26 May 2008, NORDITA, Stockholm

Excess covariance in financial returns: empirical data and theoretical models

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EU-NEST project COMPLEXMARKETS

what are markets for?

(individual optimum) x N $\neq$ *global optimum*

- markets allocate optimally resources
  (It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest. A. Smith)

- markets incorporate efficiently available information in prices (Fama)
  statistical regularities correspond to what agents are not (yet) doing

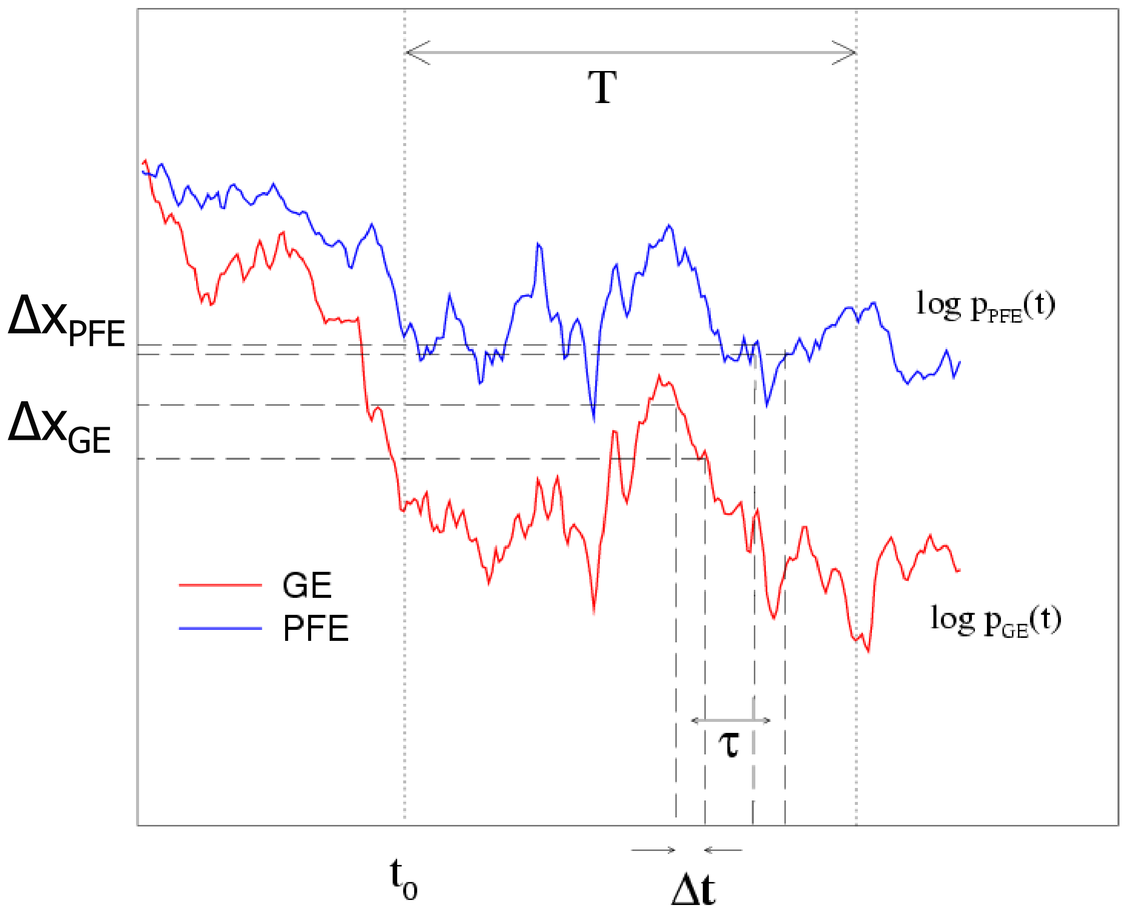
- markets allow individuals to cope with uncertainty and reduce risk
  statistical features show what traders are doing

Outline

- Data:
  Why are stock prices so correlated?
  Why are correlations so volatile?
- A phenomenological model
- A micro-economic model

correlation between assets

$$\mathrm{Cov}_{\mathrm{GE,PFE}} = E[\Delta x_{\mathrm{GE}} \Delta x_{\mathrm{PFE}}]$$



N stocks
T = window size
t_0 = initial time
Δt = time scale (1 day)
τ = time shift (=0 here)

More precisely:

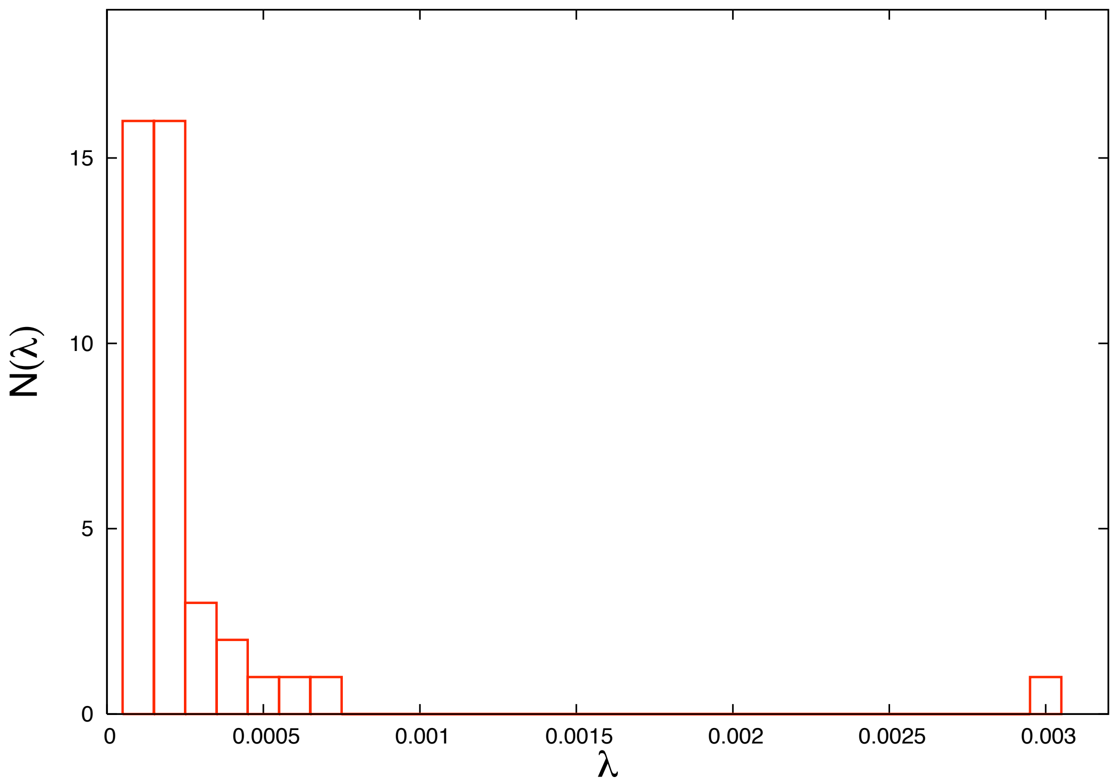
$\Delta t = 1\ day$

$i, j = 1, \ldots, N$

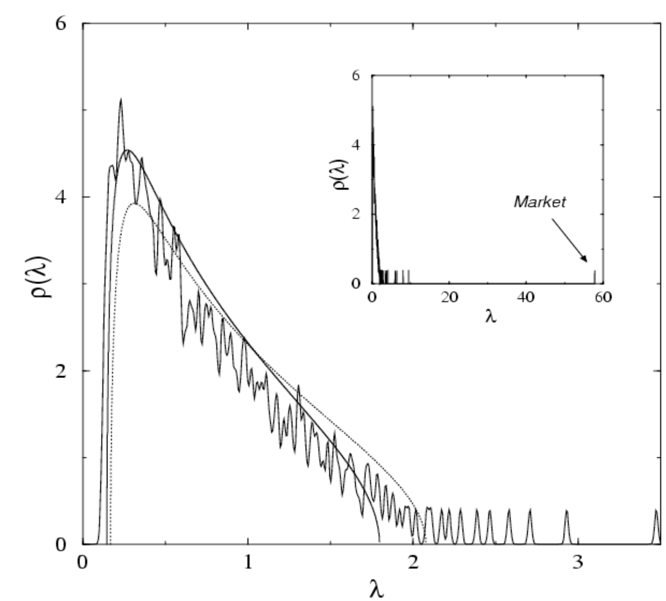
log prices	$x_i(t) = \log p_i(t)$	
log returns	$\Delta x_i(t) = x_i(t + \Delta t) - x_i(t)$	
avg return	$r_i(t) = \mu \sum_{t' \leq t} (1-\mu)^{t-t'} x_i(t')$	
covariance	$\mathrm{Cov}_{i,j}(t) = \mu \sum_{t' \leq t} (1-\mu)^{t-t'} [\Delta x_i(t') - r_i(t)][\Delta x_j(t') - r_j(t)]$	
volatility	$\mathrm{Vol}_i(t) = \mathrm{Cov_{i,i}(t)} = \mu \sum_{t' \leq t} (1-\mu)^{t-t'} [\Delta x_i(t') - r_i(t)]^2$	
correlation	$\mathrm{Corr}_{i,j}(t) = \dfrac{\mathrm{Cov_{i,j}(t)}}{\sqrt{\mathrm{Vol_i(t) Vol_j(t)}}}$	

The market mode

Histogram of eigenvalues of the covariance matrix of daily returns of N=41 stocks of NYSE on T=6910 days (1980-2007)



Same for correlation matrix (e.g. S&P500, Laloux et al.)



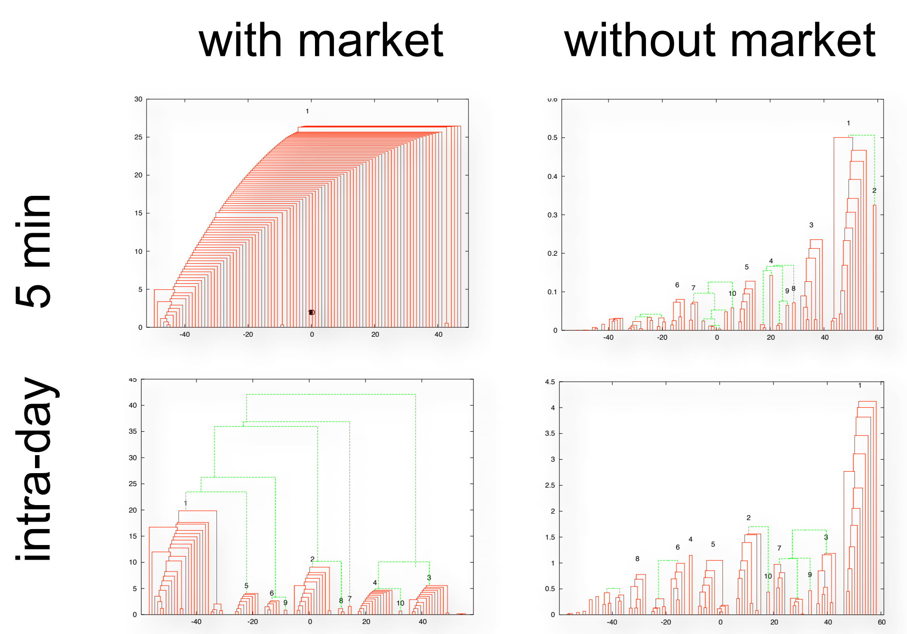
Can you really understand market dynamics by looking at a single price?

