Parameterising the WIMP speed distribution for direct (& indirect) detection experiments

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Work with **Bradley Kavanagh** and Mattia Fornasa.

- Why are uncertainties in f(v) important?
- Strategies: i) integrate out
  ii) marginalize over
- How to parameterise f(v)?

‘Model independent determination of the dark matter mass from direct detection experiments’, Kavanagh & Green, PRL, arXiv:1303.6868

‘Parameterizing the local dark matter speed distribution: a detailed analysis’, Kavanagh, PRD, arXiv:1312.1852

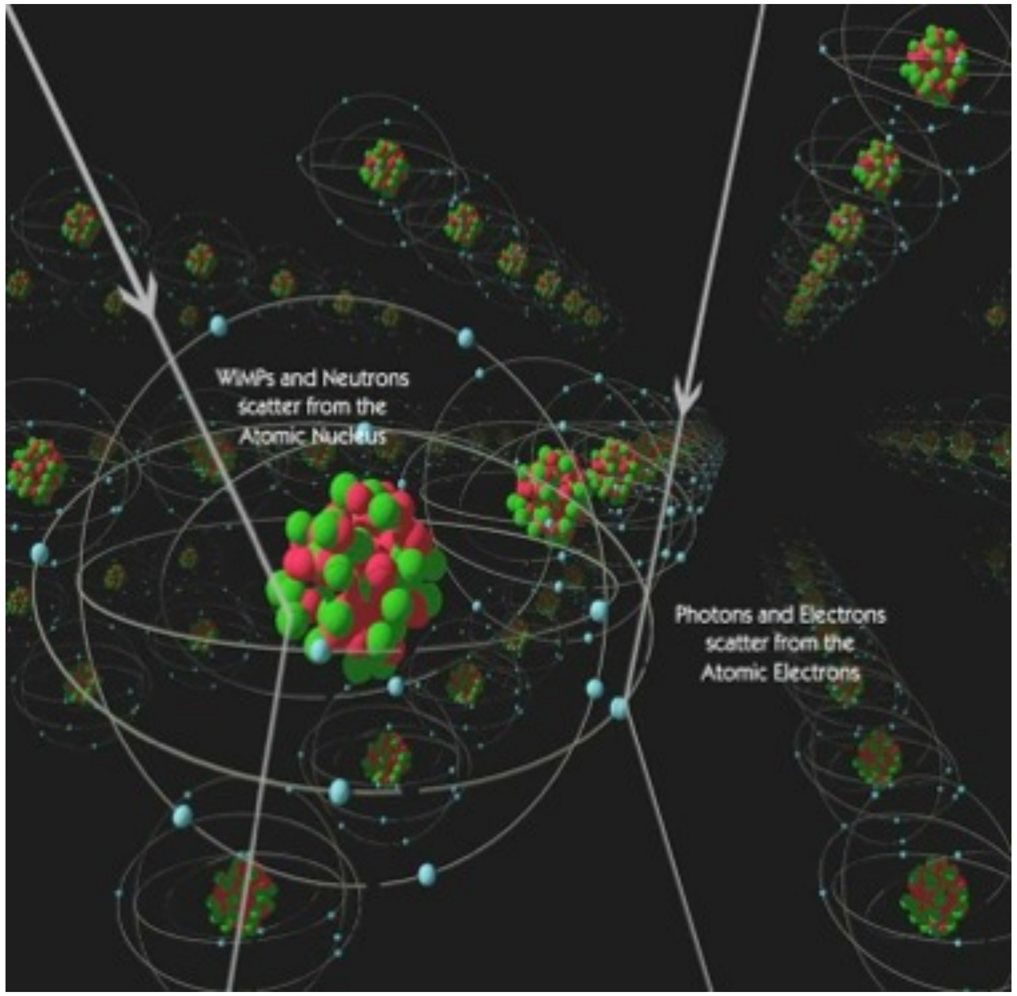
+ IceCube, Kavanagh, Fornasa & Green, in prep.

see also

‘WIMP physics with ensembles of direct detection experiments’, Peter, Gluscevic, Green, Kavanagh, Lee, arXiv:1310.7039.

Why are uncertainties in f(v) important?

Direct detection of WIMPs in the Milky Way halo via elastic scattering on detector nuclei in the lab:



$$\chi + N \rightarrow \chi + N$$

Direct detection differential event rate:

(assuming for simplicity elastic scattering, spin-independent coupling and f_p=f_n)

$$\frac{\mathrm{d}R}{\mathrm{d}E} = \frac{\rho_0 \sigma_\mathrm{p}}{2 m_\chi \mu_{\chi \mathrm{p}}^2} A^2 F^2(E) g(v_\mathrm{min})$$

Velocity integral:

$$g(v_\mathrm{min}) = \int_{v > v_\mathrm{min}} \frac{f(\mathbf{v})}{v} \mathrm{d}^3\mathbf{v}$$

Minimum WIMP speed that can cause a recoil of energy E:

$$v_\mathrm{min} = \left(\frac{E(m_\mathrm{A} + m_\chi)^2}{2 m_\mathrm{A} {m_\chi}^2} \right)^{1/2}$$

Shape of energy spectrum depends on both
particle physics parameters (WIMP mass and cross-section) and
astrophysical input (local DM density and speed distribution).

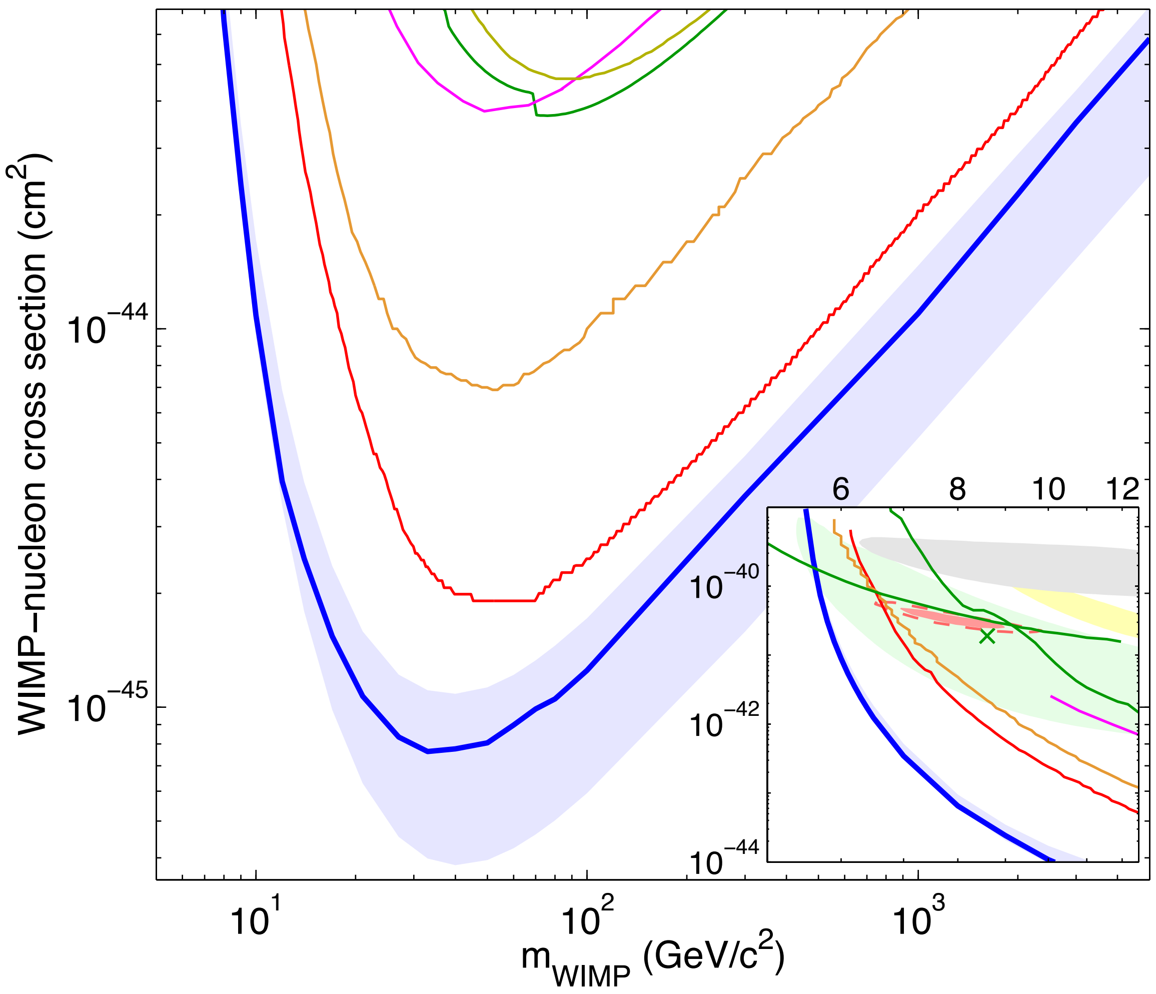
With a single experiment can't say anything about the WIMP mass without making assumptions about f(v), but with multiple experiments can break this degeneracy.
Drees & Shan; Peter

Experimental constraints on σ-m_χ plane usually calculated using ‘standard halo model’:

isotropic, isothermal sphere, with Maxwell-Boltzmann speed distribution

$$f(\mathbf{v}) \propto \exp\left(-\frac{3|\mathbf{v}|^2}{2\sigma^2}\right) \qquad \sigma = \sqrt{\frac{3}{2}}v_c$$

with v_c=220 km s^{-1} and local density ρ_0=0.3 GeV cm^{-3}.

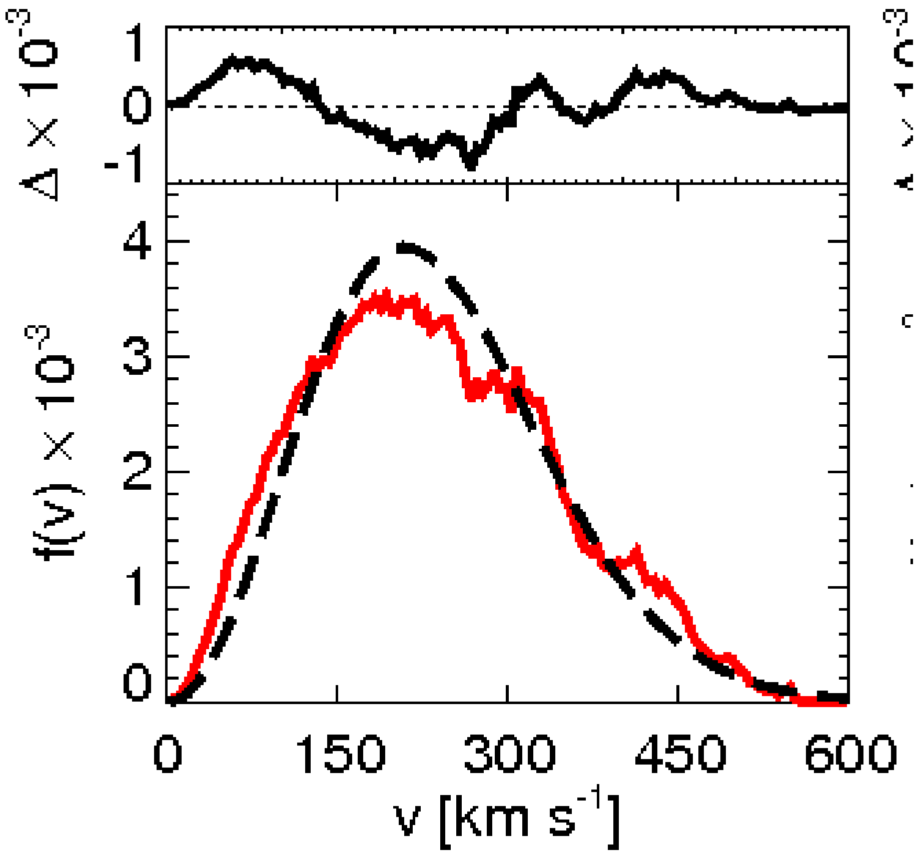


[LUX]

But simulated halos have f(v) which deviate systematically from multi-variate gaussian: more low speed particles, peak of distribution lower/flatter.

Fairbairn & Schwetz; Vogelsberger et al., Kuhlen et al.

Features in tail of dist, ‘debris flows’, incompletely phased mixed material. Lisanti & Spergel; Kuhlen, Lisanti & Spergel

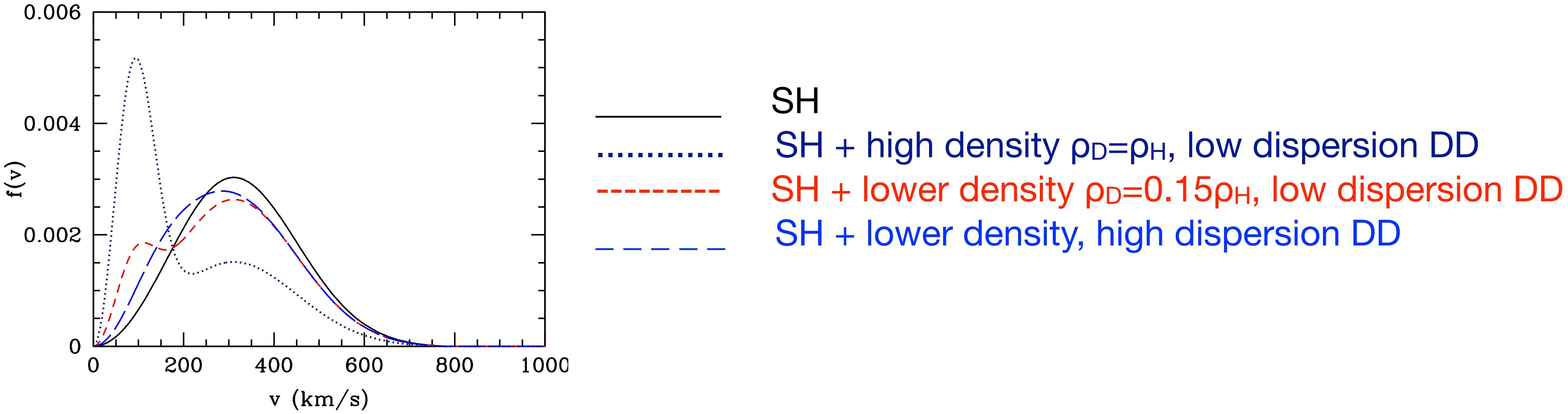


Vogelsberger et al.

red lines: simulation data,
black lines: best fit multi-variate Gaussian

Late merging sub-halos dragged into disc, where they're destroyed leading to the formation of a co-rotating **dark disc.** Read et al., Bruch et al., Ling et al.

Could have a significant effect if density is high and velocity dispersion low.

