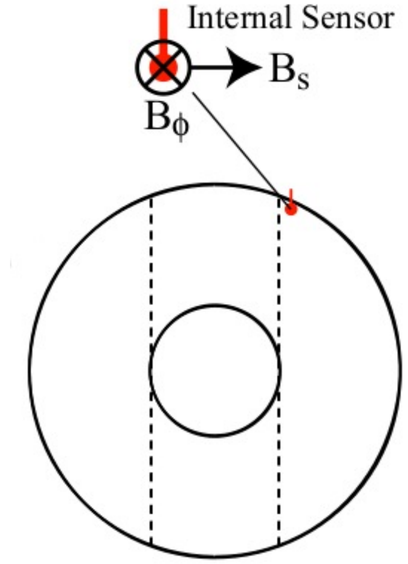
New mode, an MC mode?



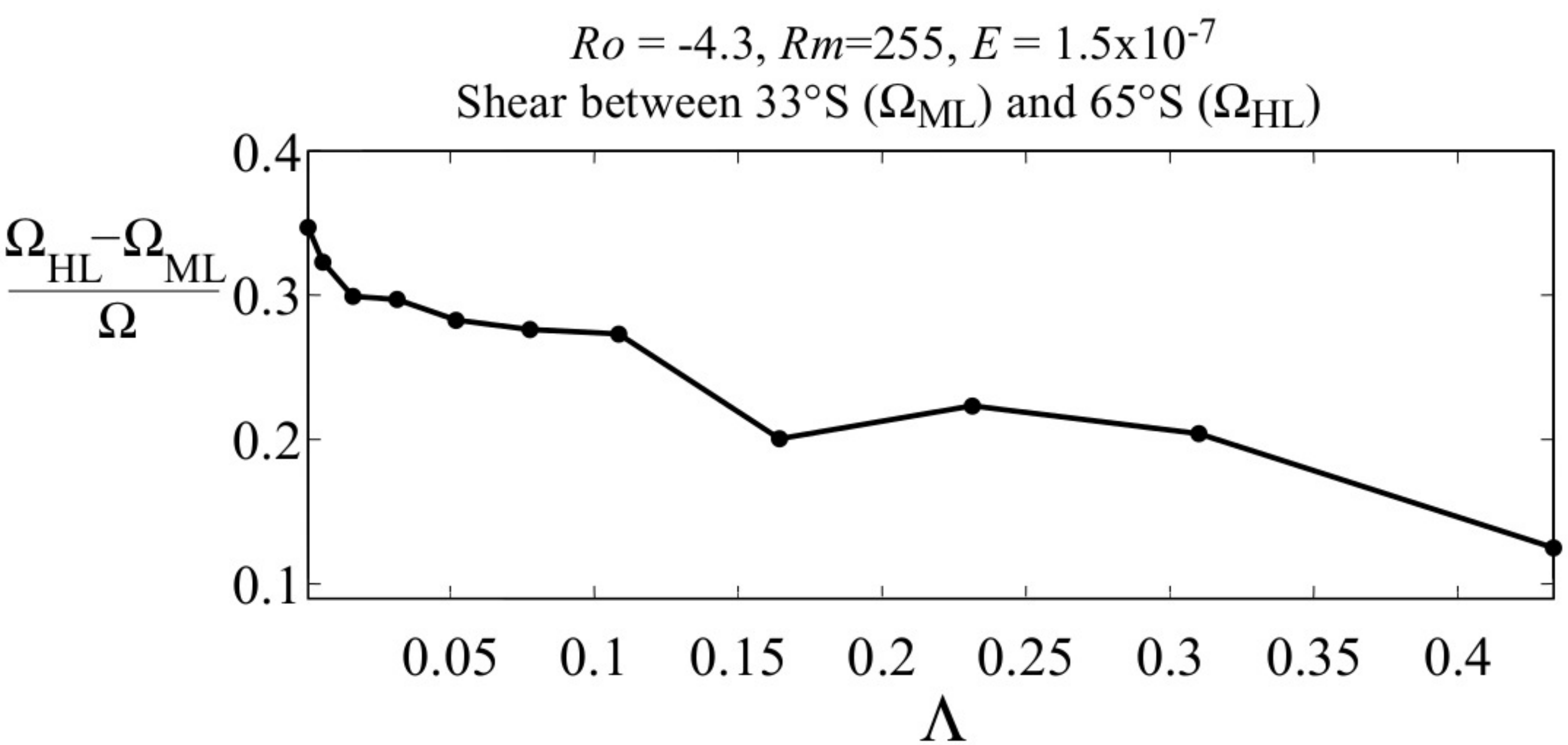


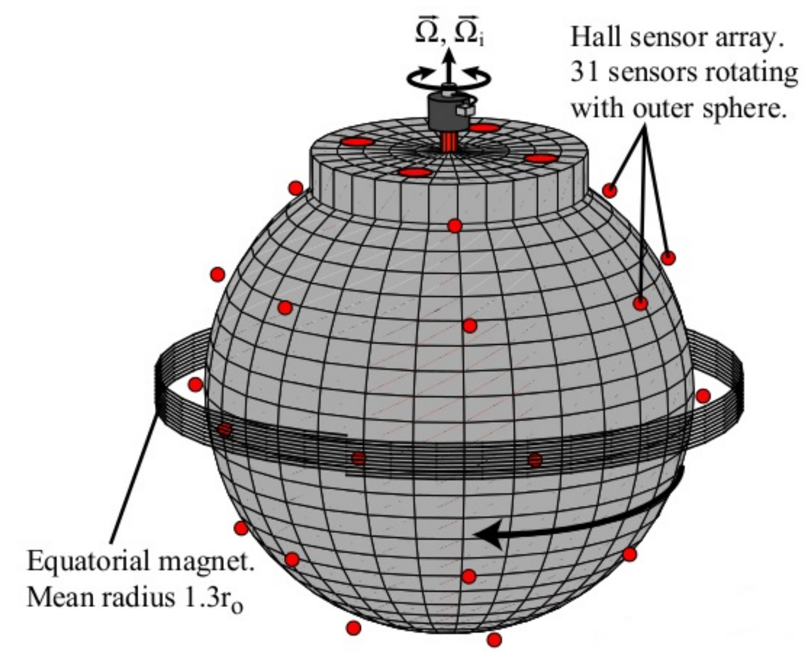






