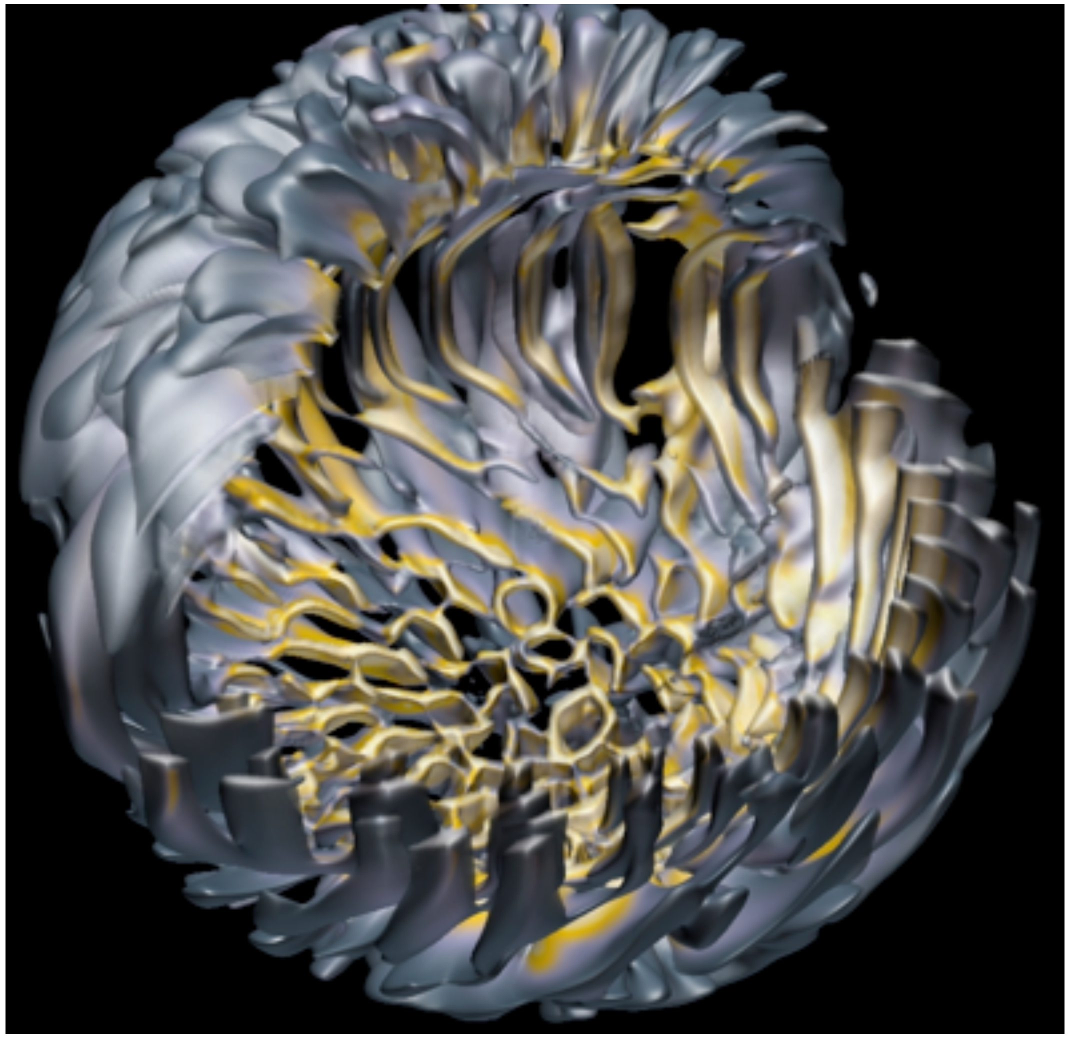
Exploring the Sun and its effects on the Earth's atmosphere and physical environment...

HIGH ALTITUDE OBSERVATORY

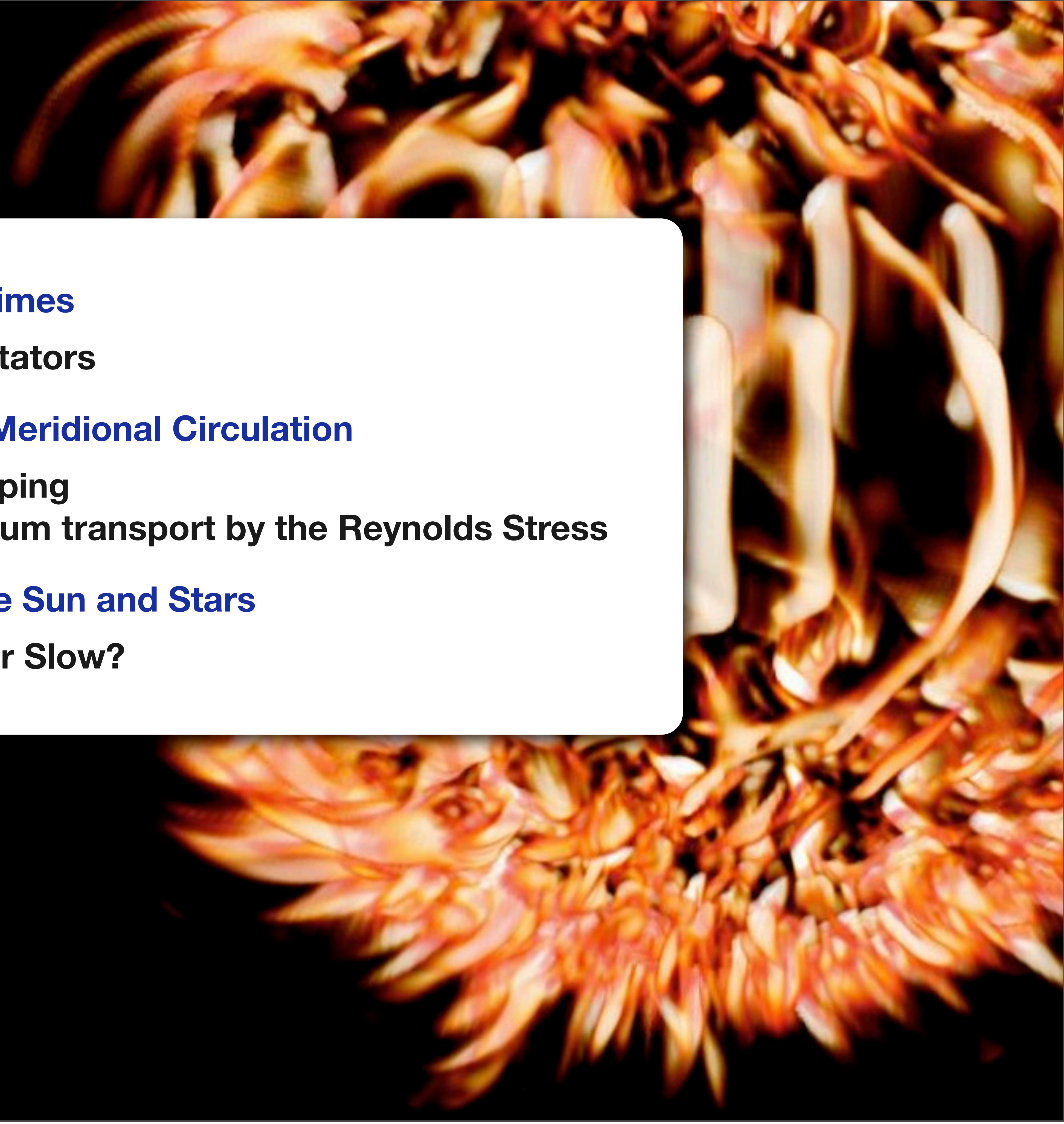


Meridional Circulation in Solar and Stellar Convection Zones

Mark Miesch
and
Nicholas Featherstone

(HAO/NCAR)

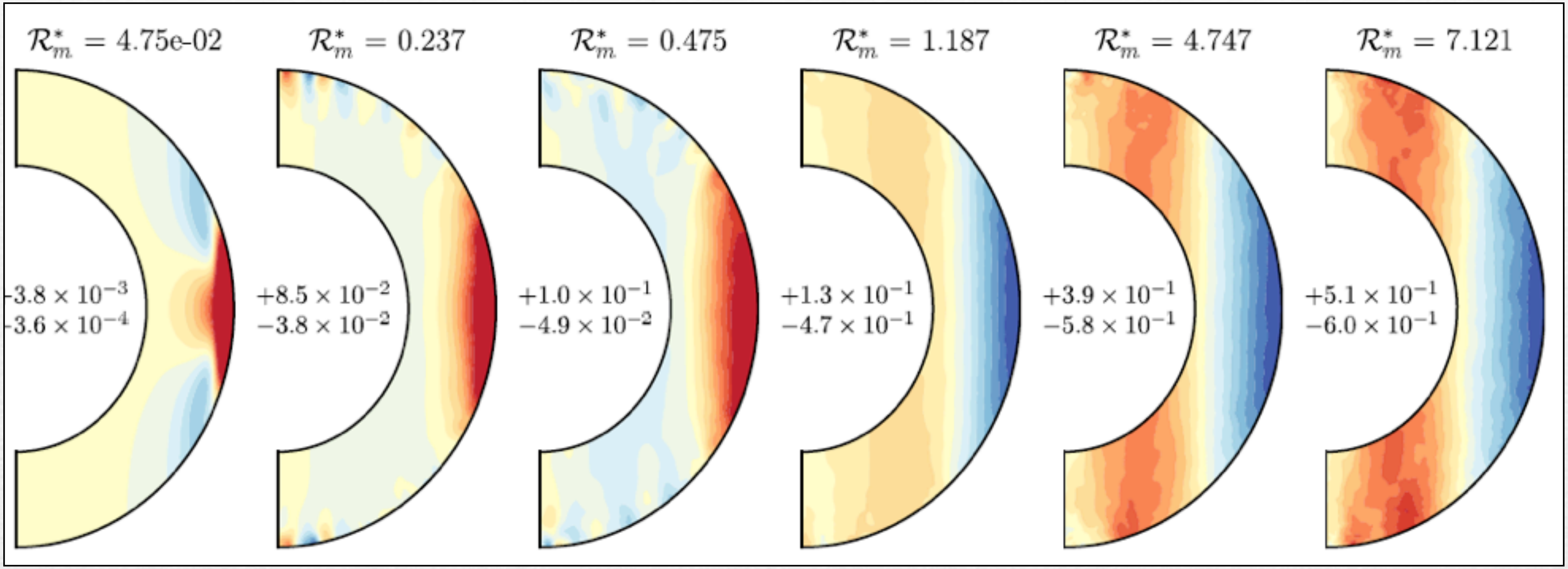
Differential Rotation and Magnetism Across the HR Diagram,
Nordita, Stockholm, Sweden, 8 April - 03 May, 2013



Outline

- **1) Mean Flows Regimes**
  - **Fast and slow rotators**
- **2) Maintenance of Meridional Circulation**
  - **Gyroscopic Pumping**
  - **Angular momentum transport by the Reynolds Stress**
- **3) Application to the Sun and Stars**
  - **Is the Sun Fast or Slow?**

