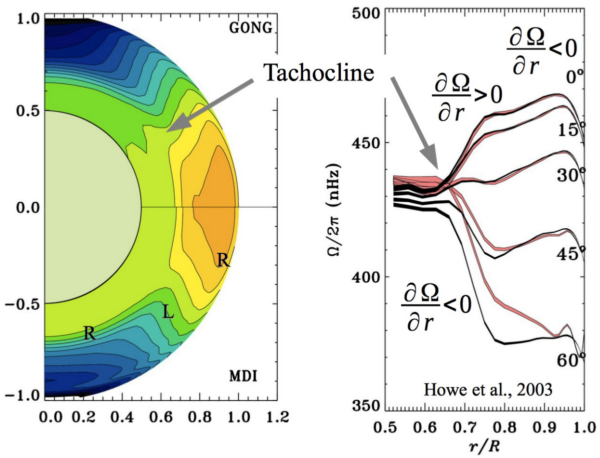
Solar differential rotation hints to obtain a near-surface shear layer



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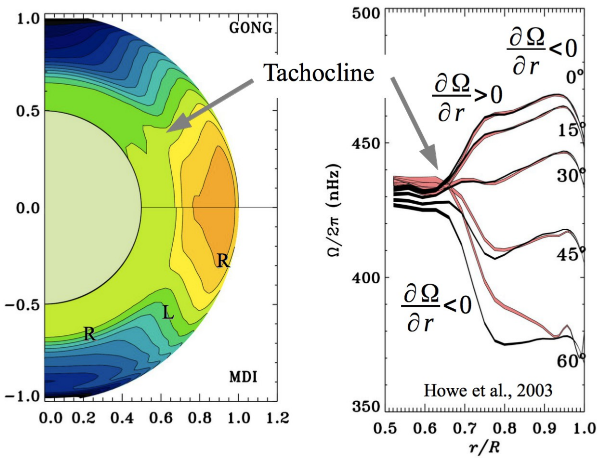
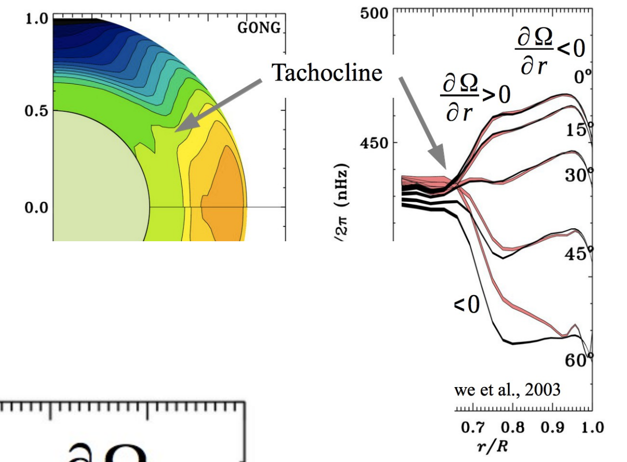
NORDITA, April. 2013

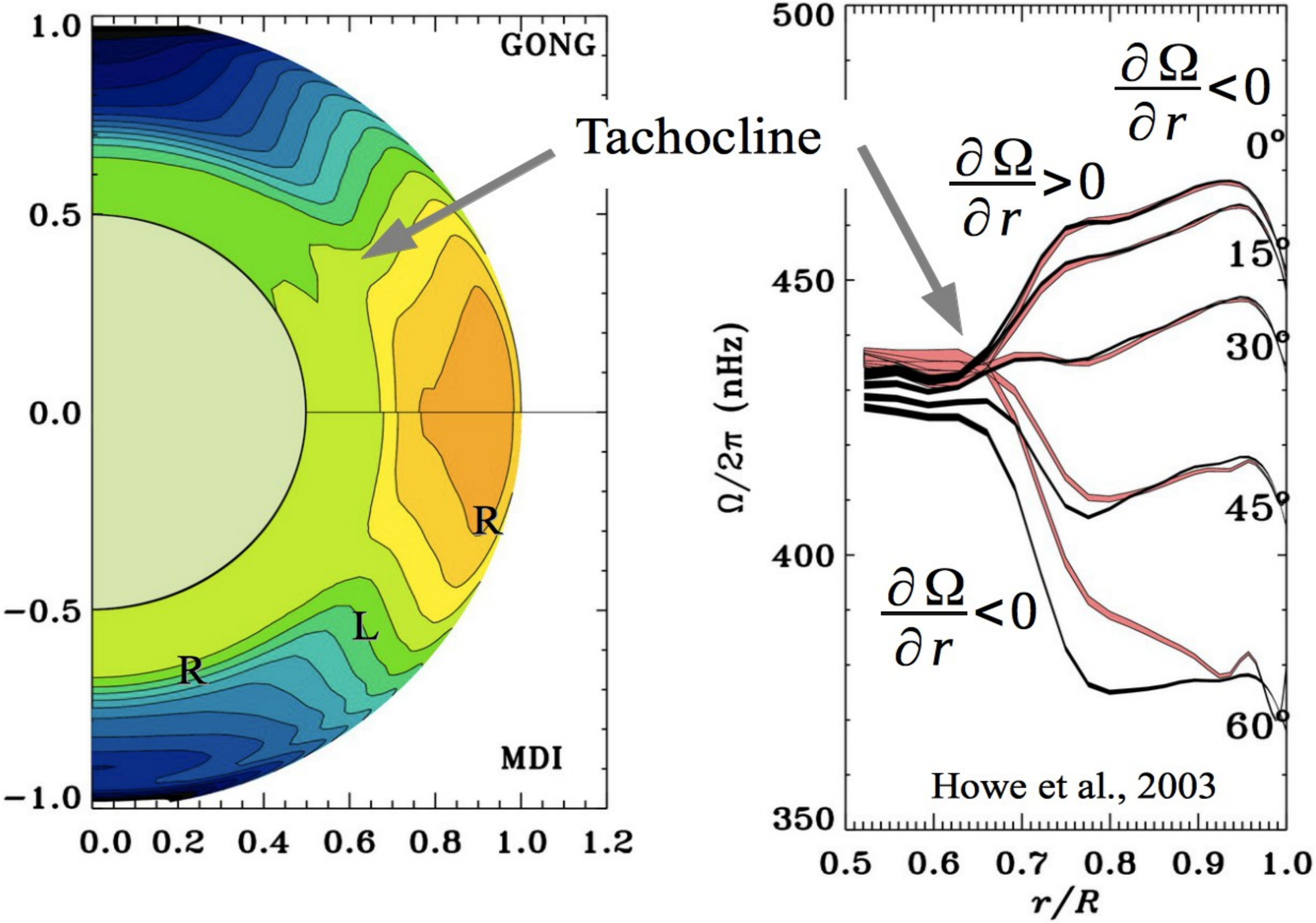
Outline

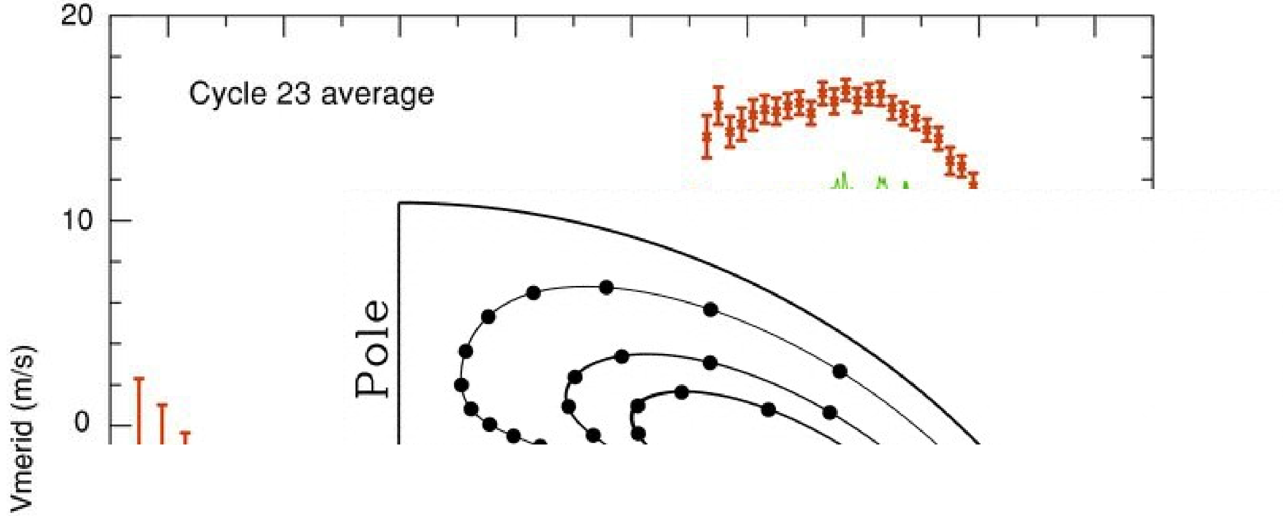


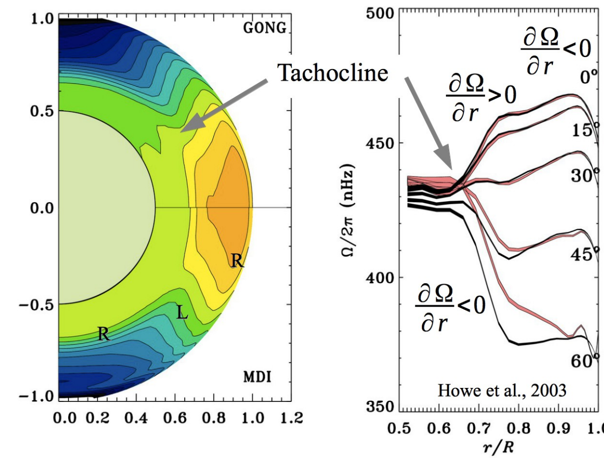
1. Large-scale flows in the solar interior
2. Mean-field models, DNS and LES
3. Anelastic simulations with EULAG
   a) Angular momentum transport
   b) Convergence
   c) Near-surface shear layer
4. Conclusions

