Self-organized Synchronization in Wireless Networks

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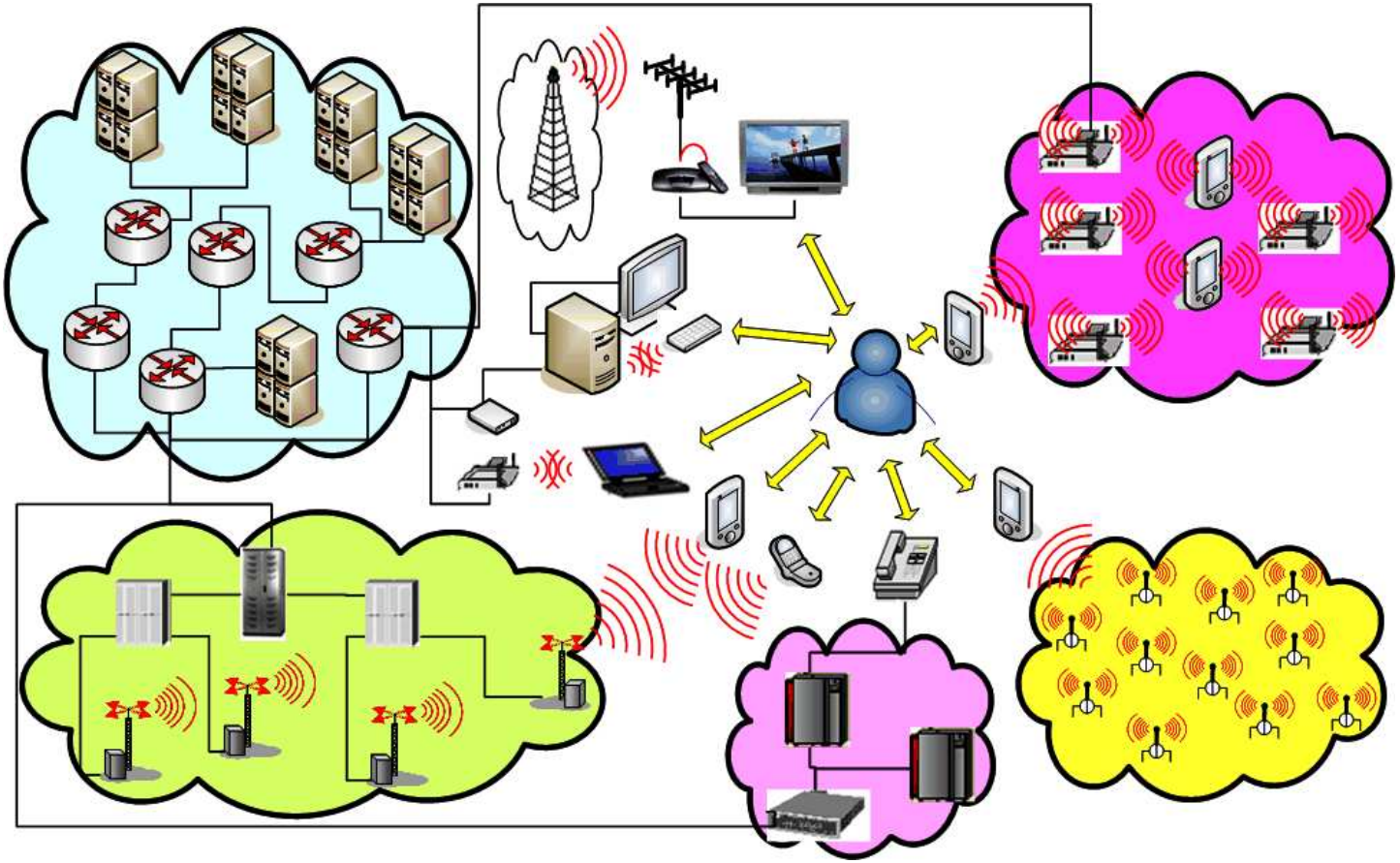
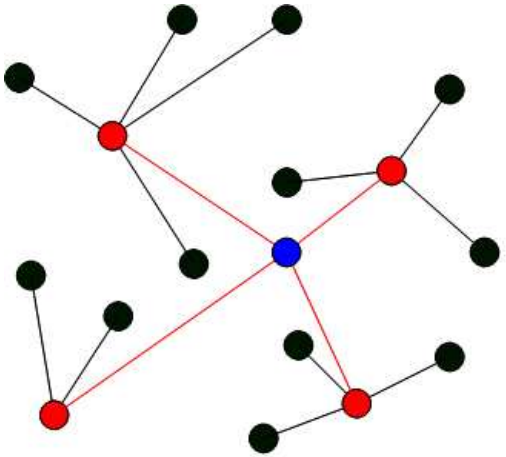
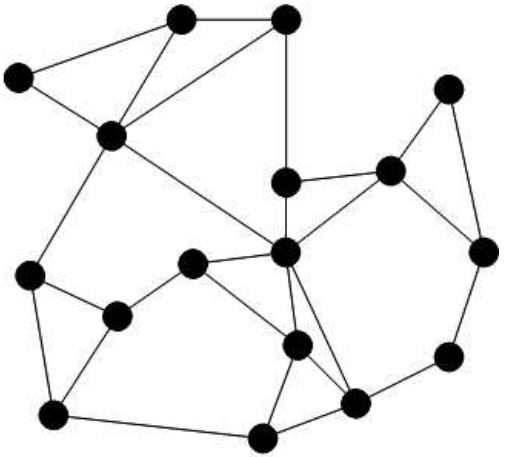
Information Networks