Internal field

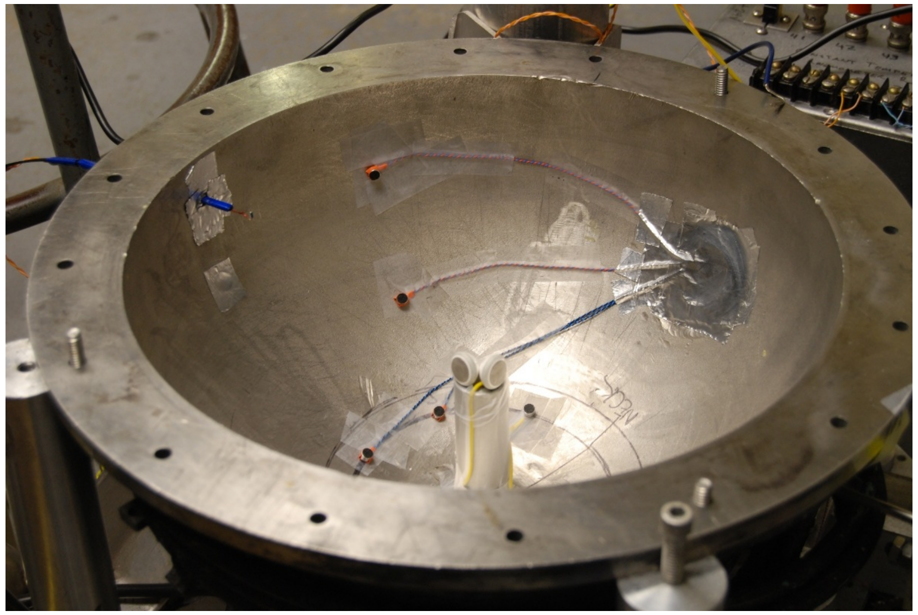


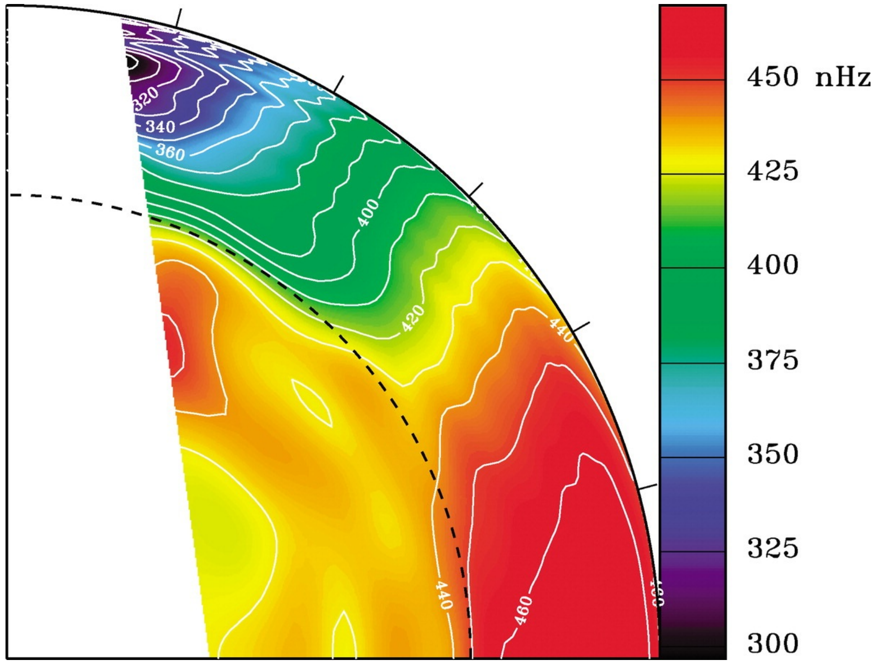


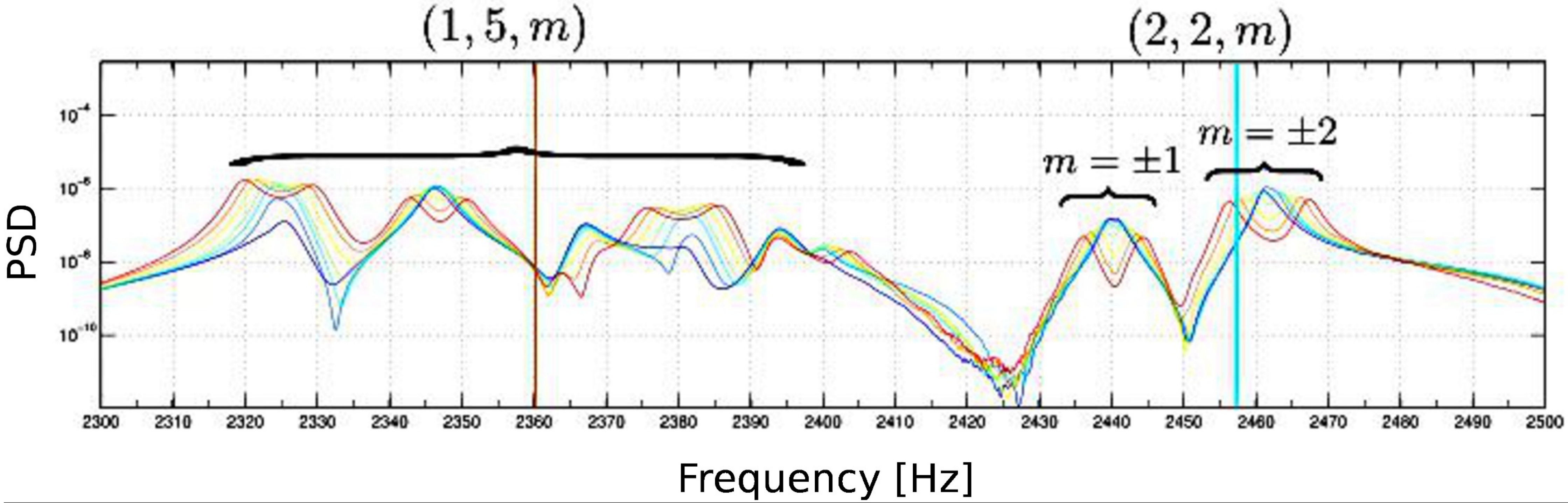
Now some acoustics:

we can use asteroseismic tools to quantify the amount of differential rotation in experiments

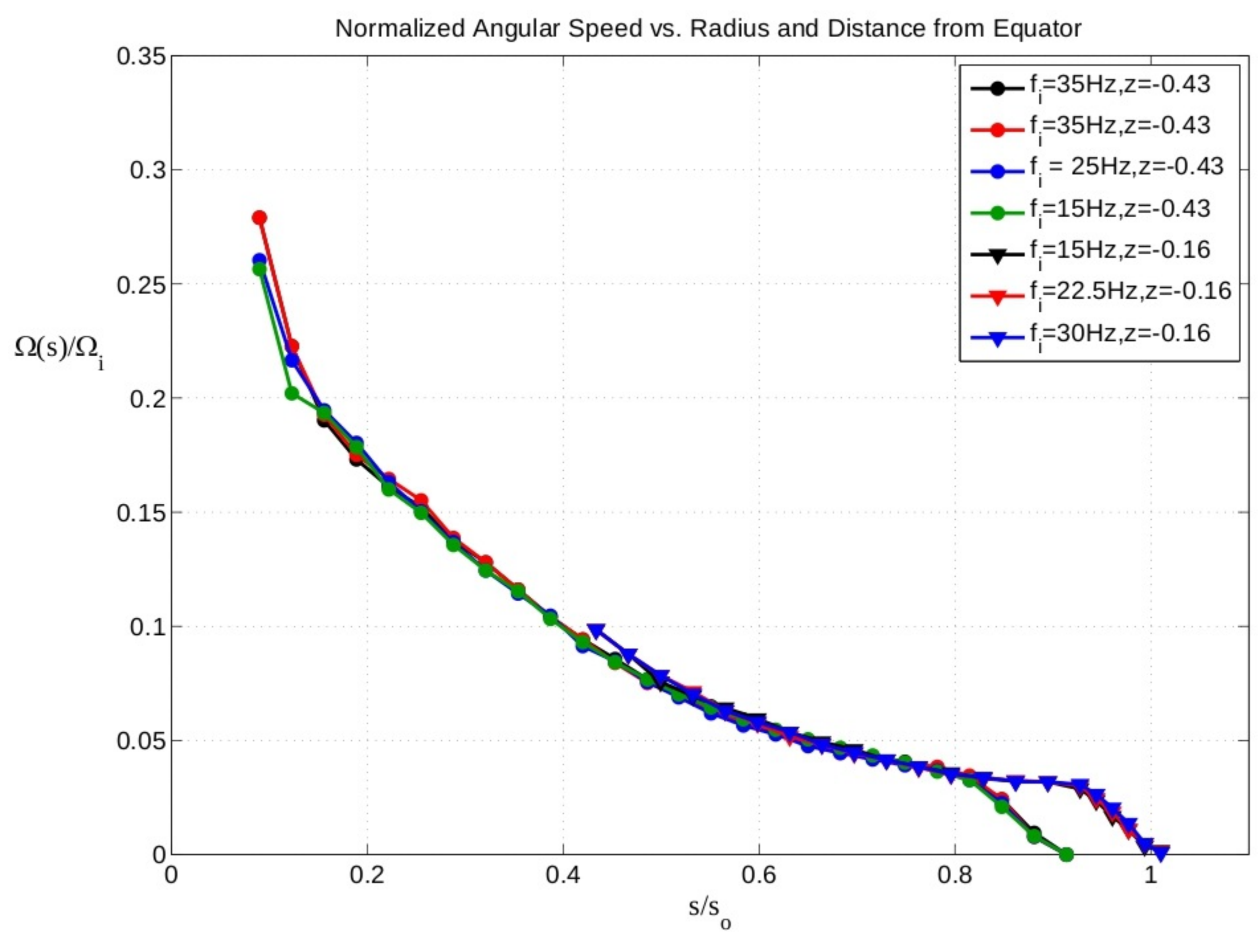
$$\delta\omega_{nlm} = m \int_0^R \int_0^{2\pi} K_{nlm}(r,\theta)\Omega(r,\theta) r \, dr \, d\theta$$