1. Large-scale motions in the solar interior

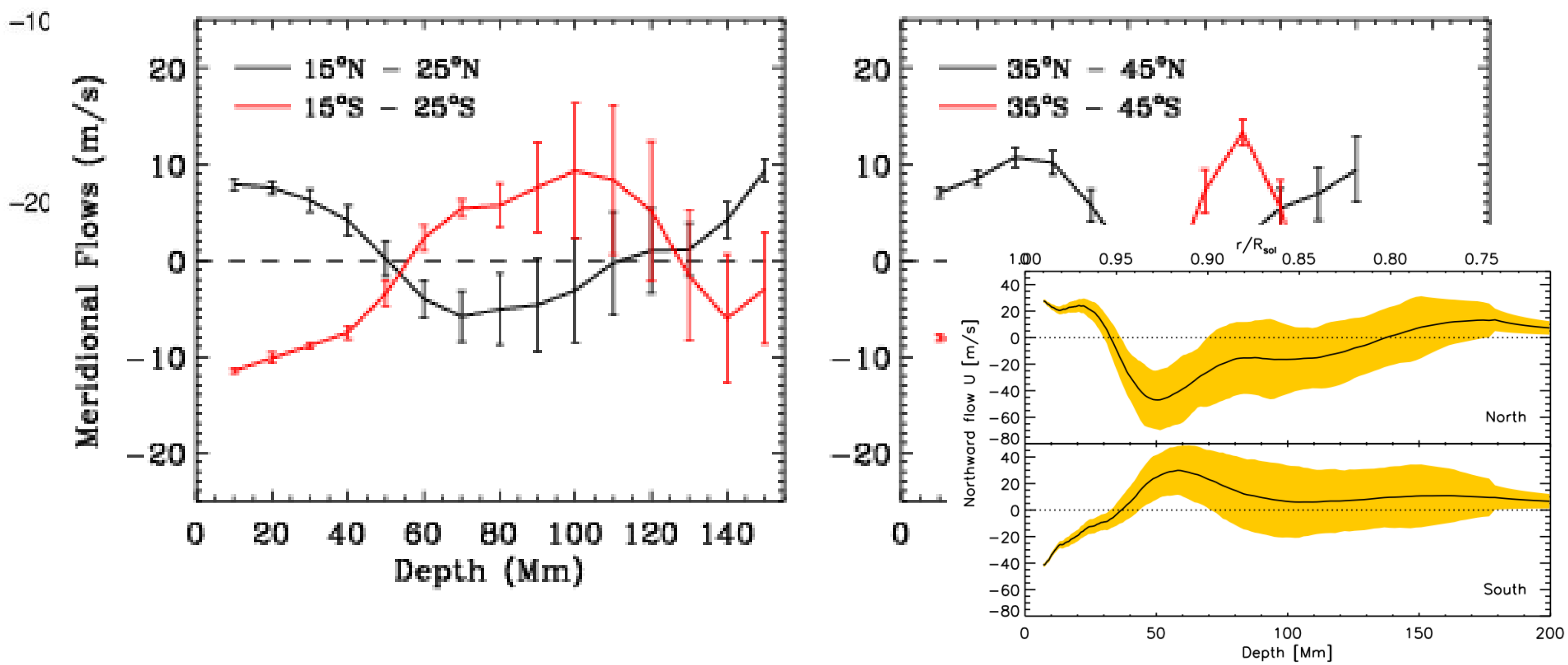






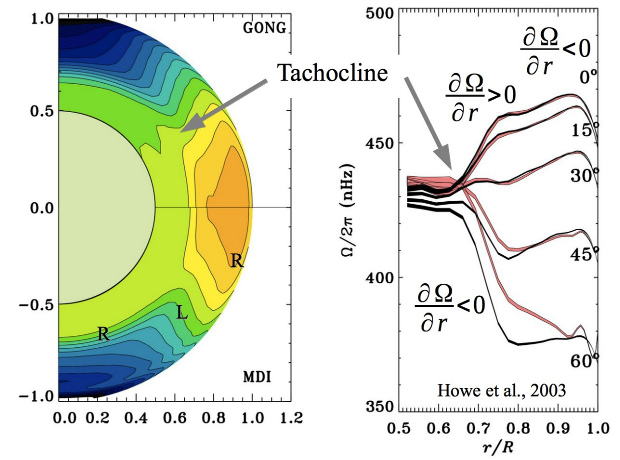


Courtesy J. Zhao



Mitra-Kraev & Thompson (2007)

2. mean-field models
DNS and LES



Large-scale motions as well as large-scale magnetic field are the result of collective effects of turbulence

$$\frac{\partial \overline{\mathbf{B}}}{\partial t} = \nabla \times (\overline{\mathbf{U}} \times \overline{\mathbf{B}} + \mathbf{E} + \eta \nabla \times \overline{\mathbf{B}})$$

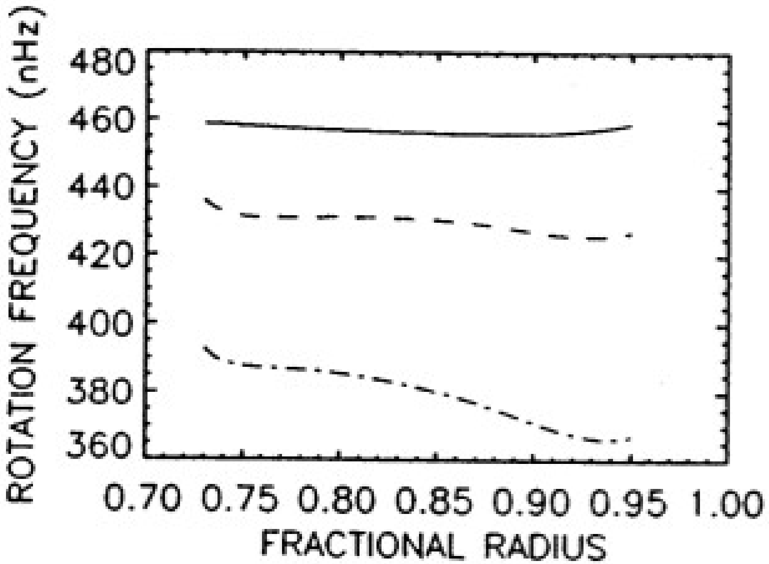
$$\mathcal{E} = \langle \mathbf{u}' \times \mathbf{b}' \rangle$$

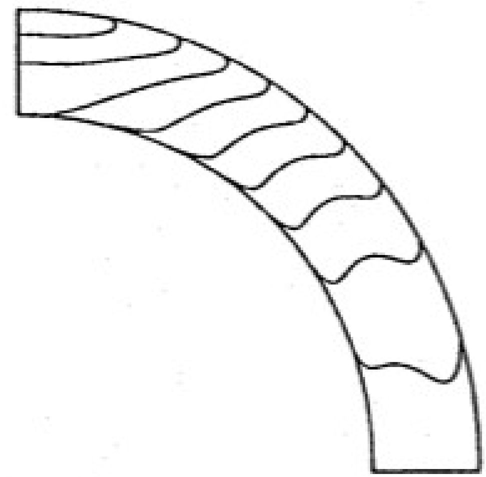
(Steenbeck et al. 66)

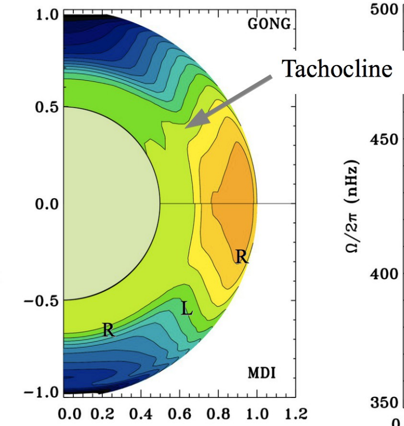
$$\frac{D\overline{\mathbf{U}}}{Dt} = -\frac{\nabla \overline{p}}{\rho} - \nabla \cdot (\rho Q) + \rho \mathbf{g} + \nabla \cdot \pi$$

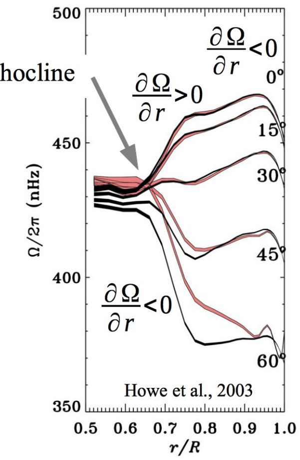
$$Q_{ij} = \langle u'_i u'_j \rangle$$

(Kitchatinov & Rudiger 93)

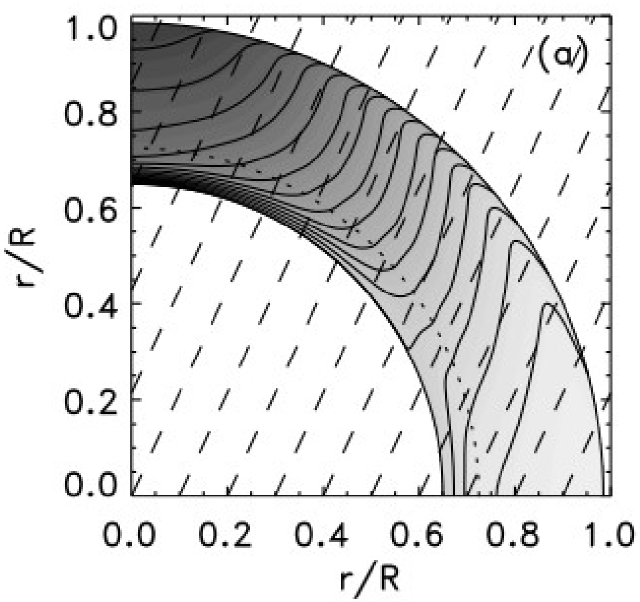


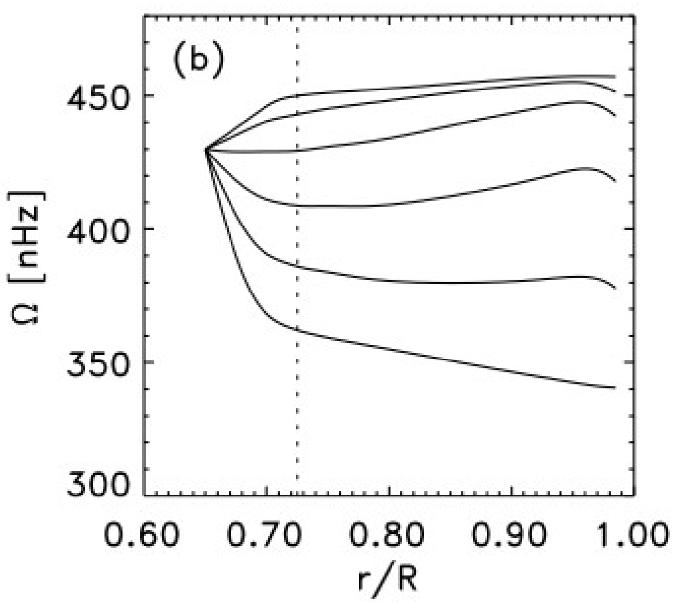


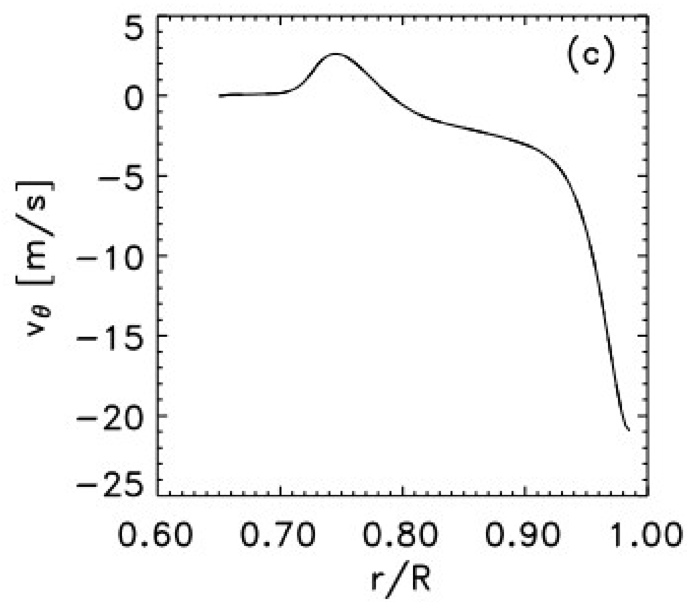




(Kitchatinov & Rudiger 95)

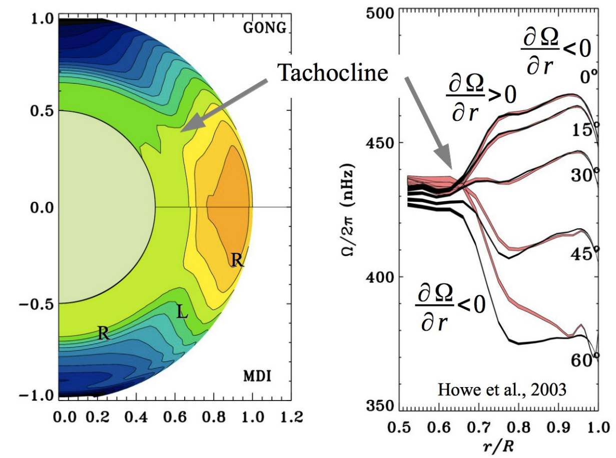






(Rempel 2005)

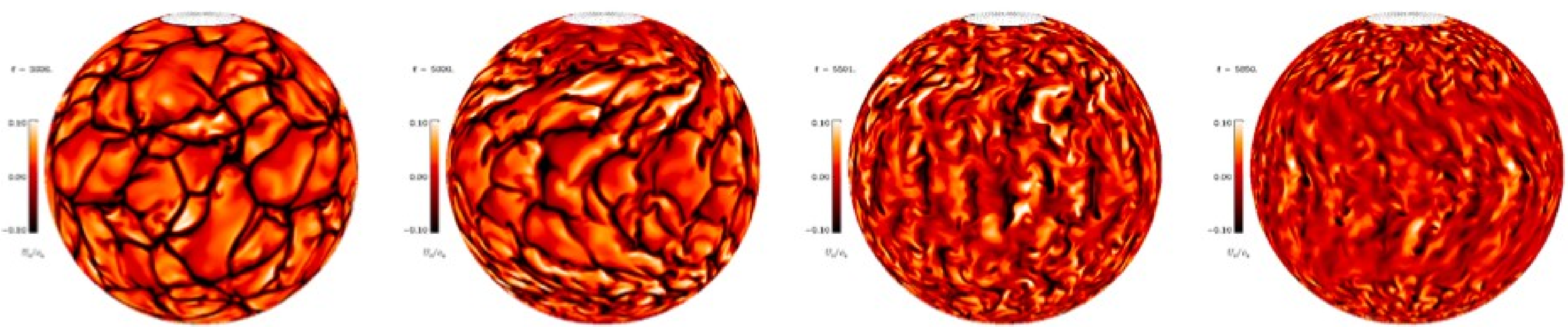
Direct Numerical Simulations (DNS)



Resolve scales down to the grid resolution. They could be global or local.

