Mean field models of Meridional flows and differential rotation in stellar convection zones

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Flux transport dynamo

- differential rotation
- α effect (helicity)
- diffusion
- meridional flow acts as conveyor belt
- cycle time determined by flow speed!

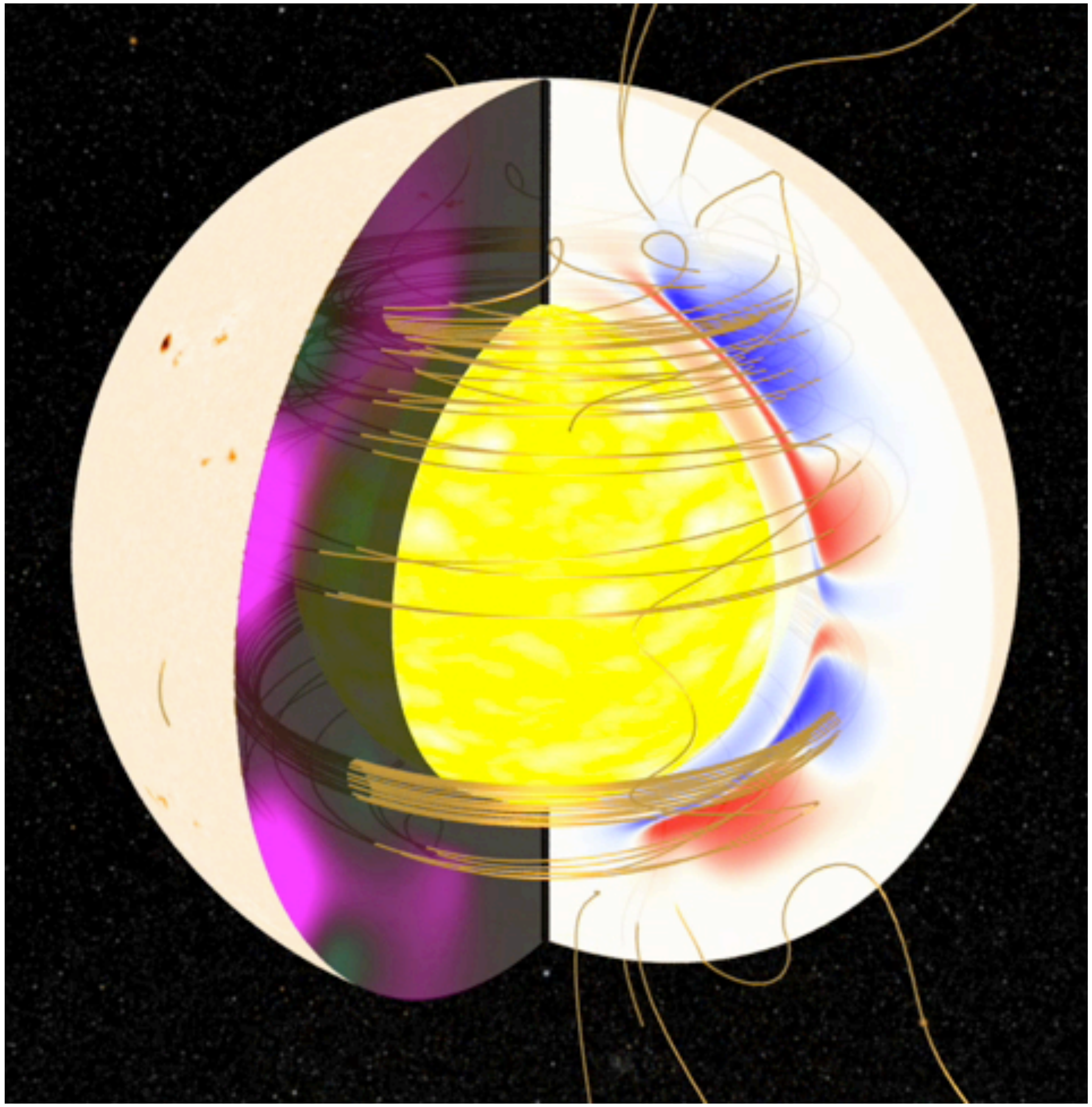
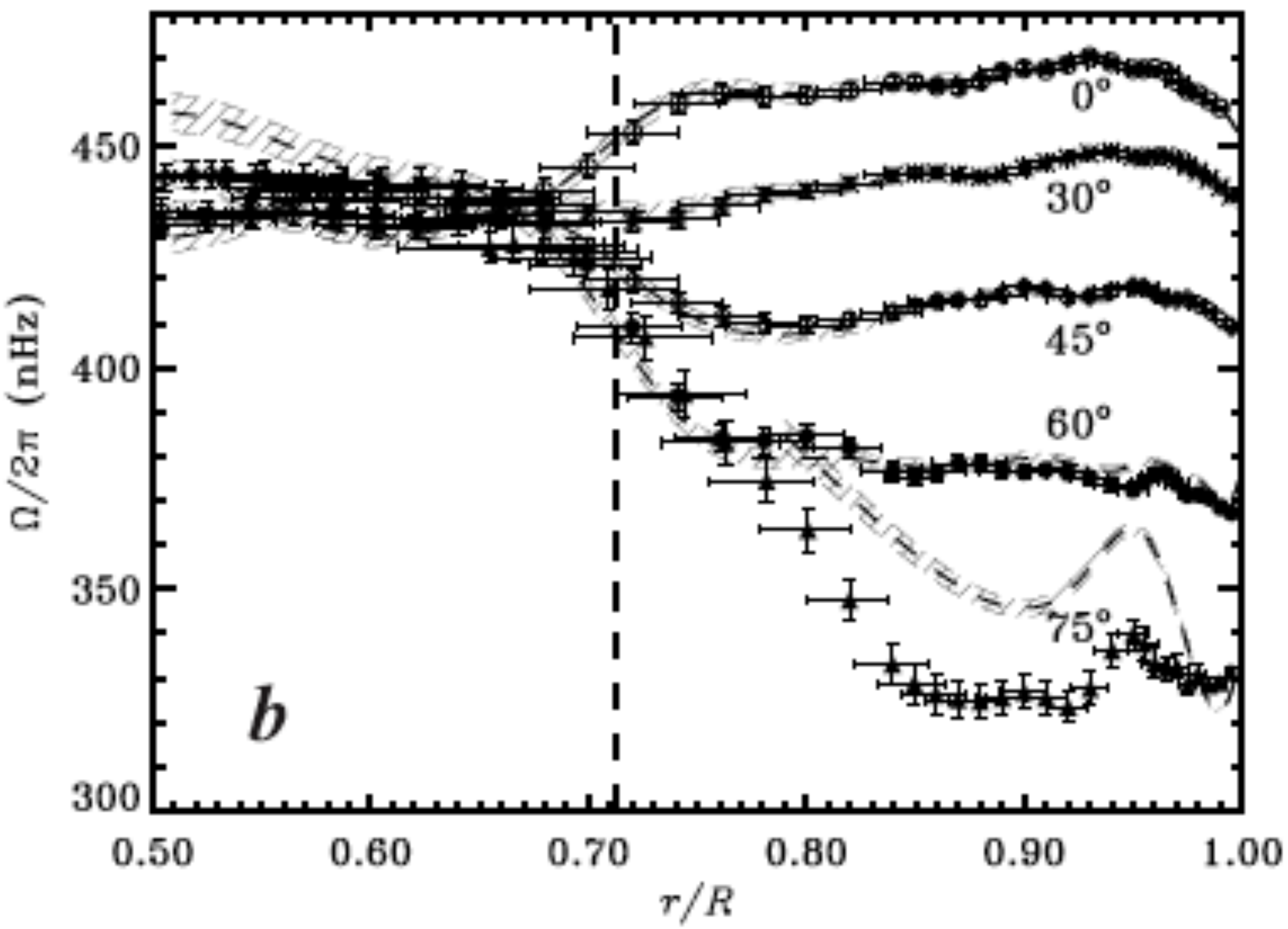
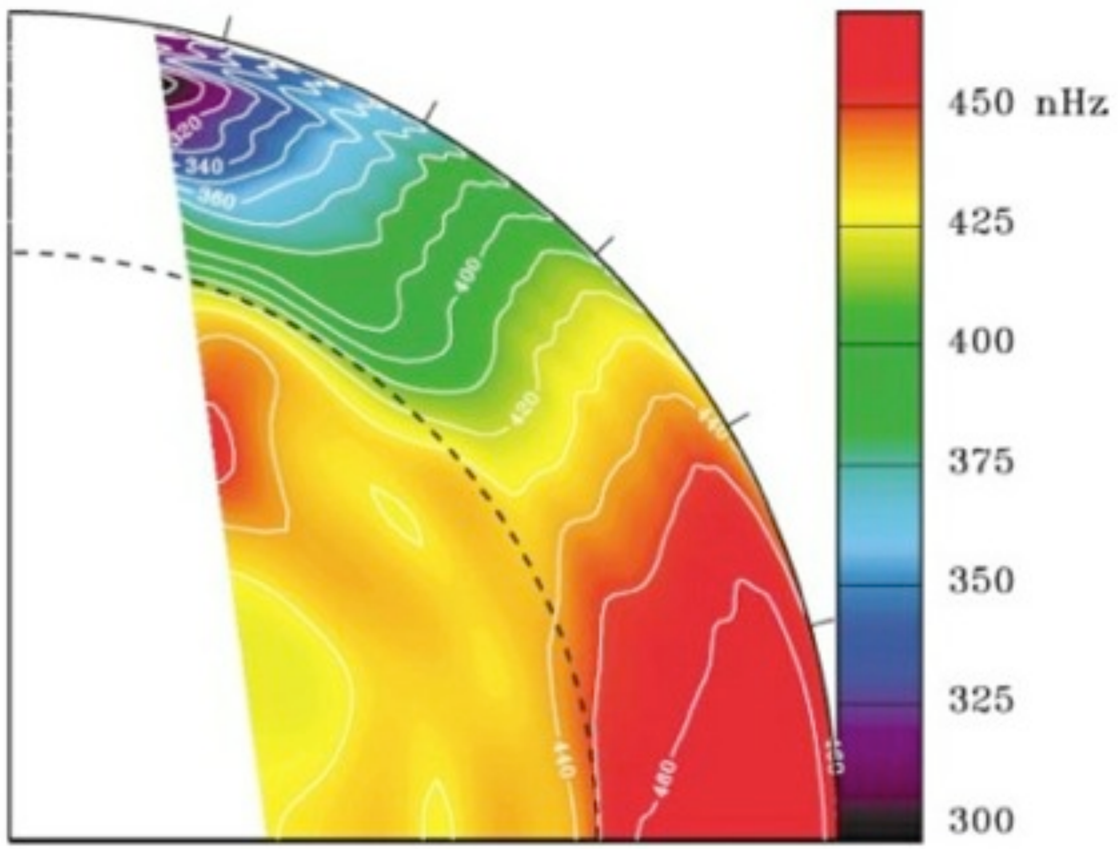


Image: NASA

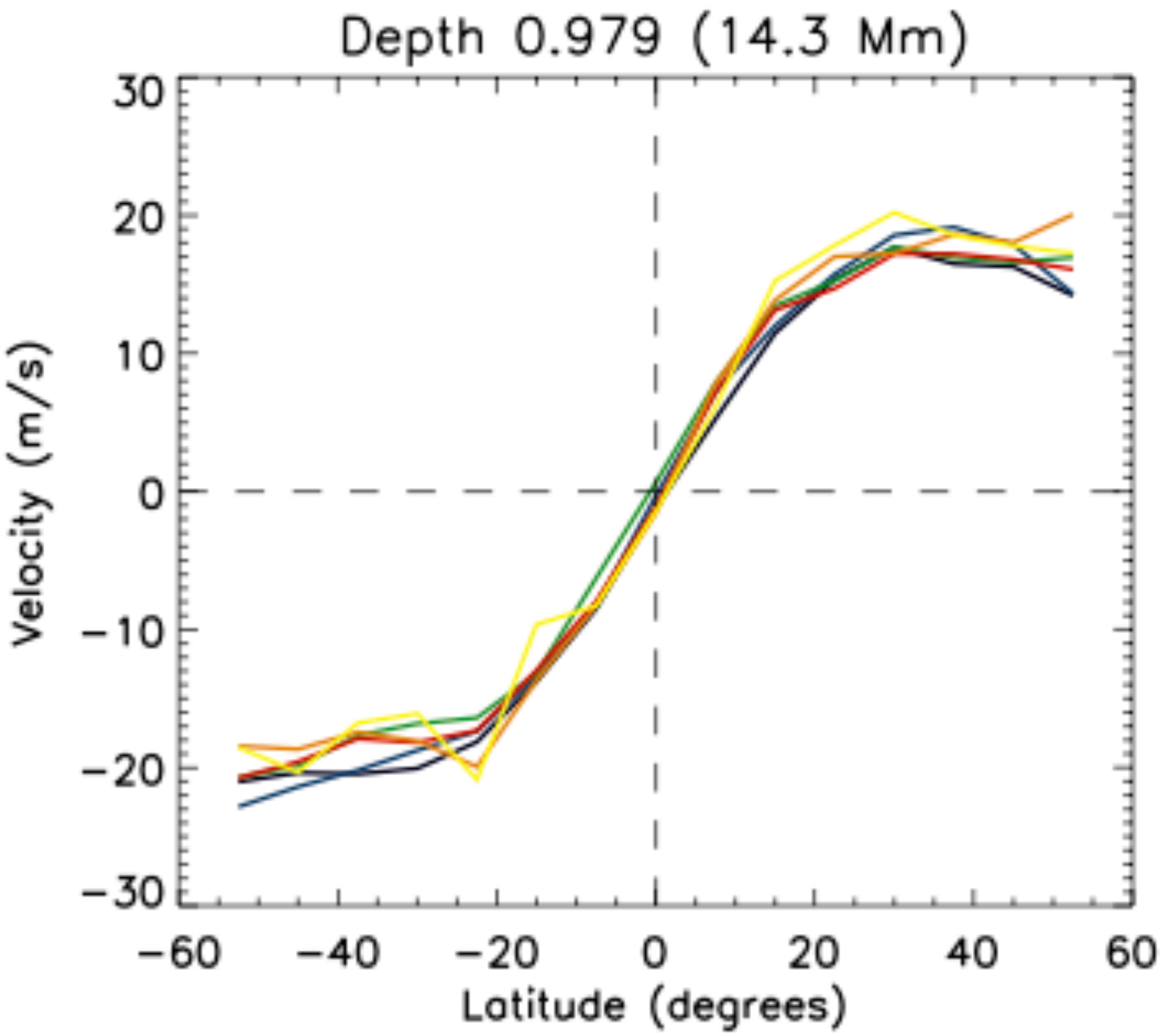
Solar differential rotation: helioseismology