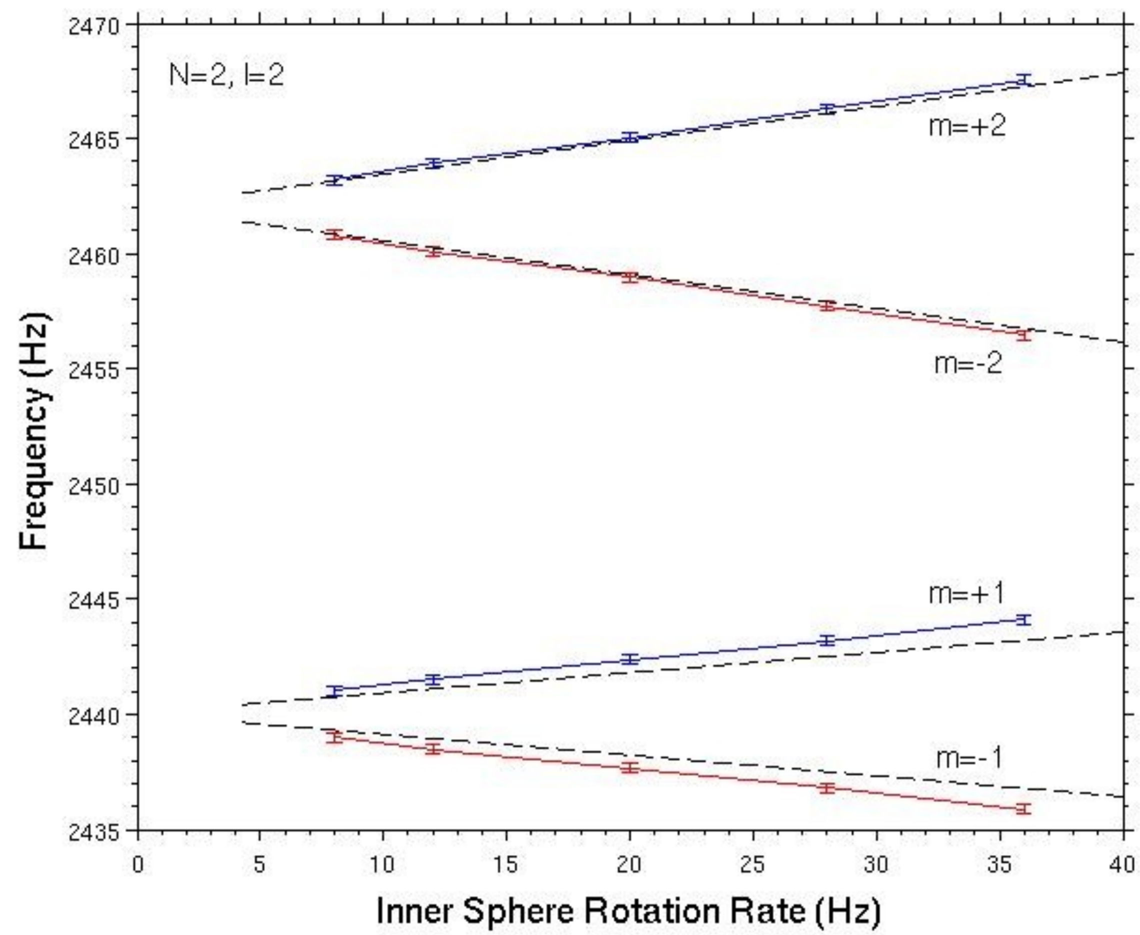




$(1, 5, m)$
$(2, 2, m)$
$m = \pm 1$
$m = \pm 2$
PSD
10^{-4}
10^{-6}
10^{-8}
10^{-10}
2300 2310 2320 2330 2340 2350 2360 2370 2380 2390 2400 2410 2420 2430 2440 2450 2460 2470 2480 2490 2500
Frequency [Hz]

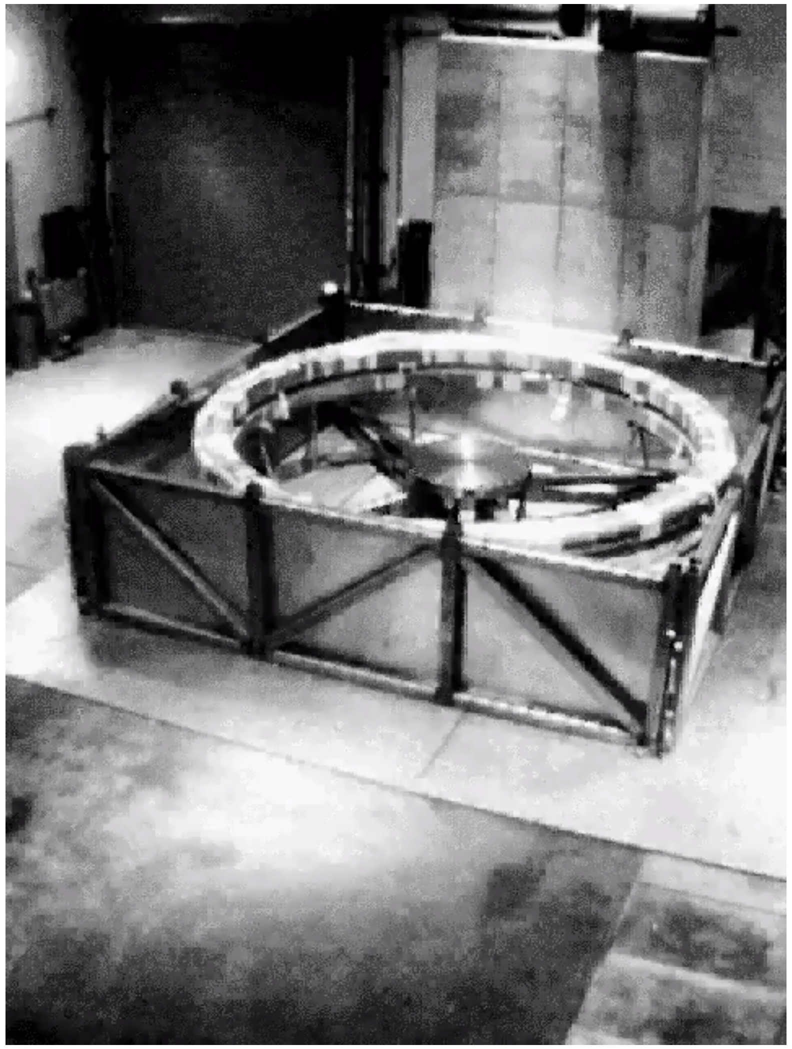


Normalized Angular Speed vs. Radius and Distance from Equator
f_i=35Hz,z=-0.43
f_i=35Hz,z=-0.43
f_i = 25Hz,z=-0.43
f_i=15Hz,z=-0.43
f_i=15Hz,z=-0.16
f_i=22.5Hz,z=-0.16
f_i=30Hz,z=-0.16
$\Omega(s)/\Omega_i$
0 0.05 0.1 0.15 0.2 0.25 0.3 0.35
0 0.2 0.4 0.6 0.8 1
s/s_o