Solar and Anti-Solar Differential Rotation

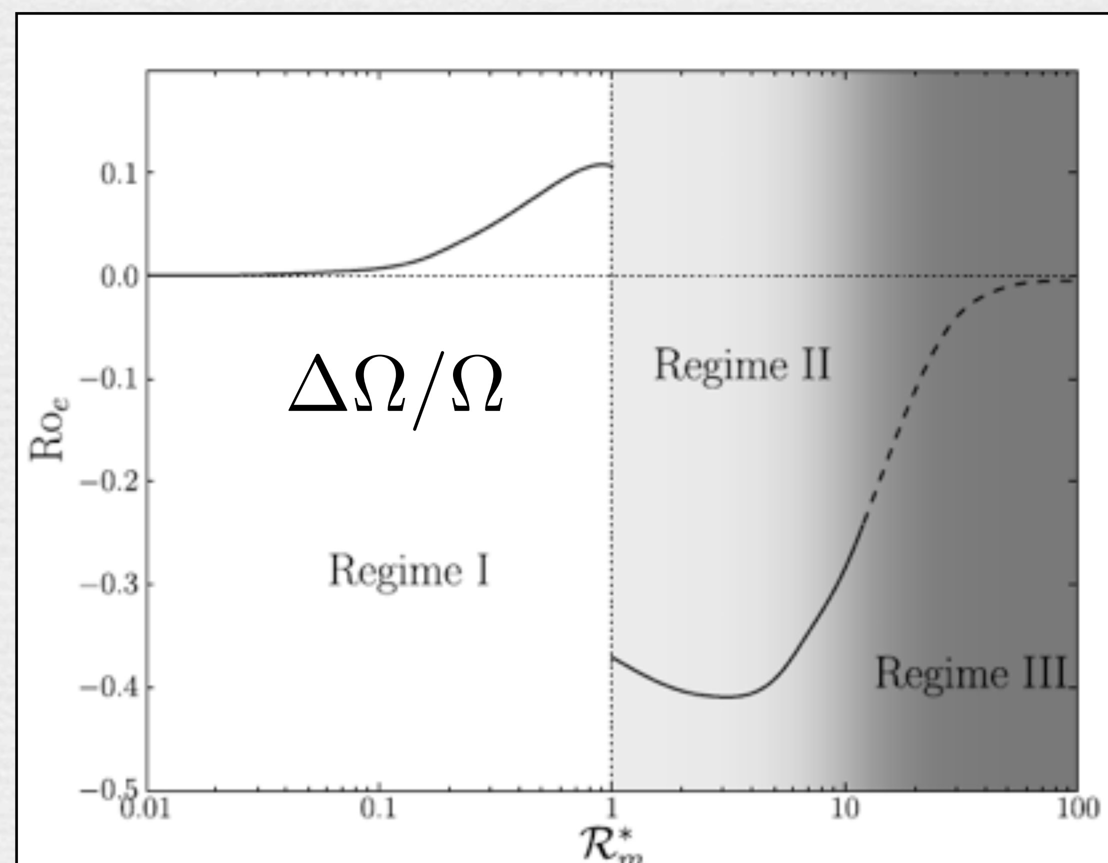


$\mathbf{\Omega}$

$$R_o = \frac{U}{2\Omega L}$$

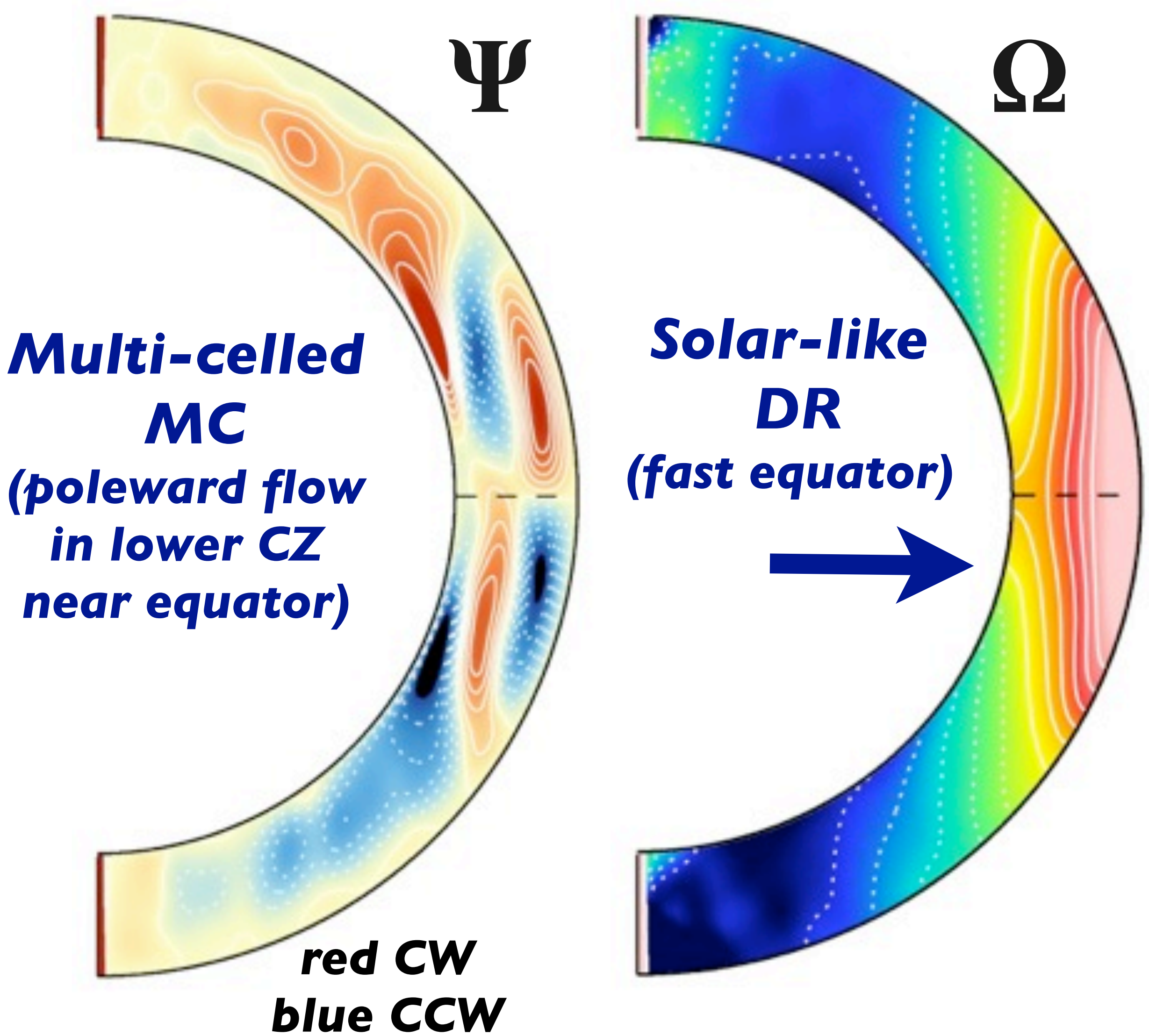
Gastine, Wicht & Aurnou (2013)

See also Gilman (1977), DeRosa et al (2002), Aurnou et al (2007), Augustson et al (2011), Kapyla et al (2011), Guerrero eta al (2013), Featherstone & Miesch (2013)

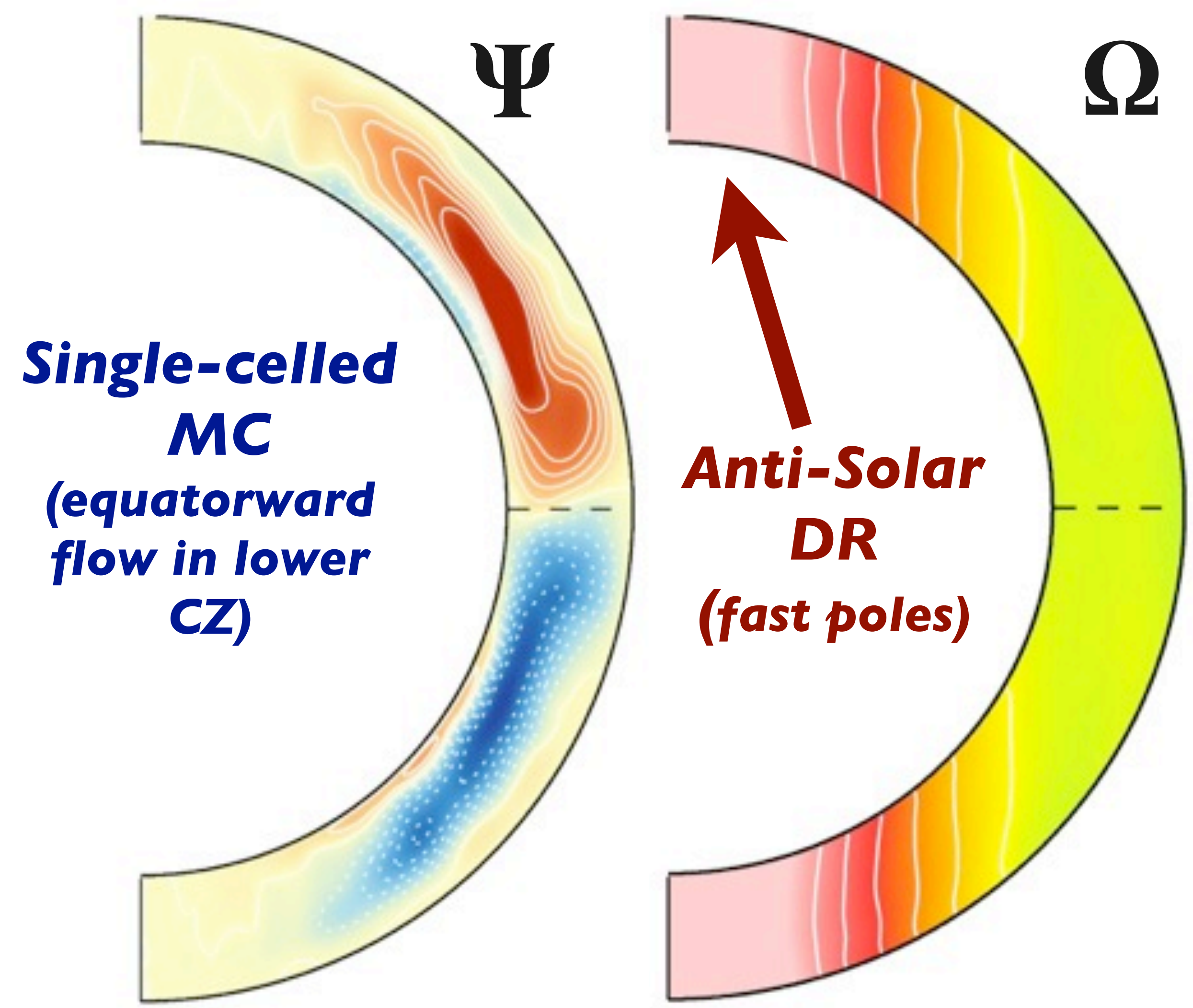


Simulations suggest two different regimes for the Mean Flows

Fast Rotation (Ro ≲ 0.3)

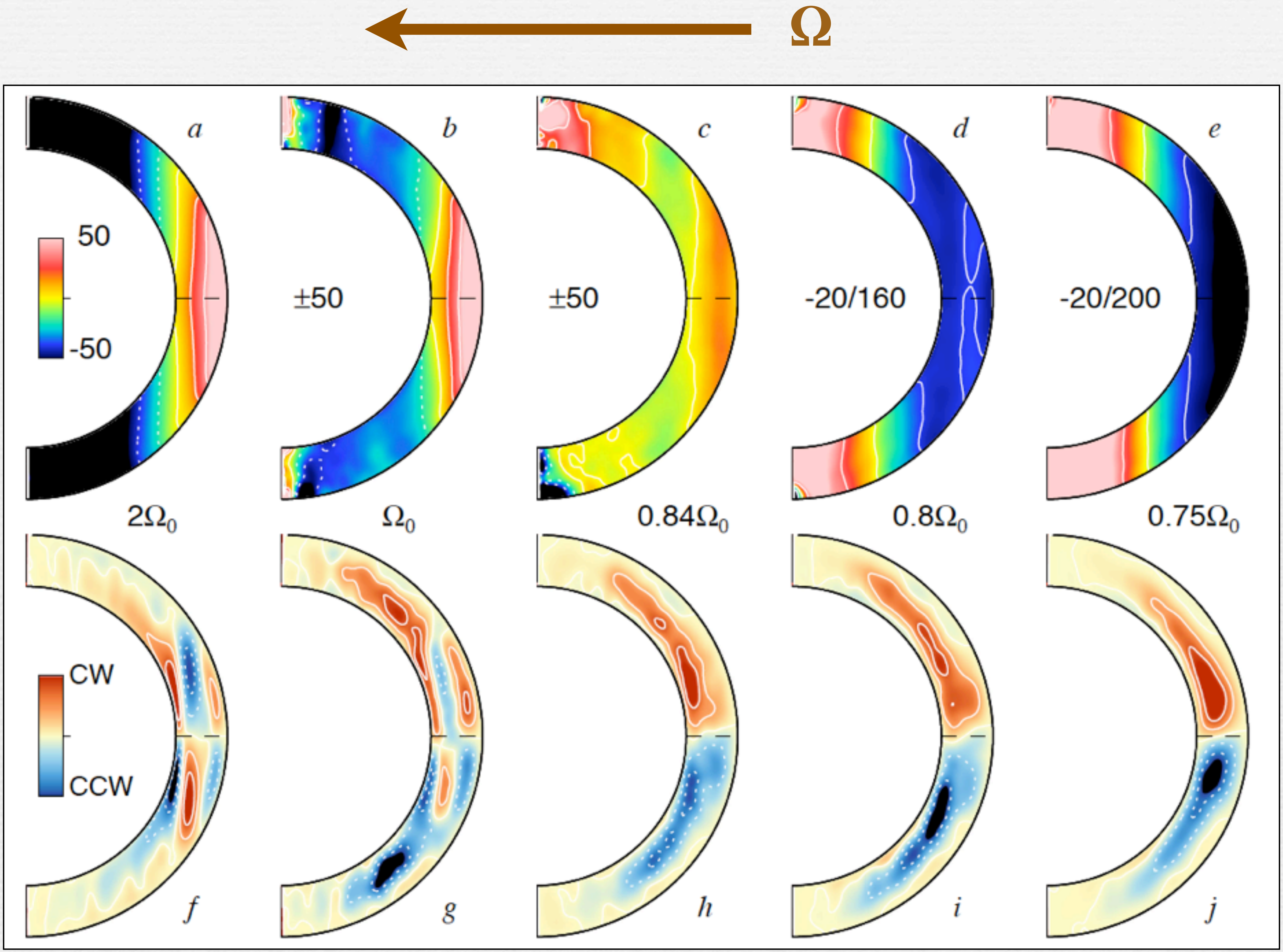


Slow Rotation (Ro ≳ 0.9)



Featherstone & Miesch (2013)

Flux-Transport dynamo models may have difficulty operating in rapidly-rotating stars

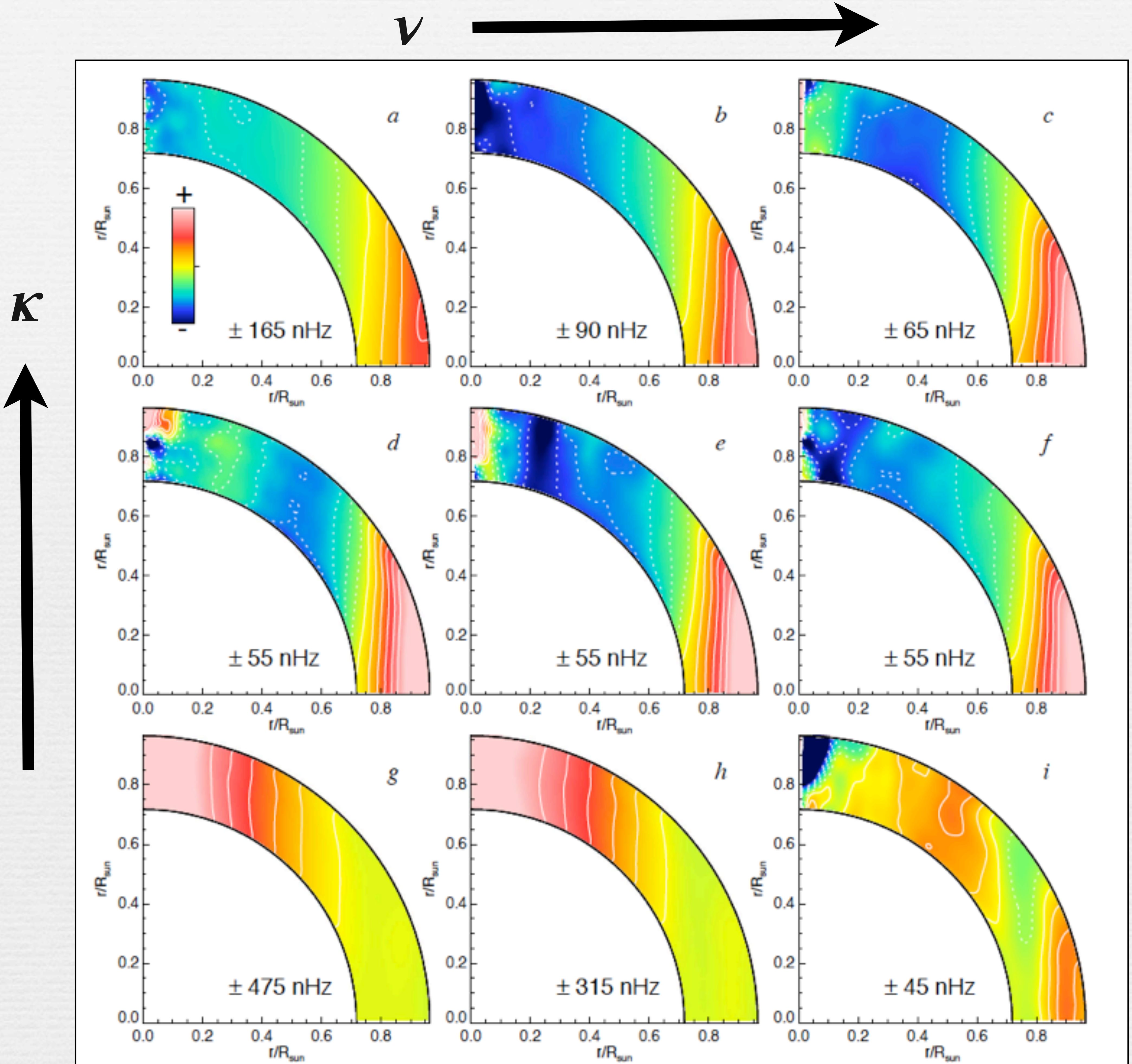

Ω
a
b
c
d
e
50
-50
±50
±50
-20/160
-20/200
2Ω0
Ω0
0.84Ω0
0.8Ω0
0.75Ω0
CW
CCW
f
g
h
i
j