Thompson et al. 2003

Solar meridional flow



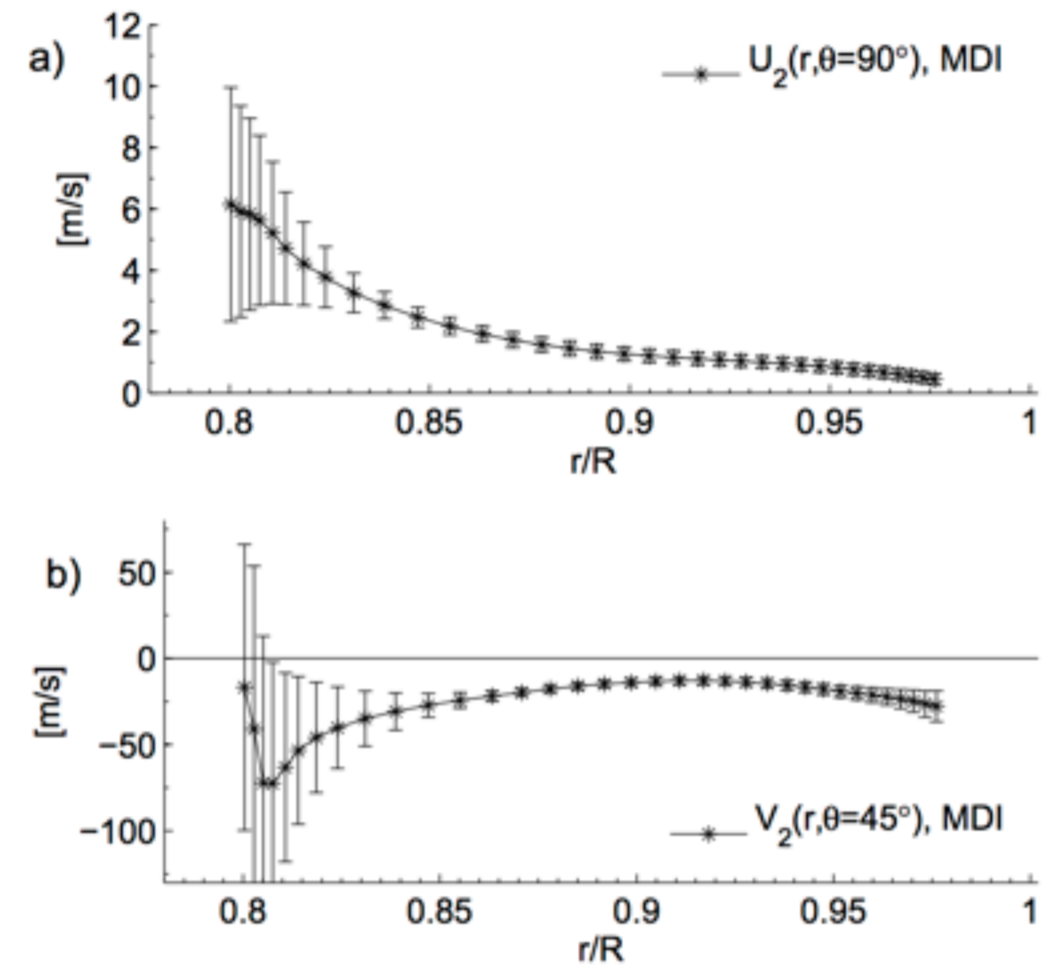


Fig. 2: *a*) Radial flow speed U_2 at the equator and *b*) horizontal flow speed V_2 at mid-latitude with 1σ-error bars as a function of r/R for the $s = 2$ component of the meridional flow estimated from data from MDI. Positive values of $U_2(V_2)$ indicate an outward (southward) directed flow.

Gonzalez-Hernandez et al. 2008

Schad et al. 2012

Mean field hydrodynamics

Mean field ansatz: $\boldsymbol{u} = \bar{\boldsymbol{u}} + \boldsymbol{u}'$ $\quad T = \bar{T} + T'$

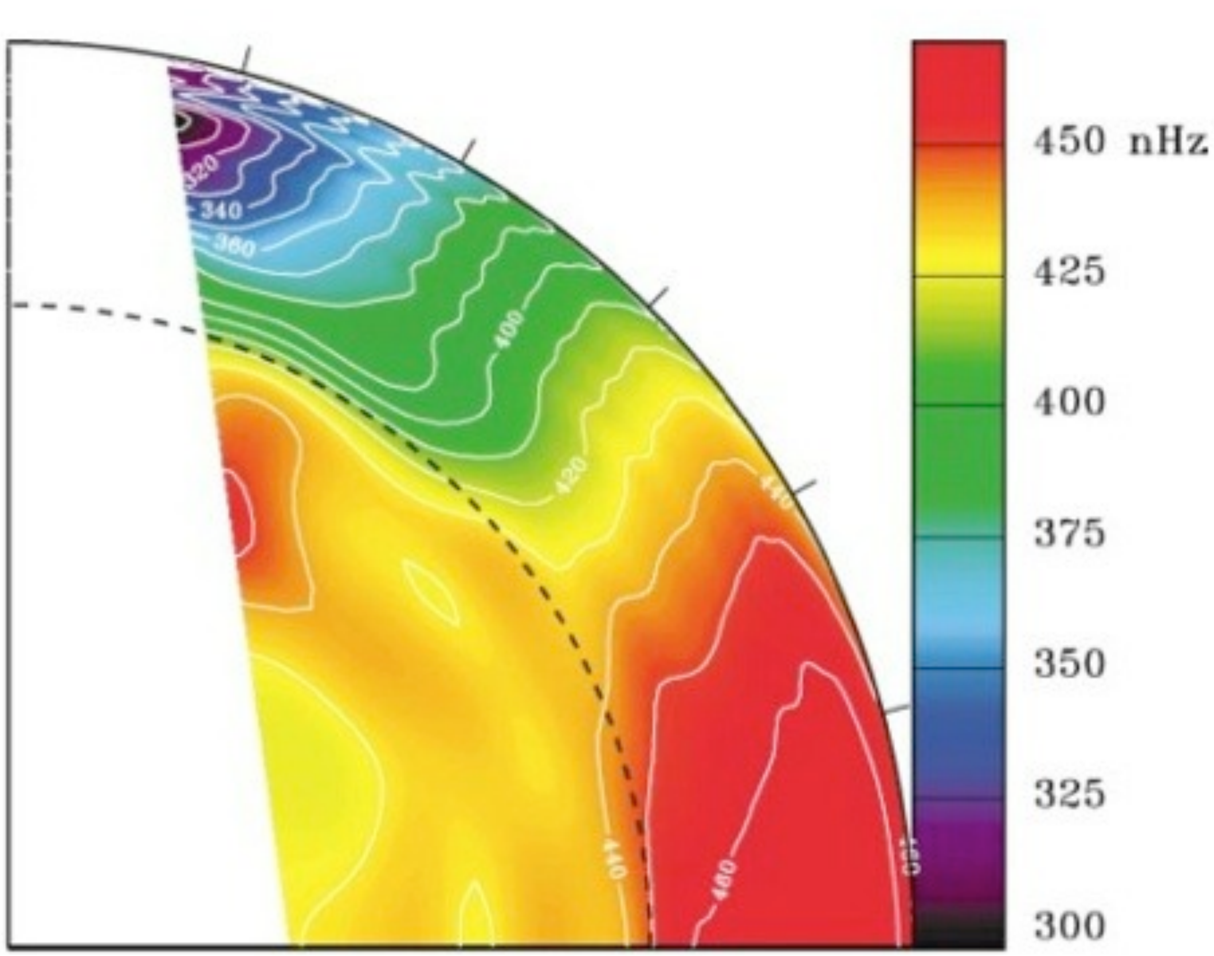
Reynolds equation: $$\rho \left[\frac{\partial \bar{\boldsymbol{u}}}{\partial t} + (\bar{\boldsymbol{u}} \cdot \nabla)\bar{\boldsymbol{u}} \right] = -\nabla \cdot \rho Q - \nabla P + \rho \boldsymbol{g}$$

Heat transport equation: $$\nabla \cdot (\boldsymbol{F}^{\mathrm{conv}} + \boldsymbol{F}^{\mathrm{rad}}) + \rho T \bar{\boldsymbol{u}} \cdot \nabla \bar{s} = 0$$

Reynolds stress: $$\begin{aligned} T_{ij} &= -\rho Q_{ij} \\ Q_{ij} &= \langle u_i' u_j' \rangle = -\mathcal{N}_{ijkl} \frac{\partial \bar{u}_k}{\partial x_l} + \Lambda_{ijk} \Omega_k \end{aligned}$$

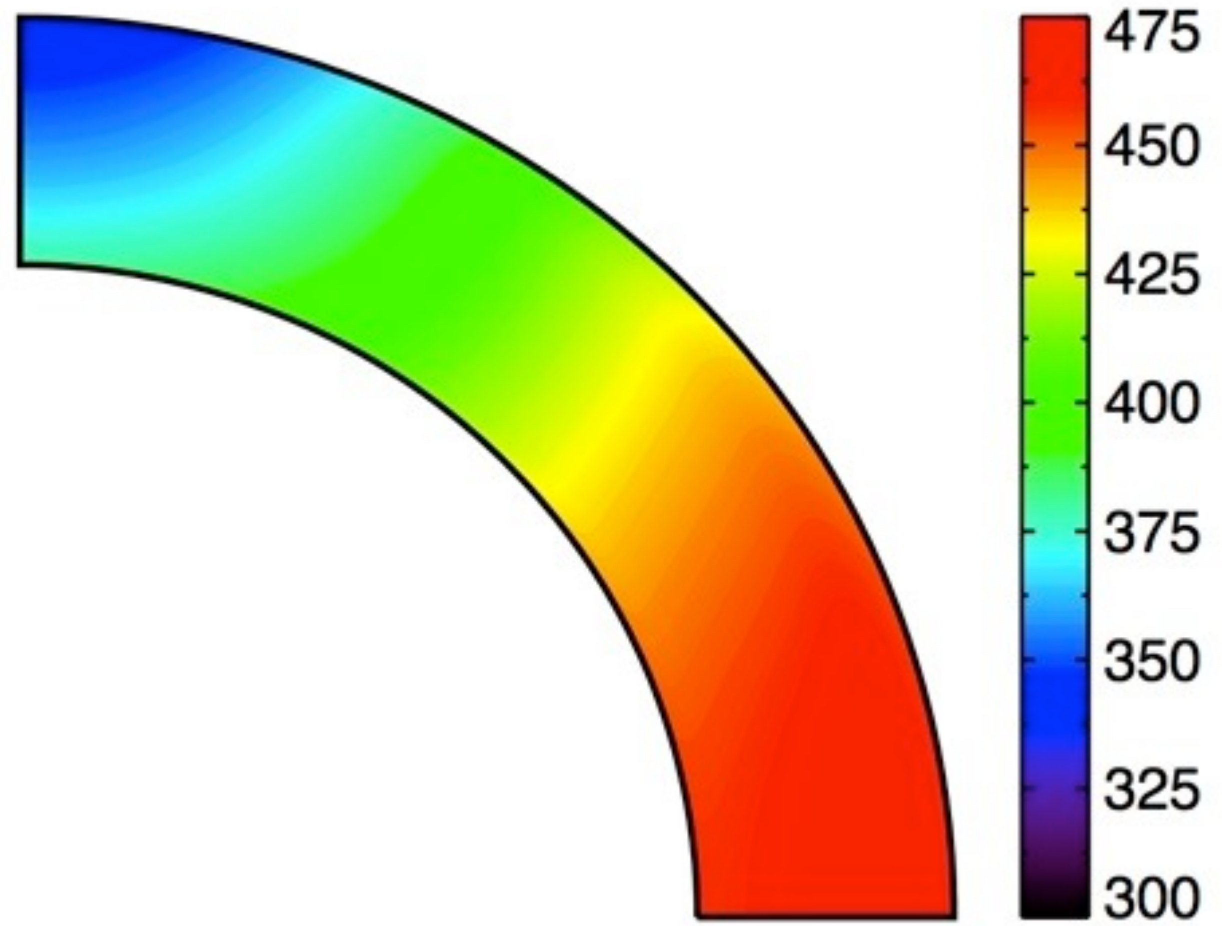
Convective heat flux: $$F_i^{\mathrm{conv}} = \rho c_p \langle u_i' T' \rangle = -\rho T \chi_{ij} \nabla_j \bar{s}$$

Solar differential rotation: observed vs. model



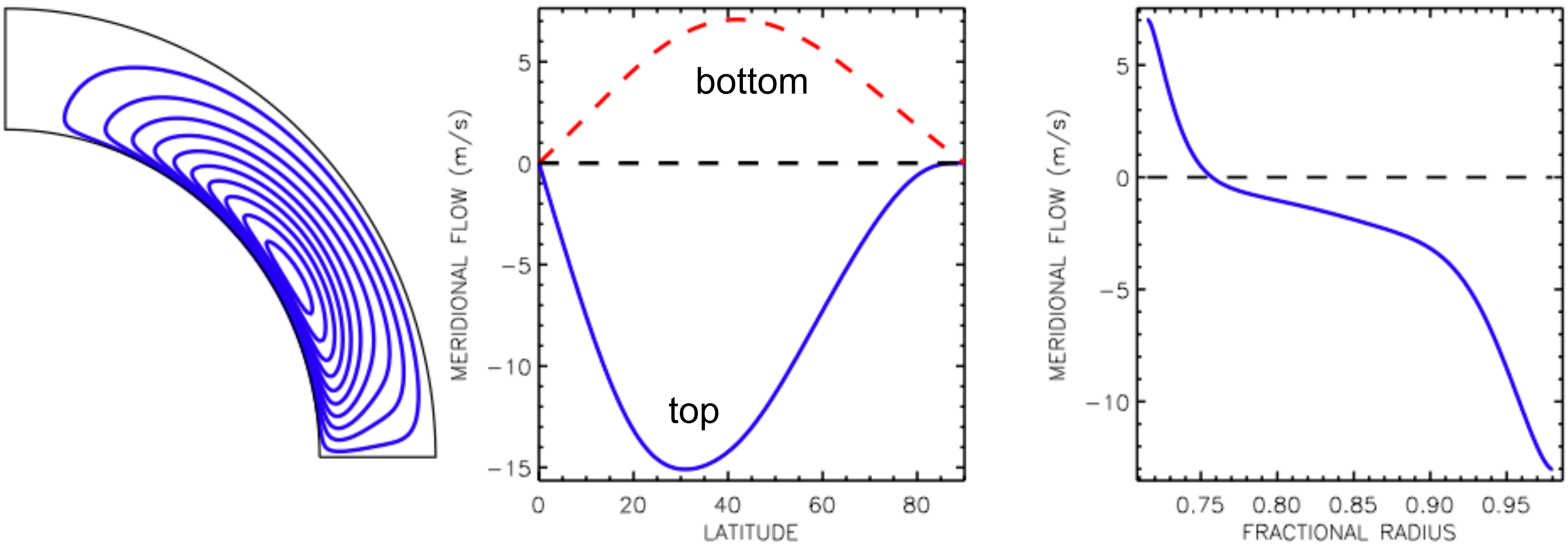
SOHO/MDI

Thompson et al. (2003)

