

Dynamic Model of Dynamo (Magnetic Activity) and Rotation

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Differential Rotation and Magnetism Across HR-Diagram, 2013

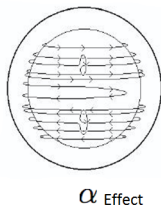
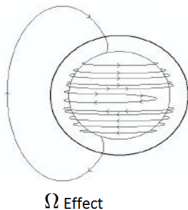
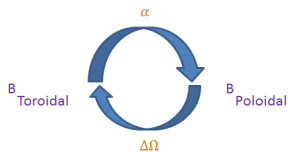
08 April, 2013

Outline

1. Motivation
2. Dynamo Model Construction
3. Reduced Fifth Order System
4. Reduced Sixth Order System
5. Conclusions

Motivation

- ▶ First successful model for solar cycle was proposed by Parker in 1955



- ▶ In $\alpha\Omega$ dynamo, regeneration of poloidal magnetic field can be due to parameter $\alpha = -\tau_c \langle u \cdot (\nabla \times u) \rangle$.
- ▶ Poloidal magnetic flux through α -effect is measured by a non-dimensional parameter $D_\alpha = \alpha R / \eta$
- ▶ Toroidal magnetic flux through differential rotation is measured by a magnetic Reynolds number $D_\Omega = \Omega' R^3 / \eta$
- ▶ Dynamo efficiency is governed by a non dimensional dynamo number D and

$$D = D_\alpha D_\Omega = \frac{\alpha R}{\eta} \cdot \frac{\Omega' R^3}{\eta},$$

$$D \propto \Omega^2$$

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- ▶ $P_{cyc} \propto D^{-\frac{1}{2}} \propto \Omega^{-1}$
- ▶ Frequency of magnetic fields $\omega \propto D^{\frac{1}{2}} \propto \Omega$
- ▶ Does this linear relation still hold in non linear dynamo theory?

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- ▶ There are at least three saturation mechanisms that have been proposed for stellar dynamo theory:
 - ▶ α quenching
 - ▶ Shear quenching
 - ▶ Magnetic flux loss

- ▶ Observationally, cycle period depends upon the stellar rotation period P_{rot} as $P_{cyc} \propto P_{rot}^n$, $n = 1.25 \pm 0.5$ [Noyes et. al(1984)].

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- ▶ Exponent $n = 0.80$ for active star and 1.15 for inactive star [Saar and Brandenburg ; 1999, Charbonneau and Saar ; 2001, Saar ; 2002]

- ▶ Can we develop a minimal dynamo model explain these observations?

Model

- ▶ Cattaneo, Jones & Weiss (1983) constructed a simple parameterized model by taking $B = (0, B(t)e^{ikx}, ikA(t)e^{ikx})$ and Lorentz force that generates the differential rotation $\frac{\partial W}{\partial z} = w_0 + w(t)\exp(2ikx)$ then dimensionless seventh order system is given as:

$$\partial_t A = 2DB - A, \quad (1)$$

$$\partial_t B = i(1 + w_0)A - \frac{1}{2}iA^*w - B, \quad (2)$$

$$\partial_t w_0 = \frac{1}{2}i(A^*B - AB^*) - \nu_0 w_0, \quad (3)$$

$$\partial_t w = -iAB - \nu w \quad (4)$$

- ▶ A is poloidal field, B is toroidal field, w_0 is mean differential rotation and w is fluctuating differential rotation.
- ▶ A, B, w are complex in nature whereas w_0 is real.

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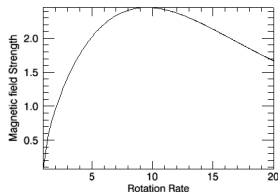
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- ▶ **Case 3:** α -quenching and flux loss i.e. $\lambda_1 = \lambda_2 = \kappa_1$ and further studied the relationship between differential rotation and rotation rate.

Case 1:

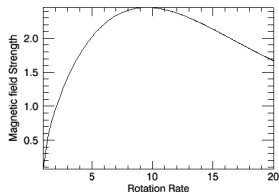
- ▶ α -quenching and no flux loss i.e. $\kappa_1 = 2.5, \lambda_1 = \lambda_2 = 0.0$



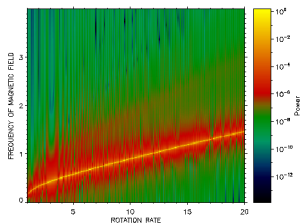
Magnetic field strength starts decreasing with increasing rotation rate.

