The appearance of discontinuities in MHD

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Contents

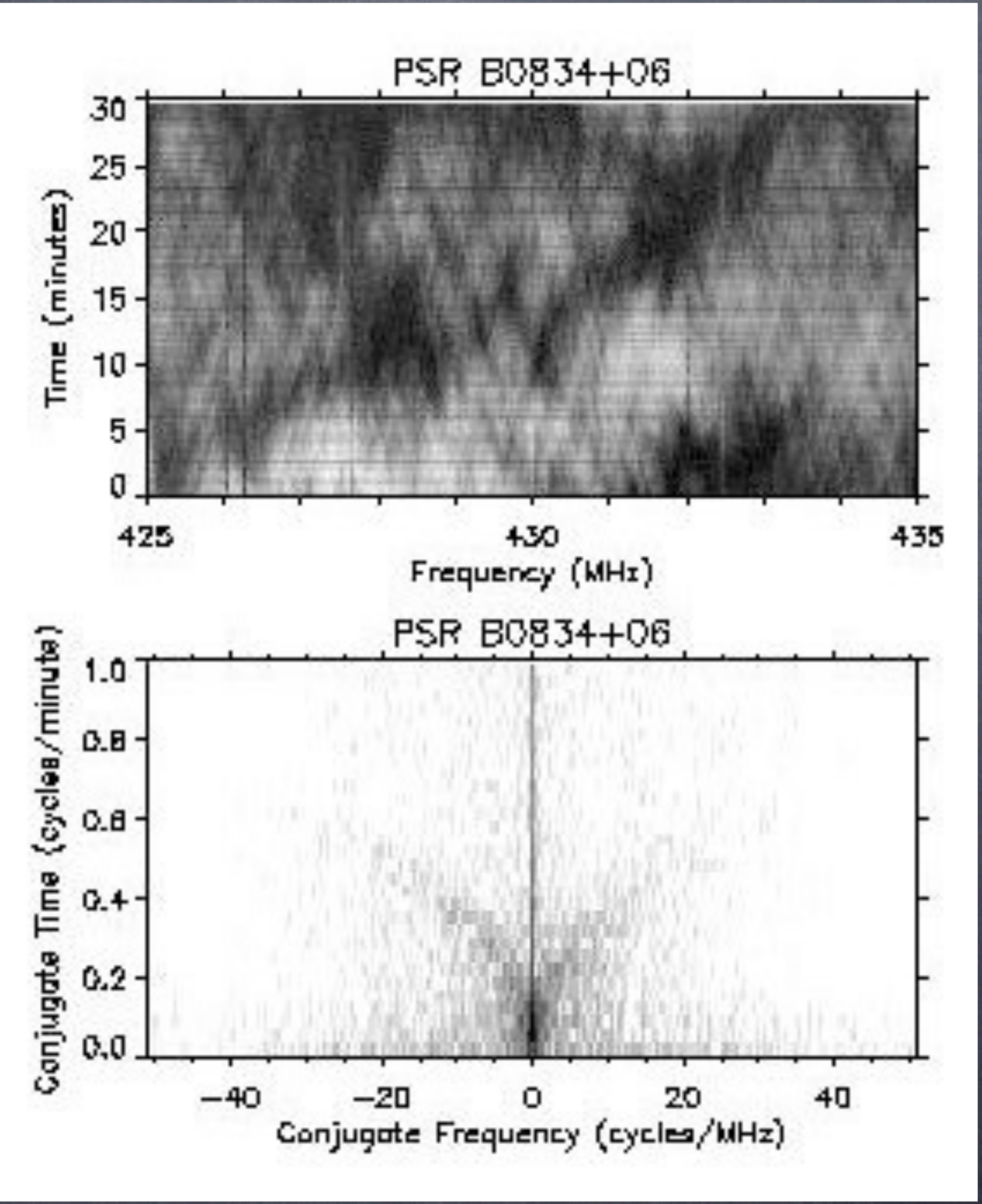
- Motivation: pulsar scintillations
- Topological considerations
- Simulations

Pulsar scintillations

- Observed with essentially all pulsars
- First reference: Scheuer 1968
- Compared to twinkling of stars:
  - also caused by varying refractive index
  - but discrete structures rather than continuous variations

Stinebring et al. 2001: „parabolic structures in the Fourier-transformed dynamical spectra of strongly scintillating pulsars"

