Differential rotation and magnetism across the HR diagram
April 12, 2013, Nordita

Effect of the turbulent pumping of the magnetic flux on the predictability of the solar cycle

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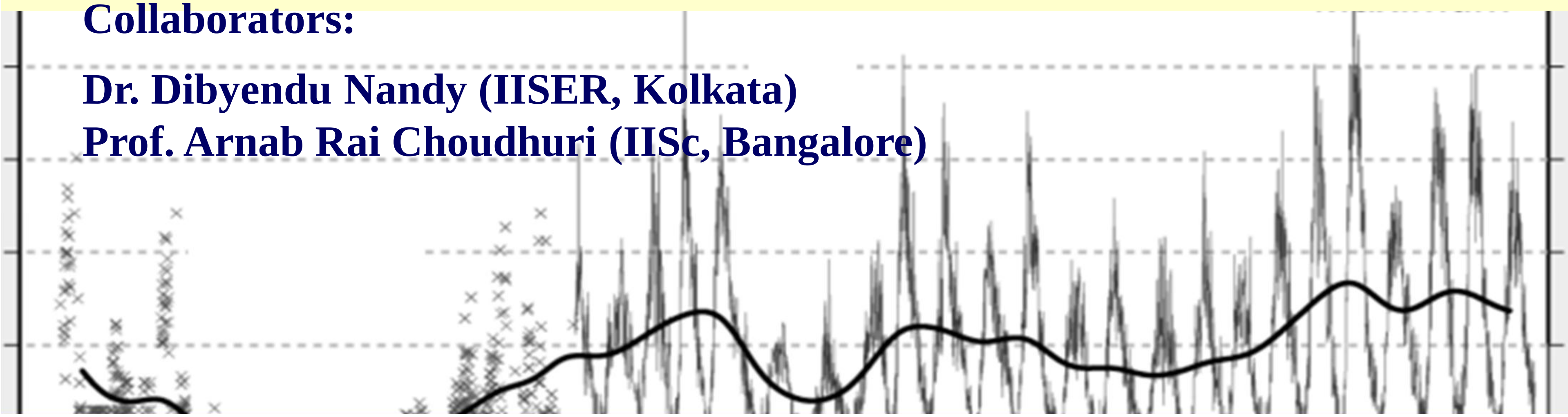


Differential rotation and magnetism across the HR diagram
April 12, 2013, Nordita

Modeling the Grand Minima of solar activity using a flux transport dynamo model

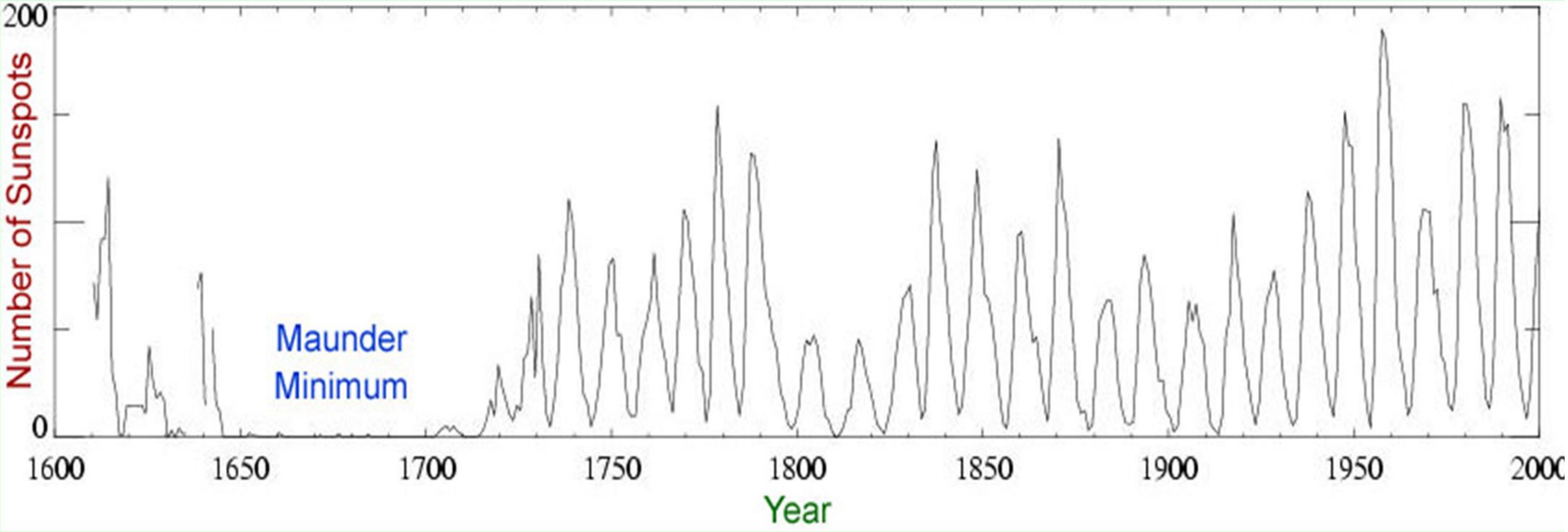
Bidya Binay Karak

&

Arnab Rai Choudhuri

Indian Institute of Science

Maunder minimum

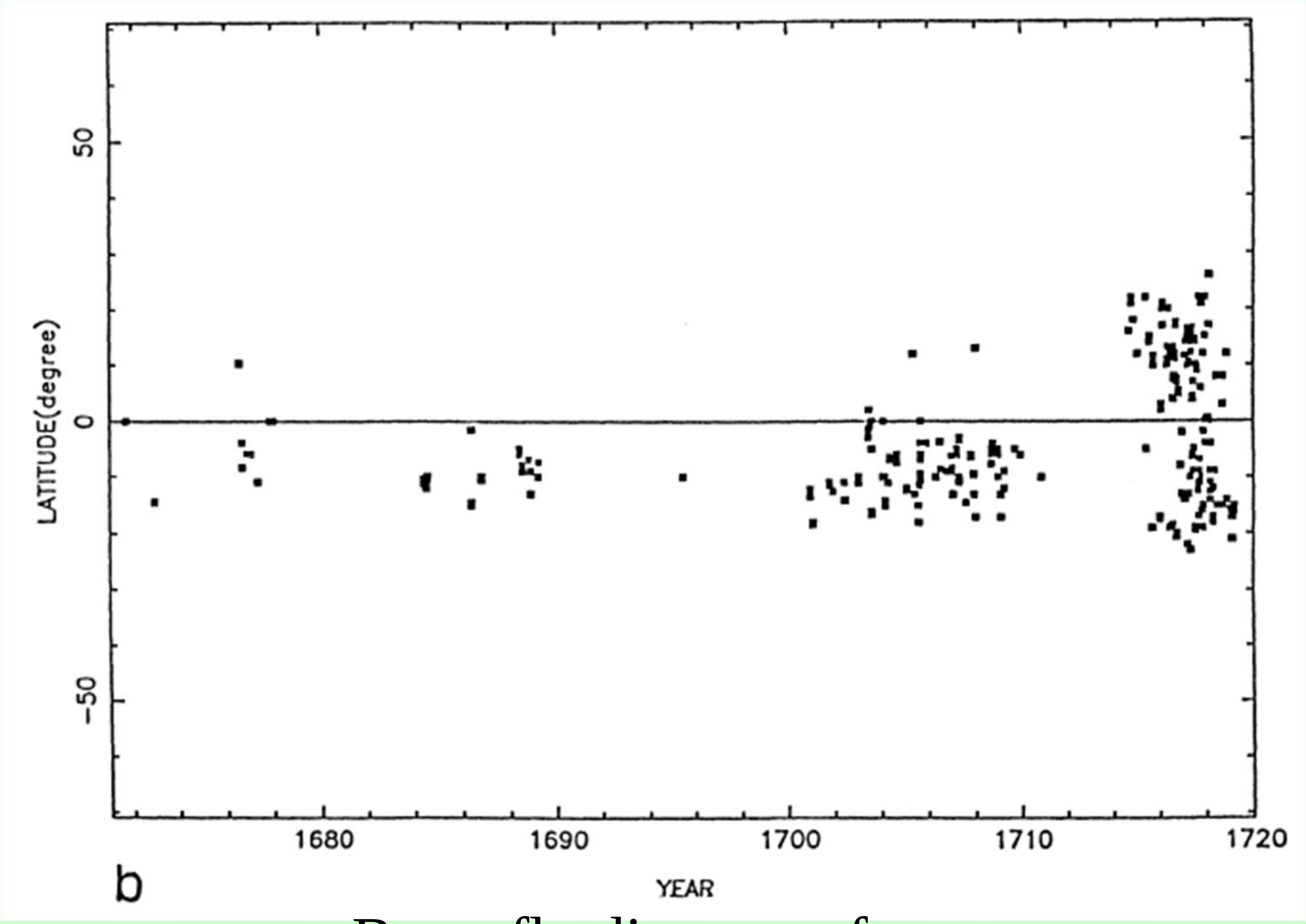


➢**Maunder minimum period = 1645 to 1715** (Eddy, 1976; Foukal, 1990; Wilson, 1994)

➢**It is a real phenomenon!** (Sokoloff & Nesme-Ribes 1994; Hoyt & Schatten 1996)

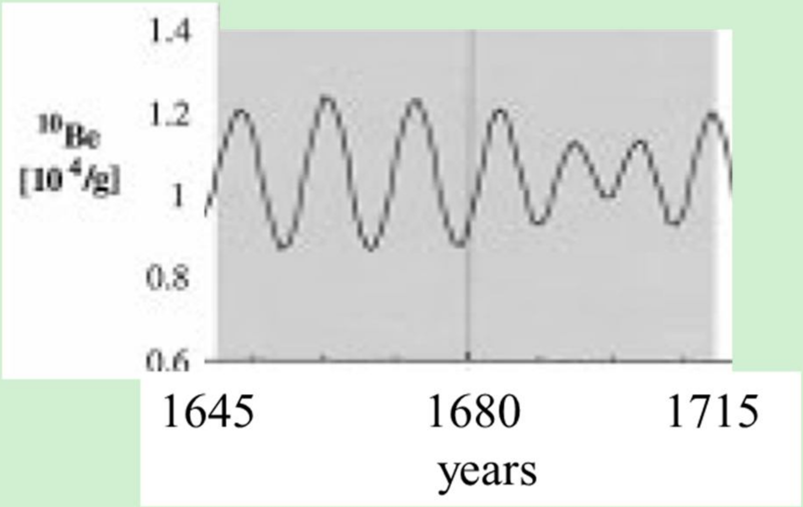
Characteristics of Maunder Minimum

Hemispheric asymmetry (Sokoloff & Nesme-Ribes 1994)



Butterfly diagram of sunspot

Cyclic solar activity (Schwab cycle) was continued. (Beer et al. 1998)



But **period** was **longer**

(Miyahara et al. 2004 Miyahara et al. 2010 Nagaya et al. 2012; Usoskin et al. 2012)

History of solar activity before telescopic records reconstructed by Beer et al. (1990,1998); Eddy (1977), Stuiver & Braziunas (1989),Voss et al. (1996), Solanki et al. 2004, Usoskin, Solanki & Kovaltsov (2007), Miyahara et al. (2004,2010), Nagaya et al. (2012); Steinhilber et al. (2012)

From Usoskin, Solanki & Kovaltsov (2007) – 27 grand minima in the last 11,000 years!

Motivation

How grand minima are produced?

Can we model the Maunder minimum or any grand minimum using a dynamo model?

If so, then how frequently it produces grand minima?

Towards flux transport dynamo model

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathrm{B} - \lambda \nabla \times \mathrm{B}),$$

with $\nabla.\mathrm{B} = 0$

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \rho \mathrm{g} + \frac{(\nabla \times \mathrm{B})}{\mu_0} \times \mathrm{B} - 2\rho \boldsymbol{\Omega} \times \mathbf{v} + 2\nabla.\nu\rho\mathrm{S},$$

$$\frac{d\rho}{dt} = -\rho \nabla.\mathbf{V},$$

$$\rho \frac{de}{dt} = -p\nabla.\mathbf{v} + \nabla.\frac{\lambda_{\mathrm{rad}}}{C_V}\nabla e + 2\nu\rho S^2 + \frac{\lambda}{\mu_0}(\nabla \times \mathrm{B})^2,$$

Kinematic model

Mean-field model

(Parker 1955; Steenbeck, Krause & Radler 1966)

$$\mathrm{v} = \bar{\mathrm{v}} + \mathrm{v}', \quad \mathrm{B} = \bar{\mathrm{B}} + \mathrm{B}', \text{ with } \bar{\mathrm{B}}' = 0 \text{ and } \bar{\mathrm{v}}' = 0$$

Mean-field induction equation:

$$\frac{\partial \bar{\mathrm{B}}}{\partial t} = \nabla \times (\bar{\mathrm{V}} \times \bar{\mathrm{B}}) + \nabla \times \varepsilon + \lambda \nabla^2 \bar{\mathrm{B}}$$

$$\varepsilon = \overline{\mathbf{v}' \times \mathbf{B}'}$$

$$\mathcal{E} = \alpha \bar{\mathrm{B}} - \beta \nabla \times \bar{\mathrm{B}}$$

where, $\alpha = -\frac{1}{3}\overline{\mathbf{v}' . (\nabla \times \mathbf{v}')}\tau$, and

$$\beta = \frac{1}{3}\overline{\mathbf{v}'.\mathbf{v}'}\tau$$

$$\frac{\partial \bar{\mathrm{B}}}{\partial t} = \nabla \times (\bar{\mathrm{V}} \times \bar{\mathrm{B}}) - \nabla \times (\alpha \bar{\mathrm{B}}) + (\lambda + \eta) \nabla^2 \bar{\mathrm{B}}$$

Axisymmetric dynamo model

$$B = B(r,\theta)e_\phi + \nabla \times [A(r,\theta)e_\phi]$$

Velocity field= $\Omega(r,\theta)r\sin\theta\, e_\phi + v_r e_r + v_\theta e_\theta$

angular frequency   meridional circulation

Substitute in

$$\frac{\partial \bar{\mathrm{B}}}{\partial t} = \nabla \times (\bar{\mathrm{V}} \times \bar{\mathrm{B}}) - \nabla \times (\alpha \bar{\mathrm{B}}) + (\lambda + \eta)\nabla^2 \bar{\mathrm{B}}$$