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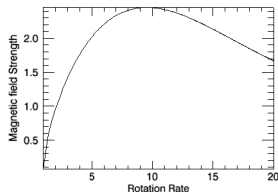
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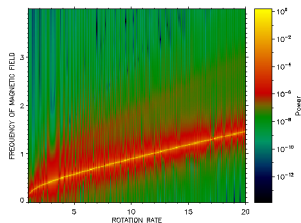
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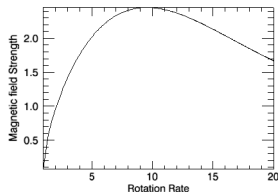
Frequency increases with rotation rate.

$$\omega \propto \Omega^\beta, \text{ where } \beta = 0.67 \text{ for } \Omega \leq 7$$

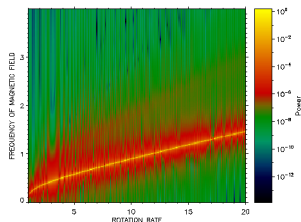
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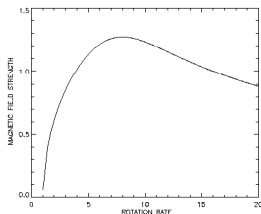
$$\omega \propto \Omega^\beta, \text{ where } \beta = 0.67 \text{ for } \Omega \leq 7$$

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- ▶ which are inconsistent with observations.

Case: 2

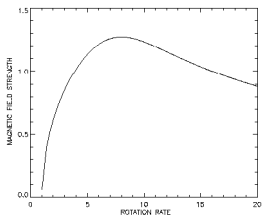
- ▶ no α -quenching and flux loss i.e. $\lambda_1 = \lambda_2 = 2.5$, $\kappa_1 = 0$



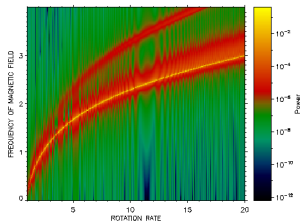
Magnetic field strength decreases with increasing rotation rate (inconsistent) .

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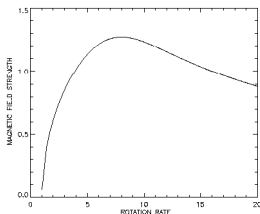
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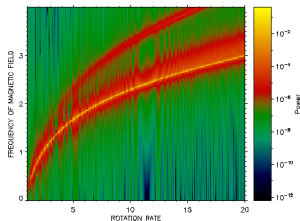
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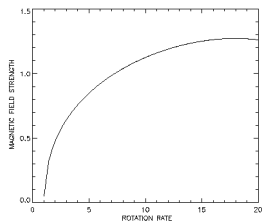


Frequency increases with rotation rate.

- ▶ scaling exponent varies with increasing rotation rate but this variation is within the observed range.

Case 3:

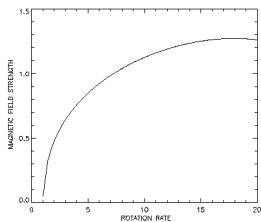
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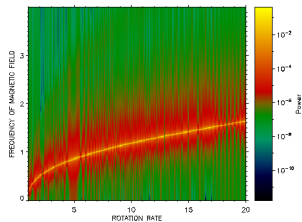
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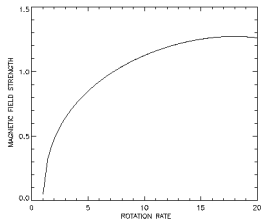
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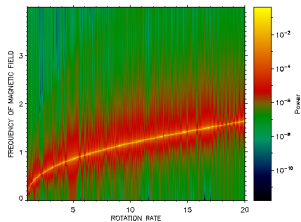
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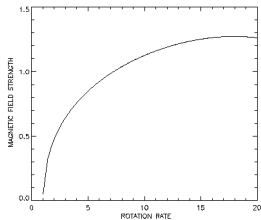


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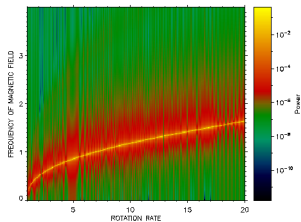
$$\omega \propto \Omega^{0.80}$$

