Comments on the velocity distribution $f(v)$ of local Dark Matter

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Motivation

Differential rate depends on the velocity distribution $f(v)$

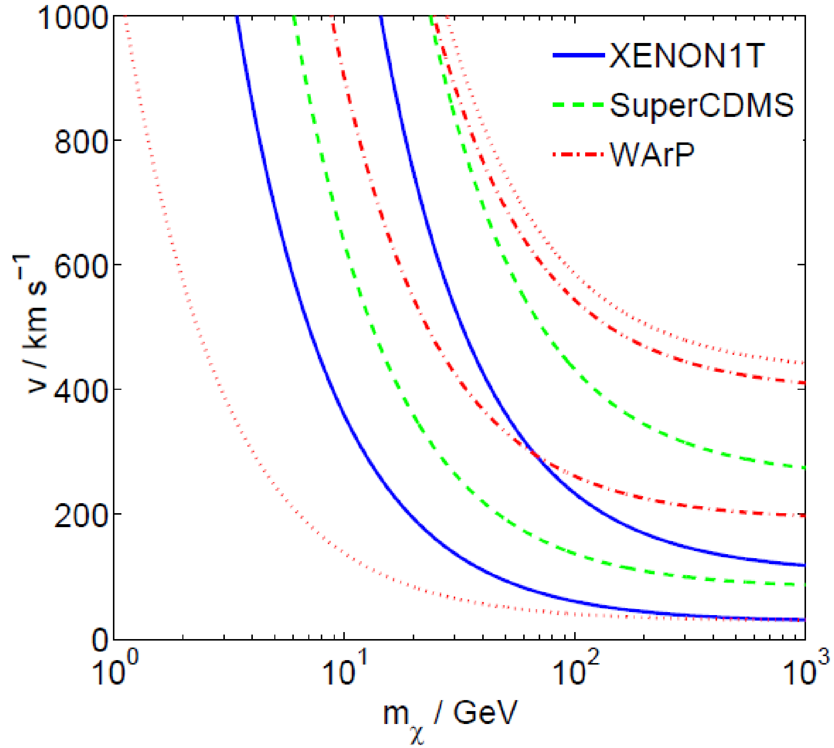
$$\frac{dR}{dE}(E) = \frac{\sigma_{p,\chi}\,\rho_\chi(R_0)}{2\mu_{p,\chi}^2\, m_\chi} A^2 F^2(E) \int_{\mathcal{V}(v_{\min})} \frac{f(\mathbf{v})}{v} d^3\mathbf{v} = \frac{\sigma_{p,\chi}\,\rho_\chi(R_0)}{2\mu_{p,\chi}^2\, m_\chi} A^2 F^2(E)\, g(v_{\min})$$

$$v_{\min} = \left(\frac{E m_A}{2\mu_{A,\chi}^2}\right)^{1/2}$$

Experimental results are usually presented assuming the Standard Halo Model (SHM), with σ related to v_c=220 km/h, v_{esc}=544 km/s and $\rho(R_0)$=0.3 GeV cm^{-3}

$$f(\mathbf{v}) \propto \exp\left(-\frac{3|\mathbf{v}|^2}{2\sigma^2}\right)$$

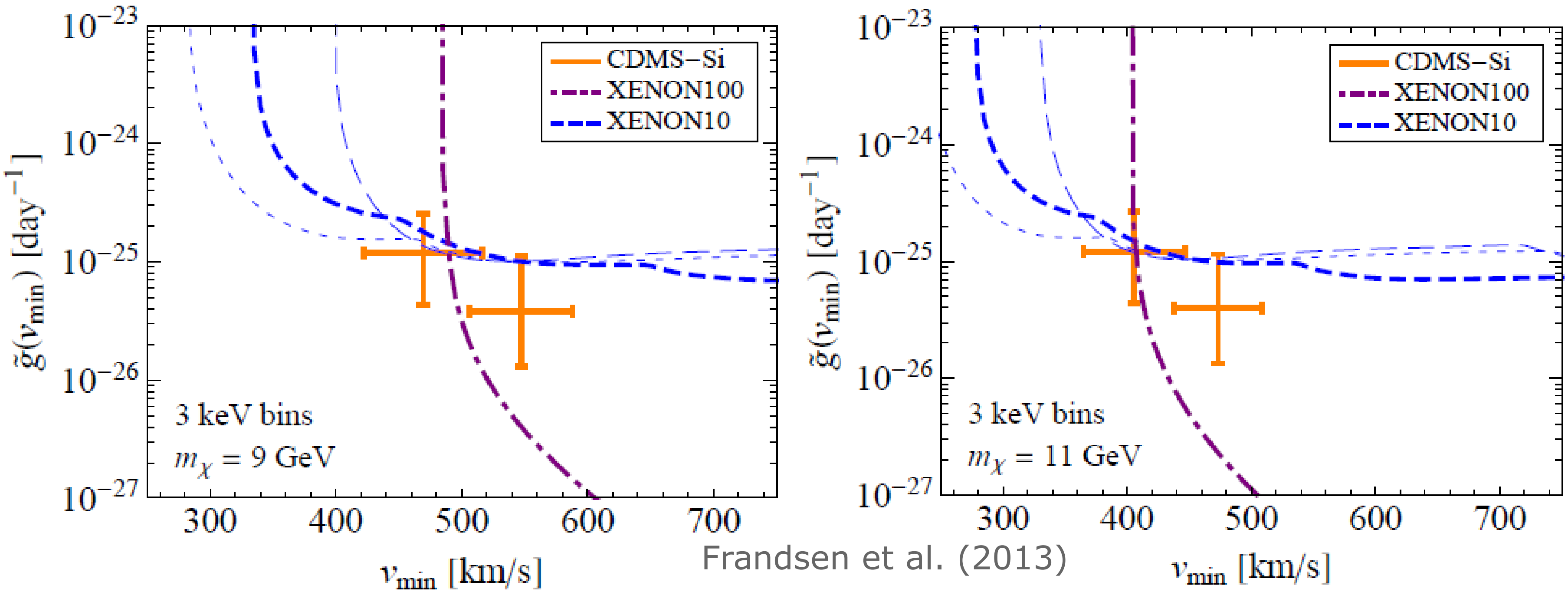
Kavanagh & Green (2012)



Motivation

Model independent way of plotting experimental results:

- extended to include energy resolution, "non-standard" interactions, annual modulation, …
- particularly useful when comparing different data sets