Regimes can be achieved by varying Ω or by varying dissipation

ν and κ

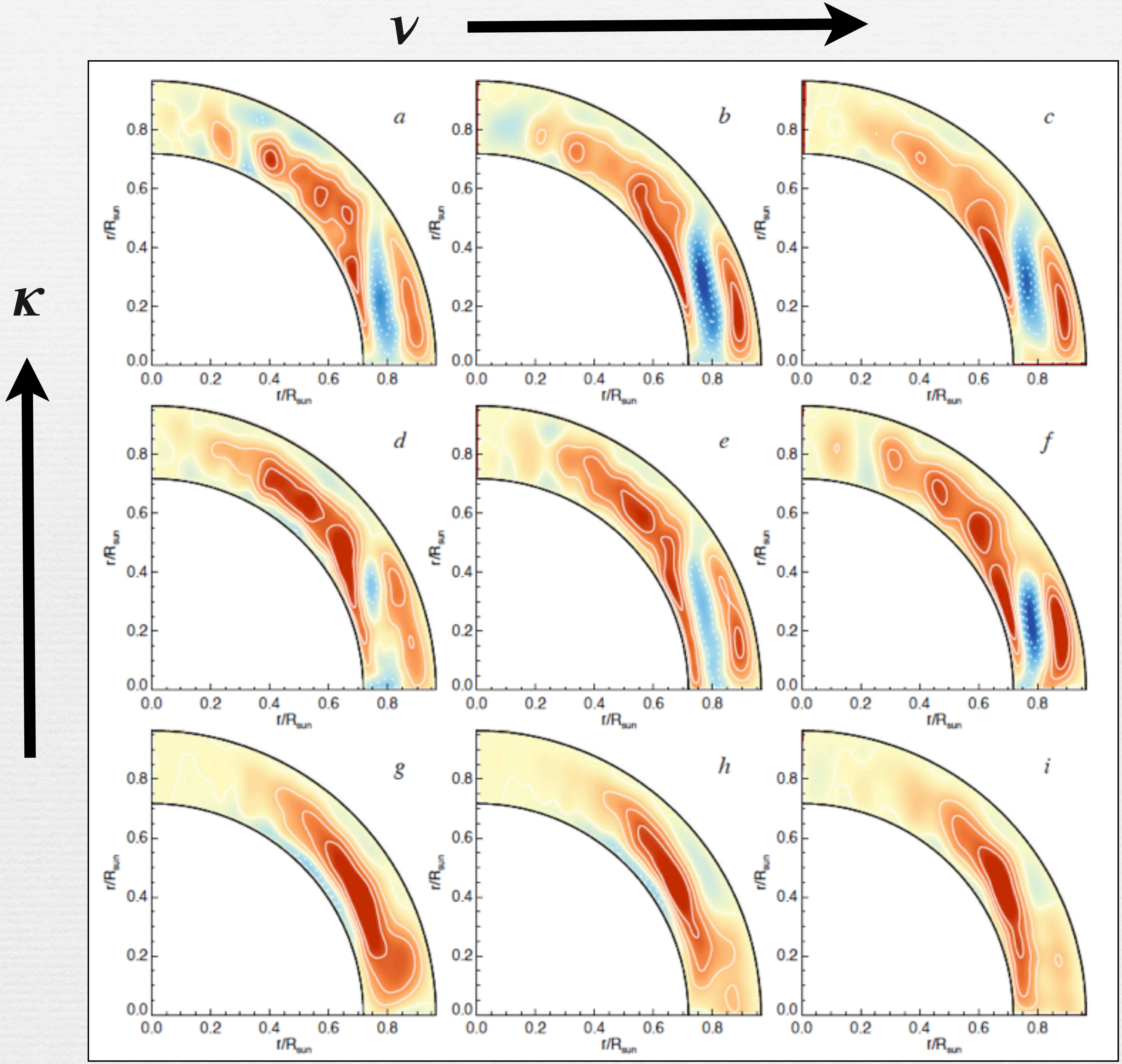
$$\kappa \left.\frac{\partial S}{\partial r}\right|_{\text{top}} \propto L$$

$$R_o = \frac{U}{2\Omega L}$$



Regimes can be achieved by varying Ω or by varying dissipation

ν and κ



$$R_o = \frac{U}{2\Omega L}$$

2) GP

Comes from the zonal component of the MHD momentum equation, averaged over longitude and time

$$\langle \rho \mathbf{v}_m \rangle \cdot \boldsymbol{\nabla} \mathcal{L} = \mathcal{F}$$

$$\frac{\partial}{\partial t}(\rho \mathcal{L}) + \langle \rho \mathbf{v}_m \rangle \cdot \boldsymbol{\nabla} \mathcal{L} = \mathcal{F}$$

No assumptions beyond basic MHD!

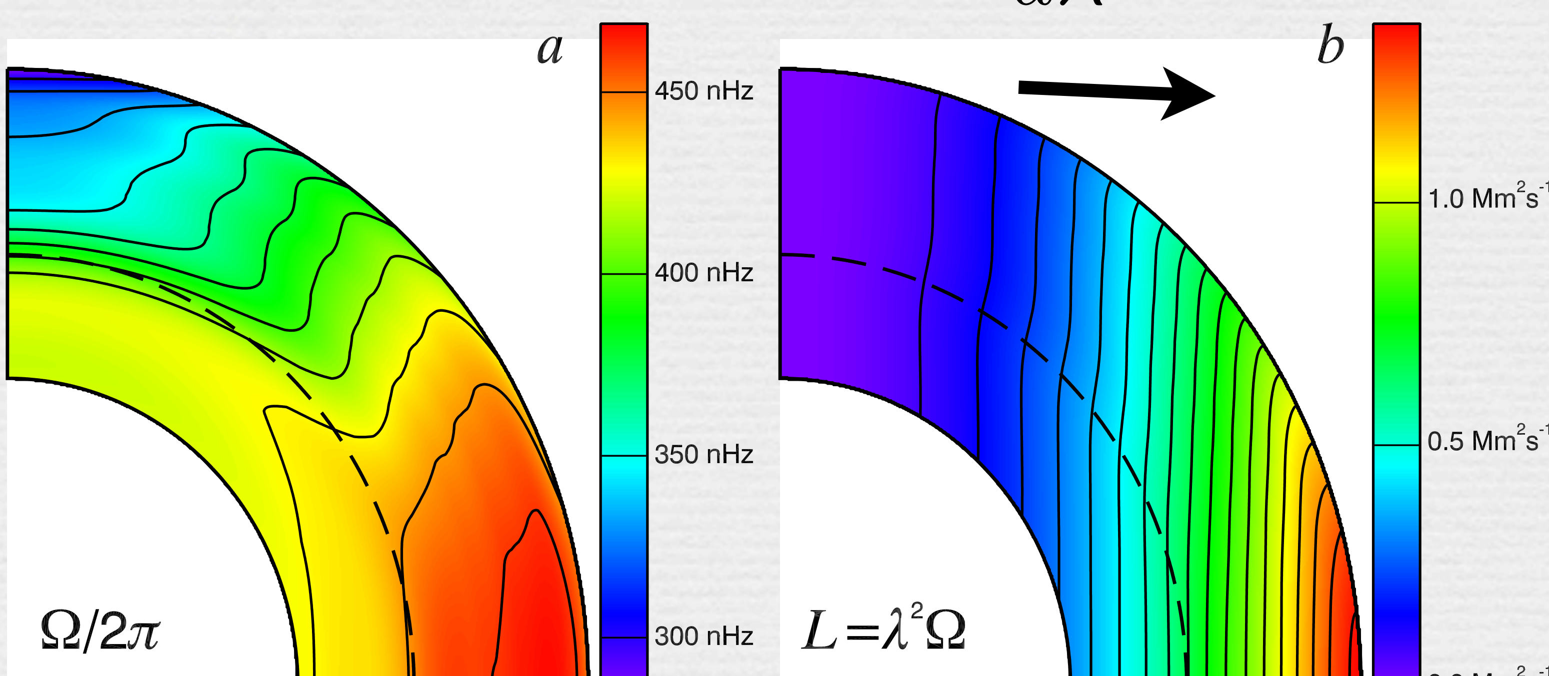
$$\mathcal{F} = -\boldsymbol{\nabla} \cdot \left[\lambda \langle \rho \mathbf{v}' v'_\phi \rangle - \lambda \langle \mathbf{B} B_\phi \rangle - \rho \nu \lambda^2 \boldsymbol{\nabla} \Omega \right]$$

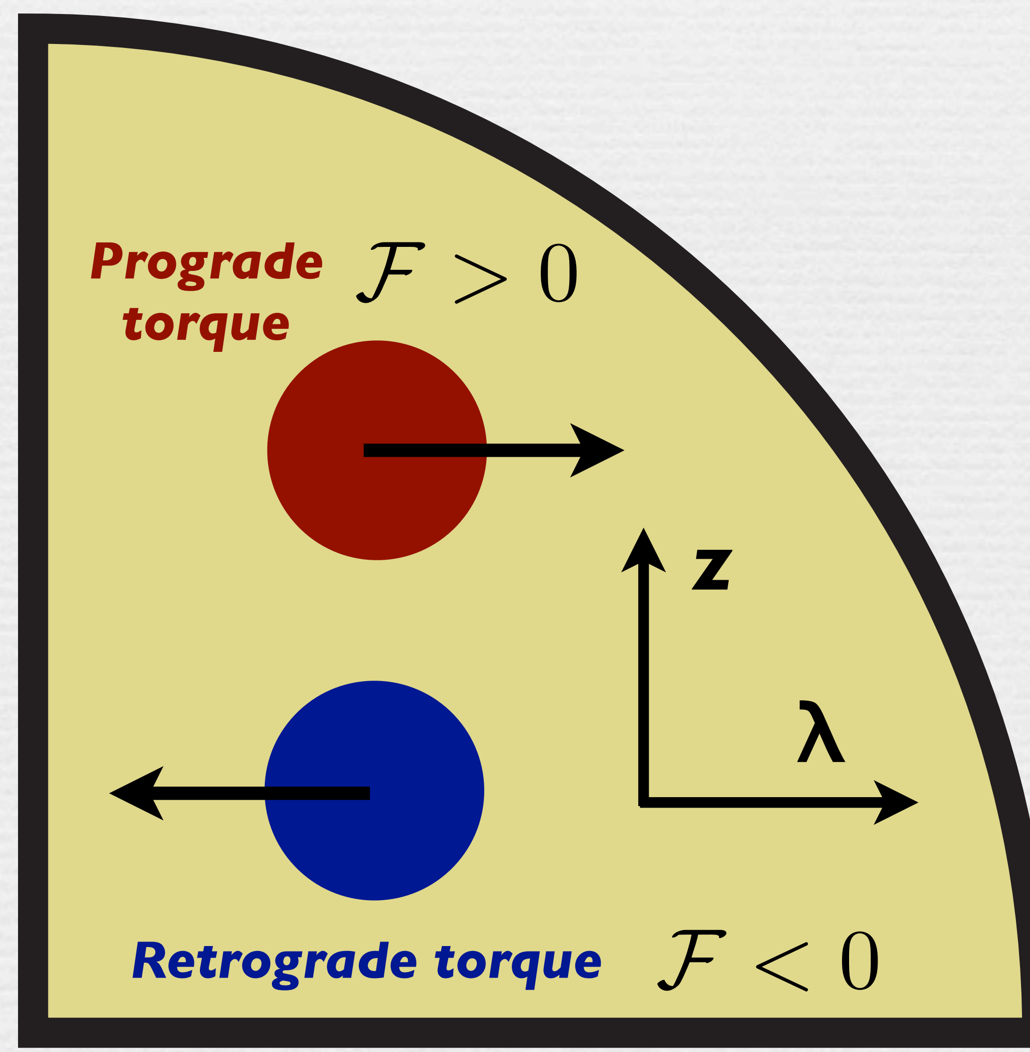
$$\mathcal{L} = \lambda^2 \Omega = \lambda \langle v_\phi \rangle$$

Reynolds stress | **Lorentz force** | **Viscous diffusion**

$$\boldsymbol{\nabla} \mathcal{L} \approx \frac{d\mathcal{L}}{d\lambda} \hat{\boldsymbol{\lambda}}$$

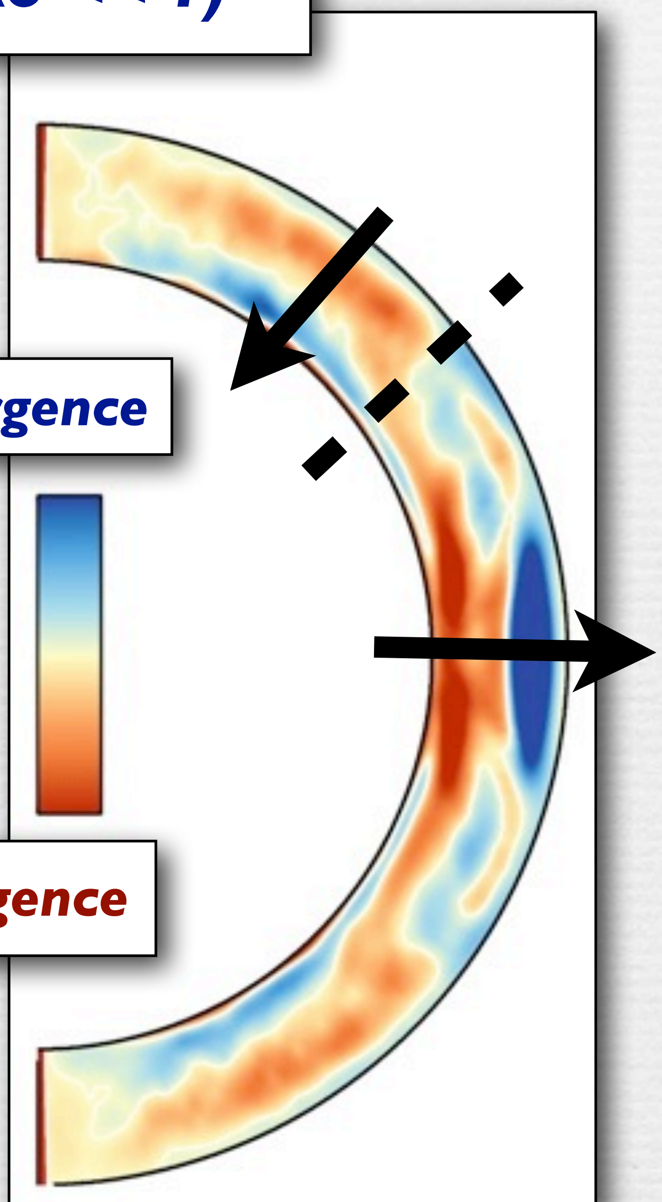
(GONG inversions, courtesy Rachel Howe)