Network Architectures



Hierarchical architecture:
Cellular



Flat architecture, no
centralized control

- Flat architectures (ad hoc networks, sensor networks...) call for self-organization features
  - routing (connectivity $\longrightarrow$ percolation)
  - clustering
  - resource sharing
  - synchronization

Wired vs. Wireless

Wireless networks are more fragile, dynamic and less orthogonal than wired

- radio propagation (worse than r^{-2}) makes node interaction short range
- network is 2-dim with nearest neighbor interaction
  - links between neighbors may be characterized with tremendous accuracy
  - here binary characterization sufficient: is or is not neighbor
  - node i has a set of nearest neighbors $\mathcal{N}_i$
  - links may be unidirectional: $i \in \mathcal{N}_j$ does not imply $j \in \mathcal{N}_i$
- network is dynamic
  - nodes may move, die and be born
  - environment (=links) may change, links may die and be born
- interference: a signal may reach a number of nodes that are not interested in it
  - orthogonal channels (dedicated interference free channels) between nodes cannot be afforded

Time Synchronicity

Synchronicity (as considered here):

- nodes have same understanding of a relative time
  - time of the day, ignoring which day

Reasons to synchronize:

- to be able to communicate effecively (protocol)
- to save power
  - news come every hour, two minutes past the hour, at other times you may sleep
- to share resources
  - when working in shifts, it is beneficial if workers clocks are in same phase

Synchronization in DIS

- Task: Align local phase variables ϕ_i at nodes based on local decisions (at nodes)
  - synchronicity: $\phi_i = \phi^* \ \forall\, i$
  - synchronicity in window w: $|\phi_i - \phi^*| < w \ \forall\, i$
- Starting point: completely disordered network
  - $\phi_i(0)$ uniformly distributed in $[0, 2\pi]$
- Information exchange between nodes:
  - node i knows $\{\phi_j | j \in \mathcal{N}_i\}$
  - with fixed/random delay, possibly up to noise/measurement error
- Nodes update their timing based on recent neighbor information
- Distributed decision making algorithms based on self-organization models adopted from physics and biology
  - neurons (firing)
  - insects (flashing)
  - spin models (cooling)