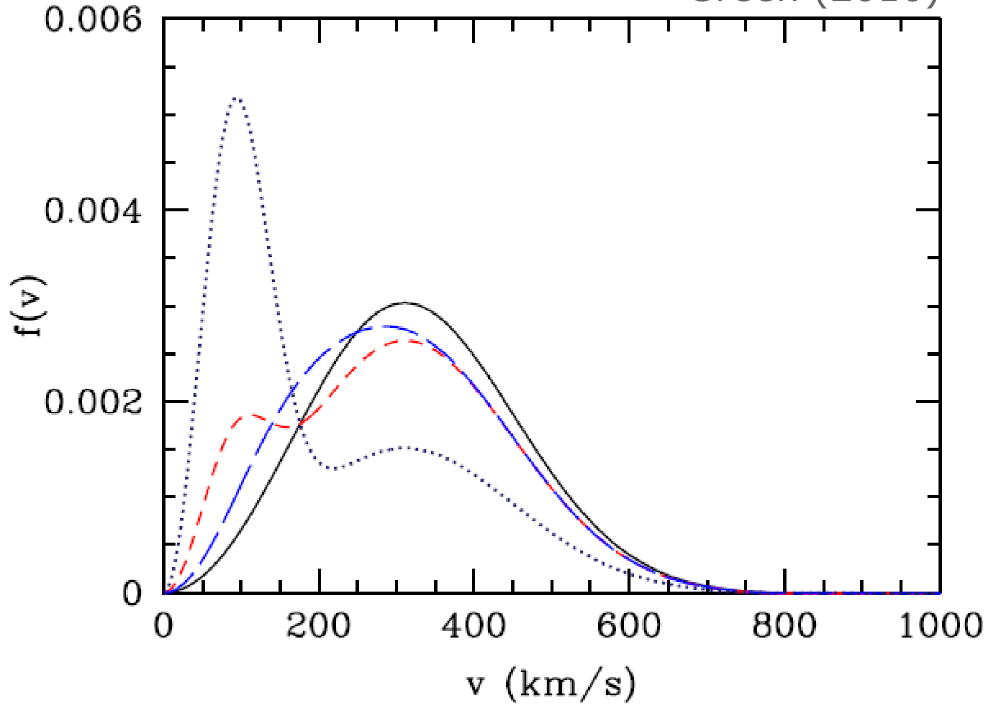
Frandsen et al. (2013)

Motivation

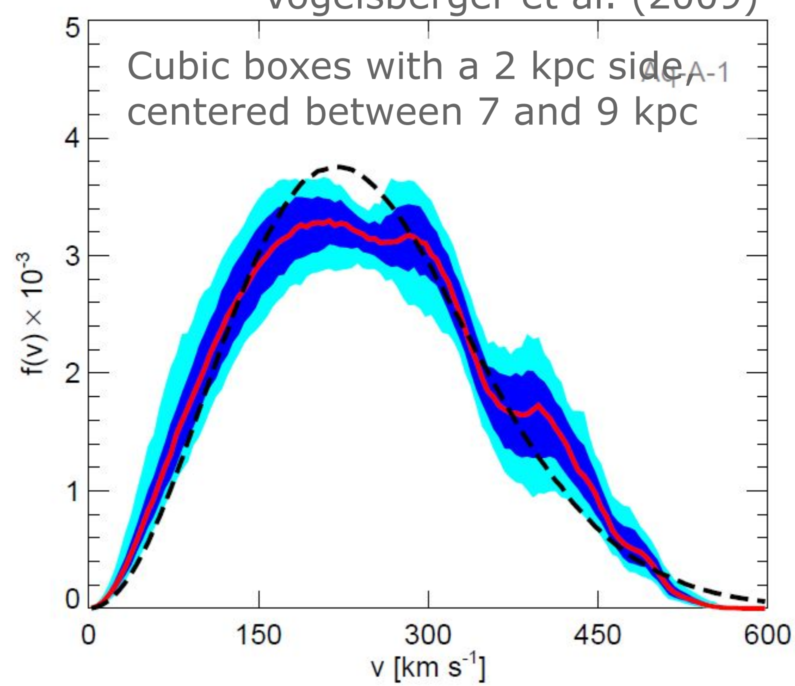
Learning something about the Milky Way through the determination of the velocity distribution.
Features in $f(v)$ can be associated to

- **dark disk**
- **merging history** of our own Galaxy

Green (2010)