Mean Flow Profiles Determined mainly by convective angular momentum transport

Fast Rotation (Ro << 1)



Convergence

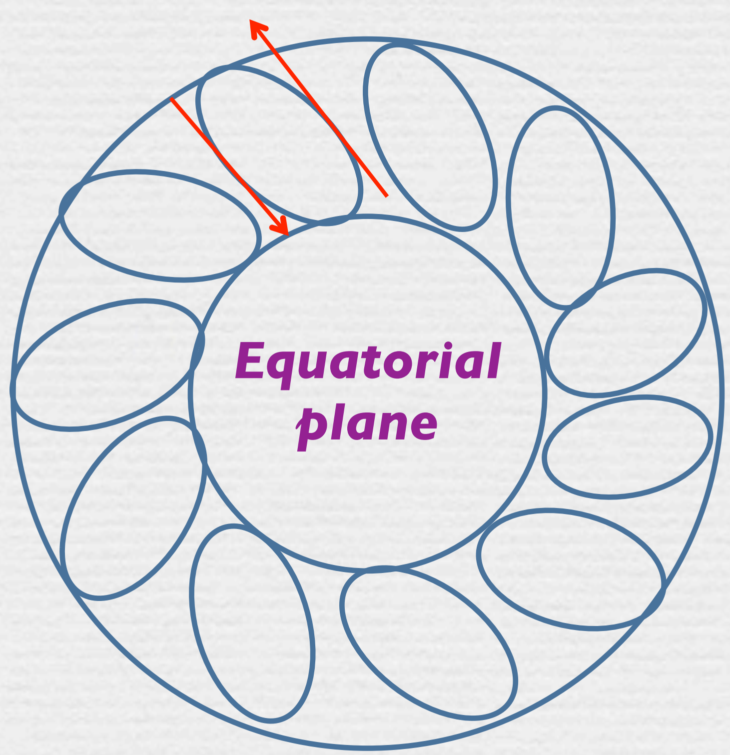
Divergence

$$\mathcal{F}_{RS} = -\nabla \cdot \mathbf{F}_{RS}$$

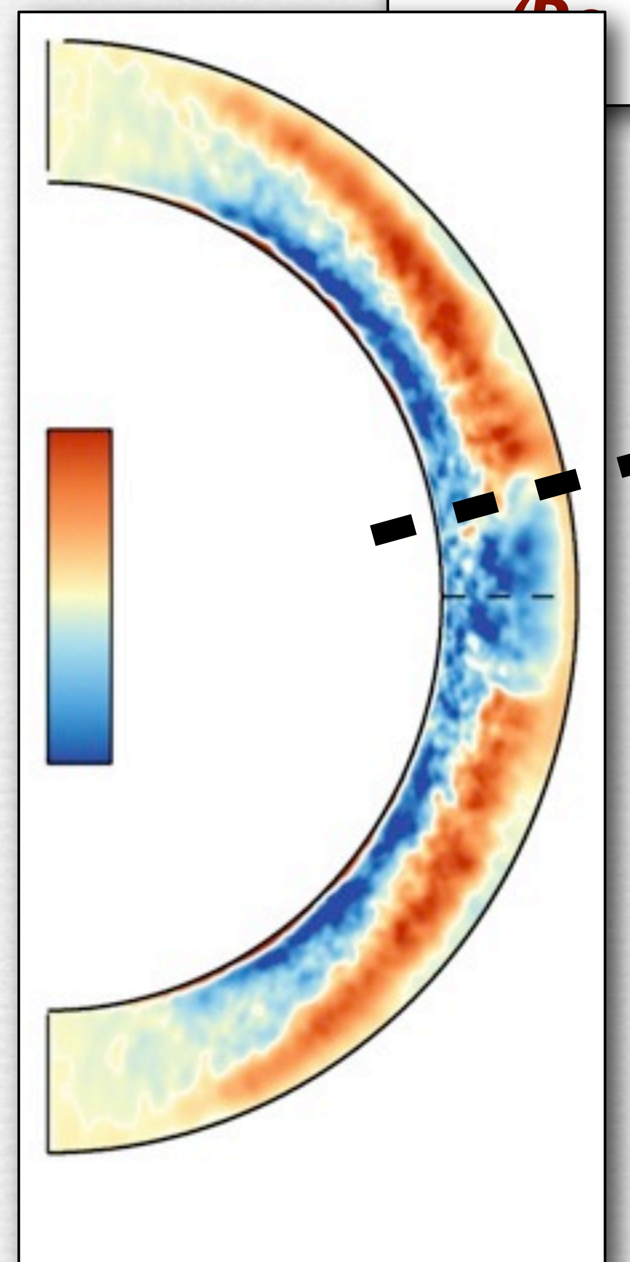
Competition between

Radially inward transport at high latitudes *(ballistic plumes)*

Cylindrically outward transport at low latitudes *(banana cells)*



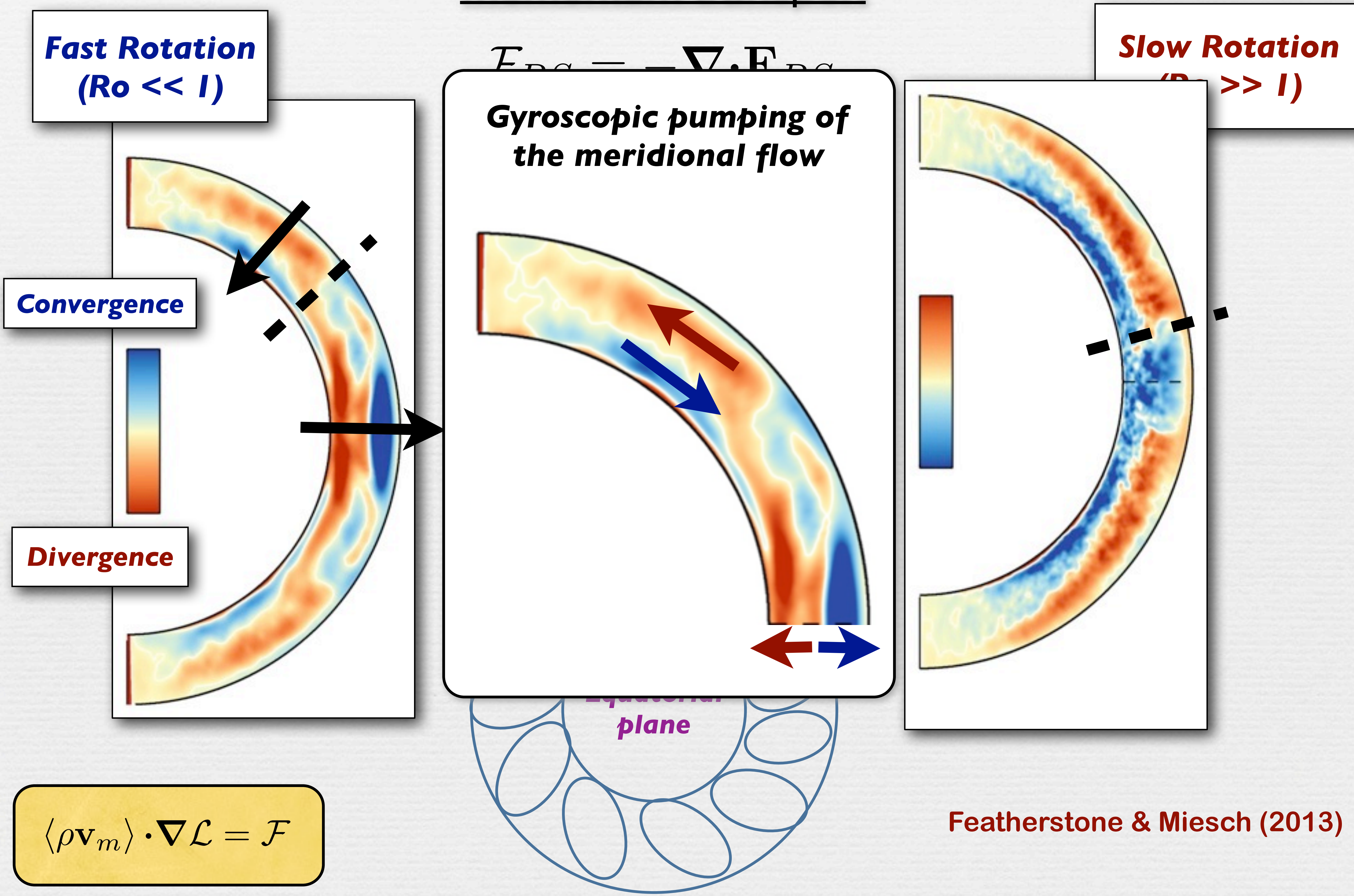
Slow Rotation (Ro >> 1)



$$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$$

Featherstone & Miesch (2013)

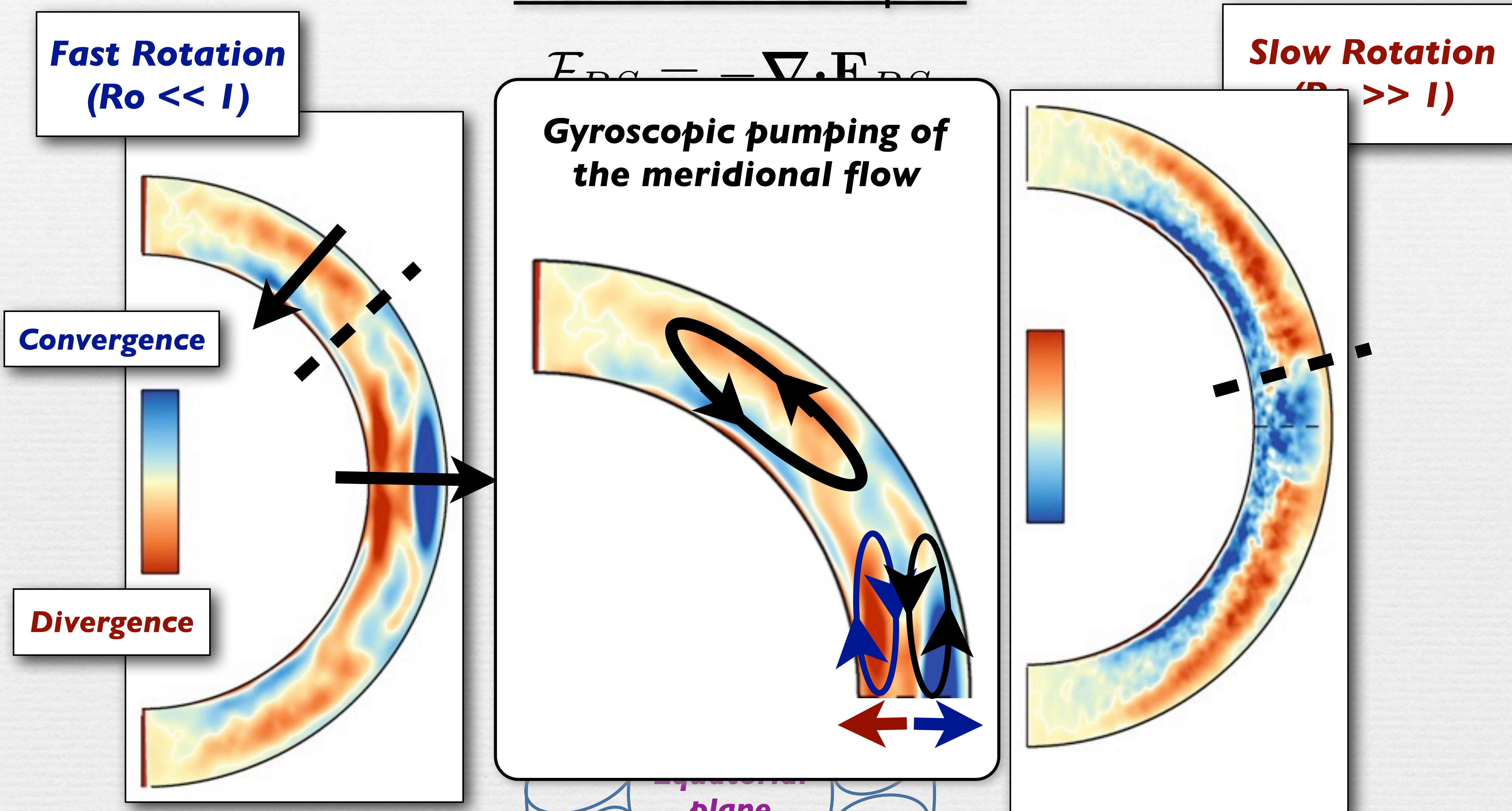
Mean Flow Profiles Determined mainly by convective angular momentum transport

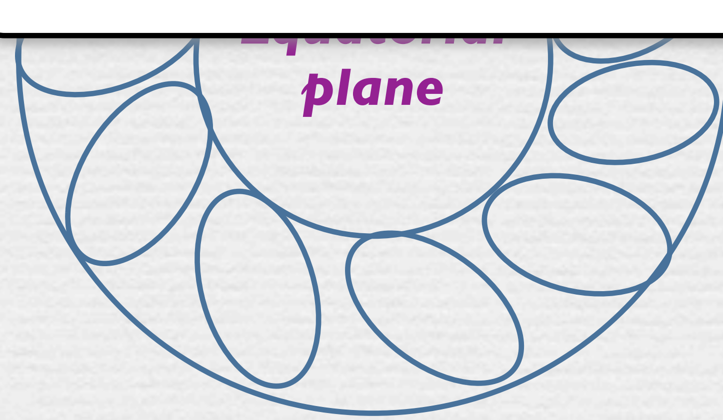


$$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$$

Featherstone & Miesch (2013)