- Dynamic spectrum (top) and its secondary spectrum
- Flux density - linear grey-scale; black indicating highest flux
- Grey-scale is logarithmic (linear in decibels) with a 48 dB range



From Stinebring et al. 2001

How to produce scintillations

- Refractive index depends on electron density
- Diffraction/refraction by clumps (clumps within sheets?) or inhomogenious sheets
- Consensus that structures are sheets nearly aligned to line-of-sight
  (Walker et al. 2004, 2008, Hill et al. 2005,
  Brisken et al. 2010, Pen & Levin 2013)

Nature of structures

- Scattering over angles 1-100 mas
- Must be almost aligned to line-of-sight: sheets more likely than filaments
- Sheets ~0.1 A.U. thick?
- Intervals of ~0.1 pc between sheets?
- Longevity uncertain
- Density contrast badly constrained, could be of order unity
- Goldreich & Sridhar (2006) and Pen & Levin (2013) suggest current sheets

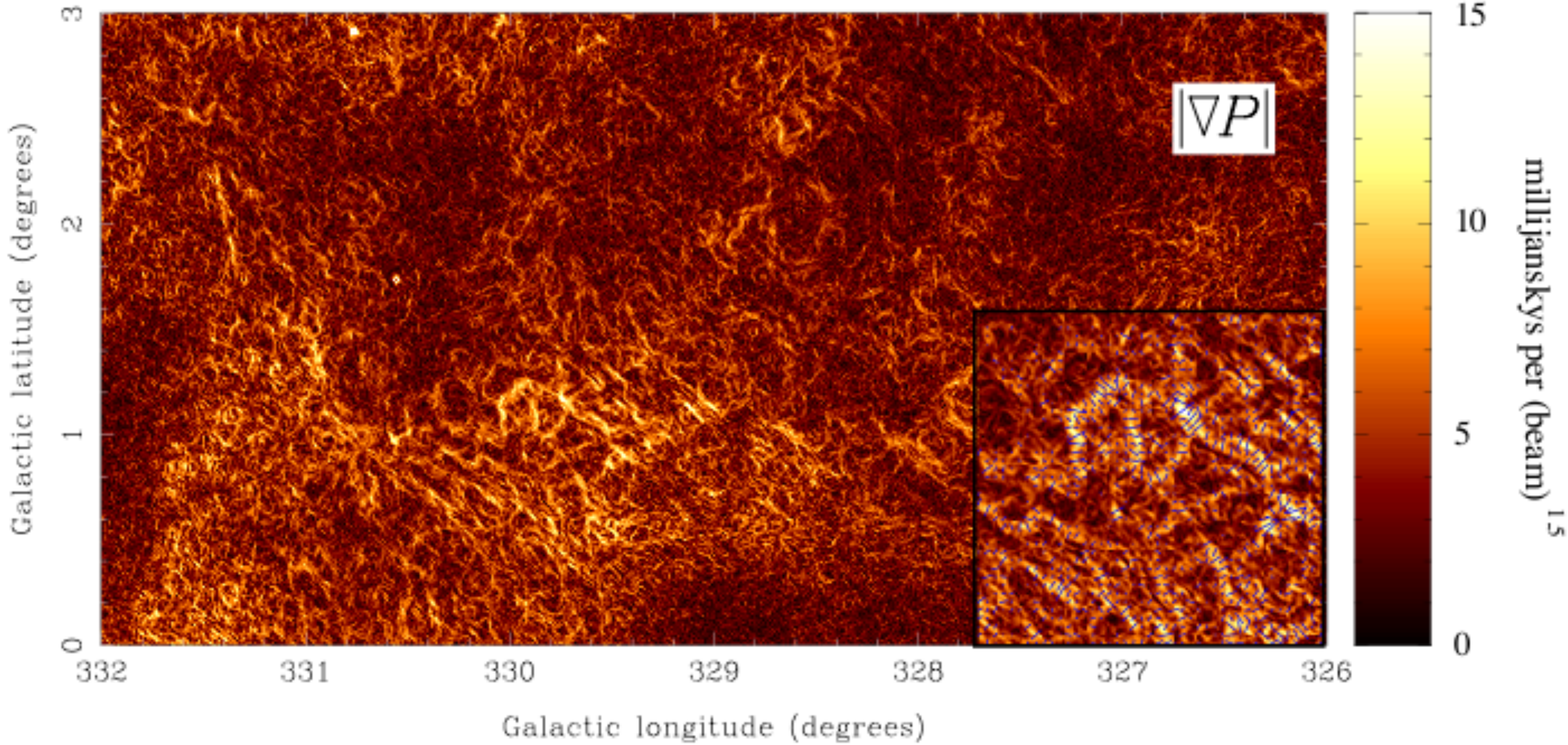


Figure 2. | The gradient image of linear polarization, $|\nabla \boldsymbol{P}|$, for an 18-deg^2 region of the Southern Galactic Plane Survey. $|\nabla \boldsymbol{P}|$ has been derived by applying Equation (1) to the Q and U images from Fig. 1; note that $|\nabla \boldsymbol{P}|$ cannot be constructed from the scalar quantity $P \equiv (Q^2 + U^2)^{1/2}$, but is derived from the vector field $\boldsymbol{P} \equiv (Q, U)$. $|\nabla \boldsymbol{P}|$ is a gradient in one dimension, for which the appropriate units are (beam)$^{-0.5}$. Because P measures linearly polarized intensity in units of millijanskys per beam, $|\nabla \boldsymbol{P}|$ has units of millijanskys per (beam)$^{1.5}$. The scale showing $|\nabla \boldsymbol{P}|$ is shown to the right of the image, and ranges from 0 to 15 mJy per (beam)$^{1.5}$. The inset shows an expanded version of the structure with highest $|\nabla \boldsymbol{P}|$, covering a box of side 0°.9 centred on Galactic longitude 329°.8 and Galactic latitude +1°.0. Plotted in the inset is the direction of $\nabla \boldsymbol{P}$ at each position, defined as $\arg(\nabla \boldsymbol{P}) \equiv \tan^{-1}\left[\mathrm{sign}\left(\frac{\partial Q}{\partial x}\frac{\partial Q}{\partial y} + \frac{\partial U}{\partial x}\frac{\partial U}{\partial y}\right)\sqrt{\left(\frac{\partial Q}{\partial y}\right)^2 + \left(\frac{\partial U}{\partial y}\right)^2} \Big/ \sqrt{\left(\frac{\partial Q}{\partial x}\right)^2 + \left(\frac{\partial U}{\partial x}\right)^2}\right]$. For clarity, vectors are only shown at points where the amplitude of the gradient is greater than 5 mJy per (beam)$^{1.5}$.

Gaensler et al. 2011, 21cm continuum

Conditions in the ISM

- Turbulence driven by galactic differential rotation, stellar winds & UV, supernovae, etc.
- Locally, in absence of driving, turbulence decays: local dynamic timescale << external driving timescale

Equilibria and the momentum equation

- Momentum equation:

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla P + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