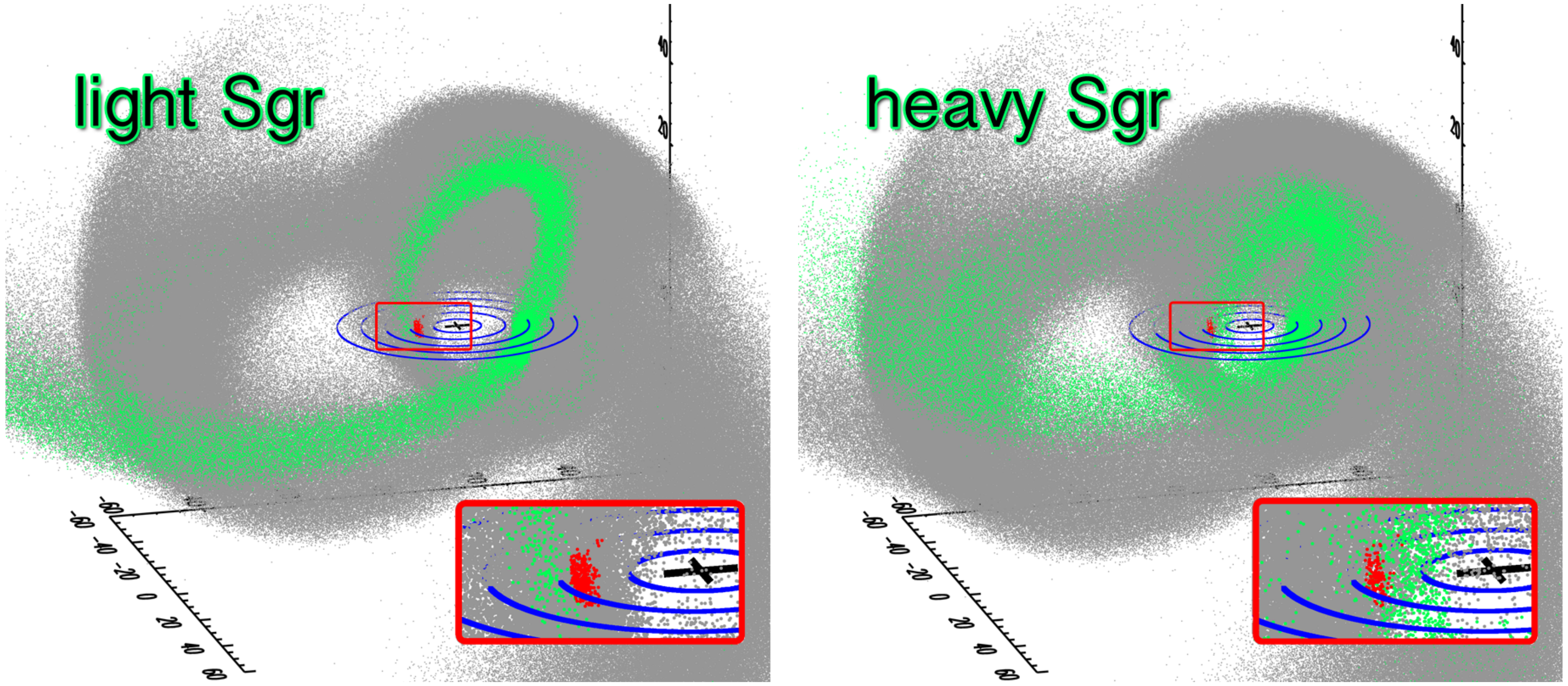


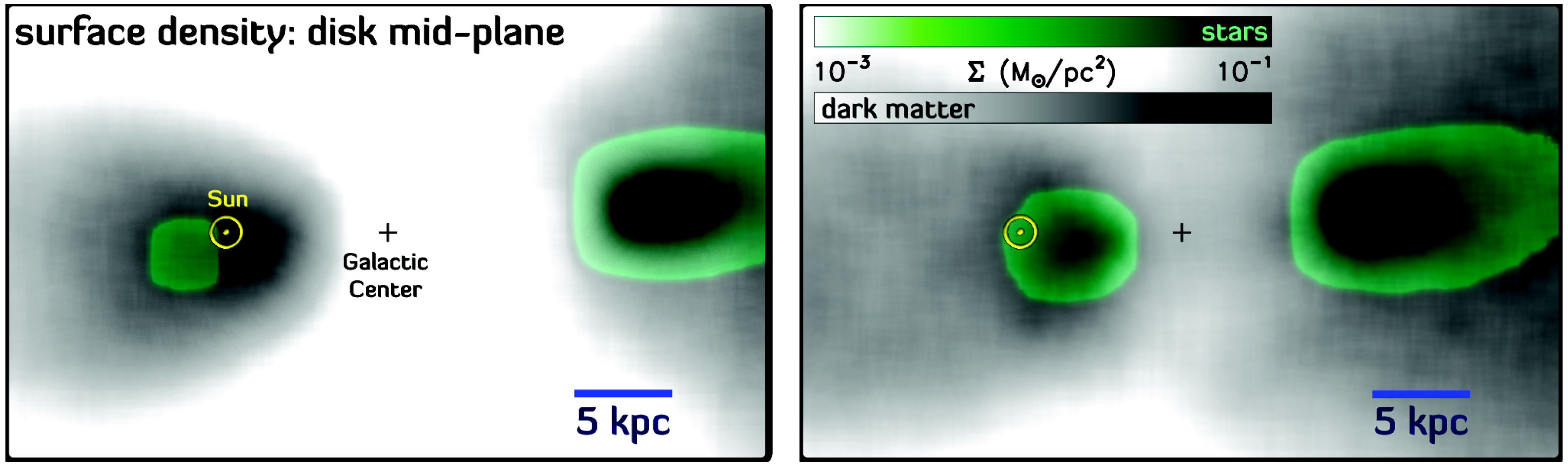
Properties of dark disc are uncertain (simulating baryonic physics and forming Milky Way-like galaxies is hard…).

To be consistent with observed properties of thick disc, MW's merger history must be quiescent compared with typical ΛCDM merger histories, hence DD density must be relatively low, $<0.2\ \rho_H$. Purcell, Bullock & Kaplinghat

DM component of Sagittarius leading **stream** may pass through the solar neighbourhood

Purcell, Zentner & Wang (as originally suggested by Freese, Gondolo & Newberg).





Eris simulation

Guedes et al.

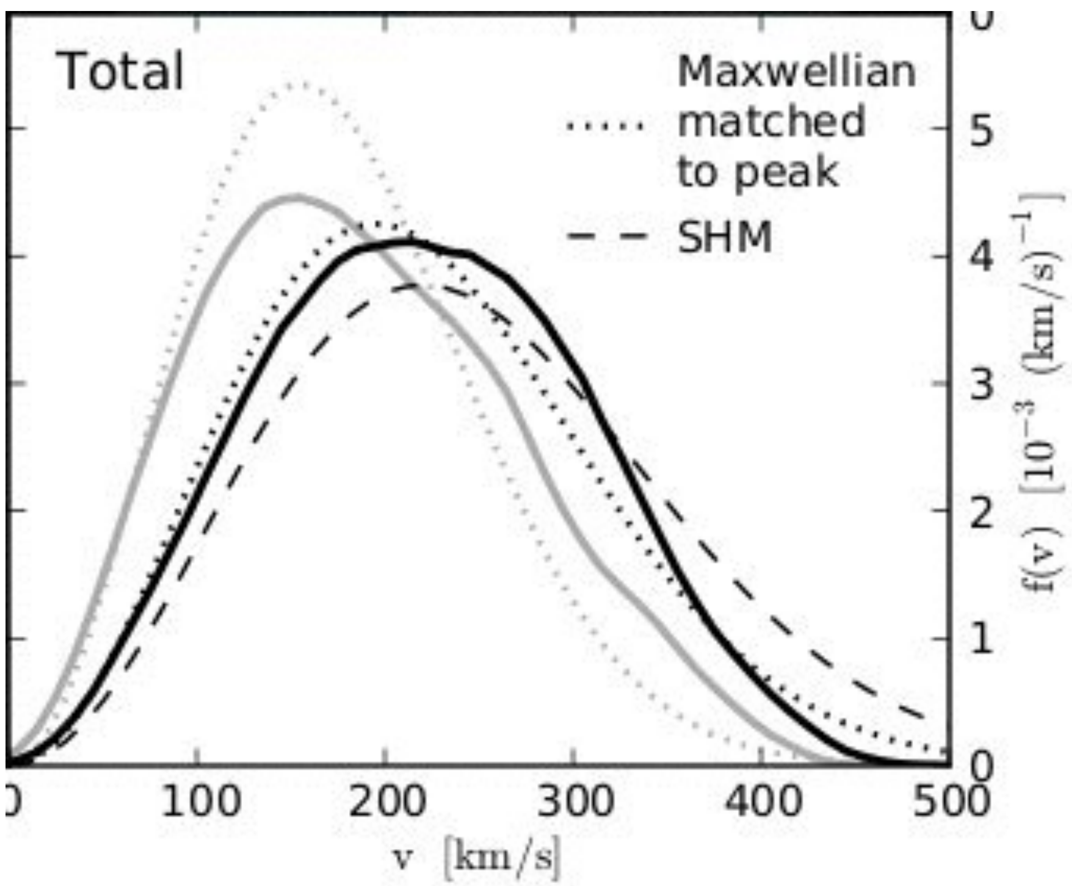
Dark disc contributes ~10% of local density.

Features in high speed tail of f(v) less pronounced than in DM only simulations.

Smoothed f(v)

DM only sim

Eris disk



Pillepich et al.

In summary:

The Maxwellian f(v) of the standard halo model (SHM) is unlikely to be a good approximation to the real f(v).

In particular there may be features in the distribution (streams, debris flow, dark disc).

Strategy: i) integrate out

Fox, Liu & Weiner

Compare experiments in terms of the renormalised velocity integral:

$$\tilde{g}(v_{\min}) = \frac{\rho_0 \sigma_{\rm p}}{m_\chi} \int_{v > v_{\min}} \frac{f(\mathbf{v})}{v} \, {\rm d}^3\mathbf{v}$$

$$v_{\min} = \left(\frac{E(m_A + m_\chi)^2}{2 m_A m_\chi^2} \right)^{1/2}$$

n.b. g($v_{\rm min}$) must be a monotonically decreasing function of $v_{\rm min}$.

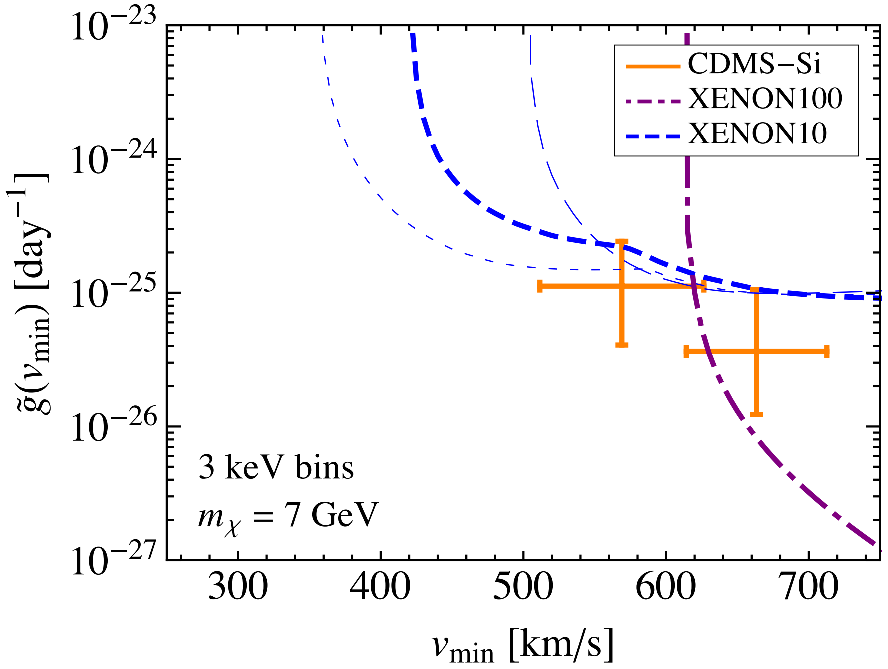
Can incorporate experimental energy resolution and efficiency Gondolo & Gelmini, annual modulation Frandsen, Kahlhoefer, McCabe, Sarkar, Schmidt-Hoberg; Herrero-Garcia, Schwetz & Zupan, unbinned data Fox, Kahn & McCullough, inelastic scattering Borzognia, Herrero-Garcia, Schwetz & Zupan, 'non-standard' interactions Del Nobile, Gelmini, Gondolo & Huh.

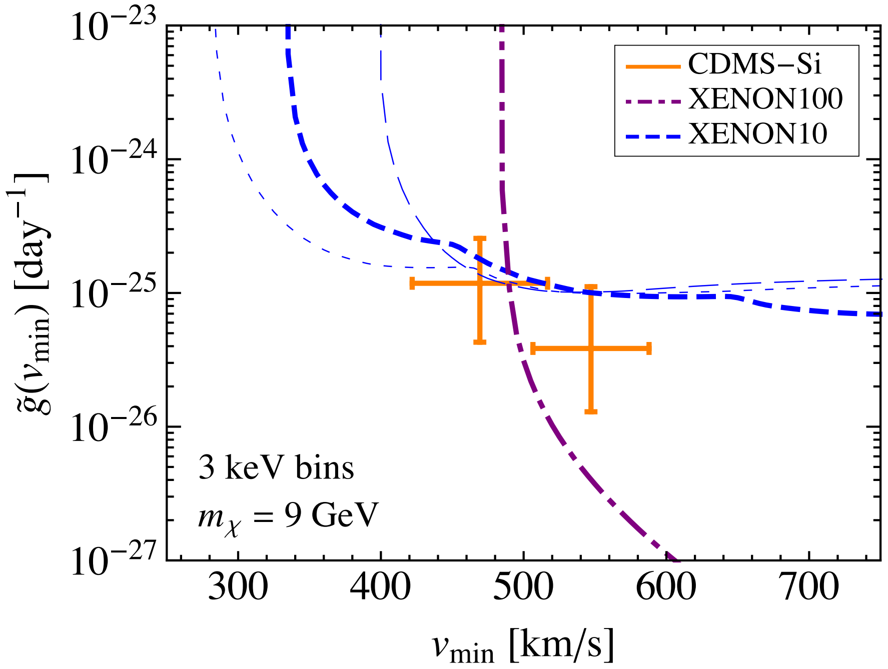
Extremely useful for checking consistency of signals and exclusion limits.

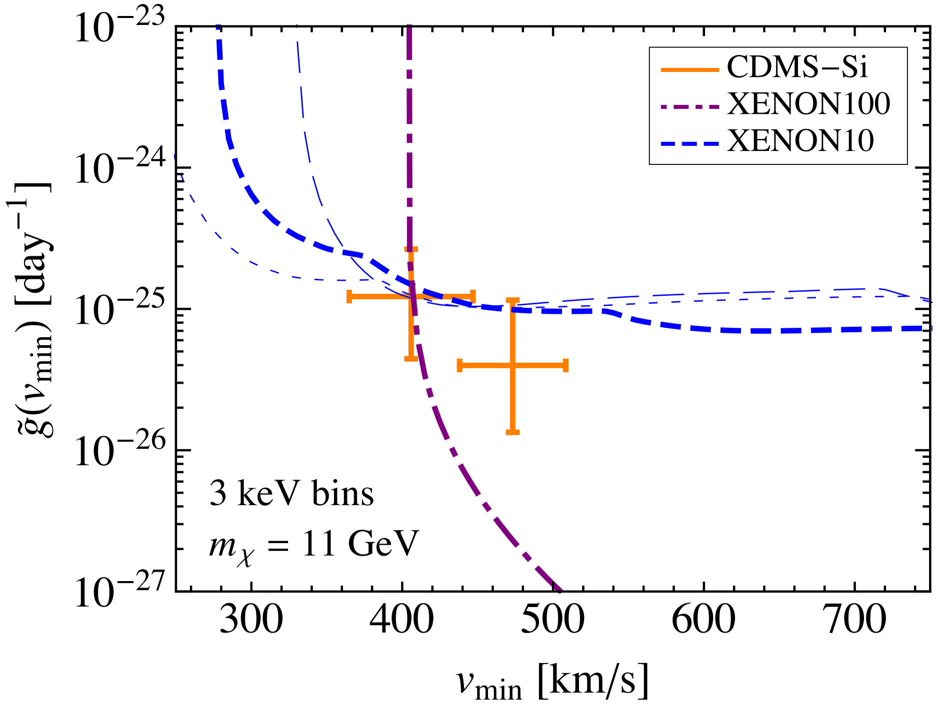
v_{min} values probed by each experiment depend on, unknown, WIMP mass, therefore need to do comparison for each mass of interest.