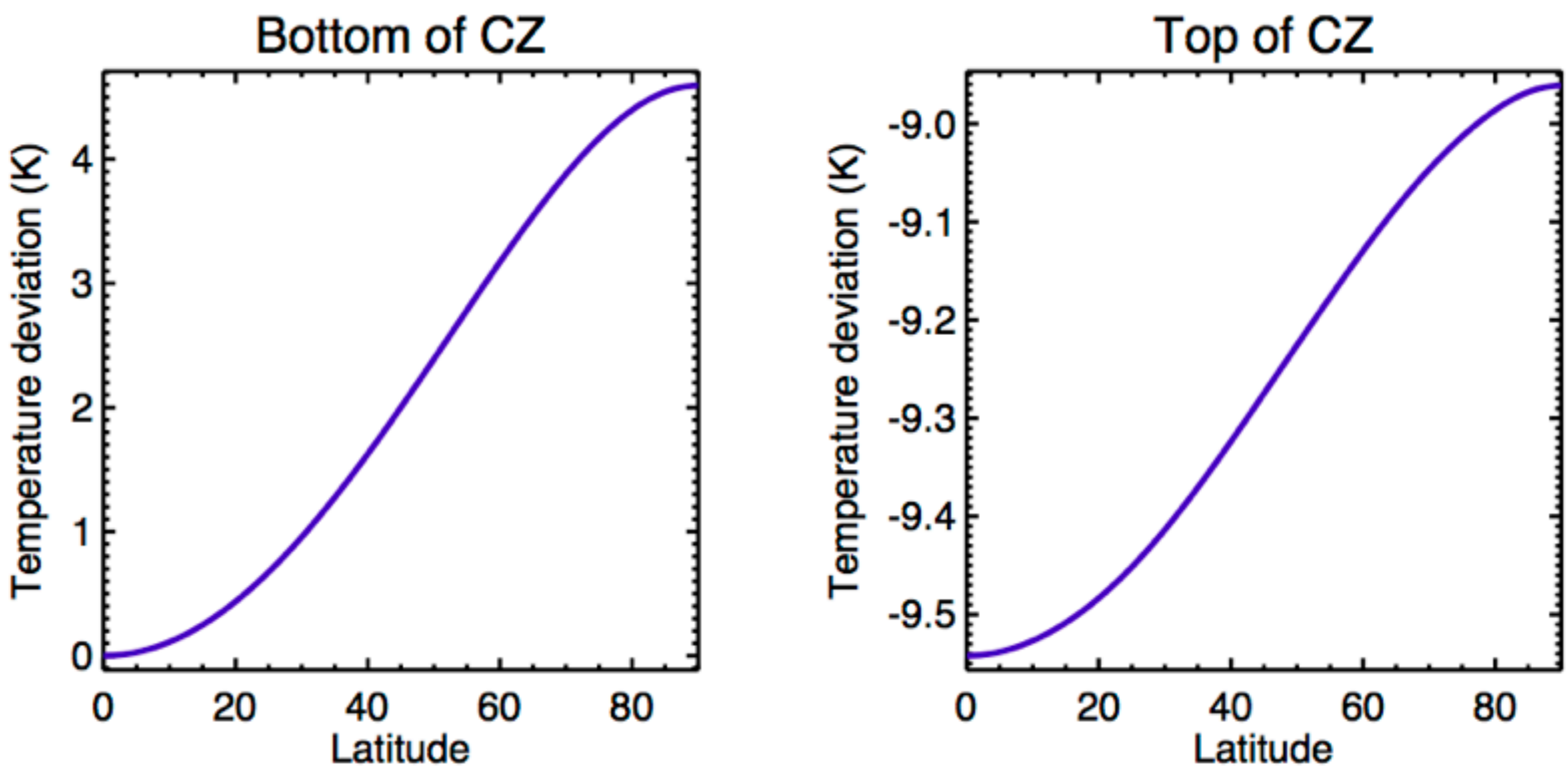


Model

Meridional flow: mean field model



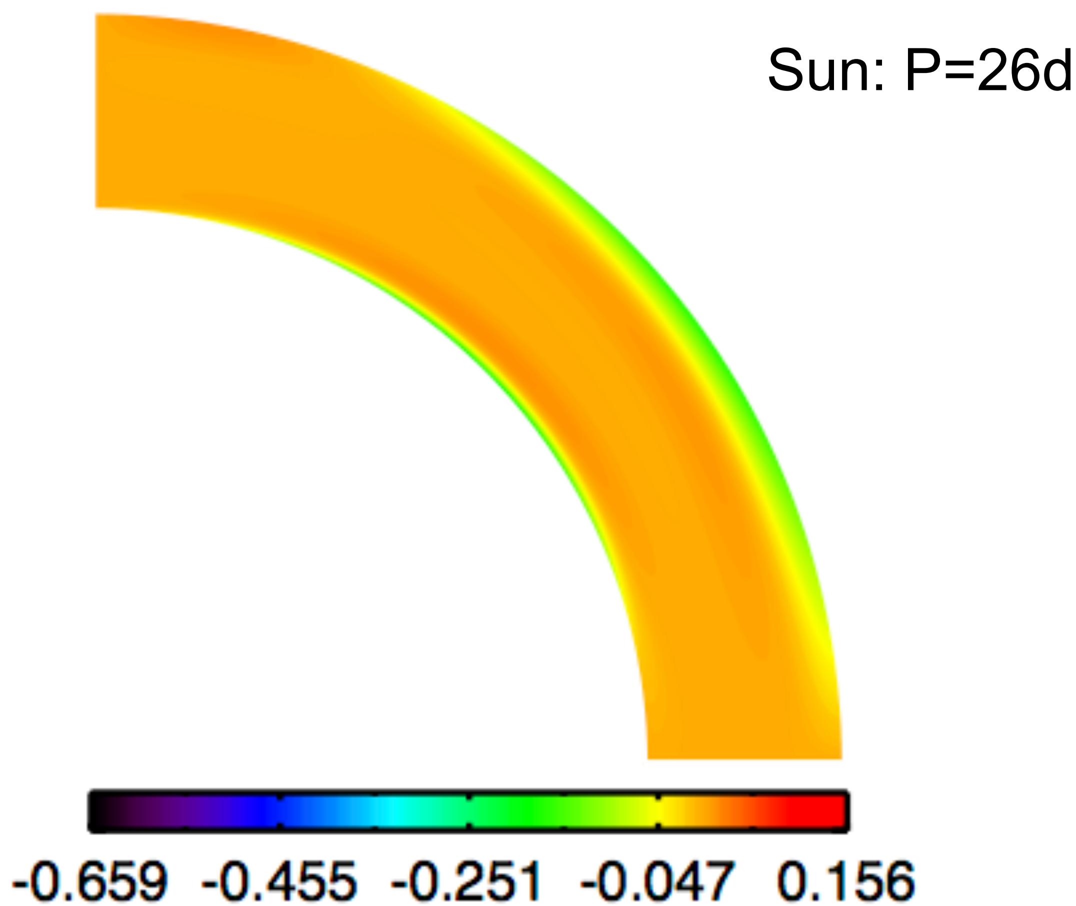
Deviation from adiabatic stratification

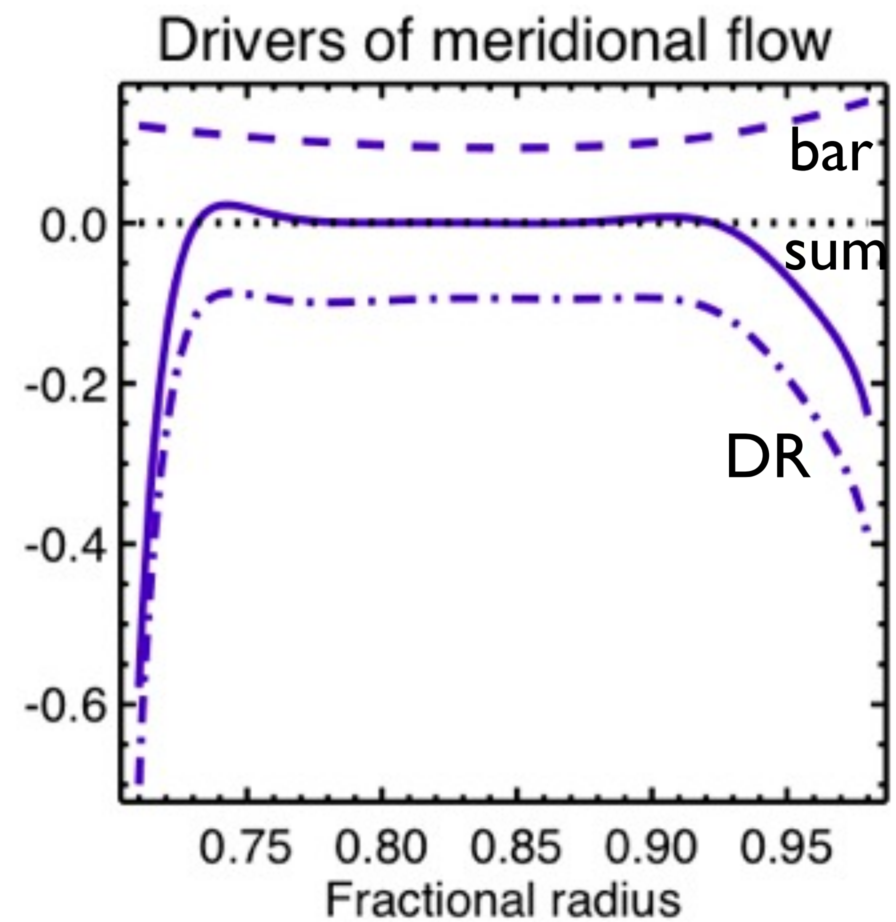


Thermal wind balance

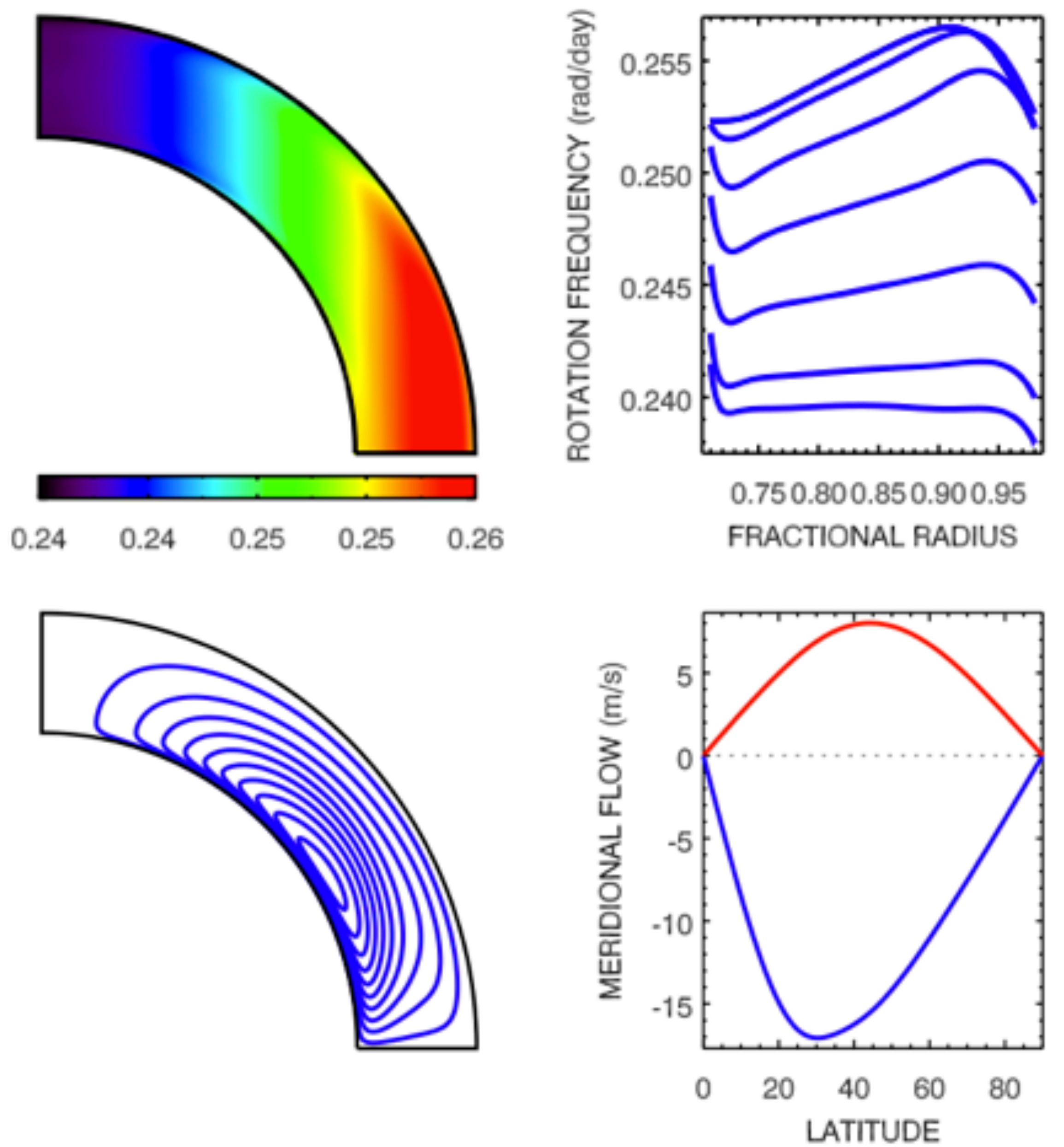
Equation of motion: $$-\left[\nabla \times \frac{1}{\rho}\nabla(\rho Q)\right]_\phi + r\sin\theta\frac{\partial\Omega^2}{\partial z} + \frac{1}{\rho^2}(\nabla\rho \times \nabla P)_\phi = 0$$

Fast rotation: $$r\sin\theta\frac{\partial\Omega^2}{\partial z} - \frac{g}{rc_p}\frac{\partial\bar{s}}{\partial\theta} \approx 0$$ not valid in boundary layers!

