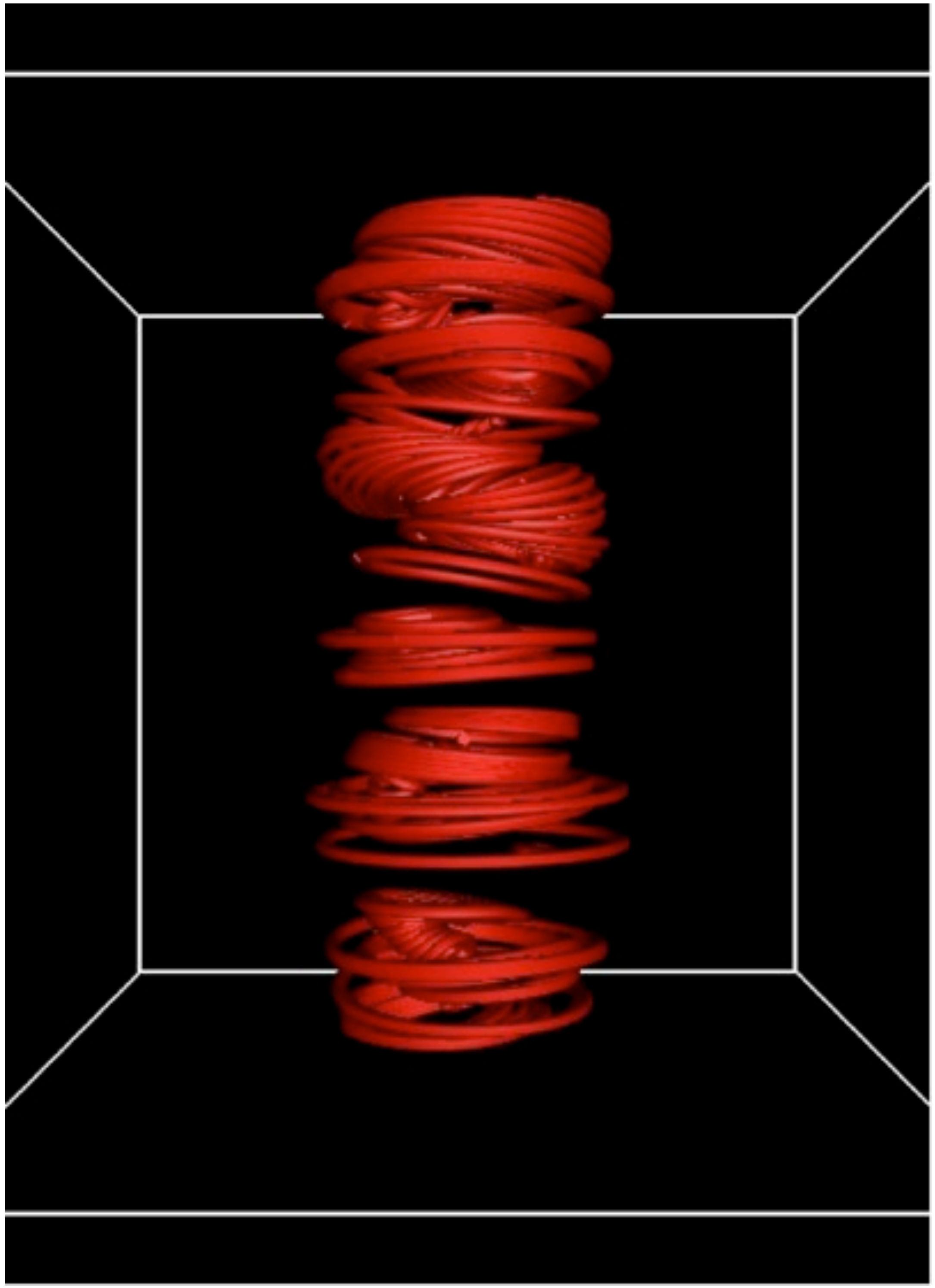
Stability of toroidal magnetic fields in non-convective stars



Jon Braithwaite
Universität Bonn

in collaboration with

Juan Ibáñez-Mejía
Institut für Theoretische Astrophysik, Heidelberg &
American Museum of Natural History, New York

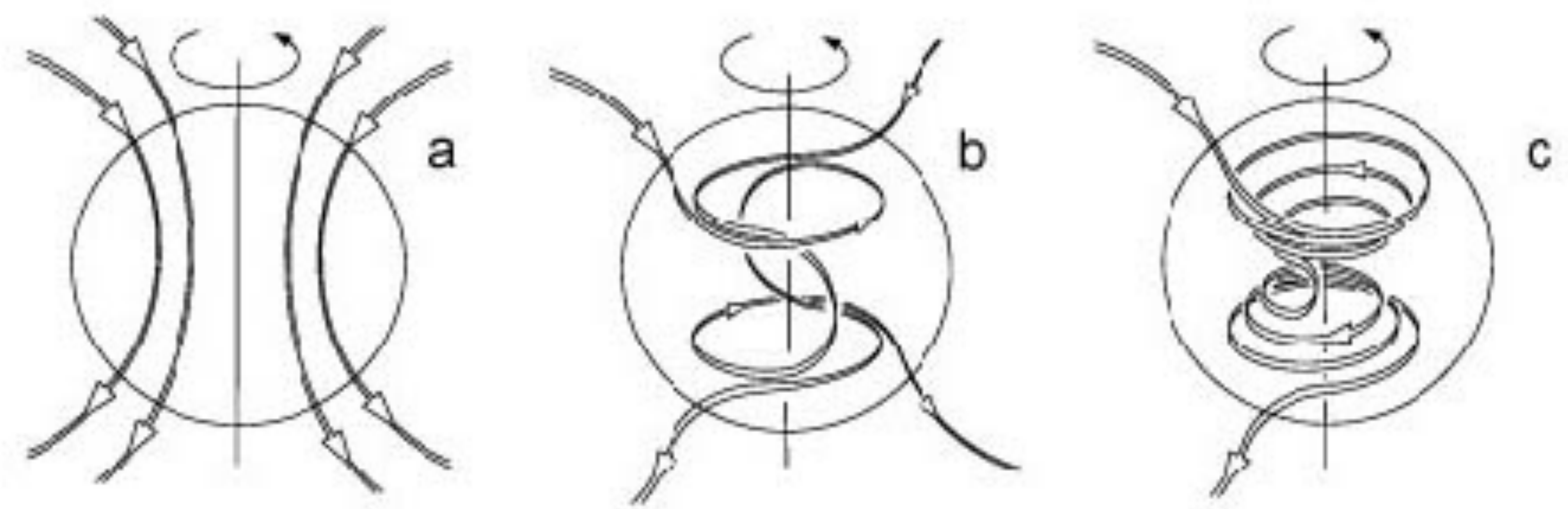
Stockholm, 11th April 2013

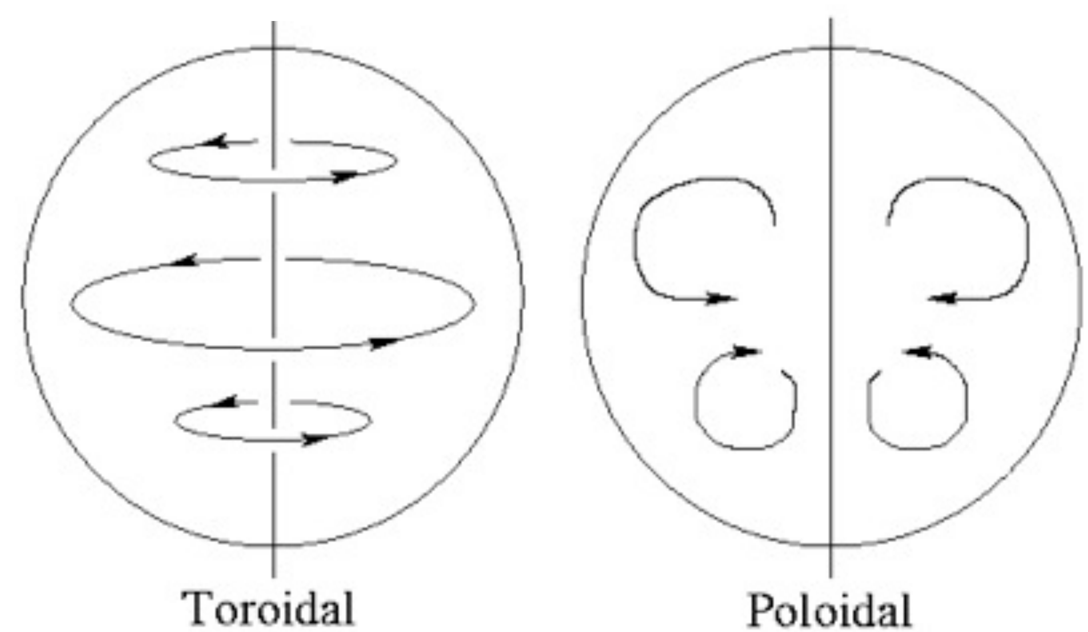
Contents

- Why is this interesting?
- The Tayler instability
    - Simulations - effects of rotation, diffusivity & weak poloidal field
    - Fossil fields
- Summary

Why study the stability of toroidal fields in stars?

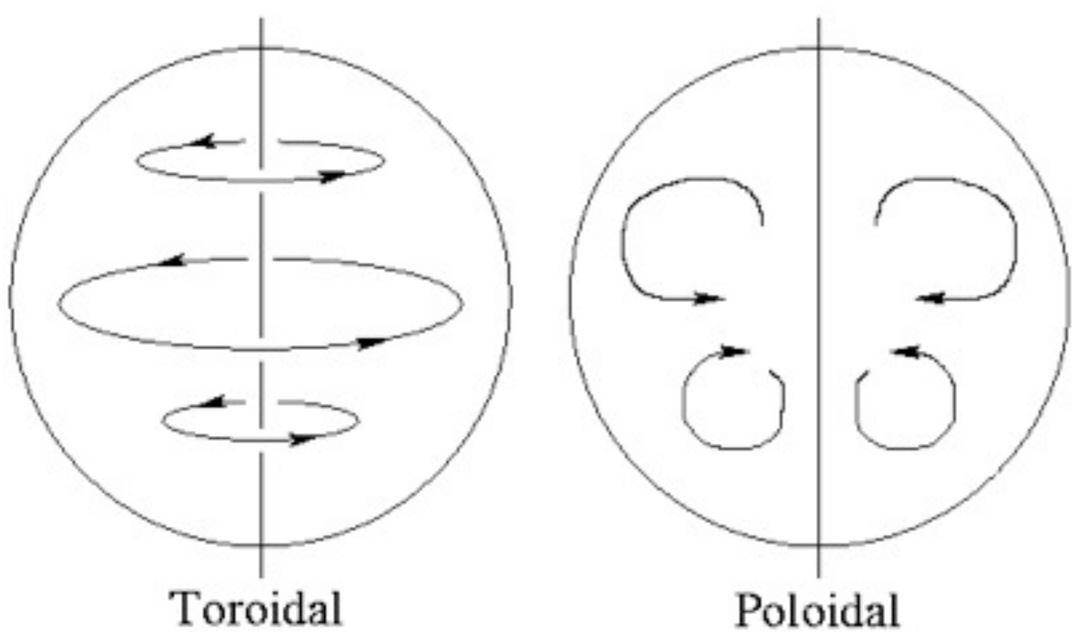
- Arbitrary seed field + differential rotation
  → predominantly toroidal field





Why study the stability of toroidal fields in stars?