inertia — pressure gradient — Lorentz force

- In fluid of infinite conductivity, field lines are 'frozen' into fluid
-

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

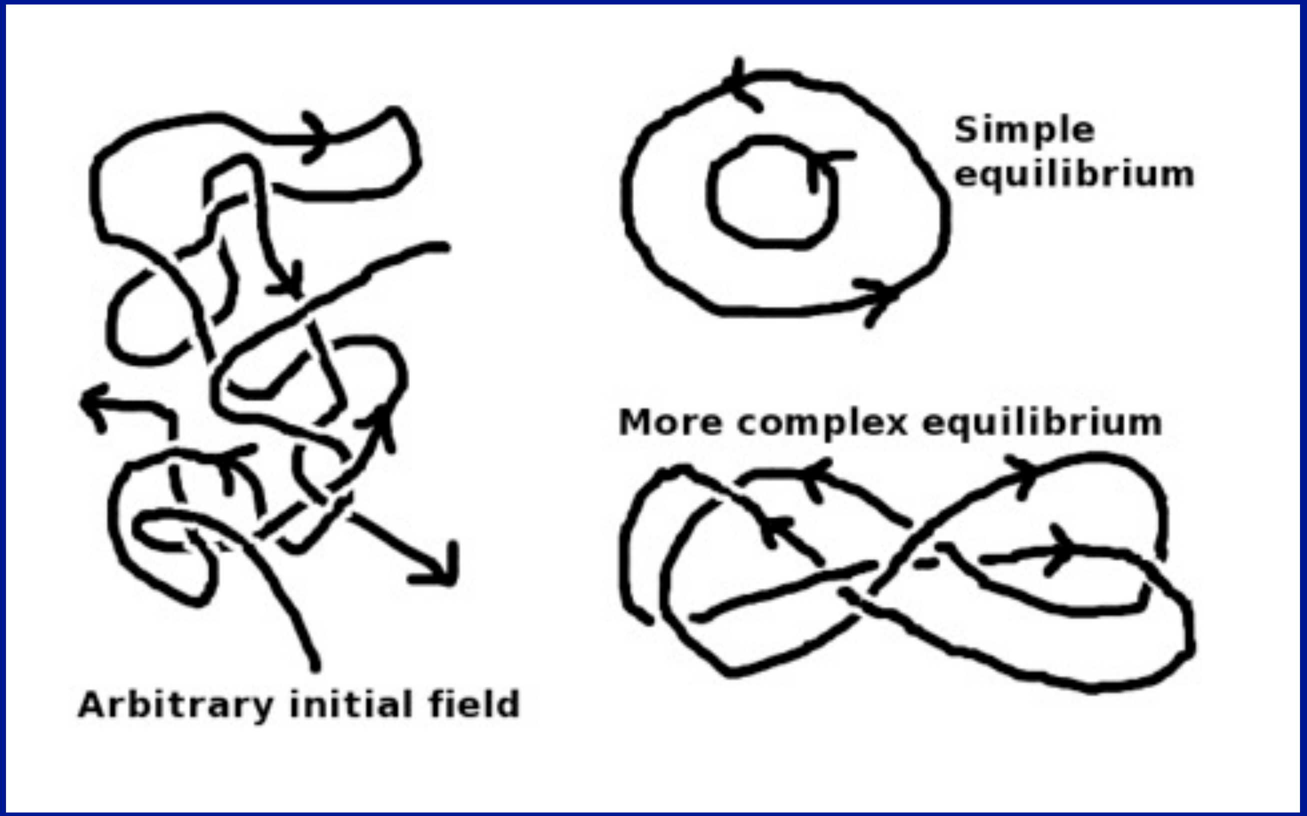
Equilibria and the momentum equation