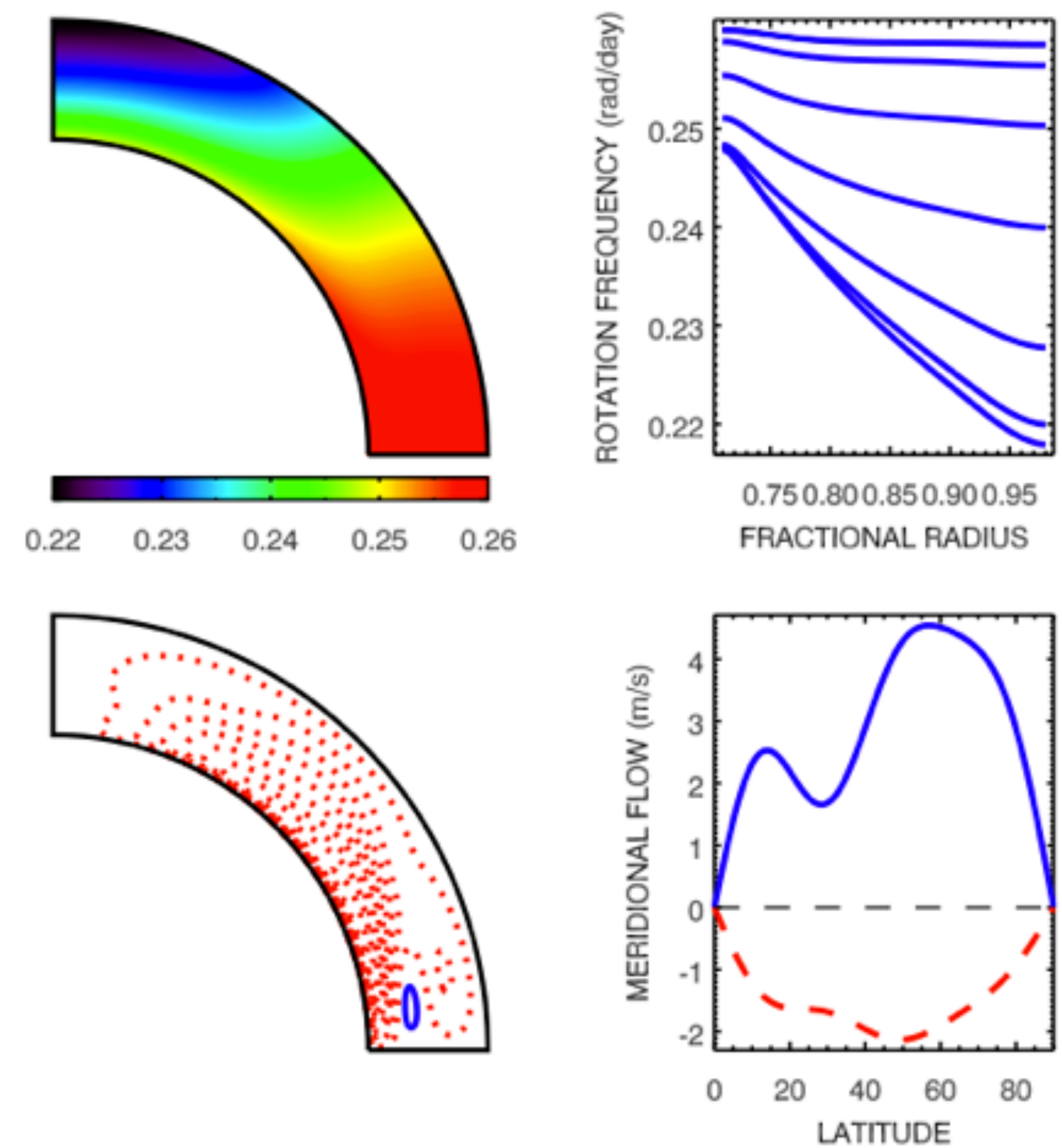




Reynolds stress vs. baroclinic flow

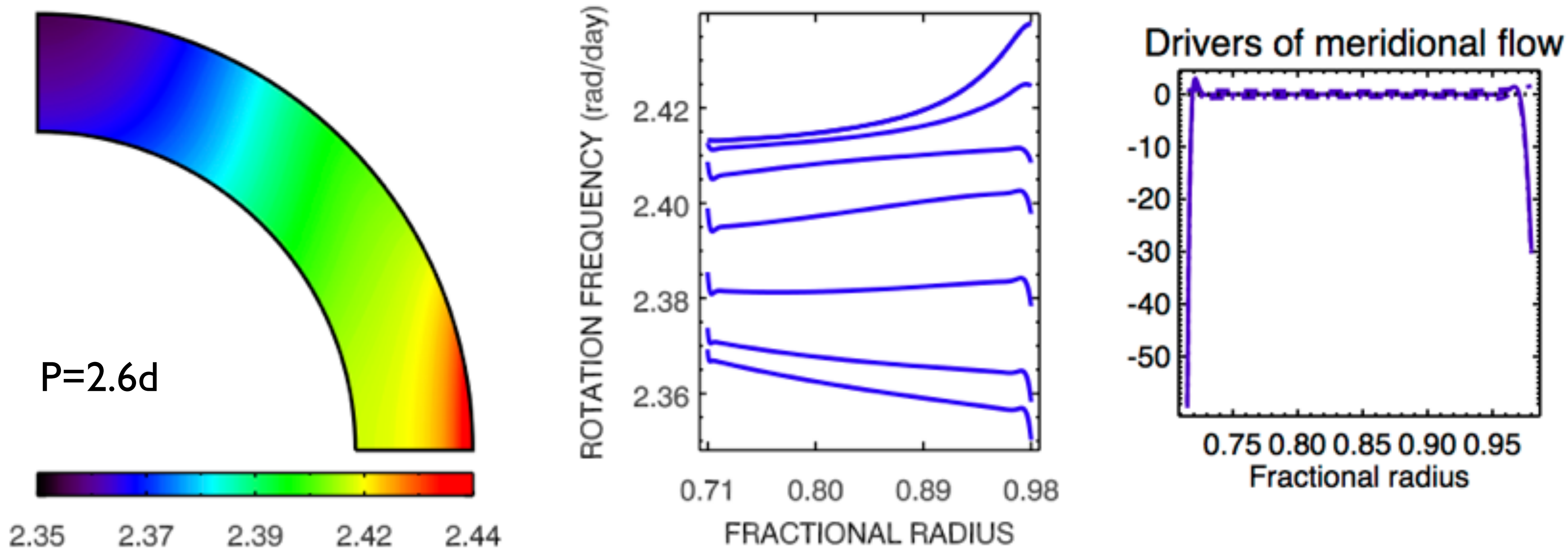


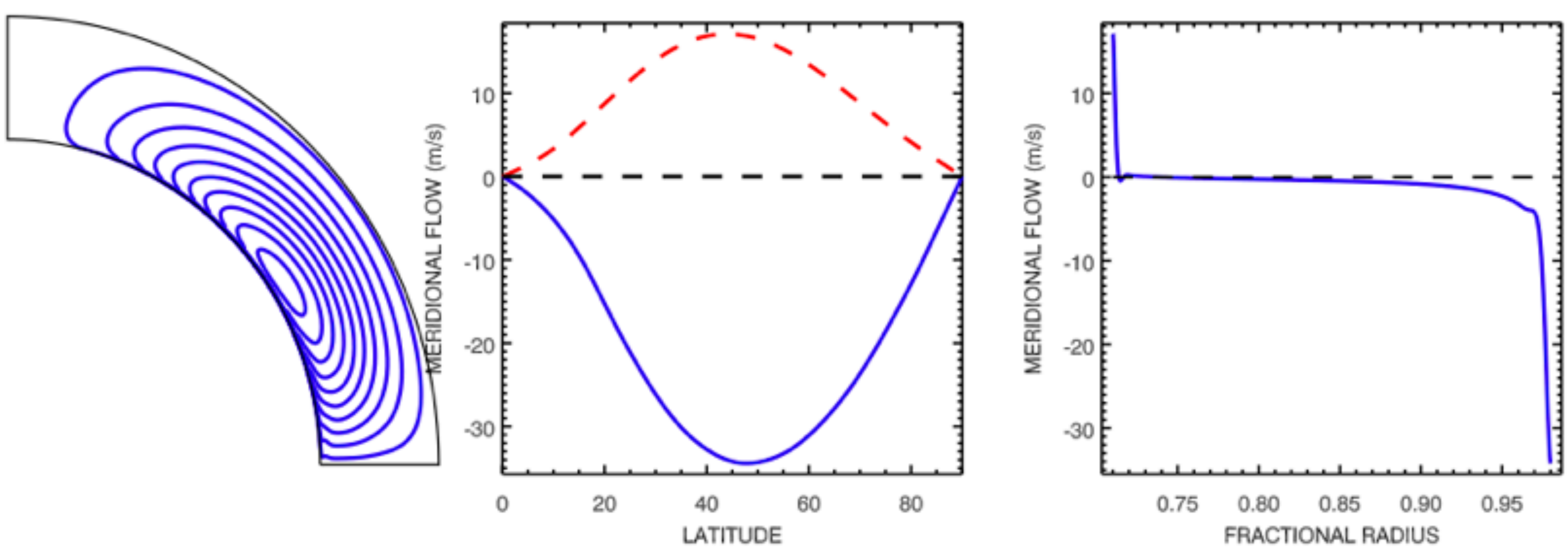
no baroclinic flow



no Λ effect

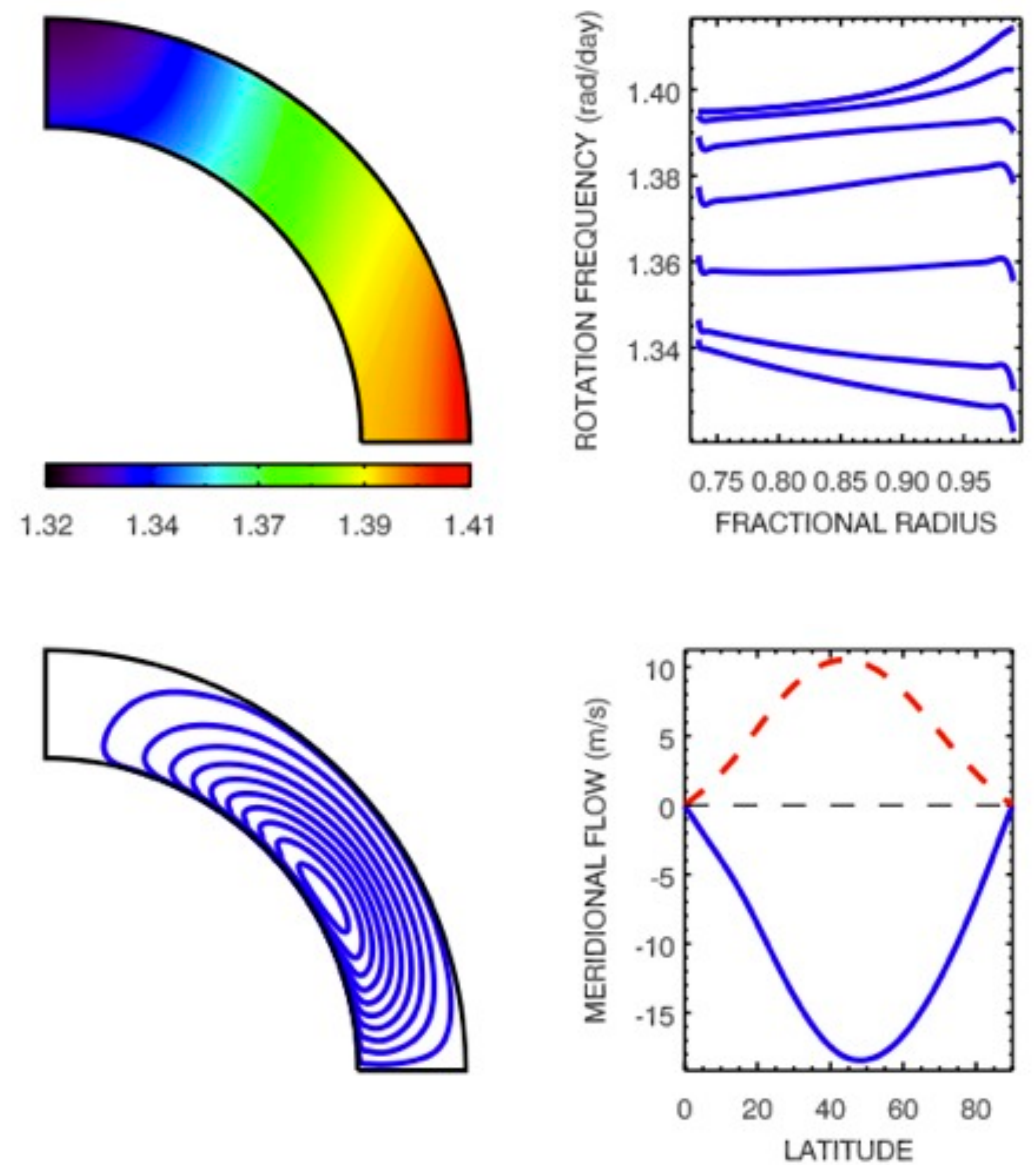
Rapidly rotating Sun



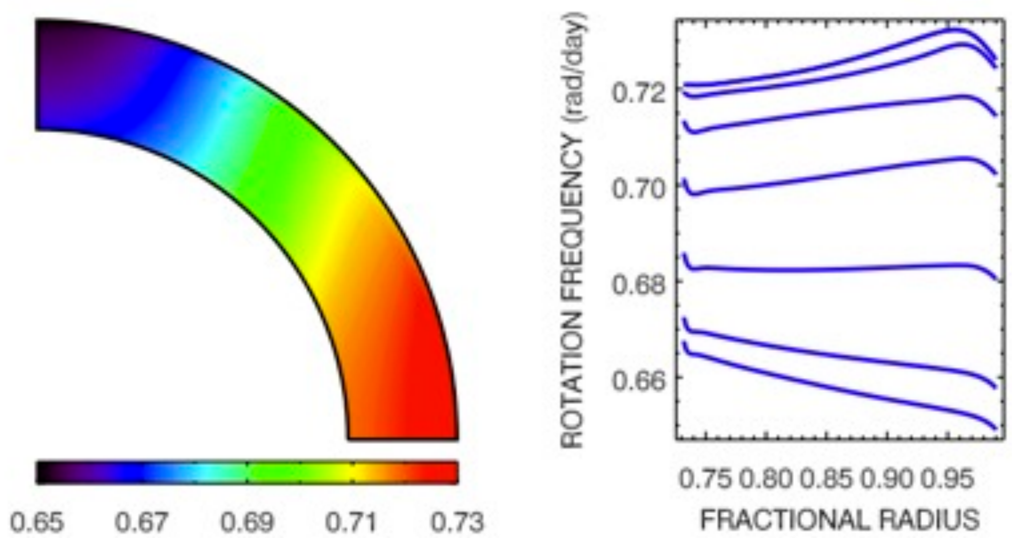


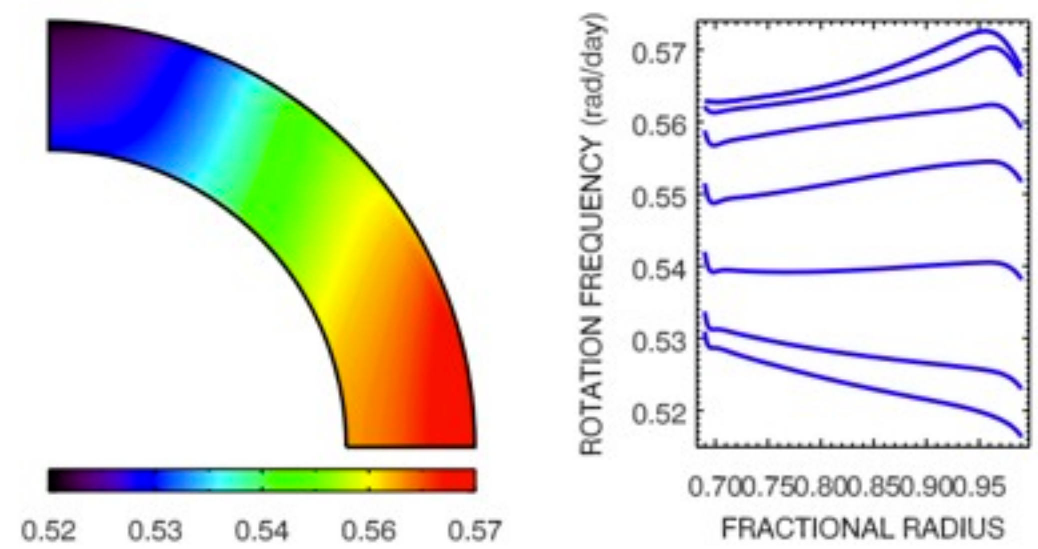
CoRoT-2a

- young solar twin (0.5 Gyr)
- rotation period: 4.5d
- spot modeling (Fröhlich et al. 2009):
  - solar-type DR
  - $\delta\Omega$=0.11 rad/day
- theory:
  - solar-type DR
  - $\delta\Omega$=0.09 rad/day
  - solar-type flow



MOST stars





κ1 Cet (G5 V)