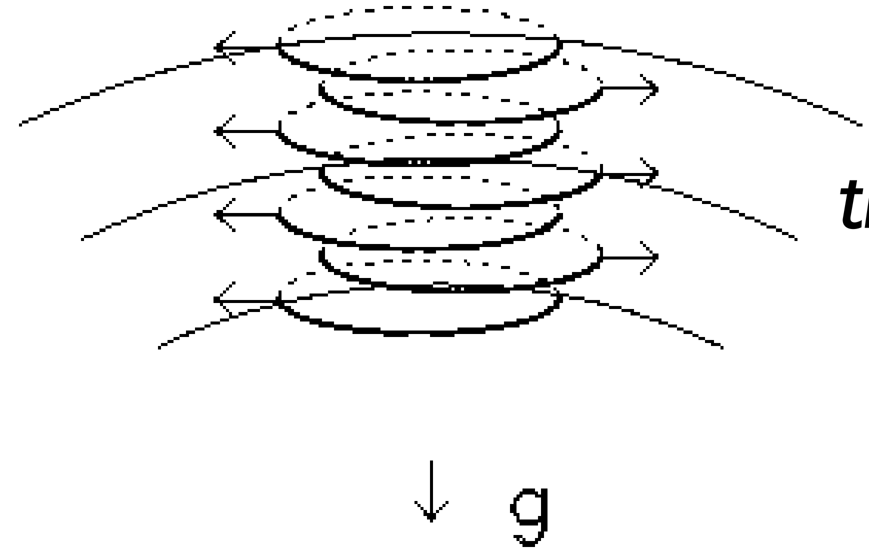
- Arbitrary seed field + differential rotation
  → predominantly toroidal field
- Toroidal fields can be unstable
- Instability → angular momentum transport & chemical mixing
- Stable (fossil) fields:
  what is possible?



Contents

- Why is this interesting?
- The Tayler instability
  - Simulations - effects of rotation, diffusivity & weak poloidal field, etc.
  - Fossil fields
- Summary

The Tayler instability

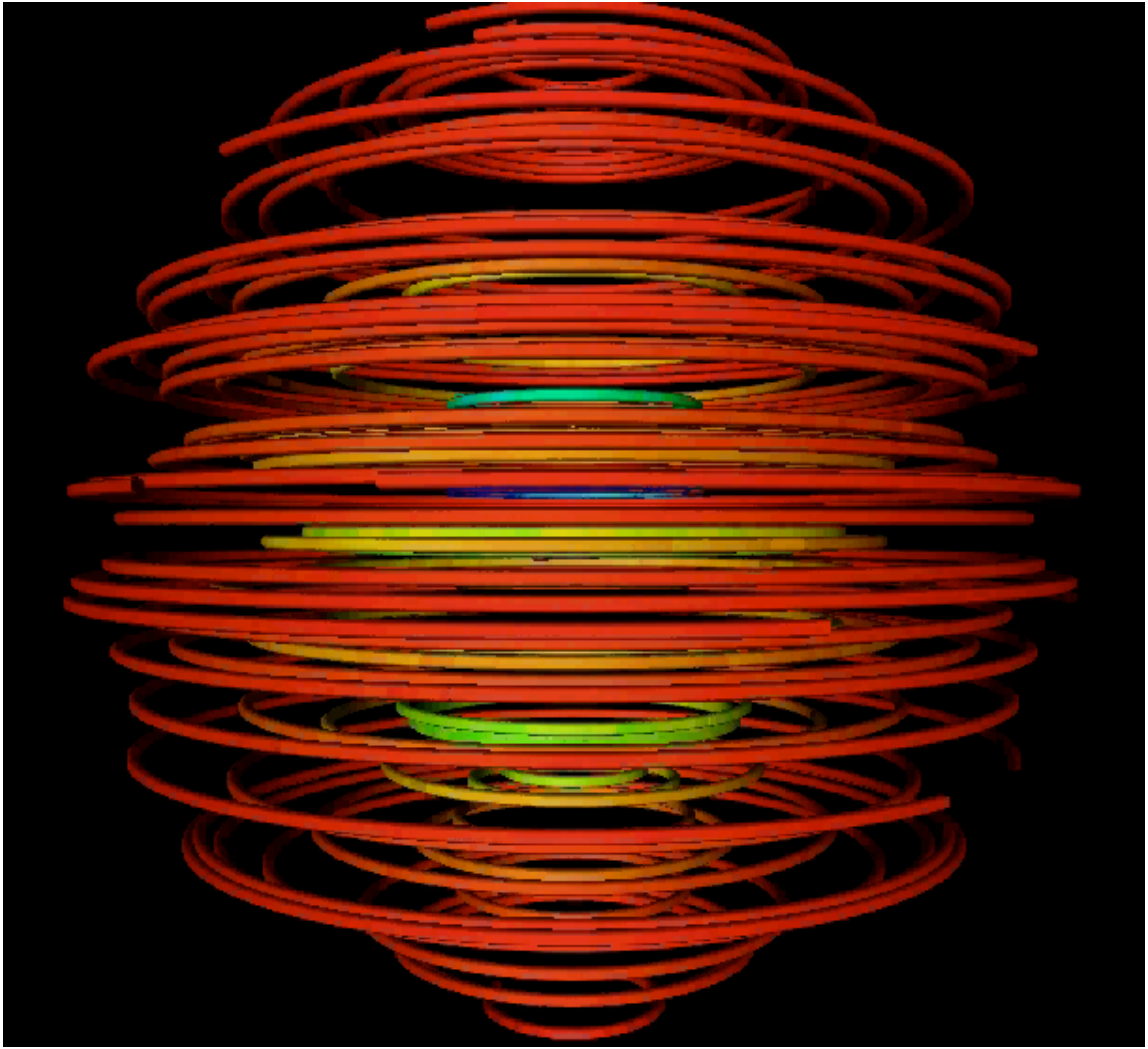


the $m=1$ mode near the pole

- Field strength increases with cylindrical radius: energy release when stronger field contracts towards axis & weaker field moves outwards.
- Instability if *(Tayler 1957, 1973)*:
  $p > m^2/2-1$ ($m \geq 1$) and $p > 1$ ($m=0$) ($p = d\ln B / d\ln R$)
  with a growth rate $\sigma \sim \omega_A$ ($\omega_A = v_A / R$)
- Low-order azimuthal modes set in first (less bending of field lines required)
- Stability conditions local in meridional plane
- Requires some radial motion!
- A bit like the fluting instability in sunspots -- curvature-driven!

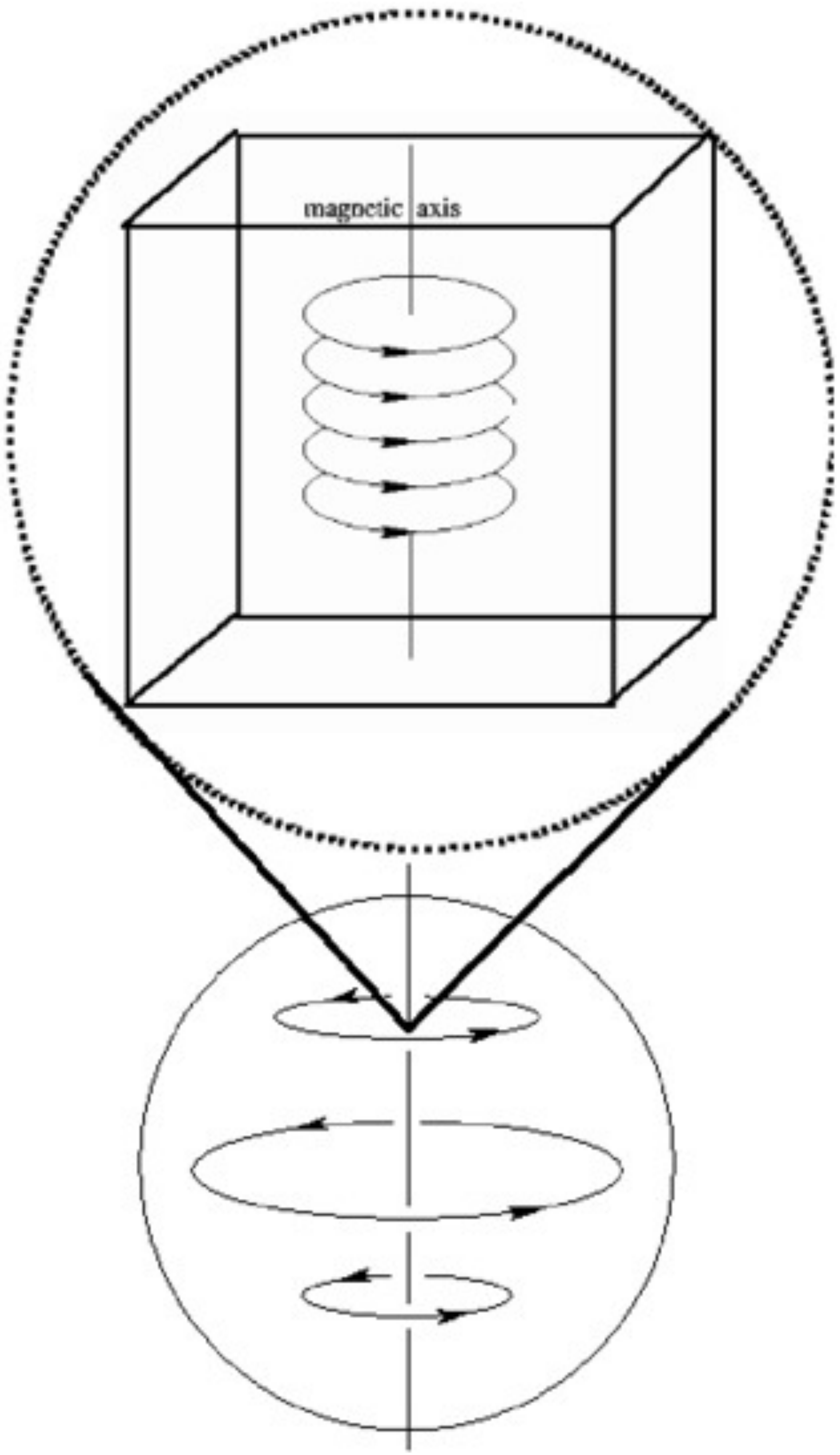
Global simulations of toroidal field instability in a star