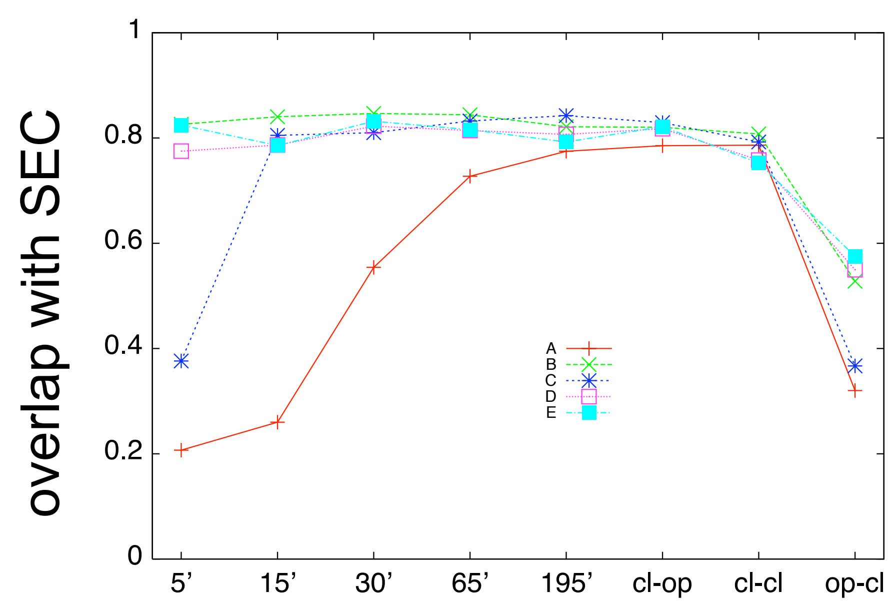
- Translation invariance:

$$p_i \to \lambda p_i, \quad \Leftrightarrow \quad x_i = \log p_i \to x_i + x_0 \ \forall \text{assets } i$$

  - center of mass - market mode (herding/non-informed trades)
  - relative coordinates - ex-market returns (informed trades)

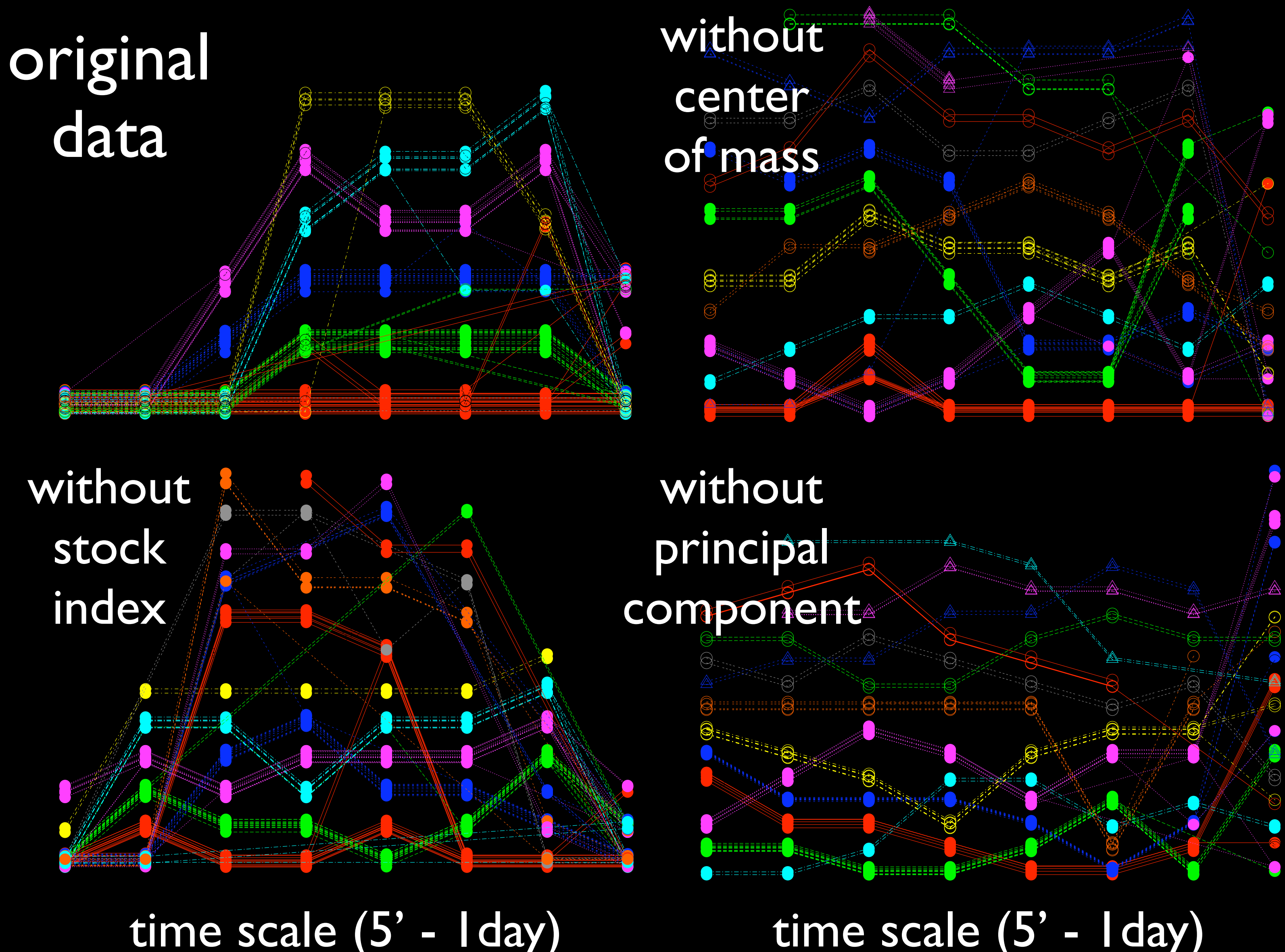
- Time-scale invariance of correlations of ex-market returns





(see C. Borghesi, MM. S. Micciche PRE 2007)

Time-horizon invariant structure without center of mass



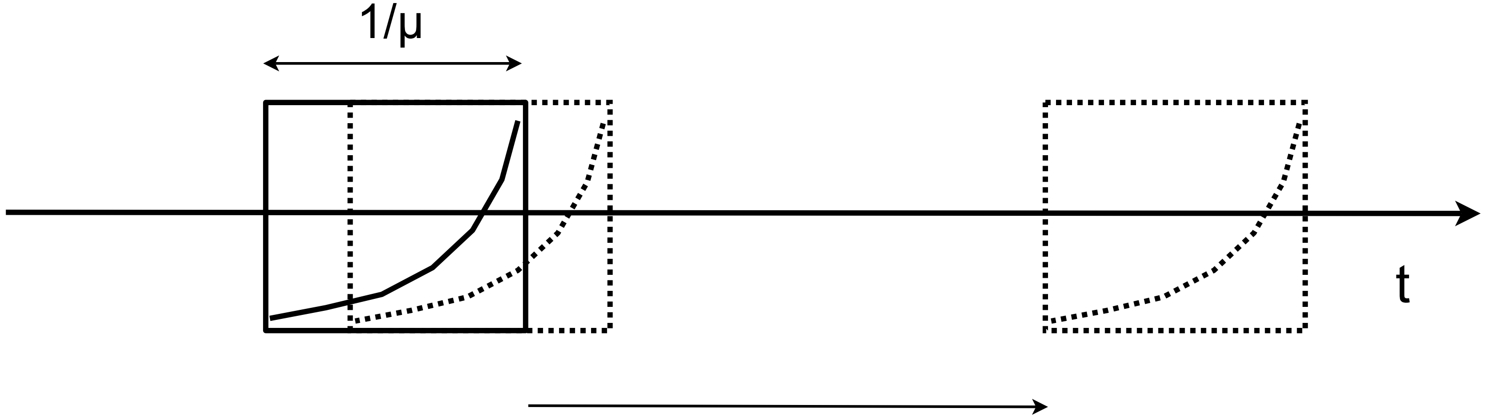
Assets in the same cluster follow the same trajectory across time-scales when center of mass is removed

The dynamics of covariance

$$\mathrm{Cov}_{i,j}(t) \;=\; \mu \sum_{t'=1}^{t} (1-\mu)^{t-t'} [x_i(t') - r_i(t)][x_j(t') - r_j(t)]$$

$$r_i(t) \;=\; \mu \sum_{t'=1}^{t} (1-\mu)^{t-t'} x_i(t')$$

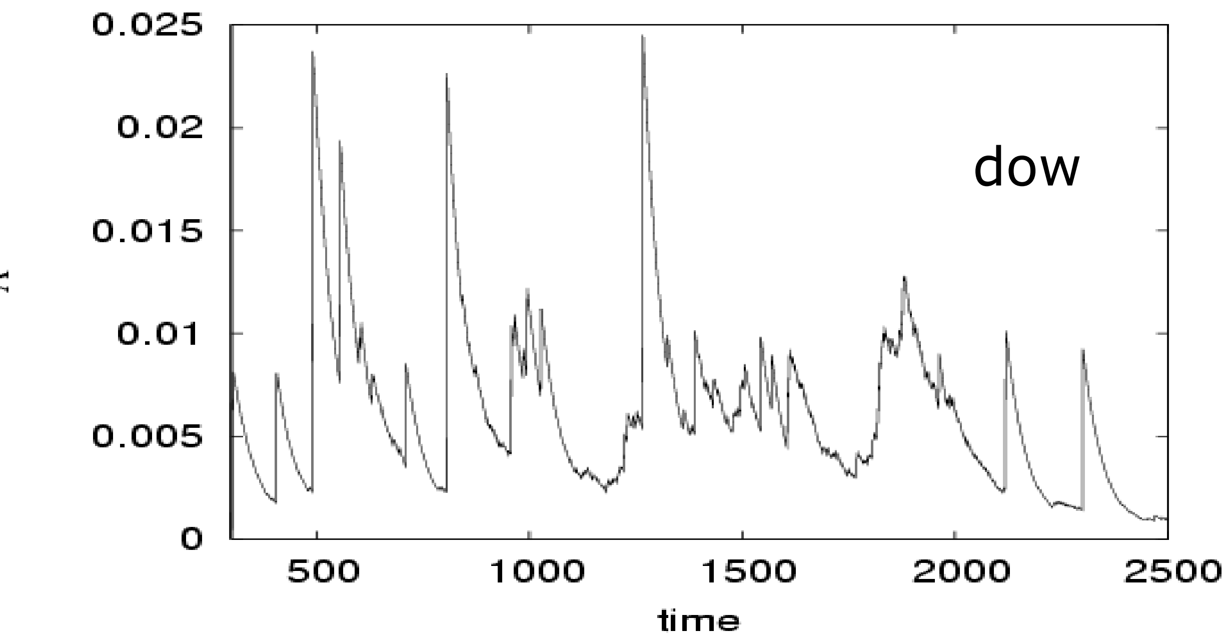
moving average on window of size 1/μ



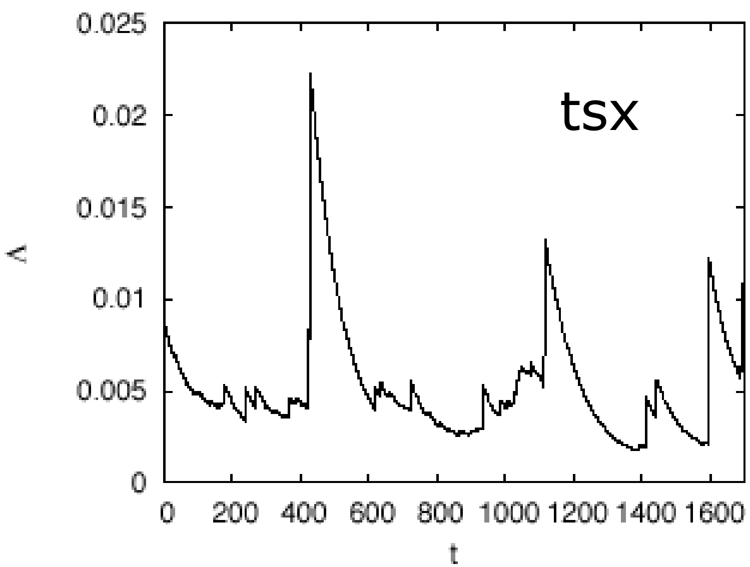
Sliding window with exponential average

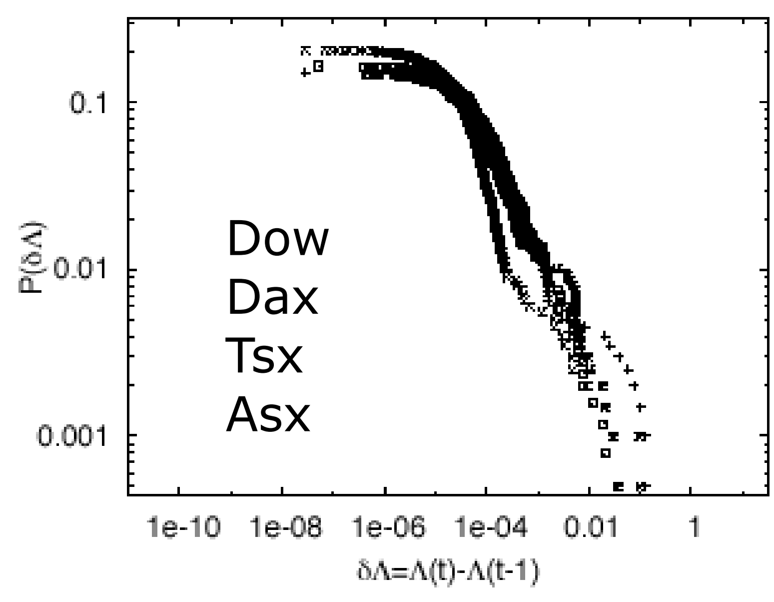
The dynamics of the market mode
(largest eigenvalue of Cov)

(see also Drozdz et al.)