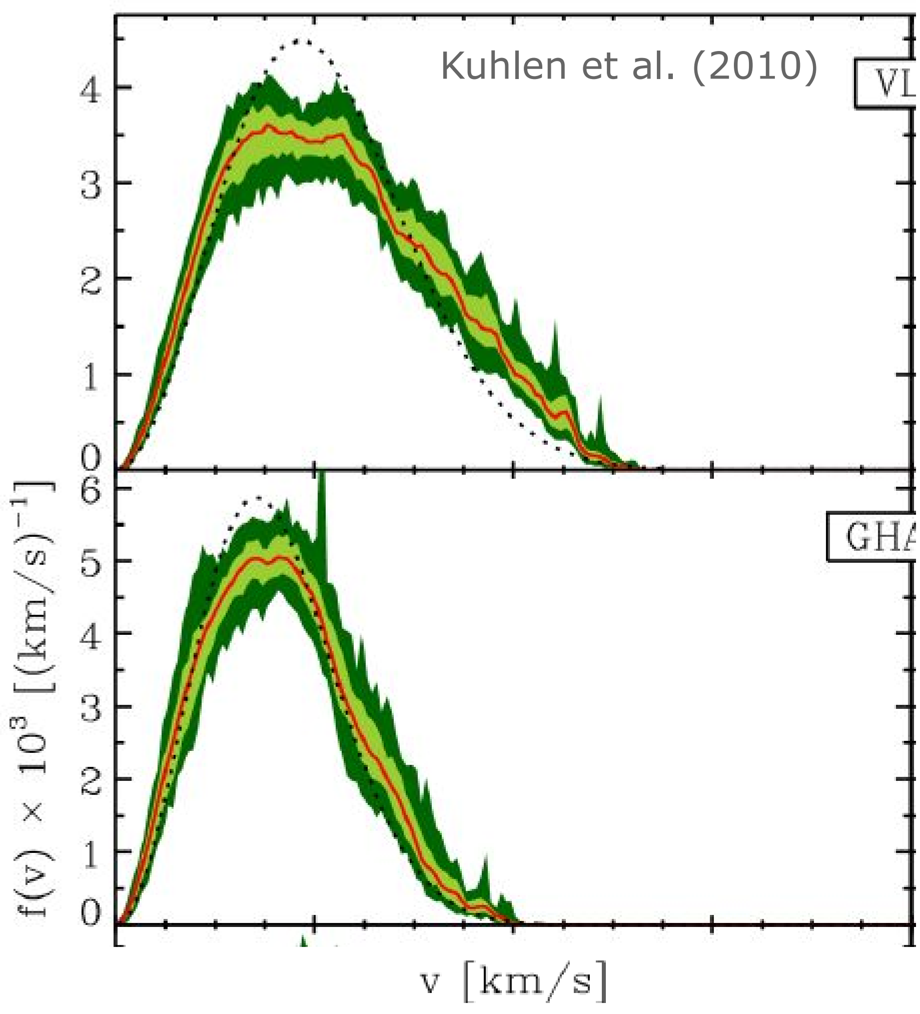


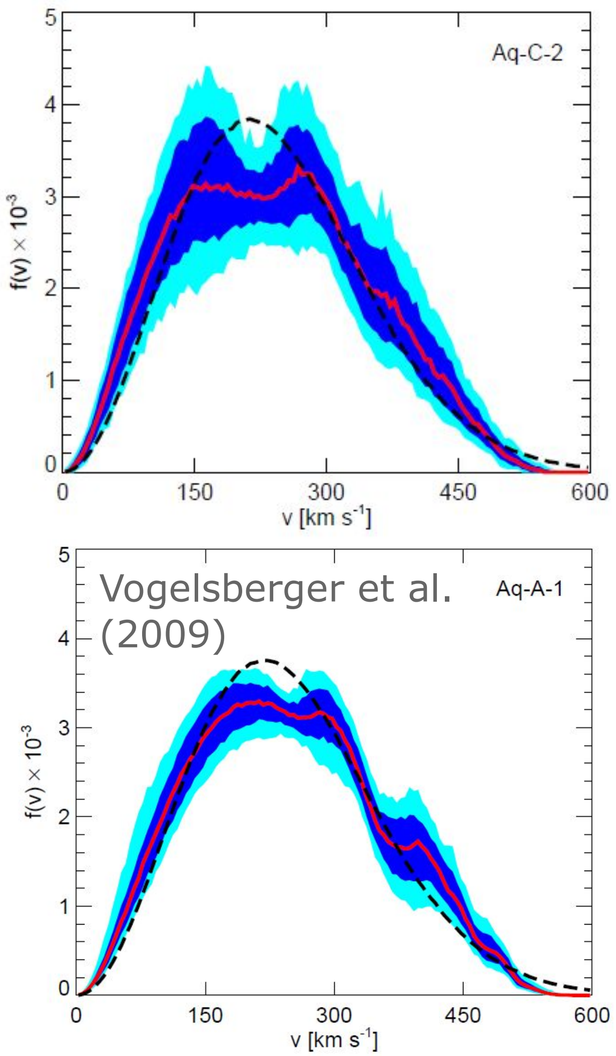
Vogelsberger et al. (2009)



N-body simulations

Deviations from SHM looking at DM-only simulations.



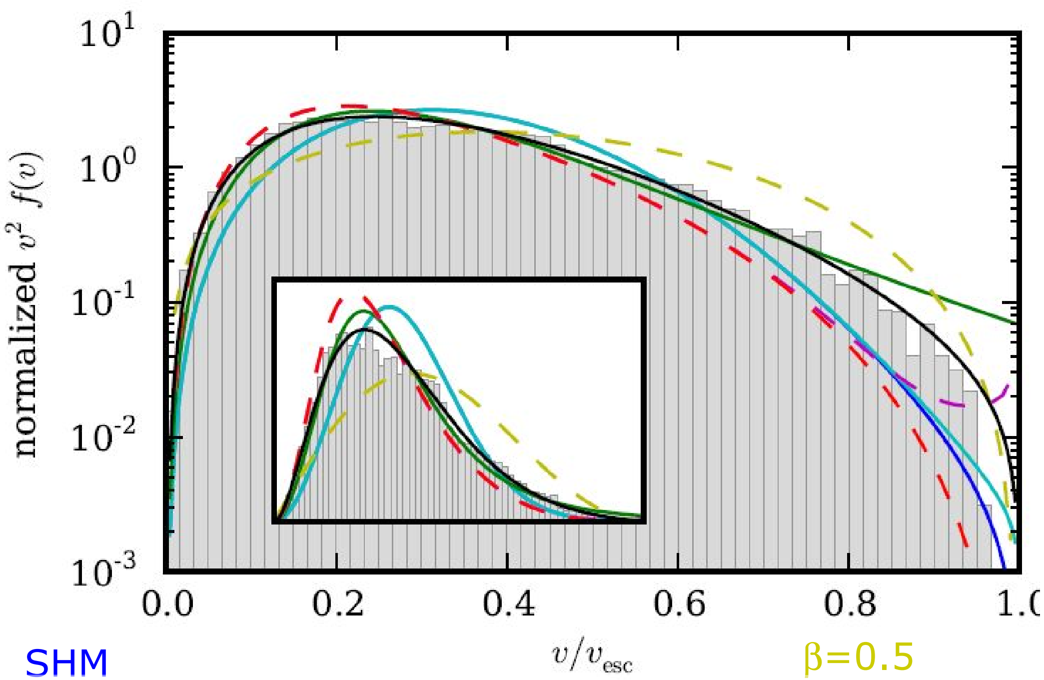


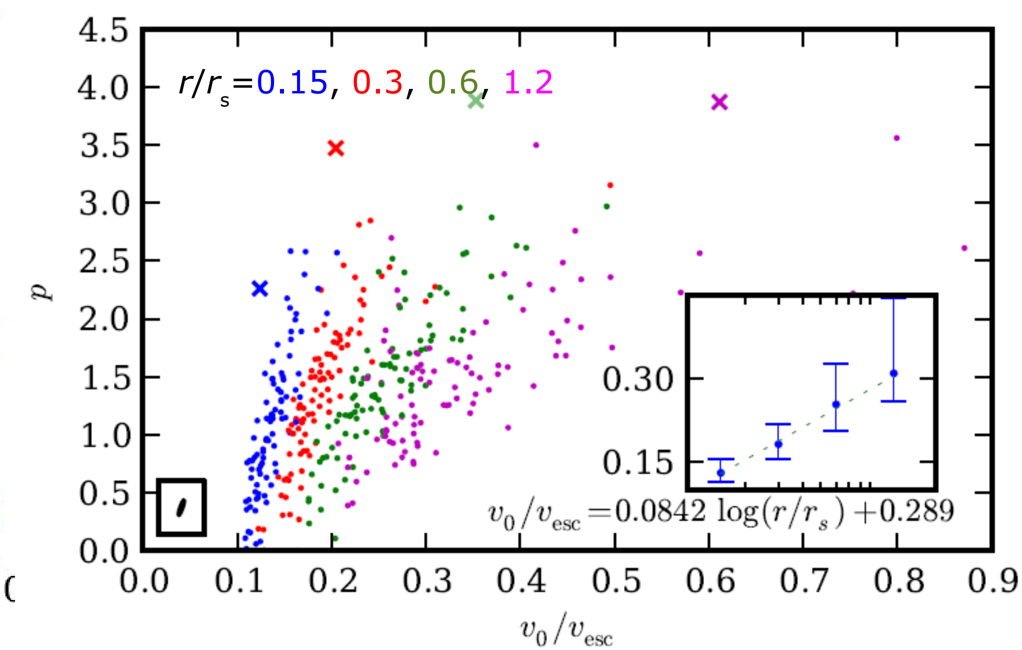
N-body simulations

Consider more DM halos and find **better parametrizations**

$$f(v) = \begin{cases} A \exp(-v/v_0)(v_{\rm esc}^2 - v^2)^p & \text{if } 0 \le v \le v_{\rm esc} \\ 0 & \text{otherwise} \end{cases}$$

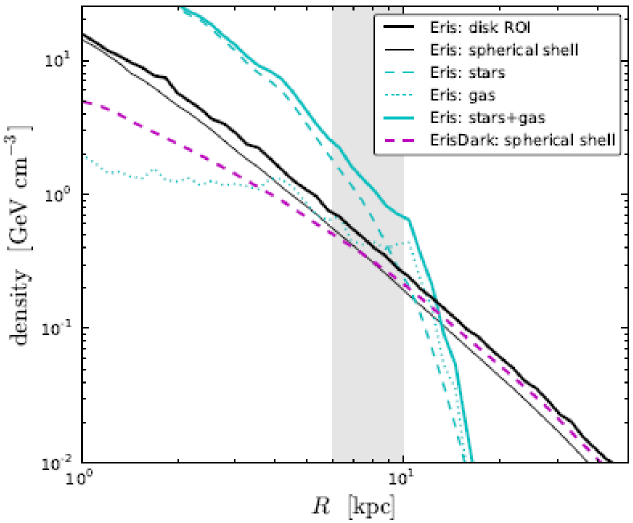
Mao et al. (2013)