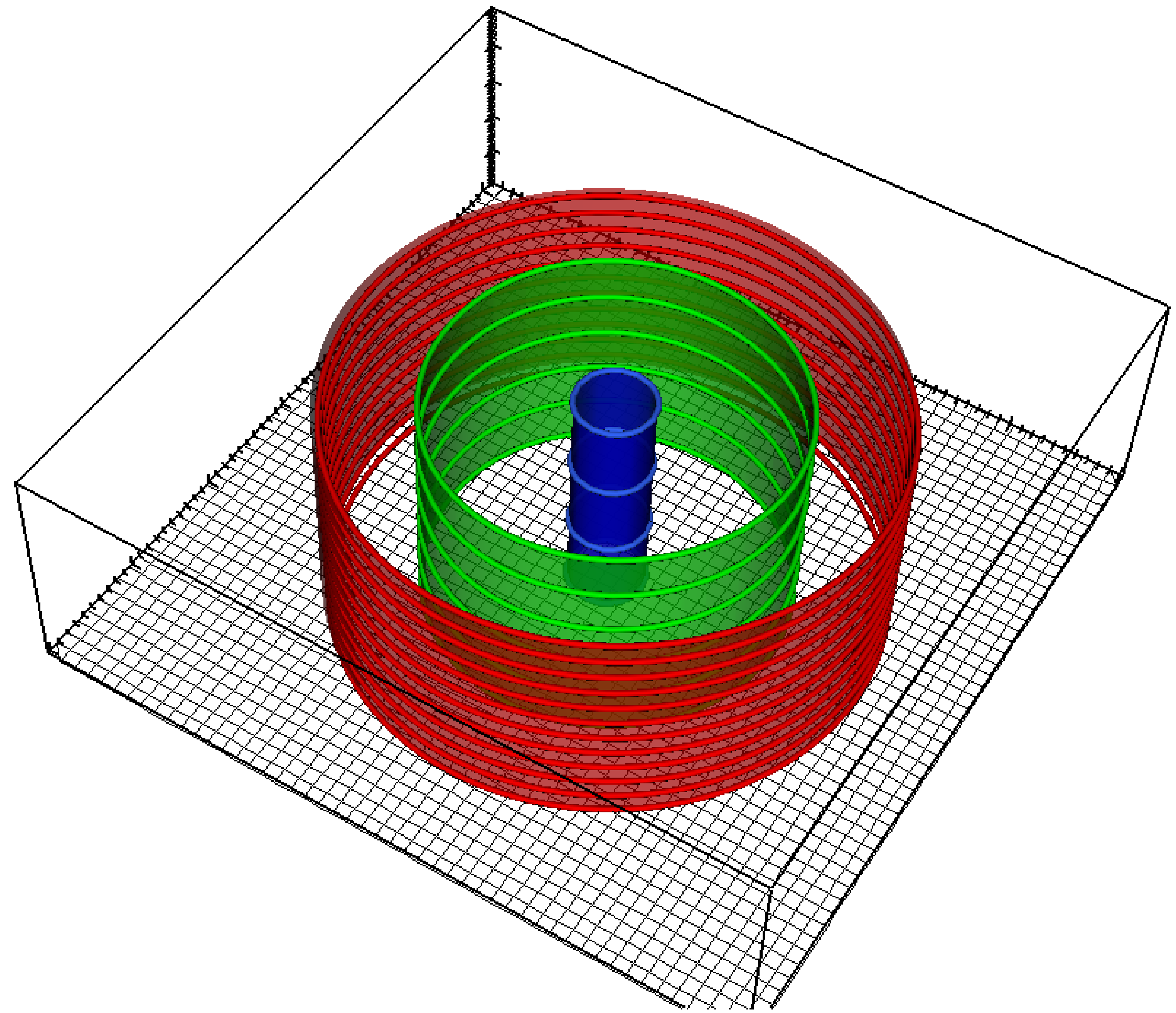
(using Åke Nordlund's stagger code)