Coupled Oscillator Synch

In literature, synchronization modeled with coupled oscillators

- Kuramoto model (Kuramoto 1975):
  $\dot{\phi}_i = \omega_i + \sum_{j \in \mathcal{N}_i} K_{ij} \sin(\phi_j - \phi_i)$
  - rich and complicated structure
- for DIS, *pulse coupled* oscillators becoming popular
  - "firefly" synchronization (Mirollo-Strogatz 1990)
  - interaction between clocks (oscillators) is not continuous
  - when oscillator reaches $\phi_i = 2\pi$, it "fires" (or "flashes"), and jumps to 0
  - other oscillators react to fire by jumping their clocks ahead
  - the closer the oscillators are to firing, the more they jump
  - reason for popularity is the firing (flashing)—oscillators need not be continuously coupled
    - signaling protocol with specific signals indicating flashes is possible

Firefly Synchronization

- Coupled differential eqns:

$$\dot{\phi}_i = \frac{1}{T} + \sum_{j \in \mathcal{N}_j} \left(\min \left[f^{-1}(f(\phi_i) + \epsilon),\ 2\pi \right] - \phi_i \right) \delta(\phi_j - 2\pi)$$

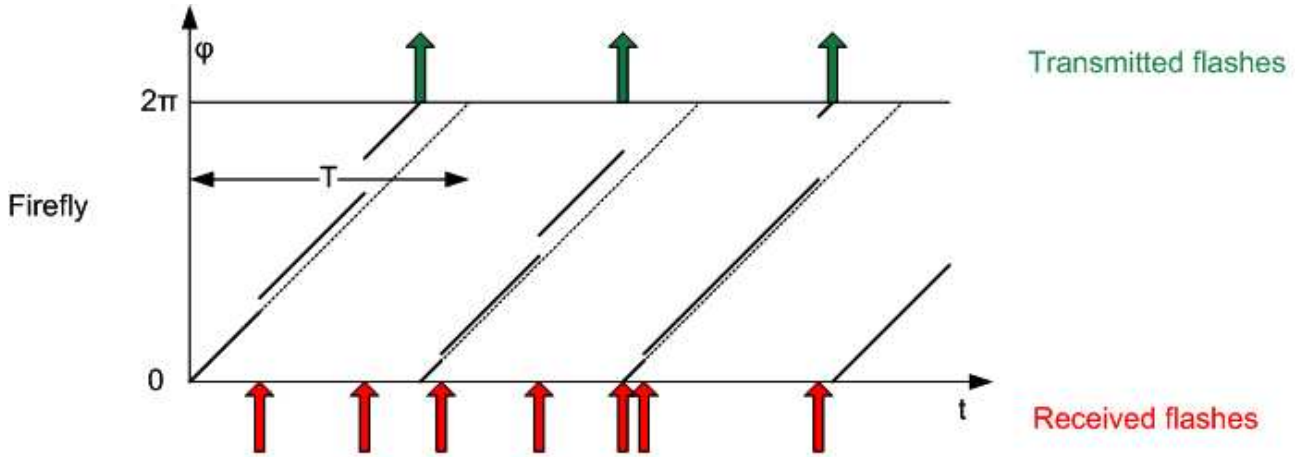
  - $\phi_i \in [0, 2\pi]$ are periodic
  - T is period of ϕ_i
  - $f(\phi)$ is a firing function, $f > 0, \quad \dot{f} > 0, \quad \ddot{f} < 0$
  - $\epsilon > 0$ is a jump constant
- Example: firing function is $f(\phi) = \ln \phi$
  - the jump at $\phi_j = 2\pi$ is