Normalised $g(v_{min})$ versus v_{min}

Frandsen, Kahlhoefer, McCabe, Sarkar, Schmidt-Hoberg







ii) marginalise over

Parameterize f(v) and/or Milky Way model and marginalise over these parameters.

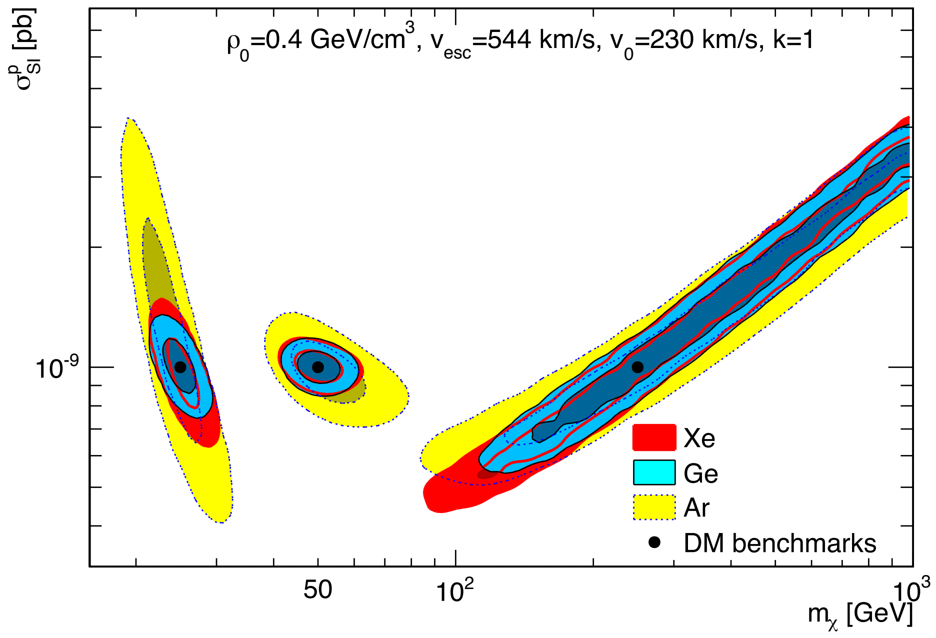
Strigari & Trotta; Peter x2; Pato et al. x2; Lee & Peter; Billard, Meyet & Santos; Alves, Hedri & Wacker; Kavanagh & Green x2; Friedland & Shoemaker

With data from multiple experiments don’t need to make strong (and possibly erroneous) assumptions about f(v).

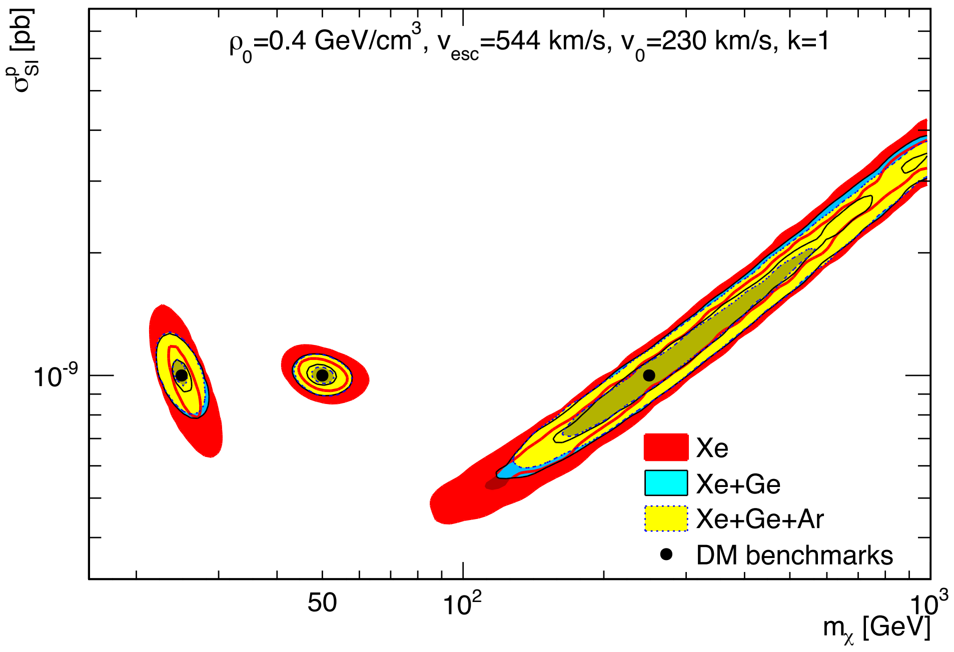
Even if f(v) were known, combining data from multiple experiments improves the determination of the WIMP parameters. Pato et al.

‘Fixed astrophysics’

Xe, Ge, Ar individually



combined

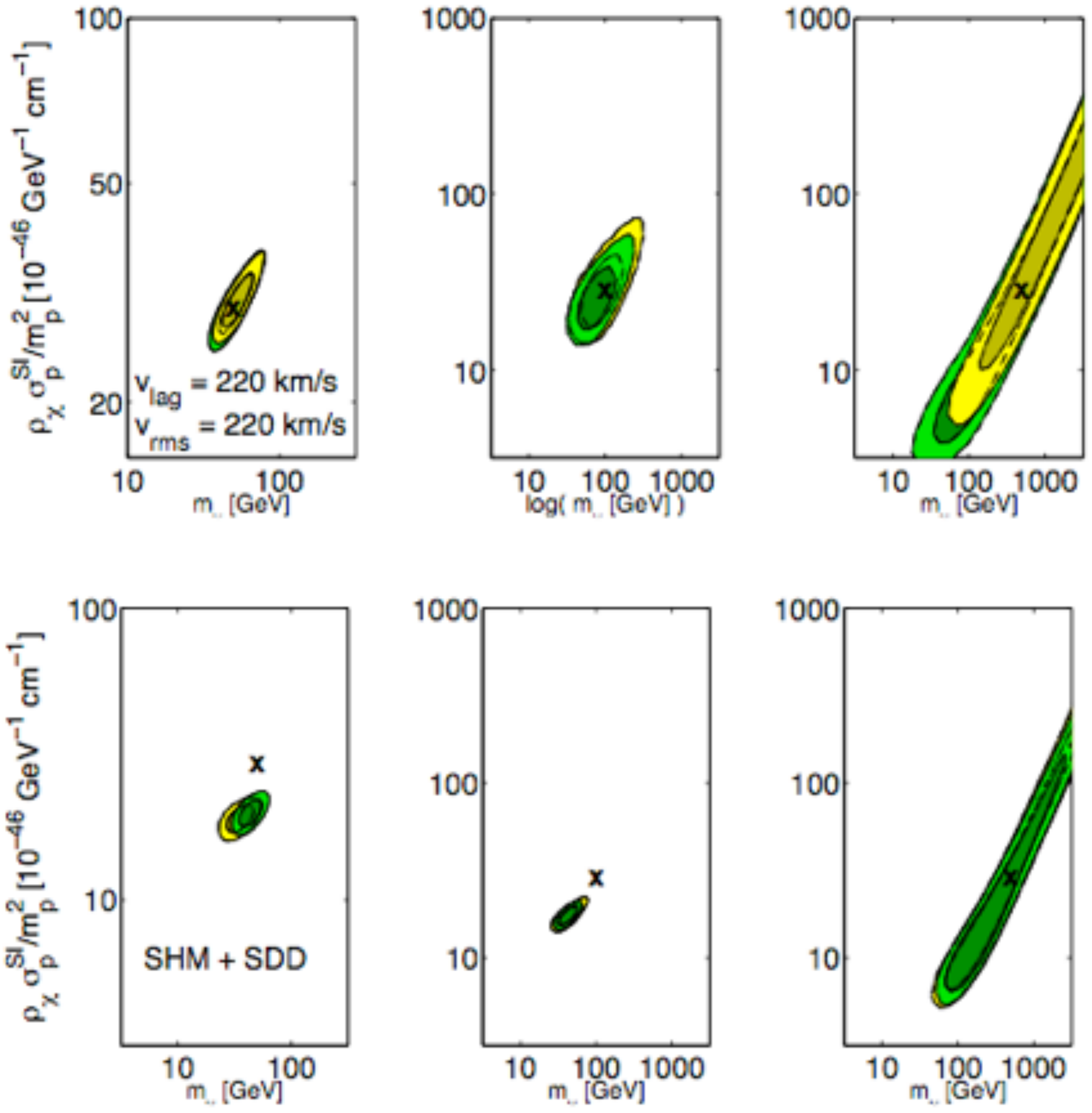


Pato et al.

If actual shape of f(v) is similar to assumed shape marginalisation approach works well, but if not can get significant biases:

$m_\chi = 50\,\mathrm{GeV}$ $100\,\mathrm{GeV}$ $500\,\mathrm{GeV}$

$$D = \frac{\rho_0 \sigma_\mathrm{p}}{m_\chi^2}$$



m_χ

Peter simulated data from future tonne scale Xe, Ar & Ge expts, analysed assuming standard halo model (allowing v_{lag} & v_{rms} to vary).

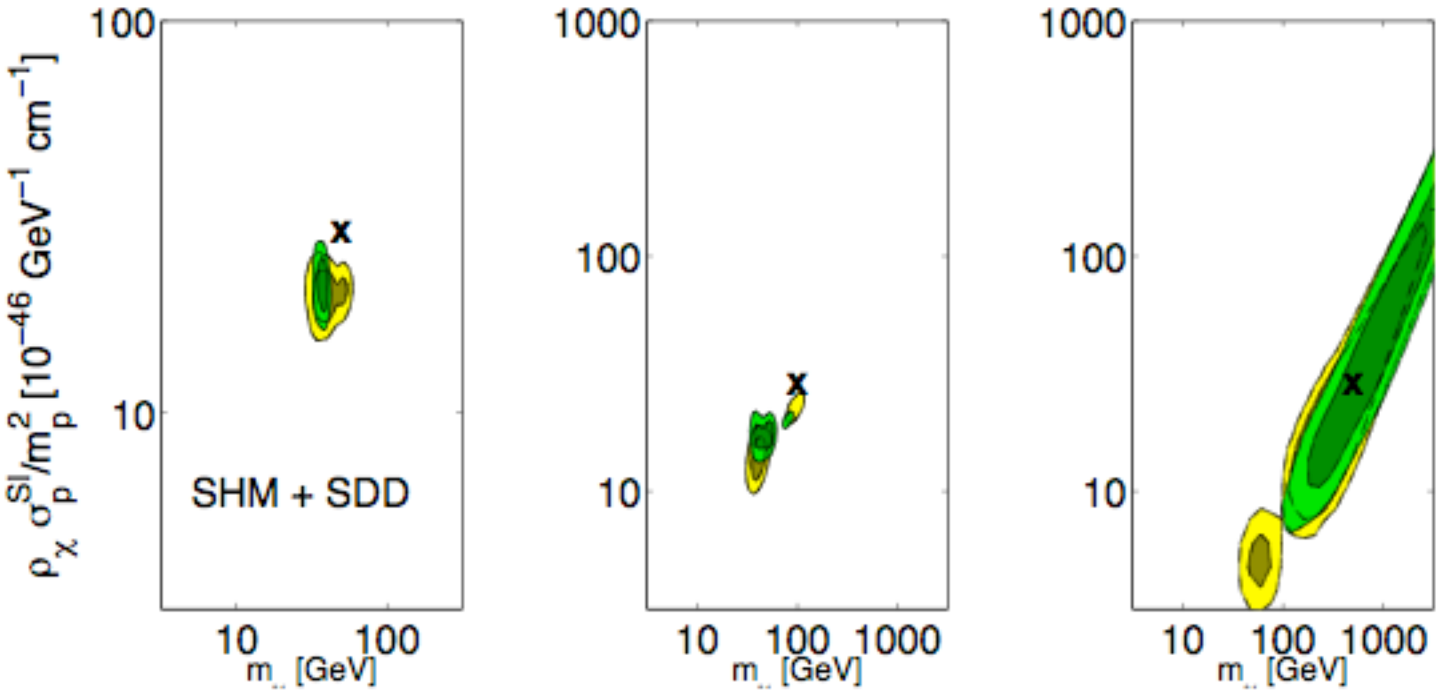
standard halo model in

standard halo model + dark disc in

How to parameterise f(v)?

Peter Use empirical parameterization of f(v), and constrain its parameters along with mass & cross-section.

First approach: piece-wise constant in bins



standard halo model + dark disc in

Better than assuming wrong f(v), but mass & cross-section both biased.

Mass: reducing m reduces width of bins in E, and enables better fit. Kavanagh & Green

Cross section: a significant (but a priori unknown) fraction of the WIMPs are below threshold. Inevitable problem when doing model independent analysis of direct detection data (but see later…)

Kavanagh & Green; Kavanagh

Want parameterisation without fixed scales, and with ability to accommodate features in speed distribution.

Since f(v) ≥ 0, parameterise log of 1d speed distribution in terms of polynomials:

$$f_1(v) \equiv \int v^2 f(\mathbf{v}) \, \mathrm{d}\Omega = v^2 \exp\left\{ -\sum_{k=0}^{N-1} a_k P_k(v/v_{\mathrm{max}}) \right\}$$

Which polynomials work best?

Orthogonal bases are *well-conditioned* (small changes in parameters lead to small changes in functions, good for parameter estimation).

Legendre and Chebysev polynomials both work well, but Chebyshev is faster.

And how many of them?