Mean-field dynamo equations

Toroidal field evolution:

$$\frac{\partial B}{\partial t}+\frac{1}{r}\left[\frac{\partial}{\partial r}(r v_r B)+\frac{\partial}{\partial \theta}(v_\theta B)\right]=\eta_t\left(\nabla^2-\frac{1}{s^2}\right)B+s(\mathbf{B}_p.\nabla)\Omega$$

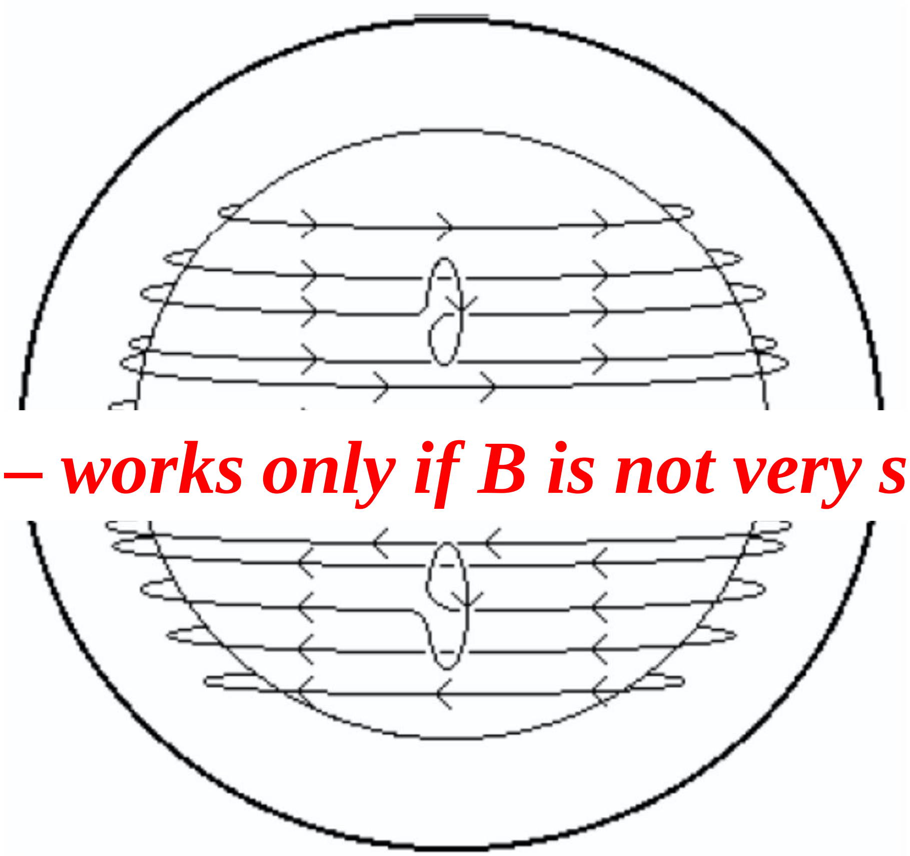
$$+\frac{1}{r}\frac{\partial \eta_t}{\partial r}\frac{\partial}{\partial r}(rB)$$

Source terms

Poloidal field evolution:

$$\frac{\partial A}{\partial t}+\frac{1}{s}(\mathbf{v}.\nabla)(sA)=\eta_t\left(\nabla^2-\frac{1}{s^2}\right)A+\alpha B$$

1. The Mean Field α-effect or classical α-effect

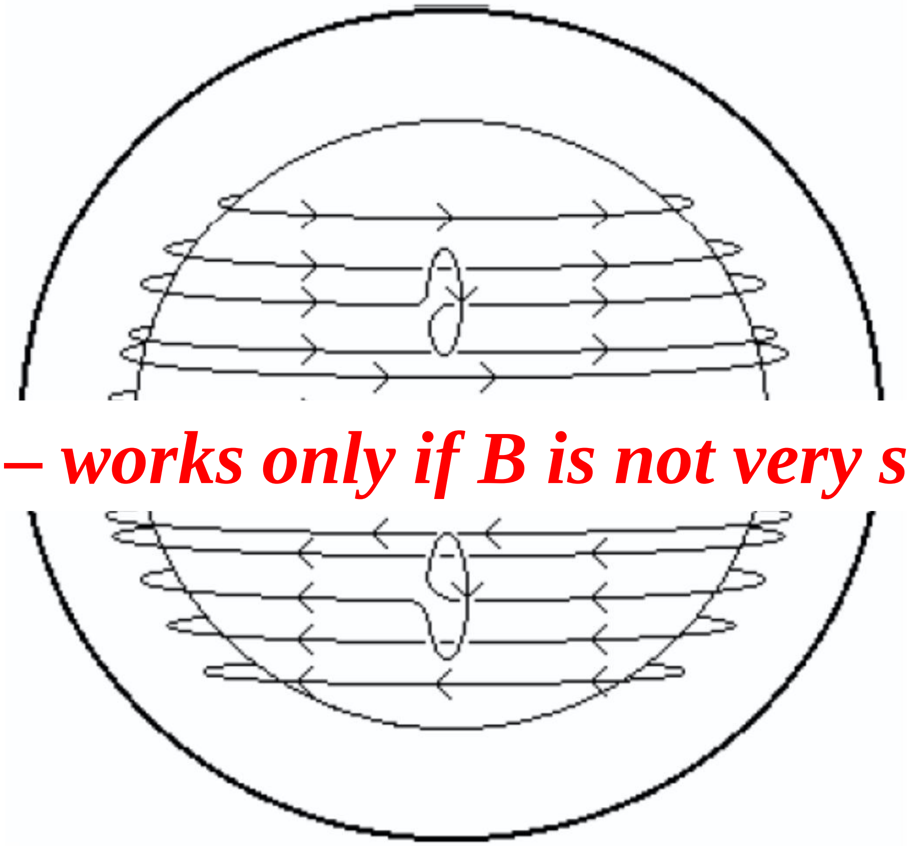


α-effect – ***works only if B is not very strong***

• Buoyantly rising toroidal field is twisted by helical turbulent convection, creating loops in the poloidal plane

• The small-scale loops diffuse to generate a large-scale poloidal field

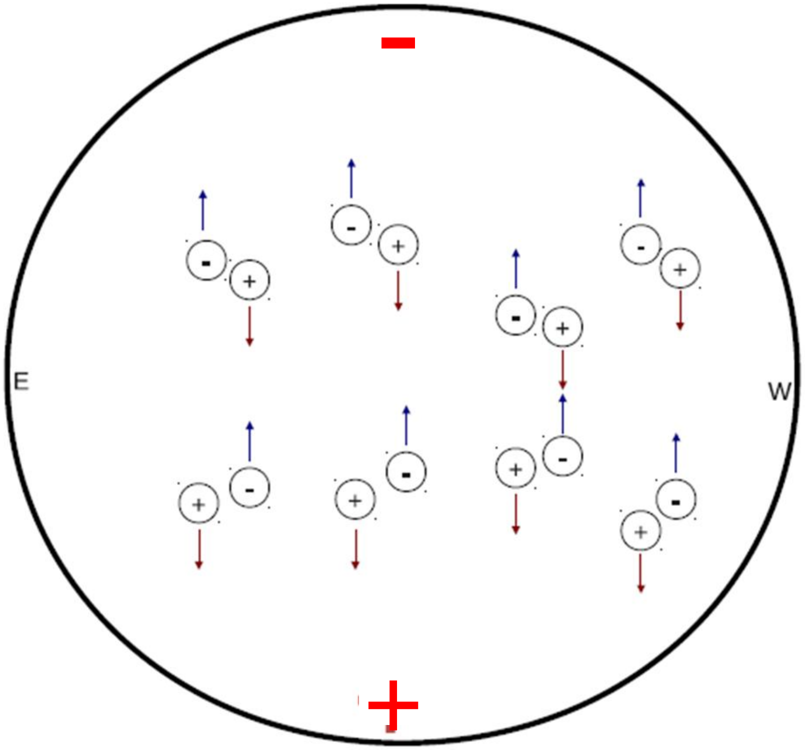
1. The Mean Field α-effect or classical α-effect

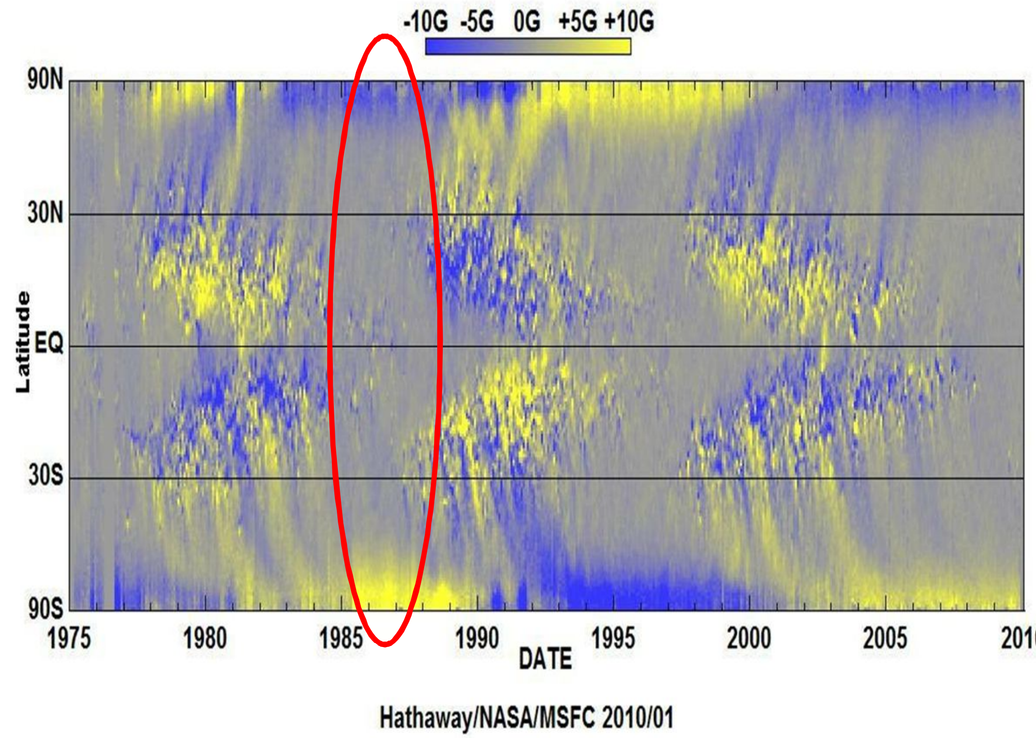


α-effect – *works only if B is not very strong*

Choudhuri 1992; Gmez & Mininni 2006; Wilmot-Smith et al. 2005; Brandenburg & Spiegel 2008; Usoskin 2009; Passos & Lopes 2011; Passos et al. 2012 – find grand minima.

2. Poloidal Field Generation:–**Babcock–Leighton alpha effect:** (Babcock 1961; Leighton 1969; Dasi-Espuig et al. 2010; Kitchatinov & Olemskoy (2011))

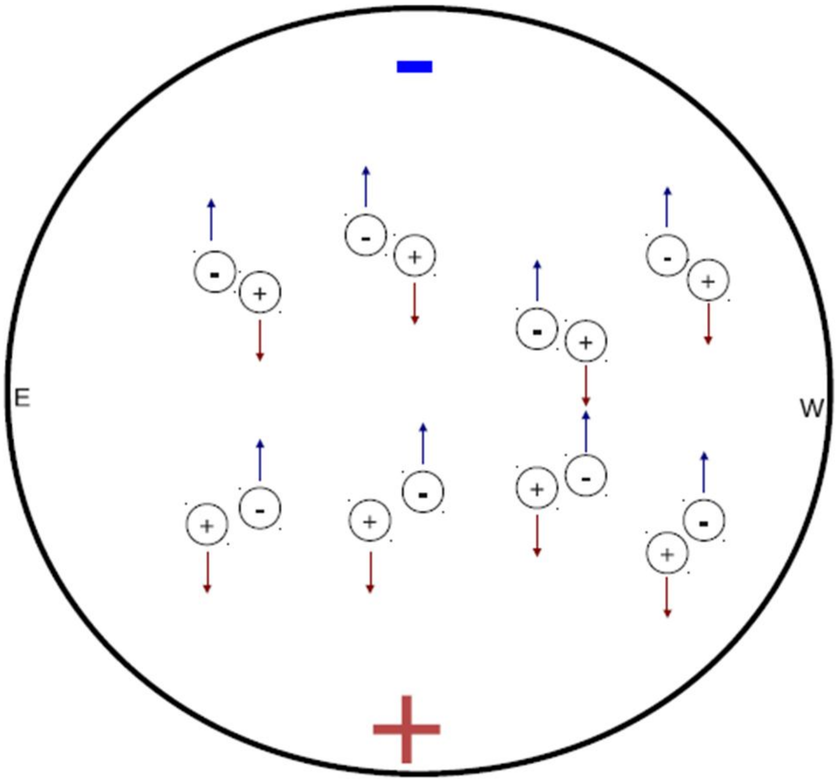




Observationally verified

Not self excited!