- Momentum equation:

$$\rho \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t} = -\nabla P + \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B}$$

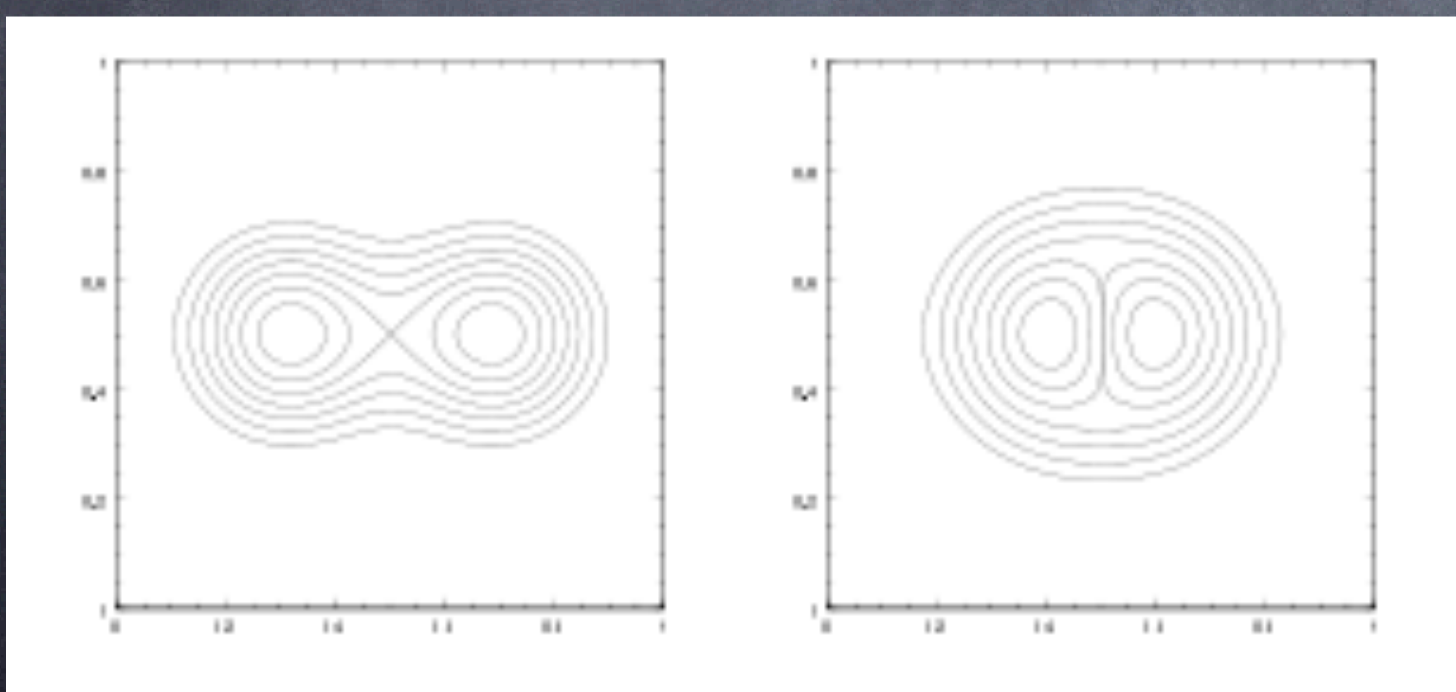
inertia | pressure gradient | Lorentz force