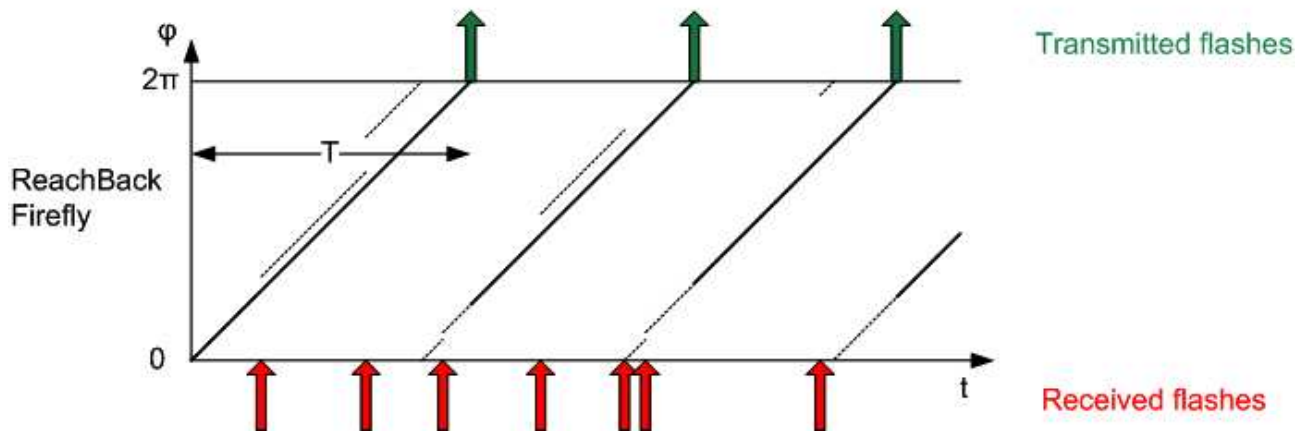
$$\Delta\phi_i = \min \left[\phi_i \, \mathrm{e}^{\epsilon},\ 2\pi \right] - \phi_i$$

Reachback Firefly Synch

- inherent (wireless) communication problems make vanilla firefly problematic
  - channel contention, processing time etc
  - reception of firings may be delayed
- *Reachback* firefly synchronization (Werner-Allen, Tewari & al. 2005) overcomes these problems
  - the effects of the pulses of neighbors are integrated until $\phi_i = 2\pi$
  - one integrated jump at $\phi_i = 0$

Firefly and ReachBack Firefly





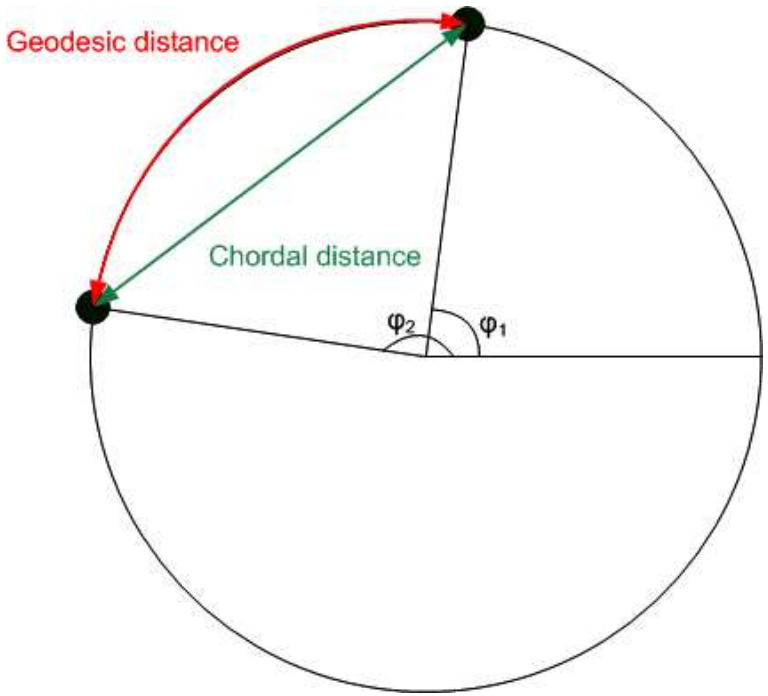
Ferromagnetic Spin Synch

- in firefly synchronization
  - whole history of neighbor timings is integrated into one variable
    - simple operation, can be implemented in analog circuits, such as insects
  - Continuity of time development is sacrificed
- In computers and network elements, more memory available
- No reason not to consider slightly more memory-heavy synchronization
- discard notion of continuous own phase development
- synchronization is inherently a ferromagnetic ordering problem
  - synchronization may be modeled as cooling of ferromagnet