Duez, Braithwaite & Mathis 2010

Local simulation around axis



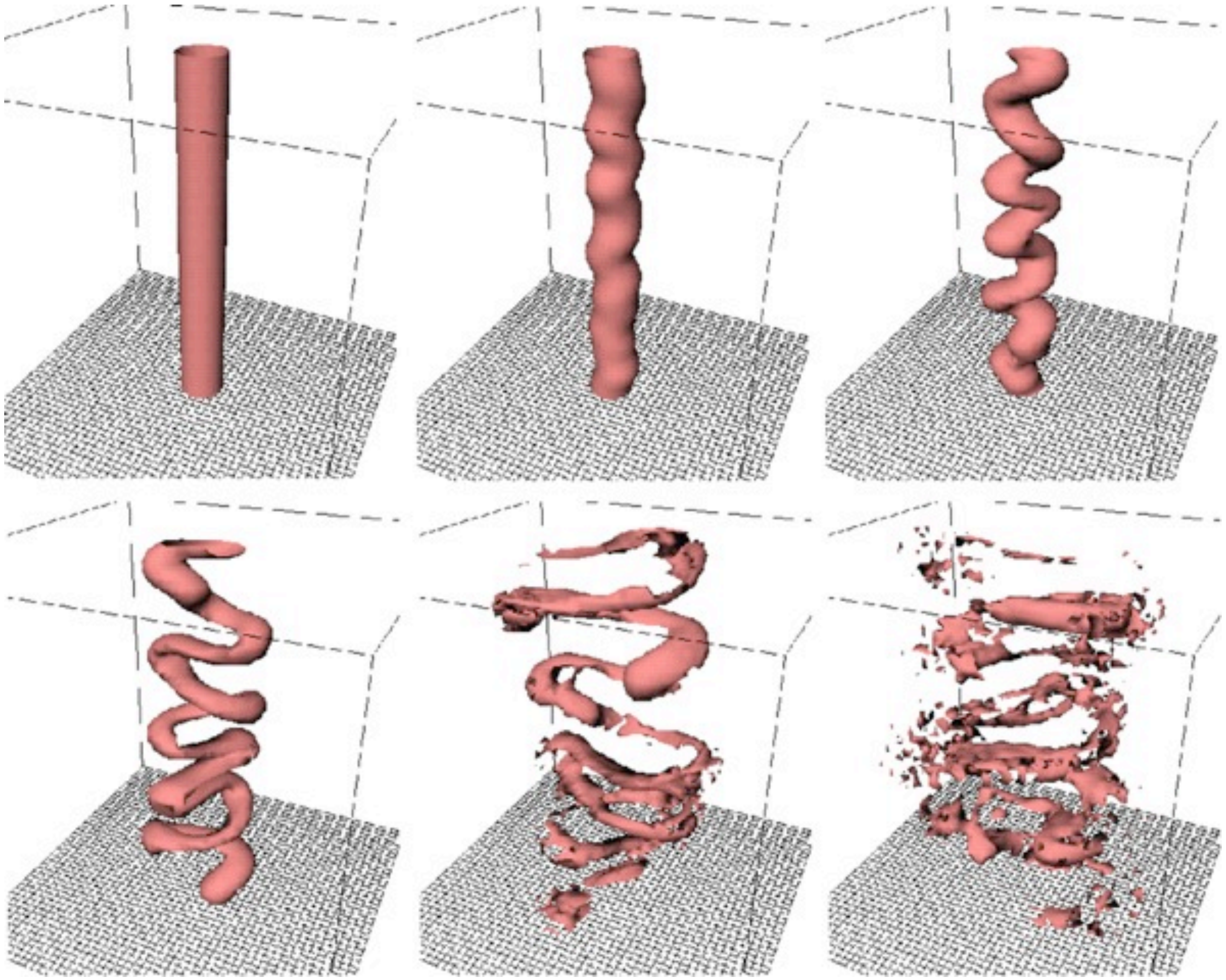


Magnetic surfaces

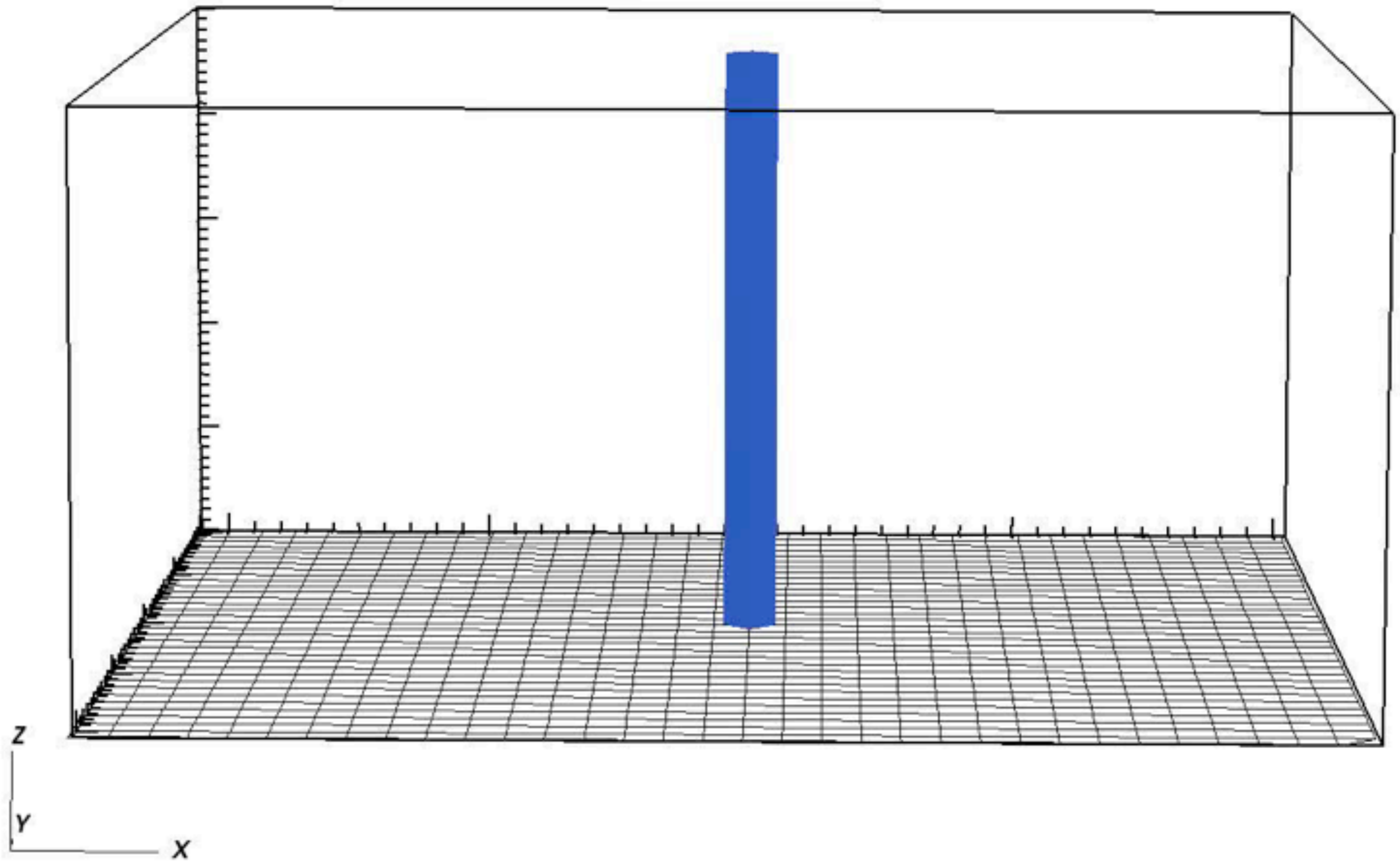
Various optional ingredients: rotation, magnetic diffusion, stratification & thermal diffusion, poloidal (axial) field

First simulations of Tayler instability

(Braithwaite 2006)



Time-series of isosurface positions



Simple simulation with toroidal field, no diffusion, no rotation, no gravity

(Ibanez & Braithwaite 2013)

Summary so far, results from simulations

- Growth rate $\sigma \simeq \omega_A$
- Dependence of stability condition on p and m:
  $p > m^2/2-1$ $(m \geqslant 1)$ and $p > 1$ $(m=0)$ $(p = \mathrm{d}\ln B/\mathrm{d}\ln R)$
- Rotational suppression in $p=1$ case
- Suppression by stratification

Contents

- Why is this interesting?
- The Tayler instability
    - Simulations - effects of rotation, diffusivity & weak poloidal field
    - Fossil fields
- Summary

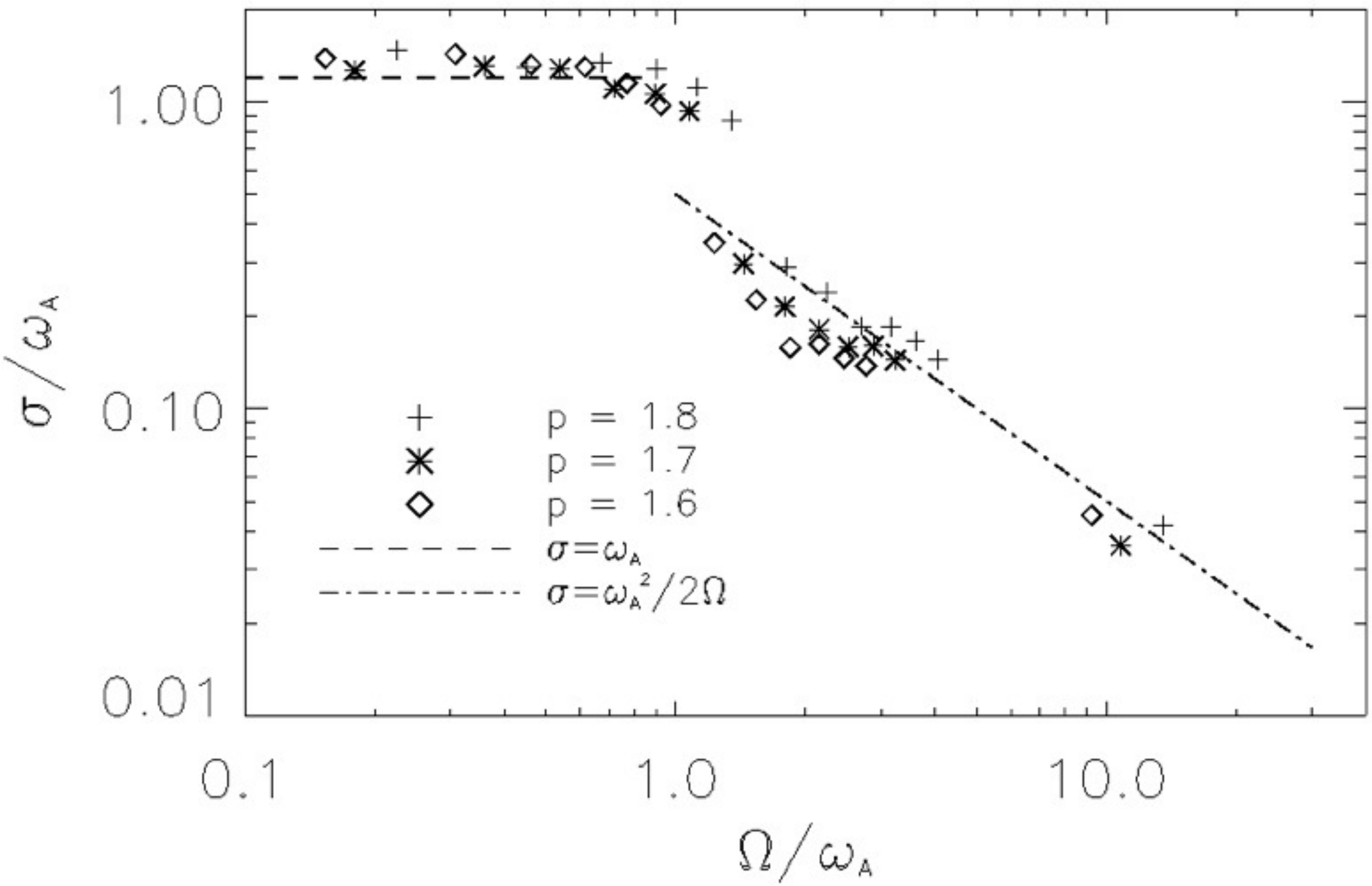
The effect of rotation
(without diffusion)

- With slow rotation ($\Omega << \omega_A$) we expect instability if *(Tayler 1957, 73)*
  $p > m^2/2-1$ $(m \geqslant 1)$ and $p > 1$ $(m=0)$
  and a growth rate of $\sigma \sim \omega_A$
- With fast rotation ($\Omega >> \omega_A$) we expect instability if *(Pitts & Tayler 1985)*
  $p > m^2/2+1$ $(m \geqslant 1)$ and $p > 1$ $(m=0)$
  and a growth rate of $\sigma \sim \omega_A^2 / \Omega$

$p>3/2$ case where $m\geq 1$ modes are unstable in both $\Omega \ll \omega_A$ and $\Omega \gg \omega_A$ regimes

$\Omega \ll \omega_A$: $\sigma \approx \omega_A$

$\Omega \gg \omega_A$: $\sigma \approx \omega_A{}^2/\Omega$



Confirmed in simulations! (Ibanez & Braithwaite 2013)

About the $\Omega \gg \omega_A$: $\sigma \approx \omega_A^2/\Omega$ result

- Almost all stars are in the $\Omega \gg \omega_A$ regime
- Visible in the momentum equation:

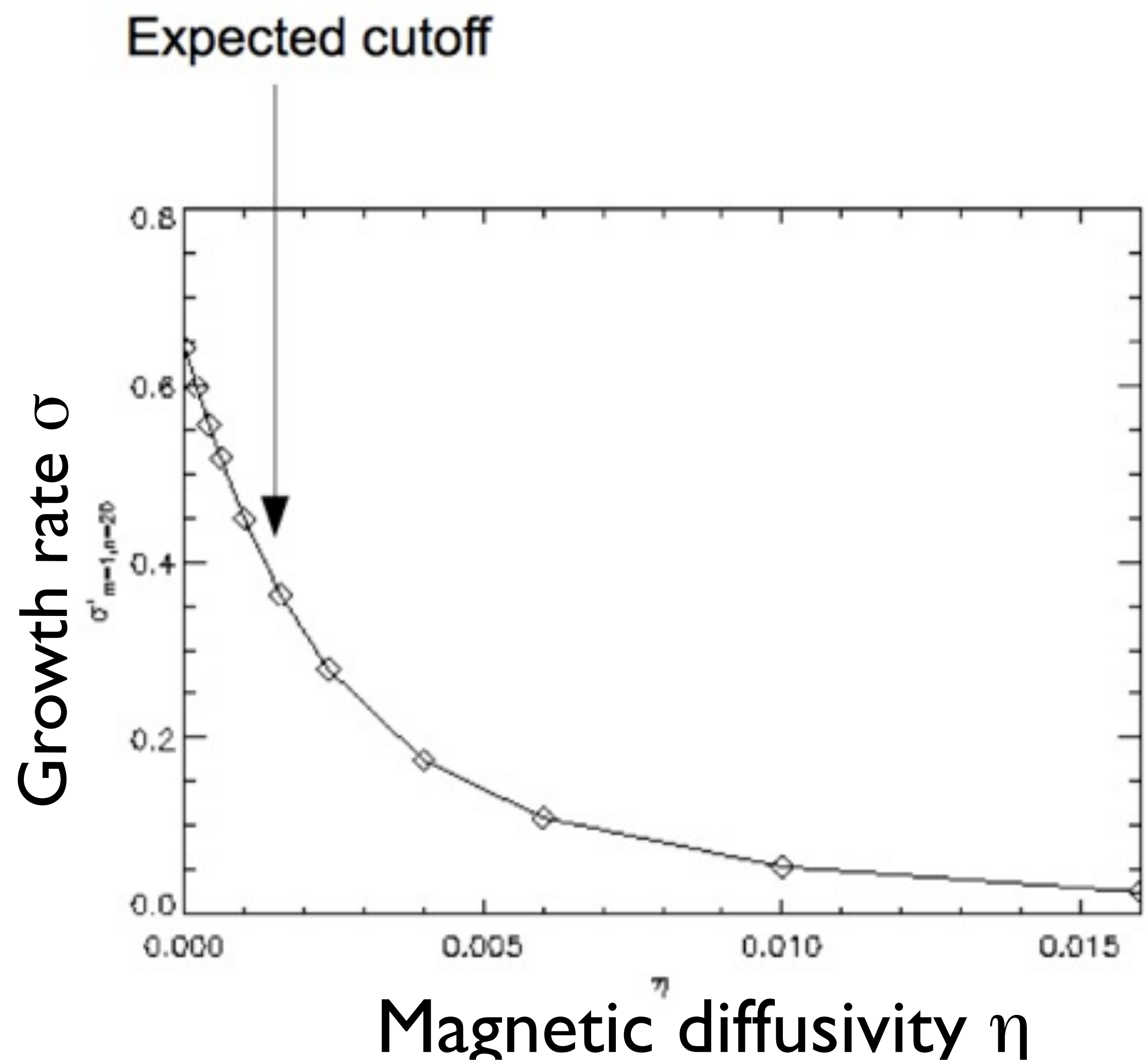
$$\frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \frac{1}{4\pi\rho}(\nabla \times \mathbf{B}) \times \mathbf{B} - 2\,\boldsymbol{\Omega} \times \mathbf{u}$$