Mean Flow Profiles Determined mainly by convective angular momentum transport

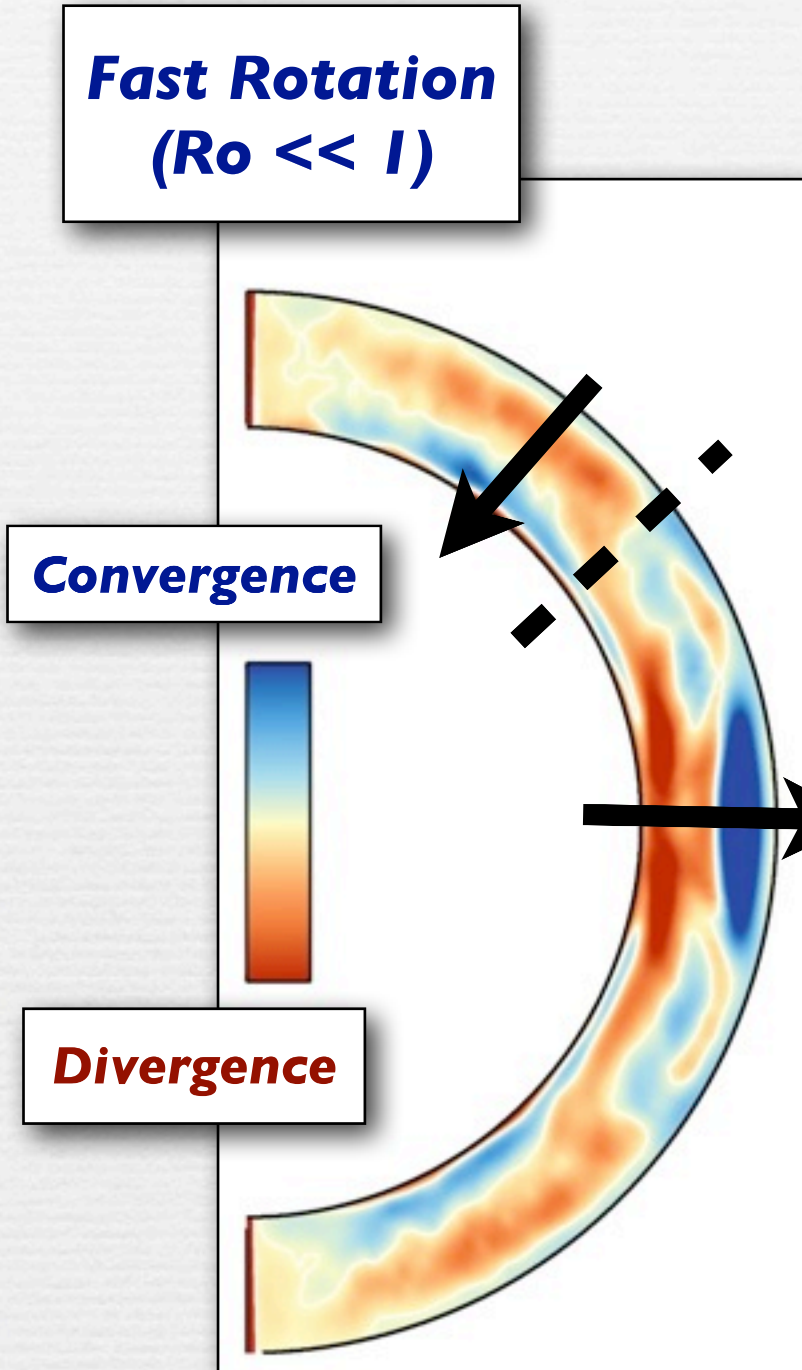


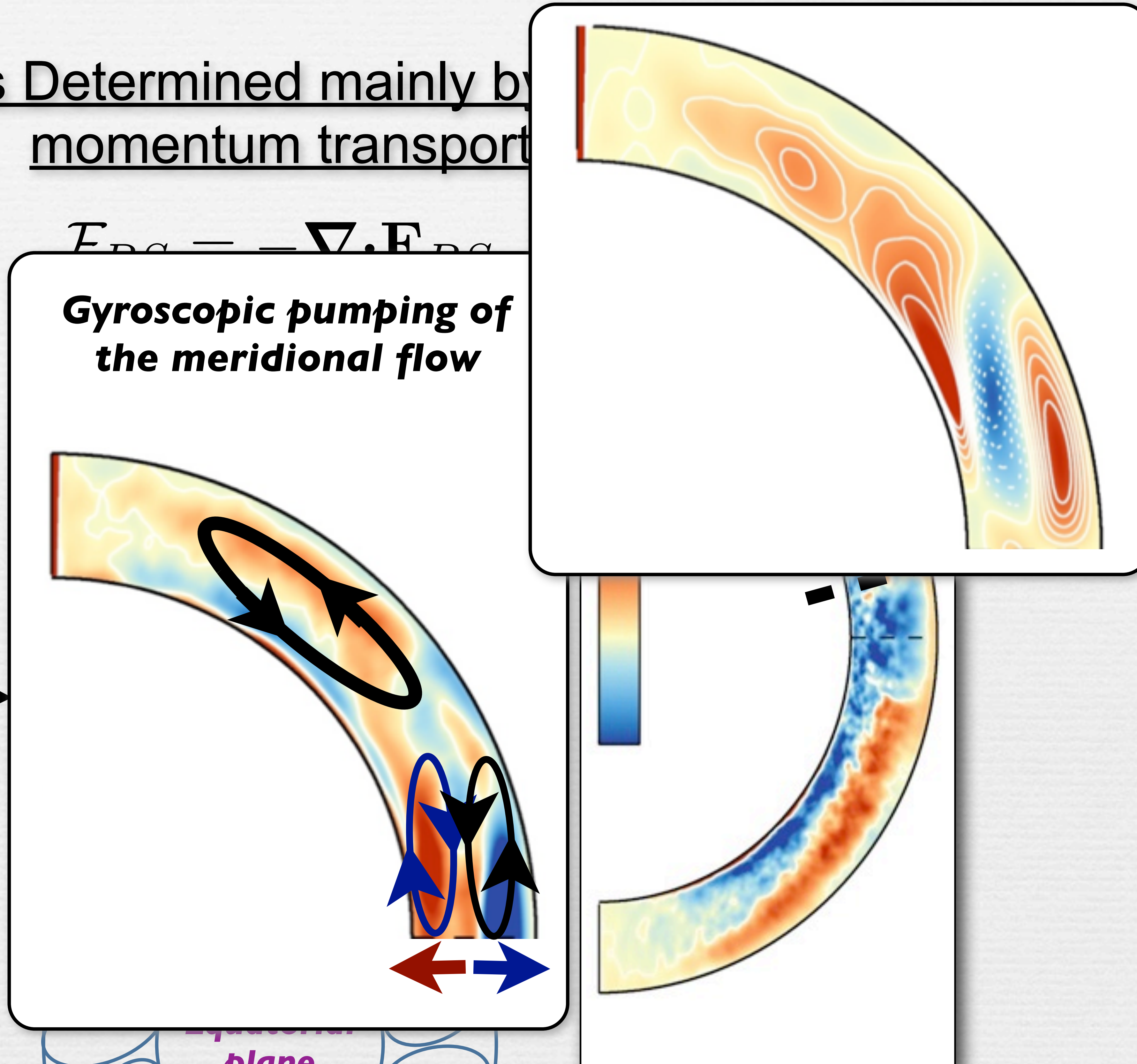


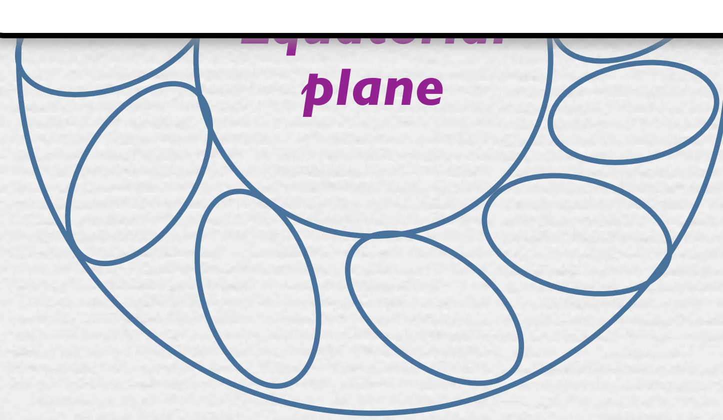
$$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$$

Featherstone & Miesch (2013)

Mean Flow Profiles Determined mainly by momentum transport







$$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$$

Featherstone & Miesch (2013)

Latitude-Dependent Rossby Number

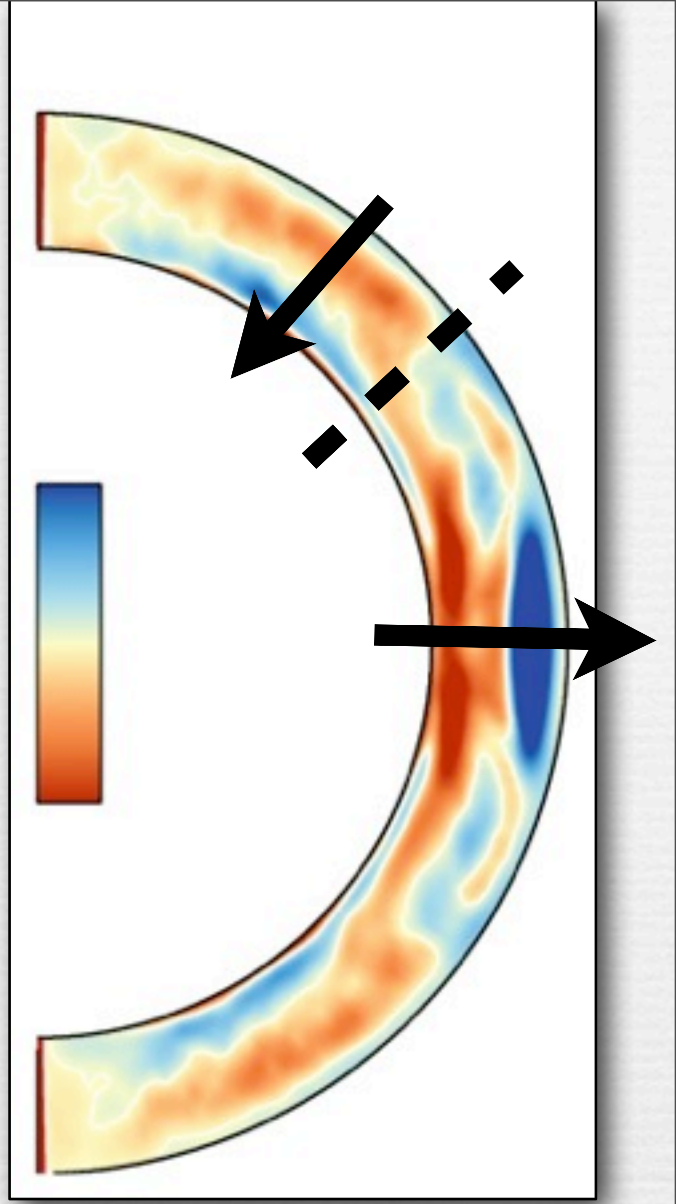
Consider Coriolis force acting on a Downward plume driven by surface cooling

$$\frac{\partial v_r}{\partial t} = 2\Omega_0 \sin\theta \; v_\phi + \ldots$$

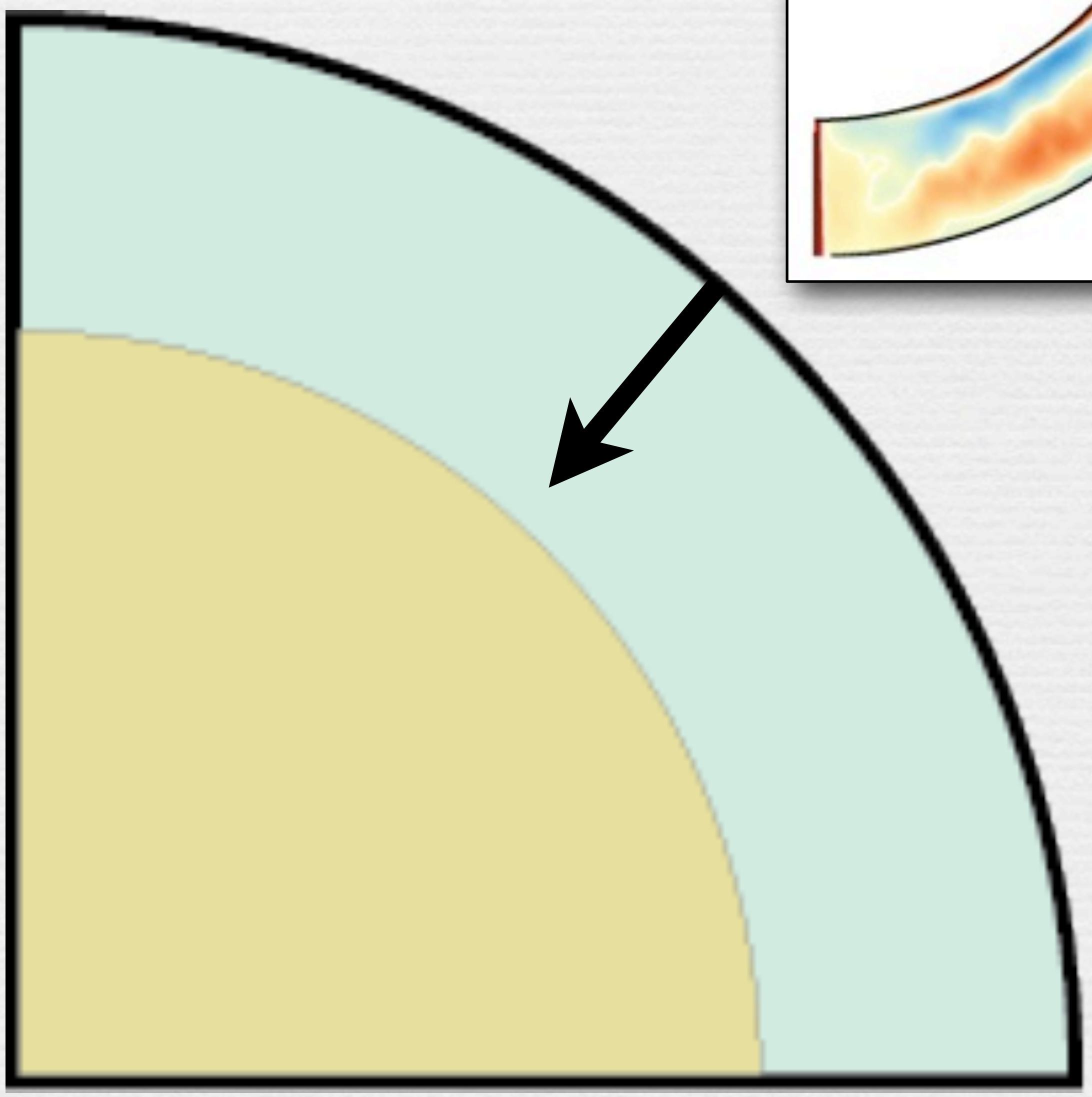
$$\frac{\partial v_\phi}{\partial t} = -2\Omega_0 \sin\theta \; v_r + \ldots$$

$$R_o = \frac{v_r}{2\Omega_0 D \sin\theta} = \frac{r_c}{D}$$

$$r_c = \left| \frac{v_r^2}{\partial v_\phi / \partial t} \right| = \frac{v_r}{2\Omega_0 \sin\theta}$$