Babcock-Leighton process



Depends strongly on average **tilt angle** --- involves randomness

(caused by convective turbulence – Longcope & Fisher 1996

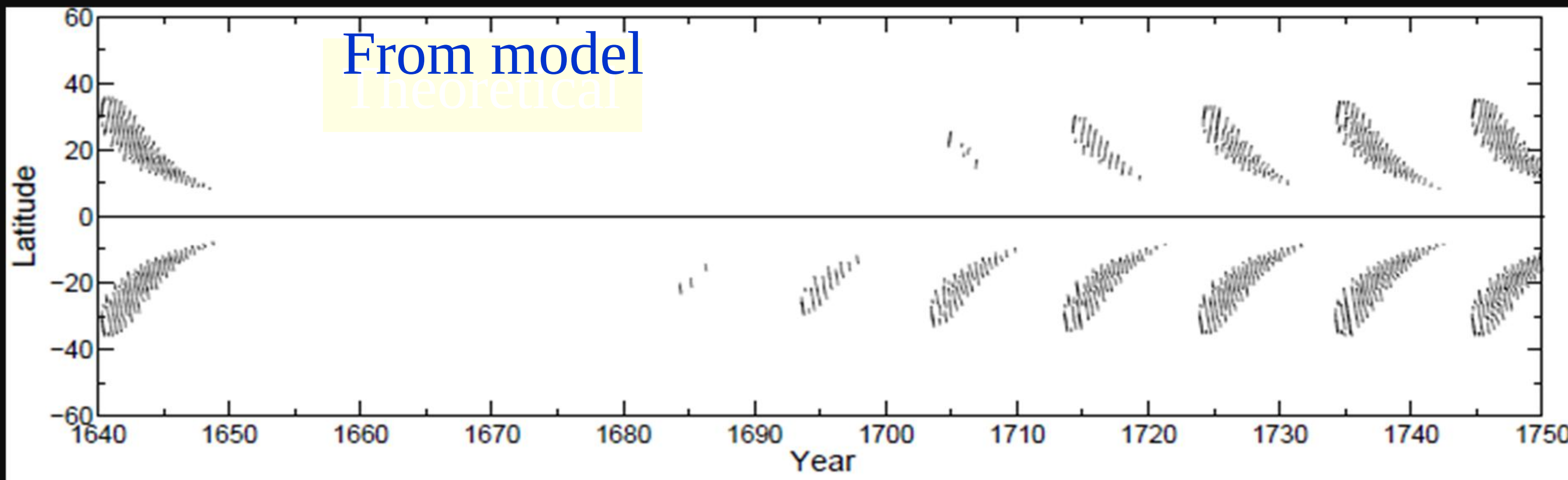
Or the inflow of the active regions)

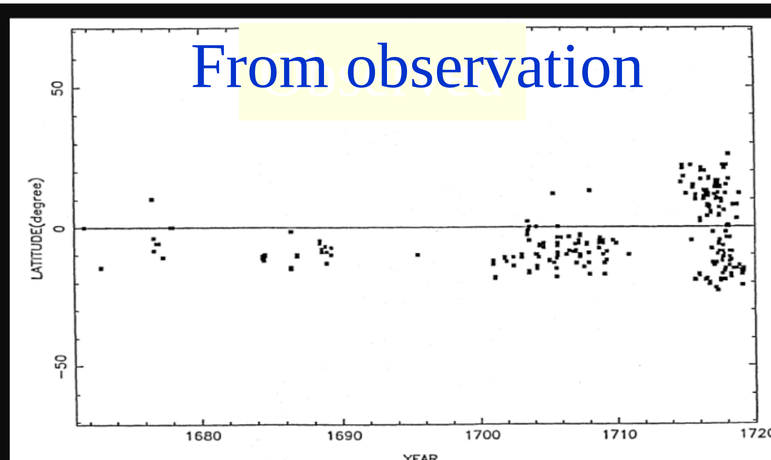
Supported by Dasi-Espuig et al. (2010) Kitchatinov & Olemskoy (2011)

Charboneau et al. (2004) - intermittencies like grand minima.

Modelling a Maunder minimum

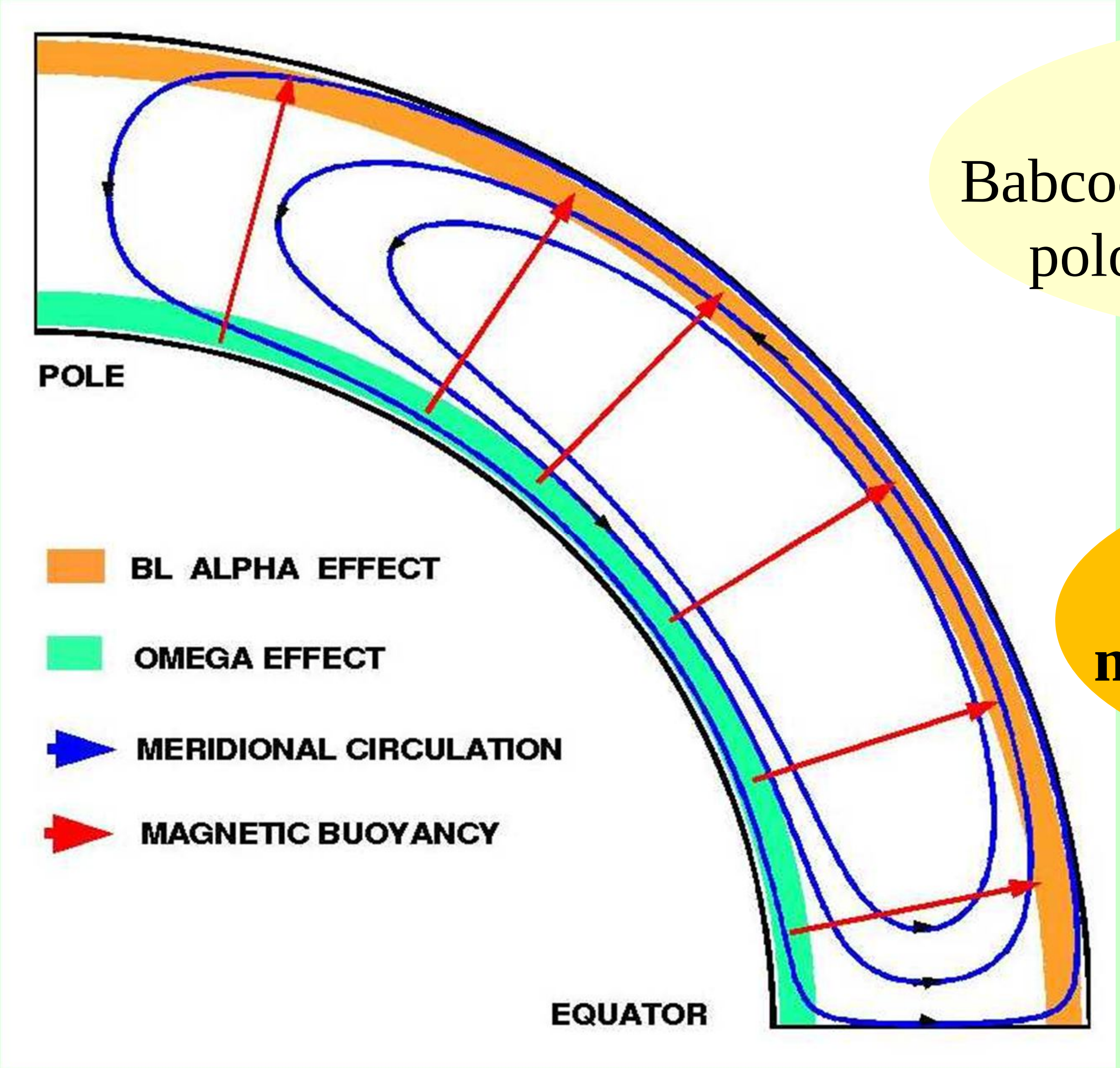
Assumption: Poloidal field drops to 0.0 and 0.4 of its average value in the two hemispheres





Choudhuri & Karak (2009)

Another source of randomness in flux transport dynamo



Fluctuations in Babcock-Leighton process of poloidal field generation

Variation in meridional circulation

Variation of meridional circulation

Indirect evidences

Wang et al. (2002)

Hathaway et al. (2003)

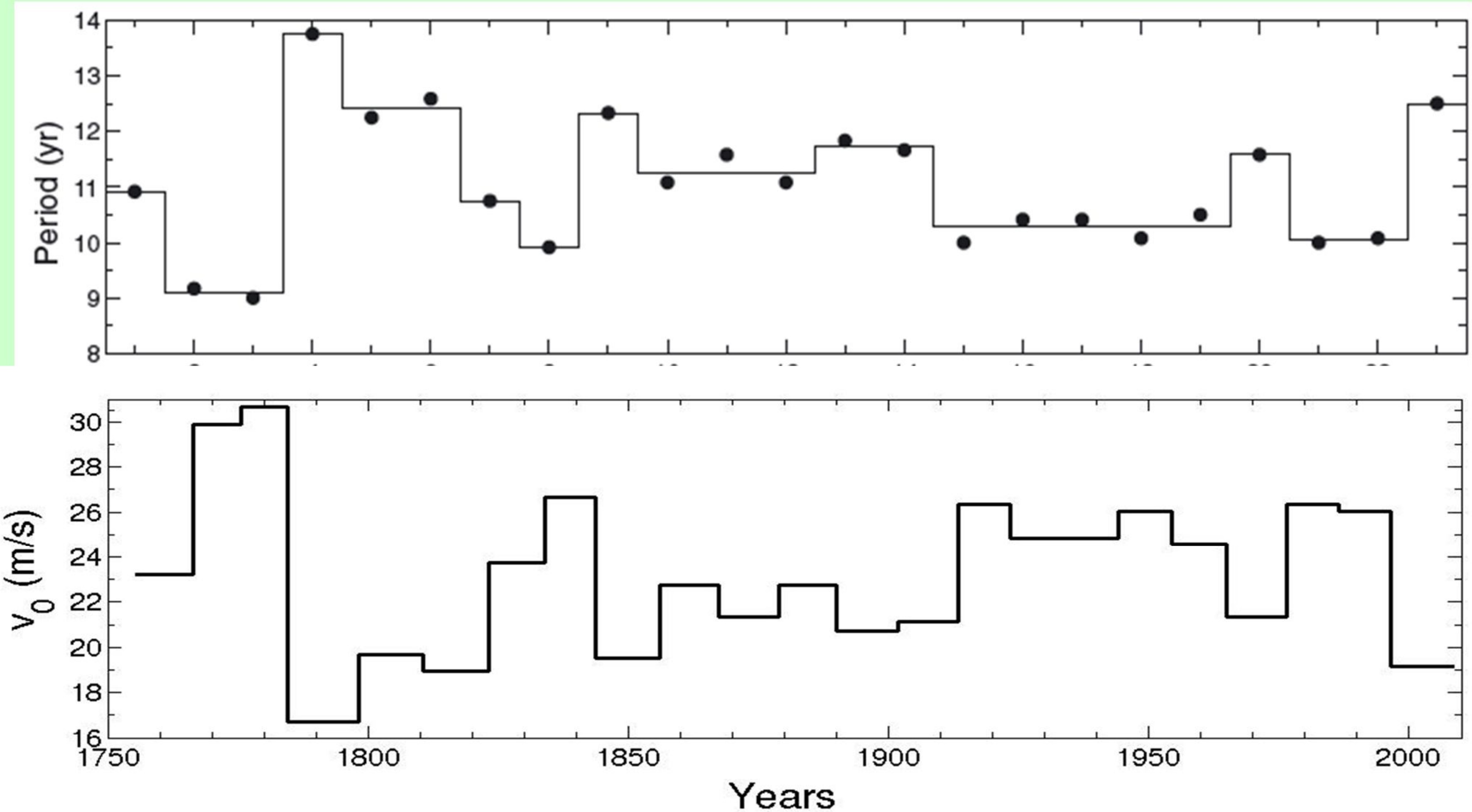
Javaraiah & Ulrich (2006)

Passos & Lopes (2008)

Georgieva & Kirov 2010)

Karak (2010)

Fluctuation of the meridional circulation



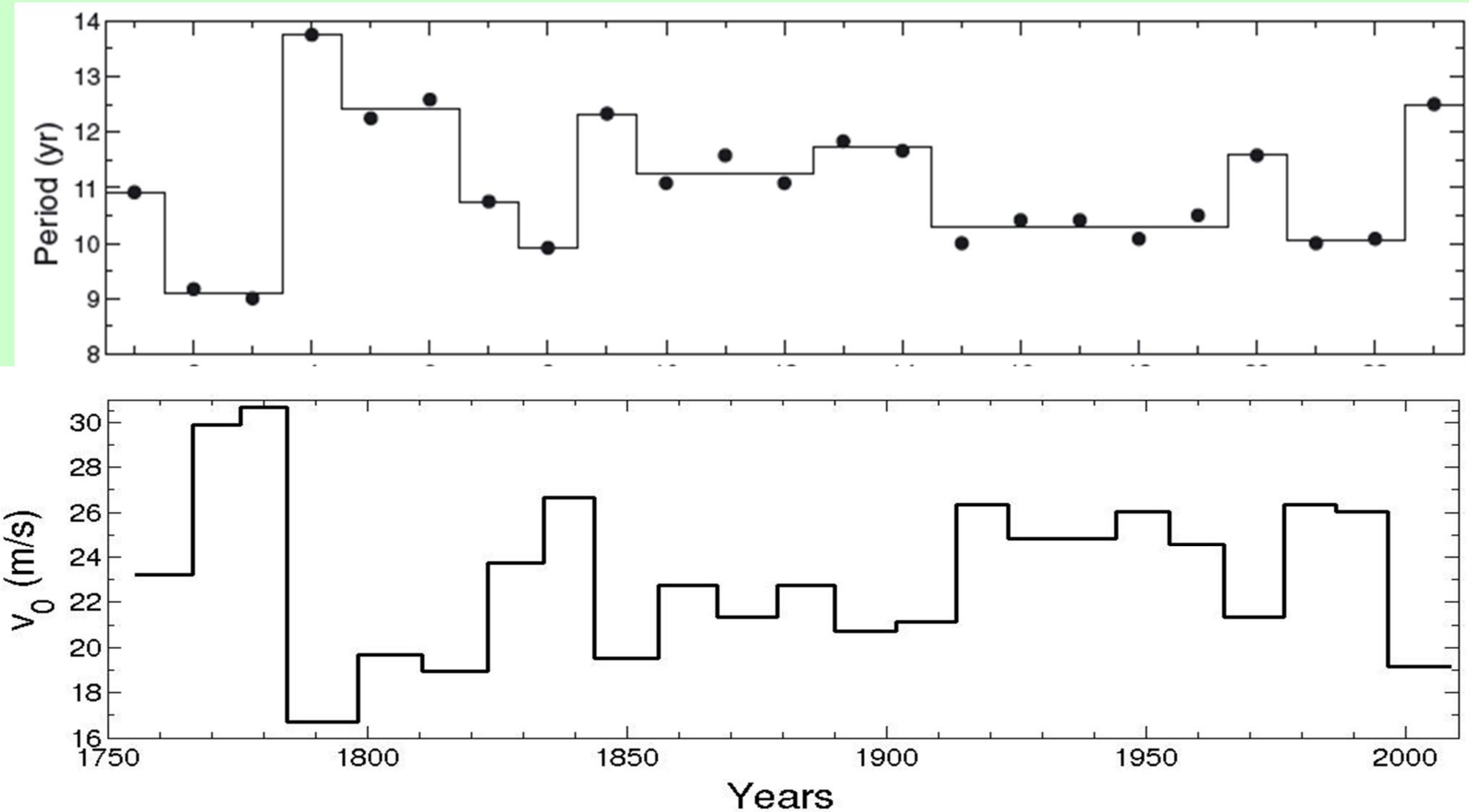
$$\text{Period} \propto \frac{1}{v_0^{0.89}}$$