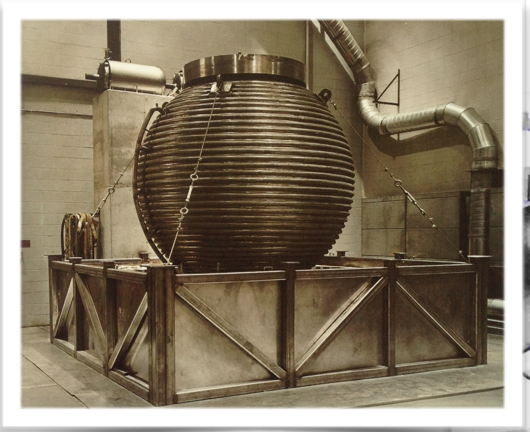


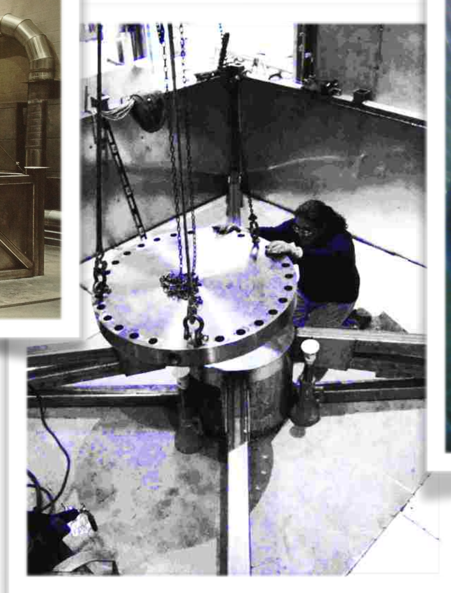
N=2, l=2
m=+2
m=-2
m=+1
m=-1
Frequency (Hz)
2435 2440 2445 2450 2455 2460 2465 2470
0 5 10 15 20 25 30 35 40
Inner Sphere Rotation Rate (Hz)