- In fluid of infinite conductivity, field lines are 'frozen' into fluid
- Arbitrary field has *volume-filling* field lines
- Equilibria have a special topology; field lines lie in 'magnetic surfaces' of constant pressure. Equilibrium field lines are *area-filling*
- To reach equilibrium, flux-freezing must break down)
- How? Diffusion time is too long
- Current sheets expected (e.g. Arnold 1986 & Gruzinov 2009)



Formation of current sheets

Demonstrated numerically in 2D
(Gruzinov 2009)



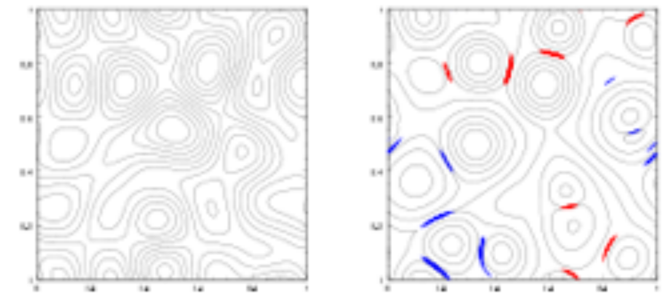


Fig. 2.— *Left*: Initial isolines of ψ. *Right*: Final isolines of ψ, also shown (thick colored lines) are the isolines of $\Delta\psi$.

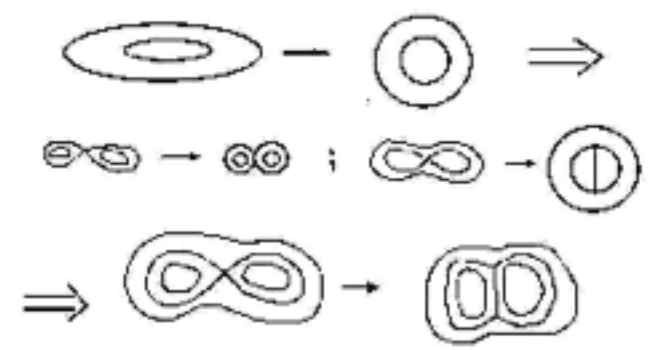


Fig. 3.— Graphical "proof" that singular current layers form from the X-points

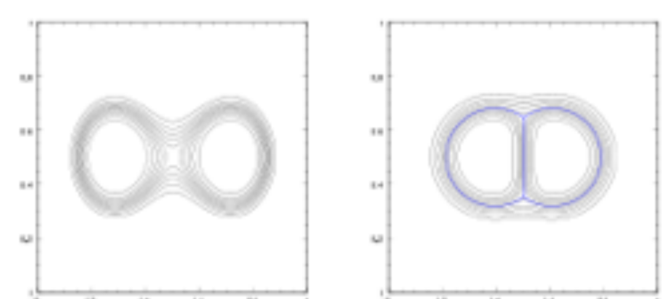


Fig. 4.— *Left*: Initial isolines of ψ. *Right*: Final isolines of ψ. Also shown (thick colored line) is the theoretical separatrix – two 251° circular arcs and a line segment.