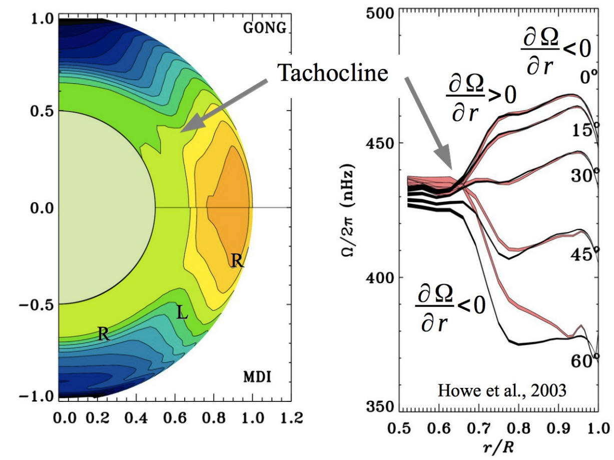
$$N^3 \geq \mathrm{Re}^{9/4}$$

$$\mathrm{Re} \simeq 10^{12} \ldots 10^{13}$$

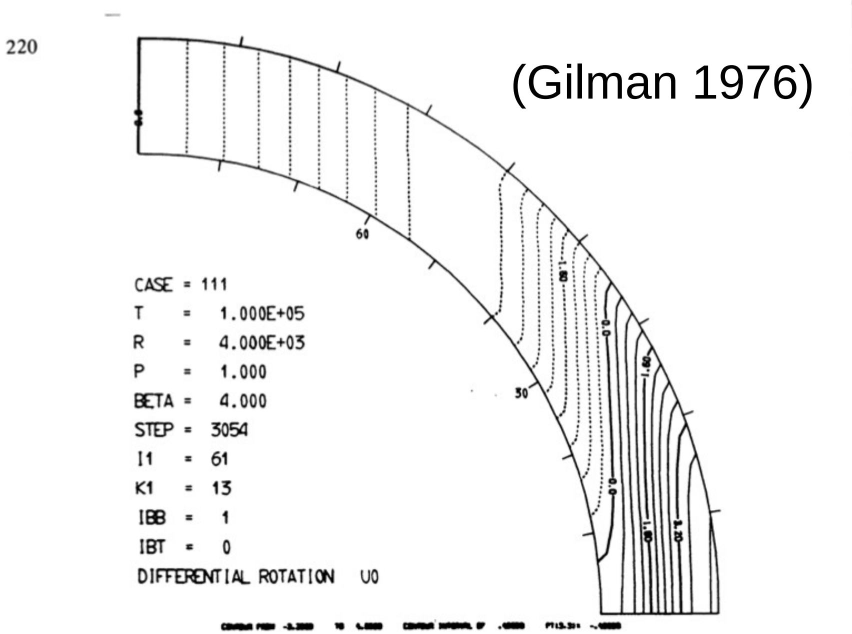


(Käpylä et al. 2011)

Large eddy Simulations (LES)



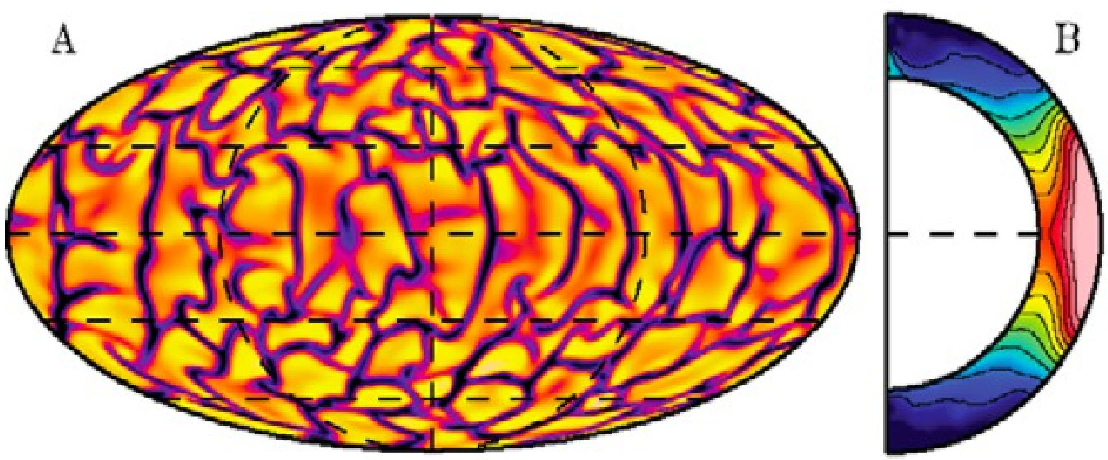
Resolve scales down to the grid resolution, the sub-grid scale (SGS) contribution is considered via a turbulent model.



(Gilman 1976)

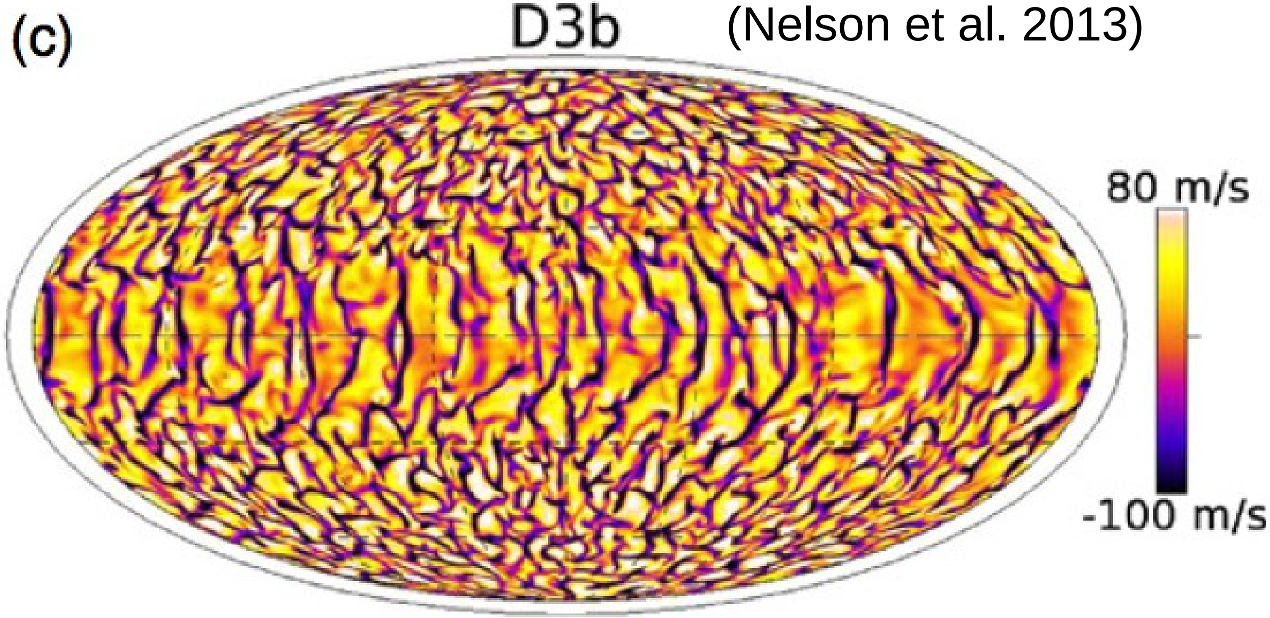
Fig. 7. Computer produced meridional cross section of linear rotation velocity u_0 relative to uniformly rotating frame, for Taylor number $T=10^5$, Rayleigh number $R=4\times10^3$, Prandtl number $P=1$. Units dimensionless, with velocity scaled by κ/d, in which κ is thermal diffusivity, d is convection zone depth. Positive u_0 indicated by solid contours; negative u_0 by dashed contours.

- ASH code, e.g. Miesch et al. (2011)
  - Anelastic approximations, LES.

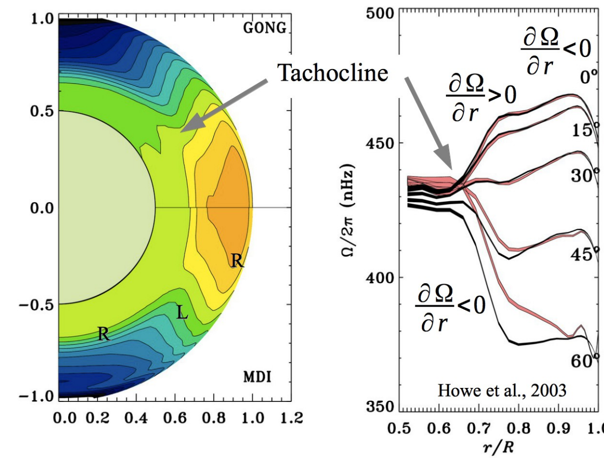


$$\nu_{sgs} = \text{cnt} \gg \nu$$

$$\nu_{SGS} \propto \rho^{-1}(\rho^{-1/2}) \gg \nu$$



(Nelson et al. 2013)

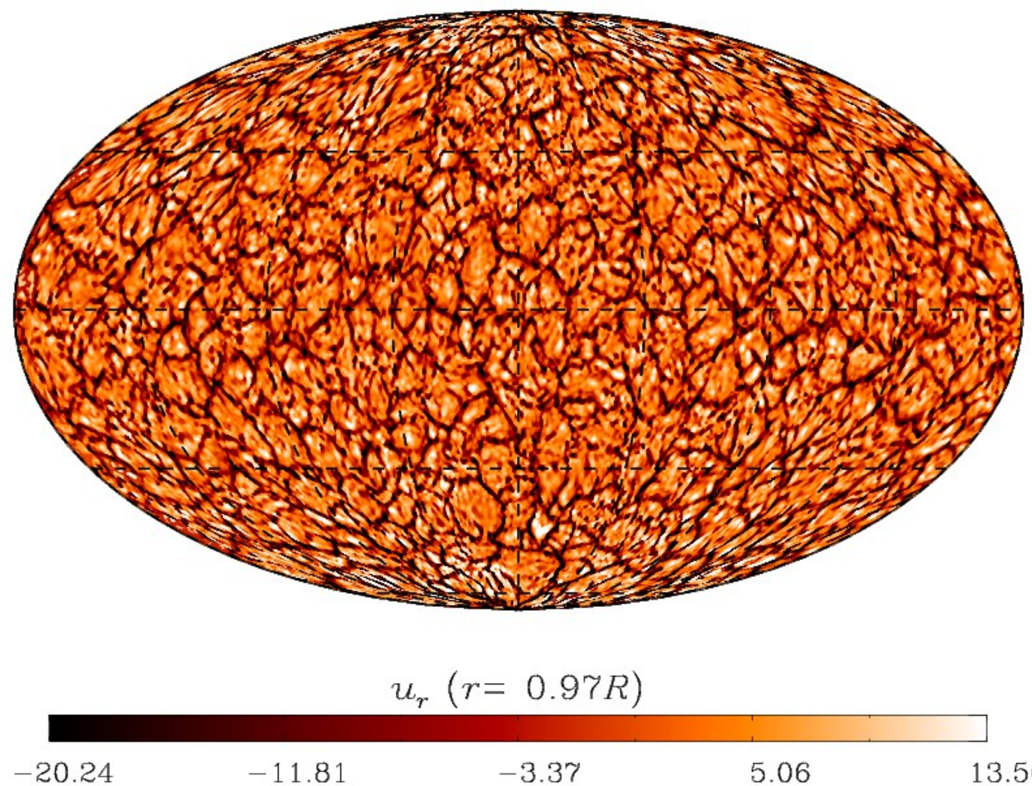


(Guerrero et al. 2013)