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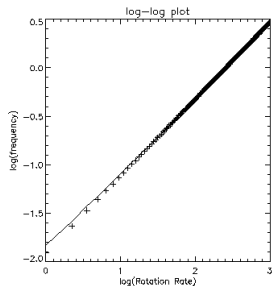
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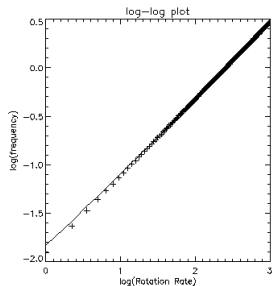
Frequency increases with rotation rate.

$$\omega \propto \Omega^{0.80}$$

- ▶ which are in agreement for active star. [Saar and Brandenburg, 1999; Charbonneau and Saar, 2001; Saar, 2002]



Frequency vs rotation rate in log-log scale.



Frequency vs rotation rate in log-log scale.

$$\omega \propto \Omega^{0.80}$$

- ▶ Among the three cases considered in this 7th order system, Case-1 and Case-2 with only alpha-quenching or flux-loss show the behavior of frequency and strength of magnetic fields in disagreement with observation.
- ▶ Agreement with observations is obtained only in Case-3 with equal amount of alpha quenching and poloidal and toroidal magnetic flux losses.
- ▶ These results thus suggest that there must be an effective balance between generation and dissipation of magnetic fields to obtain the saturation of magnetic fields at high rotation.

- ▶ Is the right balance necessary between the various non linear terms as well as various transport coefficients for a dynamo to work near marginal stability?

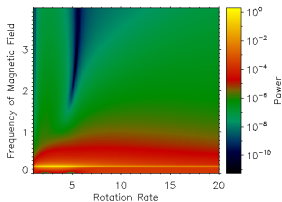
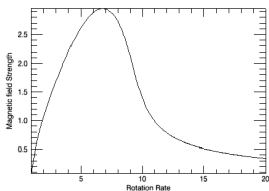
Reduced Fifth Order System

Here we consider the extreme limit where fluctuating differential rotation w is much weaker than w_0 by taking the limits of $\nu \rightarrow \infty$ and $w \rightarrow 0$ in equations (5) to (8) and reduced system is:

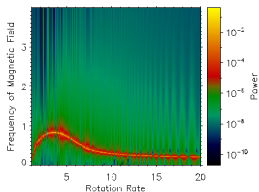
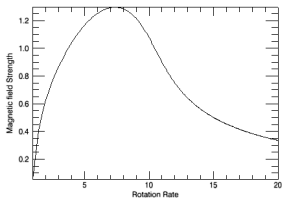
$$\partial_t A = \frac{2DB}{1 + \kappa_1(|B|^2)} - [1 + \lambda_1(|B|^2)]A, \quad (9)$$

$$\partial_t B = i(1 + w_0)A - [1 + \lambda_2(|B|^2)]B, \quad (10)$$

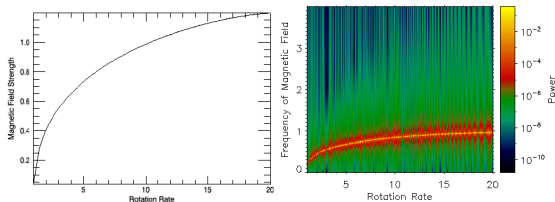
$$\partial_t w_0 = \frac{1}{2}i(A^* B - AB^*) - \nu_0 w_0. \quad (11)$$



(1) *Magnetic field strength and frequency as function of rotation rate for α -quenching and no flux loss i.e. $\lambda_1 = \lambda_2 = 0$, $\kappa_1 = 2.5$.*



(2) *Magnetic field strength and frequency as function of rotation rate for α -quenching and no flux loss i.e. $\lambda_1 = \lambda_2 = 2.5$, $\kappa_1 = 0.0$.*



(3) Magnetic field strength and frequency as function of rotation rate for α -quenching and flux loss i.e.

$$\kappa_1 = \lambda_1 = \lambda_2 = 2.5.$$

- ▶ None of the case is compatible with observations.
- ▶ Quenching in mean differential rotation tends to become too severe, possibly shutting down of dynamo for large rotation rate.
- ▶ We need fluctuating differential rotation to prevent this shear quenching.