(Dikpati & Charbonnea 1999)

$$\text{Period} \propto \frac{1}{v_0^{0.88}}$$

(Yeates, Nandy & Mackey 2008)

Fluctuation of the meridional circulation



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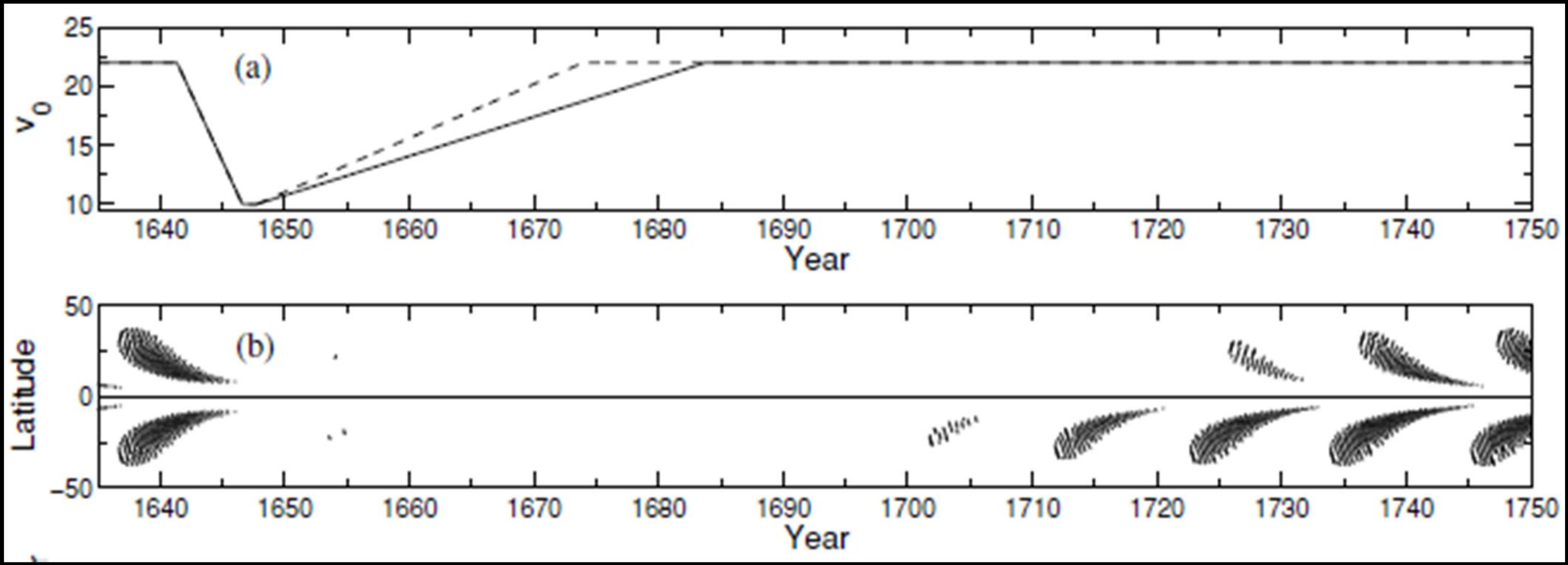
(Yeates, Nandy & Mackey 2008)

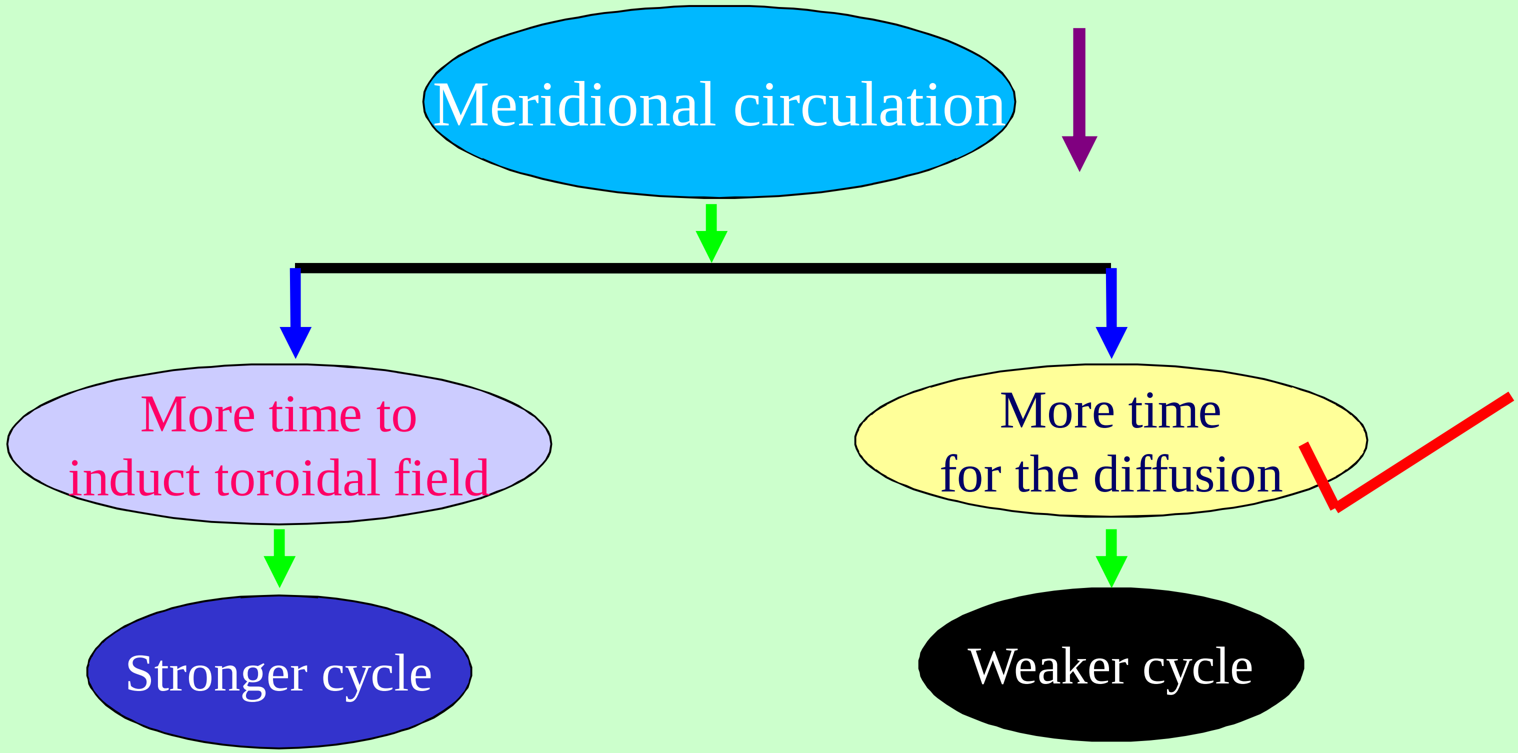
Sufficiently large decrease in meridional circulation can cause grand minimum



Karak (2010)

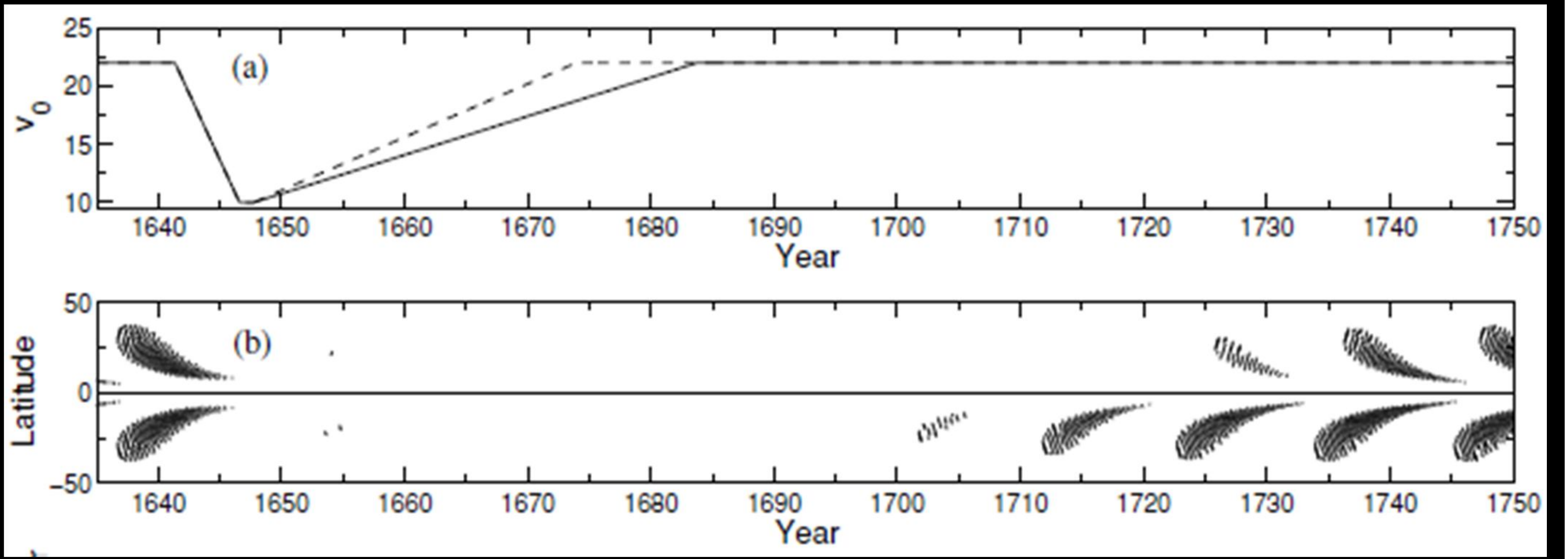
Physics of earlier result (Yeates, Nandy & Mackey 2008)

$$\frac{\partial B}{\partial t} + \ldots = \eta_t\left(\nabla^2 - \frac{1}{s^2}\right)B + s(B_p.\nabla)\Omega$$



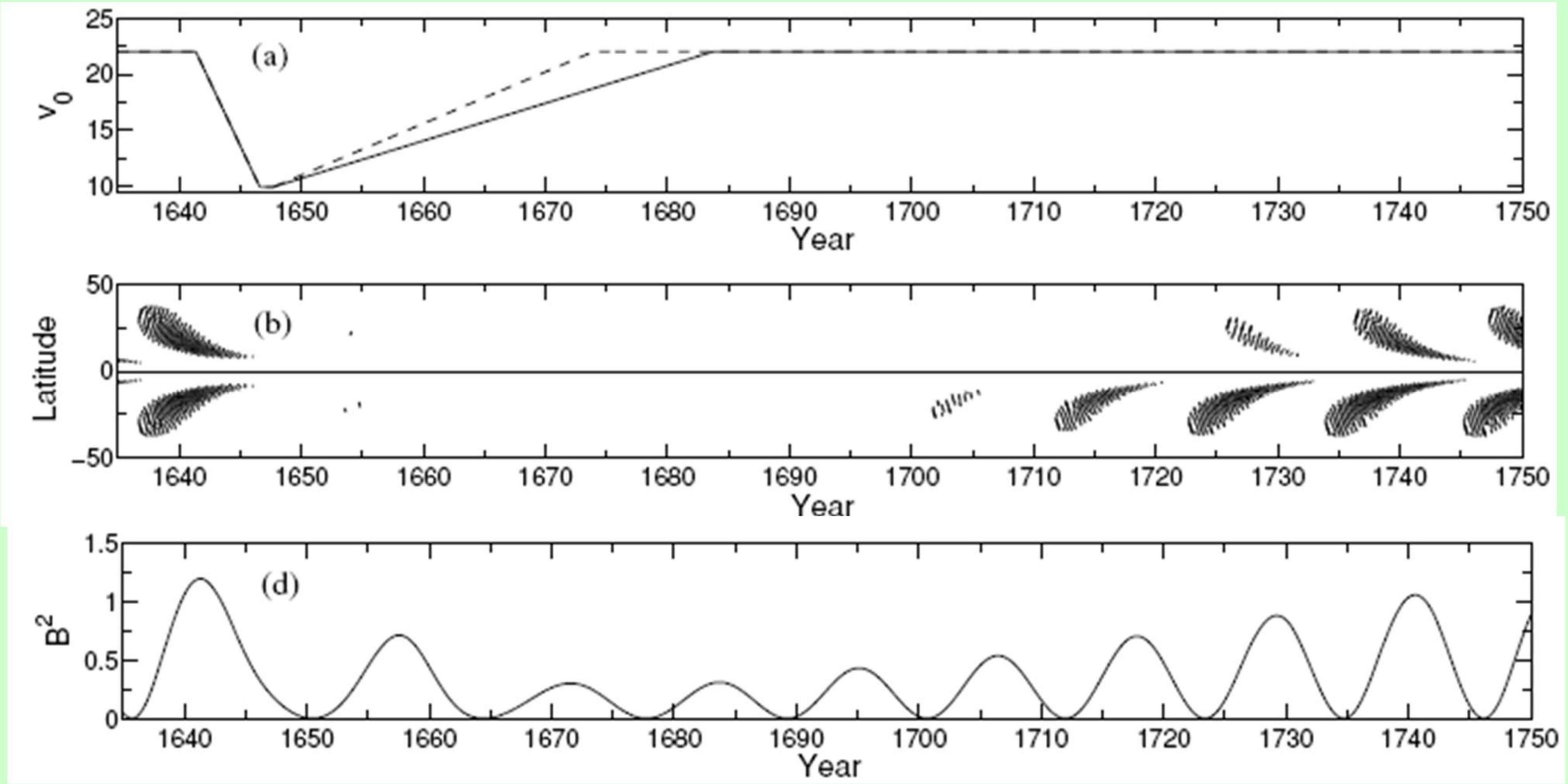
This is the case in our model!

Sufficiently large decrease in meridional circulation can cause grand minimum



Periods during grand minimum should be longer!