$$H = \frac{1}{2} \sum_i H_i$$

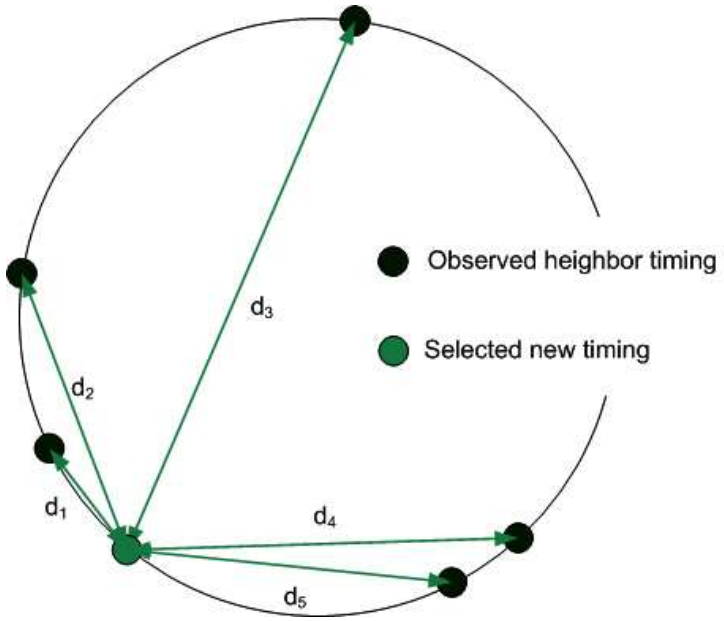
  - At each node, local potential energy $H_i = H_i \left(\phi_i; \{\phi_j\}_{j \in \mathcal{N}_i}\right)$ minimized
  - simple and natural candidates for H_i easy to construct

Natural Metrics on S^1



- Geodesic distance: $d = \min\left(|\phi_1 - \phi_2|, \pi - |\phi_1 - \phi_2|\right)$
- Chordal Distance: $d = 2\left|\sin\frac{\phi_1 - \phi_2}{2}\right|$

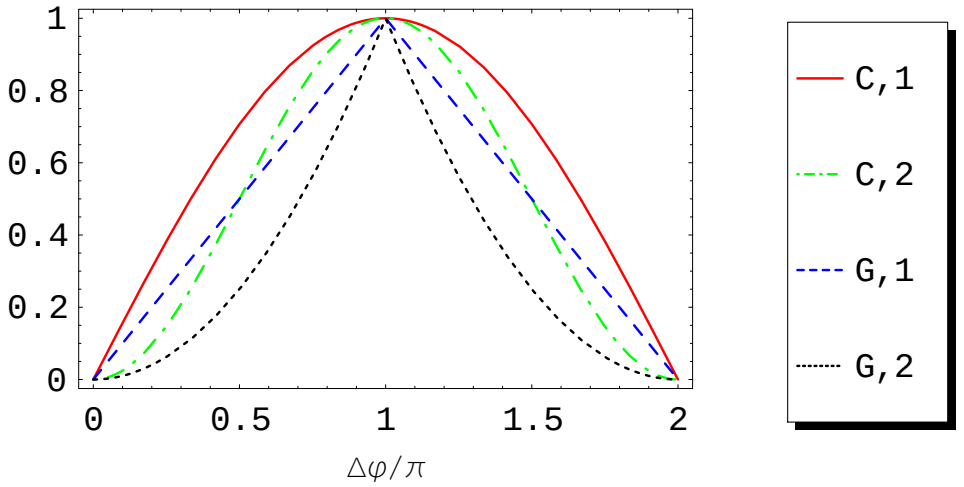
Natural Norms



- one-norm (mean): $||\mathbf{d}||_1 = \frac{1}{M}\sum_{j=1}^{M} d_j$
- two-norm (RMS): $||\mathbf{d}||_2 = \sqrt{\mathbf{d}^{\mathrm{T}}\mathbf{d}/M}$
- inf-norm (max): $||\mathbf{d}||_\infty = \max_j d_j$