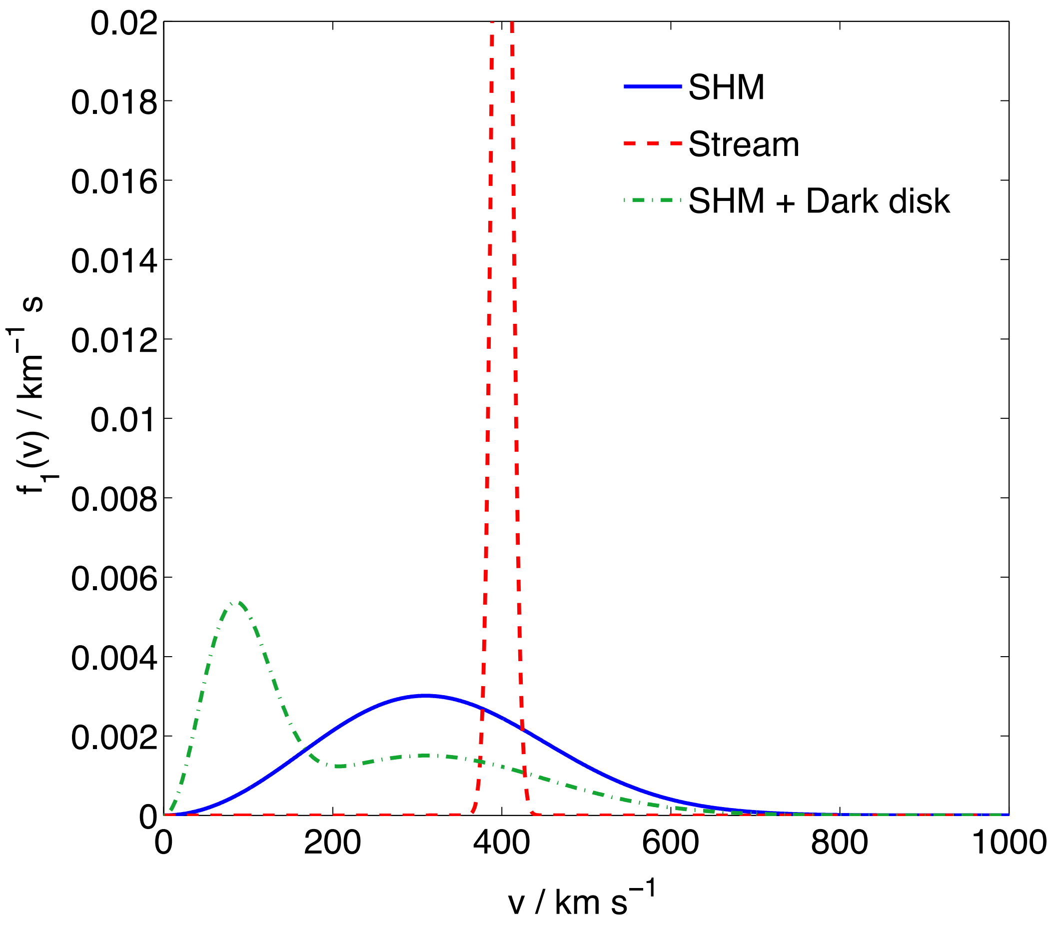
Depends on (unknown) underlying f(v). Vary N and examine goodness of fit (using e.g. Bayesian Information Criteria).

If f(v) close to Maxwellian a few is enough, with features need N~5-10.

Gives good reconstruction of WIMP mass even for extreme input f(v) (stream or dark disc).

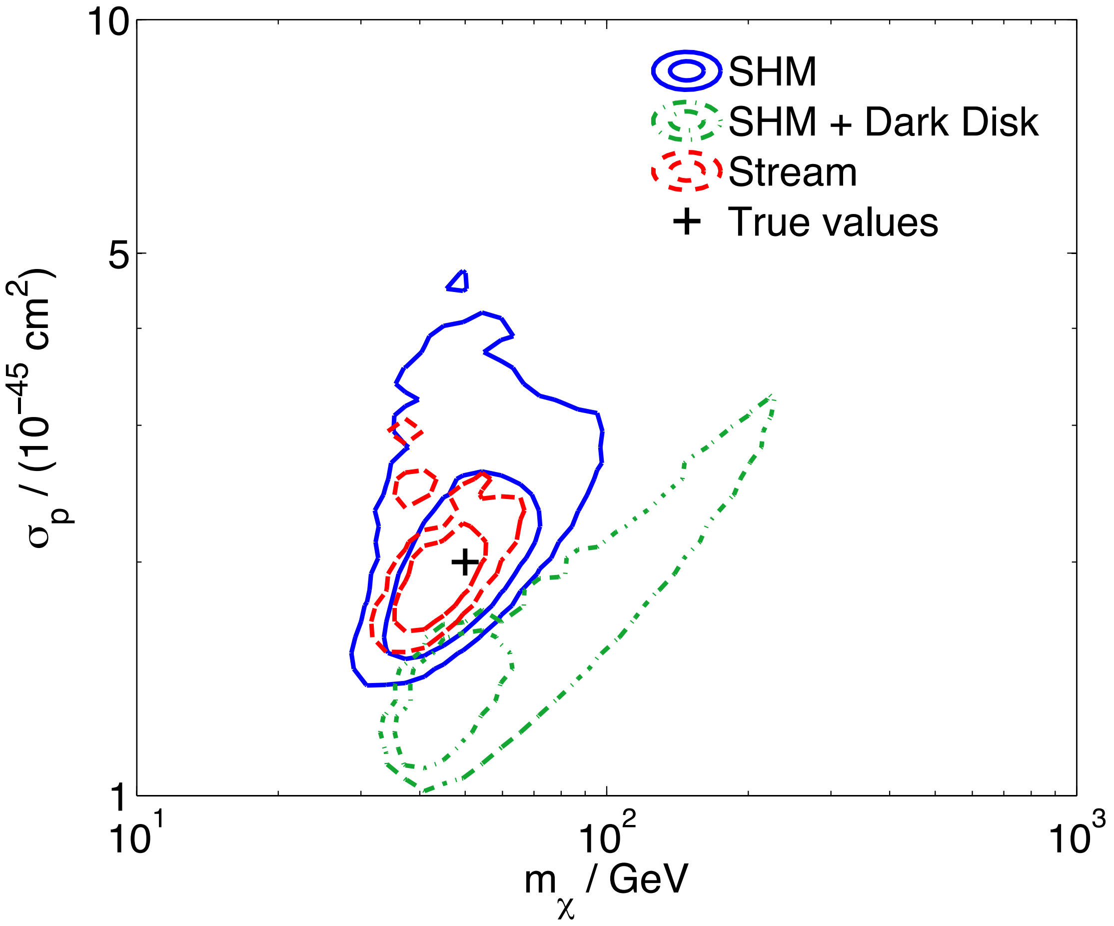
Simulated data from idealised (zero background, perfect resolution) Xenon1T, SuperCDMS & WArP like experiments, using 'Asimov' data, (i.e. ignoring effects of Poisson statistics).

input speed distributions



reconstructed m_χ and σ

68% & 95% cls from 2d profile likelihood



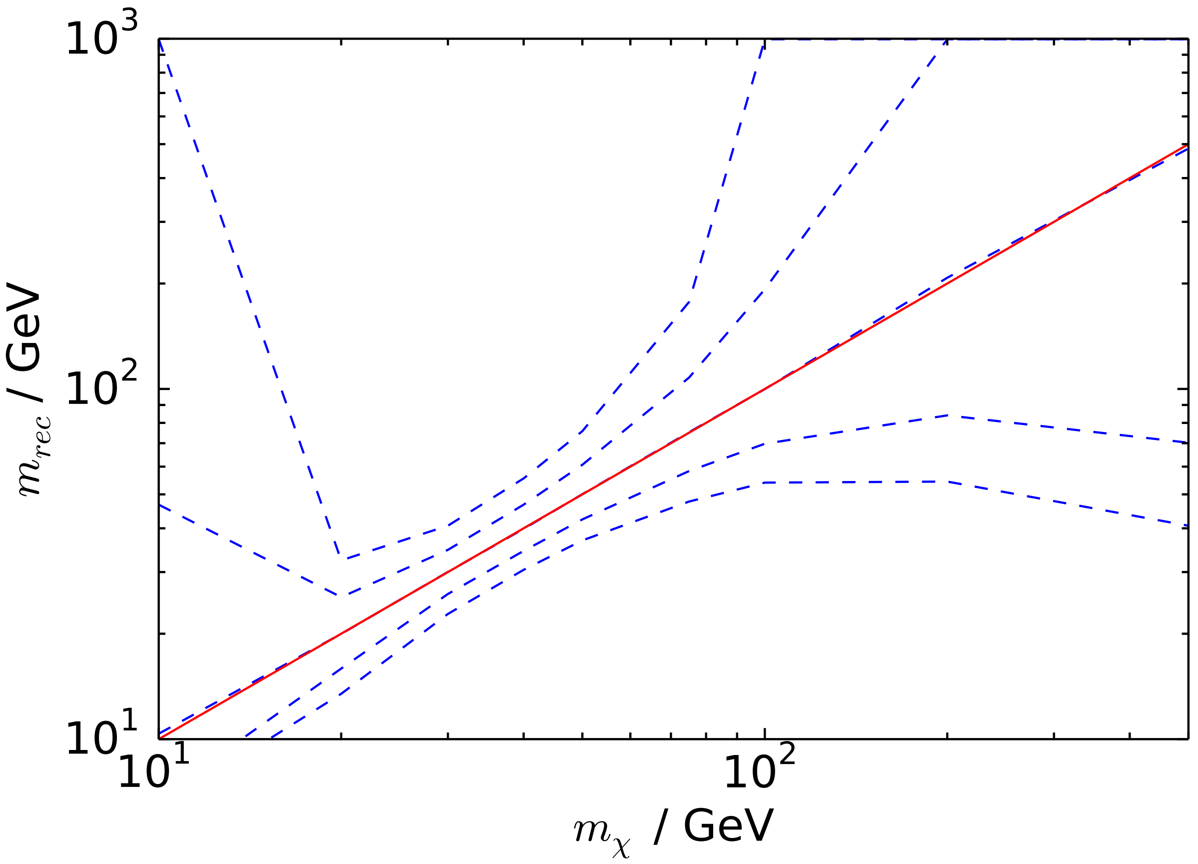
SHM

Stream

SHM + dark disc

Dependence on underlying WIMP mass

reconstructed mass
(central value + 68%
 & 95% intervals
from profile likelihood)



input mass

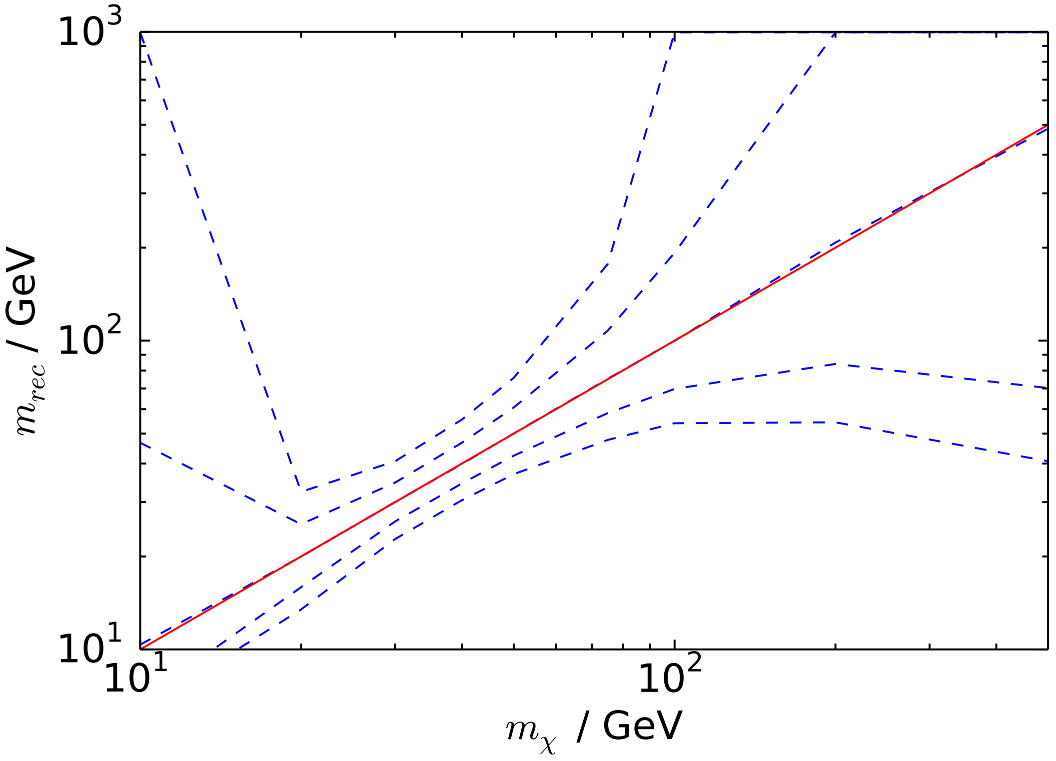
Inherent limitations in determining the mass:

at low masses event rates above thresholds small

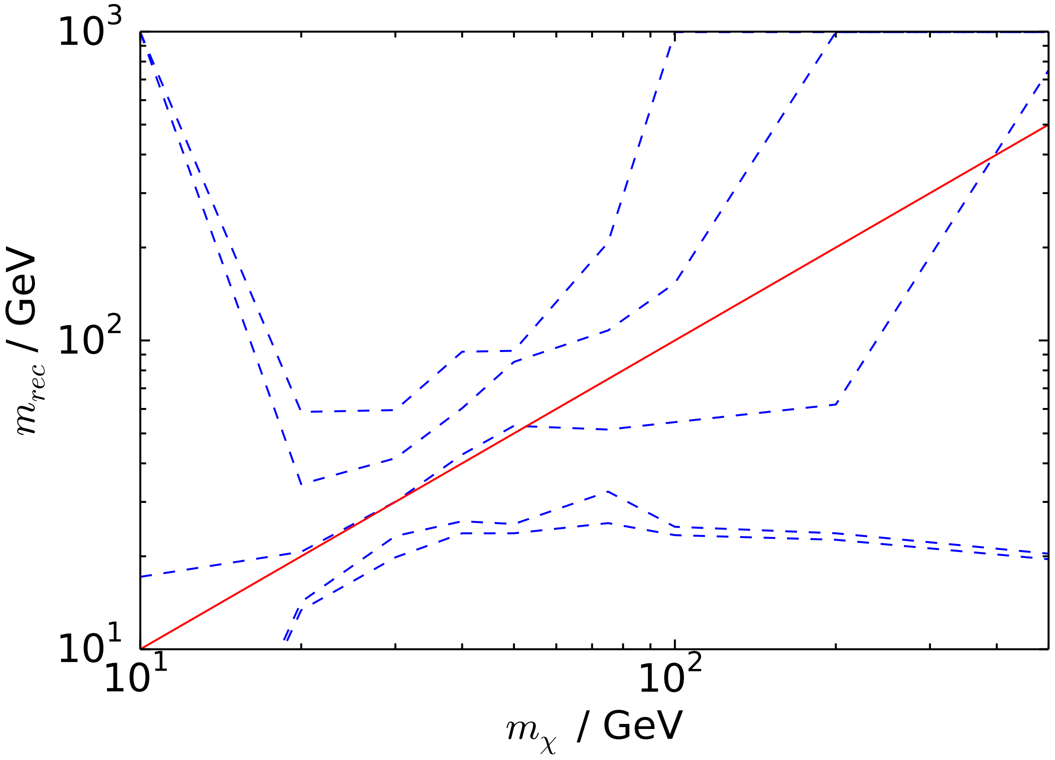
at high masses energy spectrum independent of mass

Finite background & energy resolution

Idealised experiments



1keV energy resolution
flat background, 1 /kg/day/keV



At high masses: finite resolution means shape is less well-determined

flat background spectrum can mimic heavy WIMP

Statistical fluctuations

Results from 250 realisations, including Poisson fluctuations.