Distribution of fluctuations of Λ

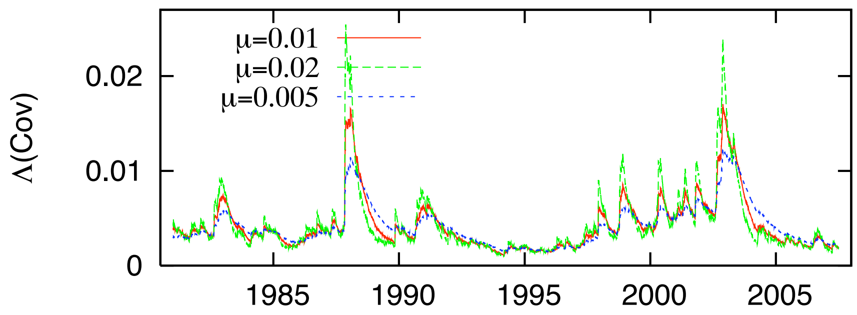




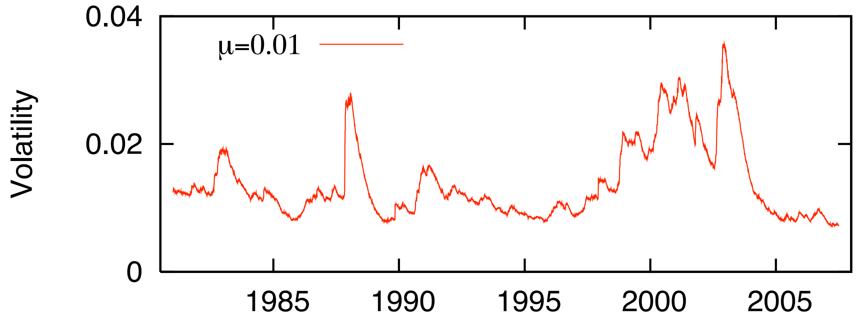
N=41 stocks of NYSE

(from yahoo.finance.com)

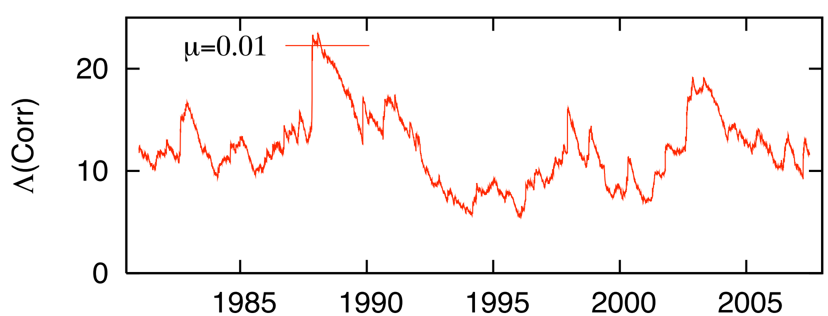
Covariance

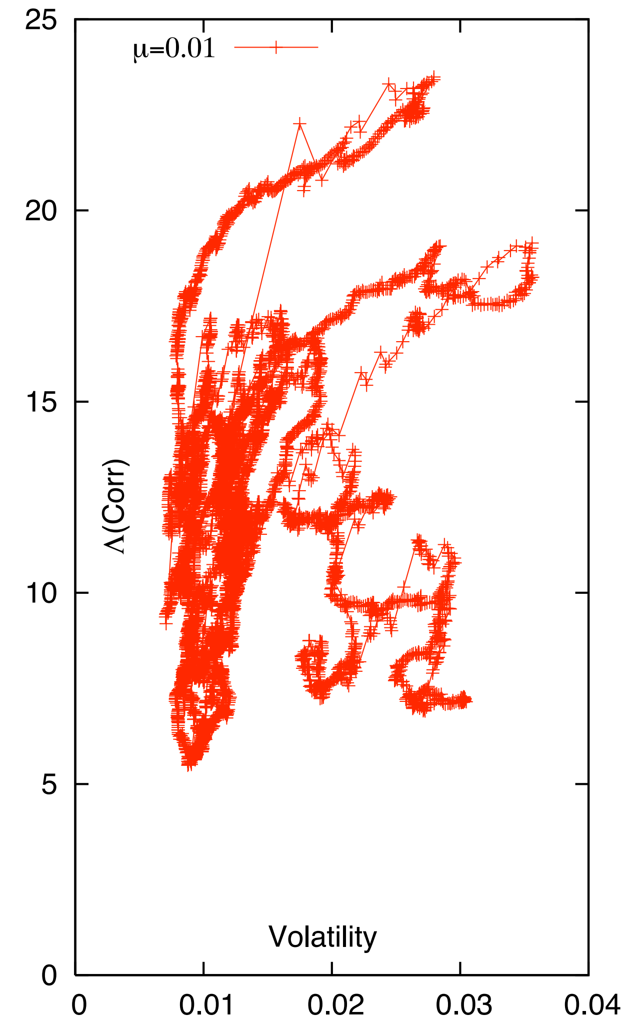


Volatility



Correlation





Where do correlations come from?

- Portfolio investment:
  agents spread investment across stocks to minimize risk (i.e. avoid correlations)
  In doing this, they invest in a correlated way in the market $\rightarrow$ they create correlations

$$\hat{C} = \hat{B} + \hat{F}(\hat{C}) + \hat{\Omega}$$

Economics Finance Noise

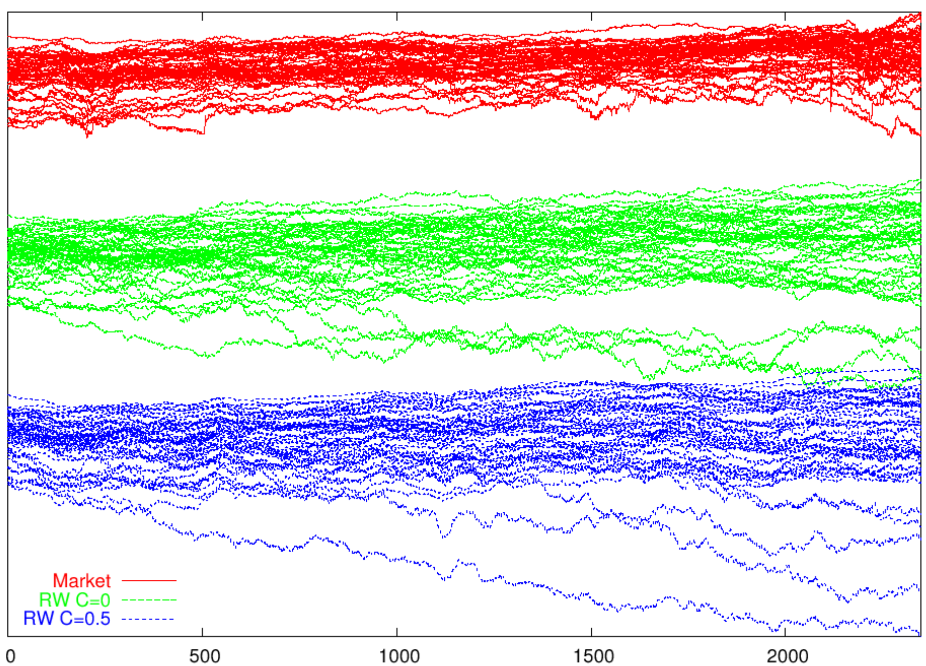
- Simple models describing this feedback

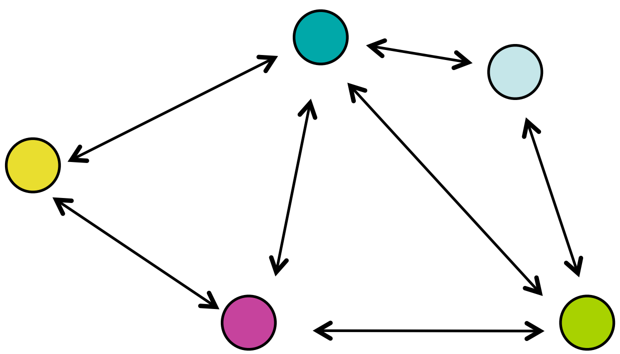
Phenomenological approach

Multi-asset markets as many "particle" interacting systems

$$x_i(t) = \log p_i(t) \qquad \begin{array}{rcl} i & = & 1, \ldots, N \\ (N & = & \# \text{ assets}) \end{array}$$

- Prices ≠ correlated random walks





✦"Bound state"

✦Collective motion

Phenomenological approach

(as little discipline as possible)

whatever Δx could depend on

$$\begin{aligned}\Delta\vec{x}(t) = \vec{x}(t+1) - \vec{x}(t) \quad &= \quad \vec{F}\left(\vec{x}(t), \partial_t\vec{x}(t), \partial_t^2\vec{x}(t), \ldots, \vec{a}(t'), \partial_t\vec{a}(t), \ldots, \vec{b}(t'), \ldots\right) \\ &= \quad \vec{F}(0,0,\ldots;t) + \left.\frac{\partial\vec{F}}{\partial\vec{x}}\right|_{0,0,\ldots} \vec{x}(t) + \left.\frac{\partial\vec{F}}{\partial(\partial_t\vec{x})}\right|_{0,0,\ldots} \partial_t\vec{x}(t) + \ldots\end{aligned}$$

1- small fluctuations → expand in powers of argument

2- high frequency → expand in time derivatives

3- eliminate terms which cannot appear

4- study the simplest non-trivial model

5- add complication

Dirac's bra-kets

- Vectors
$$|x\rangle = \begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}, \qquad \langle x| = (x_1, \ldots, x_N)$$

- Scalar product
$$\langle x|y\rangle = \sum_{i=1}^{N} x_i y_i$$

- Direct product (matrix)
$$|x\rangle\langle y| = \begin{pmatrix} x_1y_1 & x_1y_2 & \ldots & x_1y_N \\ x_2y_1 & x_2y_2 & \ldots & x_2y_N \\ \vdots & \vdots & \ddots & \vdots \\ x_Ny_1 & x_Ny_2 & \ldots & x_Ny_N \end{pmatrix}$$

- Basis
$$|k\rangle, \qquad k = 1, \ldots, N, \qquad \langle k|j\rangle = N\delta_{k,j}, \qquad \hat{I} = \frac{1}{N}\sum_{k=1}^{N} |k\rangle\langle k|$$

$$|v\rangle = \frac{1}{N}\sum_{k=1}^{N} v^{(k)}|k\rangle, \qquad v^{(k)} = \langle k|v\rangle$$

Optimal Portfolios

- Problem: Invest $|z\rangle$ $\rightarrow$ stochastic return = $\langle \Delta x|z\rangle$
  - expected return = $\langle r|z\rangle$ = R, $|r\rangle = E[|\Delta x\rangle]$
  - wealth = $\langle 1|z\rangle$ = W, $|1\rangle = (1,\ldots,1)$

  so as to minimize risk $\Sigma(|z\rangle)$
- Solution (if $\Sigma(|z\rangle) = \mathrm{Var}(\langle \Delta x|z\rangle)$):

$$|z^*\rangle = \arg\min_{|z\rangle,\lambda,\nu}\left[\frac{1}{2}\langle z|\hat{C}|z\rangle - \lambda(\langle r|z\rangle - R) - \nu(\langle 1|z\rangle - W)\right]$$

- Note:

  no impact on market, unique solution.

  What if many traders invest in this same way?

  Will this have some impact?