U^2	c_s^2	v_{ff}^2	v_A^2	$L\Omega U$
σ^2	ω_s^2	ω_{ff}^2	ω_A^2	$\Omega\sigma$

Magnetic diffusion

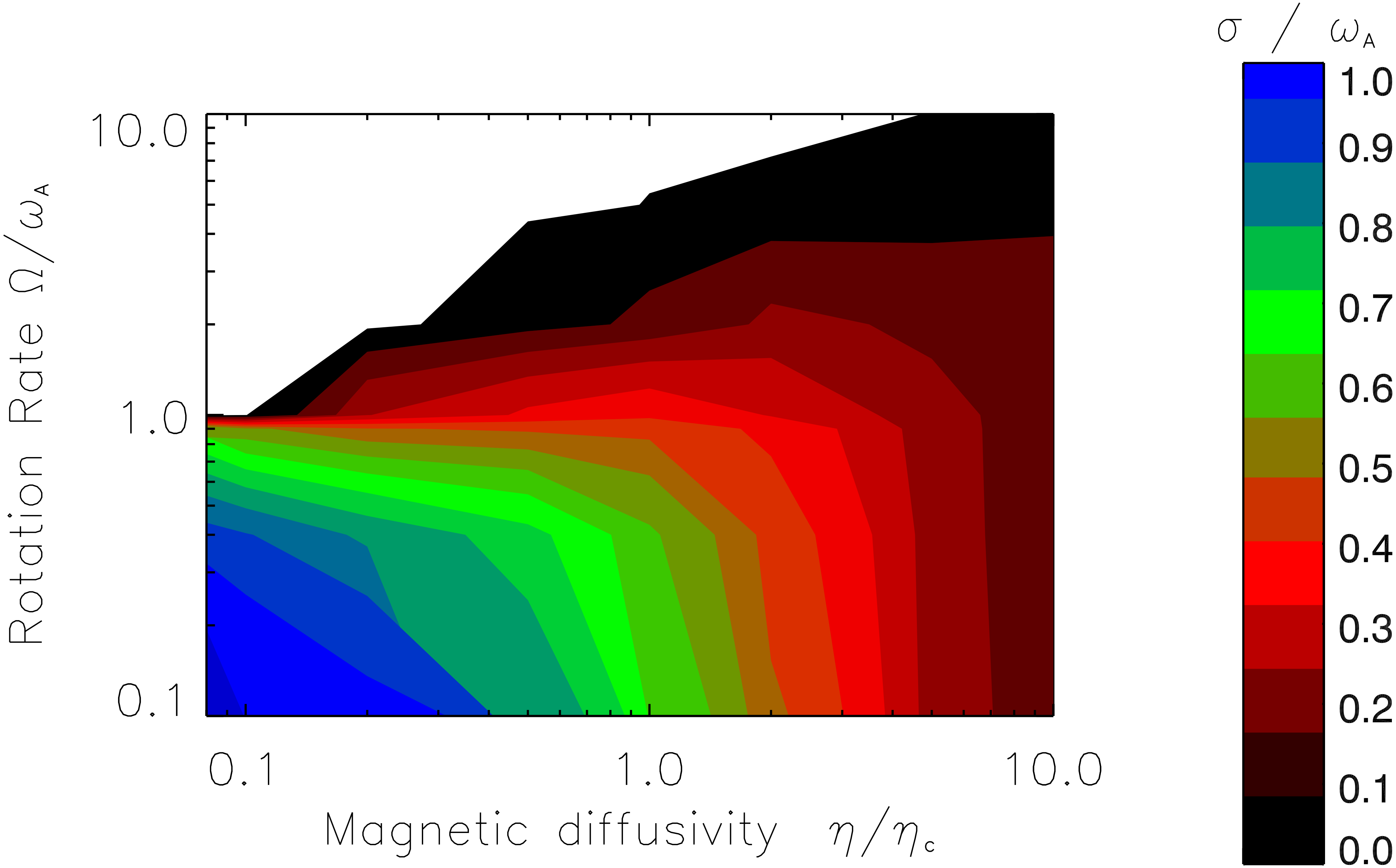
- Naive expectation:
  if diffusive frequency $\omega_{\mathrm{diff}} = \eta k_z^2 > \sigma$
  then we have stabilisation

- Reality: no suppression, just reduction in growth rate σ



p=1 case: rotation and diffusion

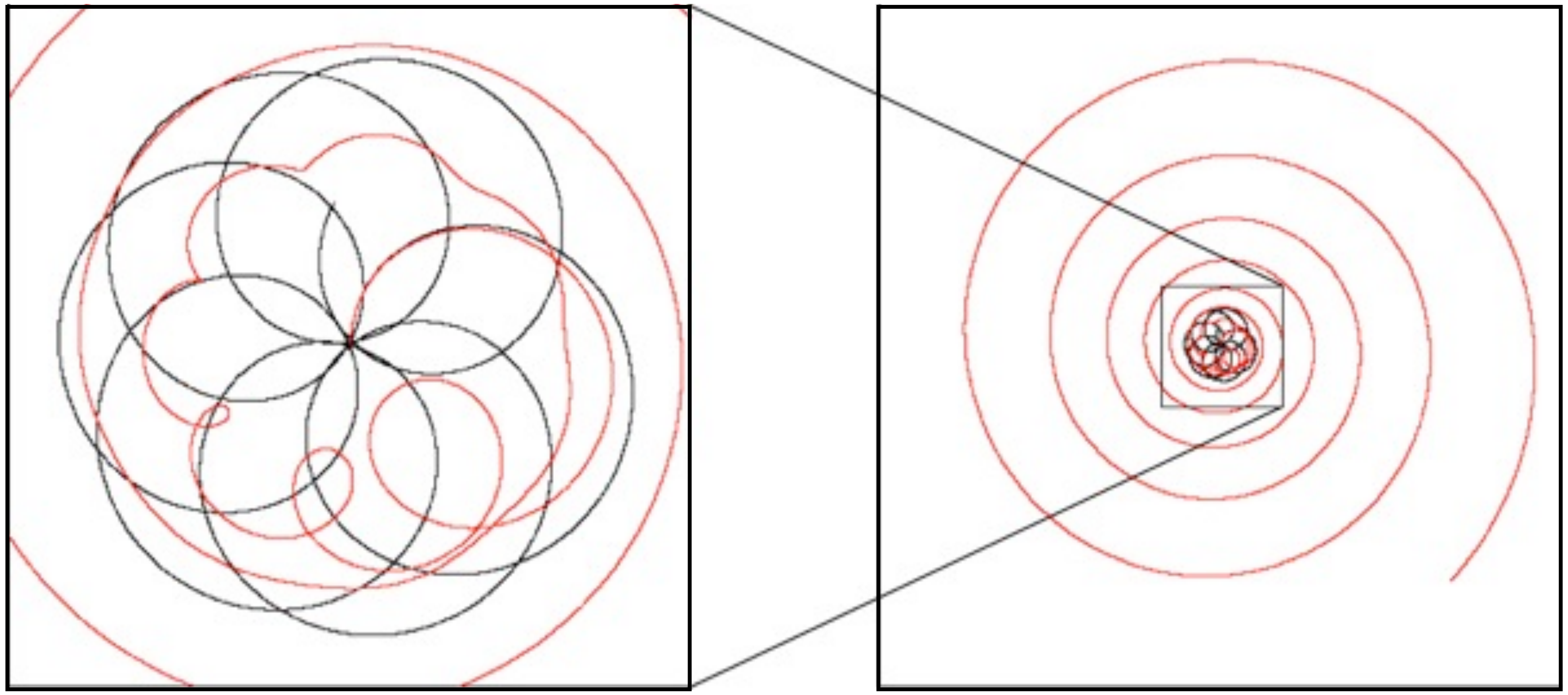
Diffusion suppresses rotational suppression!