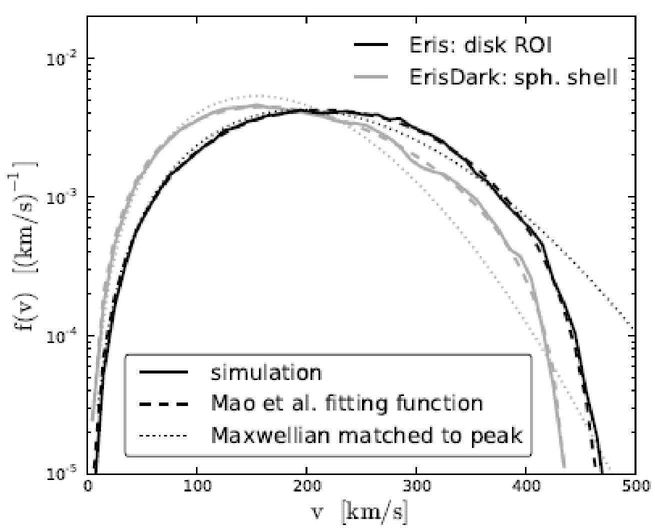


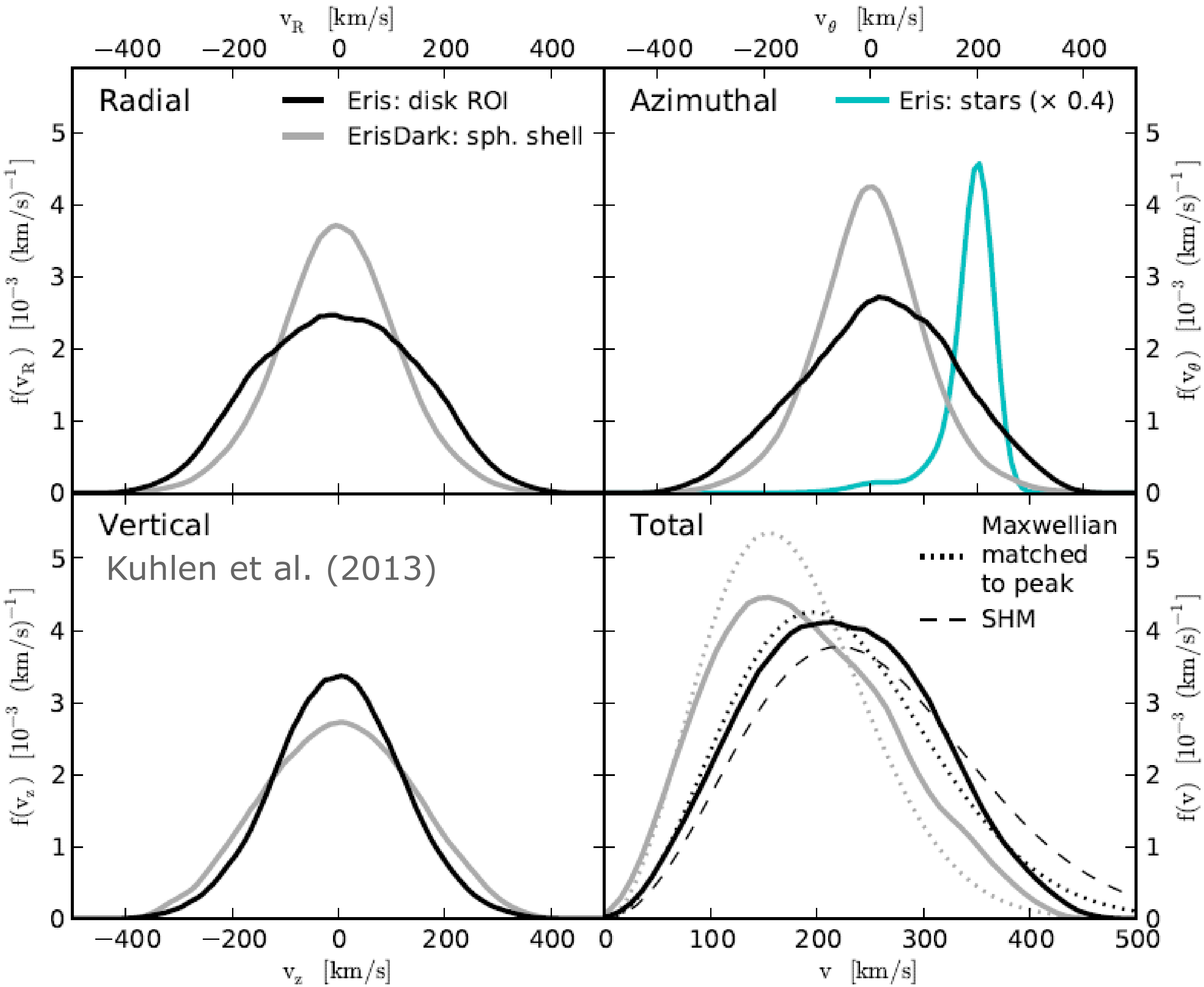


SHM
Double powerlaw
Tsallis
Eddington
Osipkov-Merritt
β=0.5
Mao et al. (2013)

N-body simulations with baryons







Self-consistent solutions

$f(v)$ comes from the 6-dimensional phase-space density distribution $F(\mathbf{x},\mathbf{v})$

$$\rho_\chi(\mathbf{x}) = \int F(\mathbf{x},\mathbf{v})\, d^3\mathbf{v} \qquad f(\mathbf{v}) = \frac{F(\mathbf{x},\mathbf{v})}{\rho_\chi(\mathbf{x})}$$

$$\langle v_i v_j \rangle = \frac{\int F(\mathbf{x},\mathbf{v}) v_i v_j\, d^3\mathbf{v}}{\rho_\chi(\mathbf{x})} = \int f(\mathbf{x},\mathbf{v}) v_i v_j\, d^3\mathbf{v}$$

Symmetries of the system are made evident once $F(\mathbf{x},\mathbf{v})$ is written as a function of integrals of motion:
$F(E,L)$ in the case of a static spherical system (anisotropic):

$$E = \Phi(\mathbf{x}) - \frac{|\mathbf{v}|^2}{2} \qquad L = r\sqrt{v_\theta^2 + v_\phi^2}$$

$$\langle v_r^2 \rangle \neq \langle v_\theta^2 \rangle = \langle v_\phi^2 \rangle \qquad \beta = 1 - \frac{\langle v_\theta^2 \rangle}{\langle v_r^2 \rangle}$$

Self-consistent solutions

Adding the assumption of an isotropic velocity dispersion tensor, the phase-space density can be written as $F(E)$: Eddington formalism.

$$\rho_\chi(r) = 4\pi \int dv\, v^2 F(E) = 4\pi \int dE\, F(E) \sqrt{2(\Phi - E)}$$

$$F(E) = \frac{1}{\sqrt{8}\pi^2} \left[\int_0^E \frac{d\Phi}{\sqrt{E - \Phi}} \frac{d^2\rho_\chi}{d\Phi^2} + \frac{1}{\sqrt{E}} \left. \frac{d\rho_\chi}{d\Phi} \right|_{\Phi=0} \right]$$

$F(E)$ can be inferred, once the gravitational potential is known.