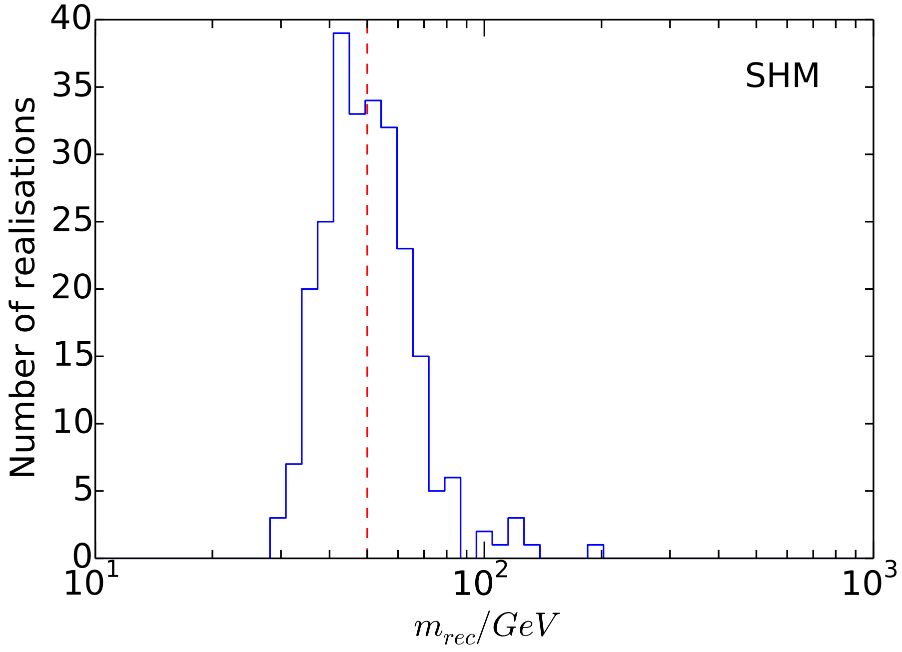
Bias: $b = \ln(m_{\rm rec}/{\rm GeV}) - \ln(m_\chi/{\rm GeV})$

Coverage: fraction of realisations in which true parameter lies within given interval

$$\langle b \rangle = 0.002 \pm 0.008$$

$$68\% \text{ coverage: } (71 \pm 3)\%$$

$$95\% \text{ coverage: } (94 \pm 3)\%$$

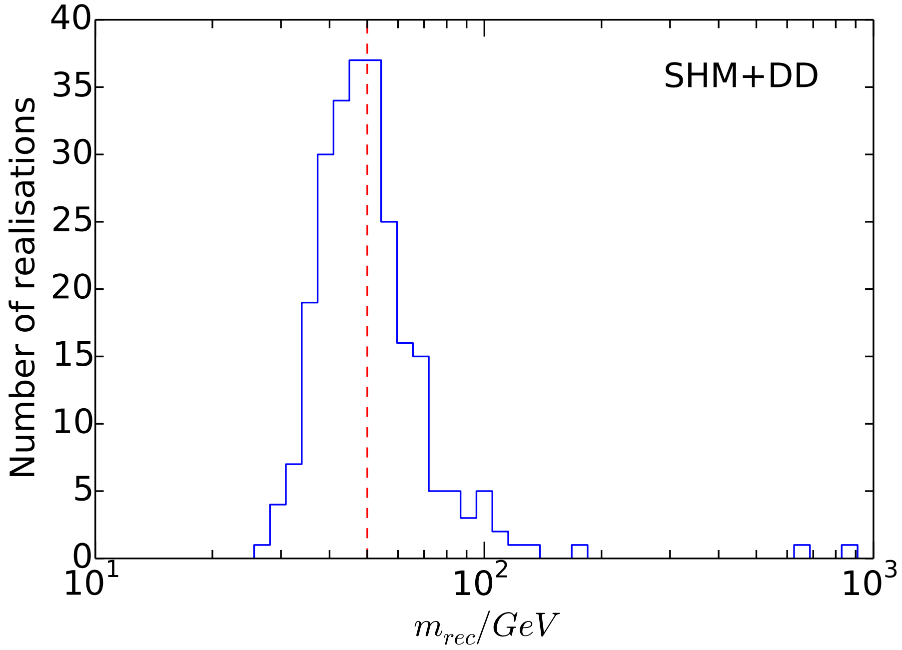


SHM

$$\langle b \rangle = 0.005 \pm 0.007$$

$$68\% \text{ coverage: } (68 \pm 3)\%$$

$$95\% \text{ coverage: } (91 \pm 4)\%$$

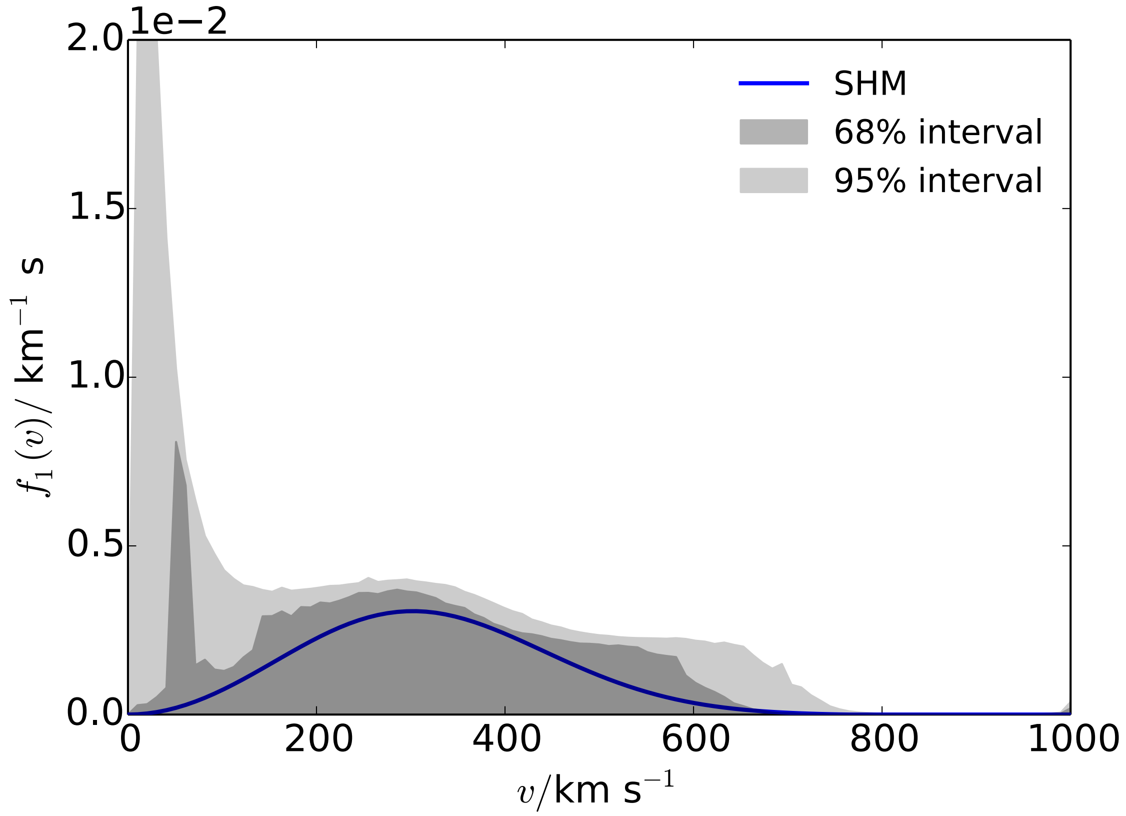


SHM+DD

Reconstructing speed distribution

Naive approach:

sample from P(**a**), get $f_1(v)$ (at each v marginalised over all other values of v).

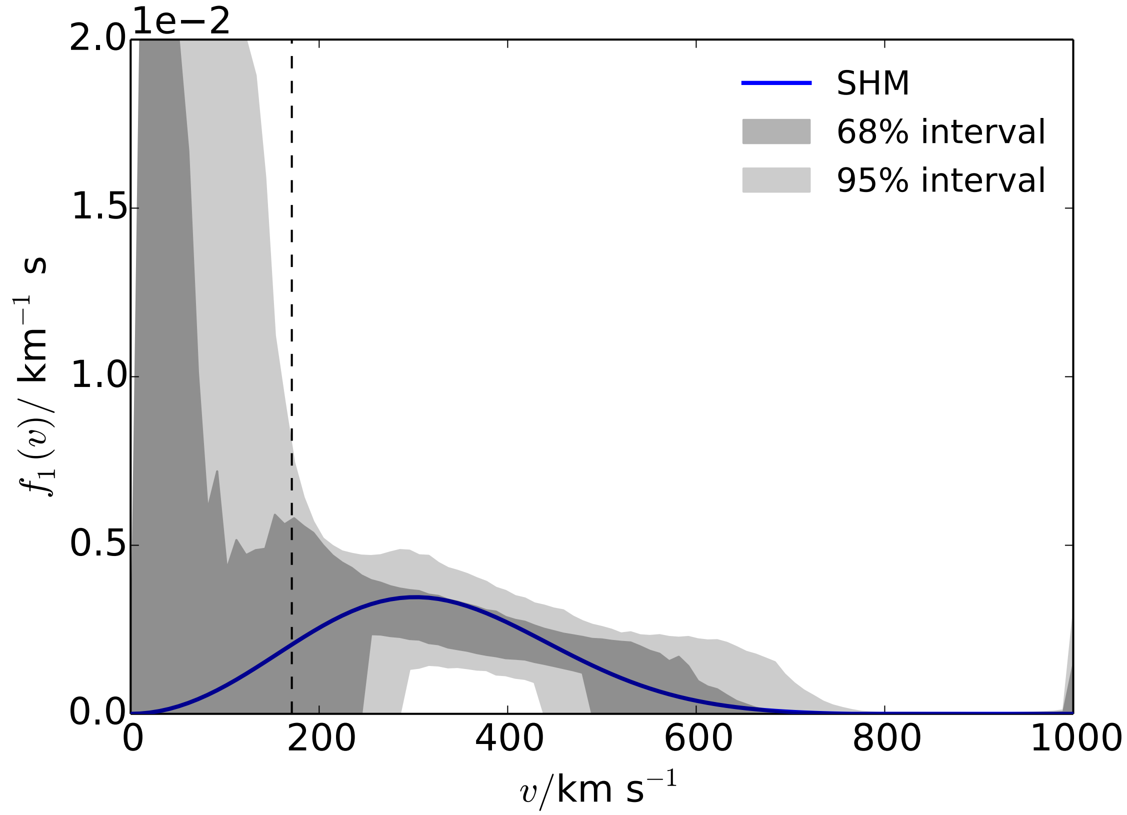


Issue:

don’t know fraction above threshold hence degeneracy between σ and $f_1(v)$.

Improved approach:

normalise $f_1(v)$ to unity above lowest speed probed.



Issue:

this speed depends on (unknown) WIMP mass, also WIMPs with speeds above experimental energy window affect reconstruction.

Better approach: reconstruct normalised velocity integral.

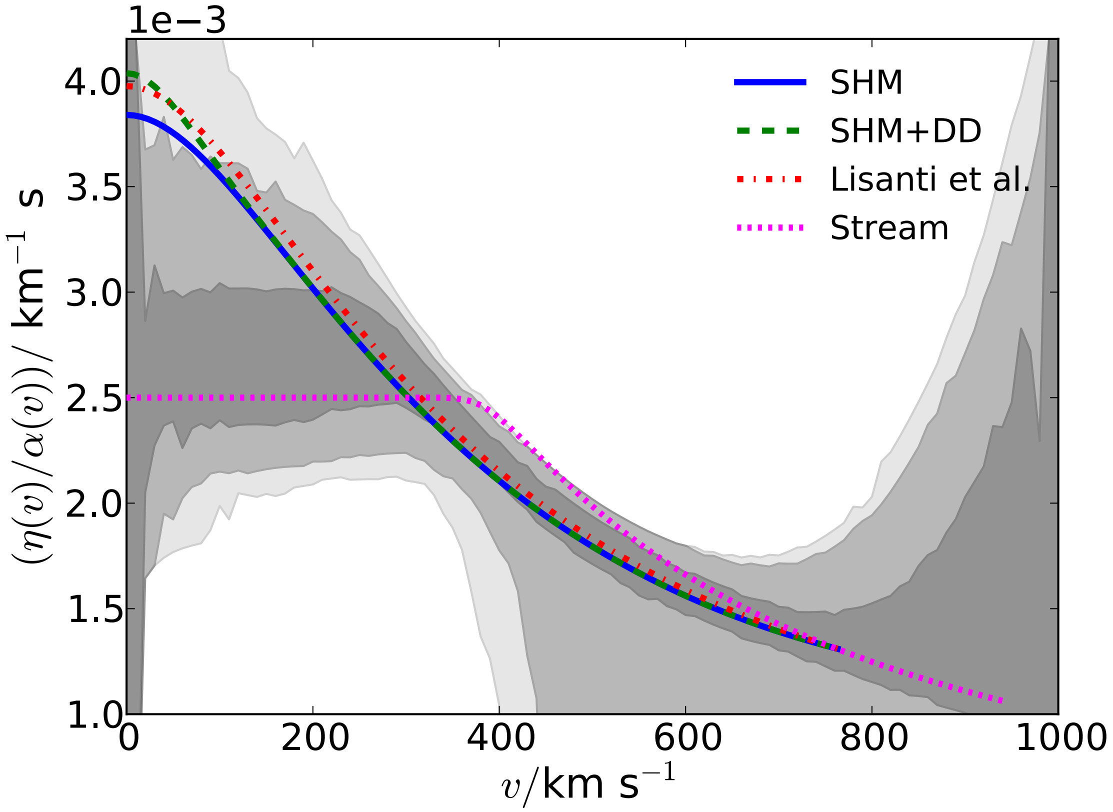
$$\eta^\star(v) = \frac{g(v)}{\alpha(v)}$$

$$g(v) = \int_{v'>v} \frac{f(\mathbf{v}')}{v'} \, \mathrm{d}^3\mathbf{v}'$$

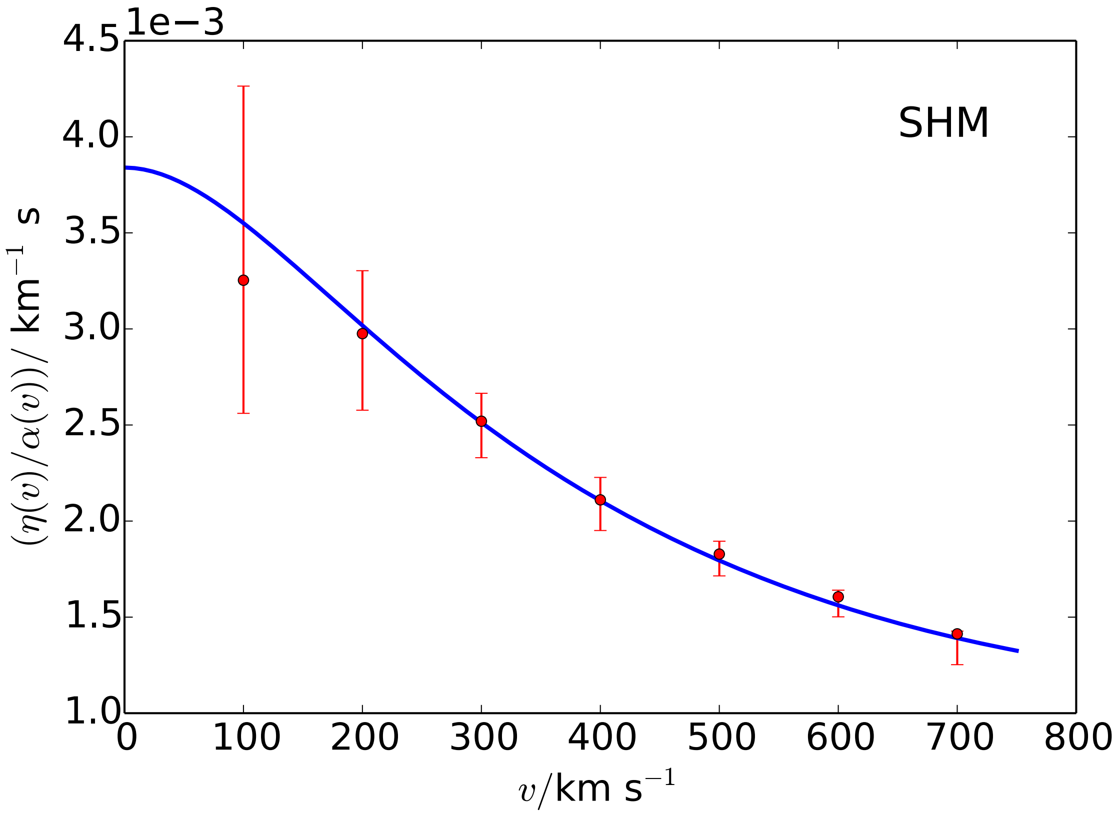
$$\alpha(v) = \int_v^\infty f_1(v') \, \mathrm{d}v'$$