Karak (2010)



Observational evidences of the longer periods during grand minima

Miyahara et al (2004; 2007; 2010) -- Maunder minimum

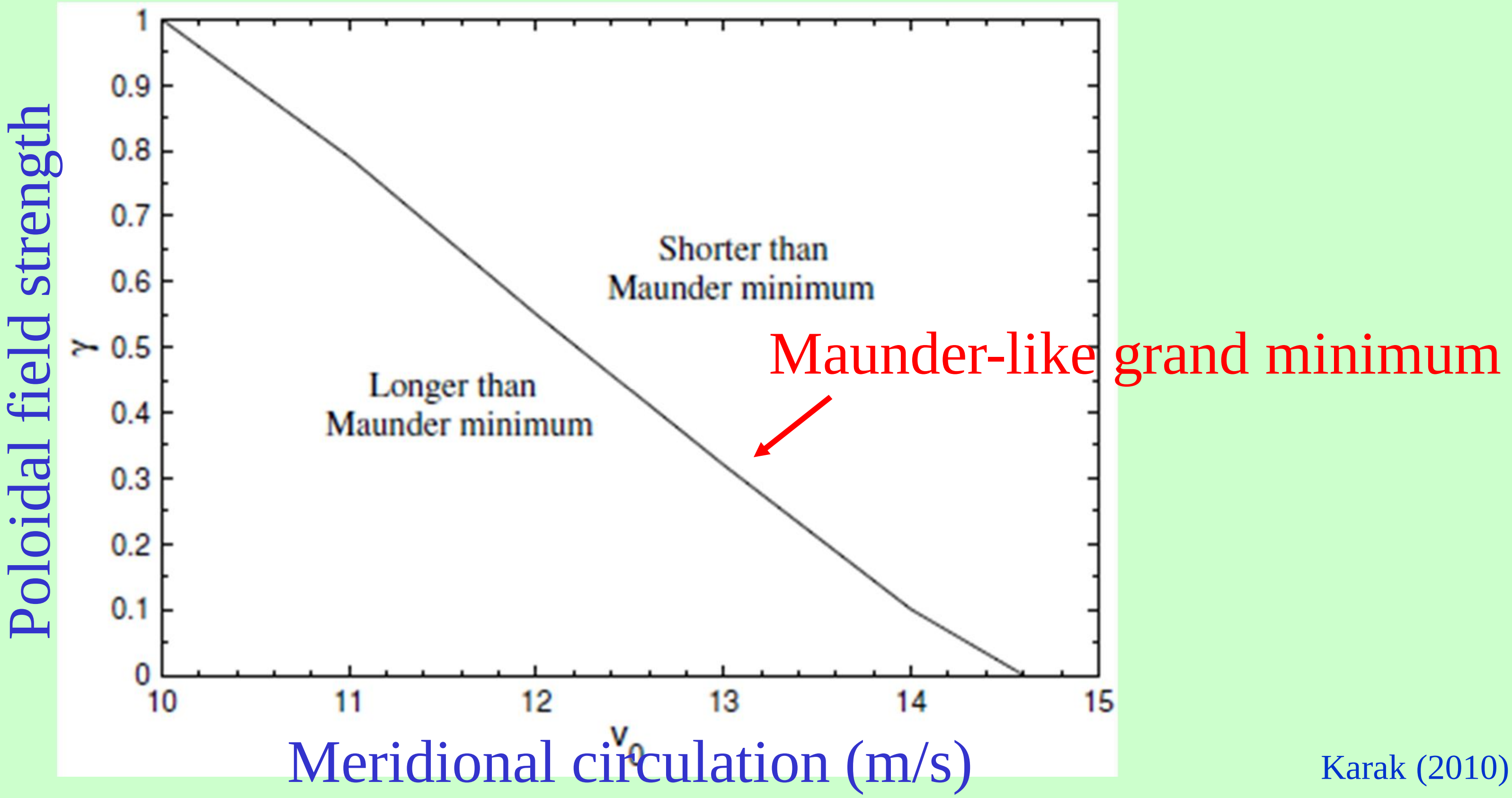
Miyahara et al (2006; 2007) -- Spoerer Minimum

Nagaya et al (2012) -- Grand minimum in the 4th Century BC

Modeling Maunder minimum

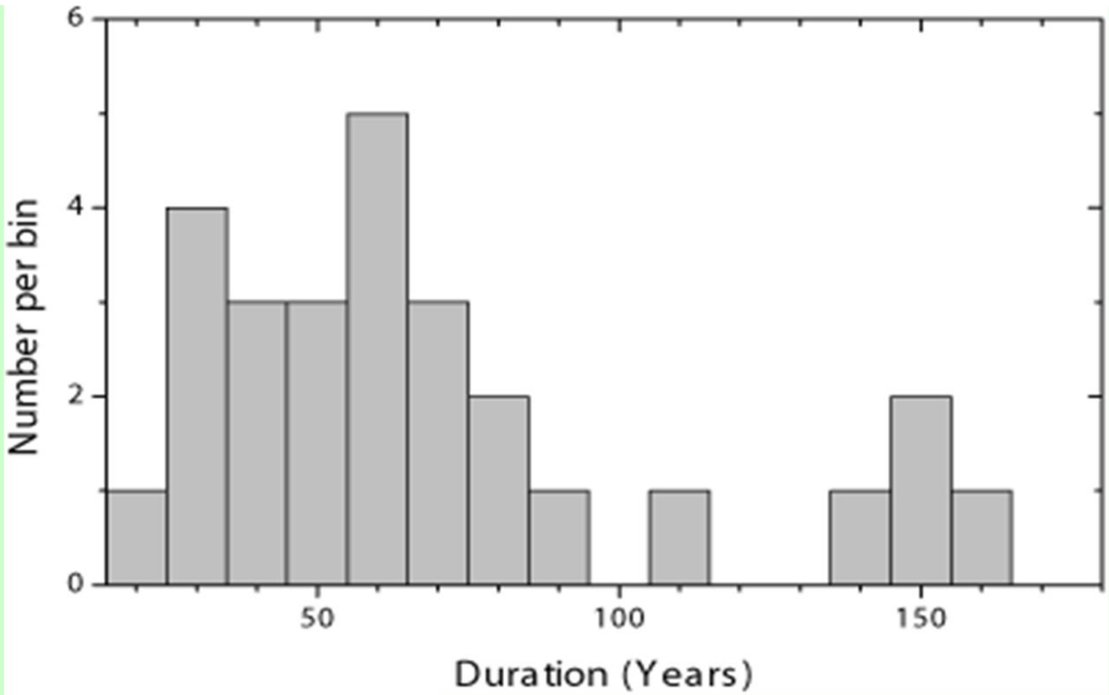
Large decrease of the poloidal field

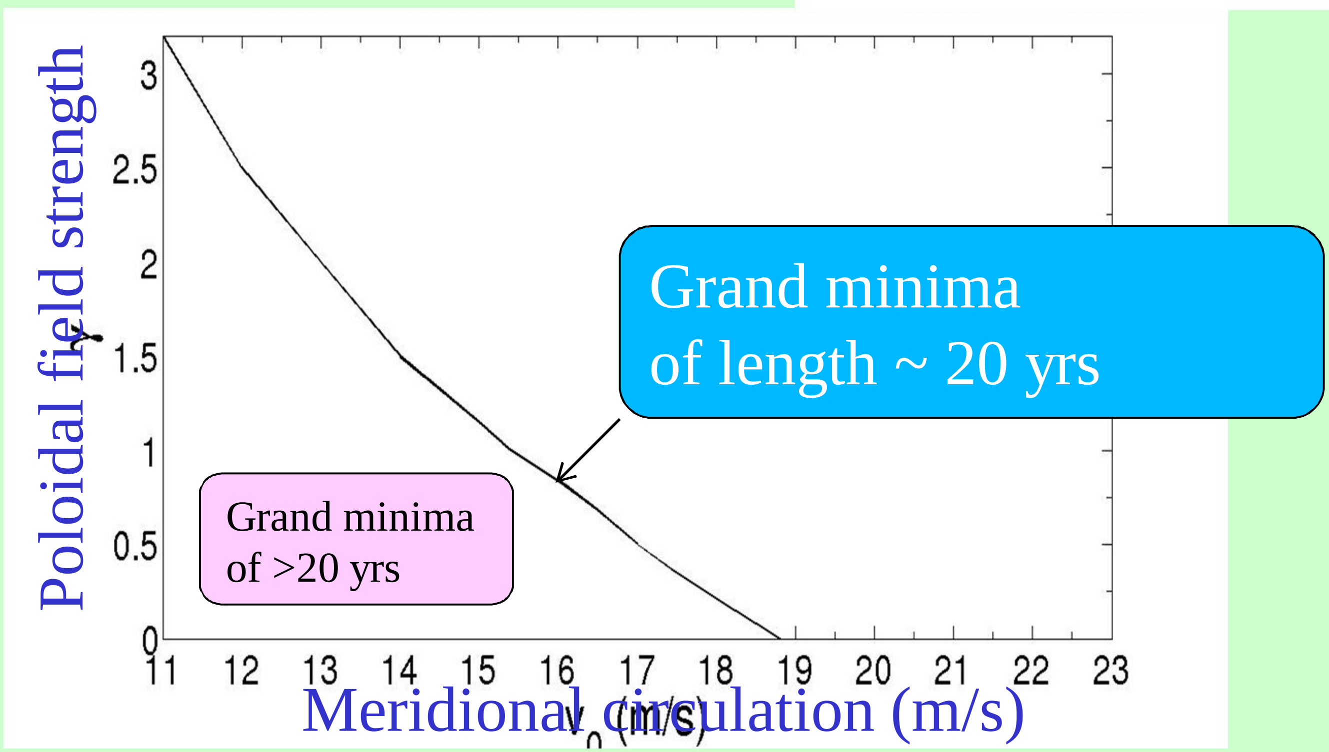
Large decrease of the meridional circulation



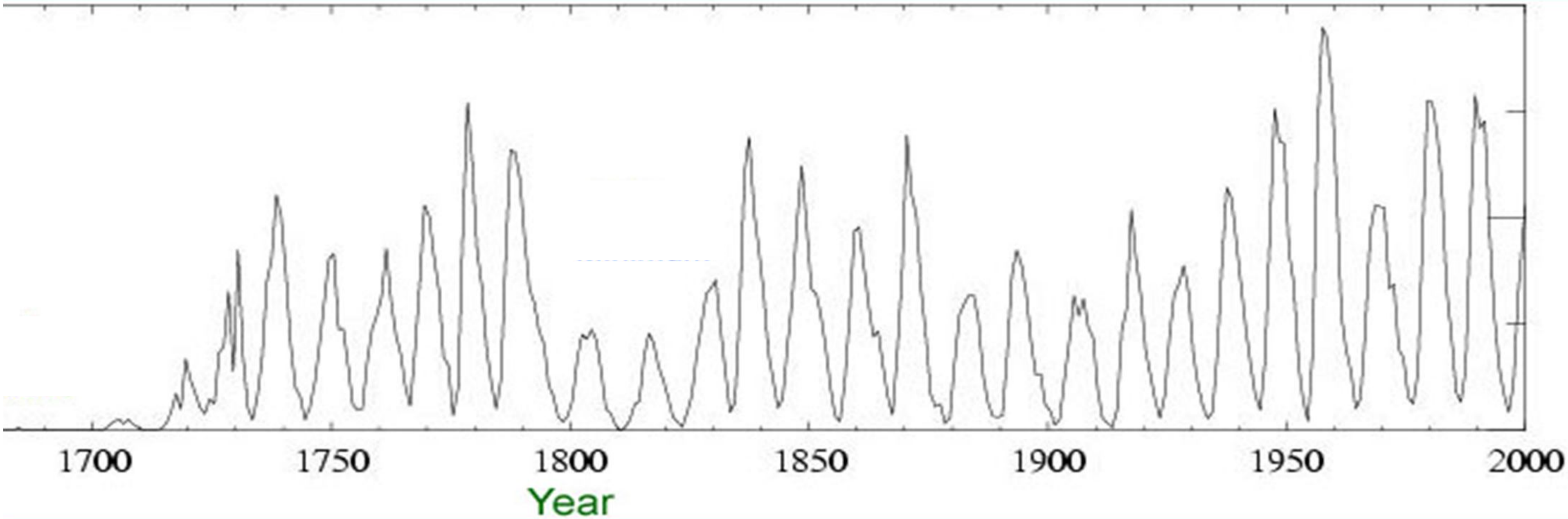
Karak (2010)

Observational data:
(Usoskin et al. 2007)





How to find out the strength of the meridional circulation in past?



In flux transport dynamo:

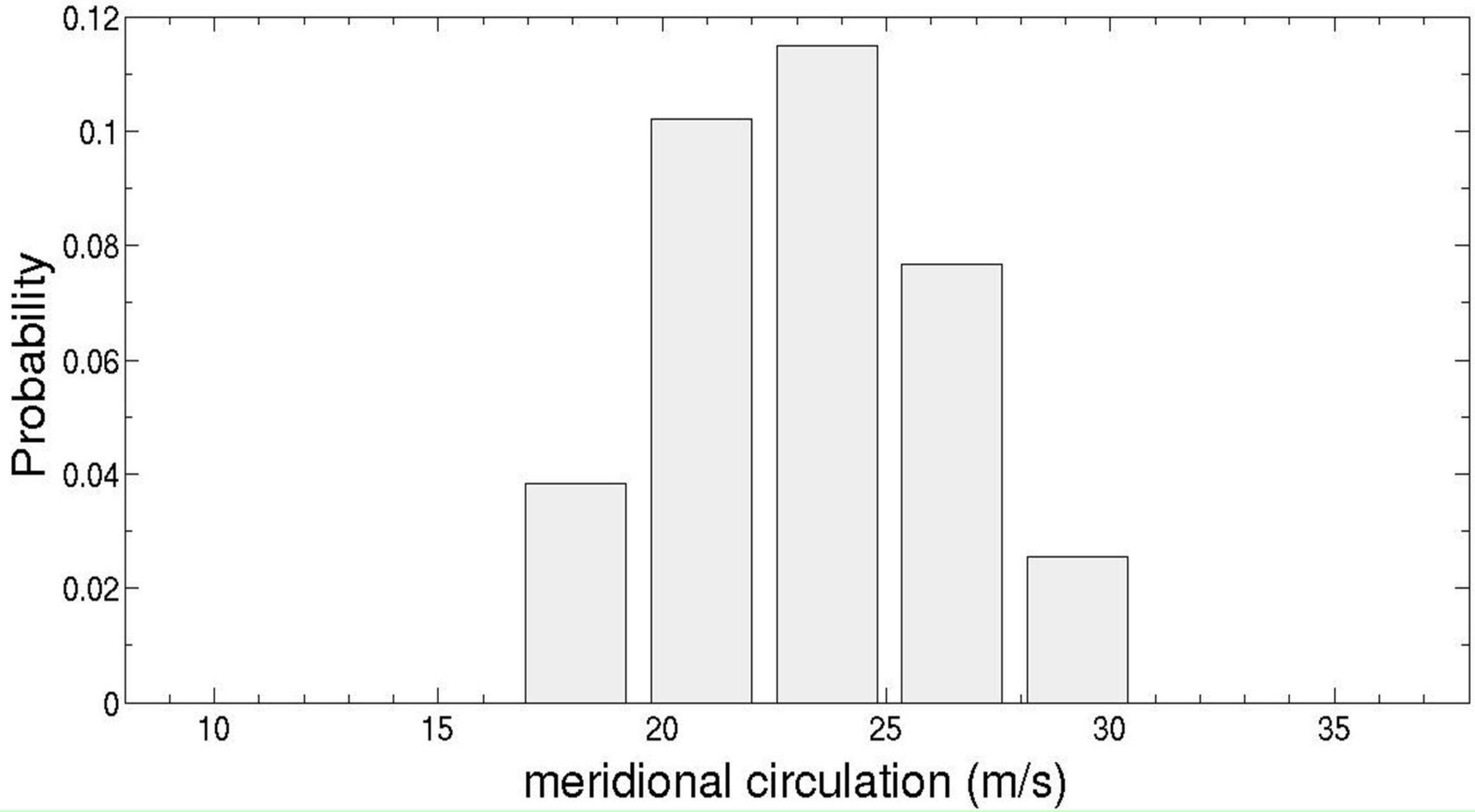
$$\text{Period} \propto \frac{1}{v_0^{0.89}}$$