Simulations of MHD relaxation

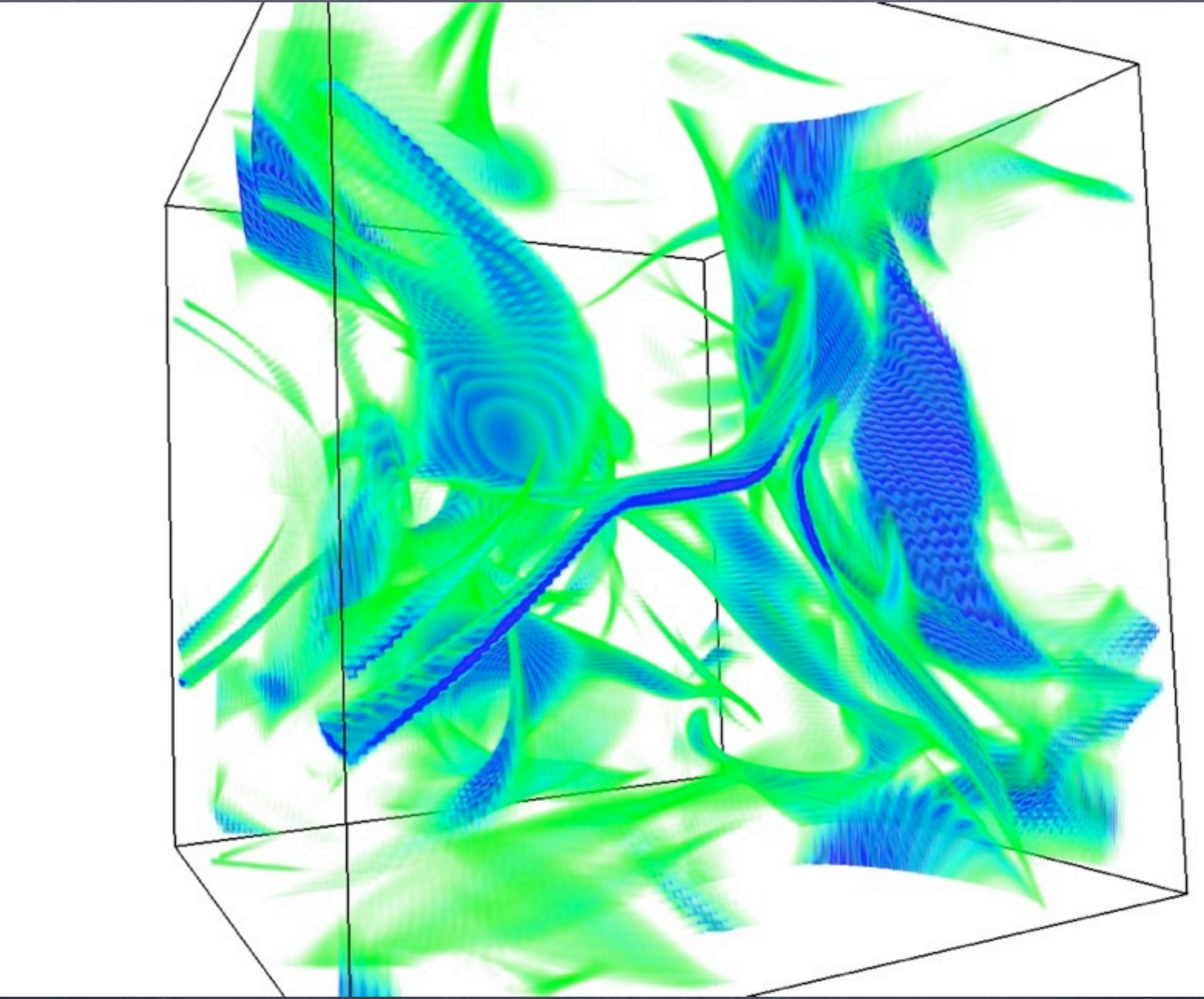
- Aim: demonstrate formation of current sheets during MHD relaxation in 3-D
- Code: Åke Nordlund's STAGGER-CODE
- Setup: cube, compressible, lowish diffusivity, Pr_m=10
- Initial conditions: constant density and pressure, smoothly varying random magnetic field with plasma-β=0.5

What happens in the simulations?