fraction of WIMPs
above v

single realisation



mean from 250 realisation



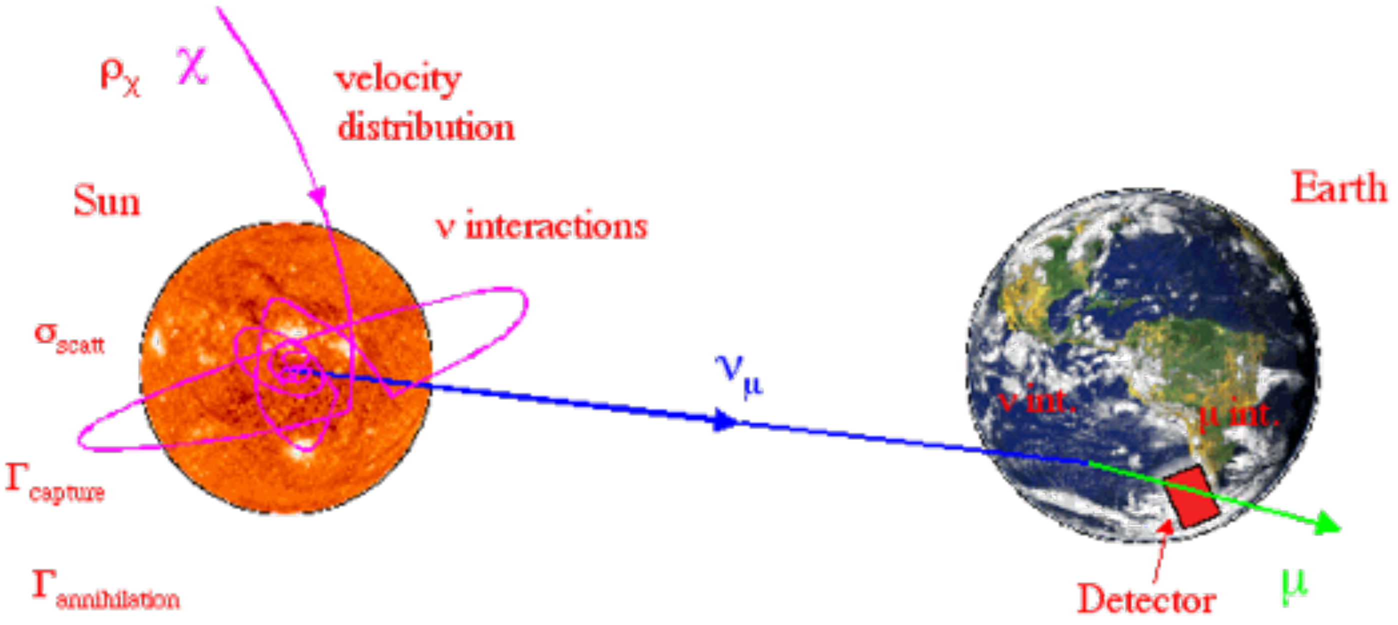
input: 50 GeV WIMP & SHM

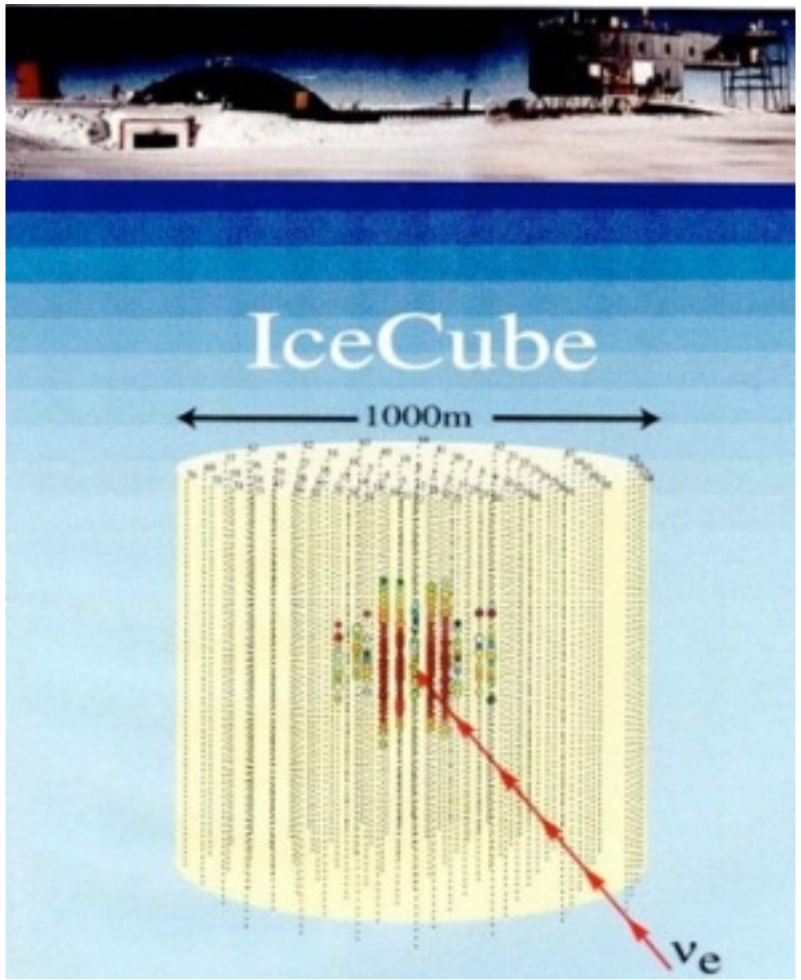
Would like to probe the entire speed distribution and get an unbiased estimate of the cross section.

Indirect detection via neutrinos:

Low speed WIMPs lose energy due to scattering are gravitationally captured in Sun then annihilate producing energetic neutrinos which escape.

Muon neutrinos produce muons which can then be detected (via Cherenkov radiation) using neutrino telescopes.



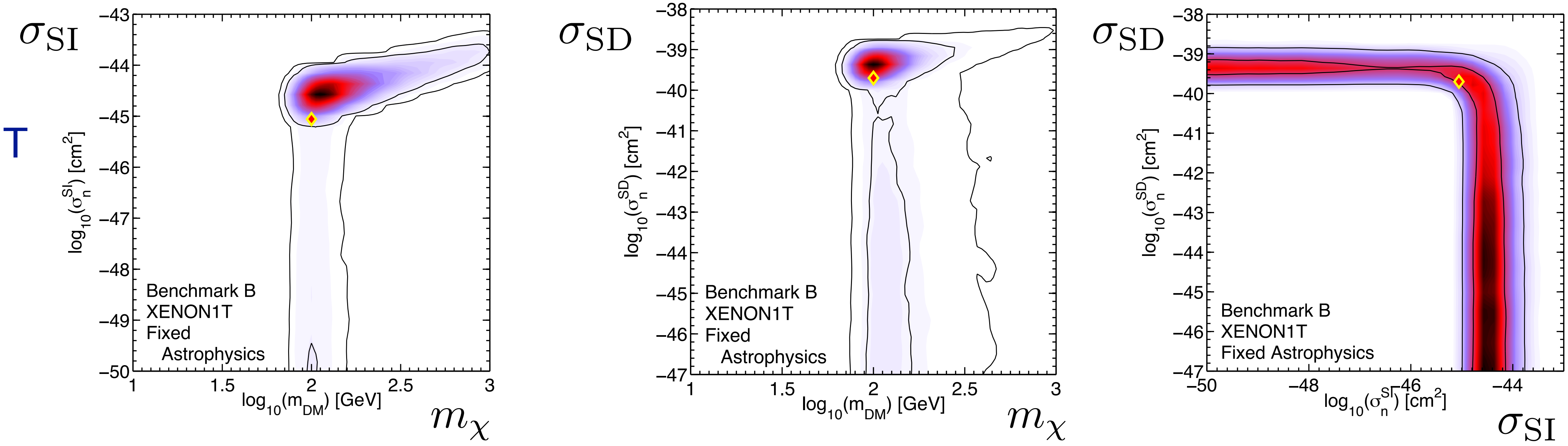


IceCube probes the low speed tail of f(v).
(up to ~200 km s^{-1} for m_χ ~ 50 GeV)

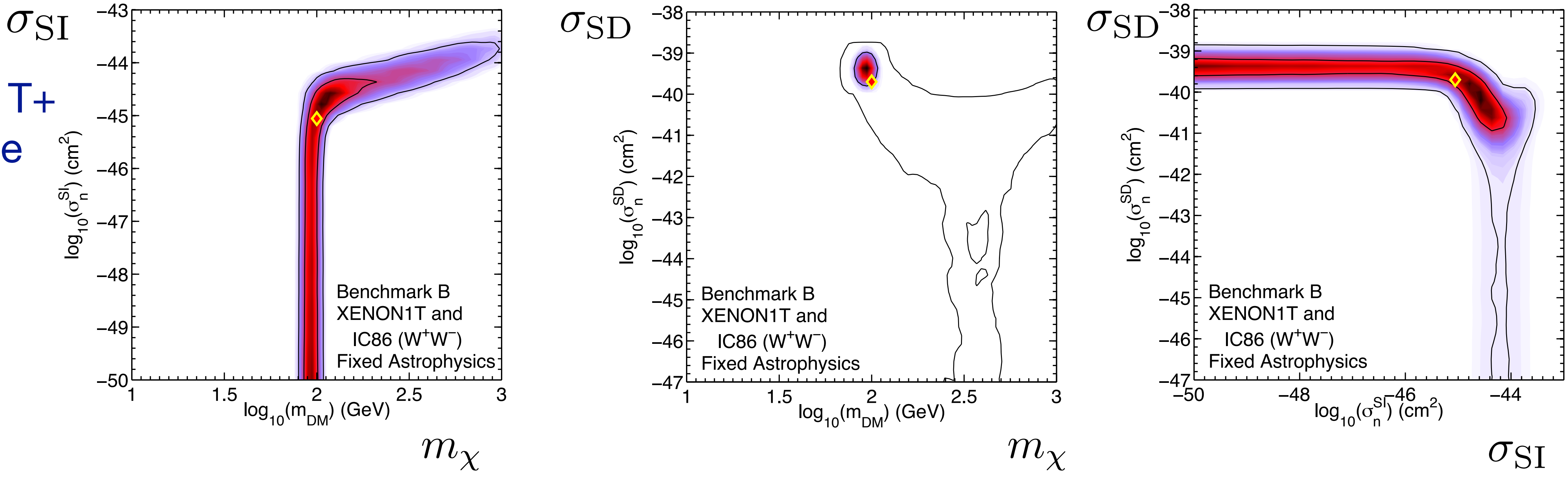
IceCube data can also break degeneracy Cerdeno, Fornasa, Huh & Peiro between spin independent and spin dependent cross-sections Arina, Bertone & Silverwood (since spin dependent scattering dominates capture).

100 GeV WIMP annihilating predominantly into W^+W^-

Xenon1T



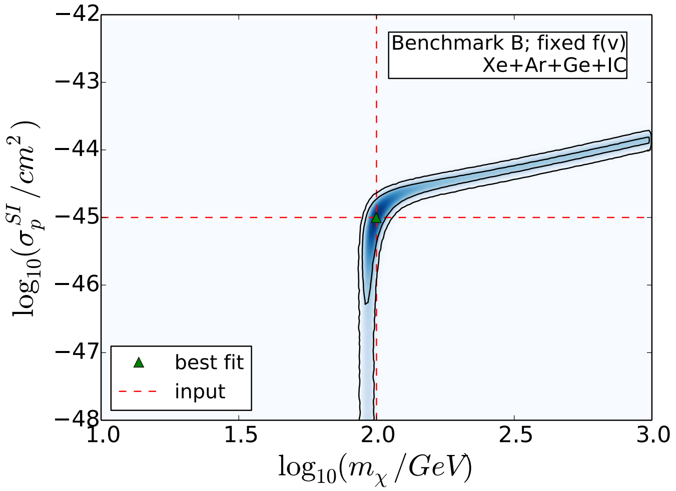
Xenon1T+
IceCube

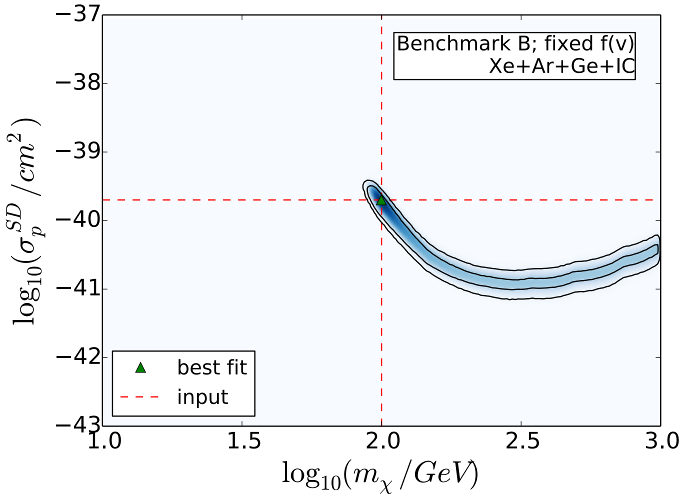


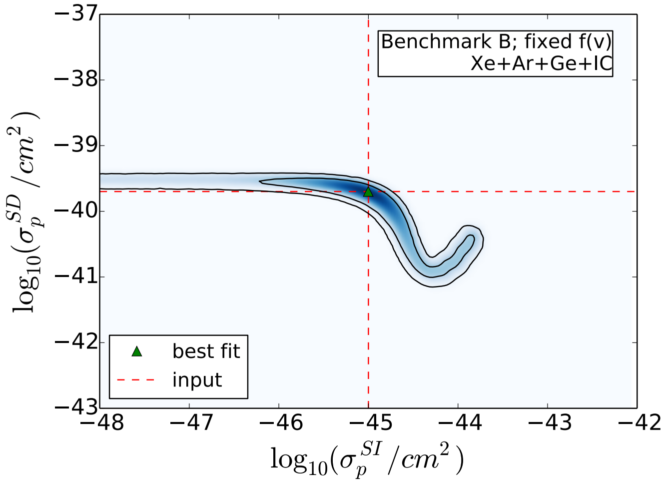
Same particle physics benchmark as Arina et al. (100 GeV WIMP annihilating predominantly into W^+W^-) but with f(v) = SHM + DD.

Simulated data from Xe, Ar & Ge direct detection experiments + IceCube.