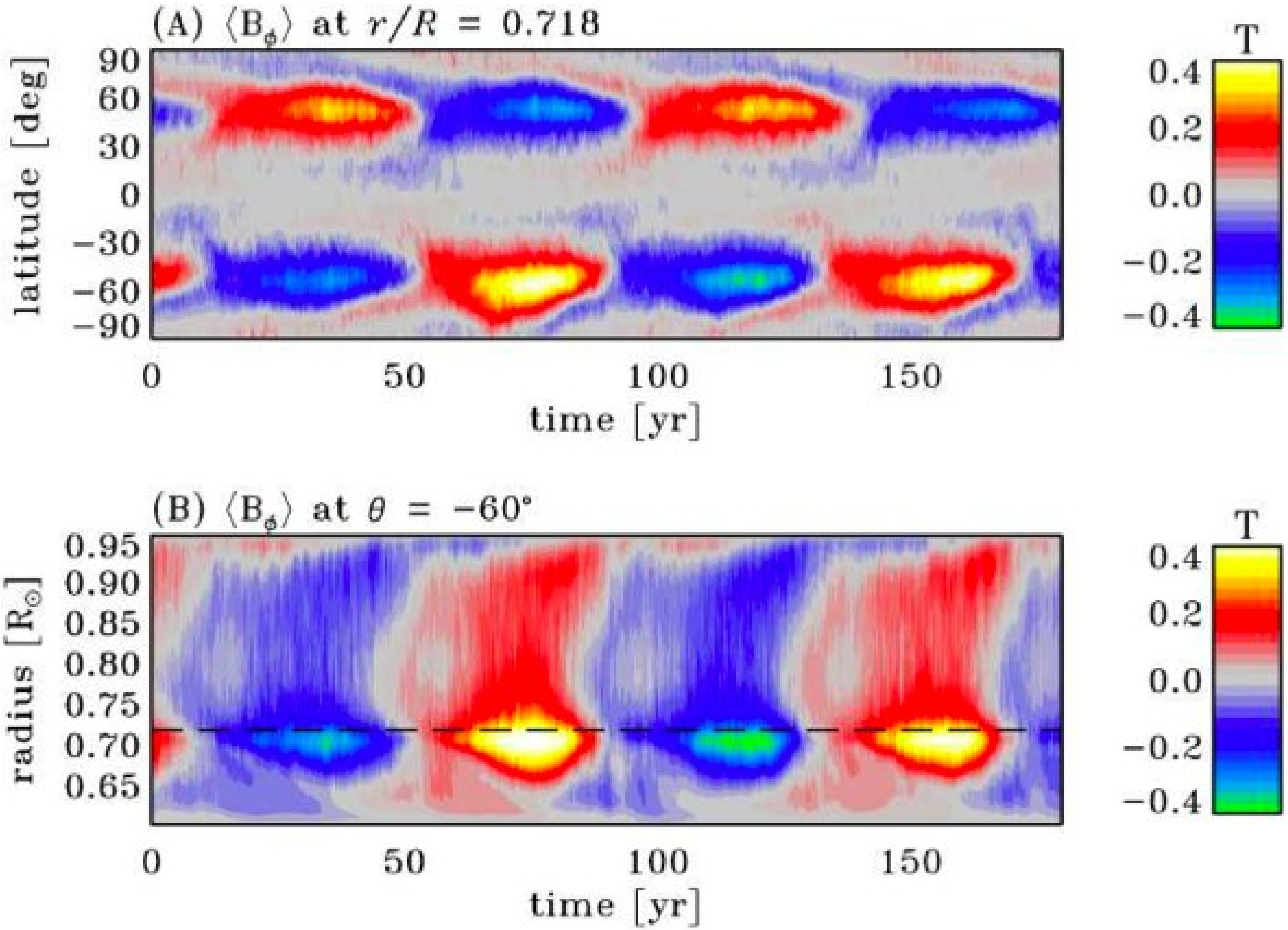
$$\nu_{SGS} = C_s \Delta (\pi_{ij} \pi_{ij})^2$$

$$C_s = \text{ctn (Smagorinsky)}$$

$$C_s = Cs(\mathbf{U}) \text{ (Dynamic Smagorinsky)}$$

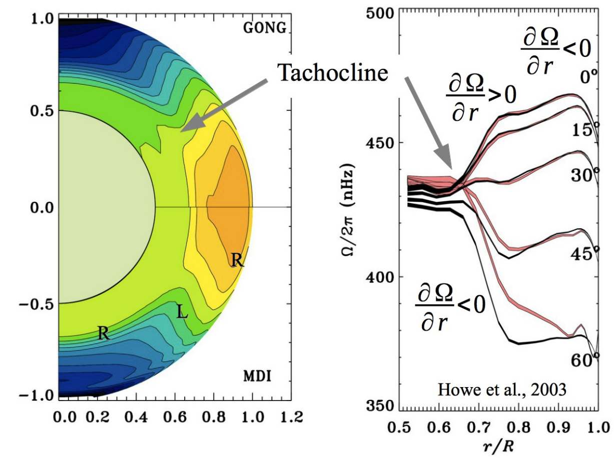


Implicit sub-grid scale modeling (ISGS, ILES): the non-linear computation of the truncation error (i.e., numerical viscosity) is identified with the SGS contribution (EULAG code), (Margolin & Rider, 2002)



Ghizaru, Charbonneau & Smolarkiewicz (2010)

3. Anelastic simulations with EULAG code



Anelastic approximation
(Lipps & Helmer 1982, Lipps 1990)

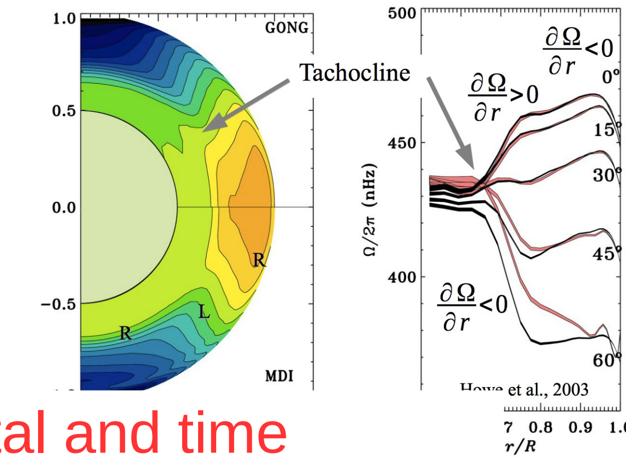
$$\nabla \cdot (\rho_0 \mathbf{u}) = 0, \quad (1)$$

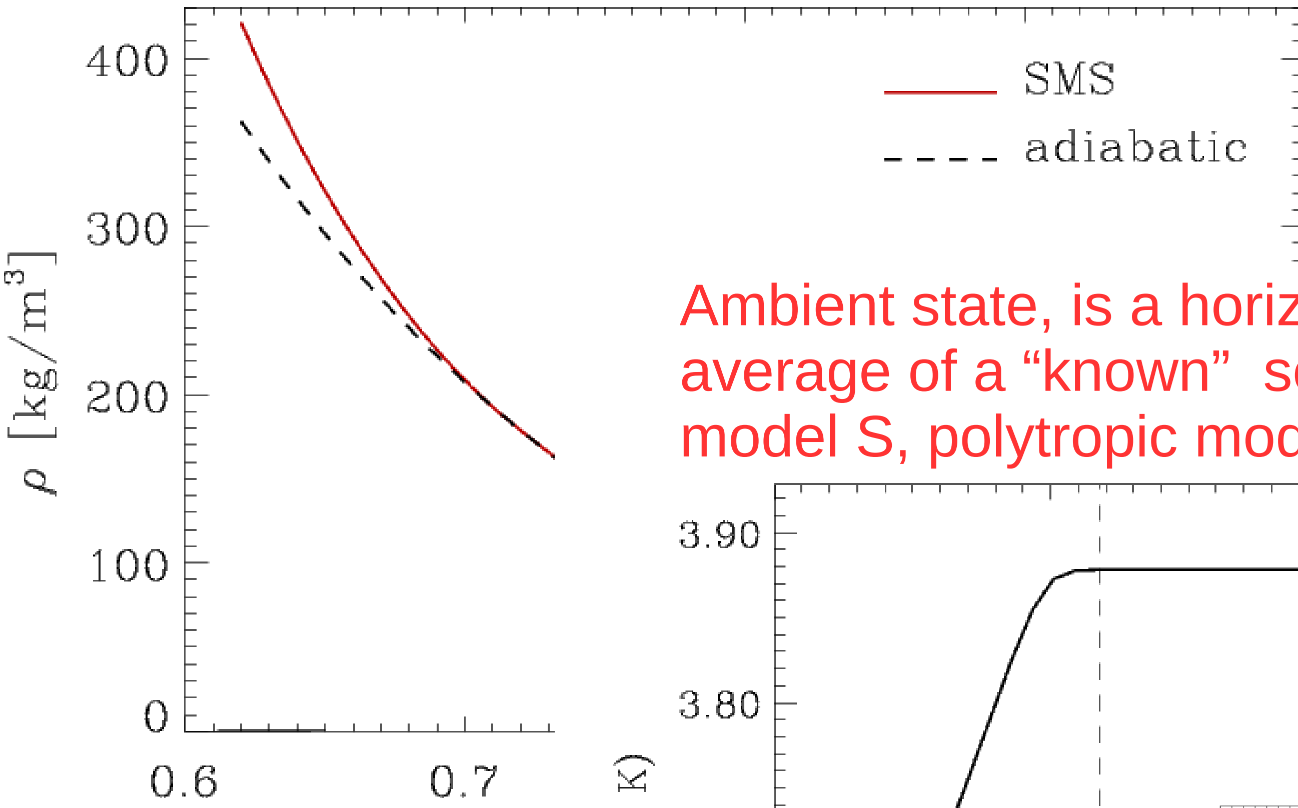
$$\frac{D\mathbf{u}}{Dt} - 2\mathbf{\Omega} \times \mathbf{u} = -\nabla\left(\frac{p'}{\rho_0}\right) + \mathbf{g}\frac{\Theta'}{\Theta_0} + \frac{\mathbf{F}}{\rho_0}, \quad (2)$$

$$\frac{D\Theta'}{Dt} = -\mathbf{u}\cdot\nabla\Theta_e + \frac{1}{\rho_0}\mathcal{H}(\Theta') - \alpha\Theta' \quad (3)$$