(Dikpati & Charbonnea 1999)

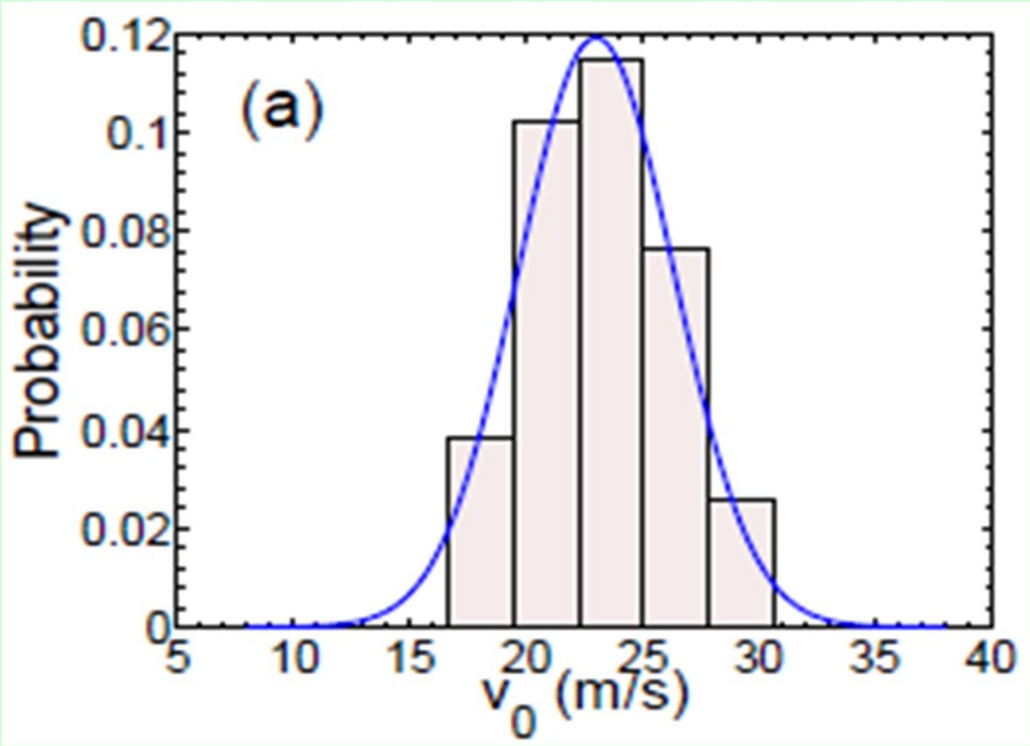
$$\text{Period} \propto \frac{1}{v_0^{0.88}}$$

(Yeates, Nandy & Mackey 2008)

Meridional circulation of last 28 cycle



Distribution of Meridional circulation



How to find the strength of the poloidal field?

.. 5, NO. 5 GEOPHYSICAL RESEARCH LETTERS

Polar field is a measure of the next sunspot cycle!

USING DYNAMO THEORY TO PREDICT

THE SUNSPOT NUMBER DURING SOLAR CYCLE 21

Kenneth H. Schatten, Philip H. Scherrer, Leif Svalgaard and John M. Wilcox

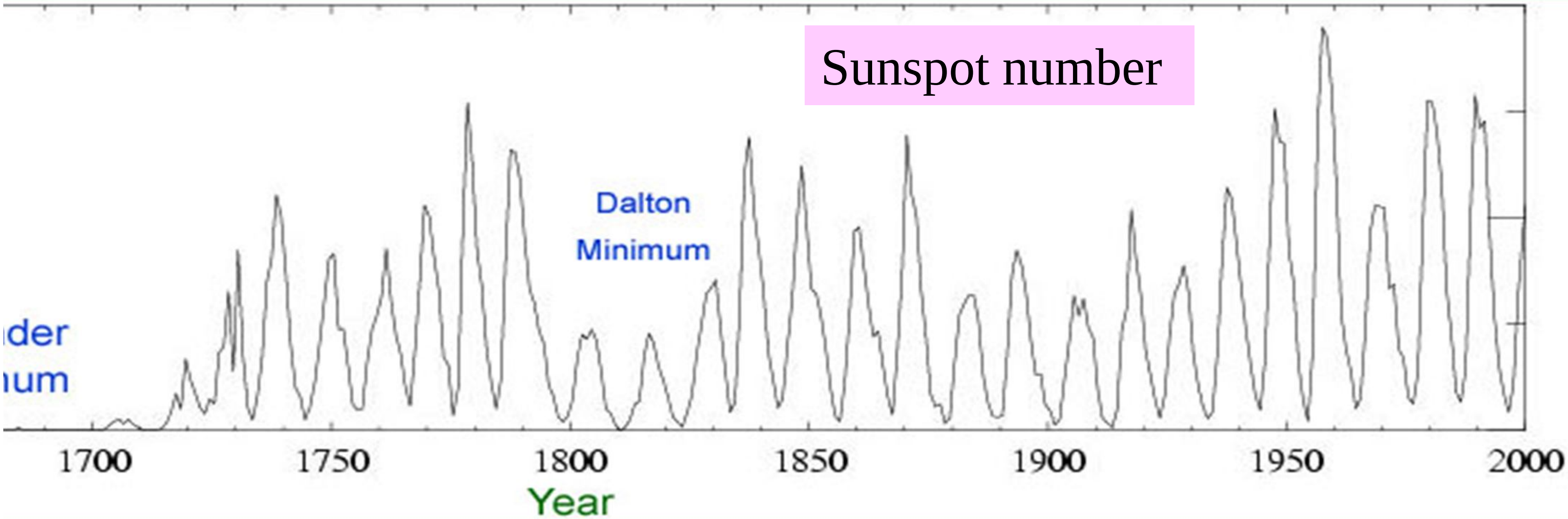
Institute for Plasma Research, Stanford University, Stanford, California

Abstract. On physical grounds it is suggested
it the sun's polar field strength near a solar
iimum is closely related to the following
:le's solar activity. Four methods of estima-
ig the sun's polar magnetic field strength near
.ar minimum are employed to provide an estimate
cycle 21's yearly mean sunspot number at solar
:imum of 140 ± 20. We think of this estimate
a first order attempt to predict the cycle's
:ivity using one parameter of physical
ɔortance based upon dynamo theory.

Polar Field Strength

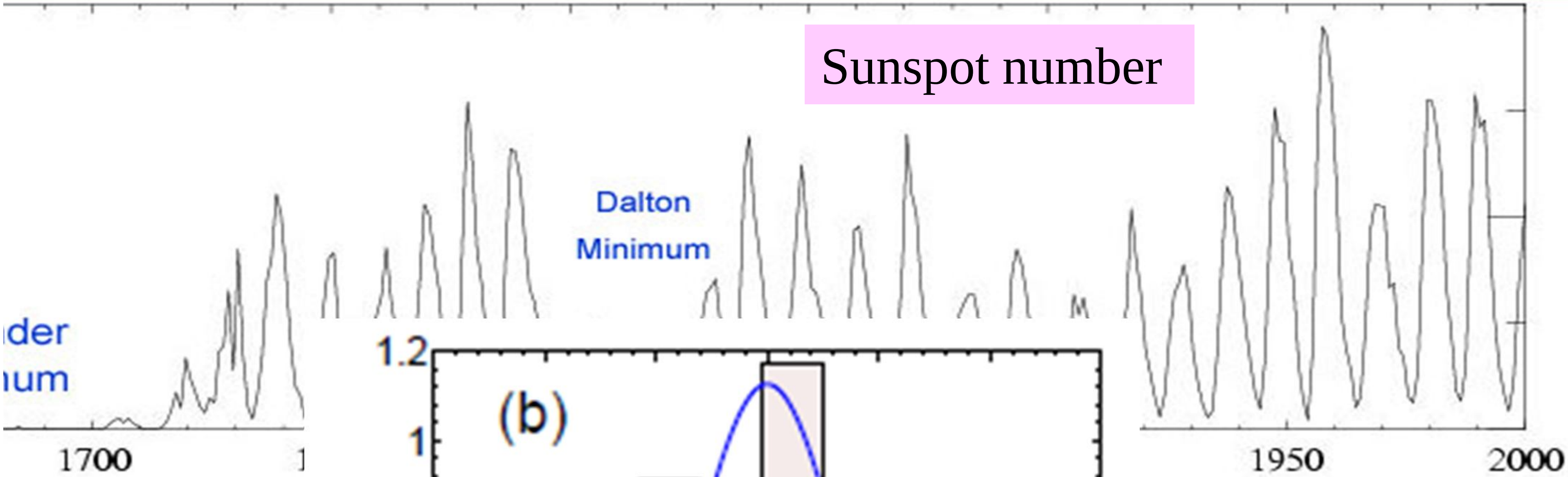
Estimates of the polar magnetic field
near sunspot minimum may be obtained fron
shape of the corona at the time of solar
or by the amount of flattening of the "wa
current sheet" at 1AU as obtained from ir
planetary magnetic field measurements ana
accordance with the methods of Rosenberg
Coleman (1969). A further and more direc
estimate of polar field strength is obtai
observing the number of polar faculae

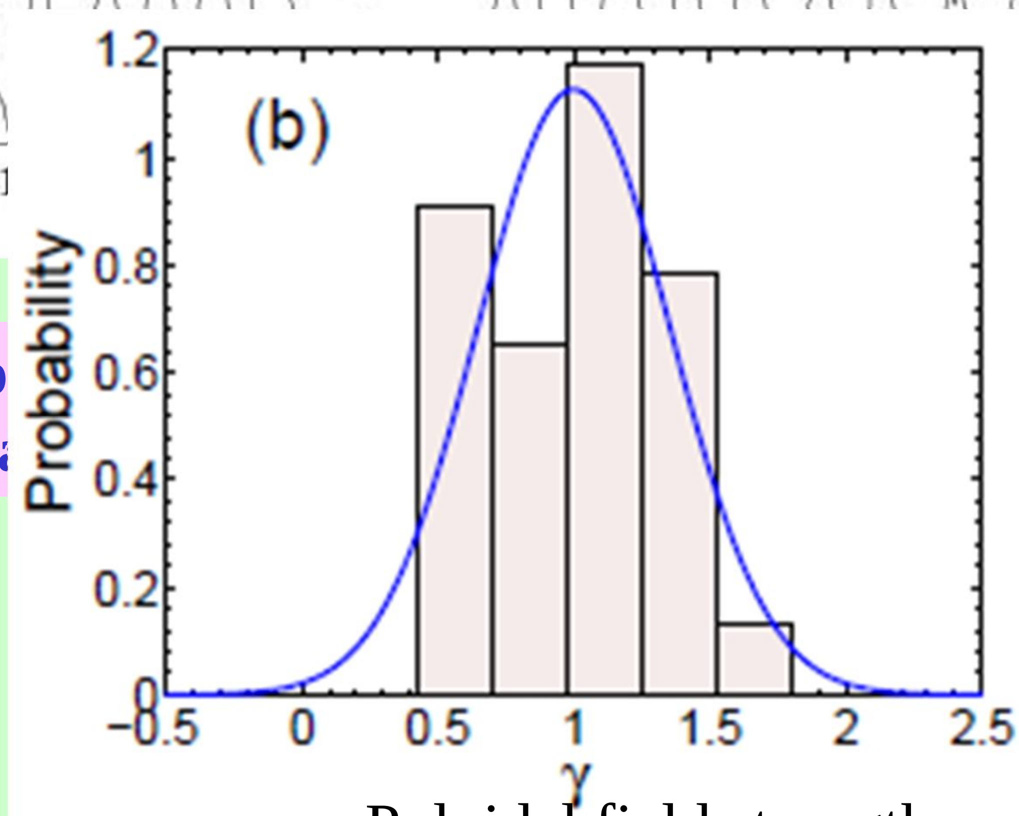
How to find out the strength of the poloidal field?



Assume a perfect correlation between the **peak sunspot number** and the **polar field of the previous cycle**

How to find out the strength of the poloidal field?