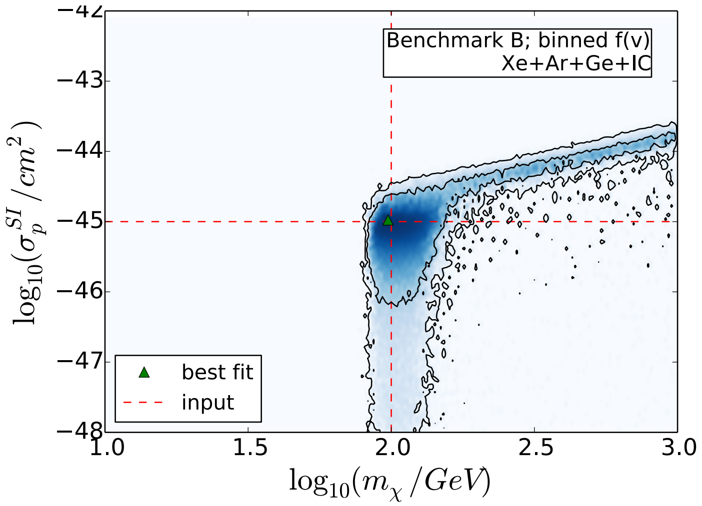
assuming SHM+DD

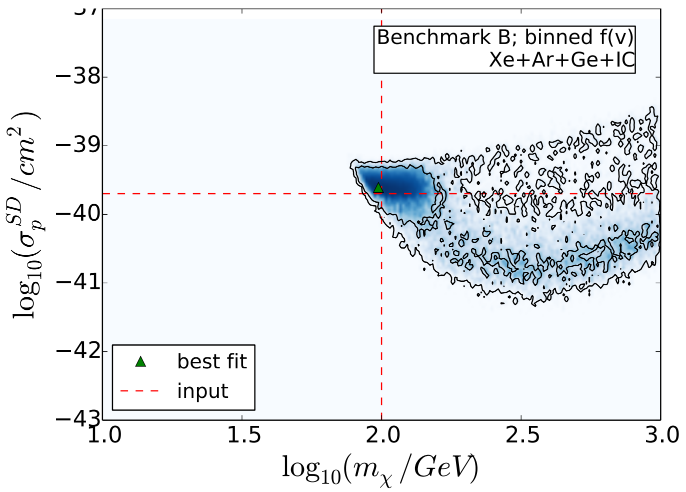


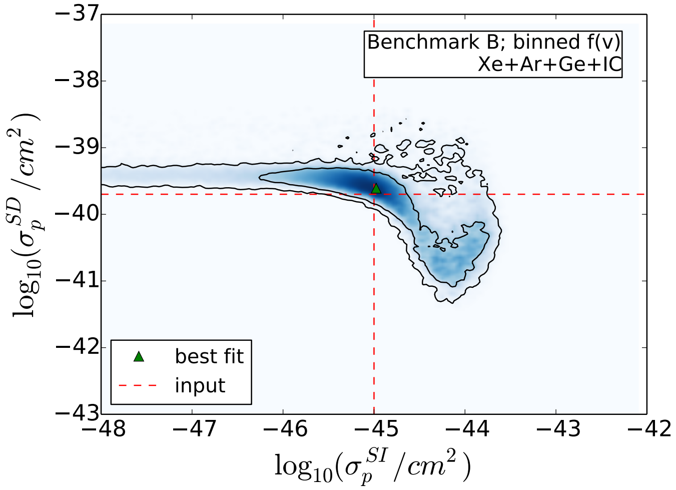




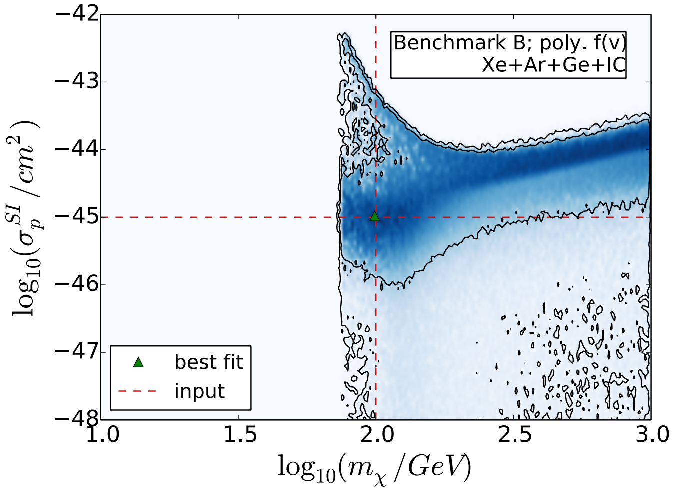
binned f(v)

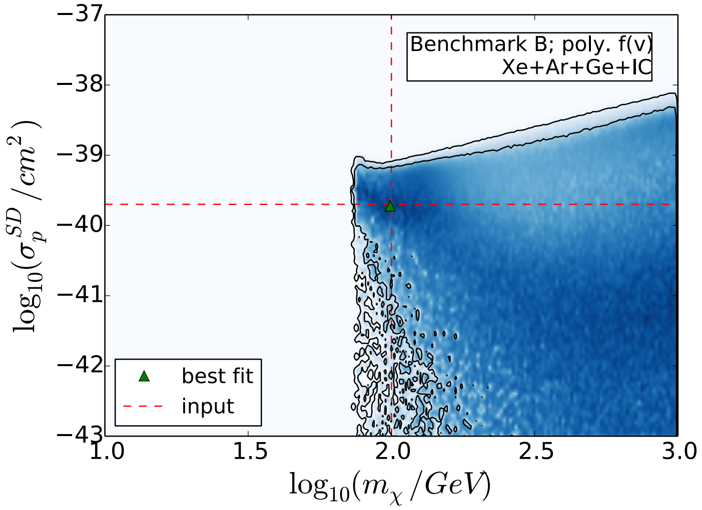


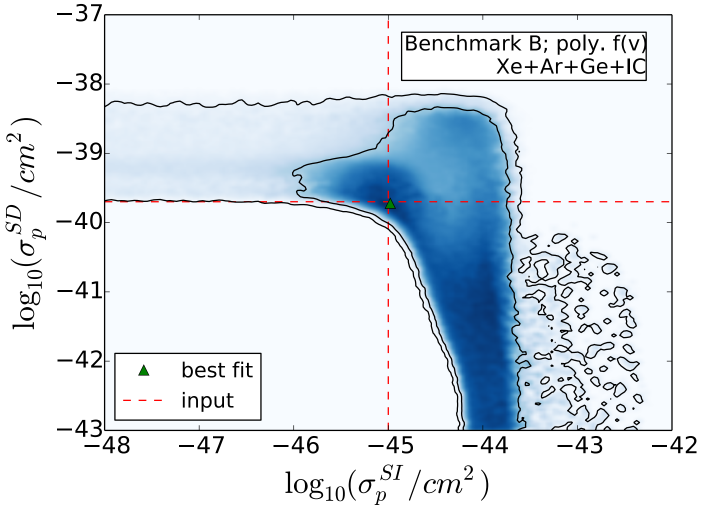




polynomial f(v)





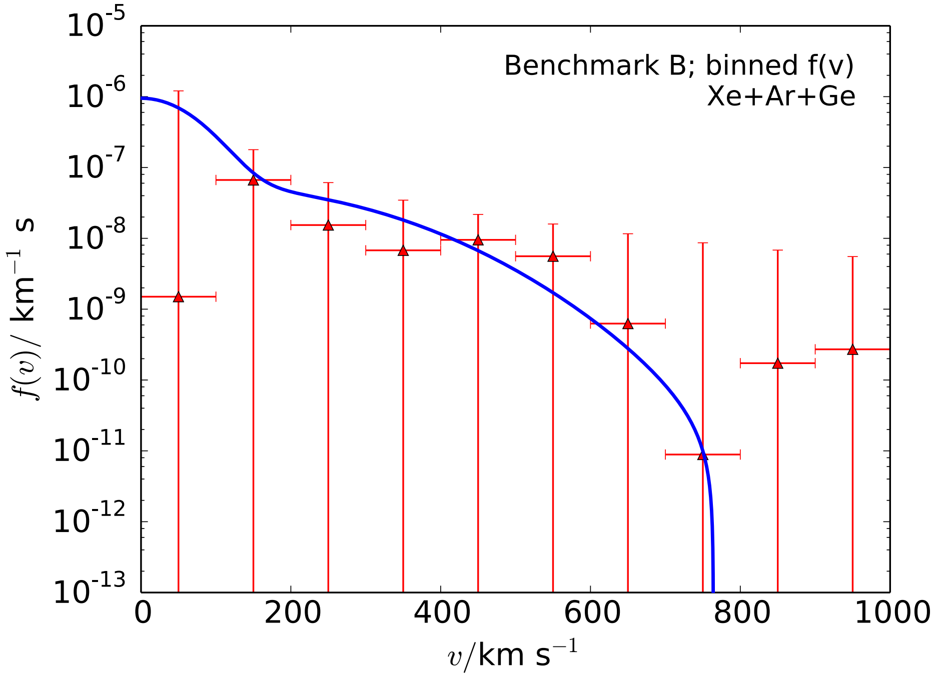


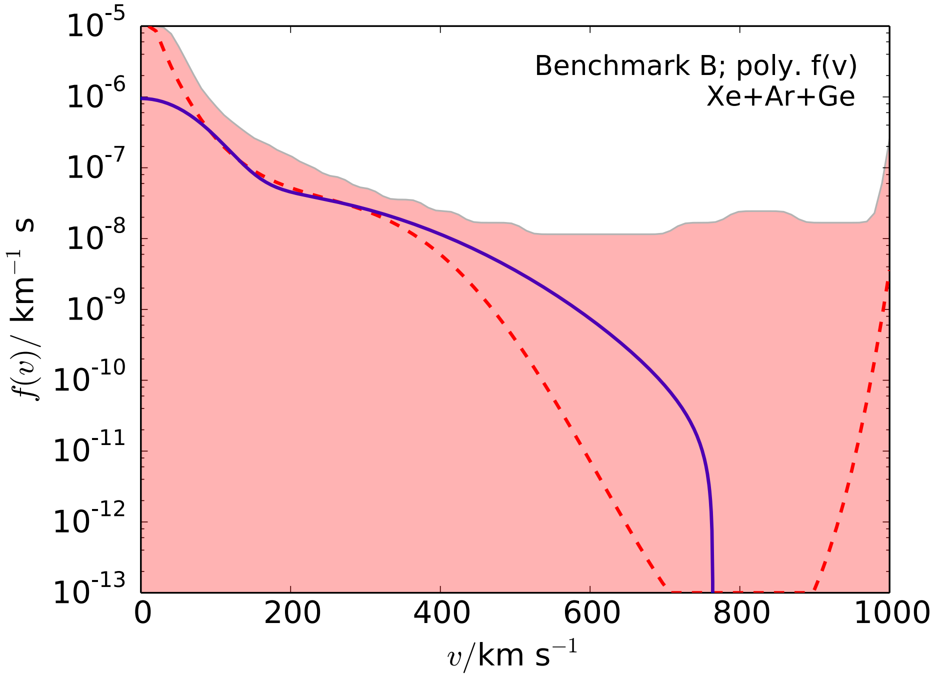
Reconstructed speed distribution

binned f(v)

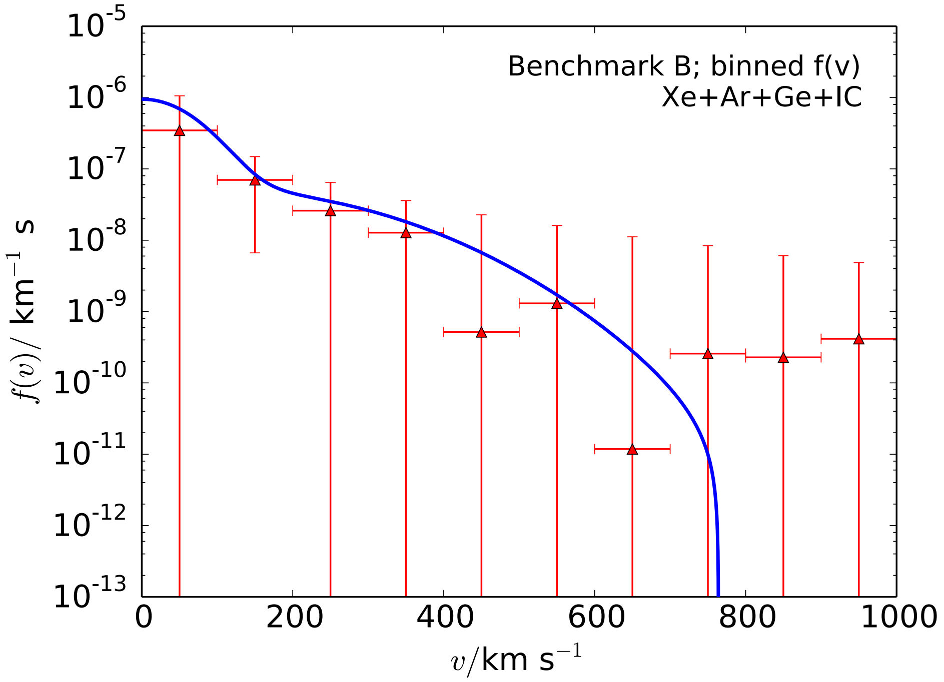
polynomial f(v)

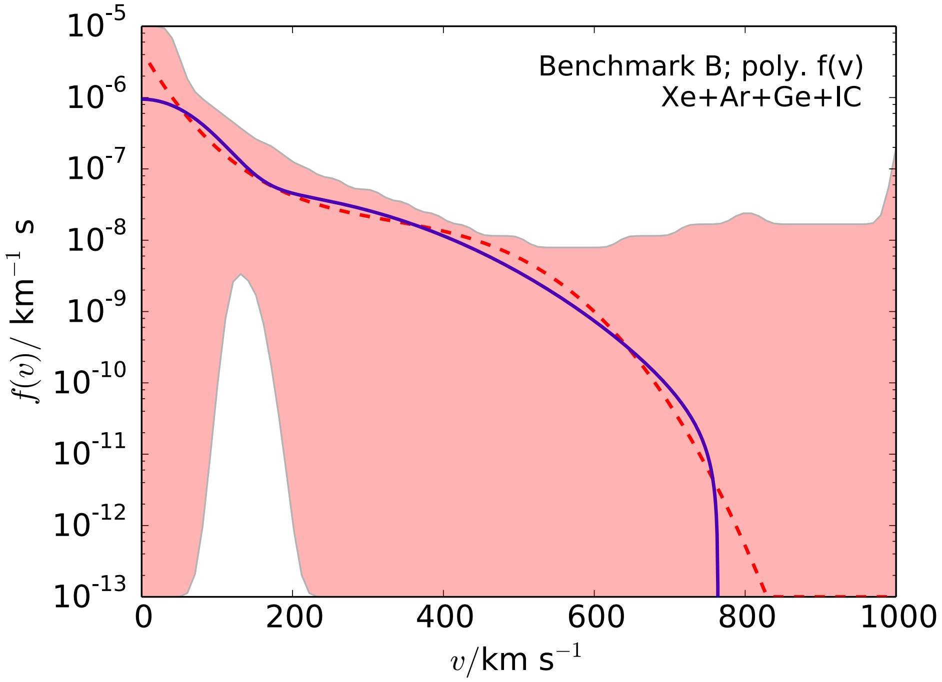
direct detection only





direct detection + IceCube





Preliminary results using 5 polynomials (may not be enough → flat likelihood function → wide contours).