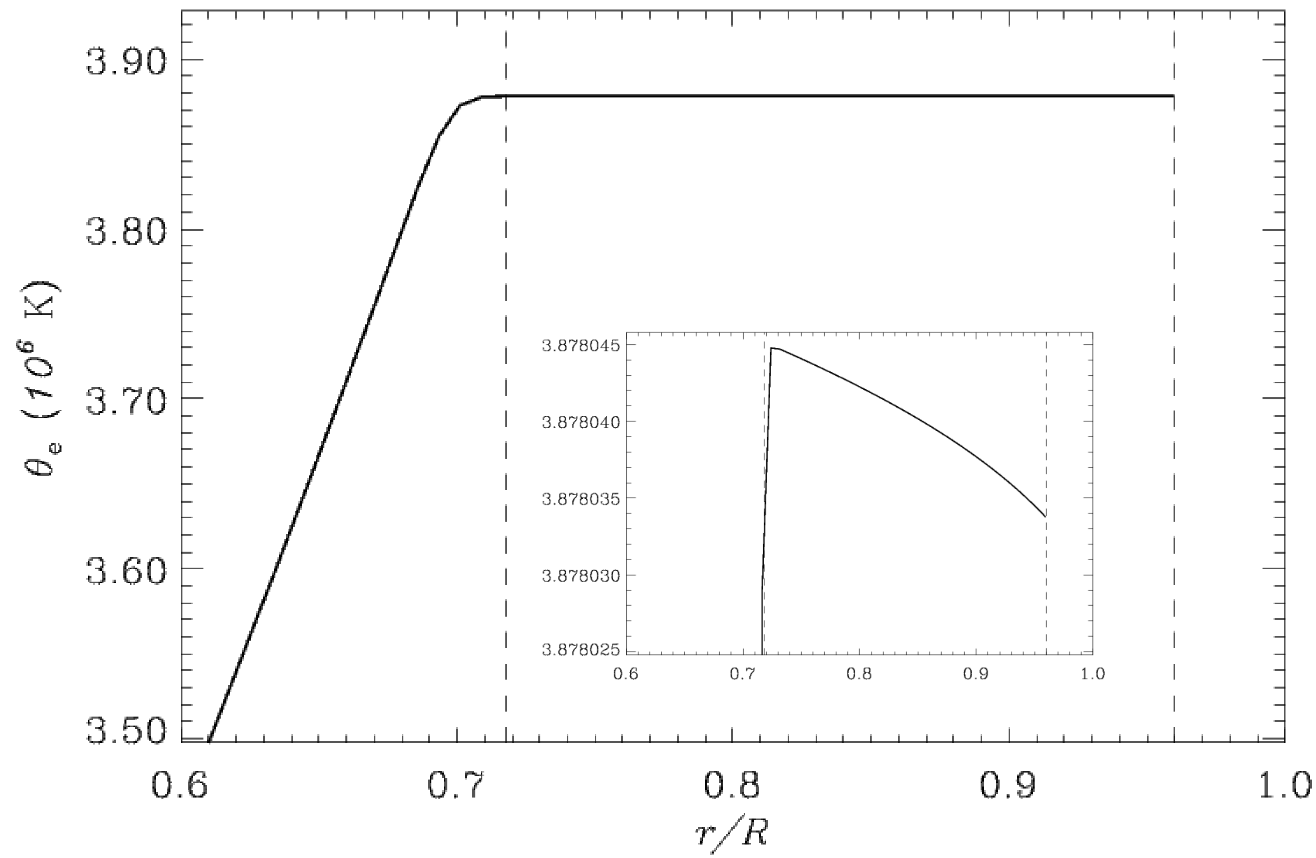
$$ds = c_p d \ln \Theta \quad (4)$$

The background state, ρ_0, p_0, Θ_0



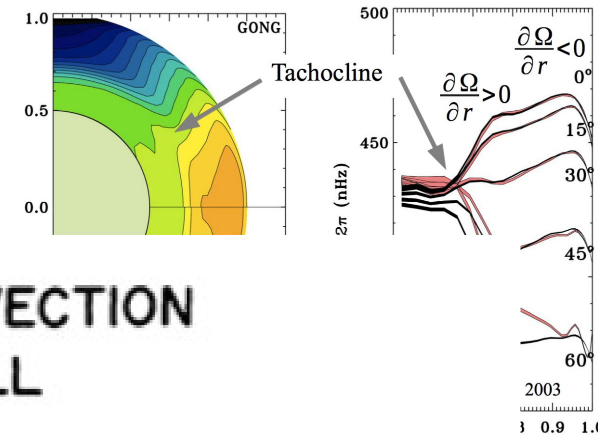


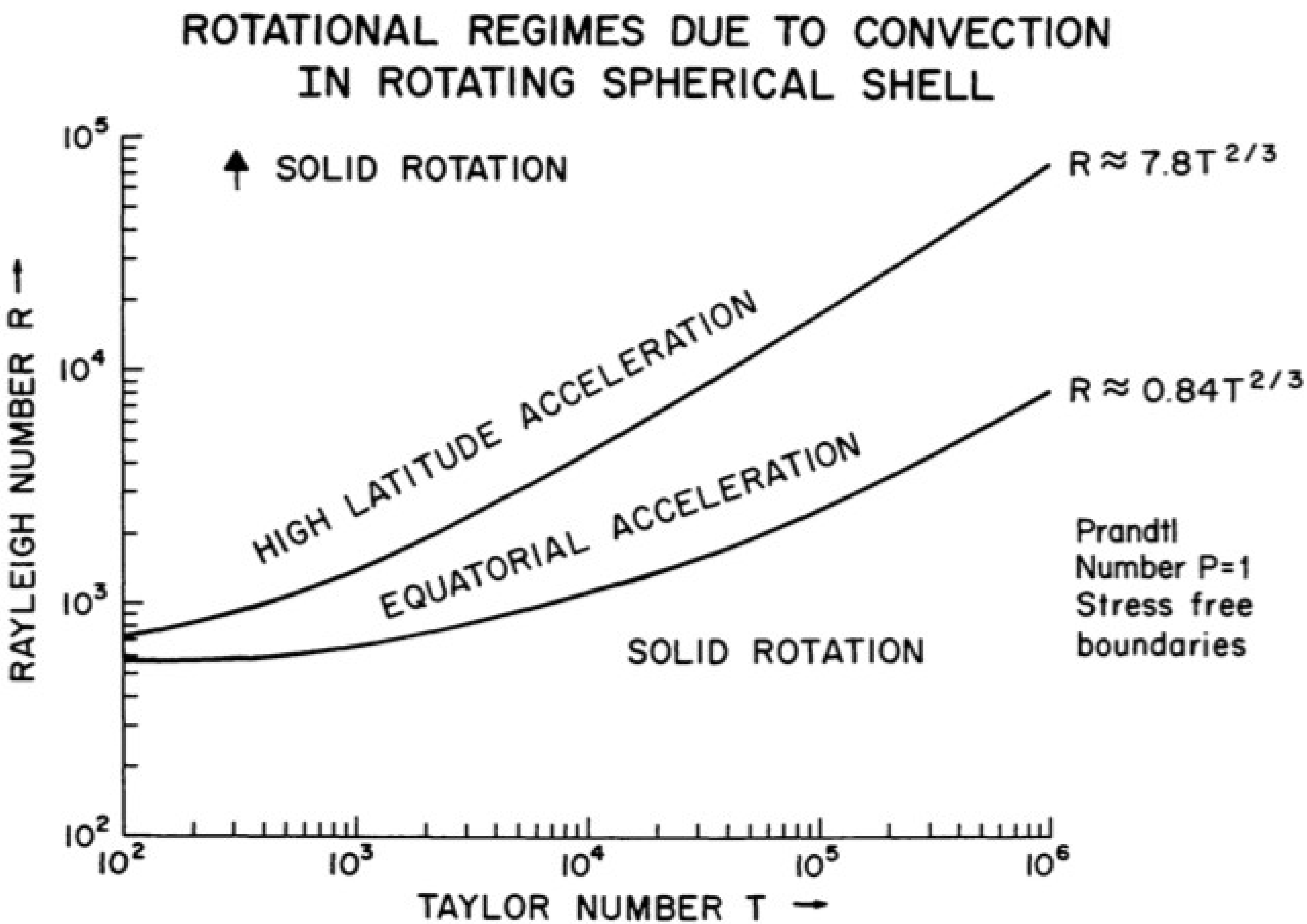
Ambient state, is a horizontal and time average of a “known” solution, e.g., solar model S, polytropic model.



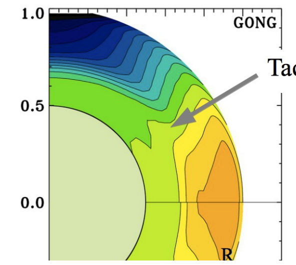
a) Angular momentum transport

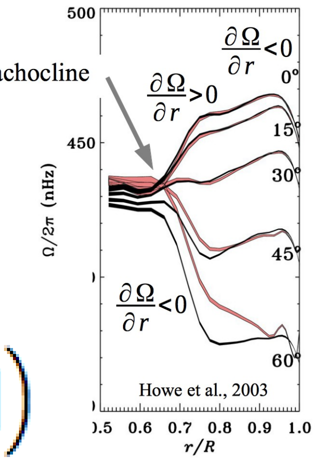
Rotation vs. Convection





Angular momentum conservation





$$\frac{\partial(\overline{\rho}\,\overline{u}_\phi)}{\partial t} = \frac{1}{r\sin\theta}\nabla\cdot\left(\overline{\rho} r\sin\theta[-\nu\nabla\overline{u}_\phi + (\overline{u_\phi} + \Omega_0 r\sin\theta)\mathbf{u} + \overline{u'_\phi \mathbf{u}'}]\right)$$

- ~~Viscous flux: $F_{vis} = -\nu\nabla\overline{u}_\phi$~~
- Meridinal circulation: $F_{MC} = (\overline{u}_\phi + \Omega_0 r\sin\theta)\mathbf{u}$
- Reynold stresses flux: $F_{RS} = \overline{u'_\phi \mathbf{u}'}$

$$(\lambda, \phi, z)$$

$$\lambda = r\sin\theta$$

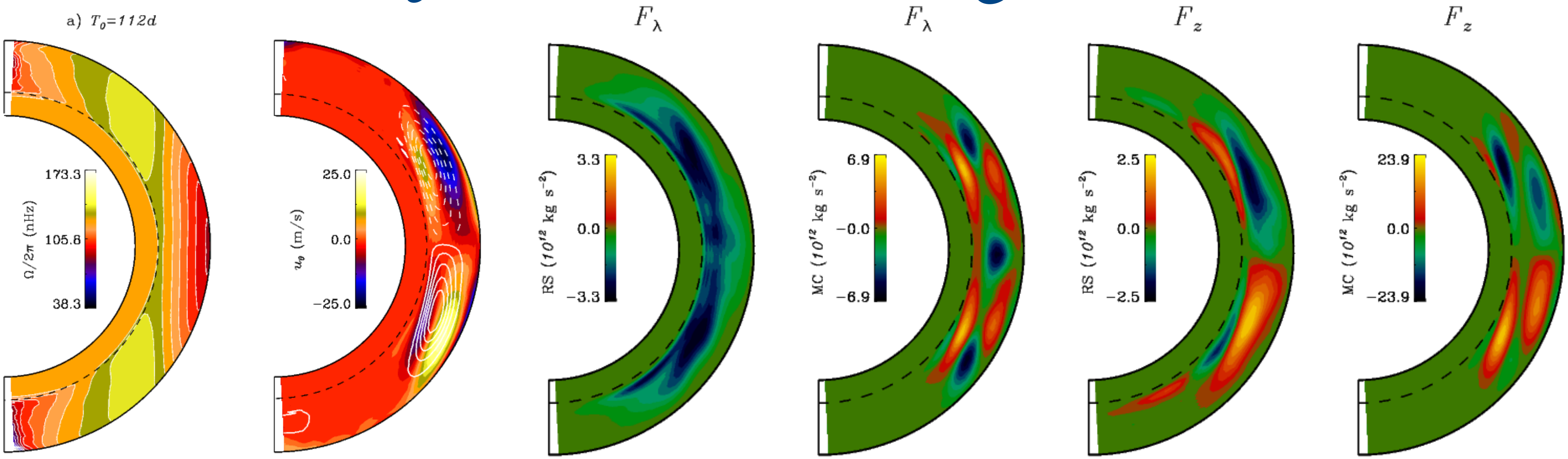
$$z = r\cos\theta$$

$$F_\lambda\hat{\lambda} = F_r\sin\theta\hat{r} + F_\theta\cos\theta\hat{\theta}$$