effective Ro large at high latitudes, smaller at low latitudes



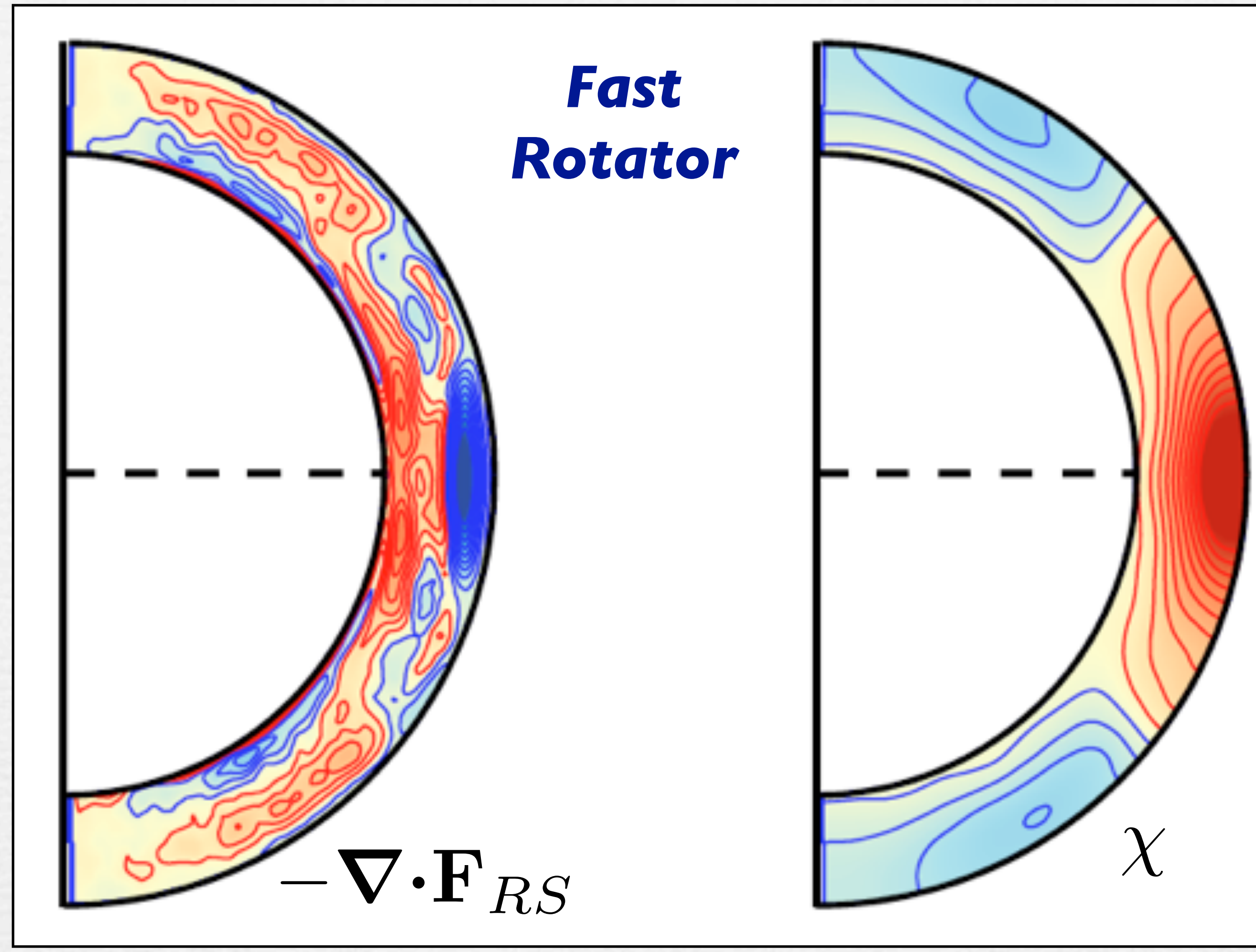
Transition between fast/slow regimes occurs as the latitude where Ro crosses 1 shifts equatorward

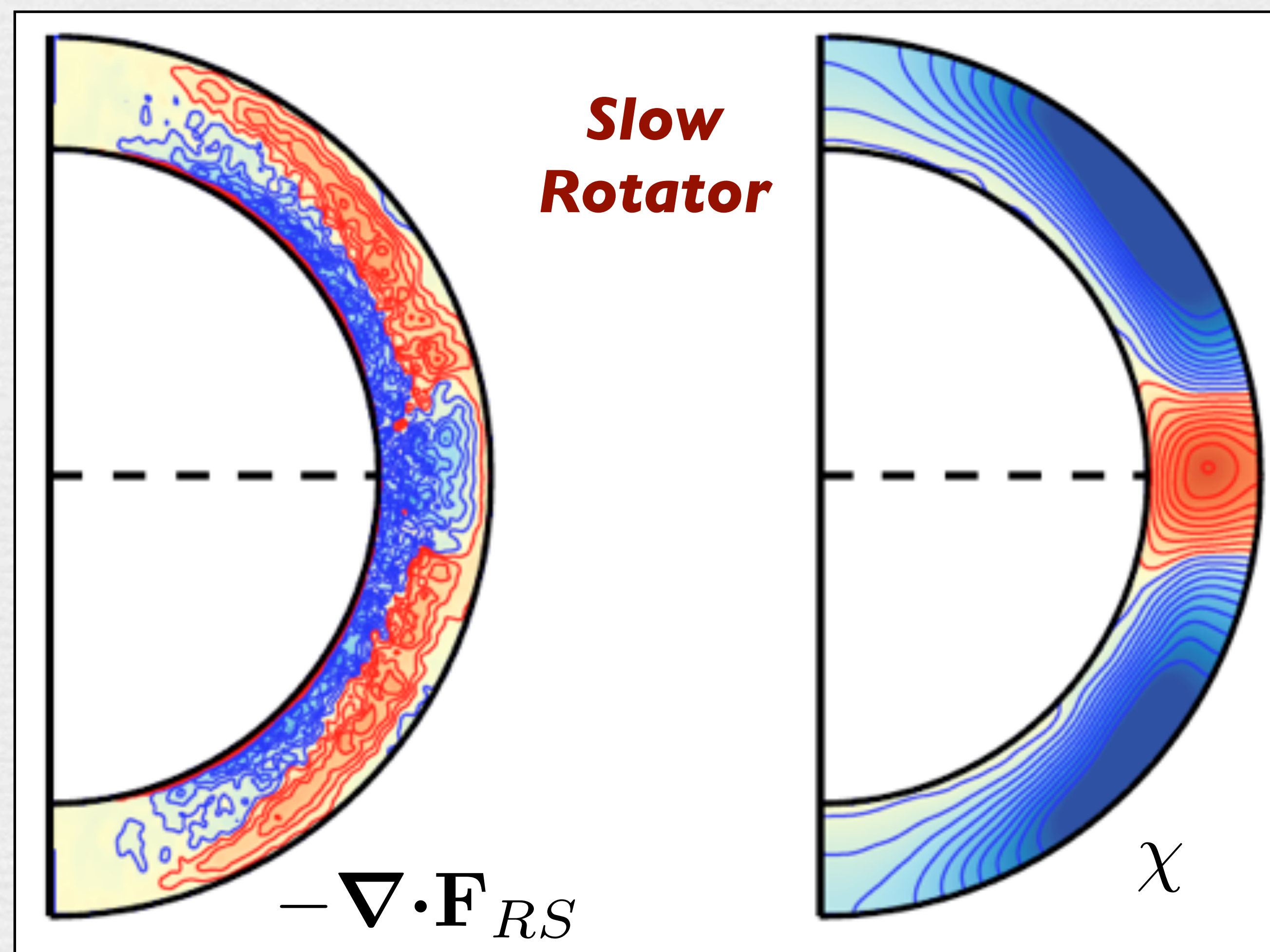
The RS transition

The transition between fast & slow rotating regimes is regulated by (the divergent component of) the [Reynolds stress]()

$$\mathbf{F}_{RS}(r,\theta) = \boldsymbol{\nabla}\chi(r,\theta) + \boldsymbol{\nabla}\times\left(\Gamma(r,\theta)\ \hat{\phi}\right)$$

$$\nabla^2\chi = -\boldsymbol{\nabla}\cdot\mathbf{F}_{RS}$$



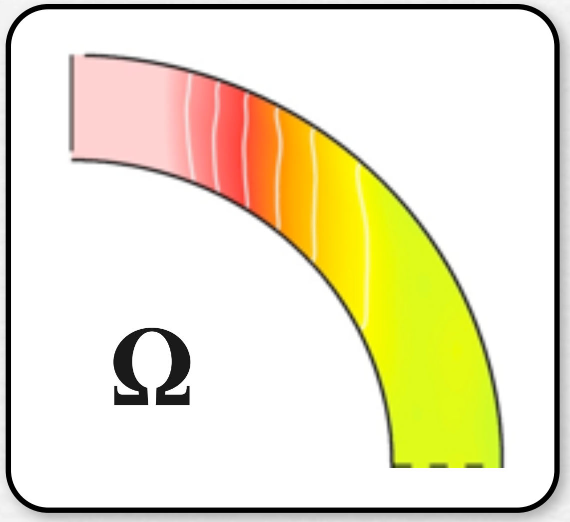


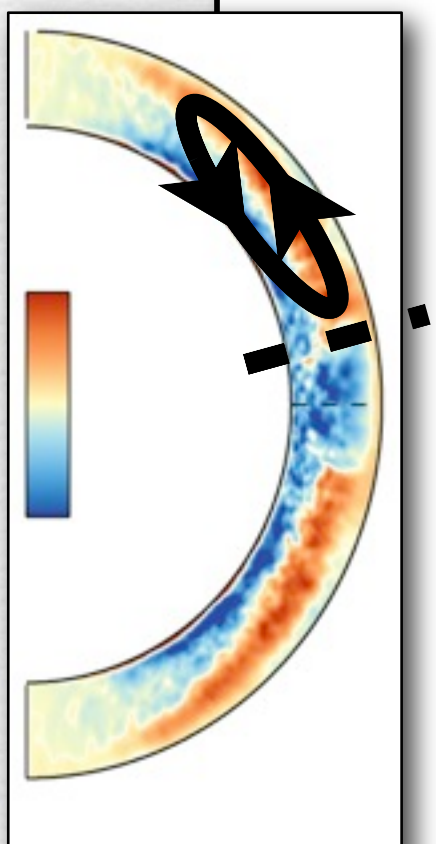
contours track increasing dominance of inward angular momentum transport at high latitudes

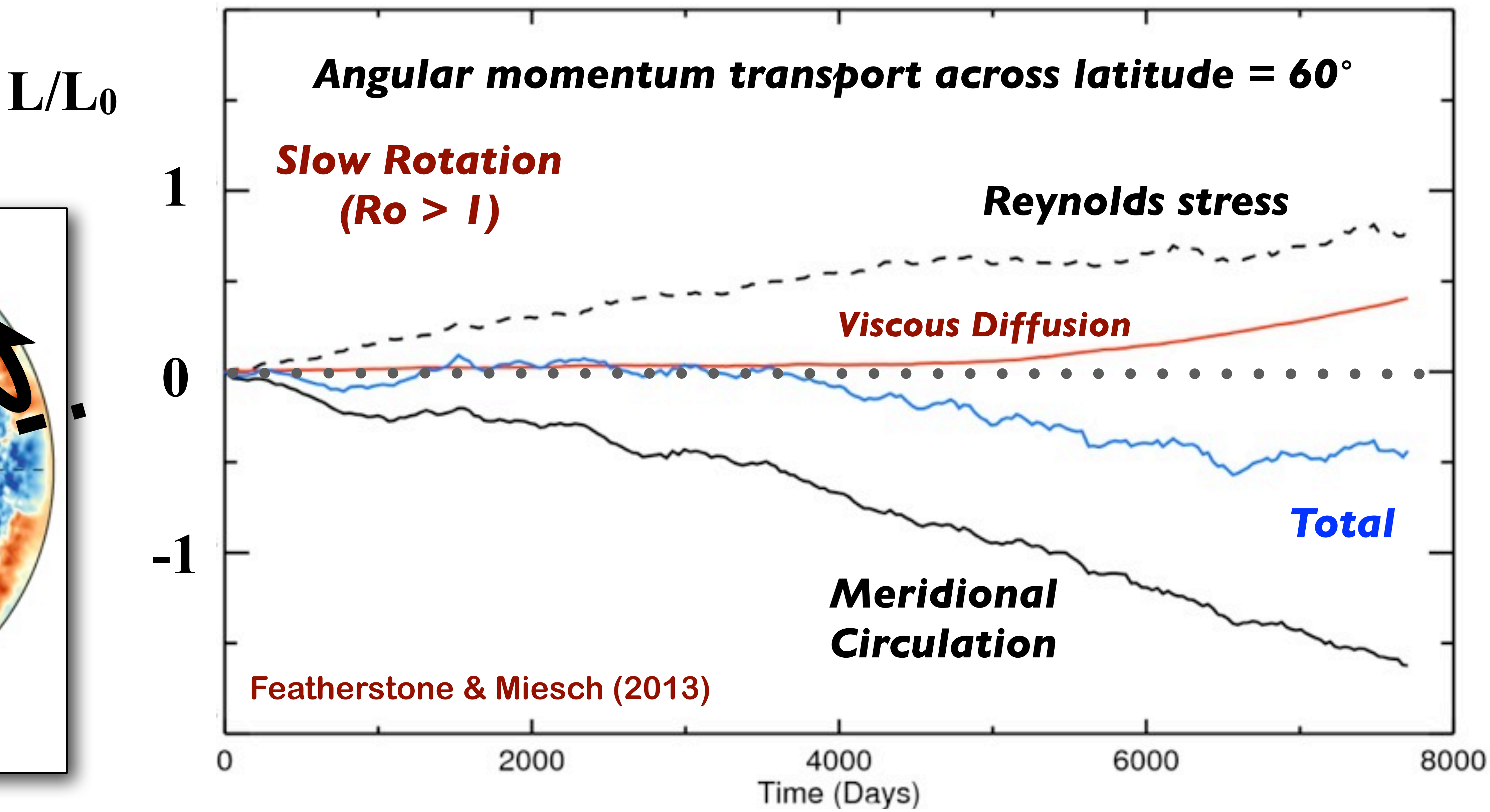
Also - indicate a shift from cylindrically outward to equatorward transport at low latitudes

It's actually the MC that spins up the poles in the slow rotators

Convective angular momentum transport establishes anti-solar DR only indirectly, by inducing a meridional flow (GP) that transports angular momentum poleward







***MC is essentially a runaway, axisymmetric convective instability** (equatorward $\nabla\langle S\rangle$)*

So Where is the Sun?