By constraining the mass model of the MW it is possible to determine $f(v)$.

Mass model for the MW

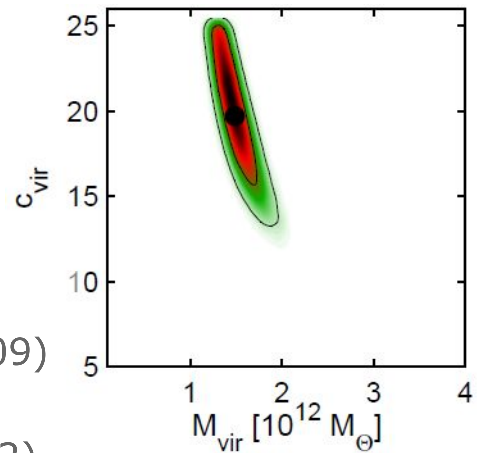
Determine a model for the different mass components of the MW:

- DM halo
- disk
- bulge/bar
- gas

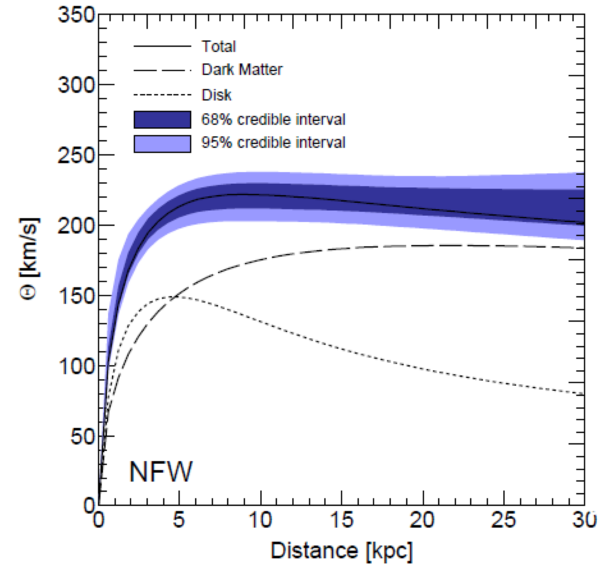
Consider a certain number of astronomical observations to constraint the model and fix the gravitational potential:

- proper motion of SgrA* (local circular velocity)
- local surface density $\Sigma(R_0, 1.1\ \mathrm{kpc})$ and $\Sigma_*(R_0)$
- rotation curve
- proper motion of masers in high-mass star-forming regions
- microlensing
- velocity dispersion at large distances

Eddington $F(E)$

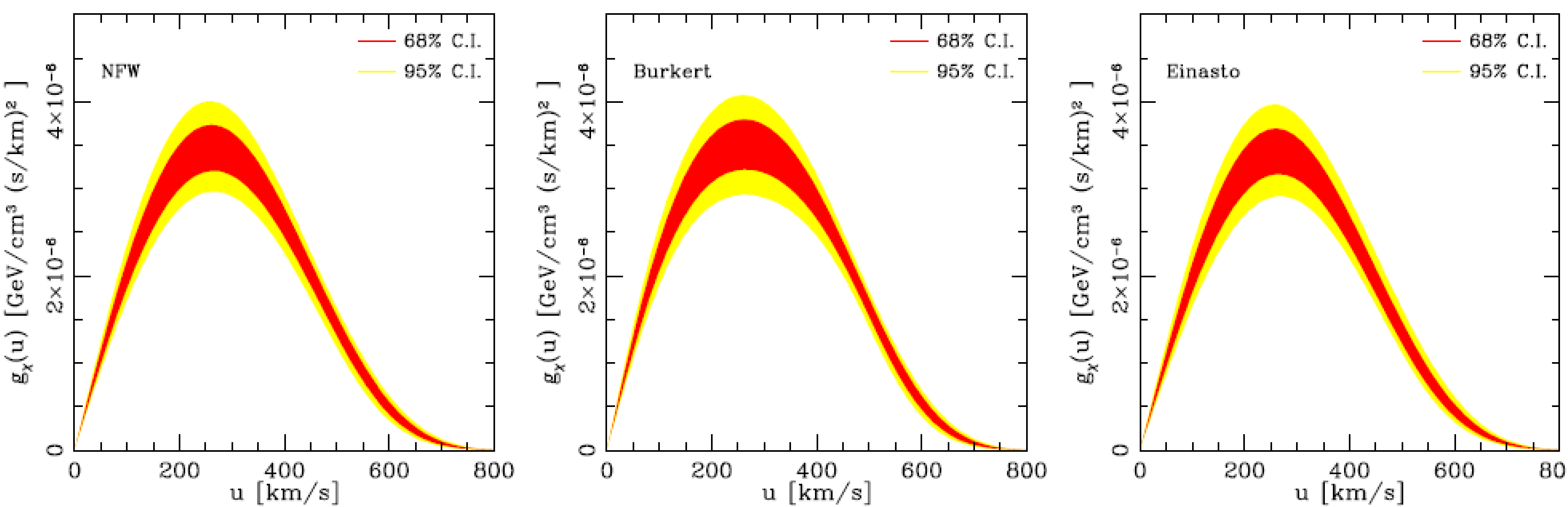


Catena & Ullio (2009)



Fornasa & Green (2013)

Catena & Ullio (2012)



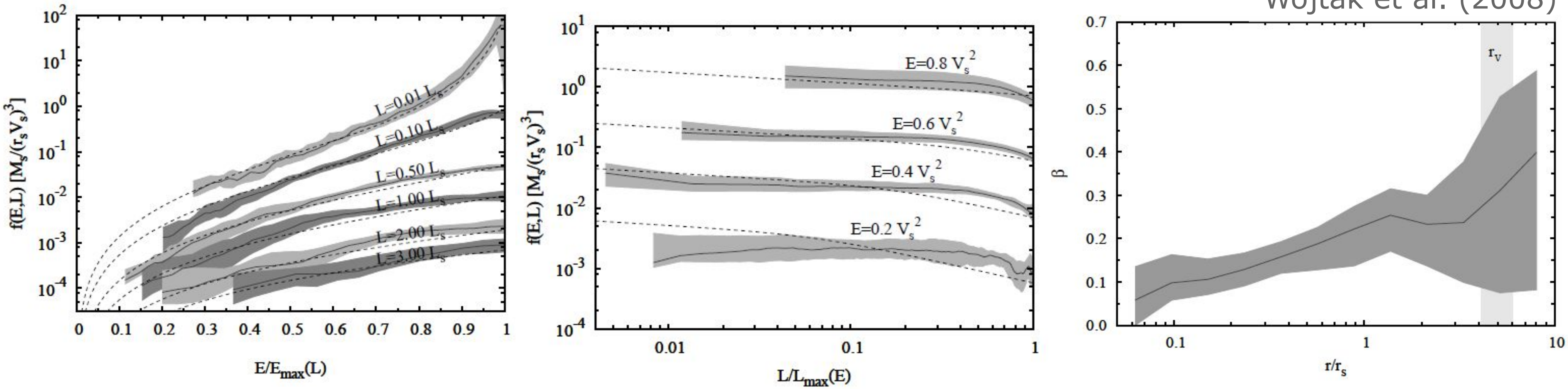
Anisotropic self-consistent $f(v)$

$$\rho_\chi(r) = 4\pi \int_0^{\Phi(r)} dE \int_0^{r\sqrt{2[\Phi(r)-E]}} dL \, \frac{F(E,L)}{r^2} \frac{1}{\sqrt{2[\Phi(r)-E]}} \frac{L}{\sqrt{1-\frac{L^2}{2r^2[\Phi(r)-E]}}}$$

$$F(E,L) = F_E(E)\, F_L(L)$$

$$F_L(L) = \left(1 + \frac{L^2}{2L_0^2}\right)^{-\beta_\infty+\beta_0} L^{-2\beta_0}$$