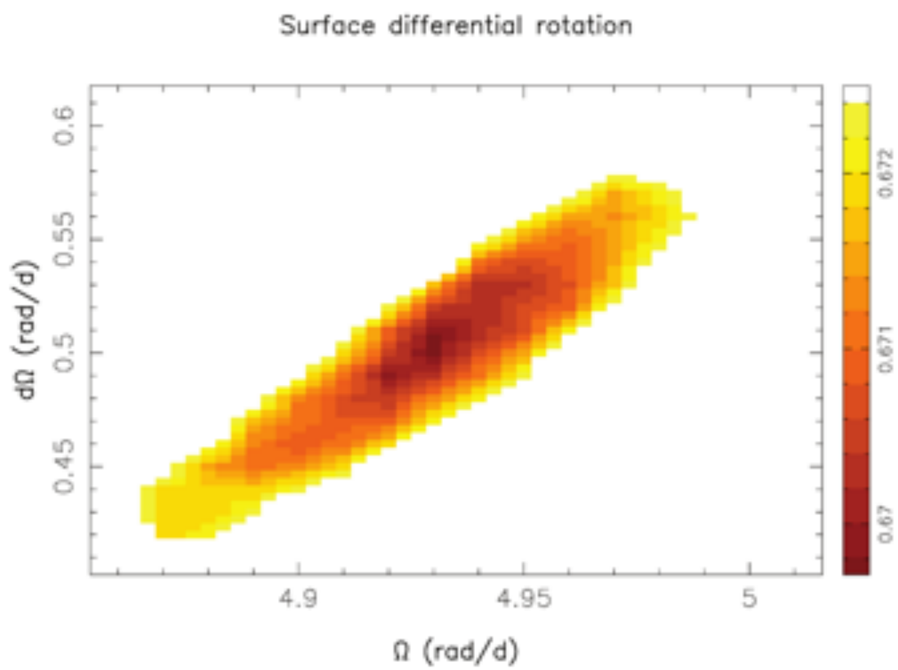
- P_{rot}=8.8d
- surface DR:
- photometry: δΩ=0.064 rad/day
- model: δΩ=0.077
- surface meridional flow speed: 19.3 m/s

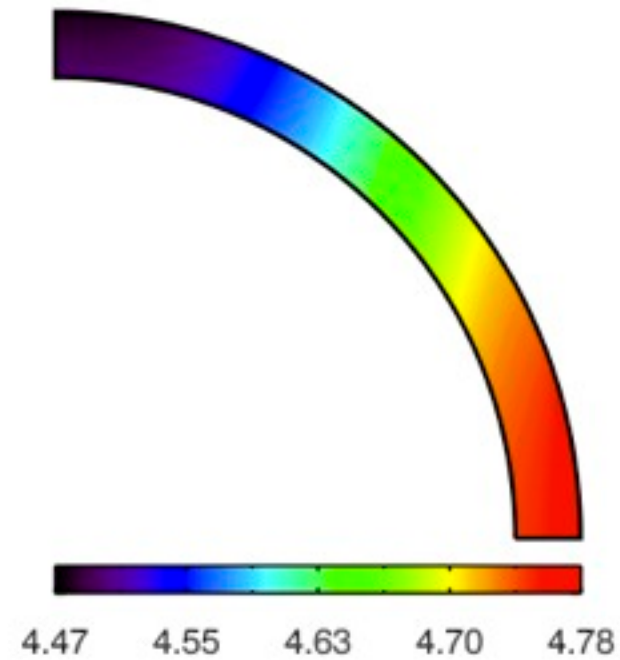
ε Eri (K2 V):

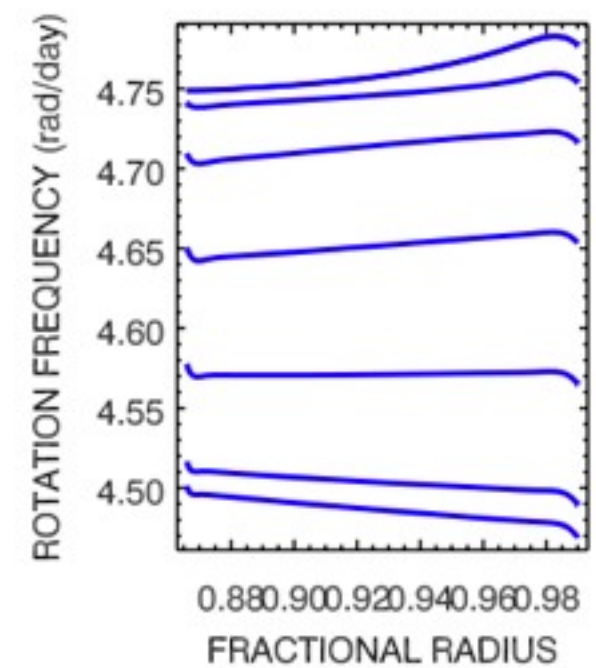
- P_{rot}=11.2d
- surface DR:
- photometry: δΩ=0.062 rad/day
- model: δΩ=0.051 rad/day
- surface meridional flow speed: 12.5 m/s

HD171488 (V899 Her)

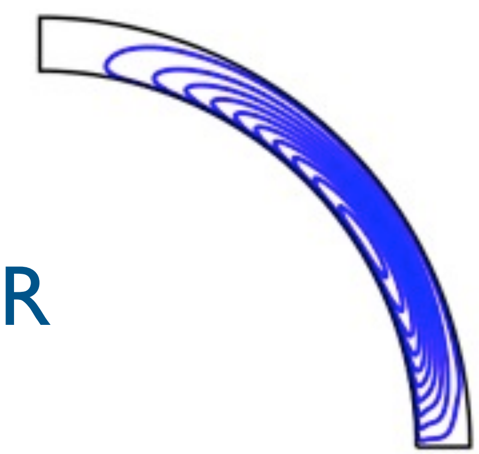
Jeffers & Donati 2008

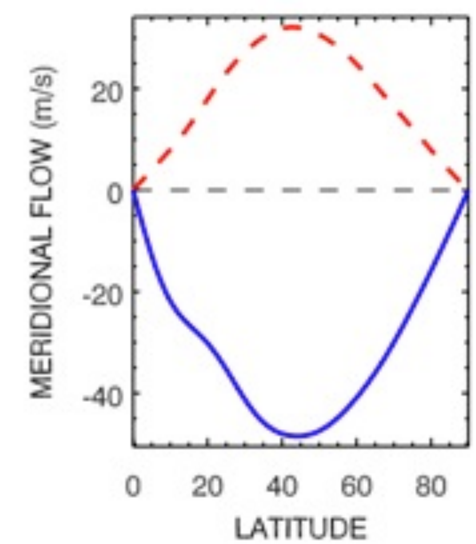






- fast-rotating young G dwarf
- 1.1 solar masses
- Doppler-Zeeman imaging: strong DR
- model: $\delta\Omega$=0.3 rad/day