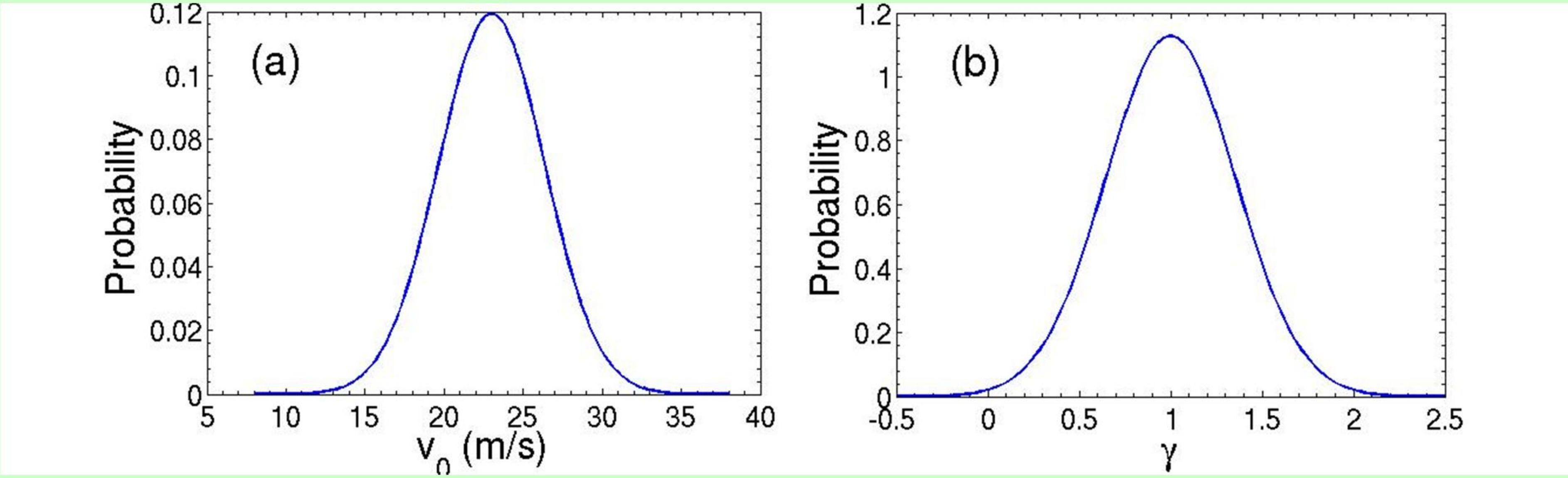


Poloidal field strength

Assume a p **spot number**
and the **pol**

Distributions of

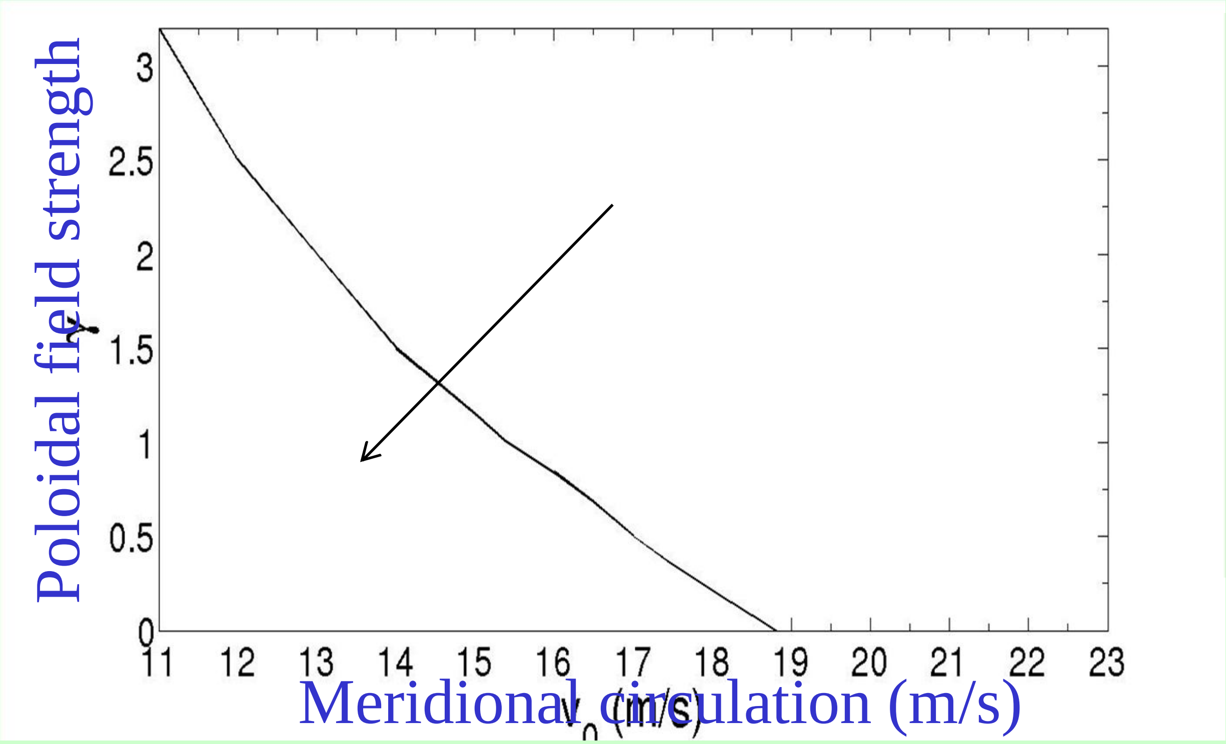
Meridional circulation **poloidal field strength**



$$P(\gamma, v_0)d\gamma dv_0 = \frac{1}{\sigma_v\sqrt{2\pi}}\exp\left[-\frac{(v_0-\overline{v_0})^2}{2\sigma_v^2}\right]\times\frac{1}{\sigma_\gamma\sqrt{2\pi}}\exp\left[-\frac{(\gamma-1)^2}{2\sigma_\gamma^2}\right]d\gamma\, dv_0$$

$$P(\gamma, v_0)d\gamma dv_0 = \frac{1}{\sigma_v\sqrt{2\pi}}\exp\left[-\frac{(v_0-\overline{v_0})^2}{2\sigma_v^2}\right]\times\frac{1}{\sigma_\gamma\sqrt{2\pi}}\exp\left[-\frac{(\gamma-1)^2}{2\sigma_\gamma^2}\right]d\gamma\, dv_0$$

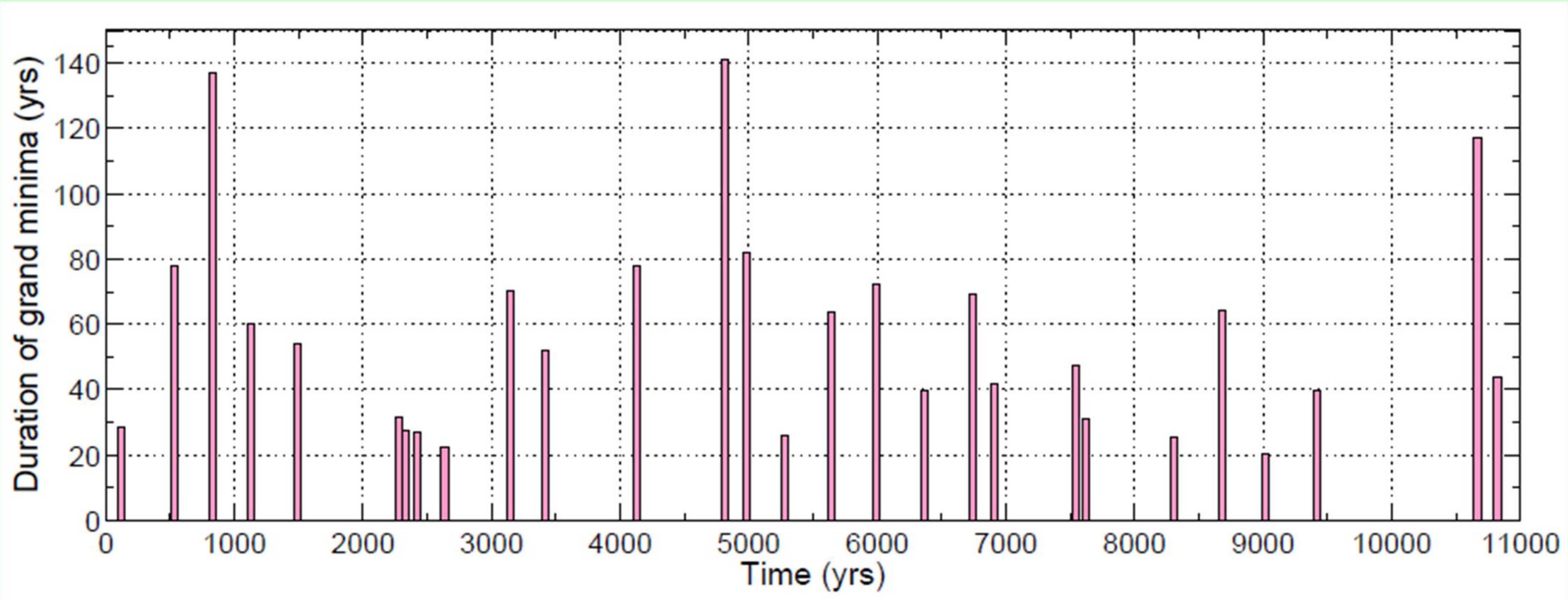
$$\int\int p(\gamma, v_0)d\gamma dv_0$$



Theoretical value = 1.8% (±0.6)

Observational value = 2.7%

Results of simulation of grand minima

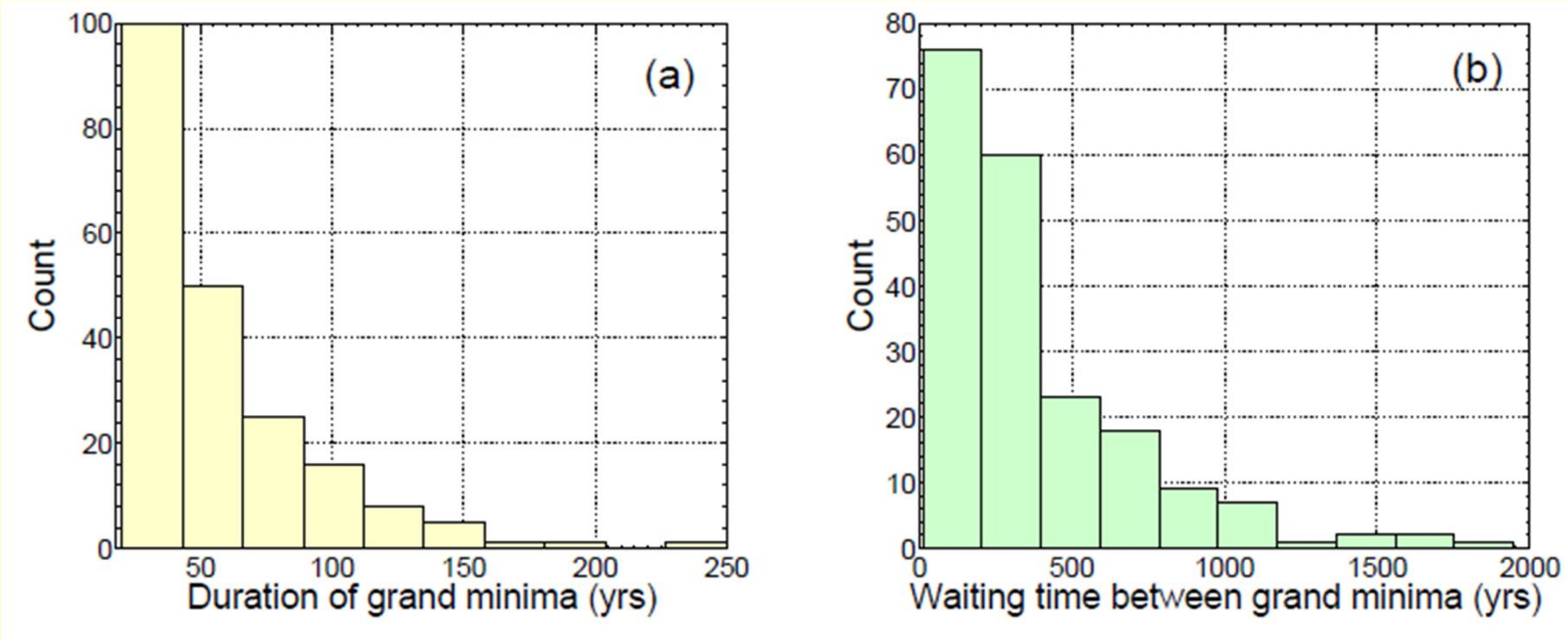


We get 20–28 grand minima in 11,000 years

Observational value = 27 (Usoskin et al. (2007)

(Choudhuri & Karak 2012)

Statistic of grand minima



Waiting times of grand minima based on 27 grand minima in last 11,400 years reported by Usoskin et al. (2007) is also exponential

-- governed by stationary memoryless stochastic processes

whereas the observed distribution of durations is not so conclusive.

Conclusions

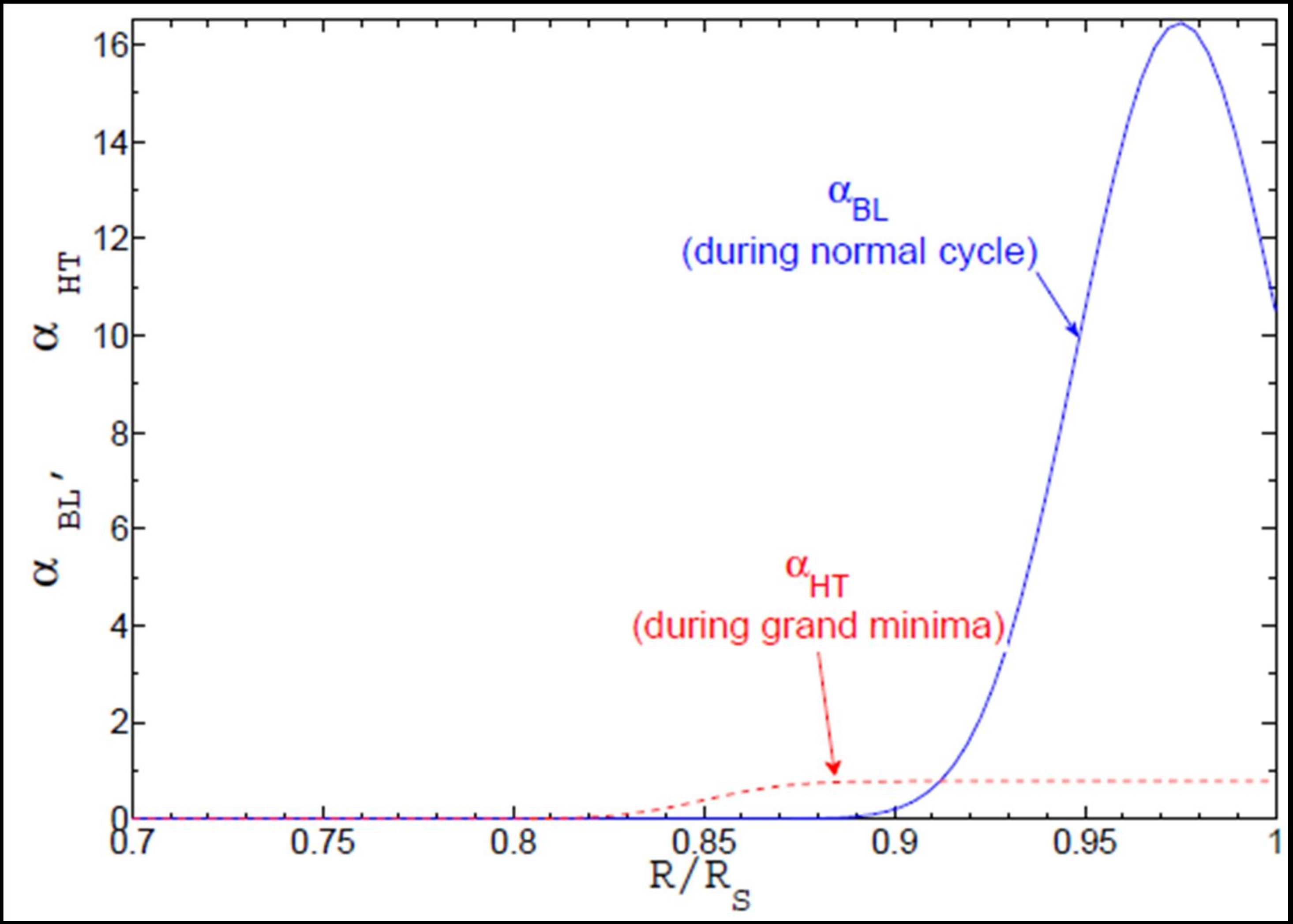
- Grand minima are probably caused by the fluctuation in the poloidal field and the fluctuation in the meridional circulation.