Natural Local Hamiltonians

- combining two metrics with three norms we get six natural local Hamiltonians
  - $H_i^{\mathrm{G},1}$, $H_i^{\mathrm{G},2}$, $H_i^{\mathrm{G},\infty}$ with the geodesic metric
  - $H_i^{\mathrm{C},1}$, $H_i^{\mathrm{C},2}$, $H_i^{\mathrm{C},\infty}$ with the chordal metric
  - $H_i^{\mathrm{G},2}$ is geodesic distance on torus
  - $H_i^{\mathrm{C},2}$ is the well-known XY-model, chordal distance on a torus
- contribution to Hamilonian of difference $\Delta\phi$ for 1- and 2-norms:



Energy Minimization

- with given $\phi_j,\ j \in \mathcal{N}_i$, select $\hat{\phi}_i$ so that H_i is minimized
- Geodesic, 1-norm:
  $H_i^{\mathrm{G},1} = \sum_{j\in\mathcal{N}_i} \min\left(|\phi_i - \phi_j|, \pi - |\phi_i - \phi_j|\right)$
  - piecewise linear, angles at $\{\phi_j\}$.
  - Minimum at $\hat{\phi}_i = \arg\min_{\phi\in\{\phi_j\}} \sum_{j\in\mathcal{N}_i} \min\left(|\phi - \phi_j|, \pi - |\phi - \phi_j|\right)$
- Geodesic, 2-norm:

$$H_i^{\mathrm{G},2} = \sum_{j\in\mathcal{N}_i} \min\left(|\phi_i - \phi_j|, \pi - |\phi_i - \phi_j|\right)^2$$

  - there are $|\mathcal{N}_i|$ branch cut alternatives
  - in interval when ϕ_i between two branch cuts

$$H_i^{\mathrm{G},2} = \sum_j \left(c_j\pi + \phi_i - \phi_j\right)^2, \quad c_j = 0, 1$$

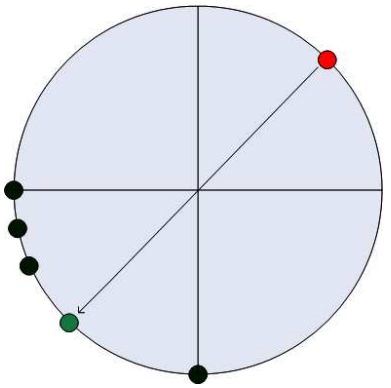
  - Minimum in branch interval at $\hat{\phi}_{ij} = \frac{1}{|\mathcal{N}_i|}\sum_j \phi_j - c_j\pi$
  - True minimum $\hat{\phi}_i = \min_j \hat{\phi}_{ij}$

Energy Minimization II

- Geodesic, ∞-norm:

$$H_i^{\mathrm{G},\infty} = \max_{j \in \mathcal{N}_i} \min\left(|\phi_i - \phi_j|, \pi - |\phi_i - \phi_j|\right)$$

  - Minimum at point that is diametrically opposite to center of largest gap between neighbor timings:



- Chordal, ∞-norm:

$$H_i^{\mathrm{C},\infty} = \max_{j \in \mathcal{N}_i} |\sin((\phi_i - \phi_j)/2)|$$

  - same solution as above

Energy Minimization III

- Chordal, 1-norm

$$H_i^{\mathrm{C},1} = \sum_{j \in \mathcal{N}_i} |\sin((\phi_i - \phi_j)/2)|$$

  - Again $|\mathcal{N}_i|$ branch intervals
  - In each interval, minimum $\hat{\phi}_{ij}$ at end points
  - or at $\hat{\phi}_{ij} = -2 \arctan \frac{\sum \pm \cos(\phi_j/2)}{\sum \pm \sin(\phi_j/2)}$ if contained in interval
  - True minimum $\hat{\phi}_i = \min_j \hat{\phi}_{ij}$