- Motion on Alfvén timescale
- Current sheets form!
- Magnetic energy drops; helicity conserved
- Energy minimum is approached:
  $E \rightarrow k_{min} H$ where $k_{min} = 2\pi / L_{box}$
- Same behaviour at high and low plasma-β

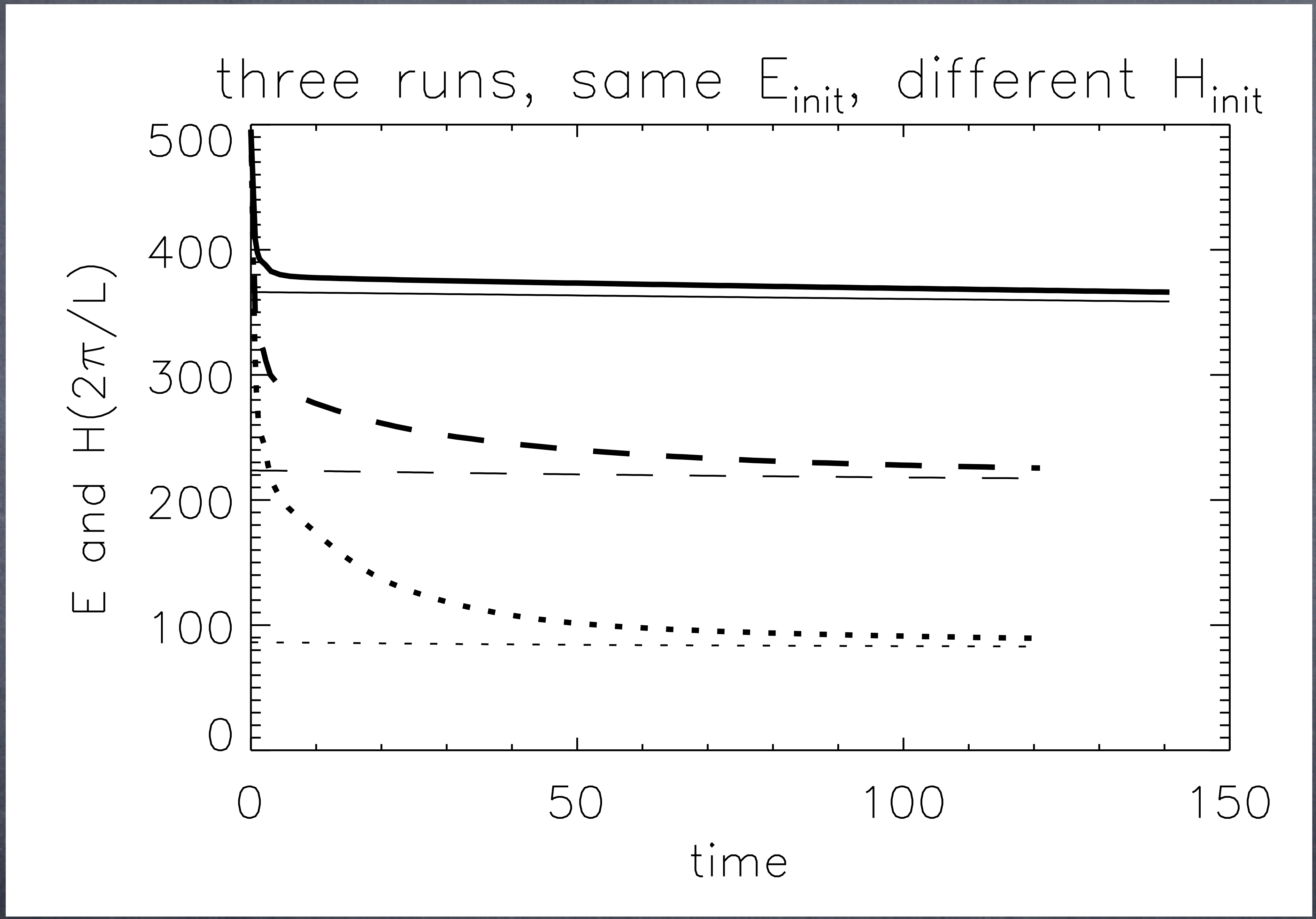
Simulation

figure shows volume-rendering of current density



Discontinuities (current sheets) appear spontaneously, allowing reconnection, i.e. topological reorganisation