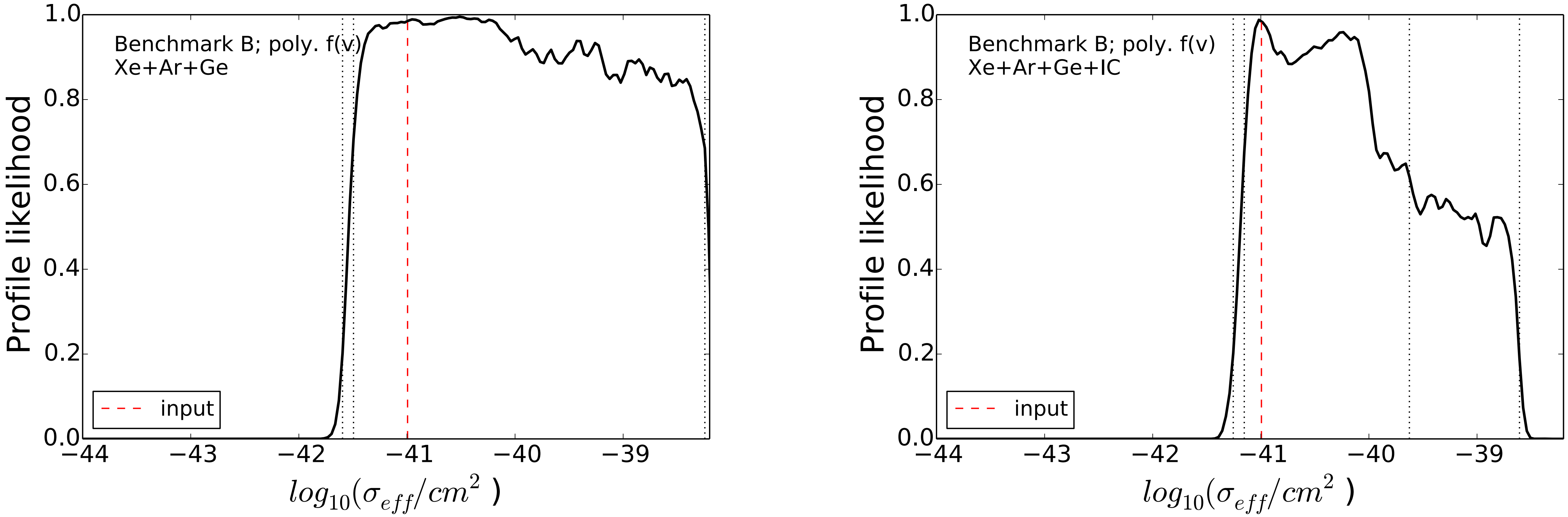
Reconstructed effective cross-section on Ge (for polynomial f(v))

$$\sigma_{\rm eff} = A^2 \sigma_{\rm SI} + a \frac{16\pi}{3} \frac{S_{00}(0)}{2J+1} \sigma_{\rm SD}$$

a=abundance of isotope with spin

direct detection only

direct detection + IceCube



With direct detection data only no upper limit on cross section (due to unknown fraction of WIMPs below threshold).

With addition of IceCube data probe low speeds and get an upper limit, plus a better lower limit.

Summary

- The direct detection energy spectrum depends on both f(v) and the WIMP mass (with a single experiment can't probe the mass without making assumptions about f(v), with multiple experiments can break this degeneracy).

- Can assess compatibility of signals/exclusion limits in speed integral, $g(v_{min})$, space ('integrating out the astrophysics').

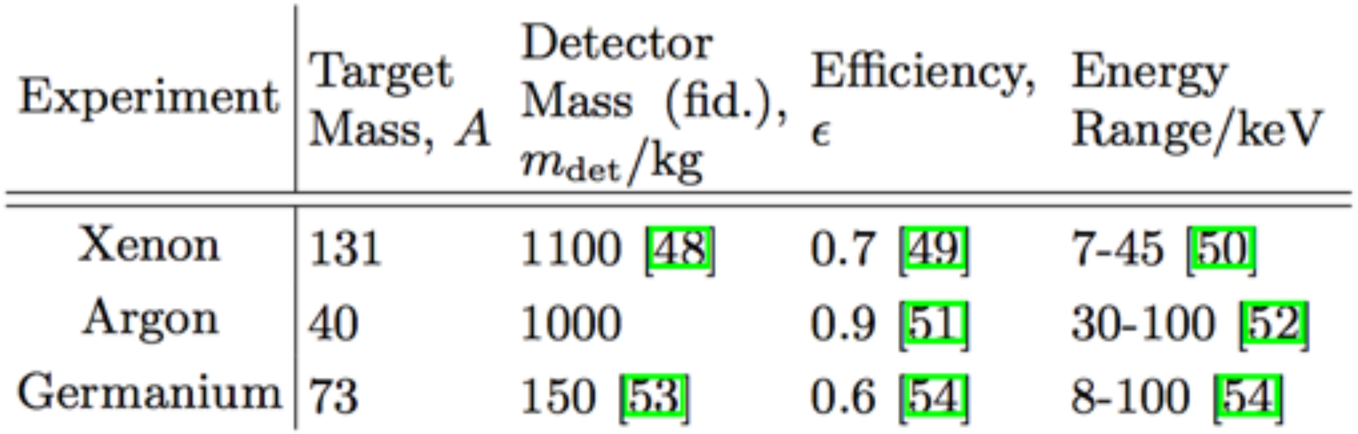
- Parameterising f(v) (or a model for the Milky Way) and marginalising works well **if** actual shape of f(v) is close to assumed shape.

- For unbiased mass measurement use a suitable empirical parameterisation (e.g. shifted Legendre or Chebyshev polynomials), and probe f(v) too.

- Combining direct detection & IceCube data allows unbiased measurement of cross-section and reconstruction of f(v).

Methodology

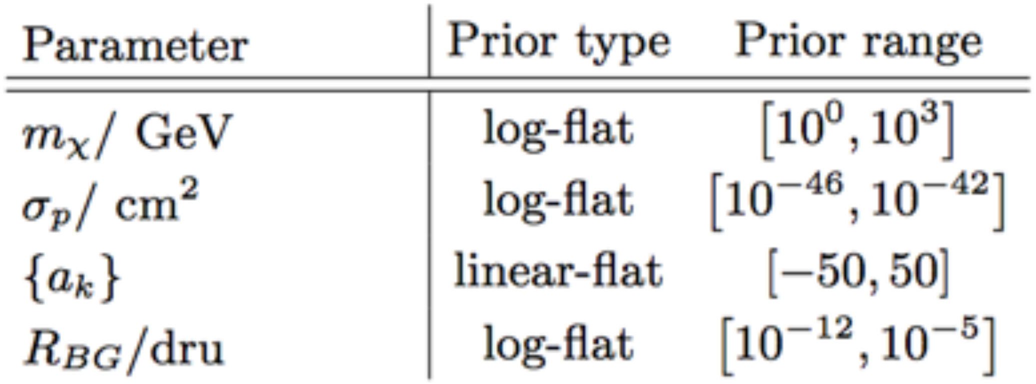
Experimental parameters



Experiment	Target Mass, A	Detector Mass (fid.), m_{det}/kg	Efficiency, ϵ	Energy Range/keV
Xenon	131	1100 [48]	0.7 [49]	7-45 [50]
Argon	40	1000	0.9 [51]	30-100 [52]
Germanium	73	150 [53]	0.6 [54]	8-100 [54]

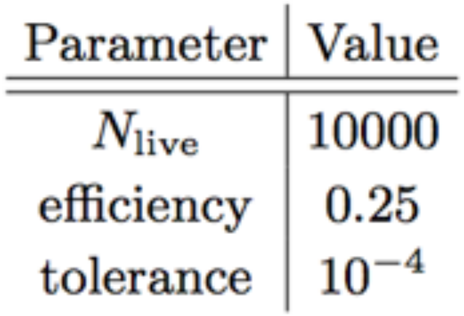
$t_{\mathrm{exp}} = 2\,\mathrm{years}$

Priors



Parameter	Prior type	Prior range
m_χ/ GeV	log-flat	$[10^0, 10^3]$
σ_p/ cm^2	log-flat	$[10^{-46}, 10^{-42}]$
$\{a_k\}$	linear-flat	$[-50, 50]$
R_{BG}/dru	log-flat	$[10^{-12}, 10^{-5}]$

Multinest sampling parameters



Parameter	Value
N_{live}	10000
efficiency	0.25
tolerance	10^{-4}

Asimov data, binned likelihood:
(1 keV bins)

$$\mathcal{L}_{\mathrm{b}} = \prod_{i=1}^{N_{\mathrm{b}}} \frac{N_{\mathrm{e,i}}^{N_{\mathrm{o,i}}} \exp\left(-N_{\mathrm{e,i}}\right)}{N_{\mathrm{o,i}}!}$$

Realisations with Poisson noise, extended Likelihood:

$$\mathcal{L} = \frac{N_{\mathrm{e}}^{N_{\mathrm{o}}} \exp\left(-N_{\mathrm{e}}\right)}{N_{\mathrm{o}}!} \prod_{i=1}^{N_{\mathrm{b}}} P(E_{\mathrm{i}})$$

IceCube: 86 string configuration including DeepCore array, calculate number of events using DarkSUSY, following Arina et al.

Reconstructed cross-section (polynomial f(v))

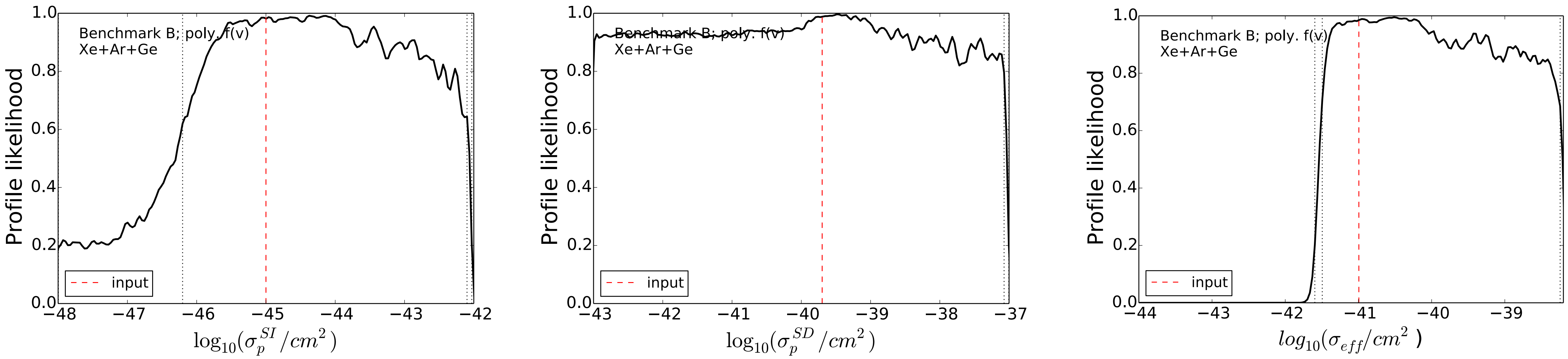
σ_{SI}

σ_{SD}

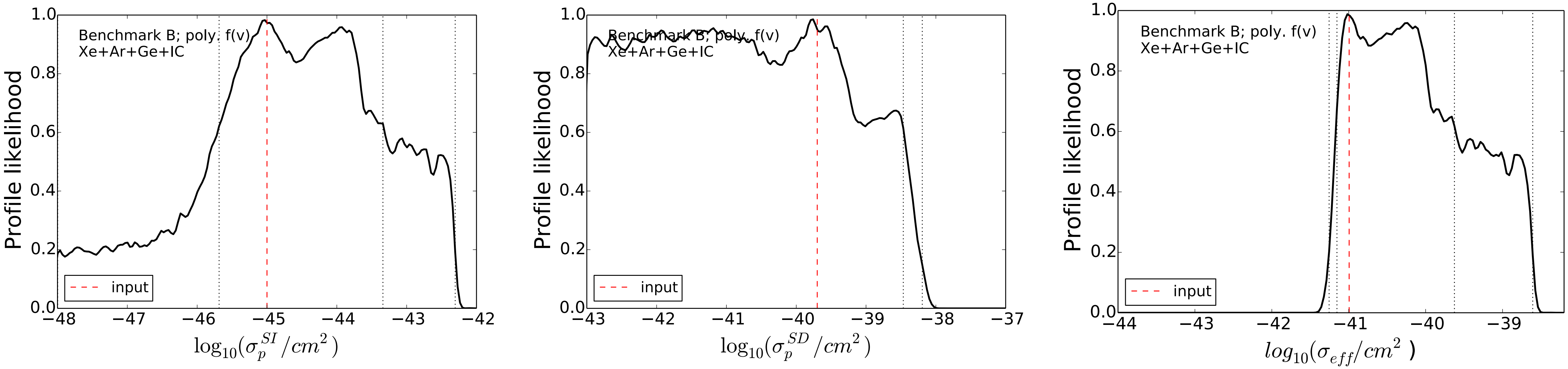
$$\sigma_{\mathrm{eff}} = A^2\sigma_{\mathrm{SI}} + a\frac{16\pi}{3}\frac{S_{00}(0)}{2J+1}\sigma_{\mathrm{SD}}$$

a=abundance of isotope with spin

direct detection only



direct detection + IceCube



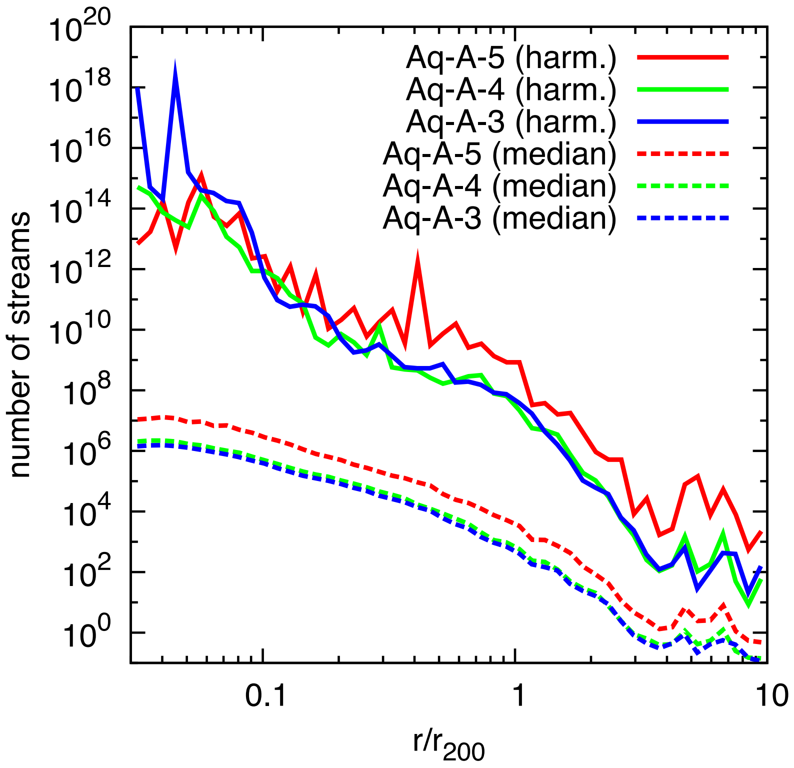
fine structure in ultra-local DM velocity distribution?

Vogelsberger & White:

Follow the fine-grained phase-space distribution, in Aquarius simulations of Milky Way like halos.

From evolution of density deduce ultra-local DM distribution consists of a huge number of streams (but this assumes ultra-local density= local density).

At solar radius <1% of particles are in streams with $\rho > 0.01\rho_0$.



number of streams as a function of radius

calculated using harmonic mean/median stream density

Schneider, Krauss & Moore:

Simulate evolution of microhalos. Estimate tidal disruption and heating from encounters with stars, produces 10^2-10^4 streams in solar neighbourhood.