A generic phenomenological model:

K components of optimal portfolios
with parameters R^k, W^k, μ^k

$$
\begin{aligned}
\Delta|x_t\rangle &= |x_{t+1}\rangle - |x_t\rangle \\
&= \left| F\left(t, |r_t\rangle, |z_t^{(1)}\rangle, \ldots, |z_t^{(K)}\rangle, \Delta|r_t\rangle, \Delta|z_t^{(1)}\rangle, \ldots, \Delta|z_t^{(K)}\rangle, \ldots\right)\right\rangle \\
&= |F(t, 0, 0, \ldots)\rangle + \\
&\quad \left.\frac{\partial|F\rangle}{\partial\langle r|}\right|_0 |r_t\rangle + \left.\frac{\partial|F\rangle}{\partial\Delta\langle r|}\right|_0 \Delta|r_t\rangle + \ldots + \\
&\quad \sum_{\ell=1}^{K}\left[\left.\frac{\partial|F\rangle}{\partial\langle z^{(\ell)}|}\right|_0 |z_t^{(\ell)}\rangle + \left.\frac{\partial|F\rangle}{\partial\Delta\langle z^{(\ell)}|}\right|_0 \Delta|z_t^{(\ell)}\rangle + \ldots\right] + \ldots \\
&= |\alpha_t\rangle + \beta_t|r_t\rangle + \tilde{\beta}_t\Delta|r_t\rangle + \ldots + \sum_{\ell=1}^{K}\left[\nu_t^{(\ell)}|z_t^{(\ell)}\rangle + \tilde{\nu}_t^{(\ell)}\Delta|z_t^{(\ell)}\rangle + \ldots\right] + \ldots
\end{aligned}
$$

low frequency expansion+
r, z^k, small → power expansion

Fundamentalists+
speculators
(noise traders)

Chartists,
trend followers

Risk managers

The simplest model:

Closed dynamical model, self-generated fluctuations/correlations

- $|x_{t+1}\rangle = |x_t\rangle + |b_t\rangle + v_t\, |z_t\rangle$

  $|b_t\rangle$ = "bare" returns

$$E\left[|b_t\rangle\right] = \bar{b}|1\rangle + \sigma|2\rangle,$$
$$E\left[|b_t\rangle\langle b_{t'}|\right] = B\hat{I}\delta_{t,t'}$$

- v_t = portfolio investment rate $E\left[v_t\right] = \bar{v}, \qquad \mathrm{Var}\left[v_t\right] = \Delta$

- Where

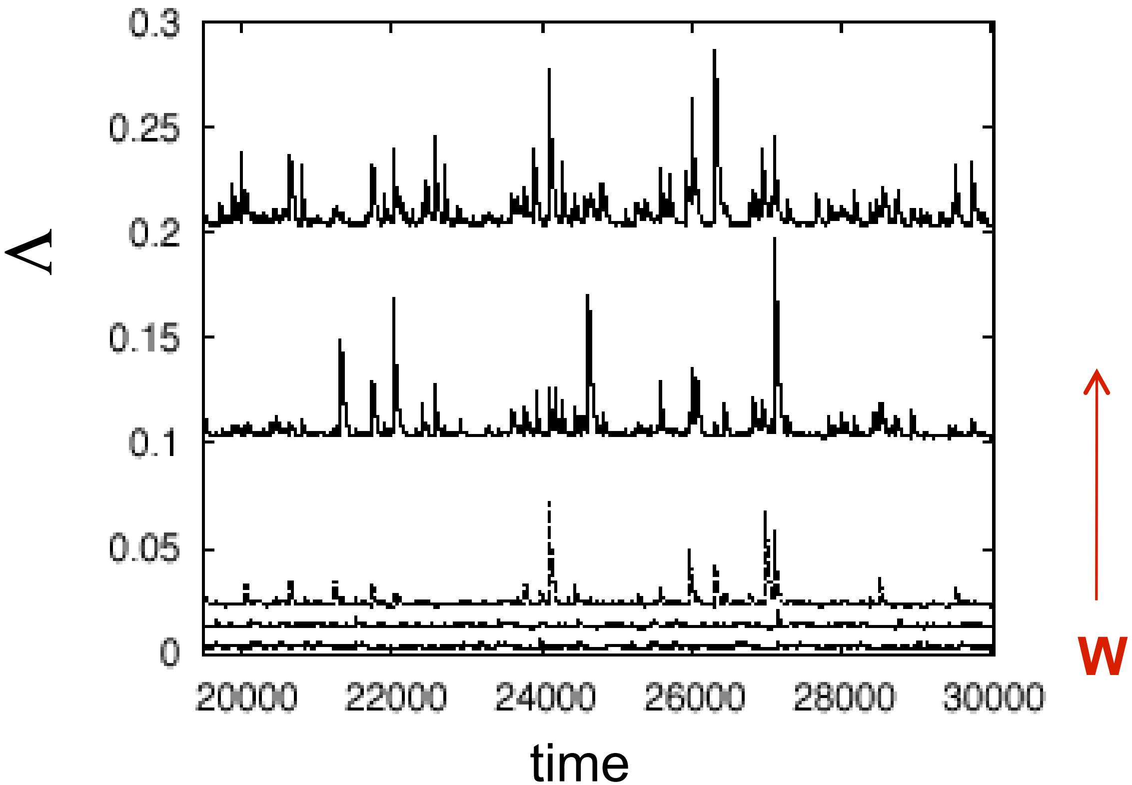
$$|z_t\rangle = \arg \min_{|z\rangle,\lambda,\nu} \left[\frac{1}{2}\langle z|\hat{C}_t|z\rangle - \lambda(\langle r_t|z\rangle - R) - \nu(\langle 1|z\rangle - W)\right]$$

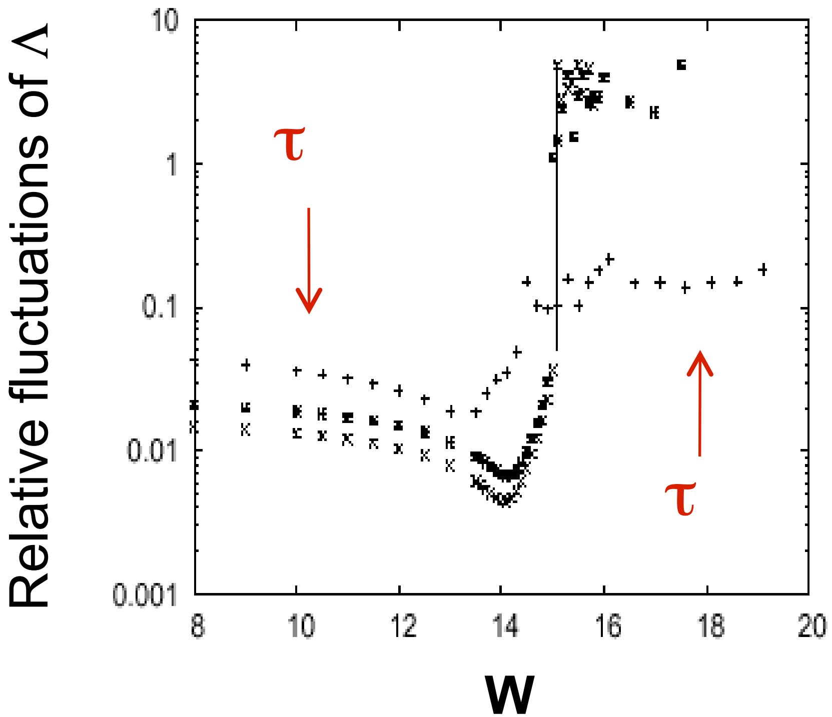
- Average return and correlation matrix ($\mu \sim 1/T_{average}$)

$$|r_{t+1}\rangle = (1-\mu)\, |r_t\rangle + \mu\, [|x_{t+1}\rangle - |x_t\rangle]$$

$$C_{t+1} = (1-\mu)\, C_t + \mu\, |\delta x_t\rangle \langle \delta x_t| \qquad |\delta x_t\rangle = |x_t\rangle - |x_{t-1}\rangle - |$$