Summary and outlook

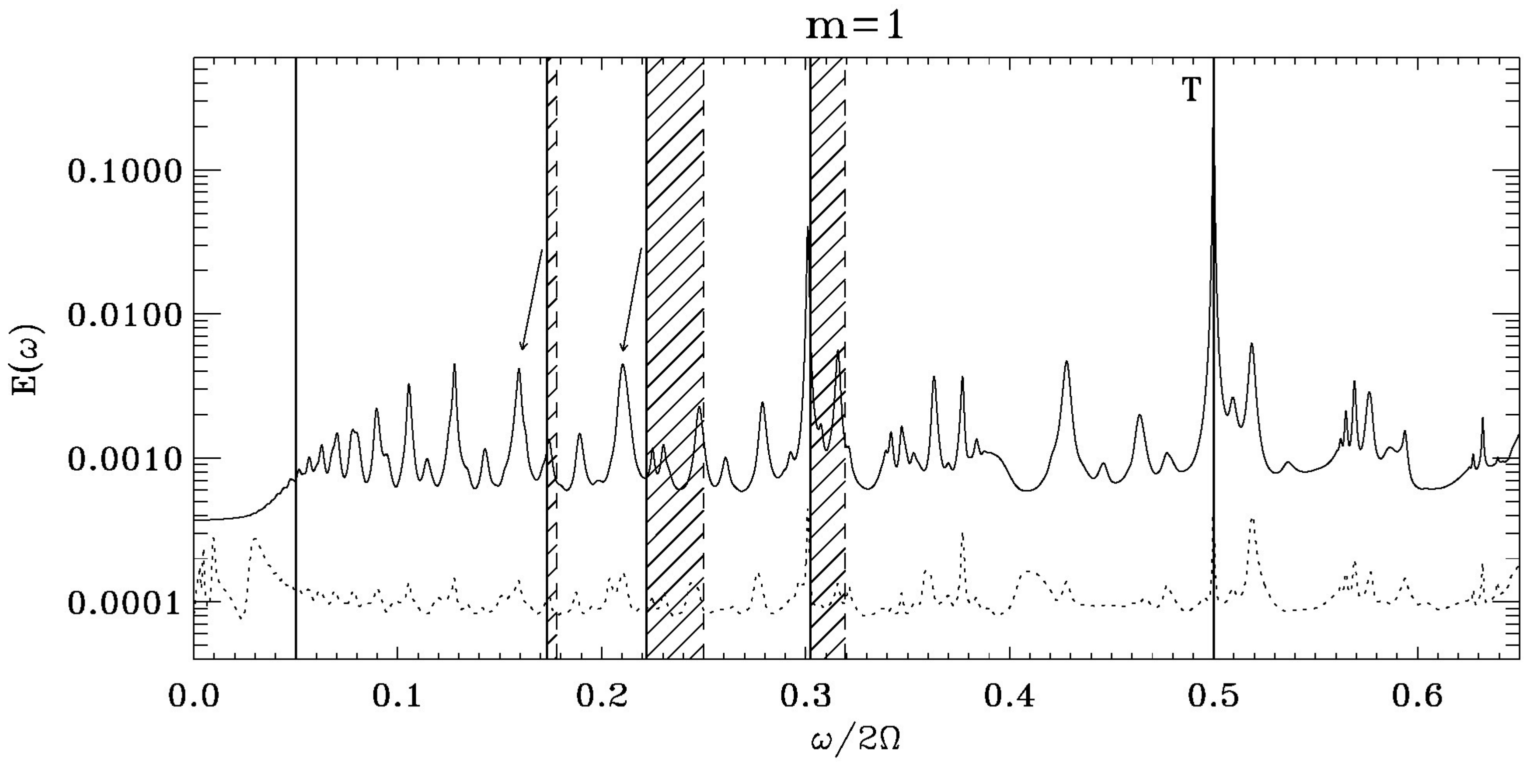


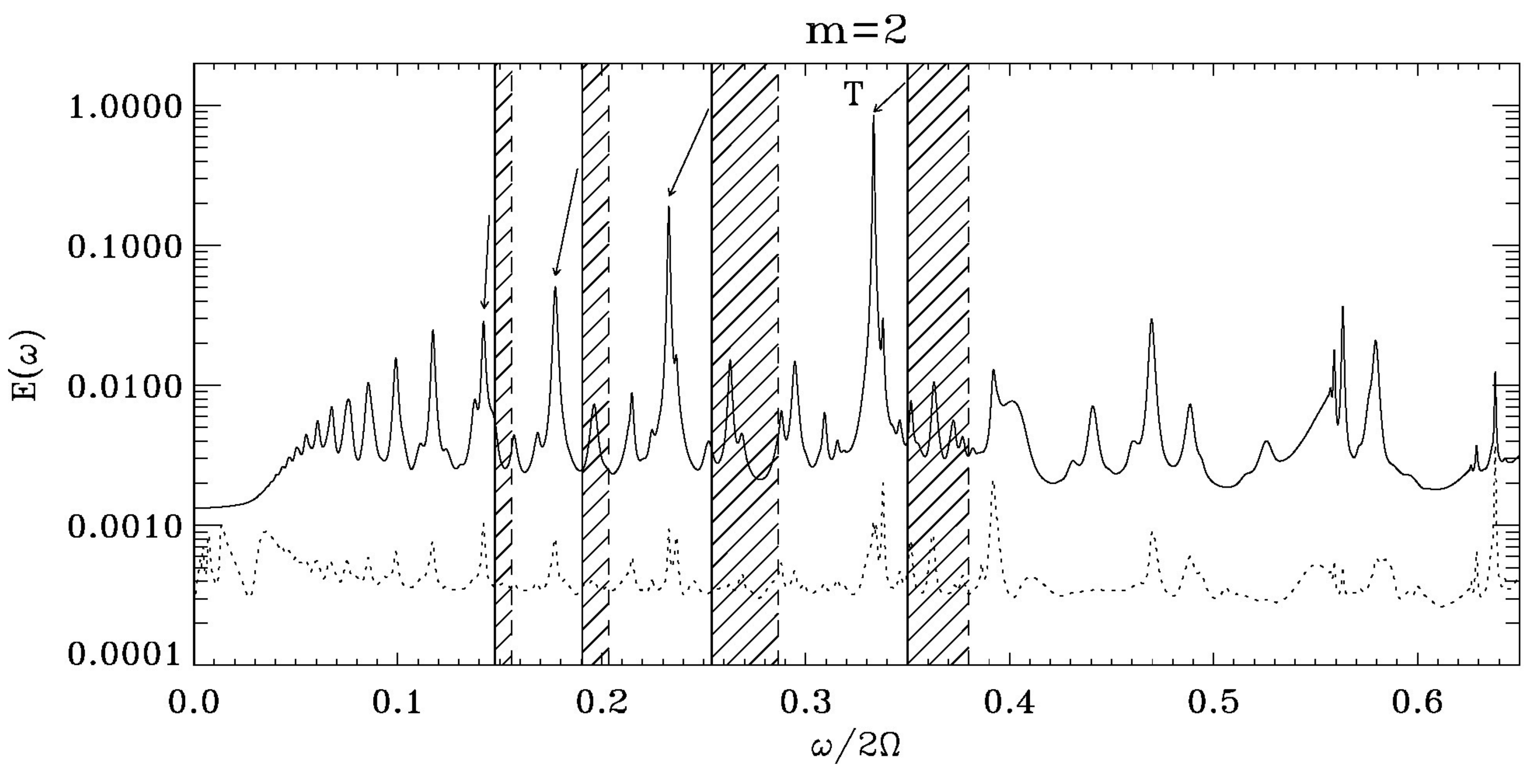
- Although dynamo action has not been achieved yet in a planetary configuration, we've learned many other things.
- Differential rotation excites inertial modes. The precise mechanism is still not well understood.
- Non-linear self interactions of inertial modes can redistribute angular momentum.
- Physics behind inertial mode excitation might help elucidate the origin of Coriolis-restored modes in stars (observed for the first time very recently).
- Gravito-inertial modes have great asteroseismic potential in rapidly rotating stars.