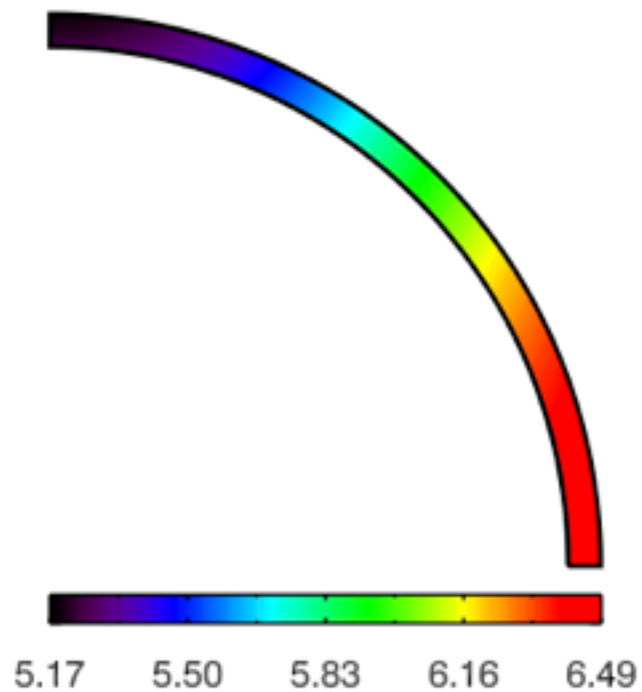


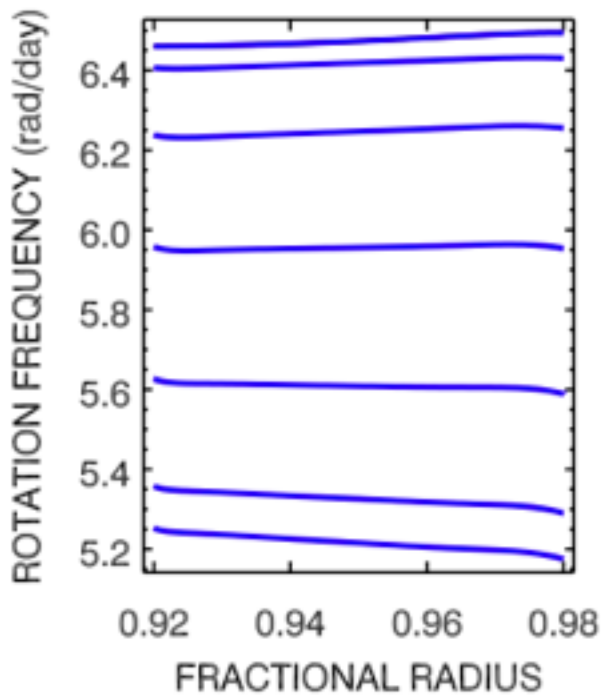


Shallow convection zone

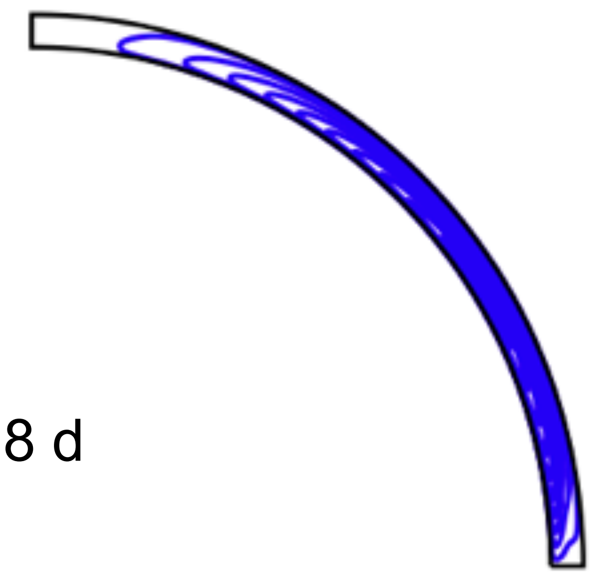
P=1d

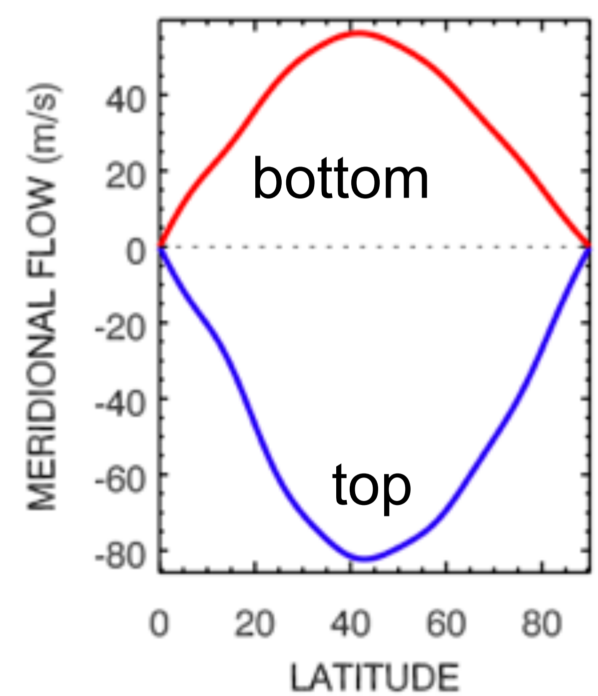
1.4Msol



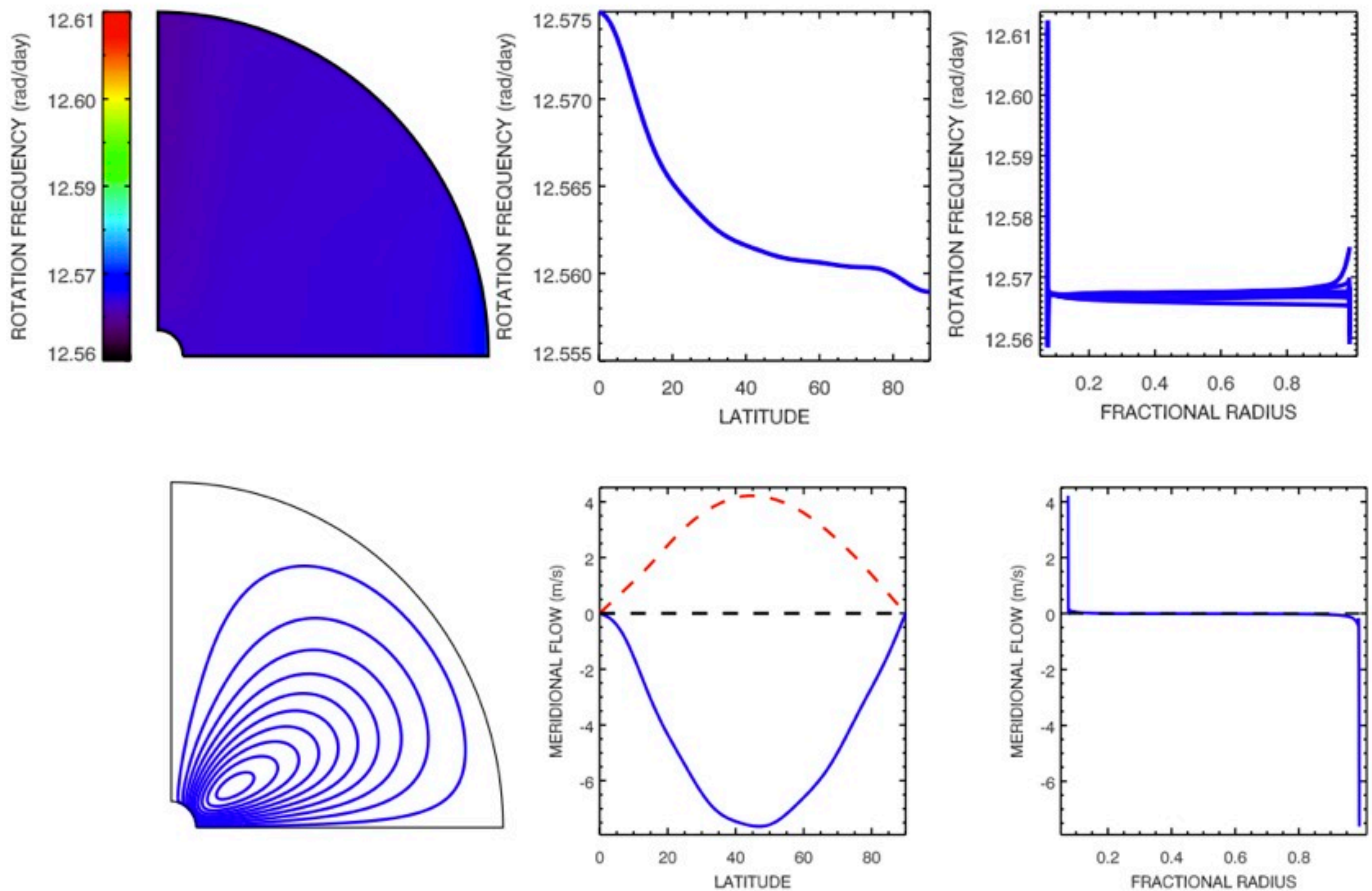


$\delta\Omega$=1.3 rad/d
lapping time=4.8 d



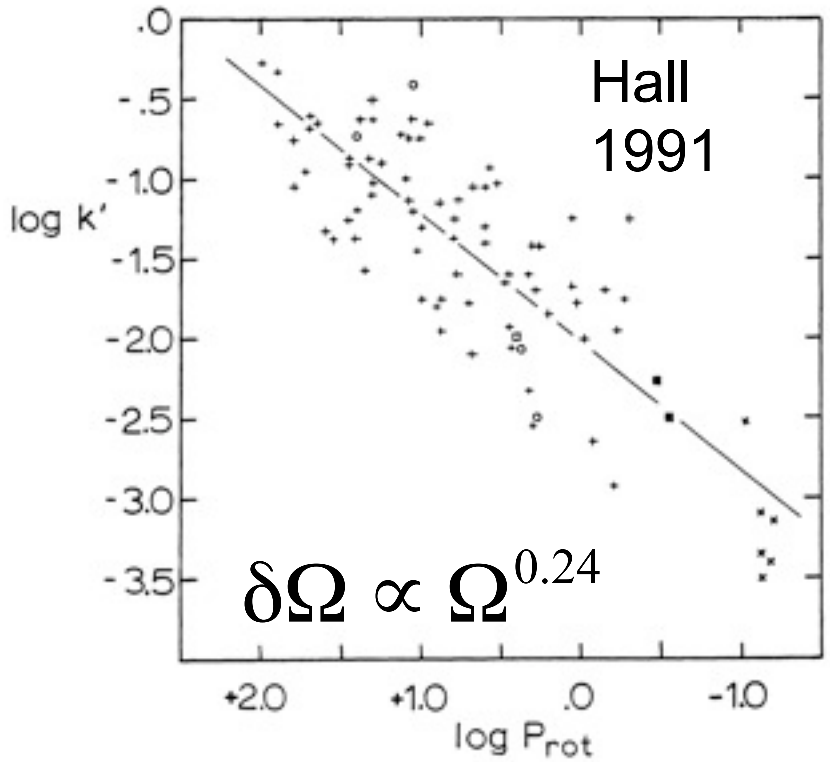


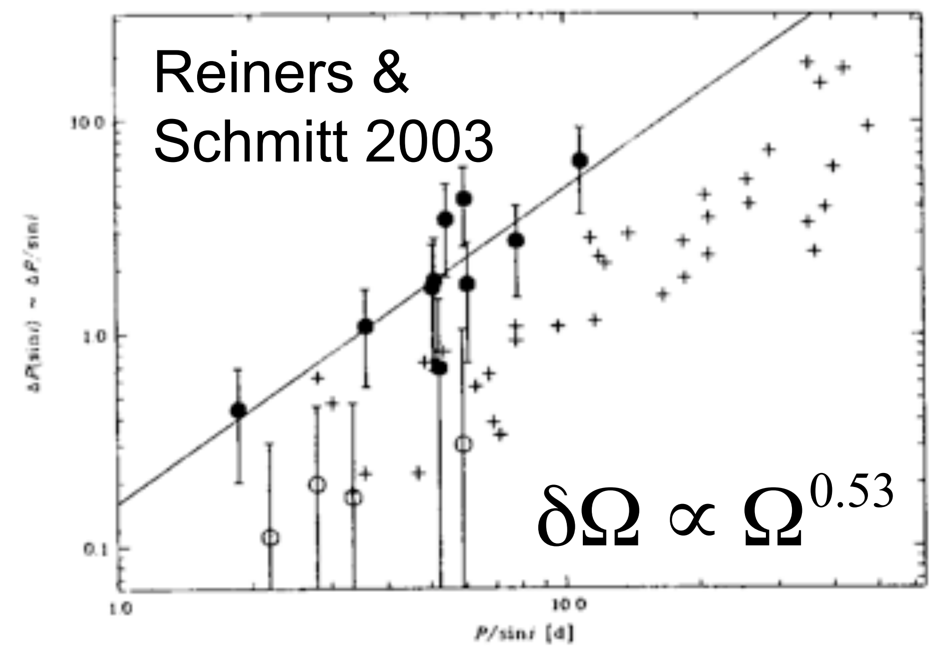
M dwarf (0.3 sol. masses)



P=0.5d

Stellar differential rotation





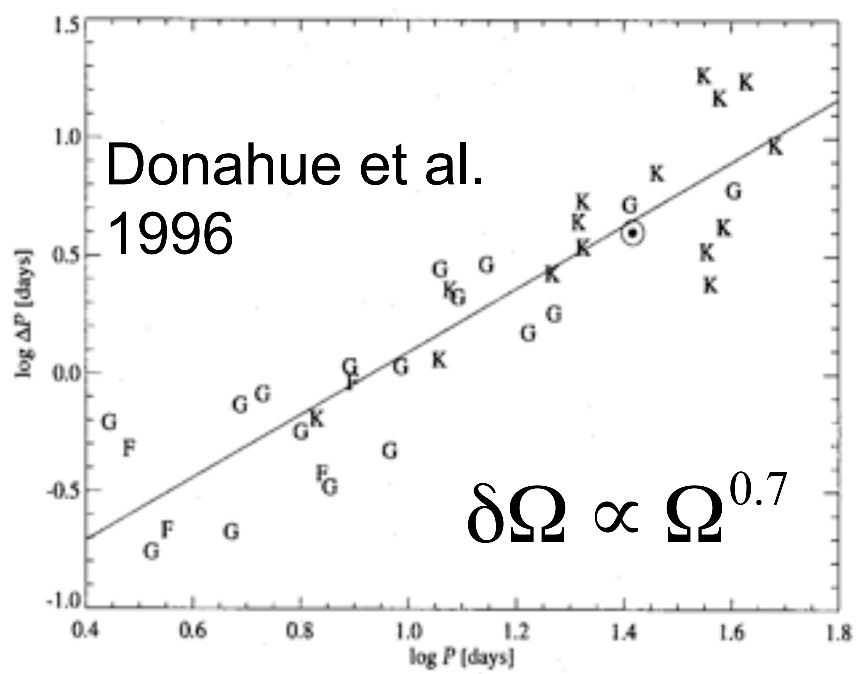
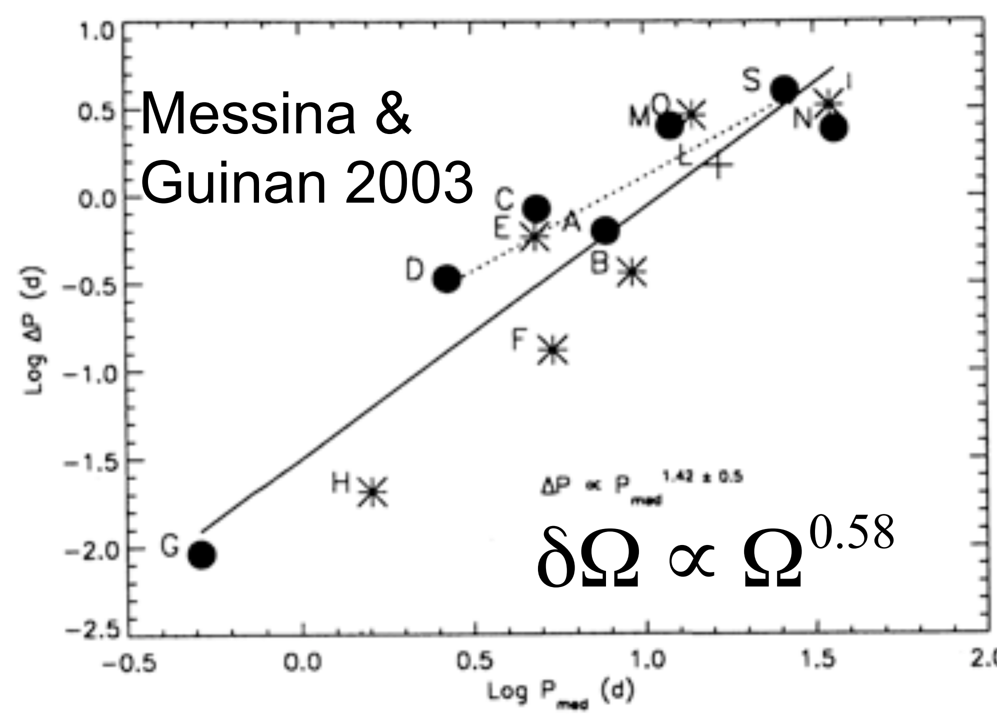
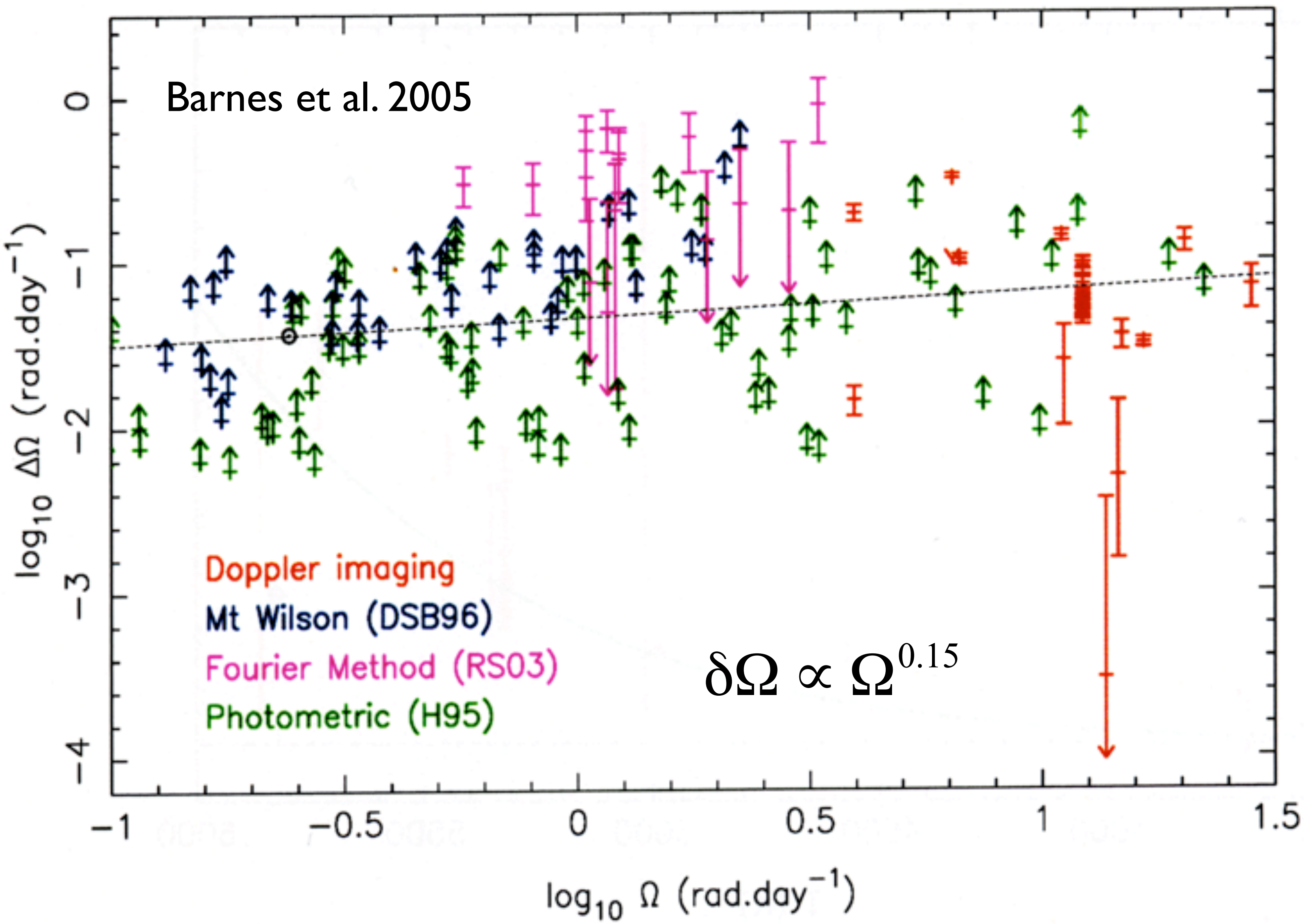


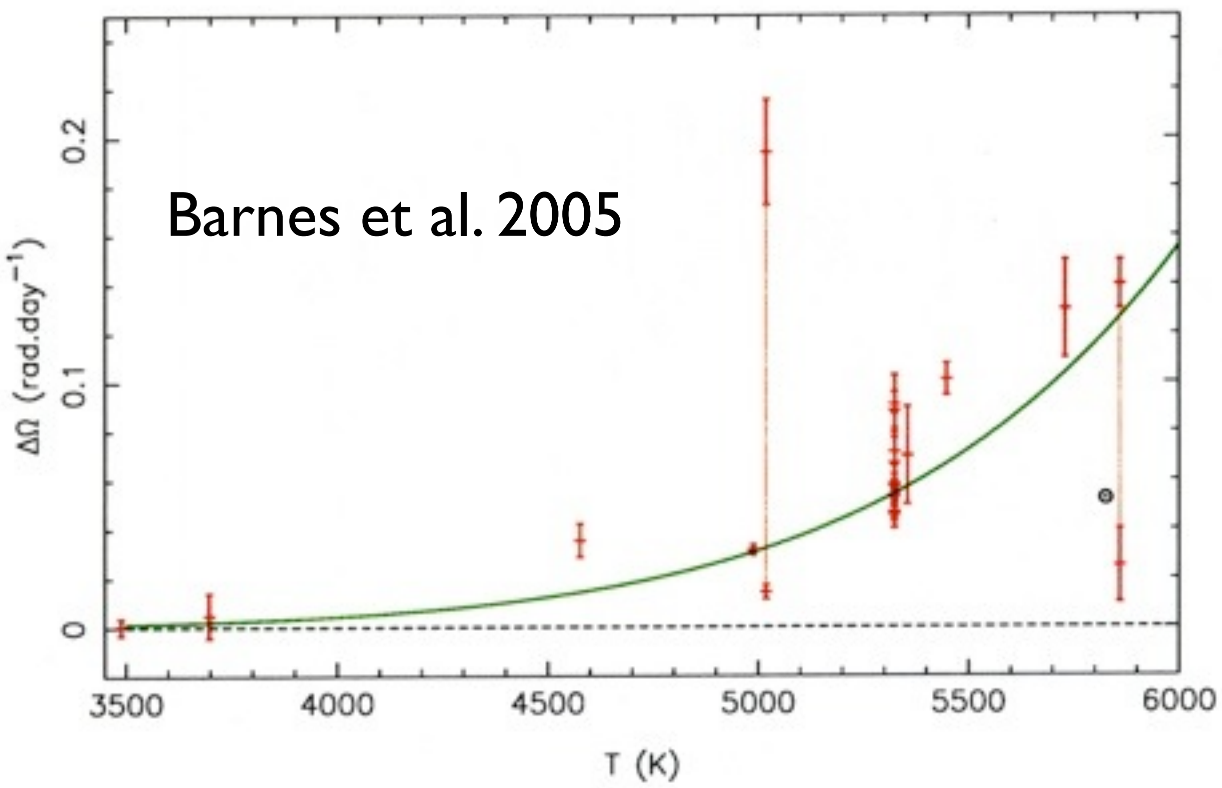
FIG. 3.—Range of observed rotation periods vs. log ⟨P⟩. Least-squares fit of these data yields $\Delta P \propto \langle P \rangle^{1.3\pm0.1}$ (correlation coefficient $r = 0.90$).