Ibanez & Braithwaite 2013

About this result

Two N body simulations of a ball pushed from the top of a hill



Black: no diffusion, undamped epicyles

Red: diffusion, energy lost →
impossible to return to equilibrium position

With additional poloidal field

Simulations with no rotation,
no diffusivity, and with p=1
so with B_z=0,
only m=1 unstable;
m=0 and m=2 marginal

Ibanez & Braithwaite 2013

With additional poloidal field

Simulations with no rotation, no diffusivity, and with $p=1$ so with $B_z=0$, only $m=1$ unstable; $m=0$ and $m=2$ marginal

Add a poloidal field B_z

From energy argument, we expect stabilisation at:

pitch number = $B_\mathrm{t} / (B_\mathrm{p}\, k_\mathrm{z} R) < 1$

Ibanez & Braithwaite 2013

With additional poloidal field

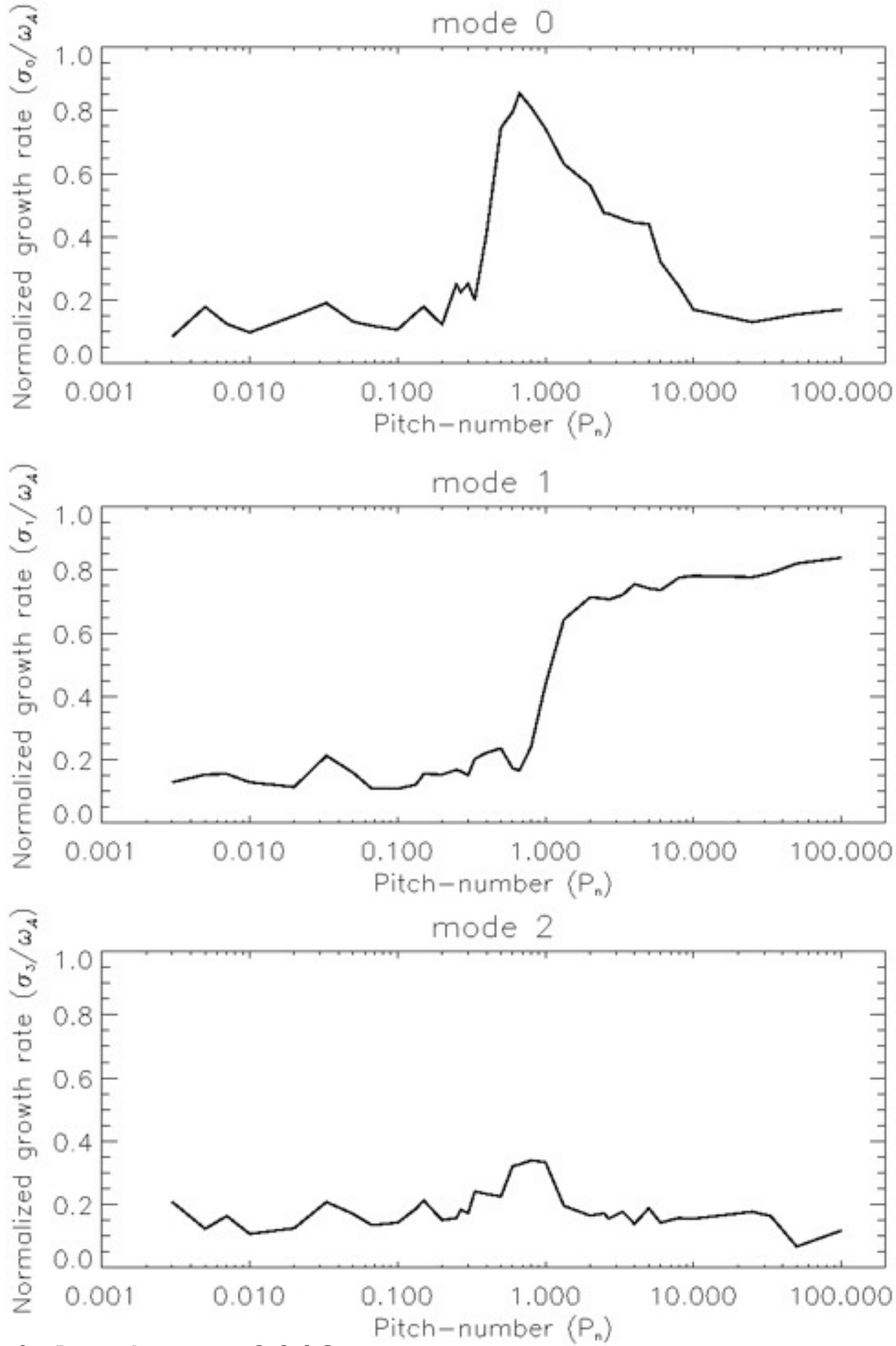
Simulations with no rotation, no diffusivity, and with $p=1$ so with $B_z=0$, only $m=1$ unstable; $m=0$ and $m=2$ marginal

Add a poloidal field B_z

From energy argument, we expect stabilisation at:

pitch number $= B_\mathrm{t} / (B_\mathrm{p} k_\mathrm{z} R) < 1$

Are there high-m modes?



Ibanez & Braithwaite 2013

New results: summary

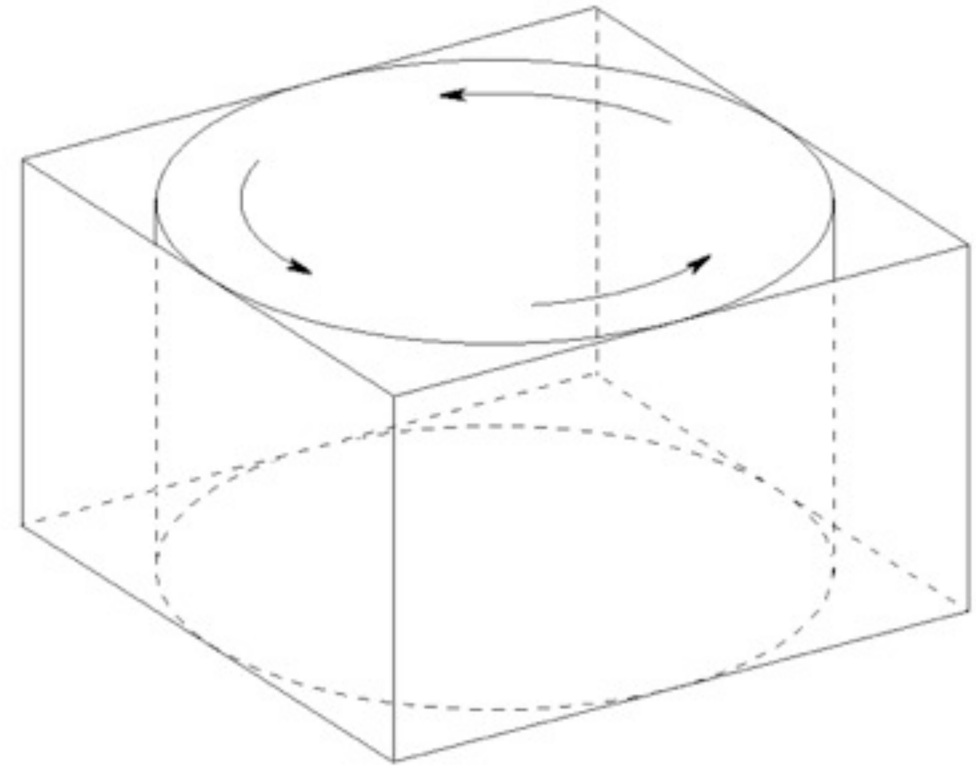
- In $p > 3/2$ case, rotation reduces growth rate to:
  $\sigma \simeq \omega_A^2 / \Omega$
- Magnetic diffusion prevents rotational suppression in $p = 1$ case
- Weak poloidal field stabilises low-m modes

The Tayler-Spruit dynamo

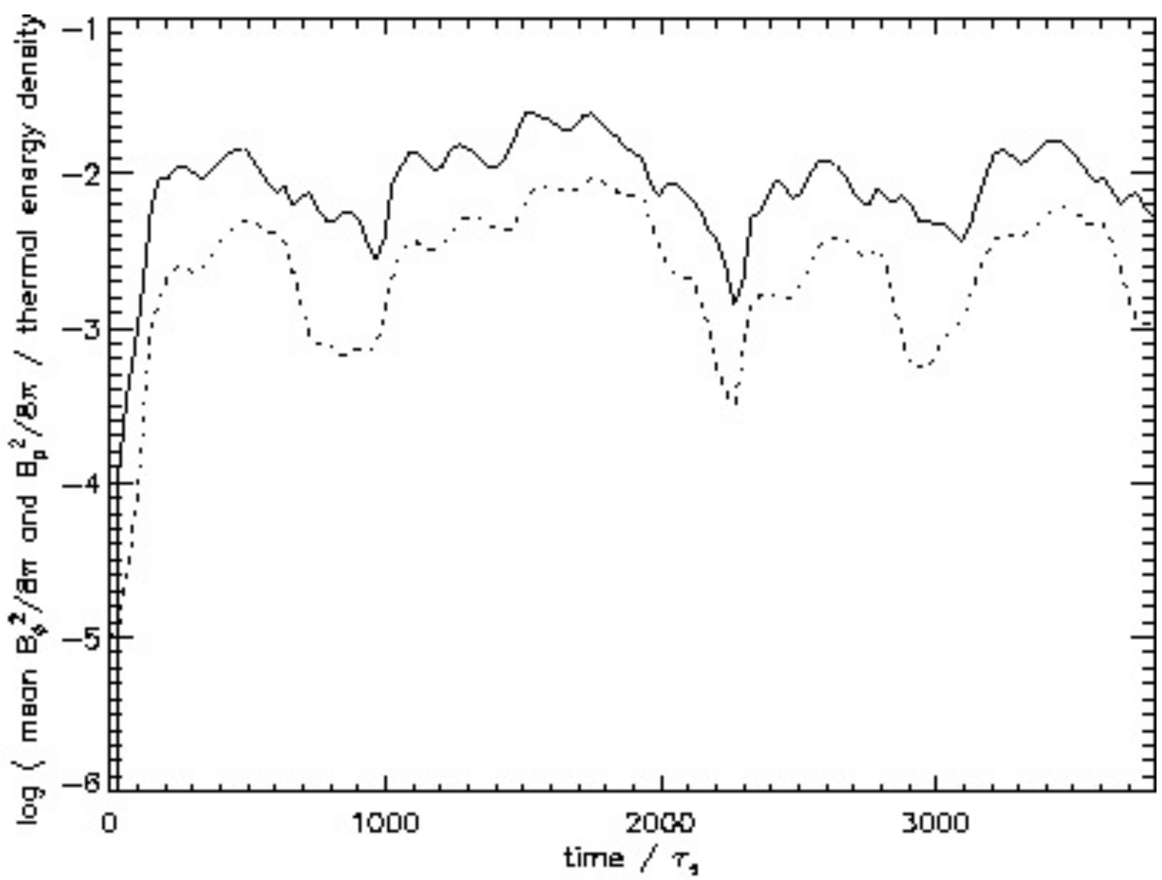
- Differential rotation winds up existing field
- Resulting toroidal field becomes unstable, creating new poloidal field
- New poloidal field gets wrapped up itself