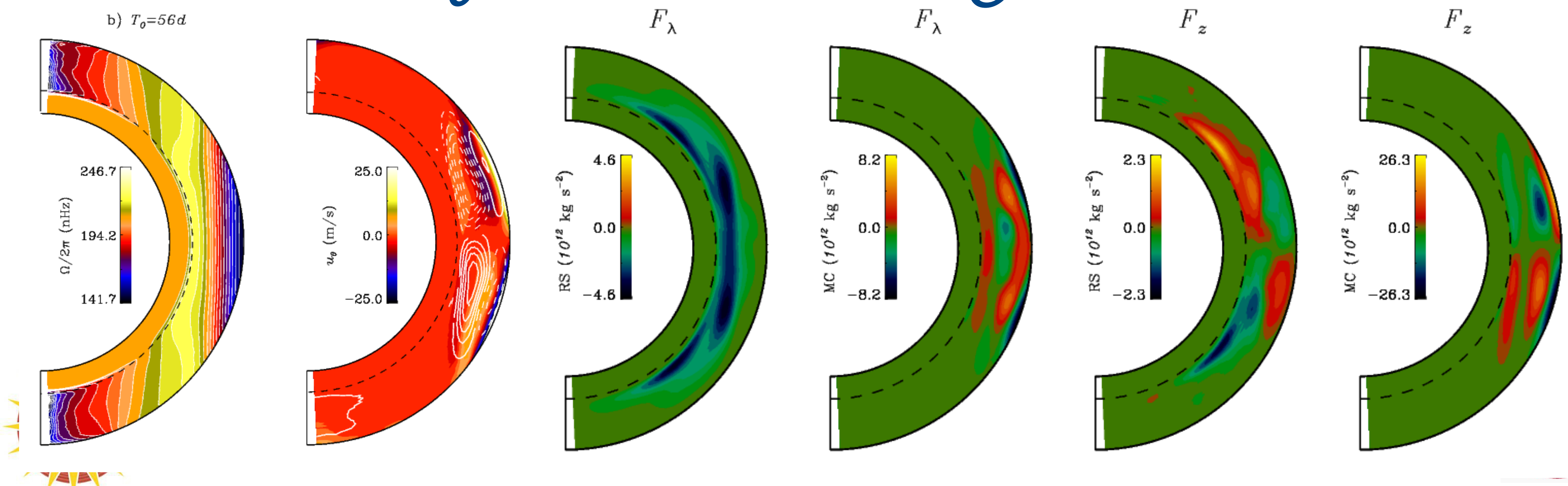
$$F_z\hat{z} = F_r\cos\theta\hat{r} - F_\theta\sin\theta\hat{\theta}$$

$$Q_{\phi r}\ ,\ Q_{\phi\theta}\ ,\ Q_{r\theta}$$

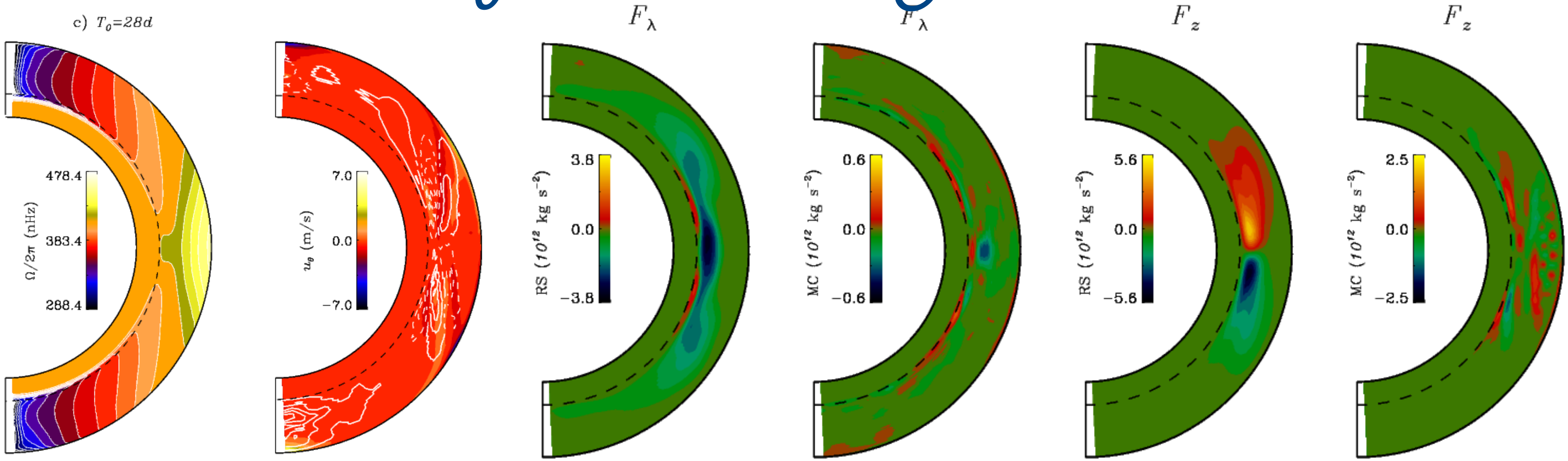
a) Ω_0=112days (1/4 $\Omega_{\odot}$, $Re \sim 60$)



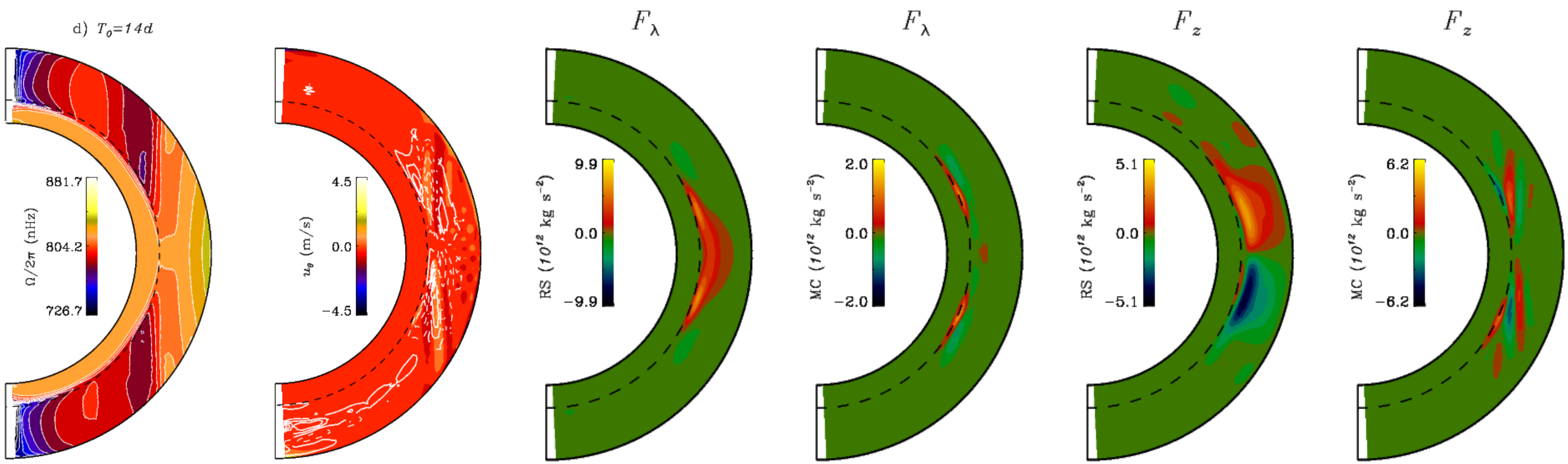
b) Ω_0=56days (1/2 $\Omega_{\odot}$, $Re \sim 60$)

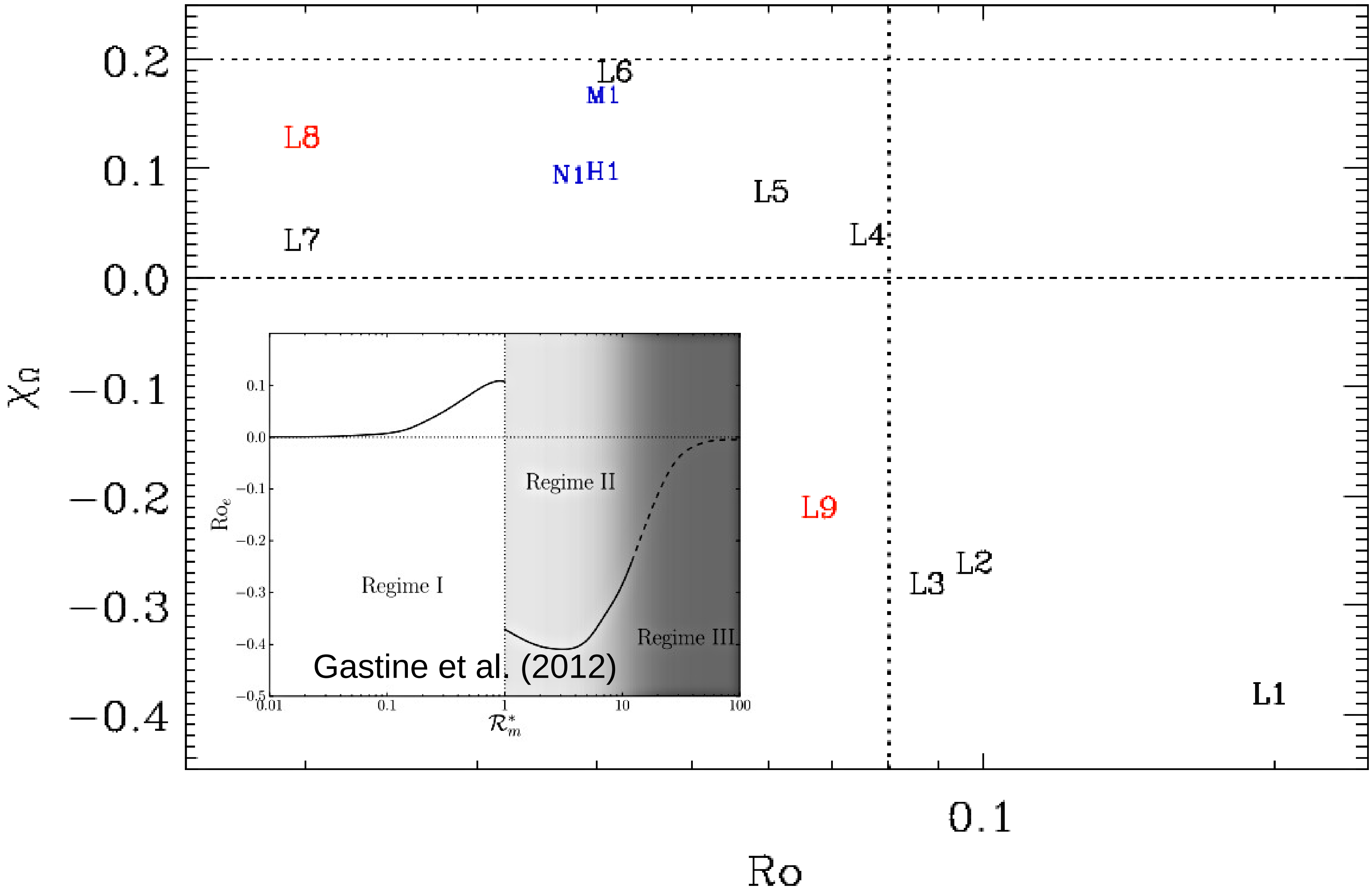


c) Ω_0=28days ($\Omega_\odot$, Re~60)