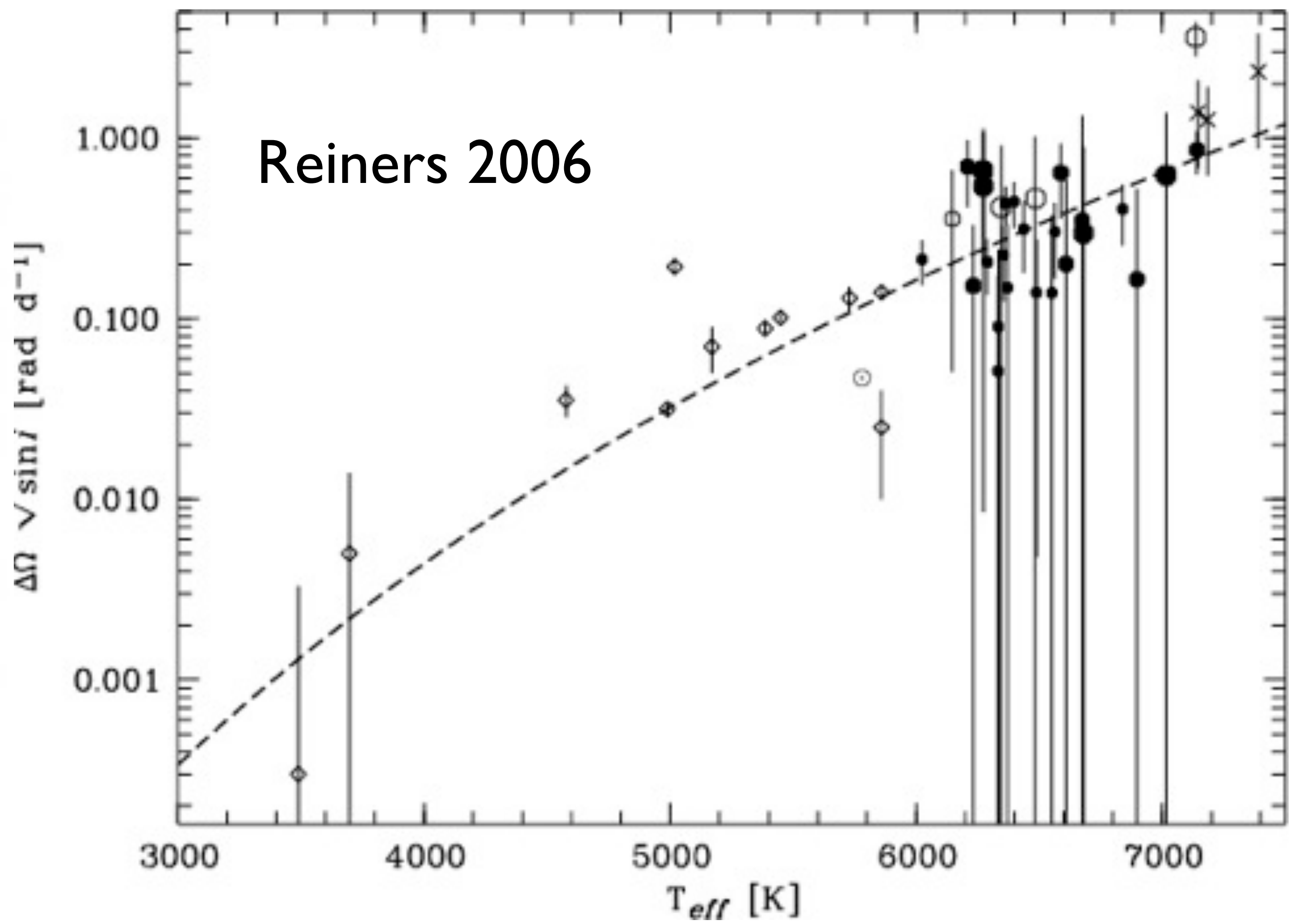


Stellar Differential Rotation: Dependence on rotation?



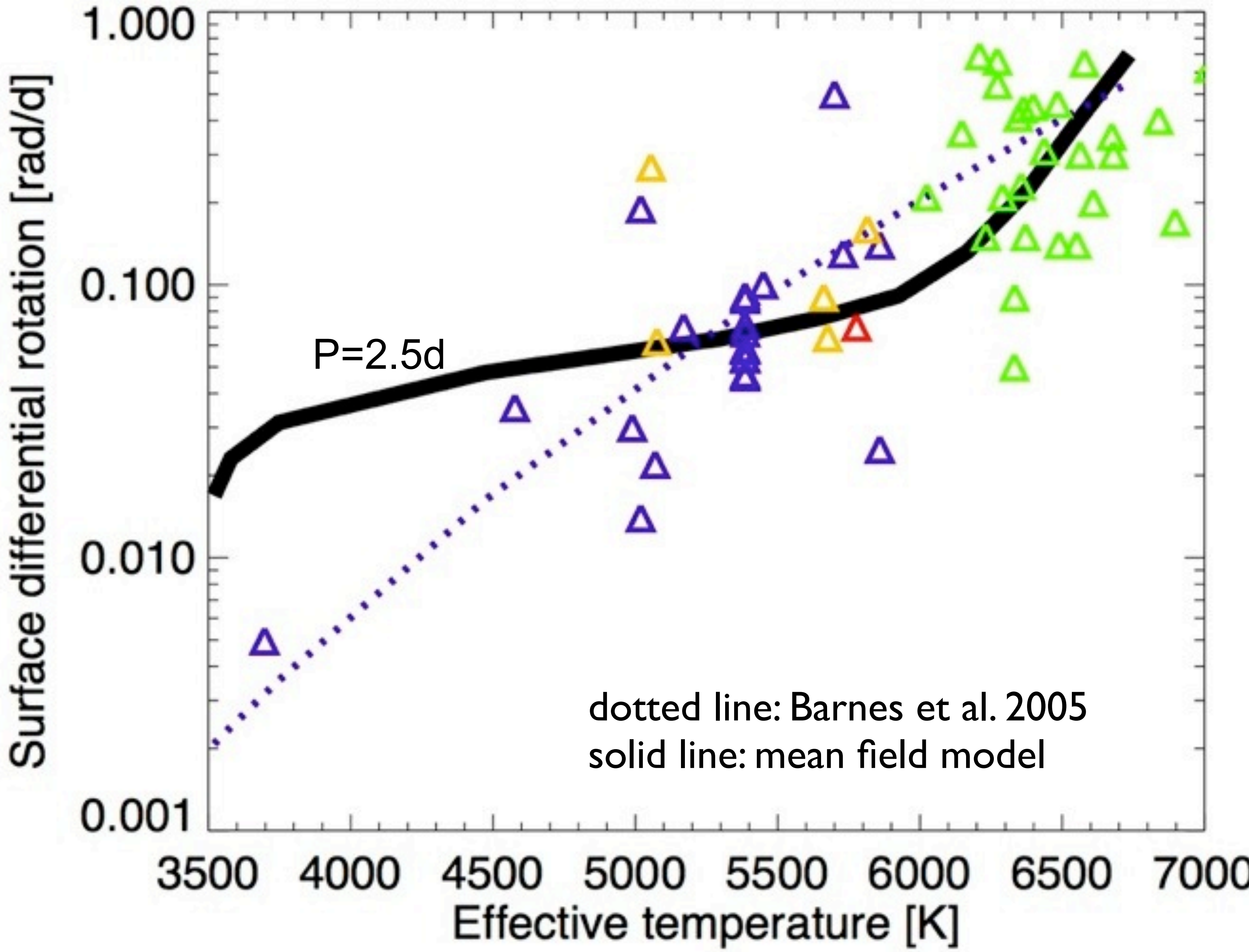
Stellar Differential Rotation: dependence on temperature





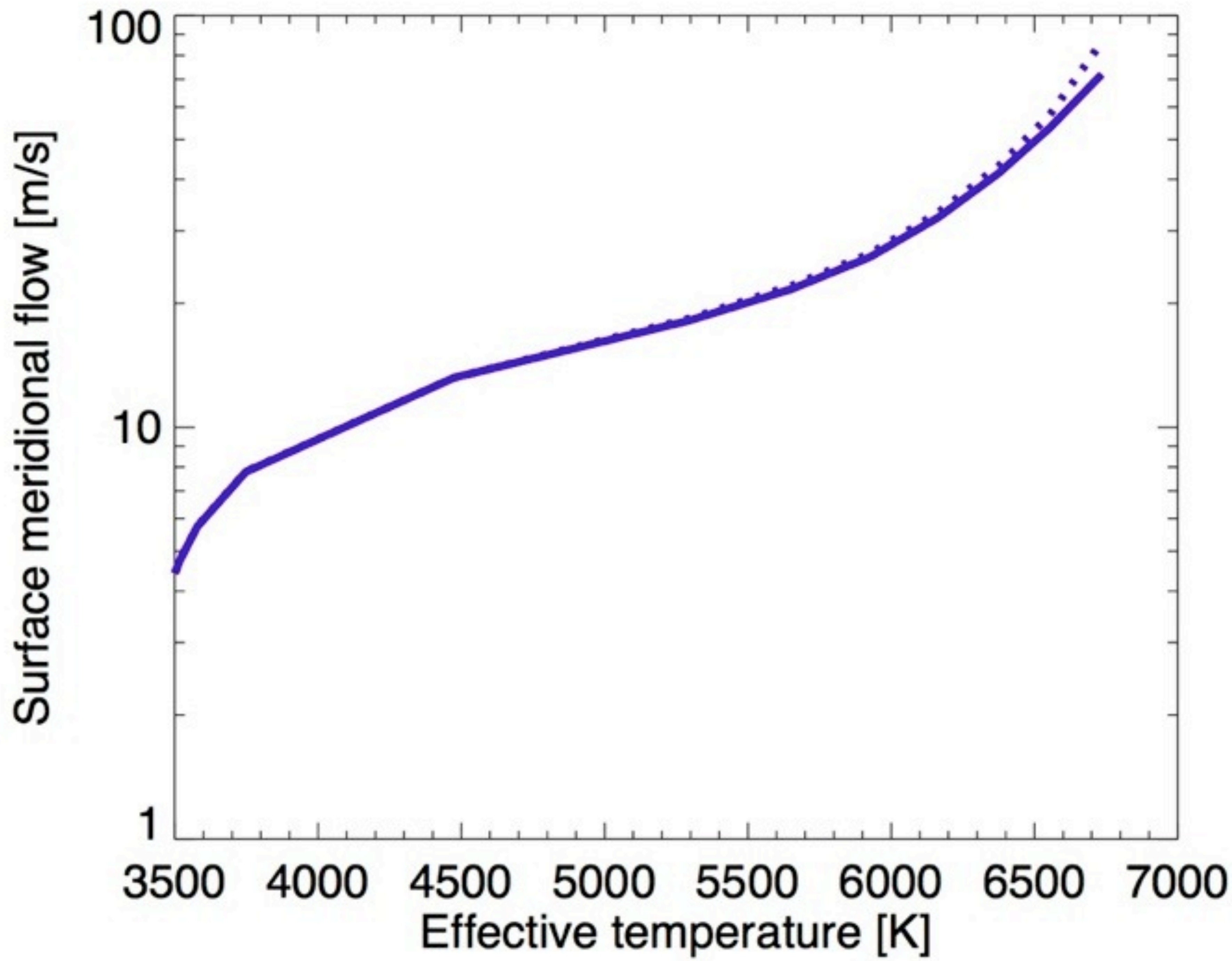
$$\delta\Omega \propto T^{8.9}$$

DR: dependence on temperature



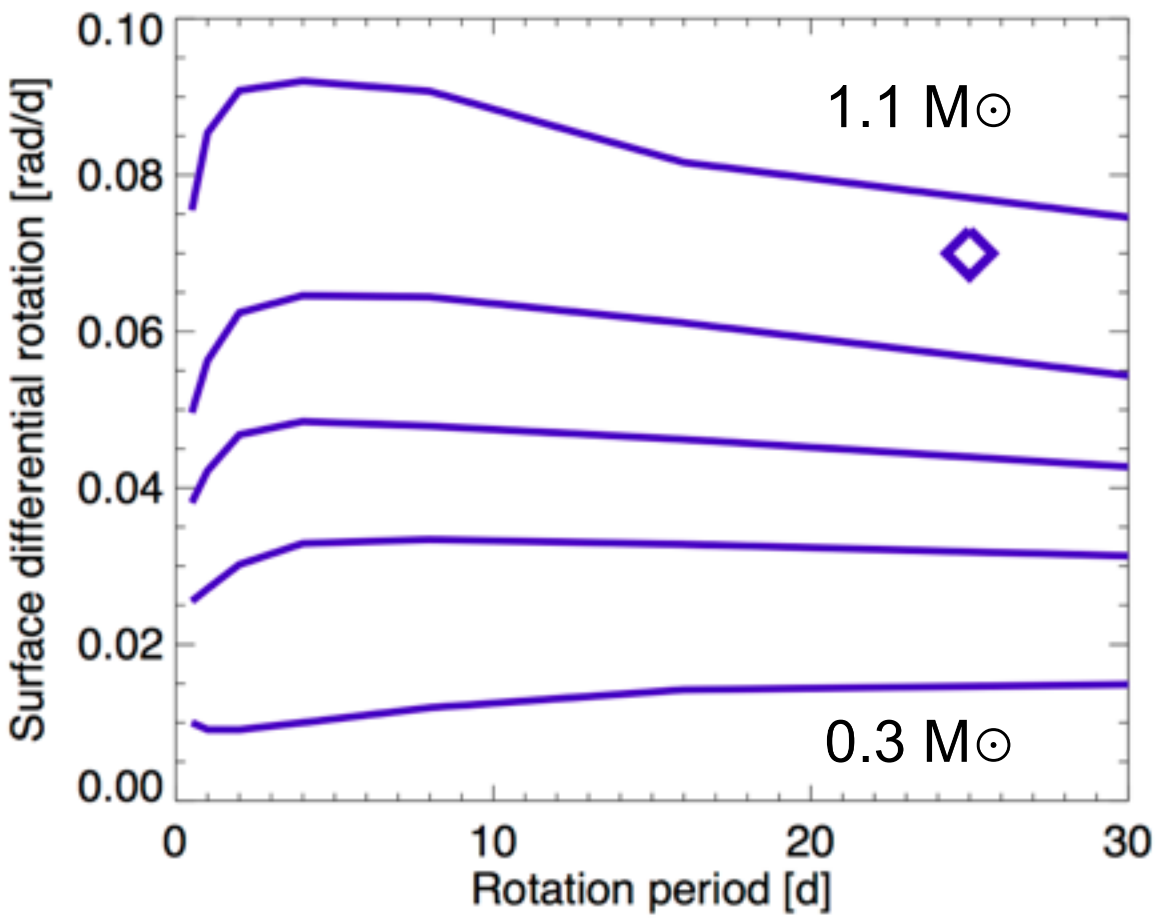
red: Sun
yellow: photometry (MOST, CoRoT, Kepler)
blue: Doppler imaging
green: Reiners 2006