The Sun ($R_o \lesssim 1$) may be in an atypical state relative to other stars

- **Faster Rotators (young stars: $R_o \lesssim 0.3$)**
  - **Cylindrical Ω profile**
  - **multi-celled MC**
- **Sun (Goldilocks: $0.3 \lesssim R_o \lesssim 1$)**
  - **Conical Ω profile**
  - **?? MC**
- **Slower Rotators (old stars $Ro \gtrsim 1$)**
  - **Anti-solar Ω profile**
  - **single-celled MC**

Minimal $\partial\Omega/\partial r$ at equator suggests there may be a convergence of the latitudinal angular momentum flux as seen in some slow rotators
$\Rightarrow$ Might produce a single-celled MC

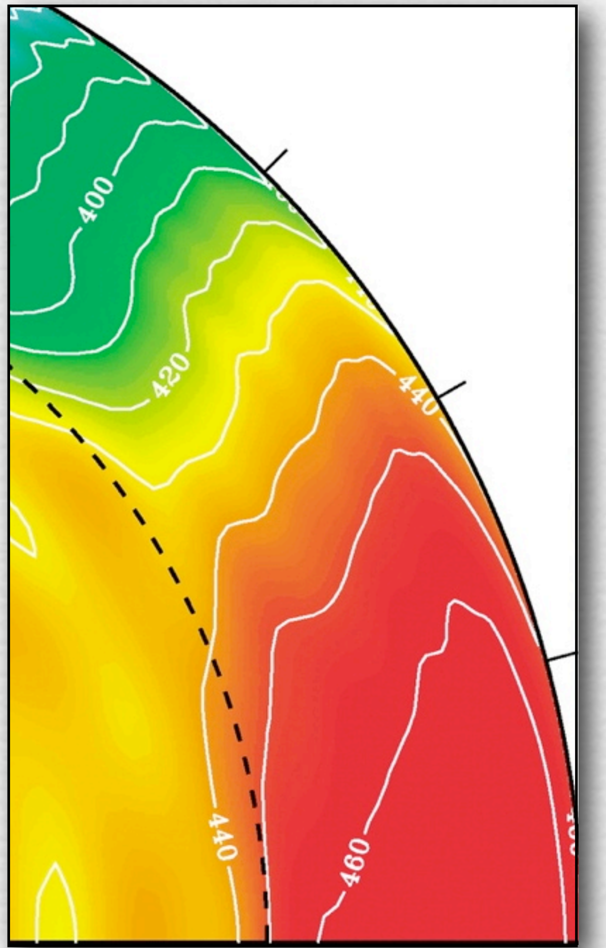
$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}$$

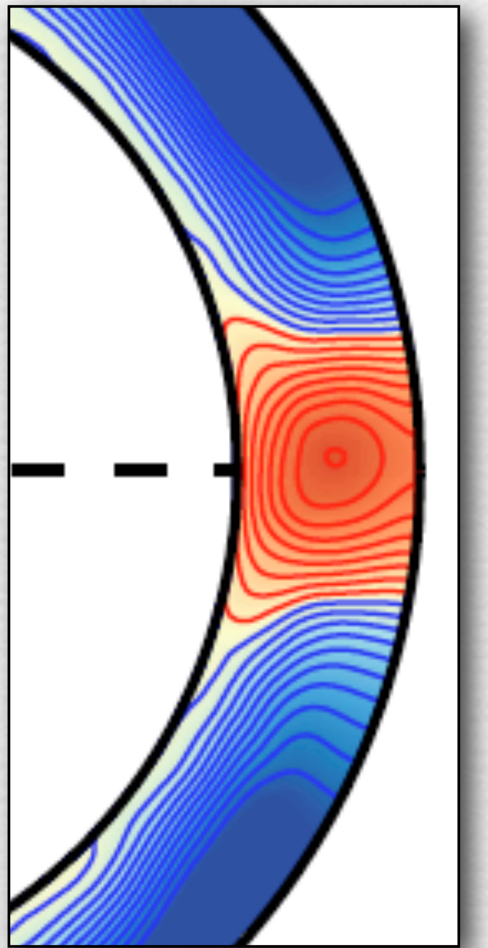
LHS scales as Ro^{-2}

RHS scales as Ro^{-1}

(Hotta & Yokoyama 2011)

(TWB Breaks down for Ro > 1)





So Where is the Sun?

The Sun ($R_o \lesssim 1$) may be in an atypical state relative to other stars

- **Faster Rotators (young stars: $R_o \lesssim 0.3$)**
  - **Cylindrical Ω profile**
  - **multi-celled MC**
- **Sun (Goldilocks: $0.3 \lesssim R_o \lesssim 1$)**
  - **Conical Ω profile**
  - **?? MC**
- **Slower Rotators (old stars $Ro \gtrsim 1$)**
  - **Anti-solar Ω profile**
  - **single-celled MC**

Minimal $\partial\Omega/\partial r$ at equator suggests there may be a convergence of the latitudinal angular momentum flux as seen in some slow rotators
⇒ Might produce a single-celled MC

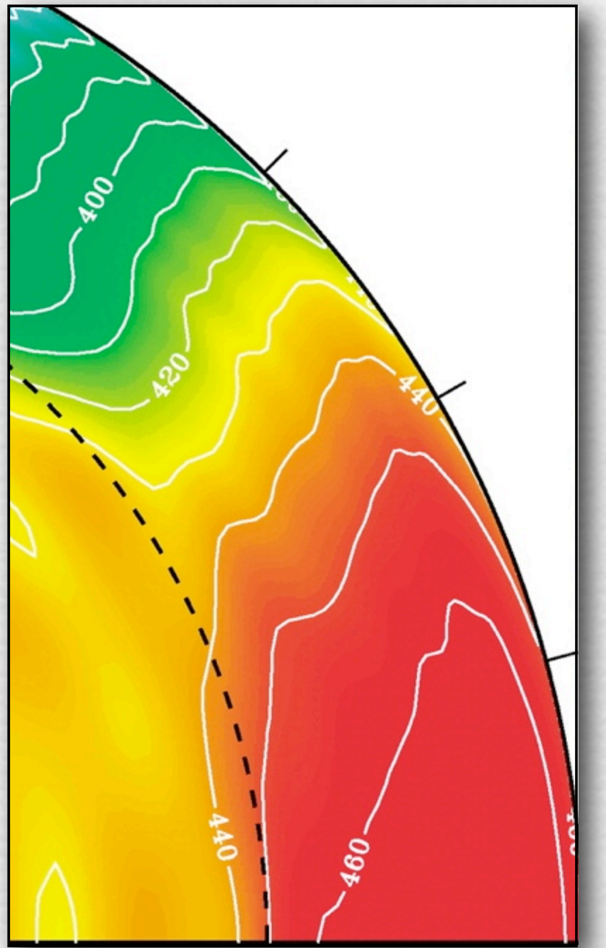
$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r \lambda C_P} \frac{\partial \langle S \rangle}{\partial \theta}$$

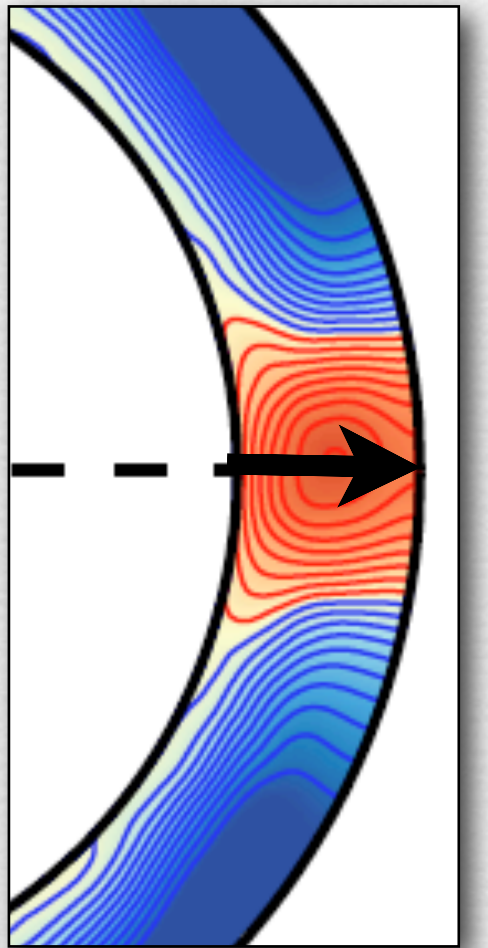
LHS scales as Ro^{-2}

RHS scales as Ro^{-1}

(Hotta & Yokoyama 2011)

(TWB Breaks down for Ro > 1)

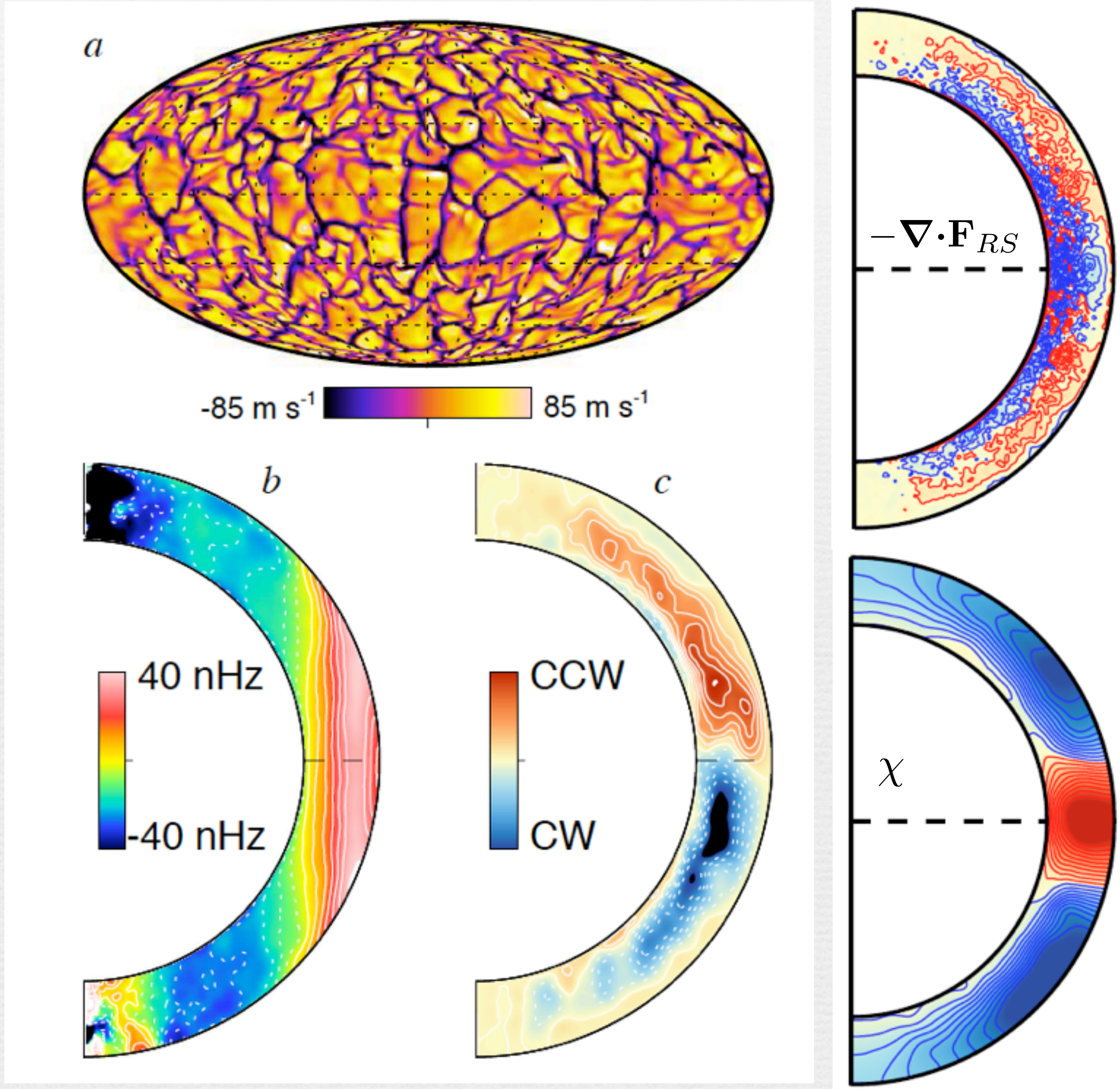




Near the Transition

Solar-like DR

but single-celled MC!

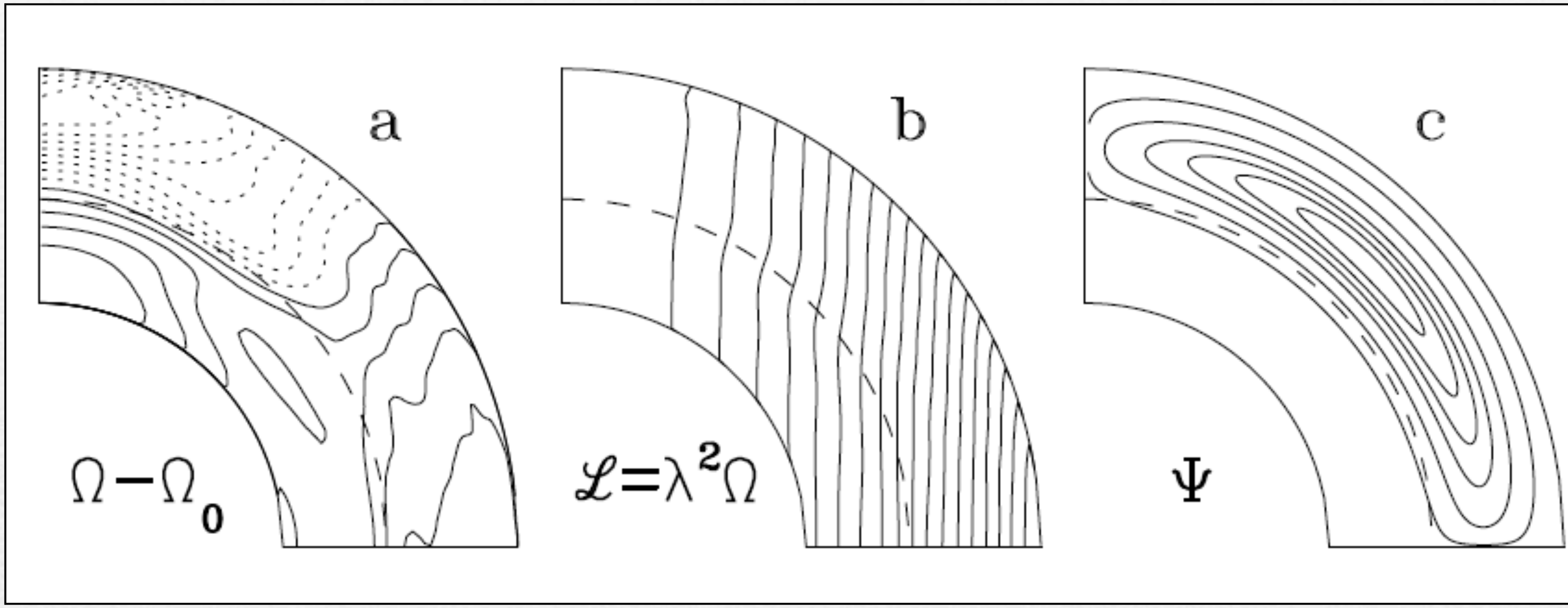


Turn the Problem around

Miesch (2005)