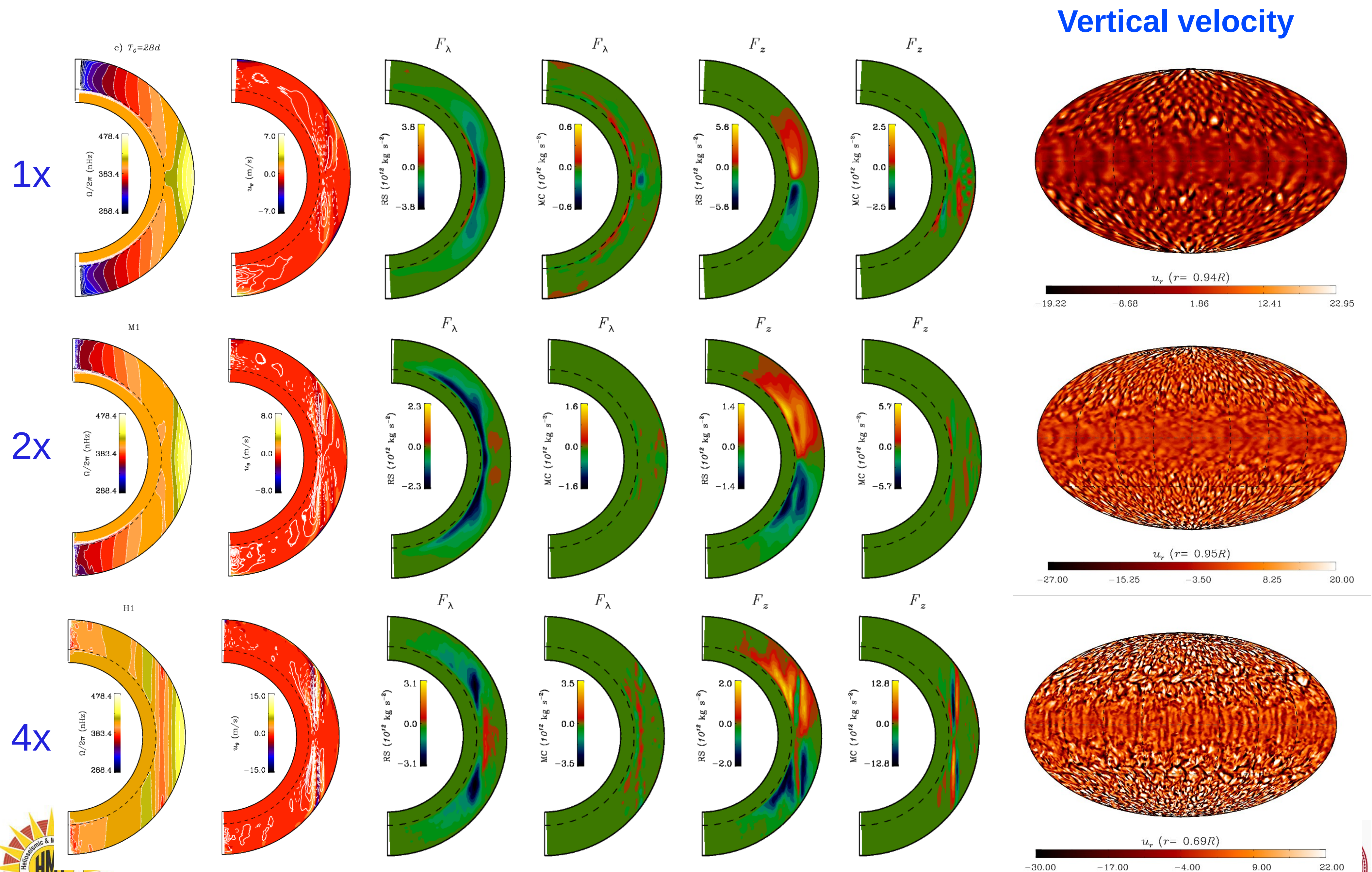


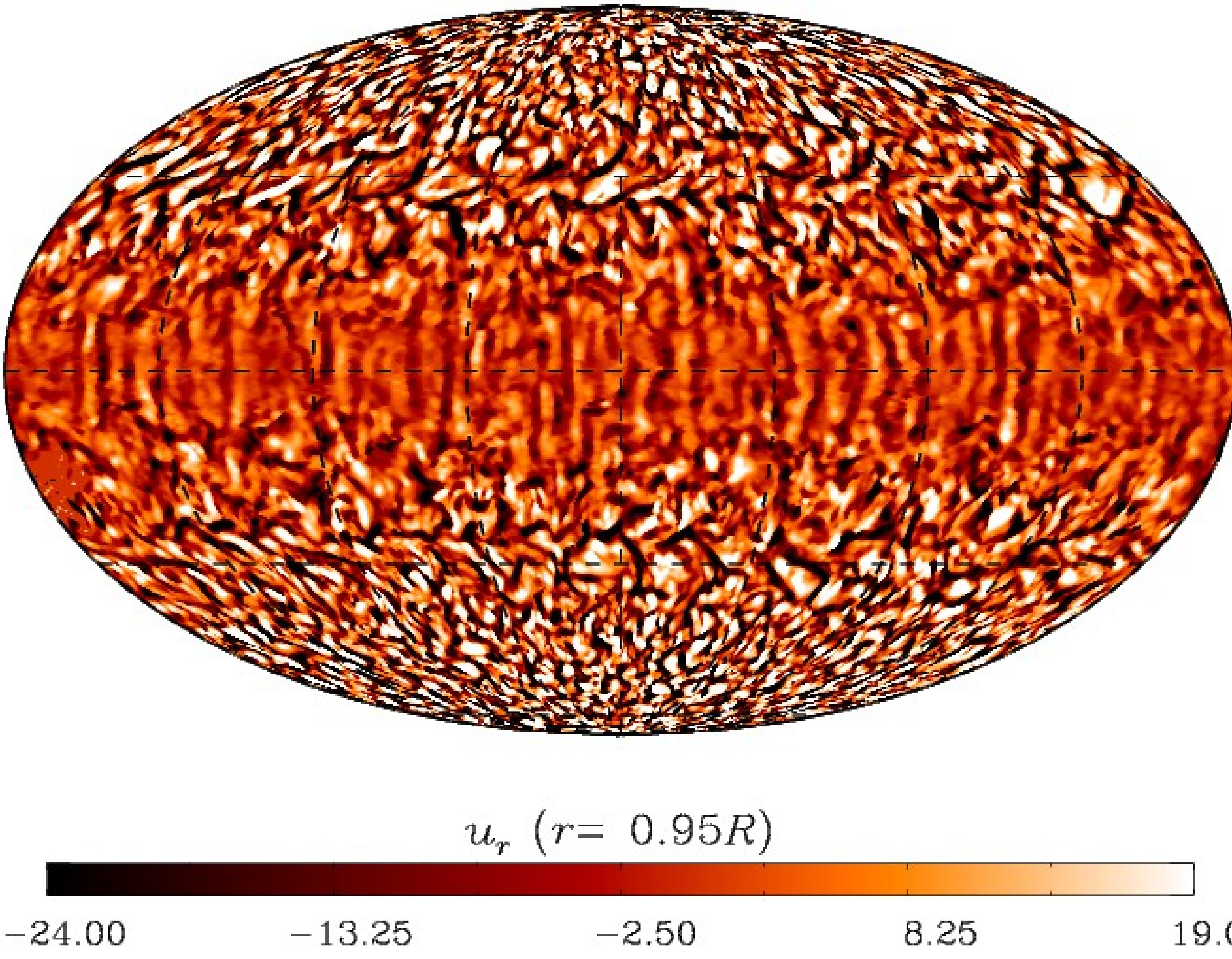
d) Ω_0=14days (2$\Omega_\odot$, Re~60)



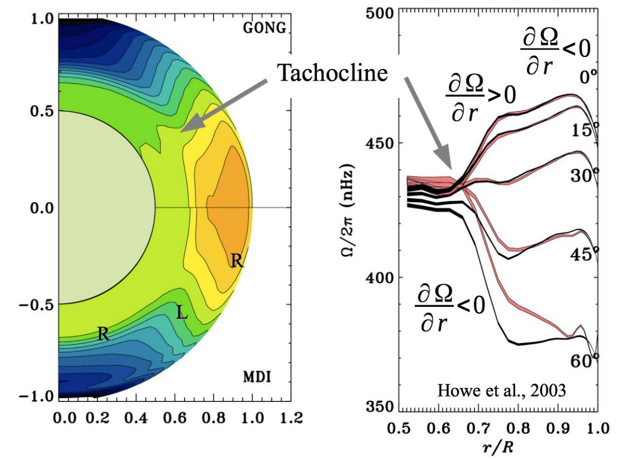

0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
χ_Ω
0.1
Ro
L1
L2
L3
L4
L5
L6
L7
L8
L9
M1
N1
H1
Regime I
Regime II
Regime III
Ro_e
$\mathcal{R}_m^*$
0.01
0.1
1
10
100
Gastine et al. (2012)