Meridional flow: temperature dependence

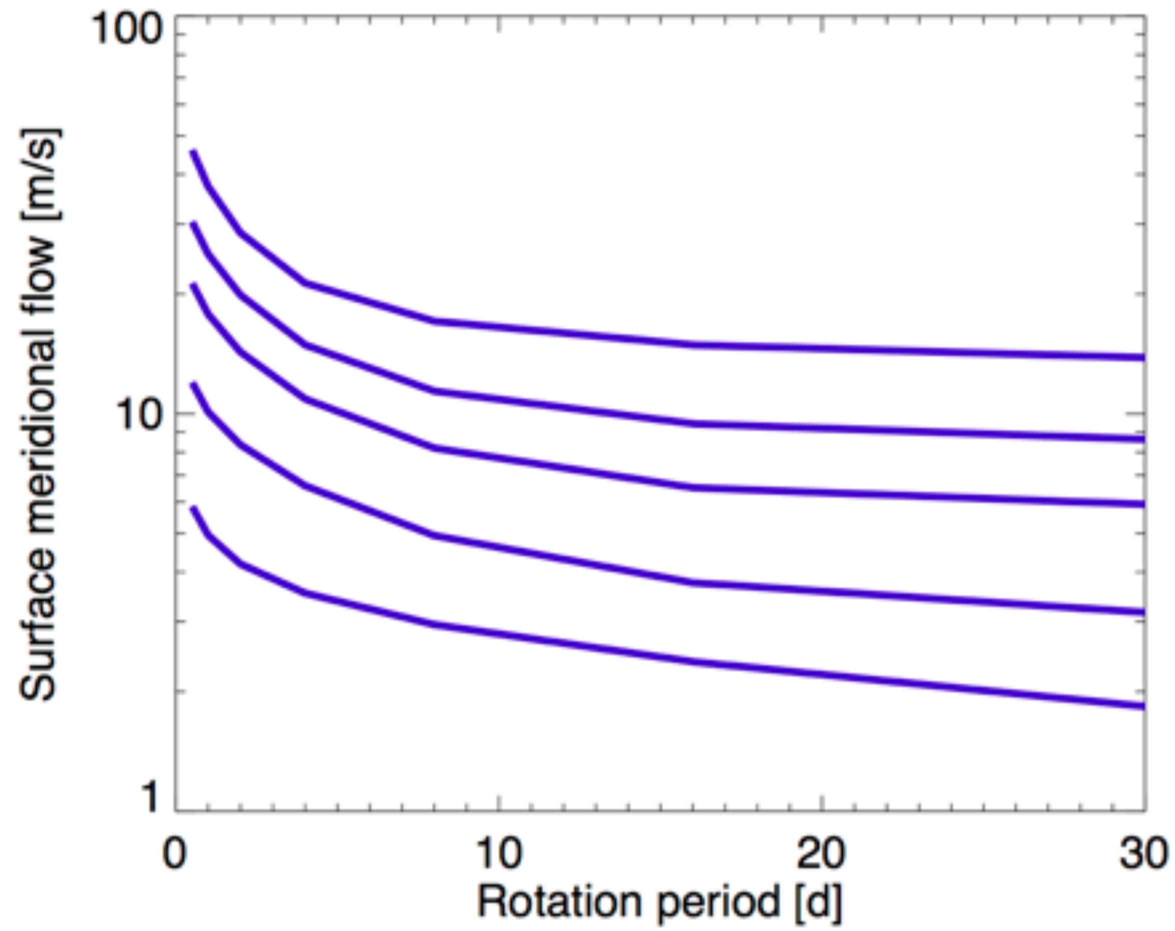


Dependence on rotation period

Differential rotation

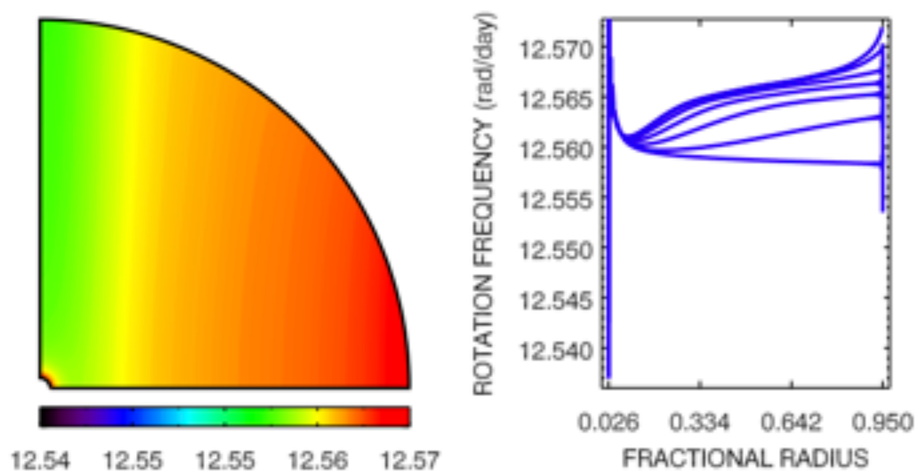


Meridional flow

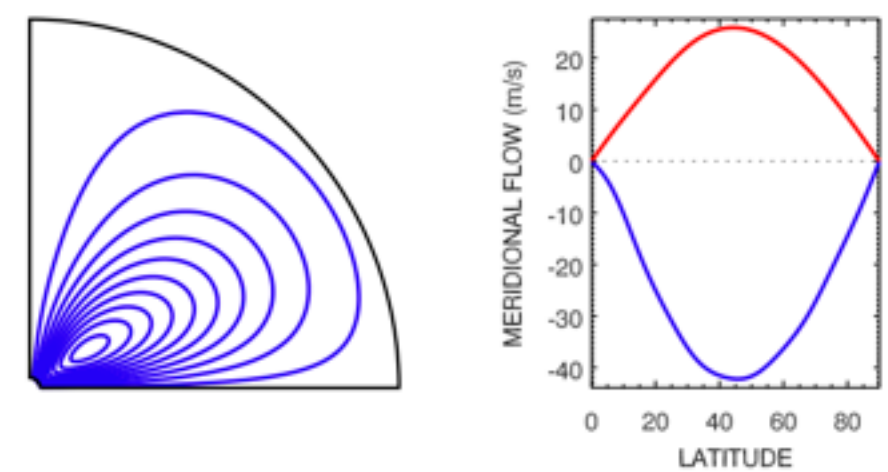


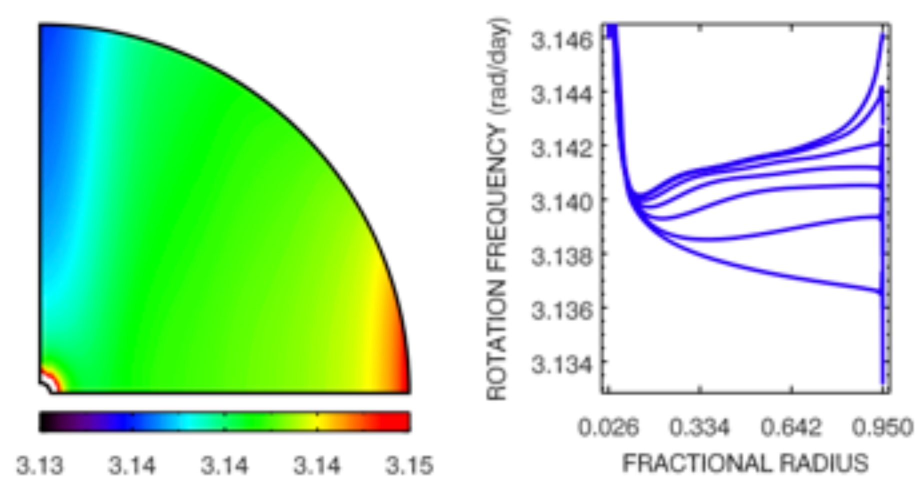
T Tauri star

1 Myr
2.33 R☉
1.94 L☉

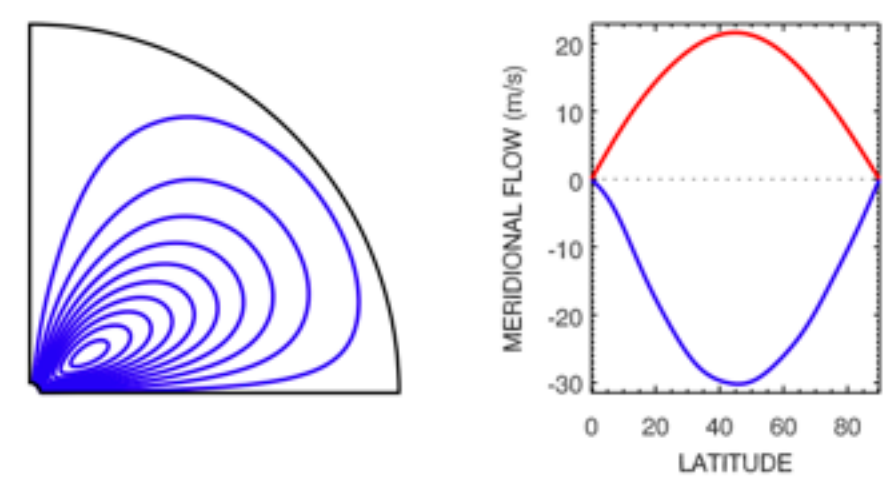


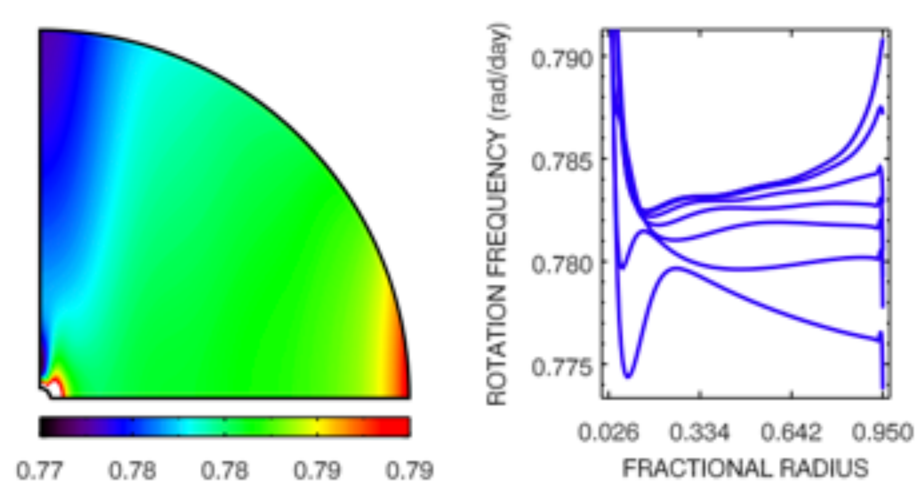
P=0.5d



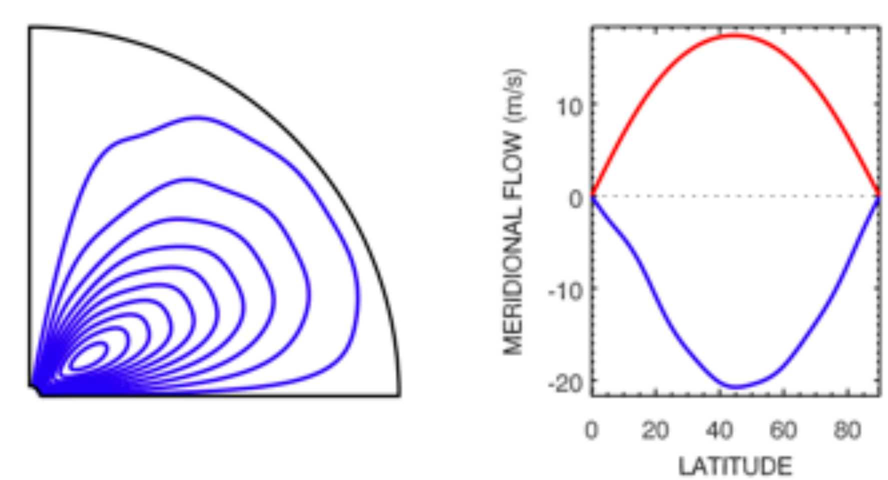


P=2d



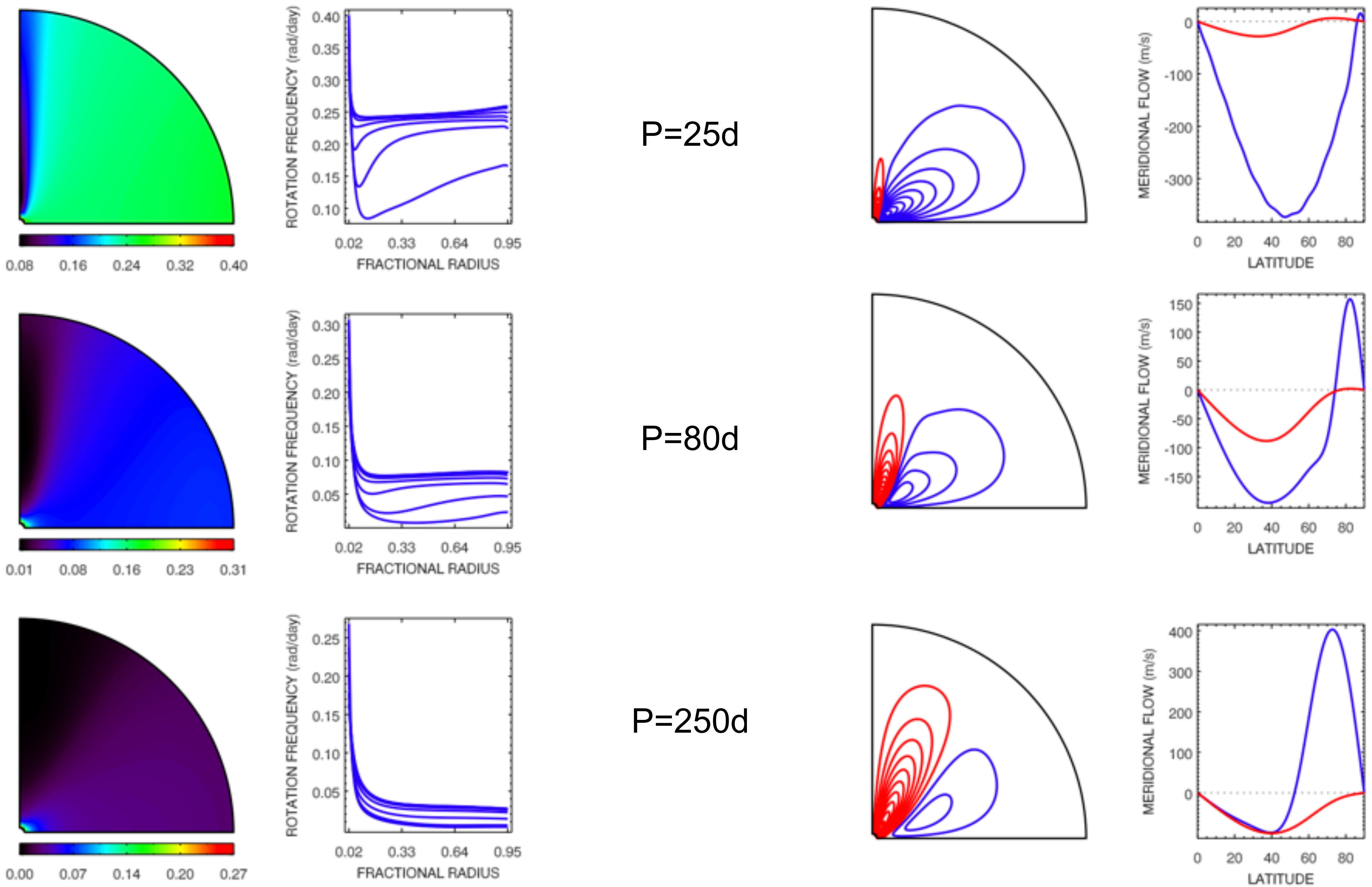


P=4d



Giant star

11.78 Gyr
37.8 $R_\odot$
386 $L_\odot$



Conclusions

- mean field model reproduces solar differential rotation and surface meridional flow
- always solar-type DR and meridional flow for MS stars
- surface DR and meridional flow of lower MS stars depend more on temperature than rotation period
- stagnation point of MF close to the bottom of CZ
- return flow not slow
- main sources of DR: Reynolds stress and baroclinic flow
- thermal wind balance does not hold near boundaries
- meridional flow in rapidly rotating convection zones is
  - driven by shear in boundary layers
  - concentrated in boundary layers
  - fast