Wojtak et al. (2008)



Anisotropic self-consistent *f*(*v*)

$$\rho_\chi(r) = 4\pi \int_0^{\Phi(r)} dE \int_0^{r\sqrt{2[\Phi(r)-E]}} dL \, \frac{F(E,L)}{r^2} \frac{1}{\sqrt{2[\Phi(r)-E]}} \frac{L}{\sqrt{1-\frac{L^2}{2r^2[\Phi(r)-E]}}}$$

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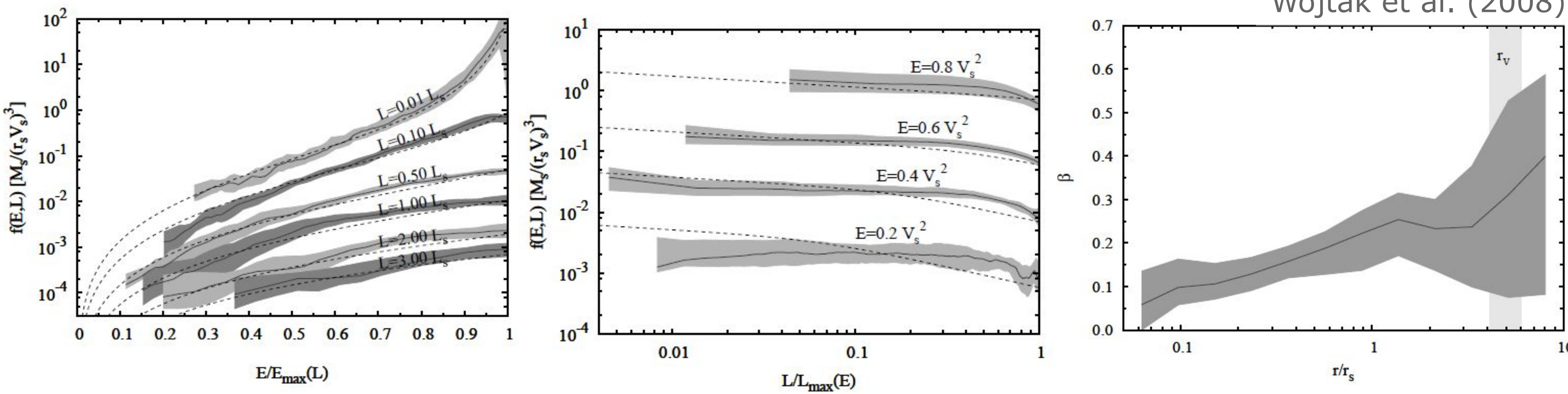
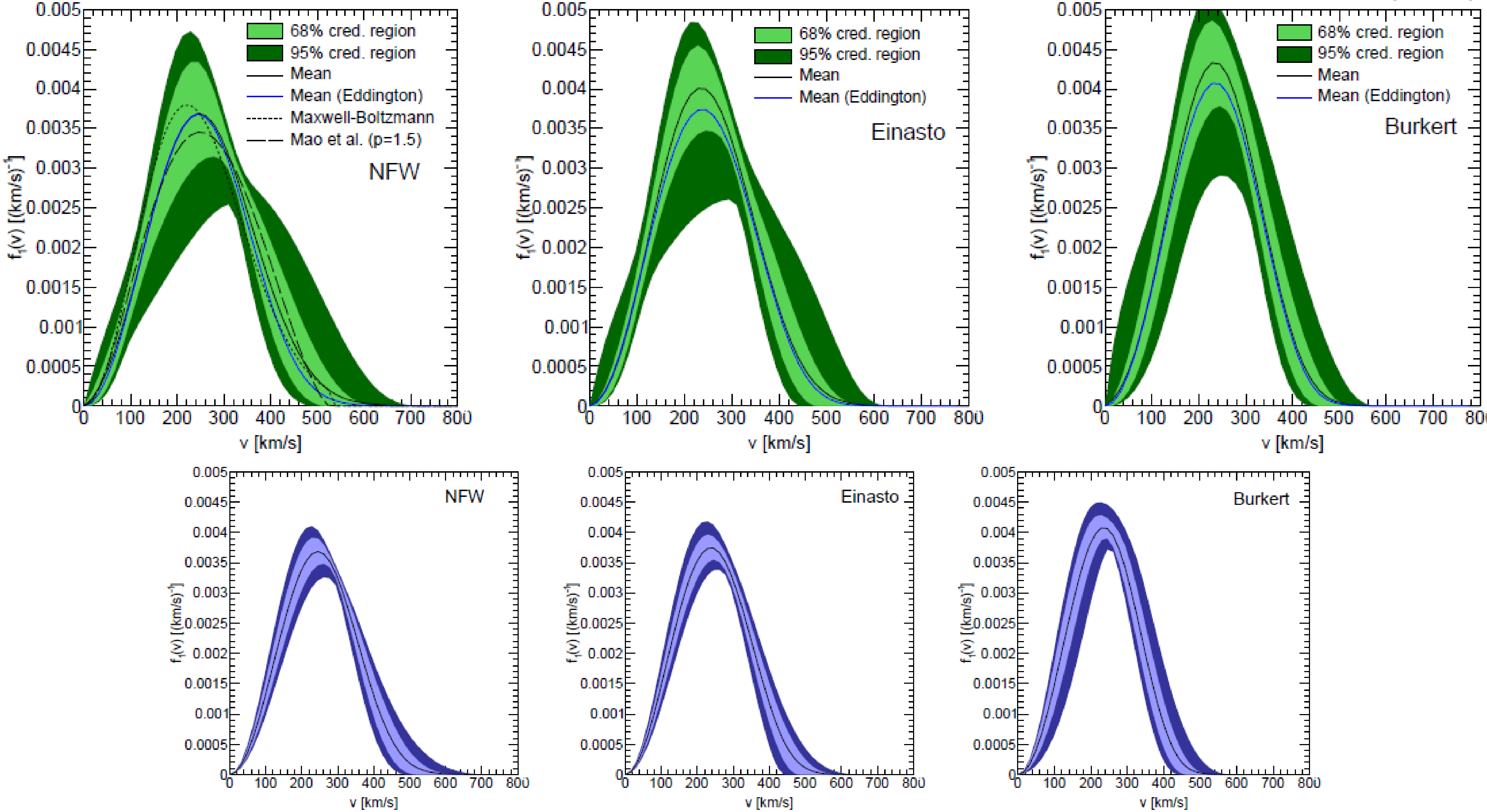


Anisotropic self-consistent $f(v)$

Fornasa & Green (2013)