b) Convergence: x=128x64x47

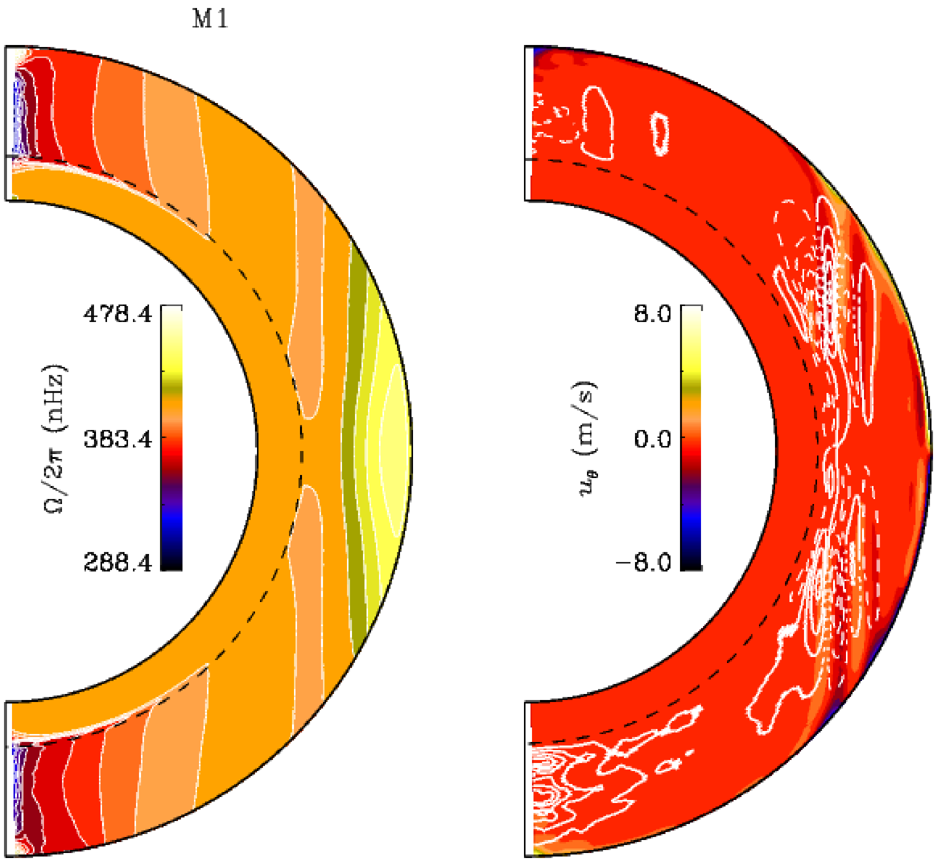



u_r (r= 0.95R)
−24.00
−13.25
−2.50
8.25
19.00

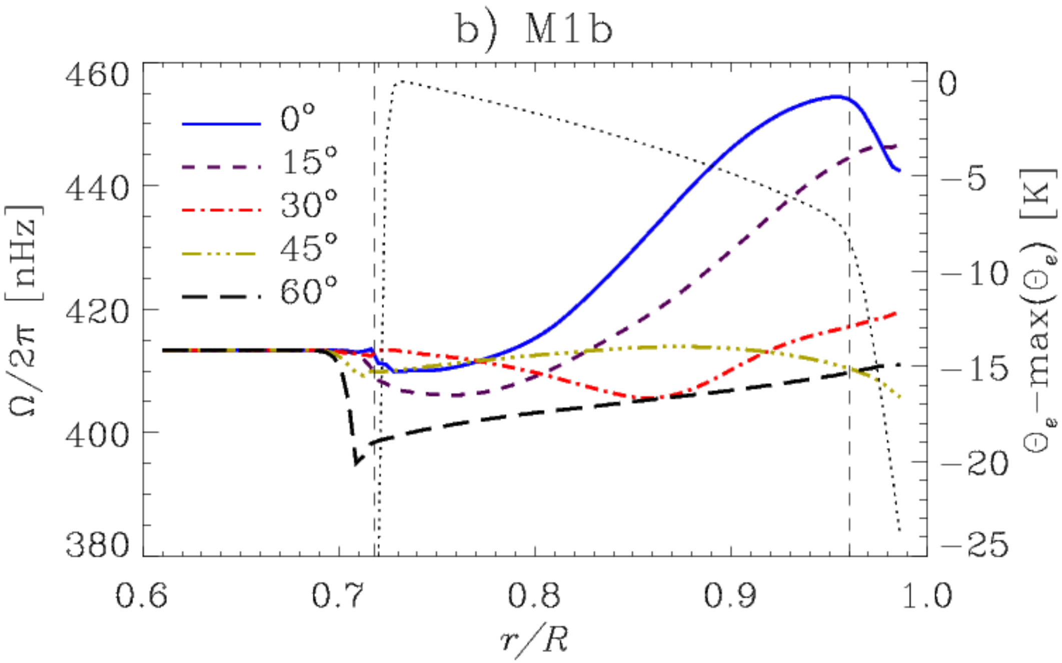
c) Near-surface shear layer



Based in models slow rotating models where convection dominates over rotation …

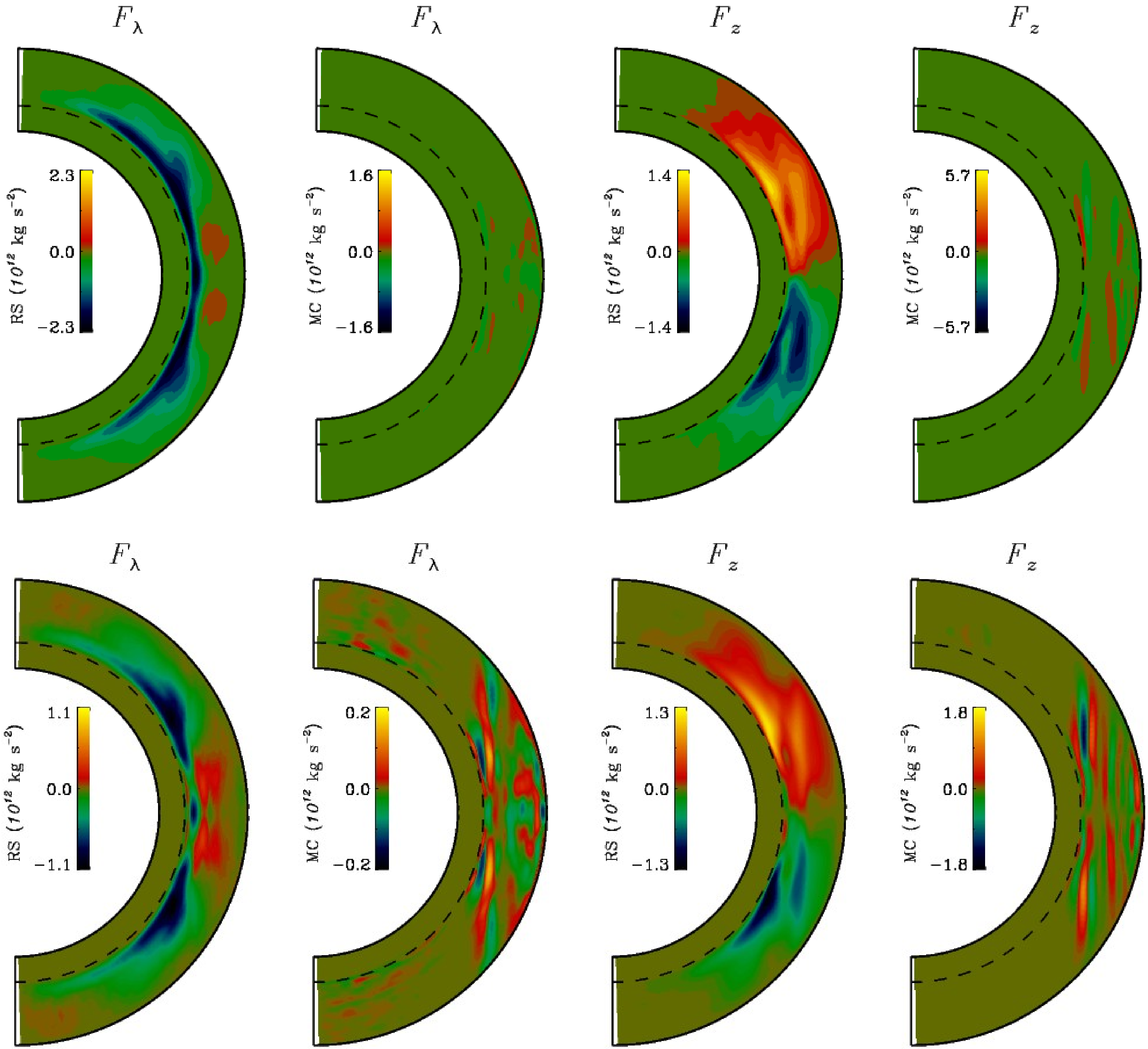


Negative radial shear appears at lower latitudes

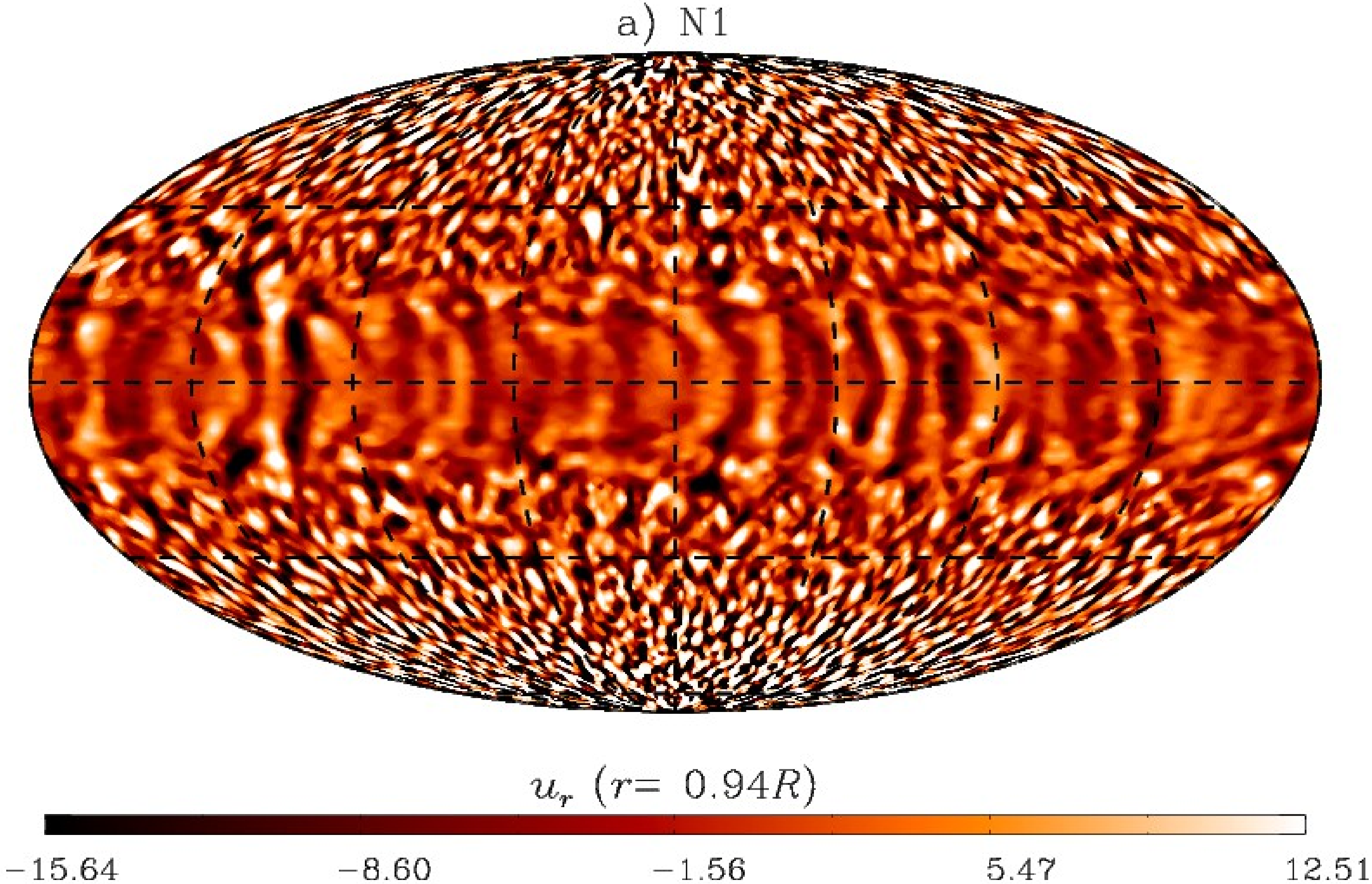


2x

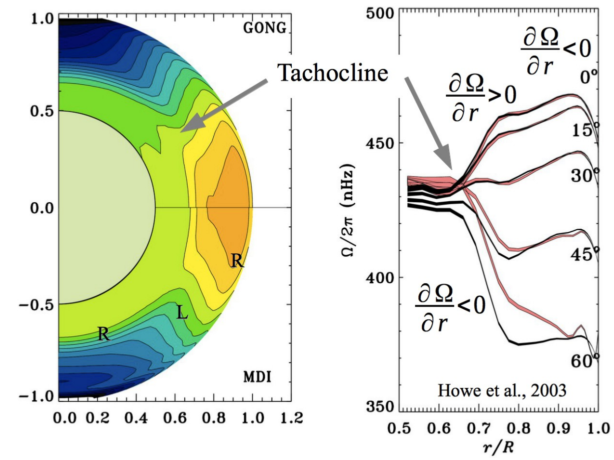
2x-nssl



Fλ
Fλ
Fz
Fz
RS (10^12 kg s^-2)
2.3
0.0
-2.3
MC (10^12 kg s^-2)
1.6
0.0
-1.6
RS (10^12 kg s^-2)
1.4
0.0
-1.4
MC (10^12 kg s^-2)
5.7
0.0
-5.7
Fλ
Fλ
Fz
Fz
RS (10^12 kg s^-2)
1.1
0.0
-1.1
MC (10^12 kg s^-2)
0.2
0.0
-0.2
RS (10^12 kg s^-2)
1.3
0.0
-1.3
MC (10^12 kg s^-2)
1.8
0.0
-1.8

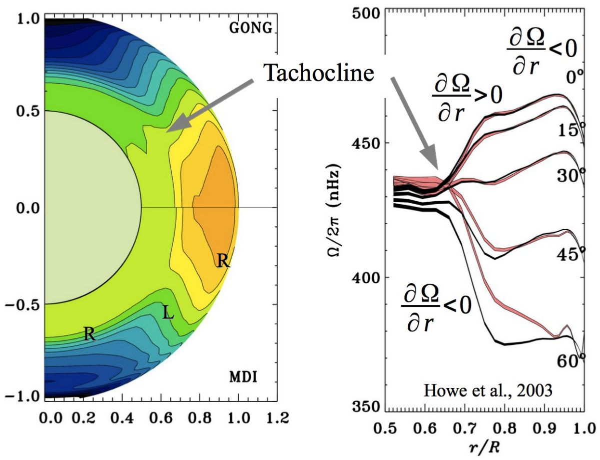

a) N1
u_r (r= 0.94R)
−15.64
−8.60
−1.56
5.47
12.51

CONCLUSIONS



• EULAG code is a powerful tool to study solar/stellar rotation and dynamo, high *Re* are obtained for relatively low resolutions

- The resulting rotation profile depends on the balance between Coriolis and buoyancy forces. For the stratification considered here solar-like rotation (fast equator) is obtained at the solar rotation rate
- Due to the highly sub-adiabatic stable region, all the simulations exhibit the formation of a tachocline
- Taylor-Proudman columnar rotation profiles are persistent, indicating deficient latitudinal turbulent heat transfer



- For all the simulations at the solar rotation rate the meridional flow is multicellular
- Vigorous convection at the upper part of the convection zone leads to the formation of a “near-surface shear layer” at equatorial latitudes