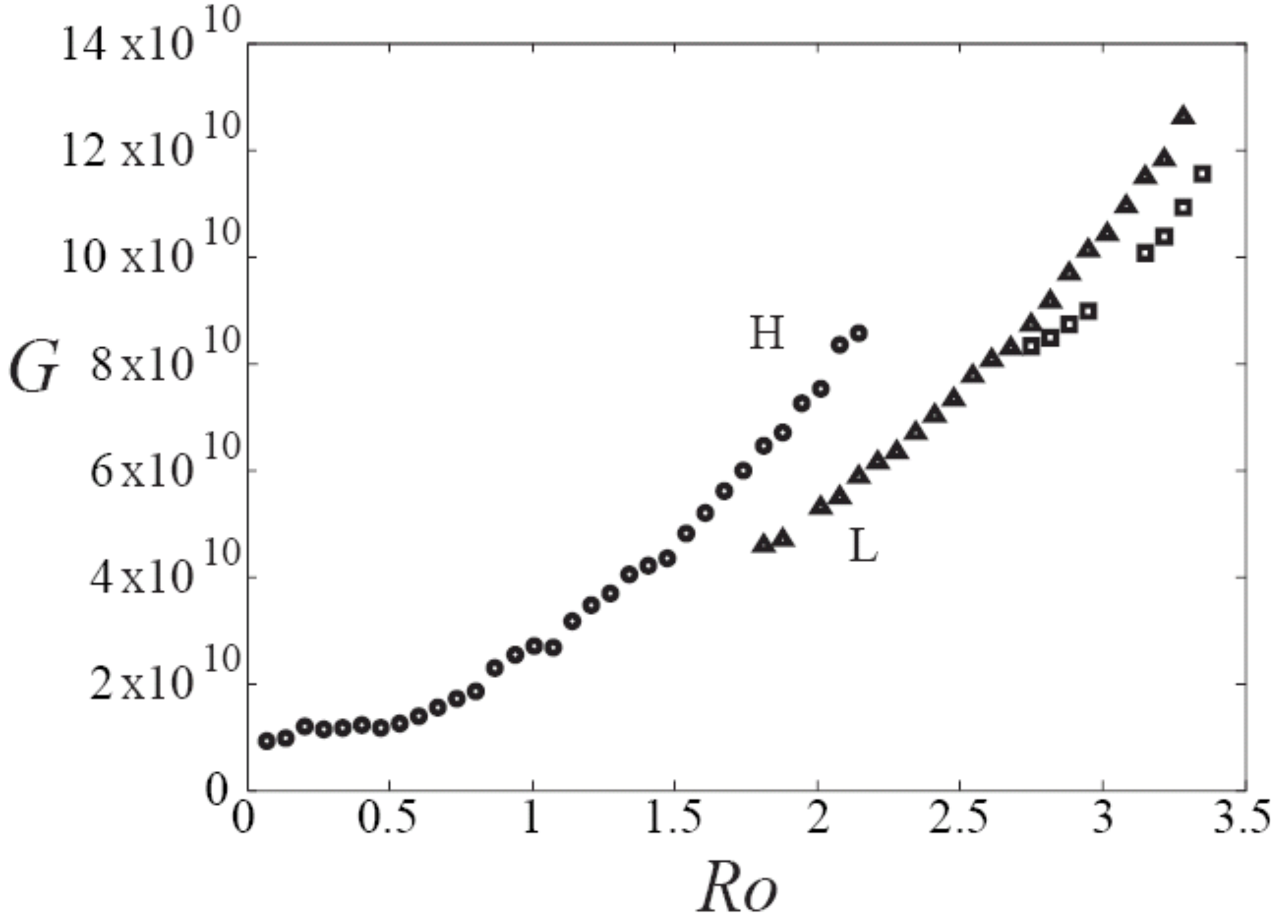
Why do we see only the ones we see?