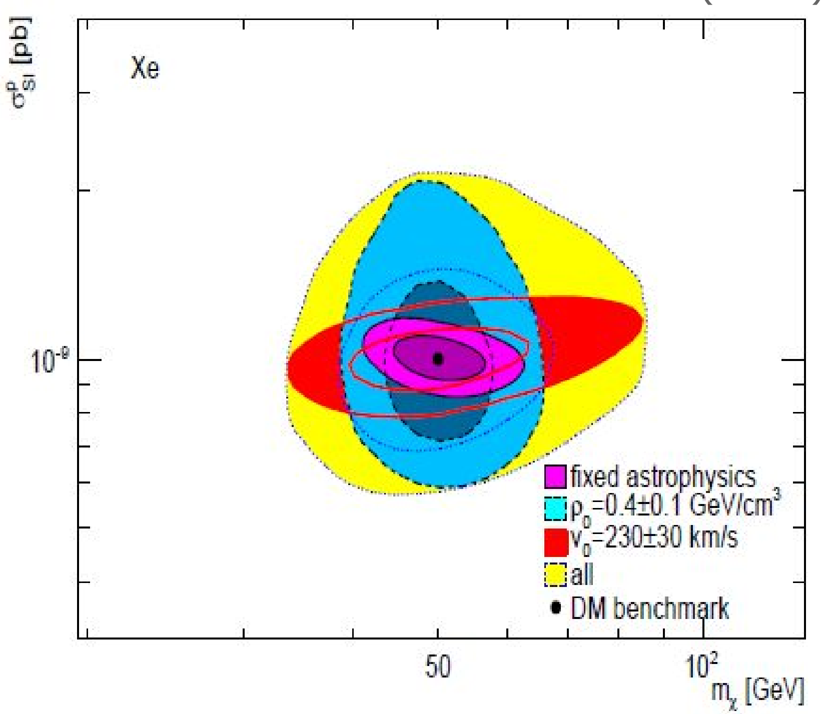
Determine *f*(*v*) via direct detection

- assuming that a certain number of events will be detected in the future
- including the parameters defining *f*(*v*) in the list of quantities to be reconstructed
- biased reconstruction may occur

Pato et al. (2011)

$$f(v) = \exp\left[-\sum_{k=0}^{N-1} a_k \tilde{P}(v/v_{\max})\right]$$

Kavanagh & Green (2013)
Kavanagh (2013)
Peter et al. (2014)



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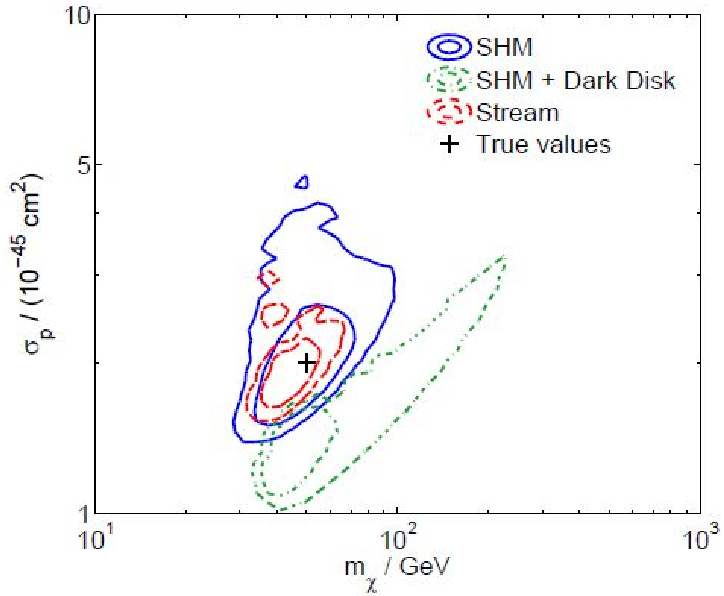
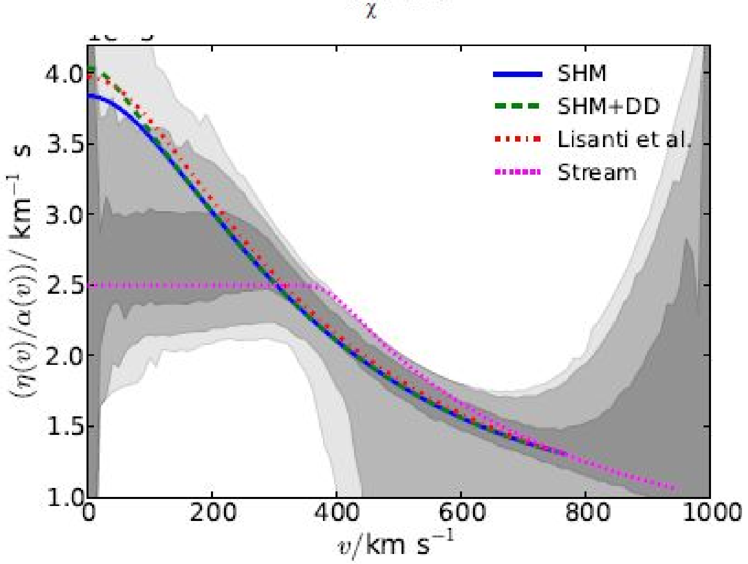
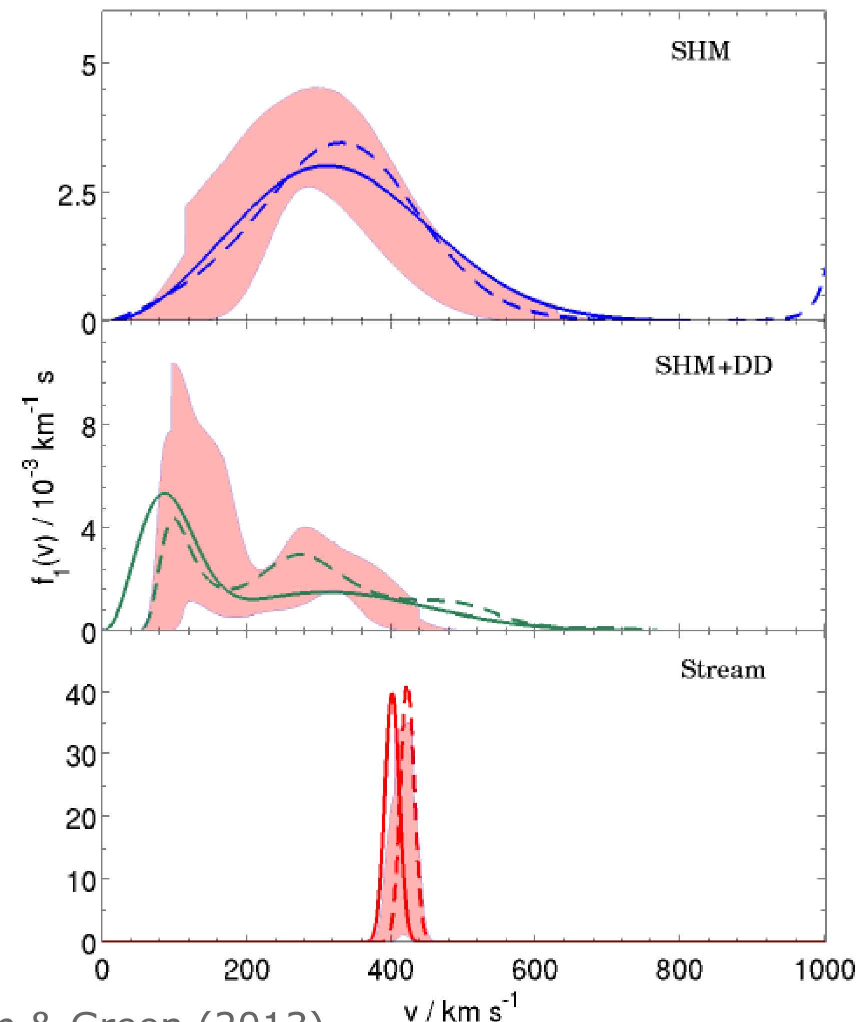