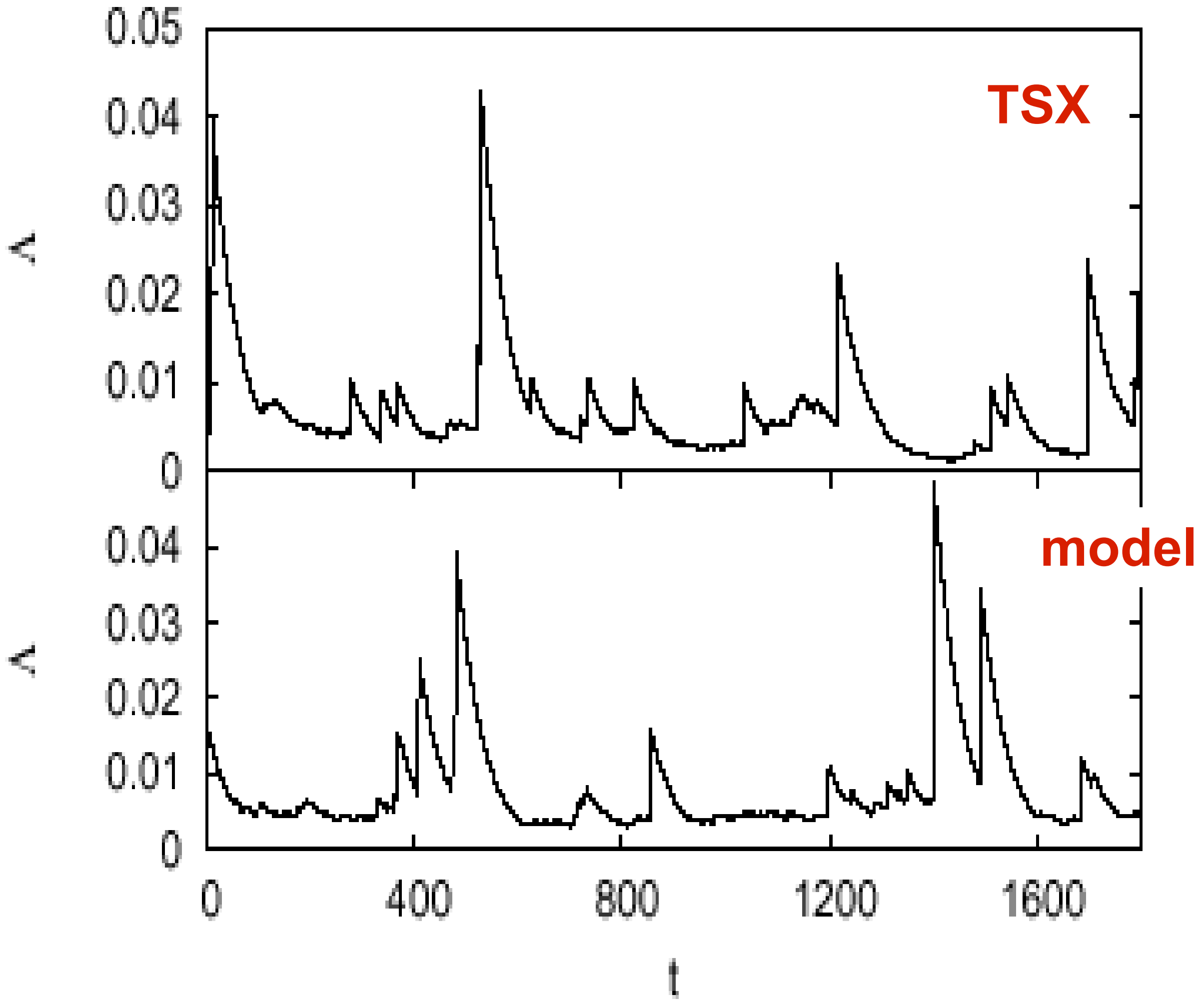
Numerical simulations

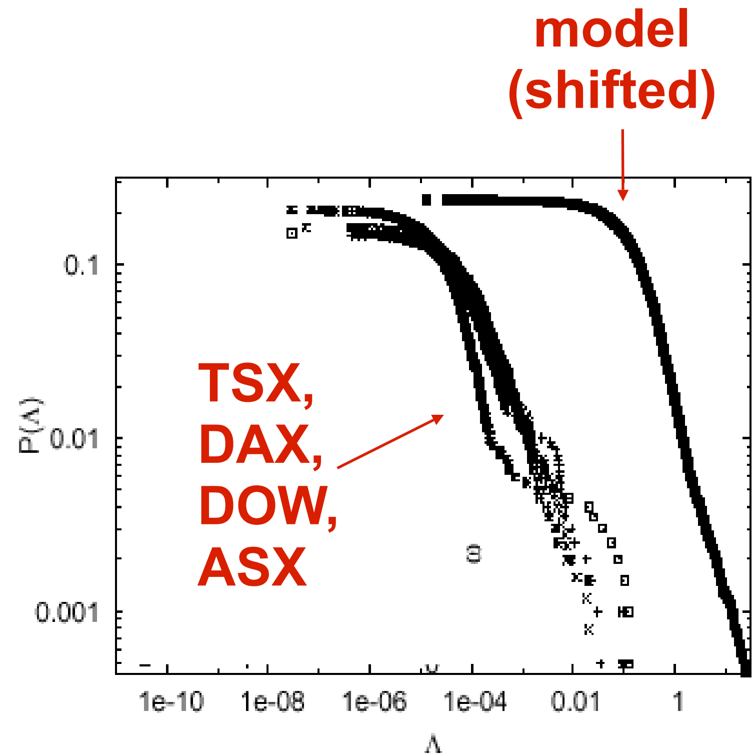


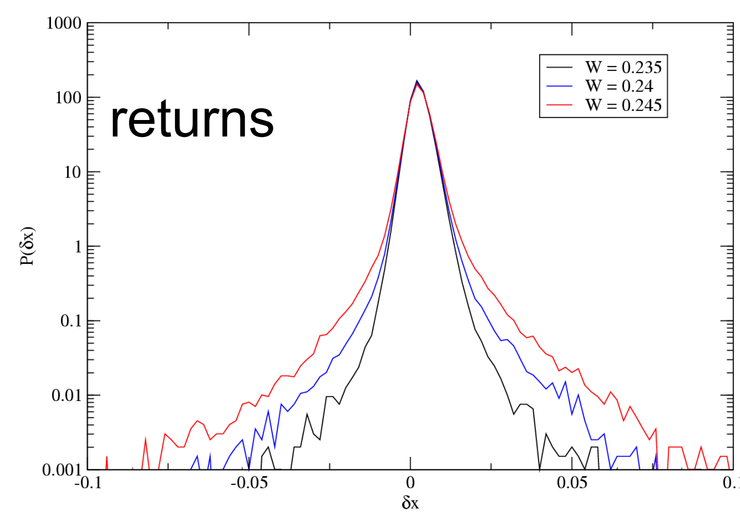


⇨ **Dynamic instability as W→W***

…and close to W*





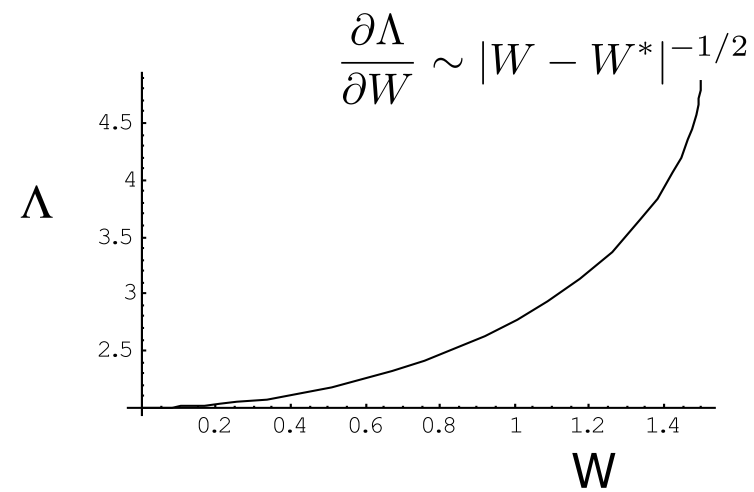


Theory: low frequency limit $\mu \to 0$

- for $\mu \to 0$

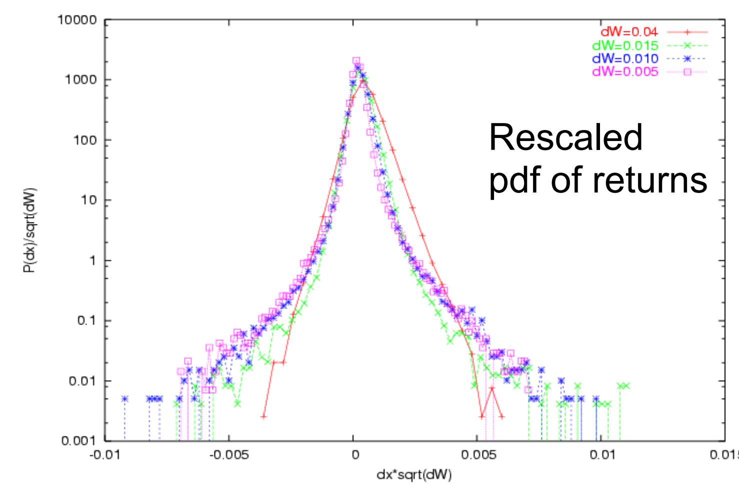
  C_t, $|r_t\rangle$ independent of t $\to$ $|z_t\rangle$ independent of t, $\Delta|z_t\rangle = 0$

- $\Lambda = B + N\,\Delta\, f(R/N, W/N, \nu)$



- Self-consistent equations
  → phase transition at W* to dynamically unstable phase

- Small μ expansion
  $\to$ scaling $\quad \frac{\delta\Lambda}{\Lambda} \sim \sqrt{\frac{\mu}{W^* - W}}$



Instability in the general model:

$$\frac{1}{4}\overline{\delta b^2} - (\bar{b} + \bar{w})\bar{w} + (1-\alpha)\bar{r} - \frac{1}{N}\vec{Z}_\perp^2 \geq 0$$

1. the phase transition is robust
   - for any risk measures of agents
   - independent of higher order derivative terms
   - noise filtering cannot help
2. the market is less stable
   - the larger the volume of trading
   - the smaller the return demanded
     (i.e. the more agents are risk averse!)
   - the stronger are trend followers
   - the larger or less diversified "bare" returns (~ dividends)
   - the more correlated are stocks a priori

$$\mu \to 0 \text{ limit}: \quad \vec{r} = \vec{b} + \alpha\vec{r} + \sum_\ell \epsilon^\ell \vec{z}^\ell$$

$$\bar{w} = \frac{1}{N}\sum_\ell \epsilon^\ell W^\ell$$

$$\bar{r} = \frac{1}{N}\sum_\ell \epsilon^\ell R^\ell$$

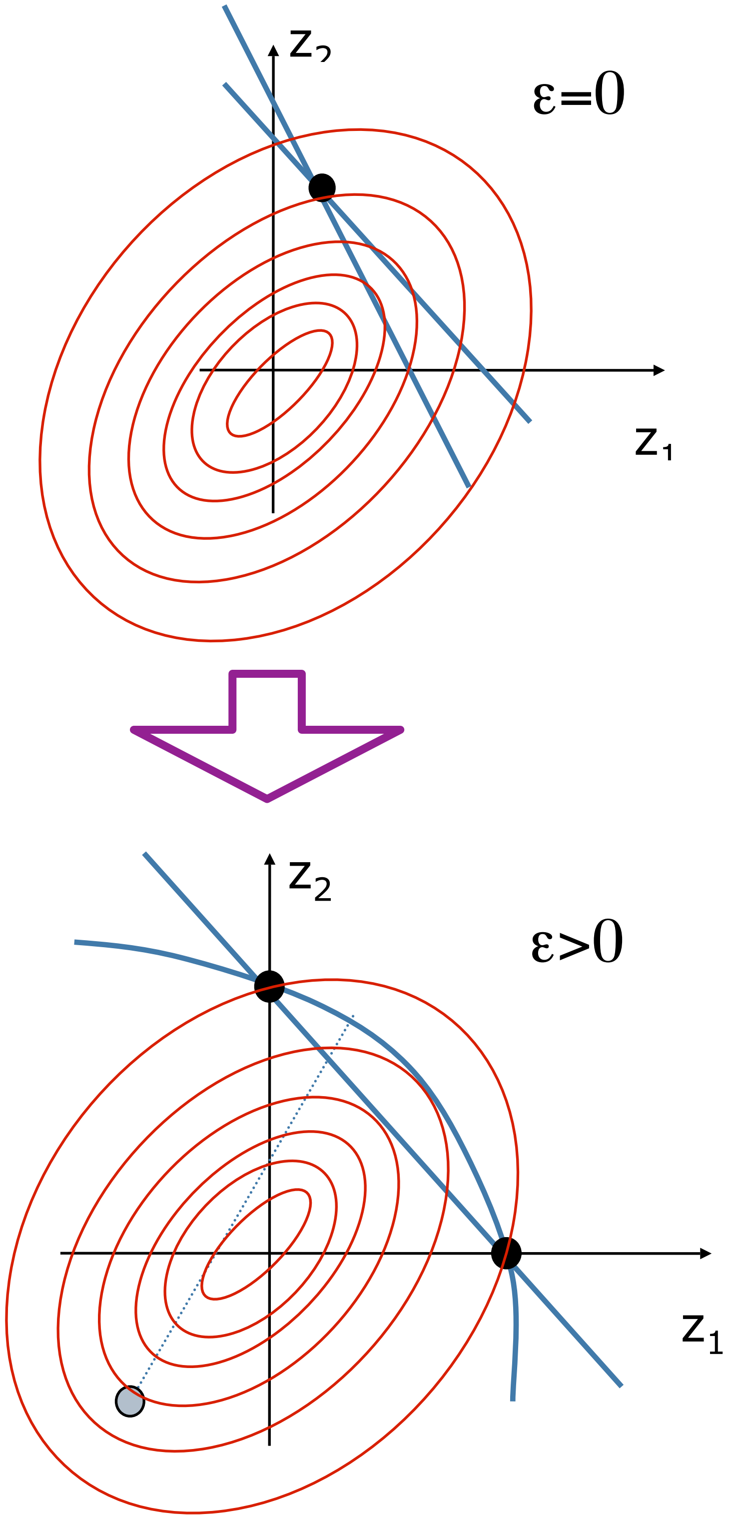
$$\vec{Z}_\perp = \sum_\ell \epsilon^\ell \vec{z}_\perp^\ell$$

$$\vec{z}_\perp^\ell \cdot \vec{b} = 0$$

What does it really depend on?

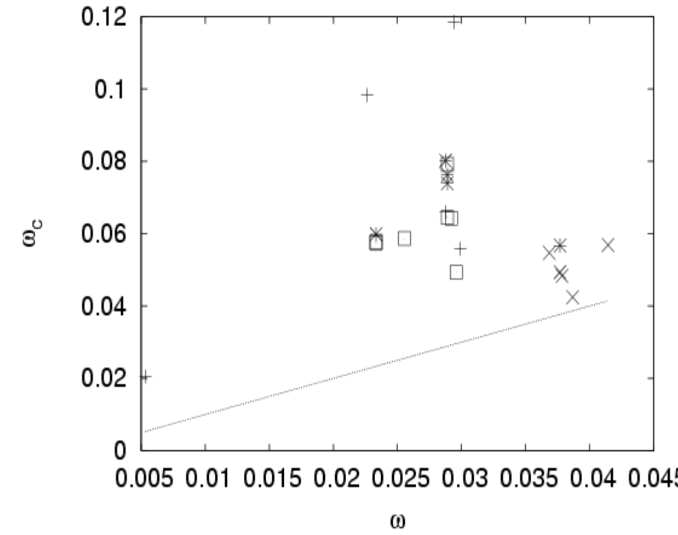
- Impact of portfolio strategies changes one constraints from hyper-plane to hyper-sphere (ε ≠ 0)

- no risk free asset



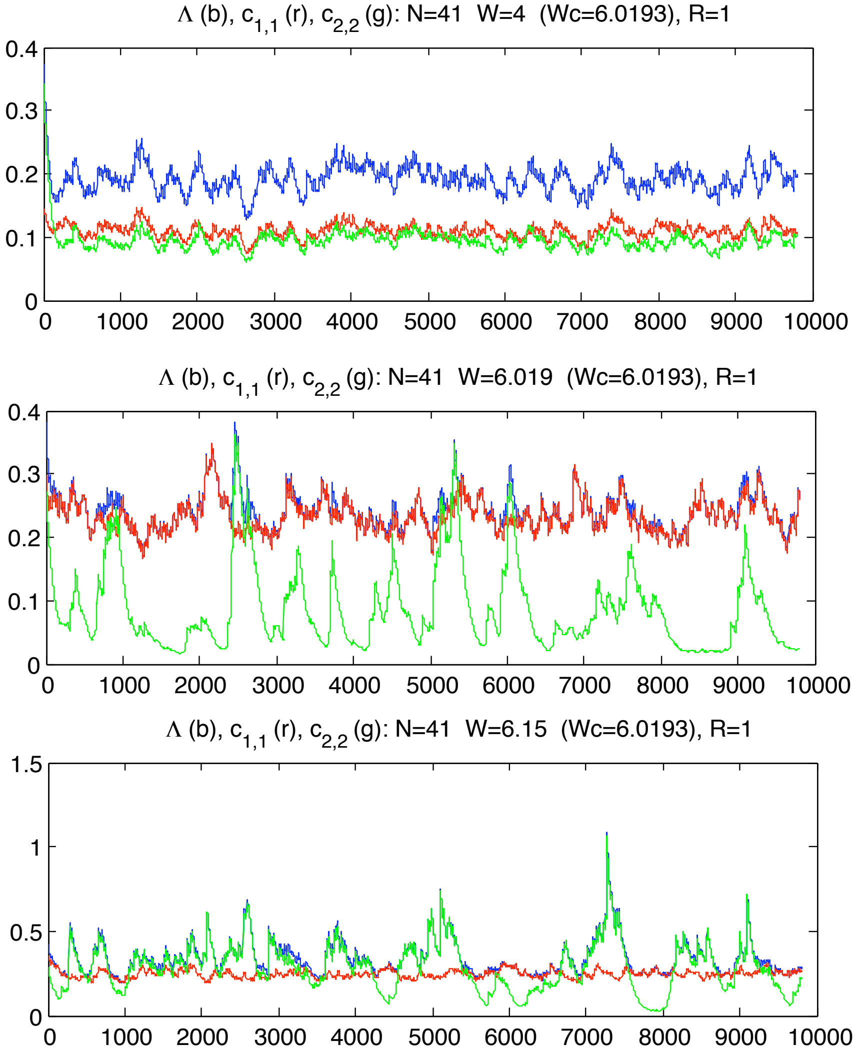
Back to real markets

1. Fit model to real market data
   1. compute likelihood
   2. maximize → parameters
   3. markets are close to instability

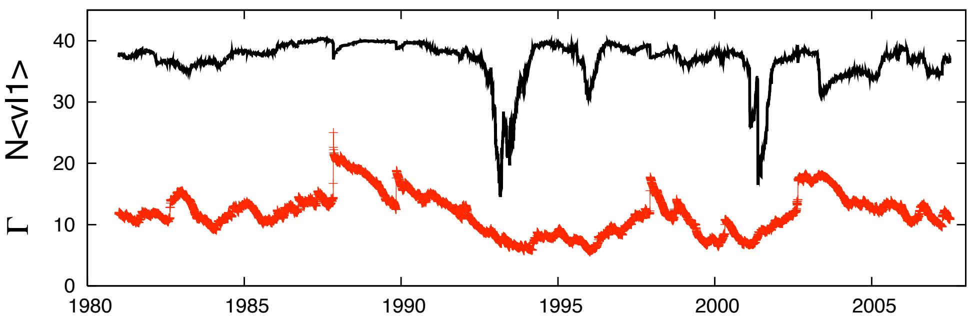


2. Compare market's and model's behaviors
   1. what picture does the model provide?
   2. does that picture agree with market data?