Reduced Sixth Order System

The effect of fluctuating differential rotation w is studied by taking $\nu_0 \rightarrow \infty$ and $w_0 \rightarrow 0$ in system (5)-(8). Reduced sixth order dynamical system in the presence of nonlinearities such as α -quenching, shear quenching and magnetic flux loss is given as follows:

$$\partial_t A = \frac{2DB}{1 + \kappa_1(|B|^2)} - [1 + \lambda_1(|B|^2)]A, \quad (12)$$

$$\partial_t B = \frac{iA}{1 + \kappa_2(|B|^2)} - \frac{1}{2}iA^*w - [1 + \lambda_2(|B|^2)]B, \quad (13)$$

$$\partial_t w = -iAB - \nu w. \quad (14)$$

- ▶ The system is studied for different combinations of $\kappa_1, \kappa_2, \lambda_1, \lambda_2$
- ▶ We find that the frequency of magnetic field is within/close to the observations.
- ▶ Magnetic field strength increases monotonically with rotation rate.
- ▶ Various saturation mechanisms are able to slow down the growth of magnetic field with rotation rate to some extent but are not sufficiently efficient to flatten the magnetic field which are inconsistent with observations.
- ▶ These results suggest that incorporation of mean differential rotation as well as fluctuating differential rotation is necessary for onset of dynamo near marginal stability.

Minimal Dynamical Model: Parameter Dependencies

- ▶ To show that seventh order system is minimal model, we investigate system (5)-(8) for nonlinear power-law dependence of α quenching and flux loss on B as follows:

$$\partial_t A = \frac{2DB}{1 + \kappa_1(|B|^m)} - [1 + \lambda_1(|B|^n)]A, \quad (15)$$

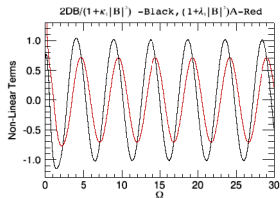
$$\partial_t B = i(1 + w_0)A - \frac{1}{2}iA^*w - [1 + \lambda_2(|B|^n)]B, \quad (16)$$

$$\partial_t w_0 = \frac{1}{2}i(A^*B - AB^*) - \nu_0 w_0, \quad (17)$$

$$\partial_t w = -iAB - \nu w, \quad (18)$$

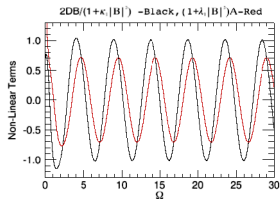
- ▶ The investigation of different cases by varying values of m , n , κ_1 , λ_1 and λ_2 systematically, we note that α -quenching power law and magnetic dissipation should increase at least quadratically

- ▶ Dynamic balance among the dissipation and generation of magnetic fields for $n = m = 2$, $\kappa_1 = \lambda_1 = \lambda_2 = 2.5$ for dynamo number $D = 2$.

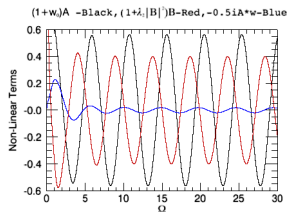


Dynamic balance between
 $\frac{2DB}{1+\kappa_1(|B|^2)}$ *and* $[1 + \lambda_1(|B|^2)]A$.

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Dynamical balance between
 $\frac{2DB}{1+\kappa_1(|B|^2)}$ *and* $[1 + \lambda_1(|B|^2)]A$.





















Dynamical balance between
 $i(1 + w_0)A, [1 + \lambda_2(|B|^2)]B$.

Results

- ▶ Detailed investigations show that seventh order system is more robust in the presence of equal combination of α quenching and magnetic flux loss.
- ▶ The linear increase in frequency and flattening of magnetic energy for higher rotation rate in this case are in agreement with observations.
- ▶ Study of reduced fifth order system and reduced sixth order system indicates that we need a right balance between mean and fluctuating differential rotation to obtain the results consistent with observations.
- ▶ We need a right balance not only in generation and destruction of magnetic fields but also in various flux transport coefficients.
- ▶ Furthermore, the linear increase in observed frequency and rotation rate could be signature of self organization.(work in progress, Sood and Kim, 2013b)
- ▶ Our model is valid for fast rotating stars.

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