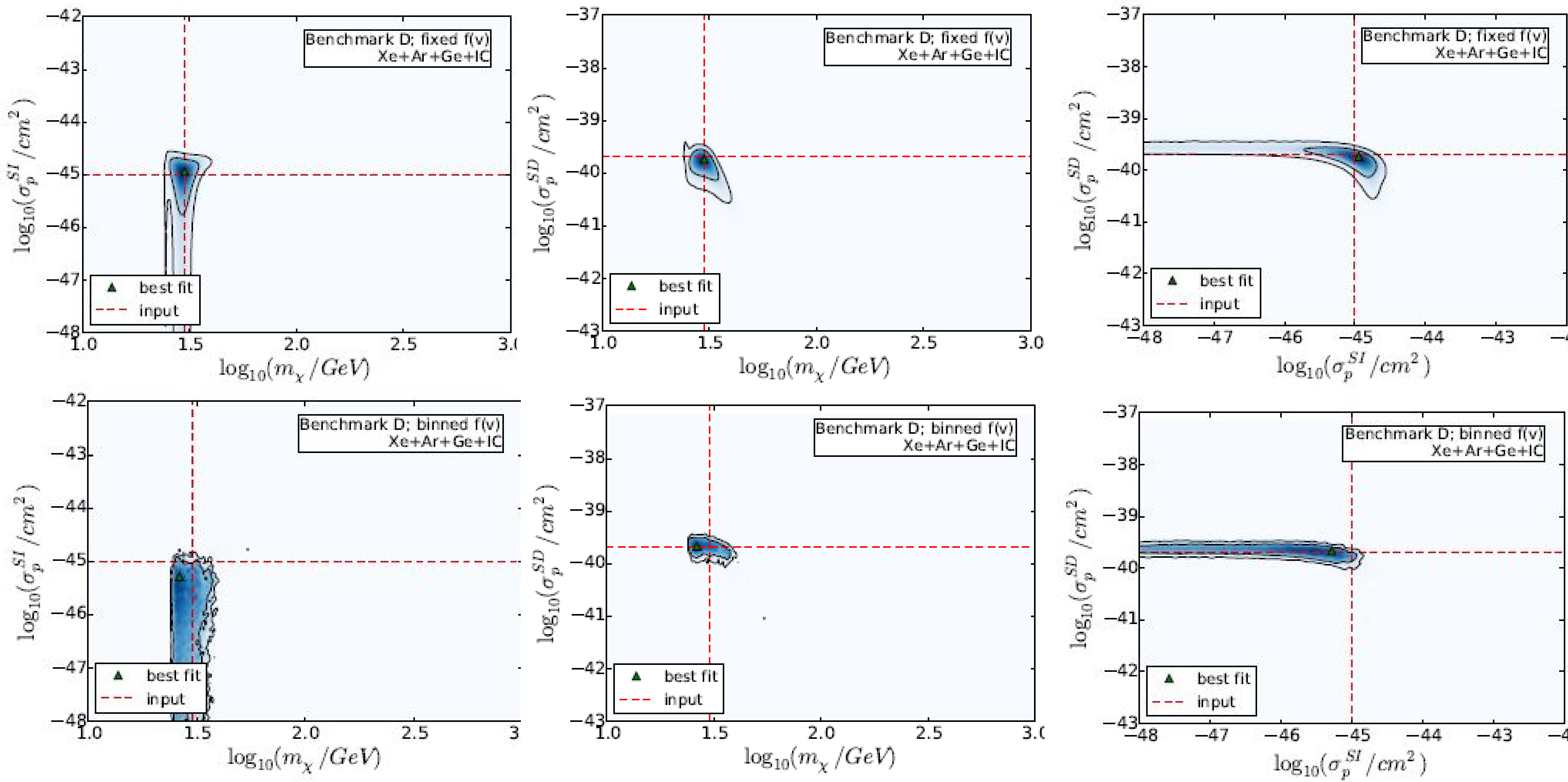




Kavanagh & Green (2013)

Adding neutrino telescopes

Number of neutrinos from the Sun depends on the **low-velocity tail** of $f(v)$.



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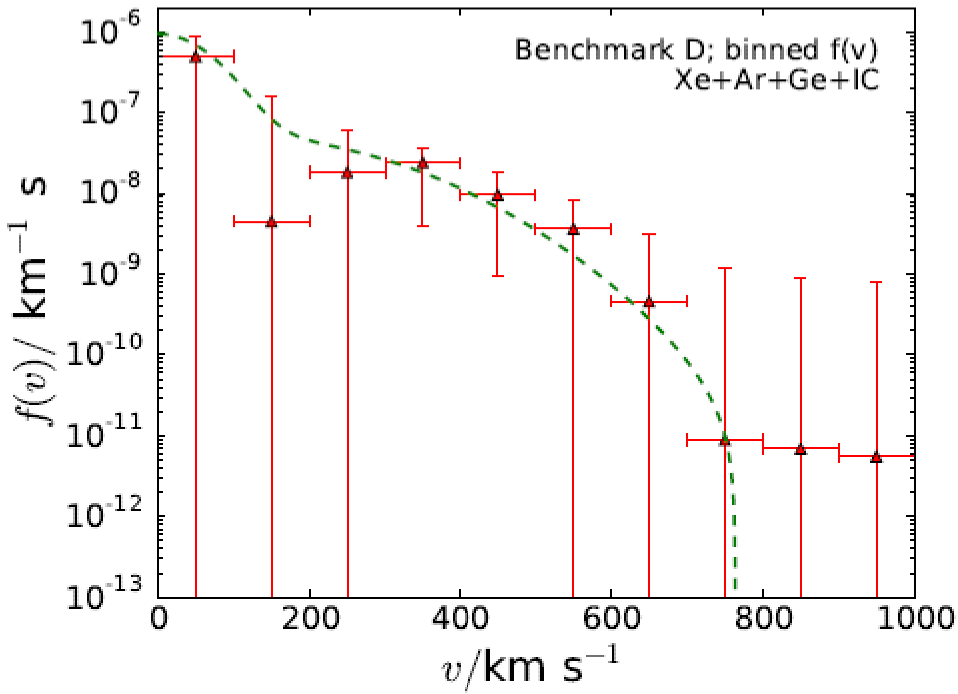

Conclusions

- constraining $f(v)$ to interpret direct detection data and learn about the composition and formation of the MW
- N-body simulations are very informative (progress expected by hydro-dynamic simulations)
- self-consistent solutions available for anisotropic models
- WIMP astronomy: direct detection and neutrino telescopes to measure $f(v)$