The model:
Switching among the two principal directions



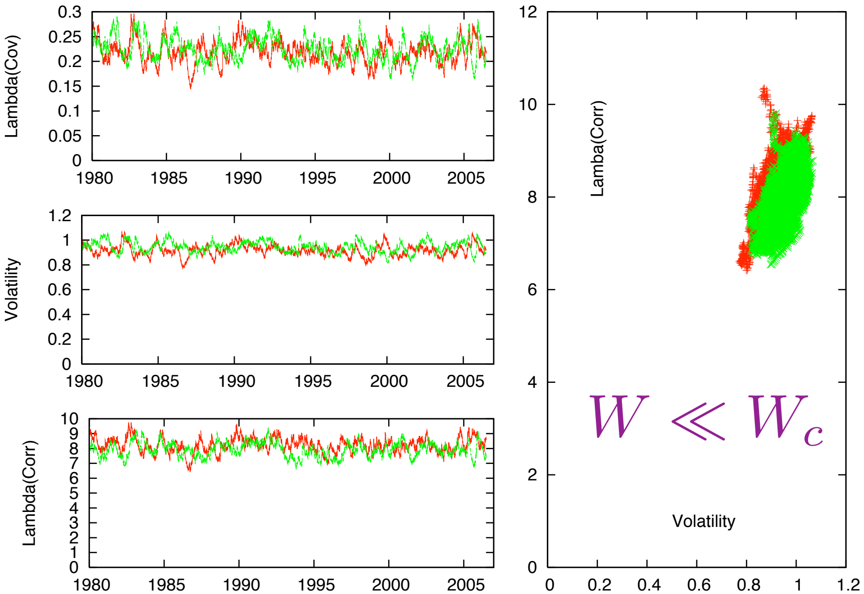
$$|b_t\rangle = \bar{b}|1\rangle + \sigma|2\rangle$$

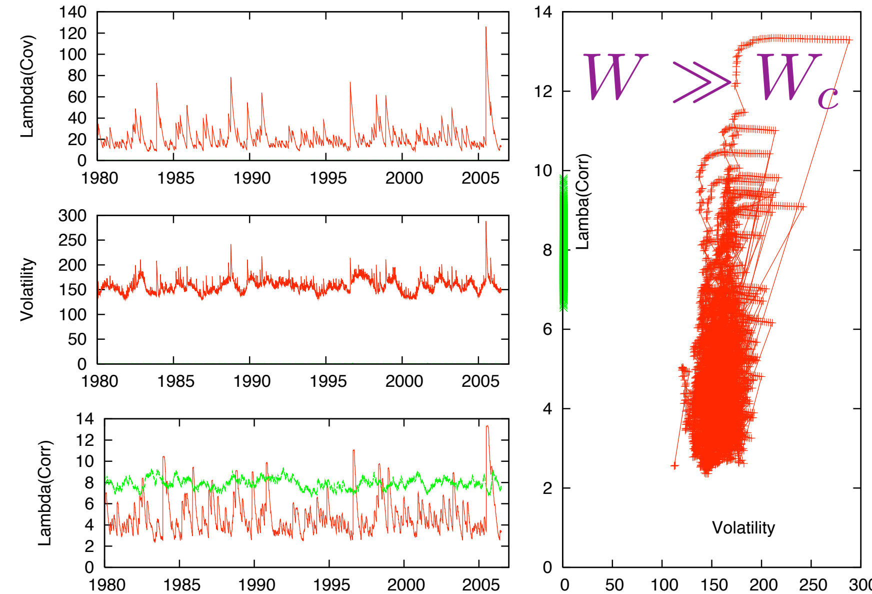


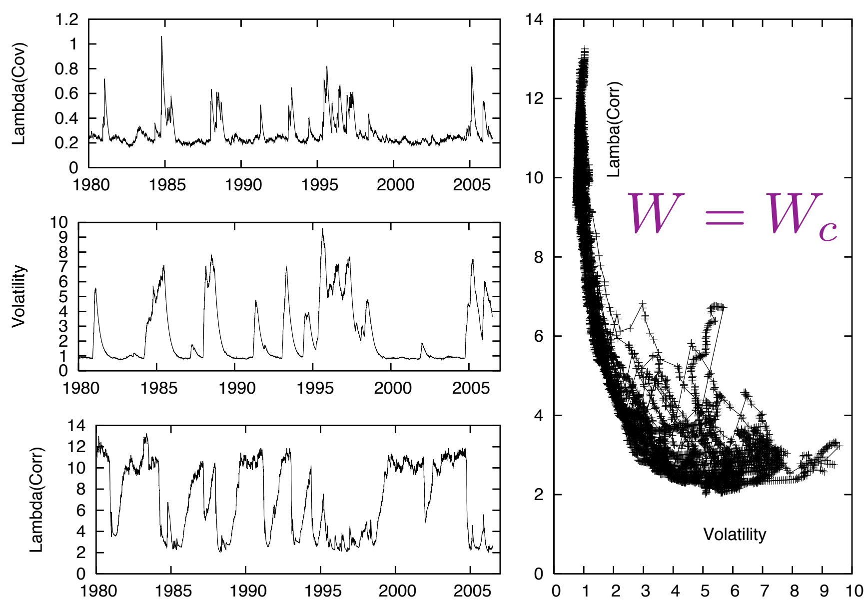
NYSE 41 stocks: Overlap on (1,1,...)

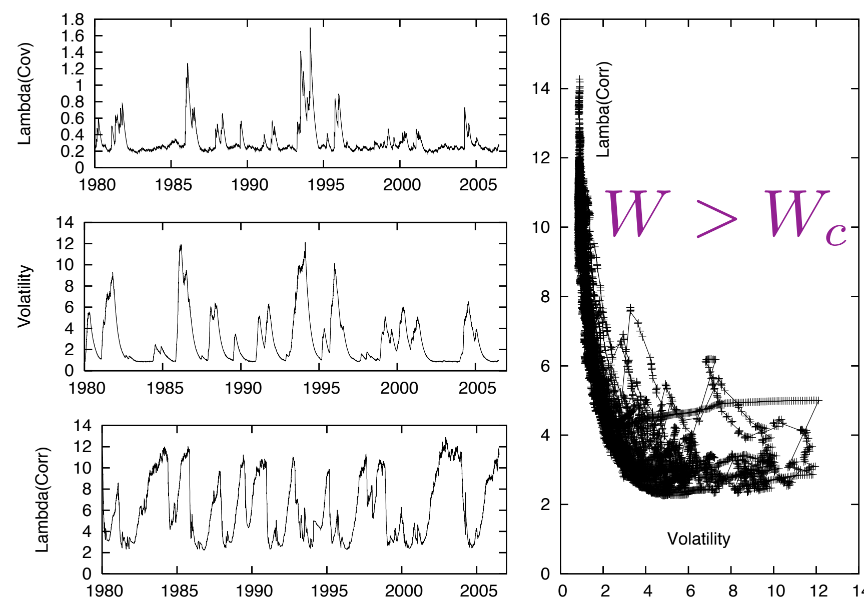
The model's picture:

Covariance, volatility and correlation







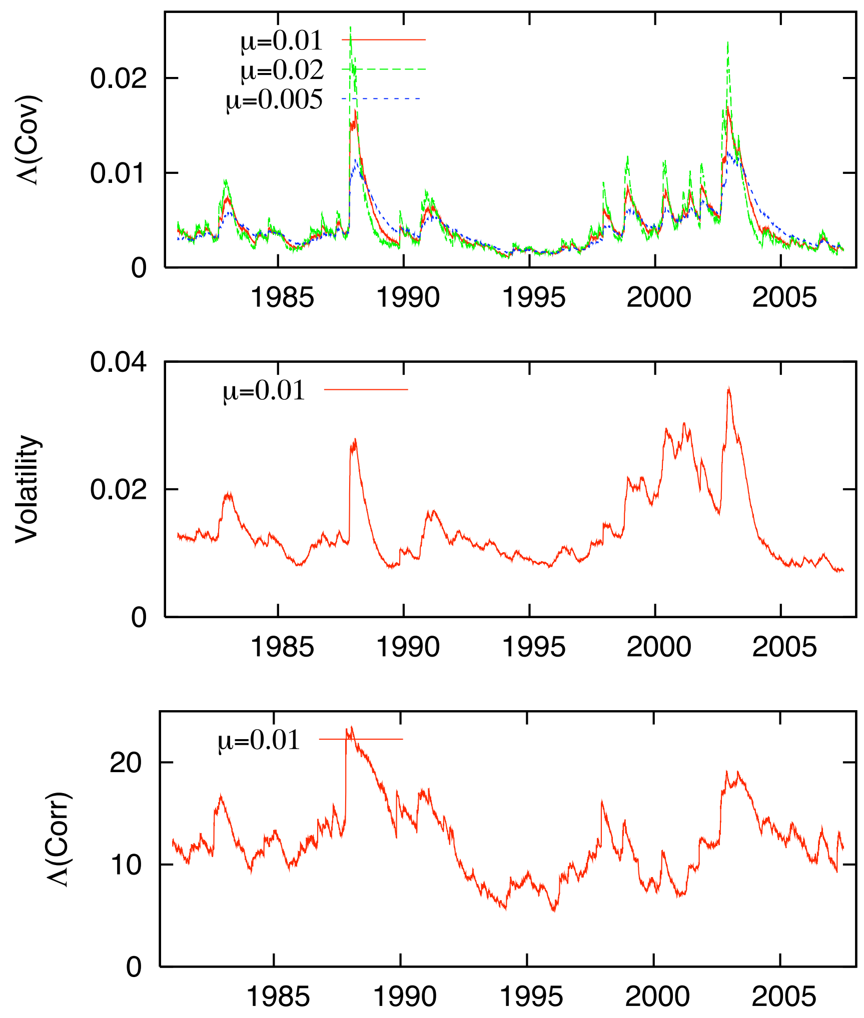


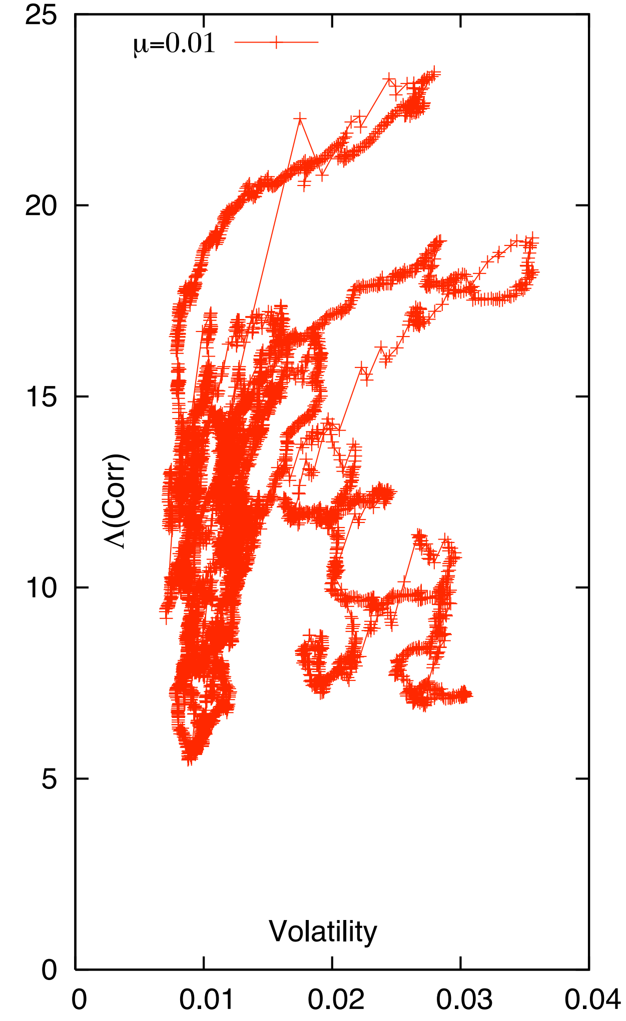
The real picture:
N=41 stocks of NYSE
(from yahoo.finance.com)

Covariance

Volatility

Correlation





Summary I

- log p translation invariance suggests different dynamics of center of mass and relative coordinates
- Complex dynamics of global correlations
- Simple model for the feedback of correlations through portfolio strategies
  - exhibit market mode
  - develops complex dynamics at a phase transition
  - suggests markets are close to a phase transition
- Real markets seem to be more complex

Economics inspired approach

(+ M. Anufriev, G. Bottazzi, P. Pin)

What micro-economics?