- By measuring the fluctuations of these we can model the observed frequency of grand minima using flux transport dynamo model.

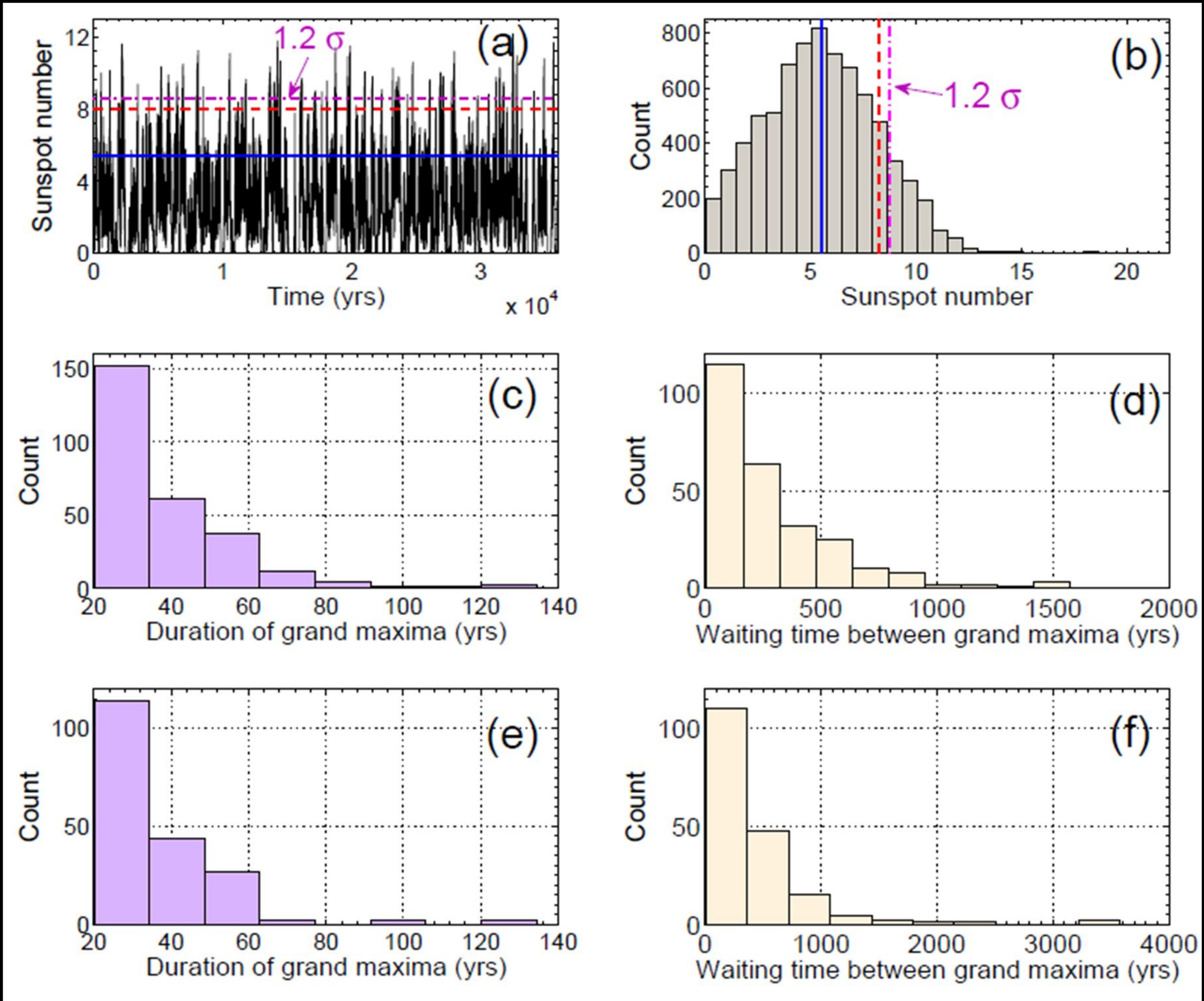
- Recovery from grand minima is less understood.

During grand minima **Babcock-Leighton mechanism may no work as there are no sunspots???**

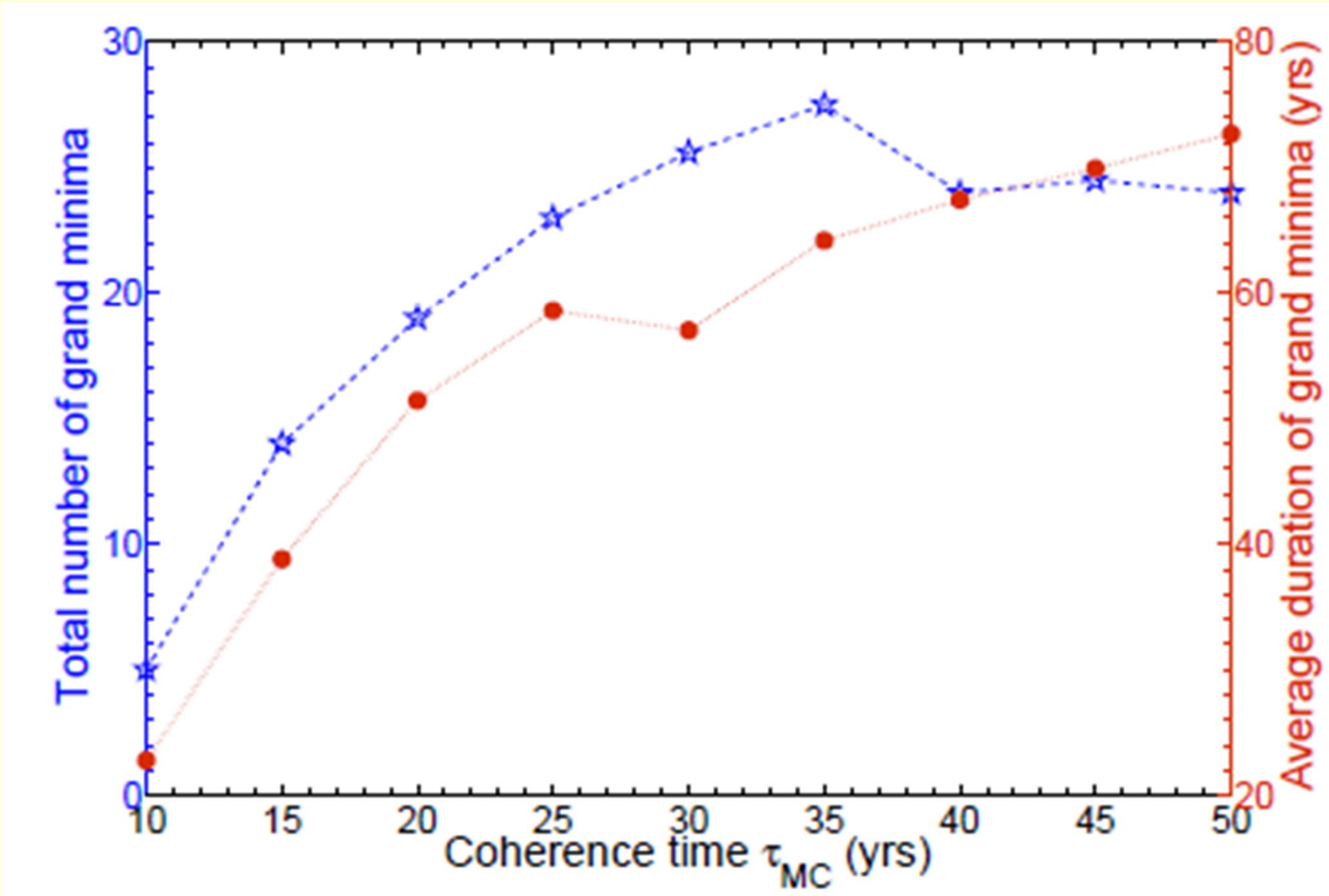
Alpha effect proposed by Parker (1955) and Steenbeck, Krause & R adler (1969) is a good candidate!



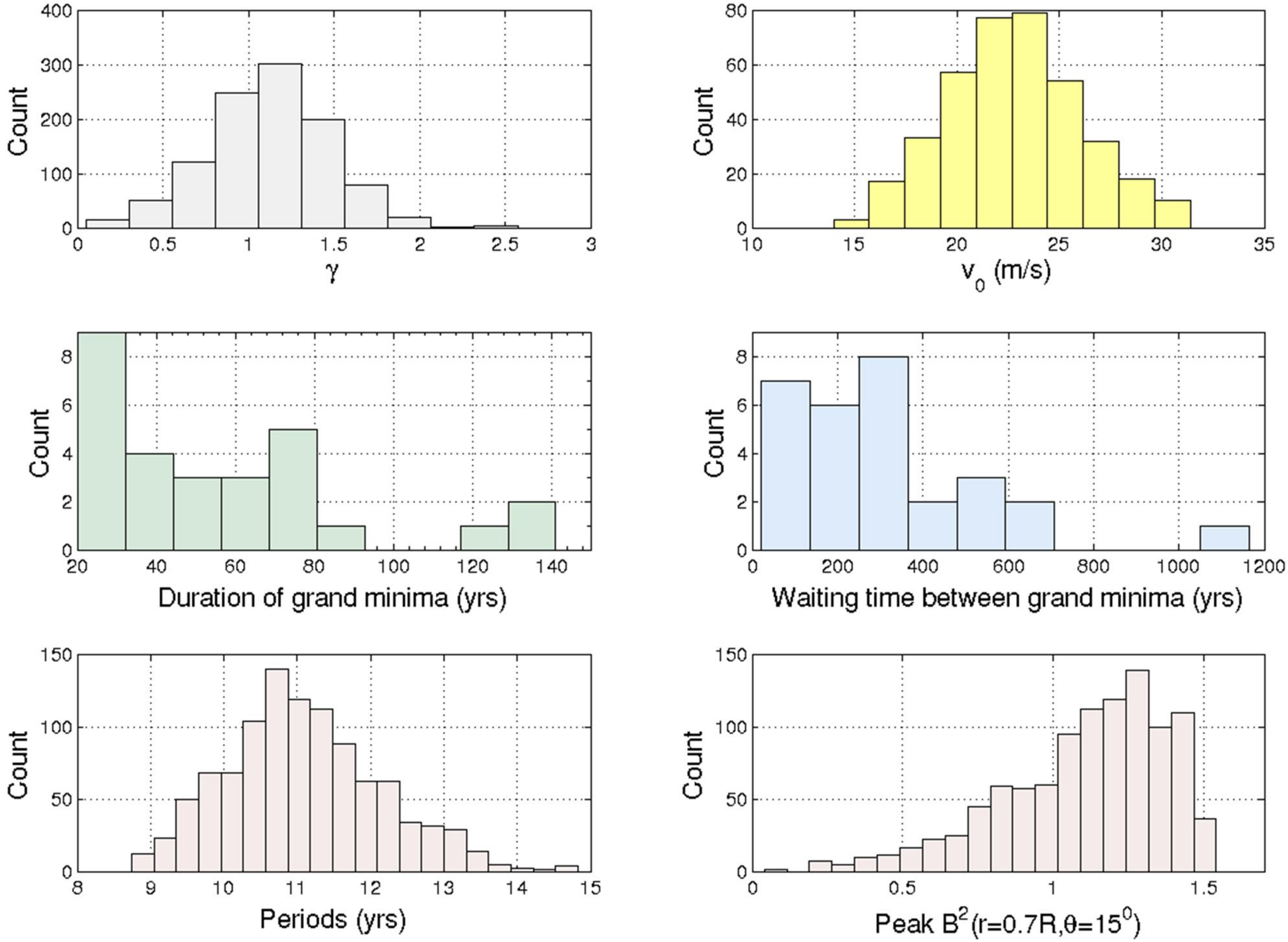
What about grand maxima



Thanks



Total number of grand minima
Average duration of grand minima (yrs)
Coherence time τ_{MC} (yrs)
0
10
20
30
20
40
60
80
10
15
20
25
30
35
40
45
50



Count
γ
Count
v_0 (m/s)
Count
Duration of grand minima (yrs)
Count
Waiting time between grand minima (yrs)
Count
Periods (yrs)
Count
Peak B^2($r=0.7R, \theta=15^0$)

α-quenching: $\alpha = \frac{\alpha_0}{1+|\overline{B}|^2}$; $\frac{\partial A}{\partial t} + \ldots = \eta_p (\nabla^2 - \frac{1}{s^2}) A + \alpha B$

Long history – Stix 1972; Ivanova & Ruzmaikin 1977; Yoshimura 1978; Schmitt & Schussler 1989, Krause & Meinel 1988; Brandenburg et al. 1989 ……….

Has a stabilizing effect instead of producing irregularities

Charbonneau, St-Jean & Zacharias (2005), Charbonneau, Beaubien & St-Jean (2007) – The odd-even effect (Gnevyshev-Ohl rule) may be due to period doubling just beyond bifurcation point

Weiss, Cattaneo & Jones (1984) found chaos in some highly truncated models

Beer, Tobias & Wiess (1998) grand minima are due to chaotic nature of the dynamo