First simulations including differential rotation

(Braithwaite 2006)



log energy

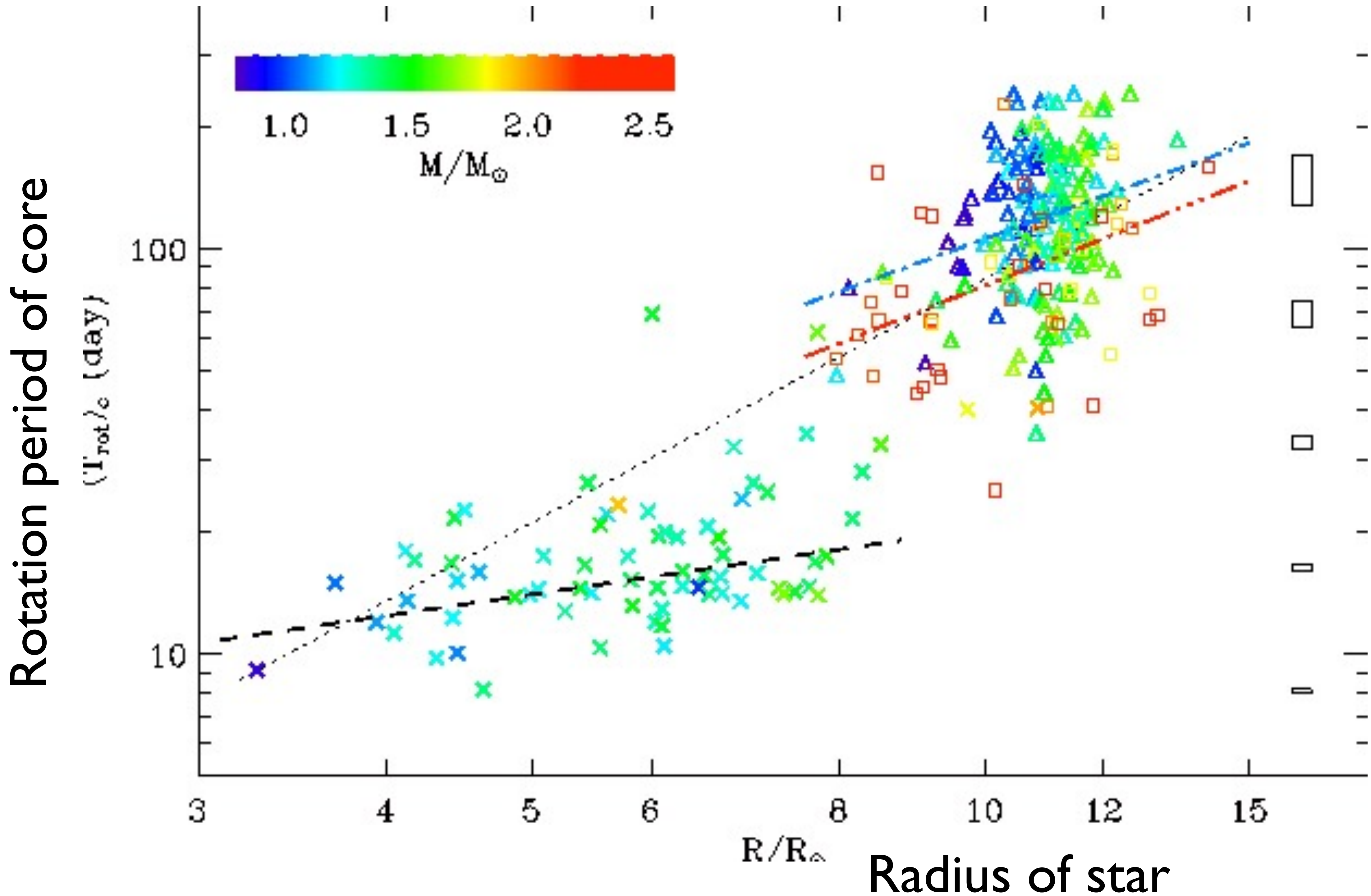


time

Simulations show self-sustaining dynamo

Necessity of angular momentum transport

- Mosser et al. (2012) show from Kepler data that red-giant cores rotate even slower than that predicted by Spruit‘s (2002) mechanism



Mean period of core rotation as a function of the asteroseismic stellar radius, in log-log scale. Crosses correspond to RGB stars, triangles to clump stars, and squares to secondary clump stars. The color code gives the mass estimated from the asteroseismic global parameters. The dotted line indicates a rotation period varying as R^2. The dashed (dot-dashed, triple-dot-dashed) line indicates the fit of RGB (clump, secondary clump) core rotation period. The rectangles in the right side indicate the typical error boxes, as a function of the rotation period. *From Mosser et al.\ 2012.*

Necessity of angular momentum transport

- Mosser et al. (2012) show from Kepler data that red-giant cores rotate even slower than that predicted by Spruit‘s (2002) mechanism
- Something missing from the theory.....
- Highlights importance of studying properties of Tayler instability

Contents

- Why is this interesting?
- The Tayler instability
  - Simulations - effects of rotation, diffusivity & weak poloidal field
  - Fossil fields
- Summary

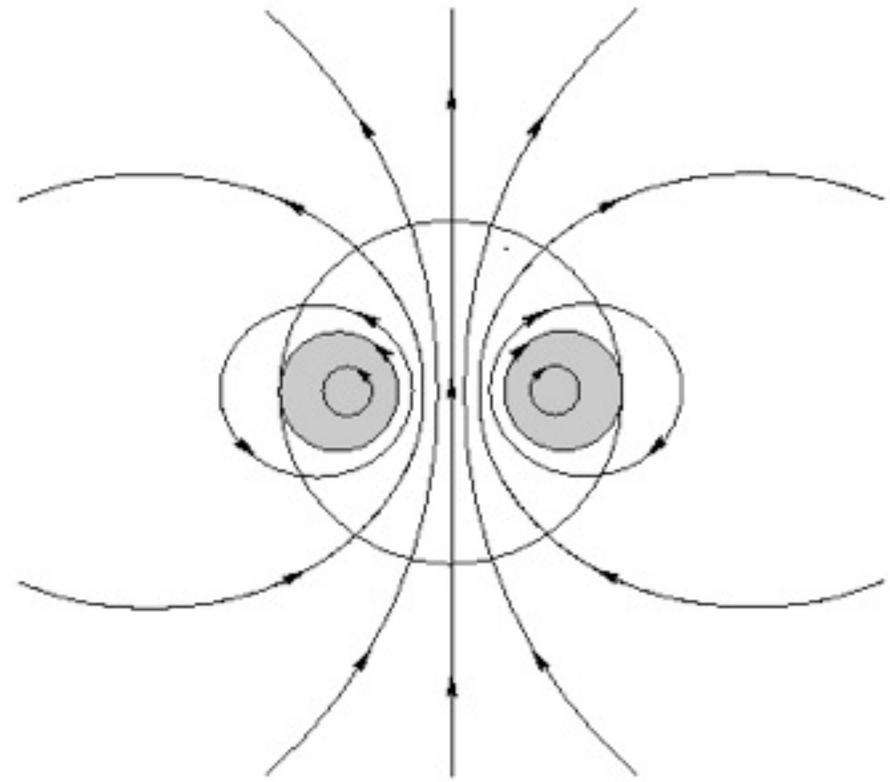
Some thoughts

- Value of parameter $p = \mathrm{dln}B/\mathrm{dln}R$ in dynamo with fluctuating toroidal field:

  $$-\infty < p < \infty$$

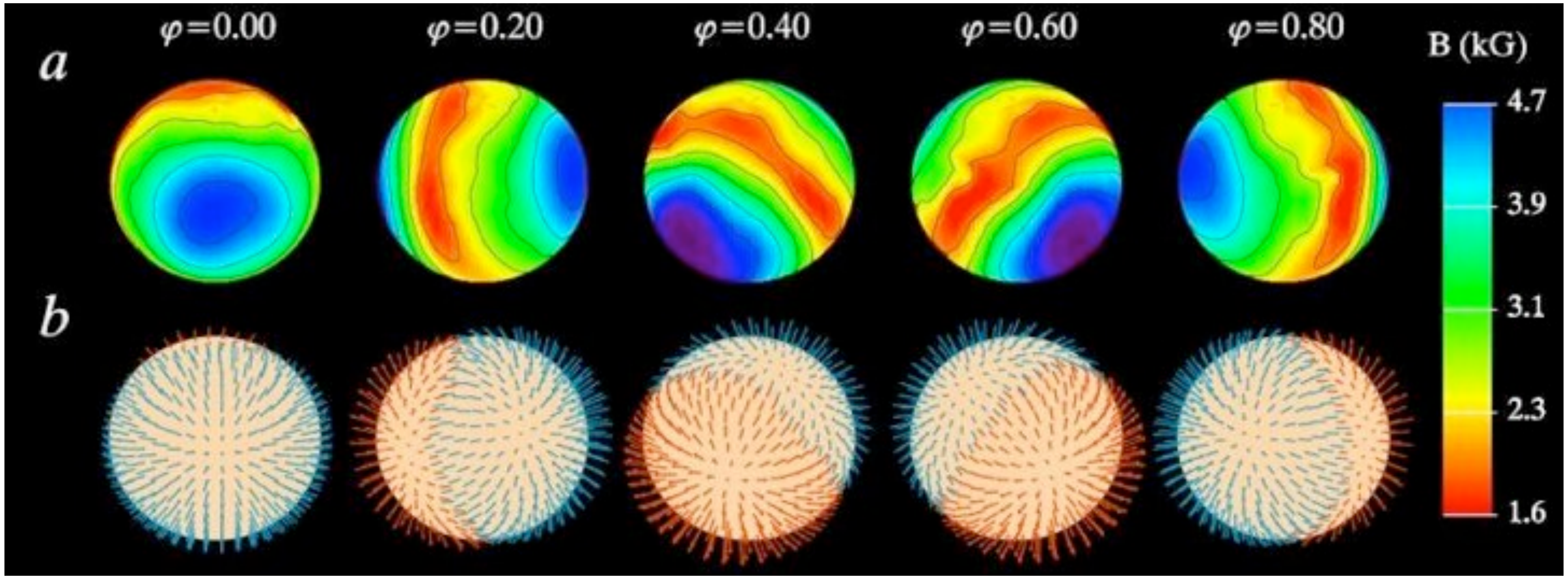
- So for Tayler-Spruit mechanism, precise value of p is of no importance
  - m=0 and m up to a large number are unstable in various regions
  - m=1 mode not really ‚dominant' in a small-scale toroidal field