- CARA models (Chiarella, Dieci, He)
  - demand independent of wealth
  - stationary equilibrium
    - price growth exogenous
    - prices can go negative

- CRRA models
  - demand proportional to wealth
  - dynamic equilibrium
    - price growth endogenous
    - positive prices

Multi-asset market: 1 agent

- N assets (1 unit) + risk free asset (return r_f)
- discrete time; t=1, 2, ...
- homogeneous investors (=1 agent): wealth w_t
  portfolio

$$|x_t\rangle = (x_t^1, \dots, x_t^N)$$

- market clearing:

$$|p_t\rangle = w_t |x_t\rangle$$

- wealth dynamics

$$w_{t+1} = w_t \left[1 + r_f + \langle x_t | r_{t+1} \rangle + \langle x_t | \delta_{t+1} \rangle\right]$$

- excess returns and dividend yields (or news arrival process)

$$r_{t+1}^a = \frac{p_{t+1}^a - p_t^a}{p_t^a} - r_f \qquad |\delta_t\rangle \in \mathcal{N}(|d\rangle, \hat{D})$$

- how is $|x\rangle$ chosen by the agent?

Equilibrium

- Time indep. portfolio $|x_{t+1}\rangle = |x_t\rangle = |x\rangle$
- Returns (ex-dividend) $|r_{t+1}\rangle = \frac{\langle x|\delta_{t+1}\rangle}{1-\langle 1|x\rangle}|1\rangle$
- Expected return $|c\rangle = \frac{\langle x|d\rangle}{1-\langle 1|x\rangle}|1\rangle + |d\rangle$
- Covariance matrix

$$\hat{C} = \hat{D} + \frac{\langle x|\hat{D}|x\rangle}{(1-\langle 1|x\rangle)^2}|1\rangle\langle 1| + \frac{|1\rangle\langle x|\hat{D} + \hat{D}|x\rangle\langle 1|}{1-\langle 1|x\rangle}$$

Note:
singularity when $\langle x|1\rangle \to 1$

Prices grow at the same rate (apart from dividends, there is a only one game)

Price volatility is ~N times higher than dividend volatility

Large eigenvector (~N) in Cov and Corr

Separation "Theorem": equilibrium is independent of r_f

“Strong” CAPM

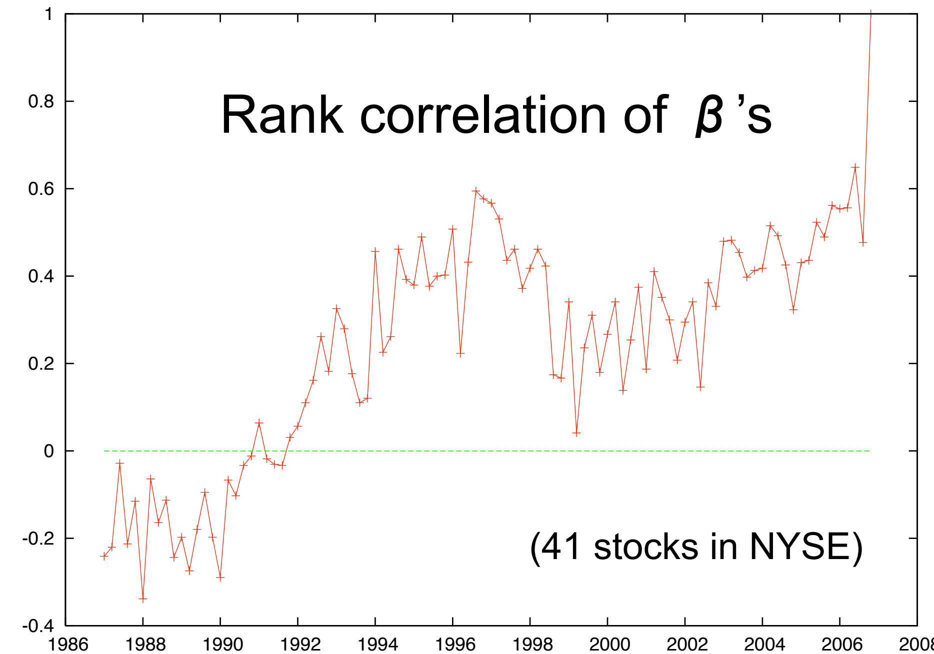
- the excess return of each asset a is given by

$$r_t^a = \frac{\langle \delta_t | x \rangle}{1 - \langle x | 1 \rangle} = \frac{1}{\langle x | 1 \rangle} \langle r_t | x \rangle$$

  - valid for all t, not just for expected values
  - same beta’s

$$\beta^a = \frac{1}{\langle x | 1 \rangle} = \frac{\mathrm{Cov}(r_t^a, \langle r_t | x \rangle)}{\mathrm{Var}(\langle r_t | x \rangle)}$$

$$= \frac{\gamma + \langle 1 | \hat{D}^{-1} | d \rangle}{\langle 1 | \hat{D}^{-1} | d \rangle}$$

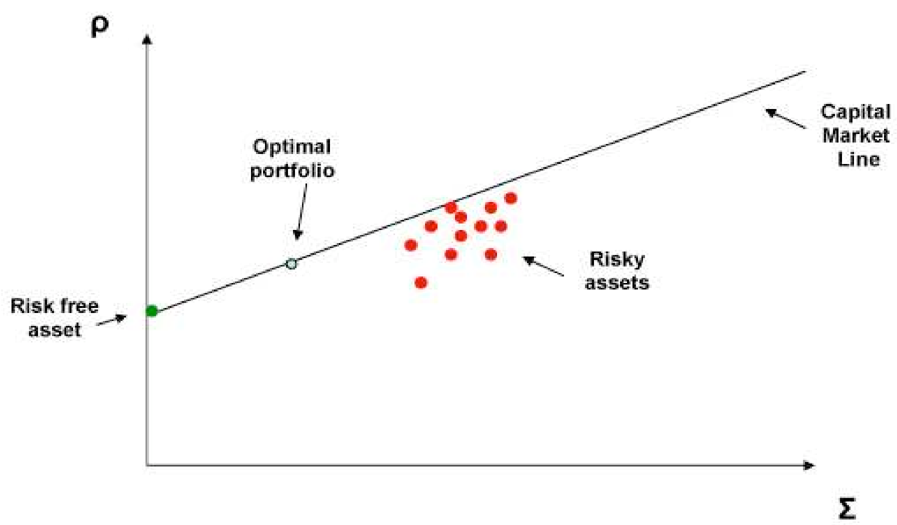


Mean–variance strategies: CRRA γ

The agents are CRRA and choose a portfolio taking into account expected first and second moments

They compute mean vector
$|c_t\rangle = E_t\left[|r_{t+1}\rangle + |\delta_{t+1}\rangle\right]$ and variance–covariance matrix
$\hat{C}_t = \mathrm{Cov}_t\Big(|r_{t+1}\rangle + |\delta_{t+1}\rangle, |r_{t+1}\rangle + |\delta_{t+1}\rangle\Big)$

They choose the portfolio
$|x\rangle = \frac{1}{\gamma}\hat{C}_t^{-1}|c_t\rangle$, where γ is a positive parameter of risk aversion.



Equilibrium:

$$|x\rangle = \frac{1}{\gamma + \langle 1|\hat{D}^{-1}|d\rangle}\hat{D}^{-1}|d\rangle$$

No short selling:
$x_i > 0 \; , \; 1 - \langle x|1\rangle > 0$

Mean–variance strategies: portfolio managers

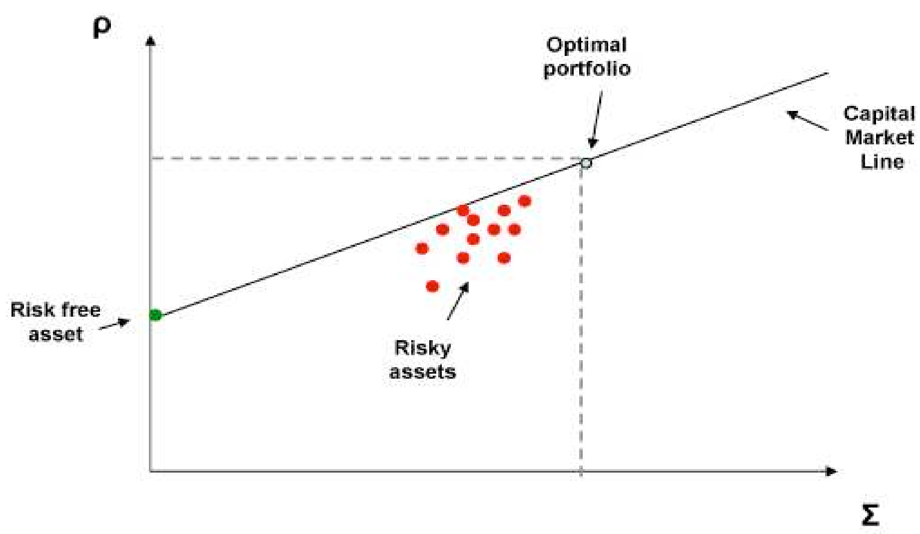
The agents are risk–neutral portfolio managers with a constraint

They also compute mean vector $|c_t\rangle$ and variance–covariance matrix $\hat{C}_t$

They fix an excess return $\rho = R = \langle x|c\rangle$, minimizing risk $\Sigma = \frac{1}{2}\langle x|\hat{C}|x\rangle$

Solution is $|x\rangle = R\frac{\hat{C}^{-1}|c\rangle}{\langle c|\hat{C}^{-1}|c\rangle}$

Analogy: γ as $\frac{R}{\langle c|\hat{C}^{-1}|c\rangle}$



Equilibrium:

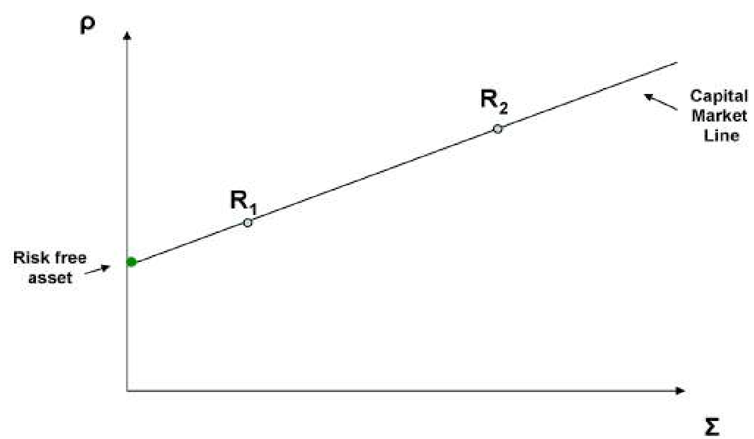
$$|x\rangle = \frac{1}{N\left(R\bar{d} + \bar{d}^2 + \sigma^2\right)} R|d\rangle$$

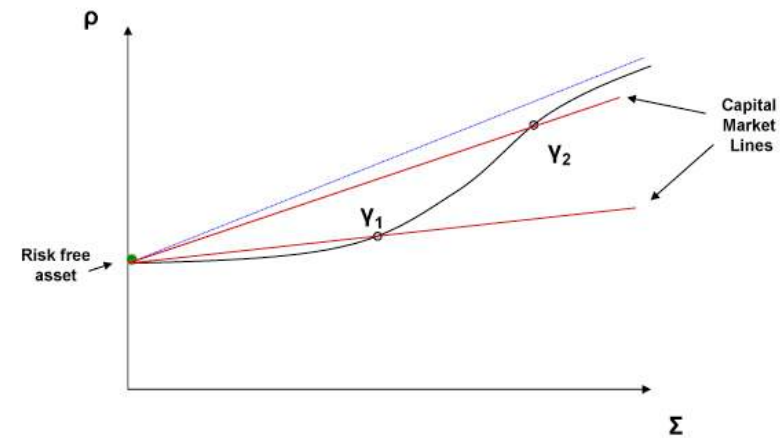
No short selling also here!