Energy & helicity



Thick lines are energy
thin lines are helicity

Time unit ~ Alfven time

Summary

- Current sheets allow topological rearrangement on dynamic timescale
- Should form spontaneously in the ISM, where turbulence is intermittant
- Could be responsible for pulsar scintillations!
- Should also form in stars(!)
- Similar to coronal heating `problem´ where current sheets form in response to motions at photosphere (e.g. Parker, various papers; Galsgaard & Nordlund 1996)

Summary

Formation of discontinuities via driving at the boundary

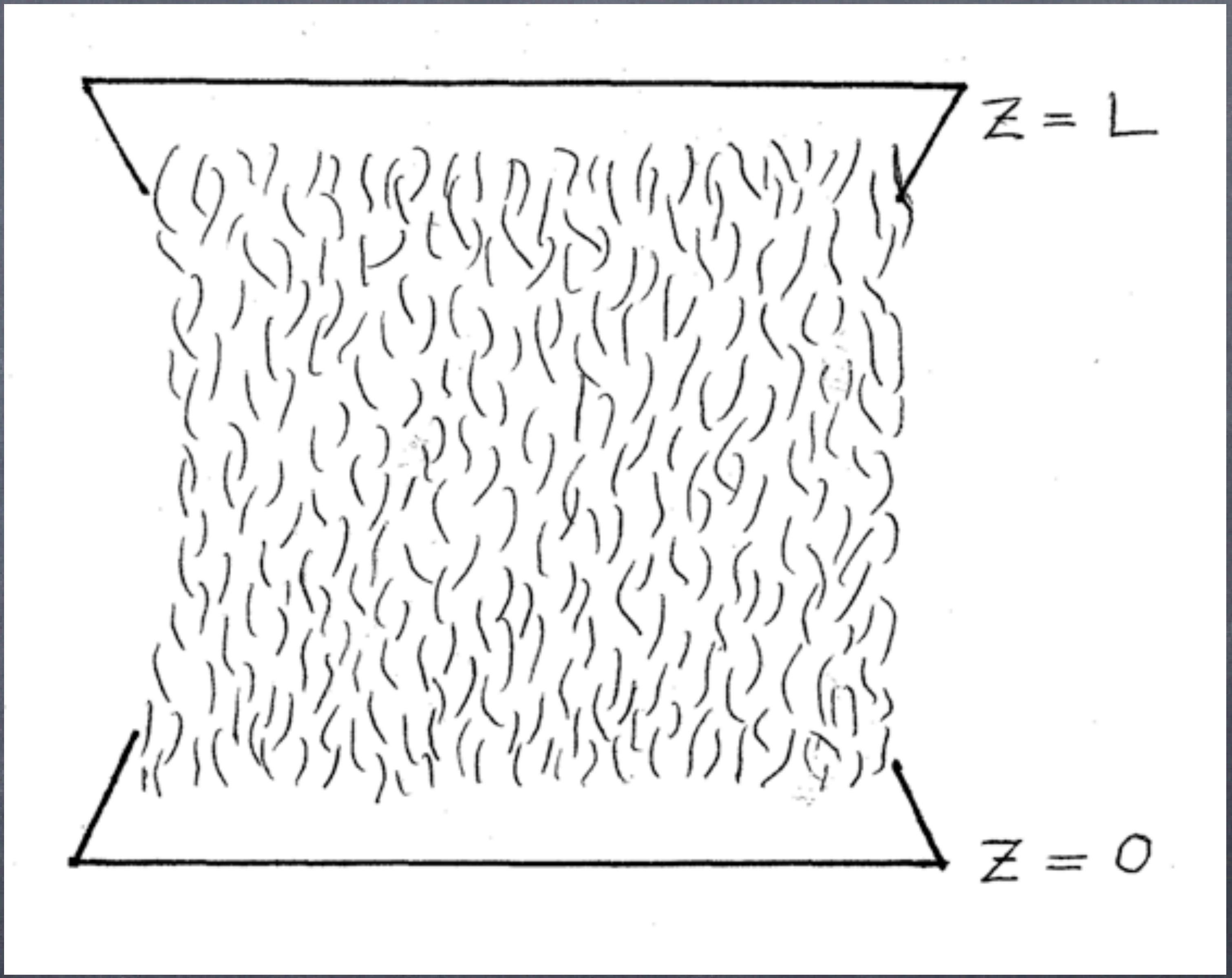


Fig. courtesy of E. Parker