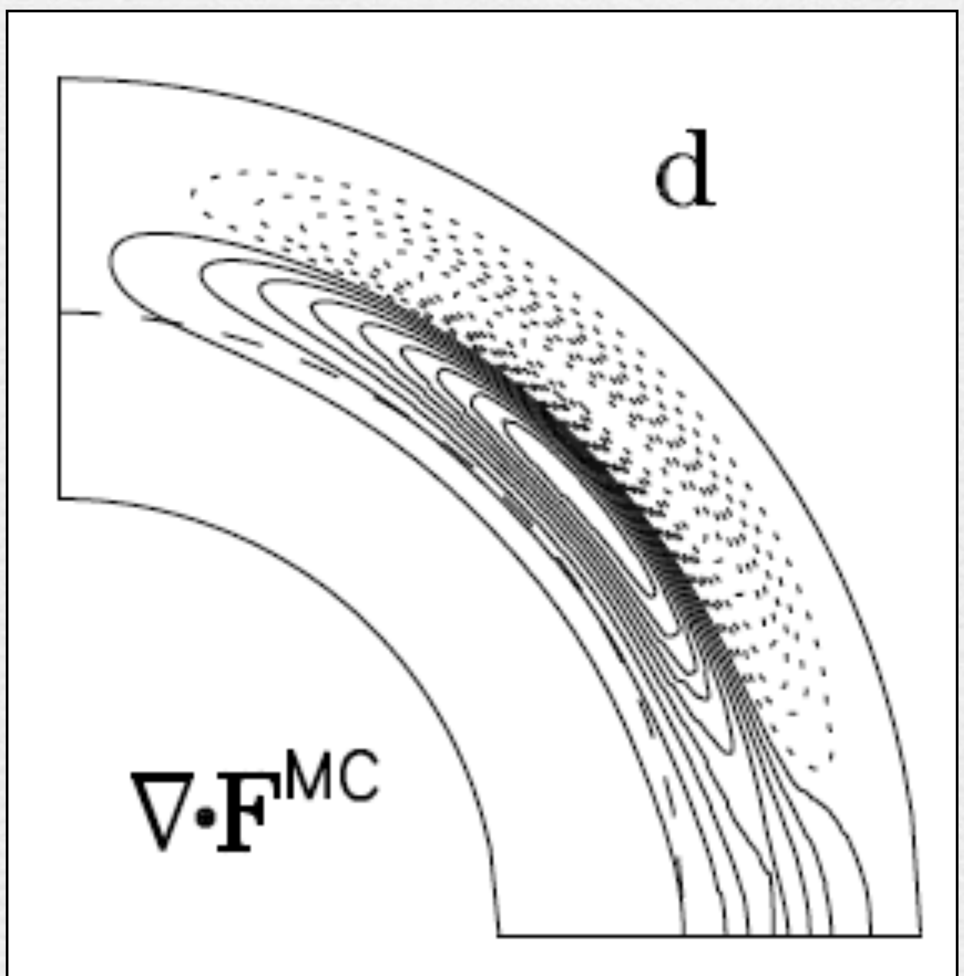
$$\langle \rho \mathbf{v}_m \rangle \cdot \nabla \mathcal{L} = \mathcal{F}$$

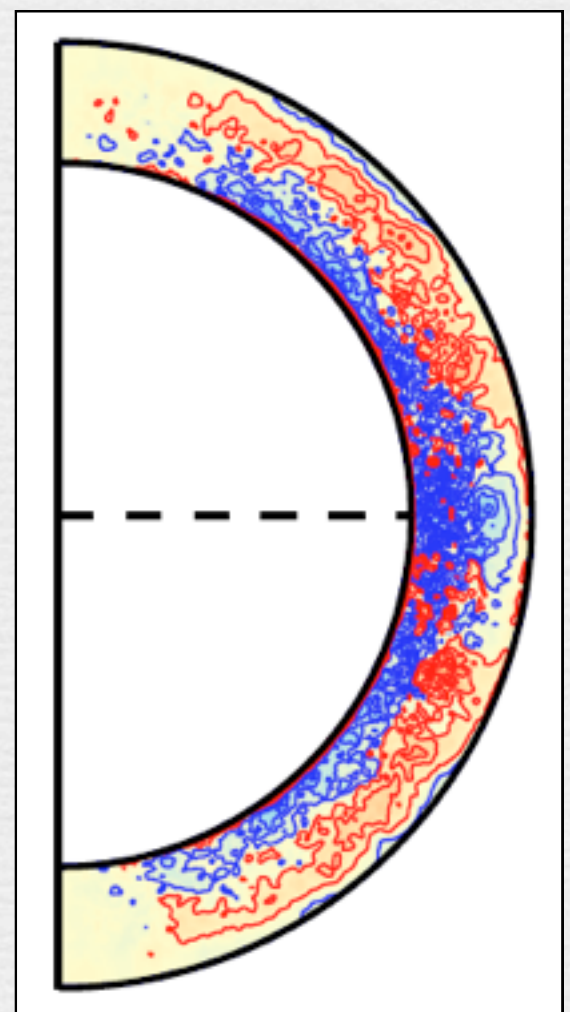
Take solar Ω profile and ask what RS would be needed to sustain a single-cell profile

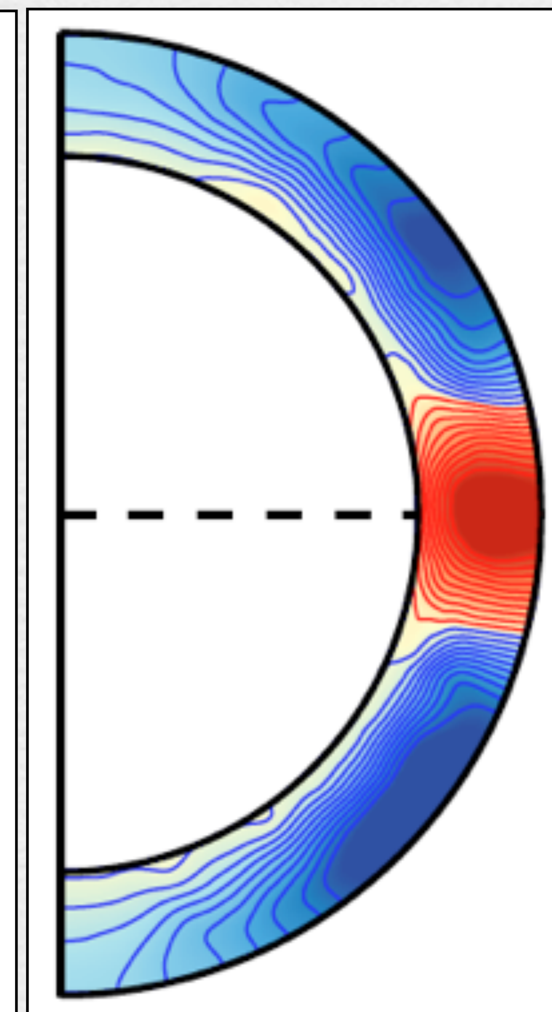


Summary: Inconclusive!

The sun is close to the transition between single and multiple-celled MC profiles; could go either way







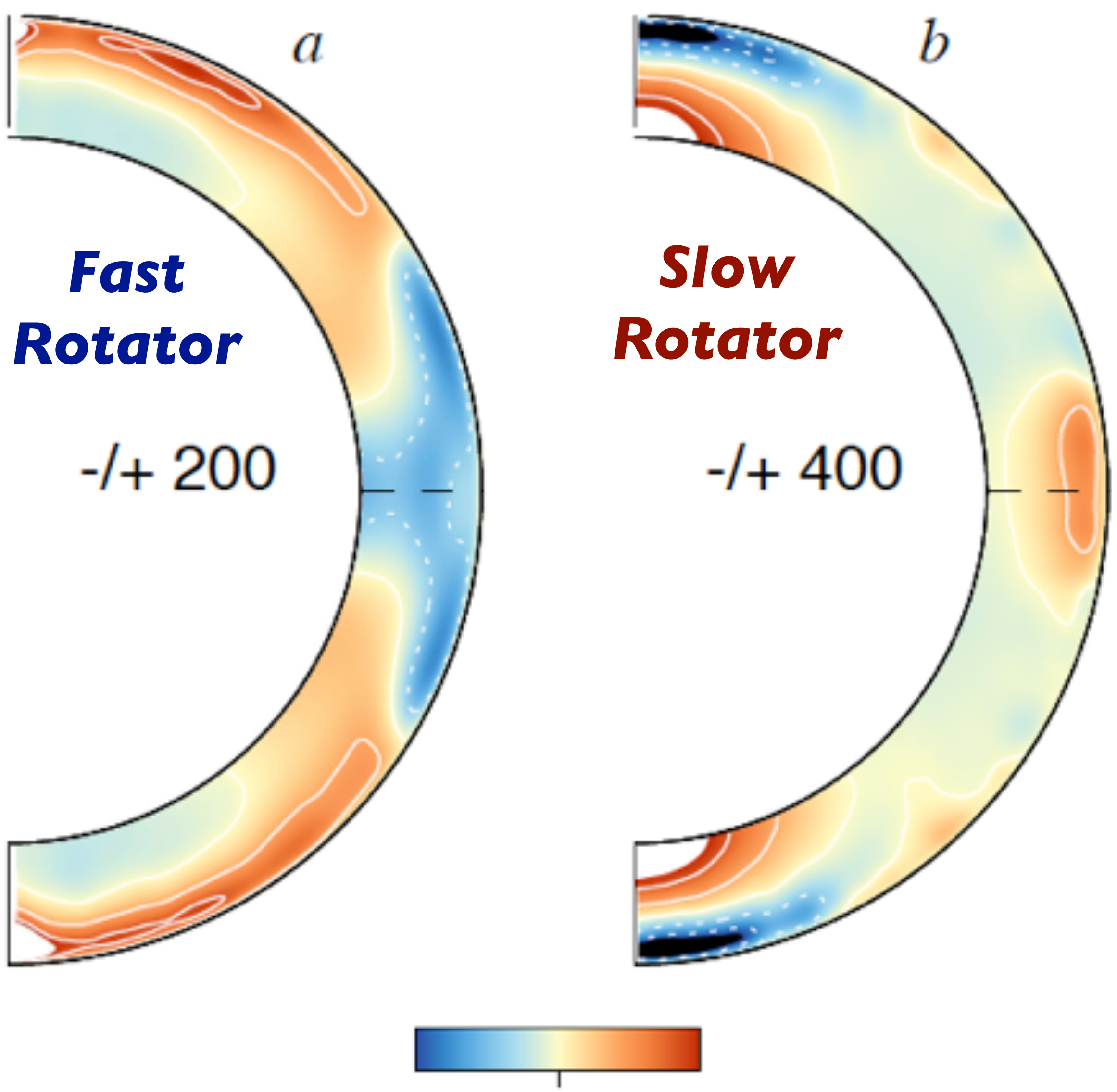
Summary

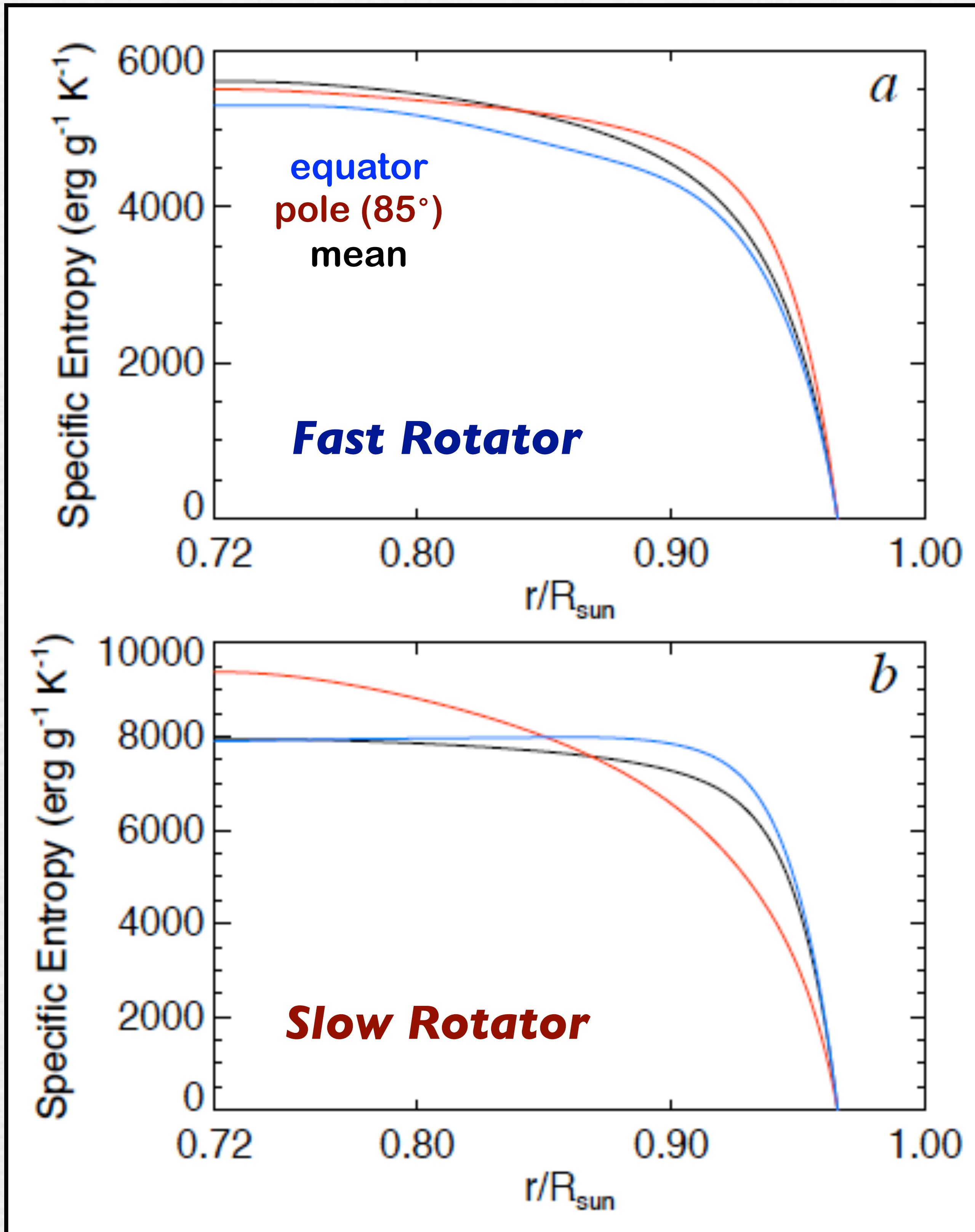
- **Mean Flow regimes**
  - **Fast rotators: solar-like DR, multi-celled MC**
  - **Slow rotators: anti-solar DR, single-celled MC**
    - **Sun may be near the transition**
- **Angular momentum transport by the RS**
  - **Ballistic plumes at high latitudes (radially inward)**
  - **Banana cells at low latitudes (cylindrically outward)**
  - **Latitude where effective Rossby number crosses O(1) regulates fast/slow transition**
  - **Polar spin-up in slow rotators attributed to MC**
- **Maintenence of MC**
  - **zonal forcing (RS)**
  - **Gyroscopic pumping**

Mean flow regimes reflected in stratification (specific entropy S)

$$\frac{\partial \Omega^2}{\partial z} = \frac{g}{r\lambda C_P}\frac{\partial \langle S \rangle}{\partial \theta}$$

residual entropy





Polar vortex acts as a transport barrier for slow rotators