Fossil fields

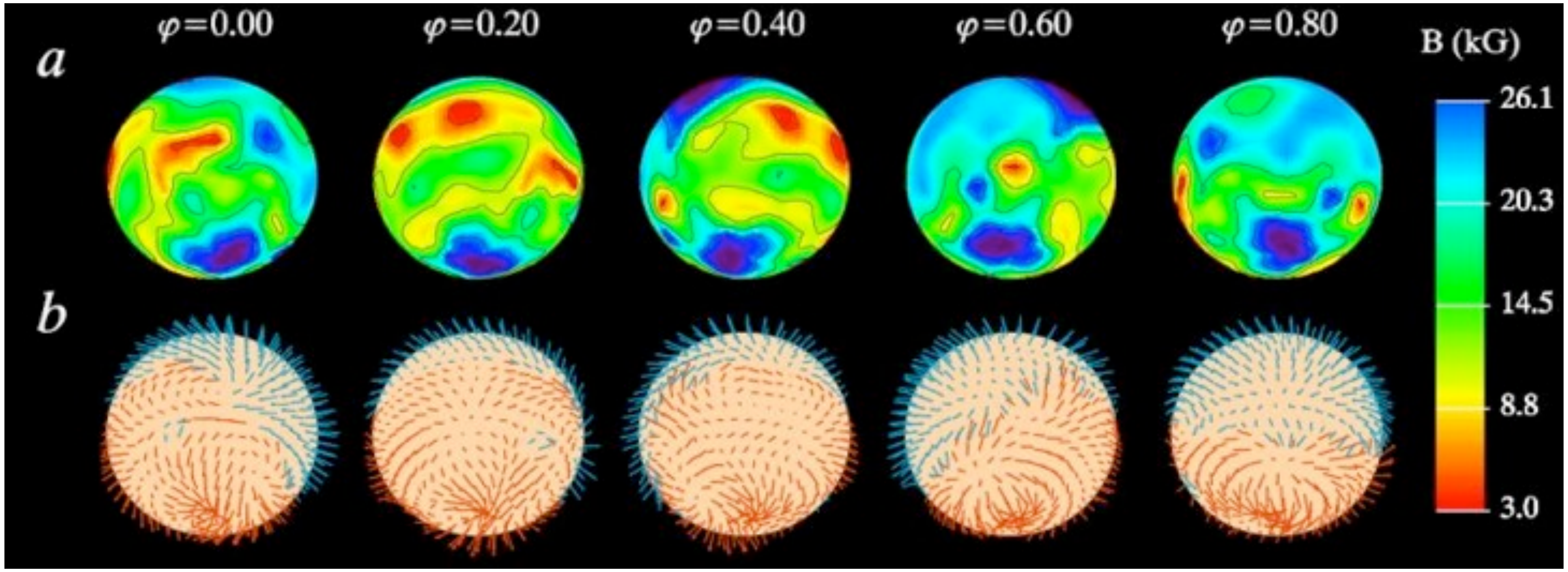


- Non-convective stars can contain fossil equilibria
- Configuration must be free from instability
- Axisymmetric equilibria: what range of poloidal-toroidal ratios possible?

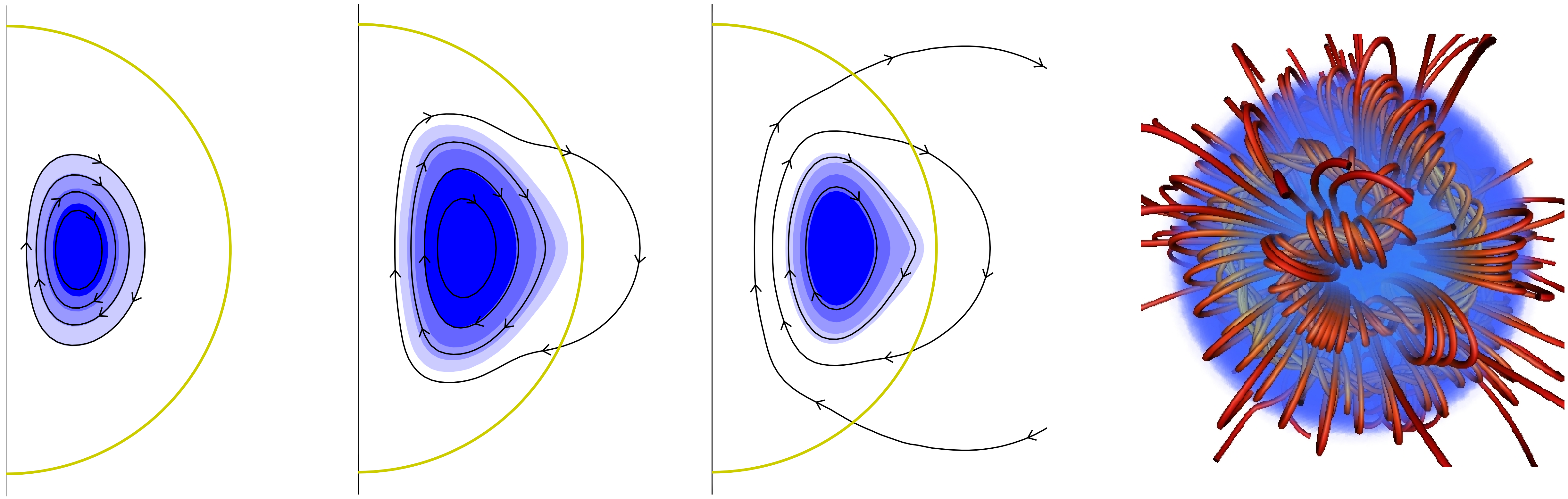
Fossil fields can be axisymmetric or not...



α2 CVn
(Kochukhov et al 2002)



53 Cam
(Kochukhov et al 2004)



- Equating energy released by instability to energy required to bend poloidal lines (confirmed here):
  $E_{\mathrm{pol}} / E_{\mathrm{tot}} > a\, E_{\mathrm{tot}} / |E_{\mathrm{grav}}|$ (a is a factor of order unity)
  (Braithwaite 2009, Akgun et al. 2013]

- Much stronger toroidal field possible! And likely? (interesting particularly for gravitational wave people: millisecond magnetars?)

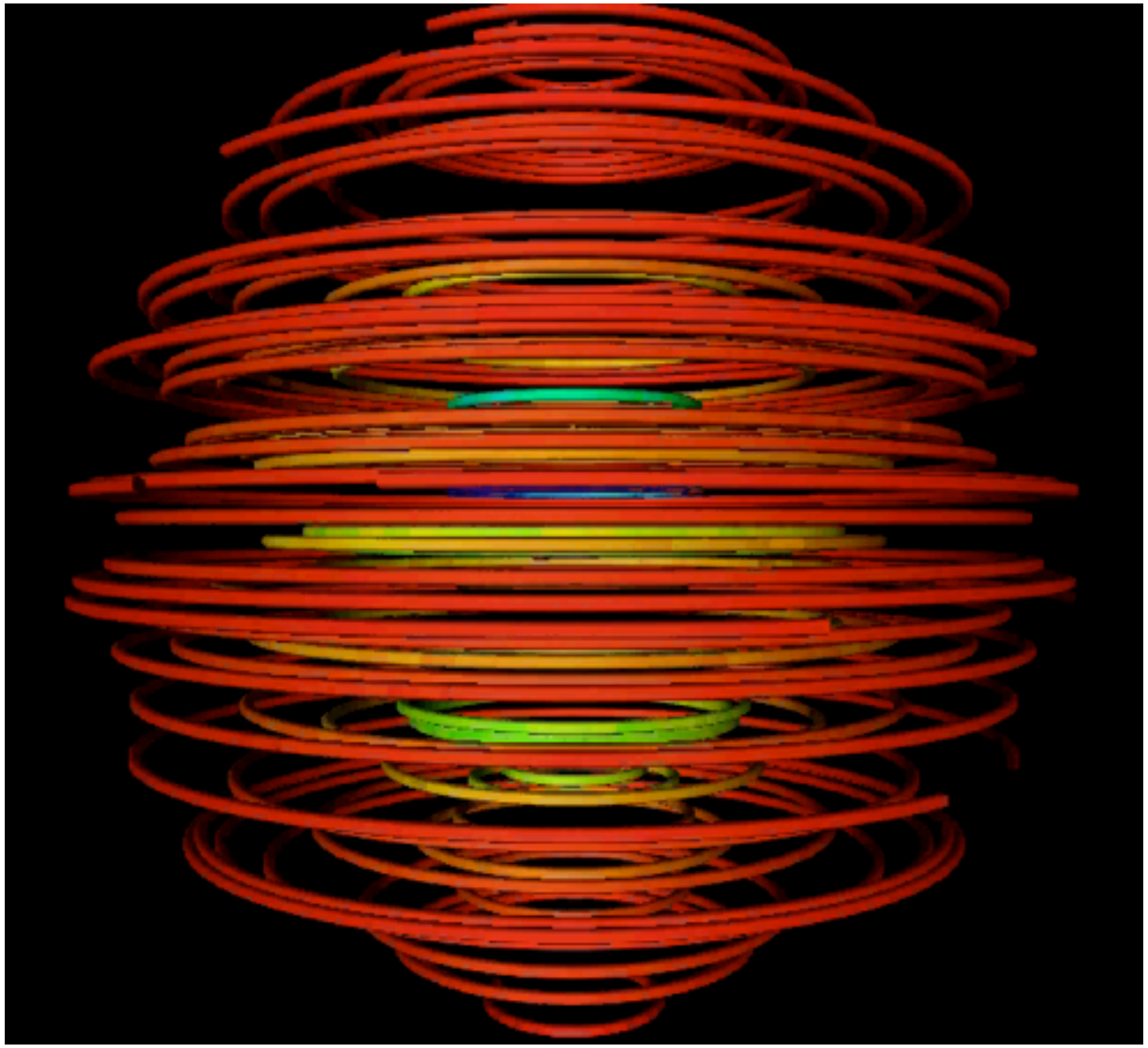
Contents

- Why is this interesting?
- The Tayler instability
  - Simulations - effects of rotation, diffusivity & weak poloidal field
  - Fossil fields
- Summary

Summary

- Various properties of Tayler instability looked at: rotation, diffusion, poloidal field
    - most expectations confirmed
- Fossil fields might contain strong toroidal field, hidden from the observer
    - Grav. waves from millisecond magnetars?

What next?



- Linear stability conditions are local in meridional plane; in non-linear regime local only in radius, so...
- Do global or thin-spherical-shell simulations
- Include differential rotation