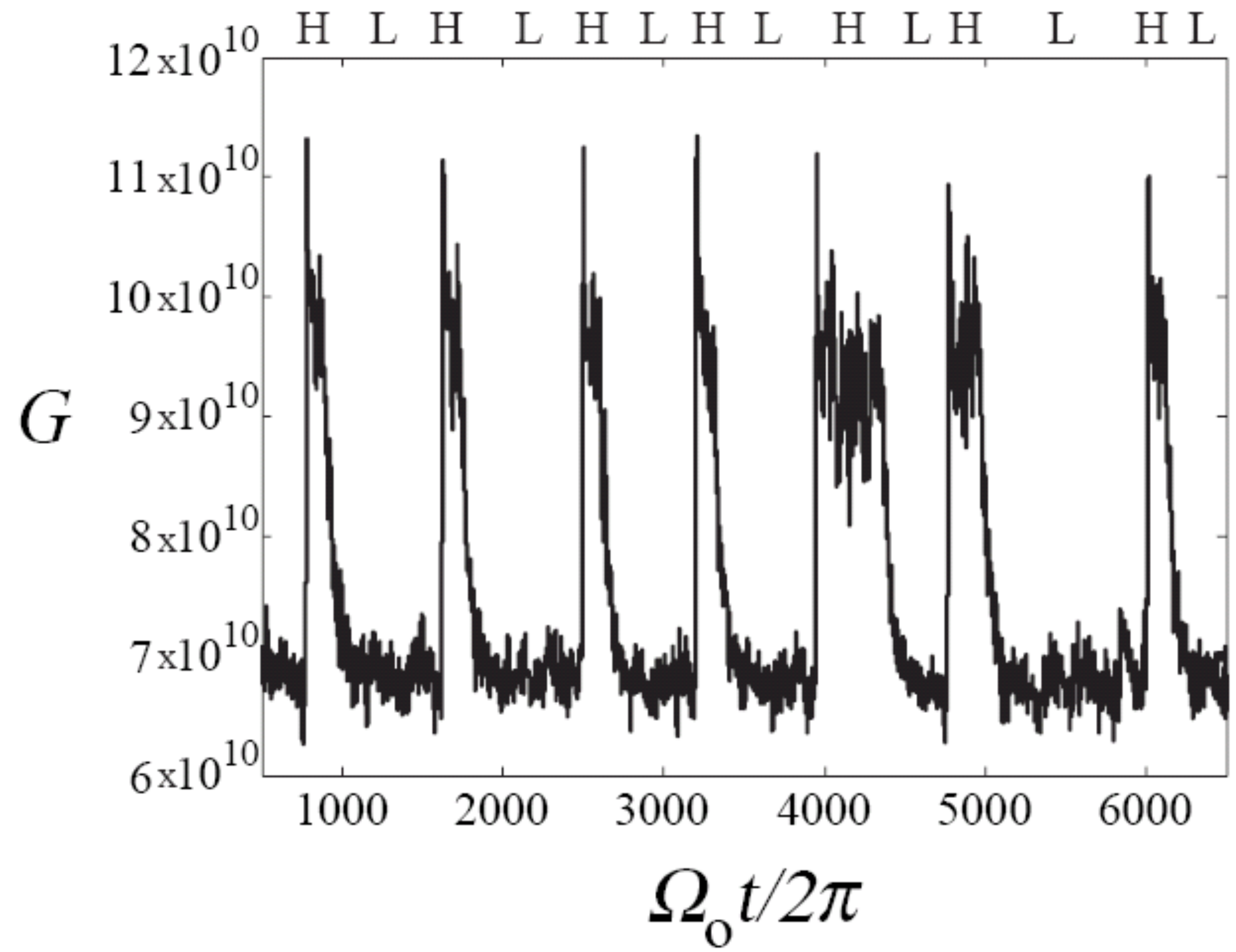




They seem to be the least damped in a shell

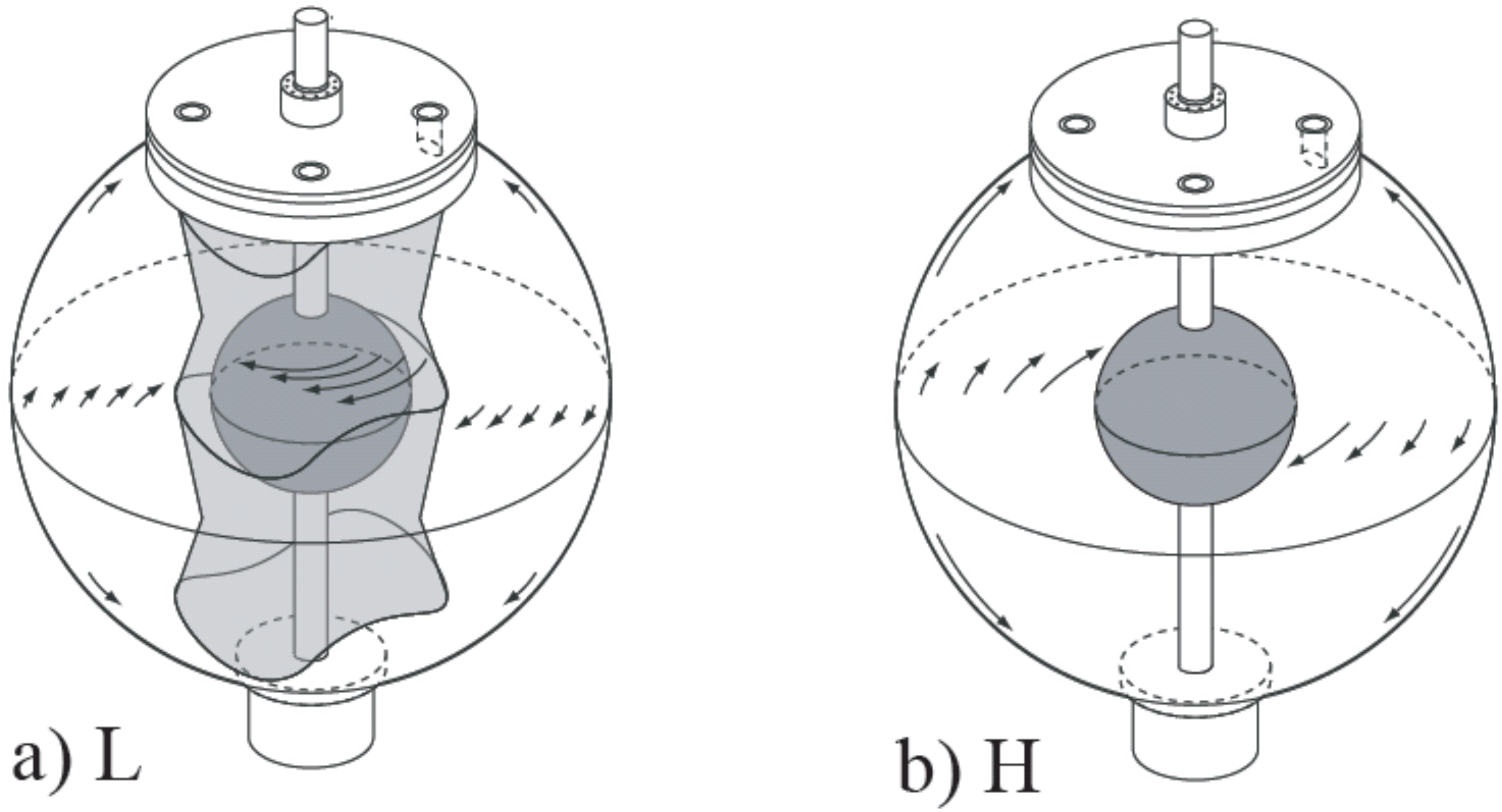
More water experiments: Bistability





$$G = \frac{T}{\rho \nu^2 a}$$

More water experiments: Bistability



Zimmerman, Triana & Lathrop, PoF 23, 065104, 2010

Lets look at the equations of motion for an electrically conducting fluid:

$$\frac{\partial \mathbf{u}}{\partial t} + Ro(\mathbf{u} \cdot \nabla)\mathbf{u} + 2\Omega \times \mathbf{u} = -\nabla p + E\nabla^2 \mathbf{u} + \frac{\Lambda}{Rm}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

$$\frac{1}{Ro}\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{Rm}\nabla^2 \mathbf{B}$$

These are dimensionless,
with no reference to a specific fluid system.
(The Buckingham-Pi Theorem)

Ekman number	$E \equiv \dfrac{\nu}{\Omega L^2}$	$\dfrac{\text{Viscous forces}}{\text{Coriolis forces}}$
Rossby number	$Ro \equiv \dfrac{U}{\Omega L}$	$\dfrac{\text{Inertial forces}}{\text{Coriolis forces}}$
Magnetic Reynolds	$Rm \equiv \dfrac{UL}{\eta}$	$\dfrac{\text{Fluid advection}}{\text{Magnetic diffusivity}}$
Magnetic Prandtl	$Pm \equiv \dfrac{\nu}{\eta}$	$\dfrac{\text{Viscosity}}{\text{Magnetic diffusivity}}$
Elsasser number	$\Lambda \equiv \dfrac{\sigma B_o^2}{\rho\Omega}$	$\dfrac{\text{Lorentz forces}}{\text{Coriolis forces}}$