- Chordal, 2-norm (XY-model)

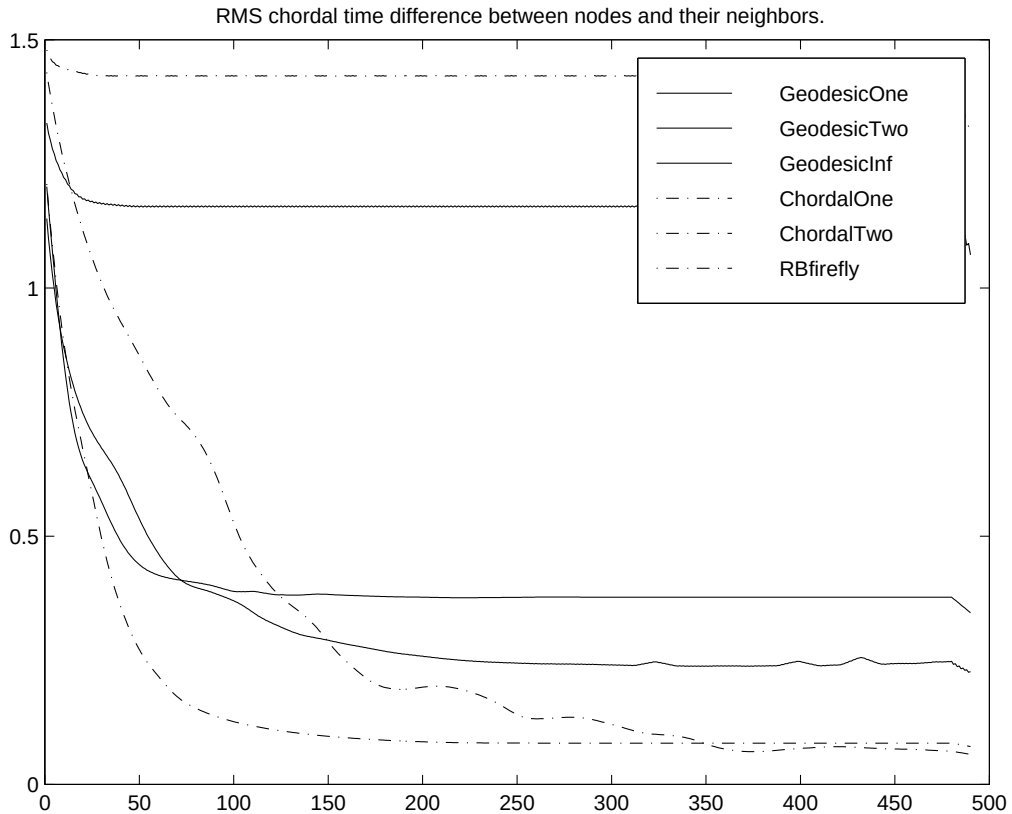
$$H_i^{\mathrm{C},2} = \sum_{j \in \mathcal{N}_i} (\sin((\phi_i - \phi_j)/2)^2$$

  - minimum at $\hat{\phi}_i = \arctan \frac{\sum \sin \phi_j}{\sum \cos \phi_j} + \frac{\pi}{2} \pm \frac{\pi}{2}$

Cooling Dynamics

- Each node updates at a specified instance according to internal clock
- The data that the update is based on may be delayed
- e.g. a “Reachback” firefly -like update algorithm can be considered
- distance from synchrony may be measured by the same norms/metrics as the local Hamiltonians
- rate and speed of convergence towards synchrony are global measures of interest
  - In firefly, rate of convergence scales with ϵ for small ϵ, for large ϵ convergence is lost (Lucarelli-Wang, 2004)
- global and local Hamiltonians may be different
- modeled with discrete d.o.f (1000 states)

Convergence



Summary

- work very much in progress
- With descrete d.o.f, spin cooling does not converge to final synchrony in finite size system
- adding “temperature” helps