How does the market depends on agent's behavior?

We can normalize expected return and variance by the risky portfolio

$$\rho = \frac{\langle x|c\rangle}{1-\langle x|1\rangle} + r_f \;, \quad \text{and} \quad \Sigma = \frac{\sqrt{\langle x|\hat{C}|x\rangle}}{1-\langle x|1\rangle} \;.$$





Explicit curves for optimal portfolios are:

- R portfolio managers: $\Sigma = \sqrt{\frac{D}{N(\bar{d}^2+\sigma^2)}}\,(\rho - r_f)$
- CRRA γ: $\Sigma = \sqrt{\frac{D}{N(\bar{d}^2+\sigma^2)}}\sqrt{1+\frac{\bar{d}^2+\sigma^2}{\bar{d}(\rho-r_f)}}\,(\rho - r_f)$

Learning to be a mean-variance investor

μ updating and ν inertia

$$
\begin{aligned}
i) \quad |x_t\rangle &= (1-\nu)|x_{t-1}\rangle + \tfrac{\nu}{\gamma}\hat{C}_{t-1}^{-1}|c_{t-1}\rangle \\
ii) \quad r_t^i &= \tfrac{\langle x_{t-1}|\delta_t\rangle x_t^i + (1+r_f)(1-\langle x_{t-1}|1\rangle)x_t^i}{(1-\langle x_t|1\rangle)x_{t-1}^i} - 1 - r_f \\
iii) \quad |c_t\rangle &= (1-\mu)|c_{t-1}\rangle + \mu E_\delta\Big[|r_{t-1}\rangle + |\delta_{t-1}\rangle\Big] \\
iv) \quad \hat{C}_t &= (1-\mu)\hat{C}_{t-1} + \mu\mathrm{Cov}(|r_{t-1}\rangle + |\delta_{t-1}\rangle)
\end{aligned}
$$

$\hat{D} = D\hat{I}$

The expectation over $|\delta_t\rangle$ could be simply equal to last observation

Stability in the limit $\mu \to 0$

Assuming $\mu \to 0$, $\nu \to 0$ (randomness $\to 0$),

and $\frac{\nu}{\mu} \equiv \lambda$, we can take continuum limit ($\tau \equiv \mu t$)

$$\frac{d|x\rangle}{d\tau} = -\lambda|x\rangle + \frac{\lambda}{\gamma}\hat{C}^{-1}|c\rangle$$

$$\frac{d|c\rangle}{d\tau} = -|c\rangle + \frac{\langle x|d\rangle}{1-\langle x|1\rangle}|1\rangle + |d\rangle$$

$$\frac{d\hat{C}}{d\tau} = -\hat{C} + \frac{\langle x|\hat{D}|x\rangle}{(1-\langle x|1\rangle)^2}|1\rangle\langle 1| + \frac{|1\rangle\langle x|\hat{D}+\hat{D}|x\rangle\langle 1|}{1-\langle x|1\rangle} + \hat{D}$$

$\hat{D} = D\hat{I}$: we consider only projections on $|1\rangle$ and $|2\rangle$
(as in equilibrium)

We have a 7–dimensional system: its Jacobian has all negative eigenvalues

For the R portfolio managers case we change $\frac{d|x\rangle}{d\tau}$: it is still stable

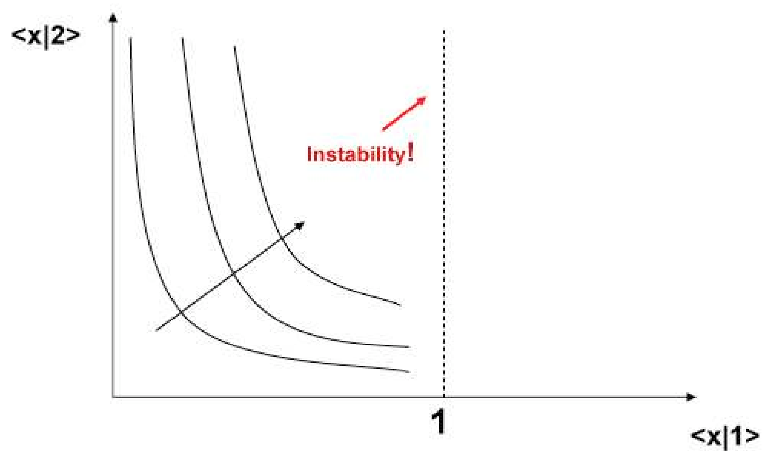
Evolutionary (in)-stability

A part of the population changes slightly from $|x\rangle$ to $|x+\epsilon\rangle$ (still in the $|1\rangle$–$|2\rangle$ plane)

We can compute the gradient of a variation that brings higher expected wealth growth

$$\frac{\partial E\left[\log\left(\frac{w_{t+1,\epsilon}}{w_{t,\epsilon}} \Big/ \frac{w_{t+1}}{w_t}\right)\right]}{\partial\langle\epsilon|} = E\left[\frac{|\delta_{t+1}\rangle + r_{t+1}|1\rangle}{1+r_f+r_{t+1}+\langle\epsilon|\delta_{t+1}\rangle + r_{t+1}\langle\epsilon|1\rangle}\right]$$

$$\simeq E\left[\frac{|\delta_{t+1}\rangle + r_{t+1}|1\rangle}{1+r_f+r_{t+1}}\right]$$



Since yields $|\delta_{t+1}\rangle$ are all positive:

Evolutionary pressure brings towards instability

Many (M) agents, many (N) assets

- Portfolios $|x_{t,a}\rangle, \quad a = 1, \ldots, M$
- Wealths

$$w_{t+1,a} = w_{t.a}\left[(1+r_f)(1-\langle x_{t,a}|1\rangle + \langle x_{t,a}|\delta_t\rangle\right] + w_{t,a}\langle x_{t,a}|R_{t+1}\rangle$$

gain from risk free asset and dividends — capital gain

- ex-dividend returns

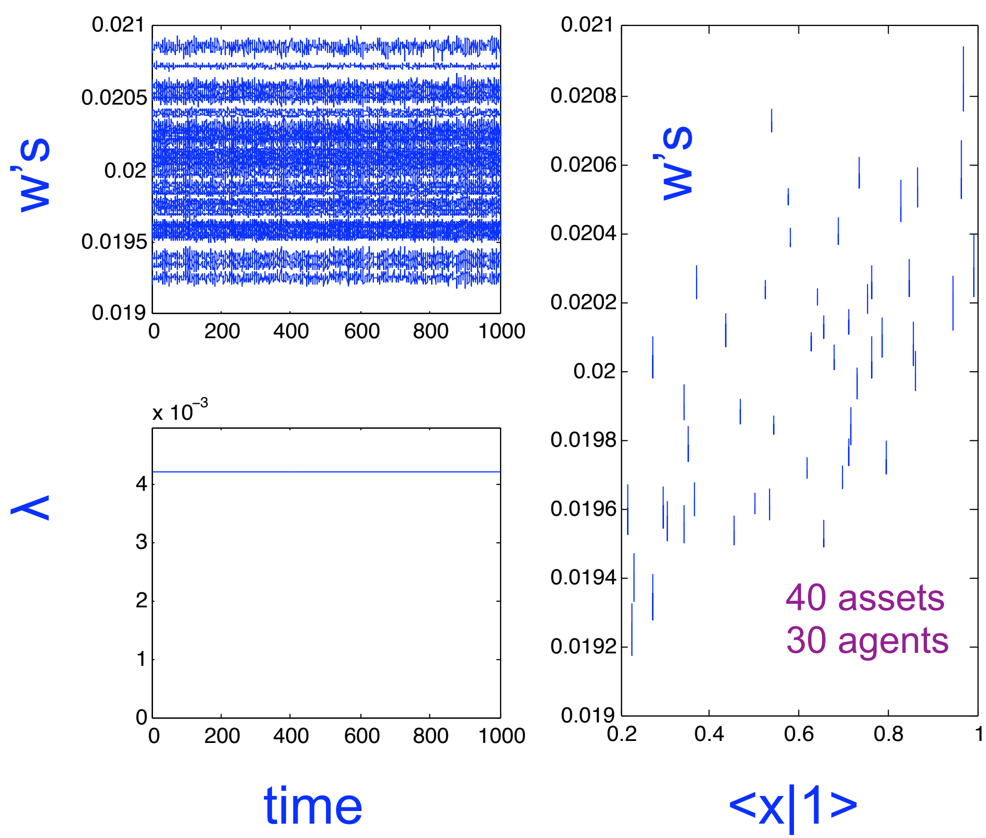
$$R^i_{t+1} = \frac{p^i_{t+1}}{p^i_t} = \frac{\sum_a w_{t+1,a} x^i_{t+1,a}}{\sum_a w_{t,a} x^i_{t,a}}$$

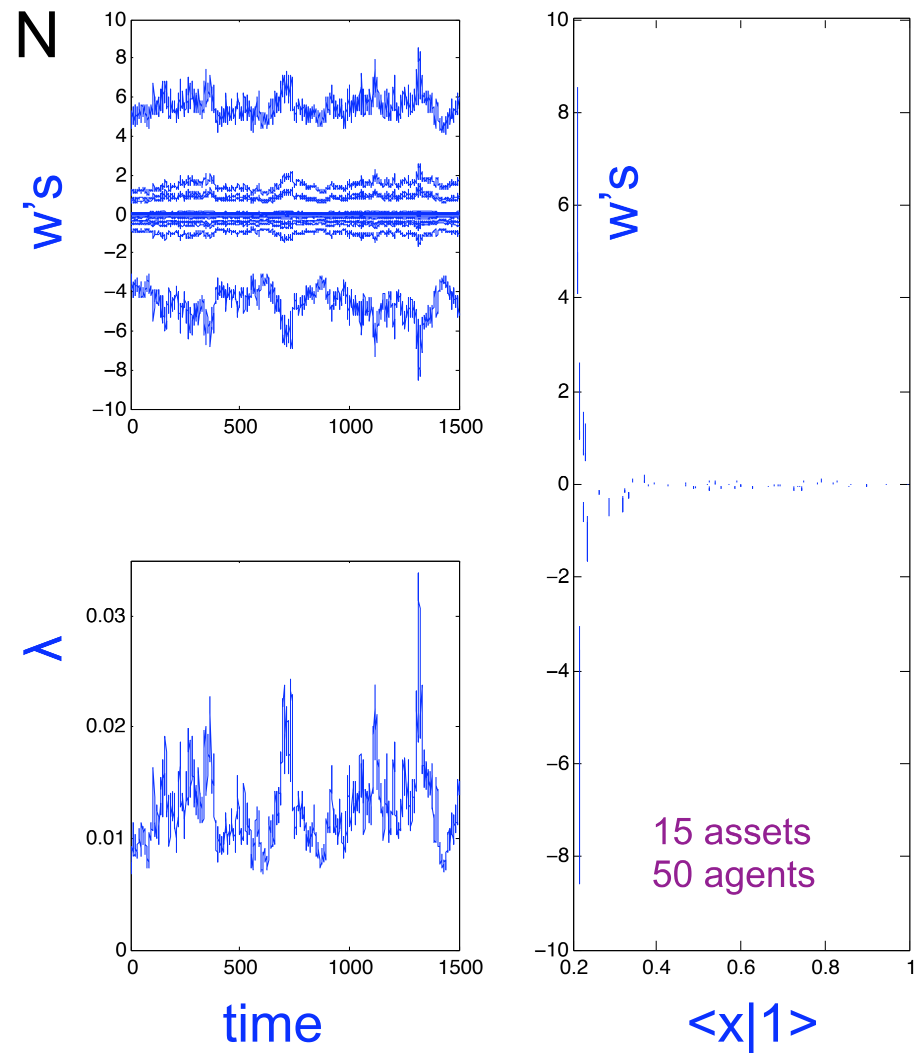
- non-linear coupled dynamics

Numerical simulations

i) more than one agent survives!

ii) non-trivial fluctuations when M > N





Summary

- Multi-asset model:
  - structure of correlations (market mode)
  - Single agent
    - picture of how agents shape the market
    - stability of learning dynamics
    - evolutionary pressure towards instability
    - stable correlations: no non-trivial fluctuations
  - Many agents
    - stable correlations with few types
    - non-trivial fluctuations with many types (numerical)

Thanks

C. Borghesi, MM. S. Micciche PRE 2007

G. Raffaelli, MM JSTAT L08001 (2006)

MM, B. Ponsot, G. Raffaelli submitted JECD

M. Anufriev, G. Bottazzi, MM, P. Pin forthcoming

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