创寰宇学府 育天下英才

Study of prisoner's dilemma game, snowdrift game, and naming game on complex network

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INTRODUCTION

How natural selection can lead to cooperative behavior?

- **This question has fascinated evolutionary biologists for several decades**

INTRODUCTION

* Evolution is based on a fierce competition between individuals.

* Evolution reward only selfish behavior.

* Every gene, every cell, and every organism should be designed to promote its own evolutionary success at the expense of its competitors.

* Why we still observe cooperation on many levels of biological organization?

INTRODUCTION

Genes cooperate in genomes. Chromosomes cooperate in eukaryotic cells. Cells cooperate in multicellular organisms. There are many examples of cooperation among animals

INTRODUCTION

Humans are the champions of cooperation: From hunter-gatherer societies to nation-states, cooperation is the decisive organizing principle of human society. No other life form on Earth is engaged in the same complex games of cooperation and defection.

Emergence of cooperation and evolutionary stability ?

- Hence, to explain the evolution of cooperation by natural selection has been a major goal of biologists since Darwin.

- Here we shall study evolutionary game dynamics in finite populations with network structure.

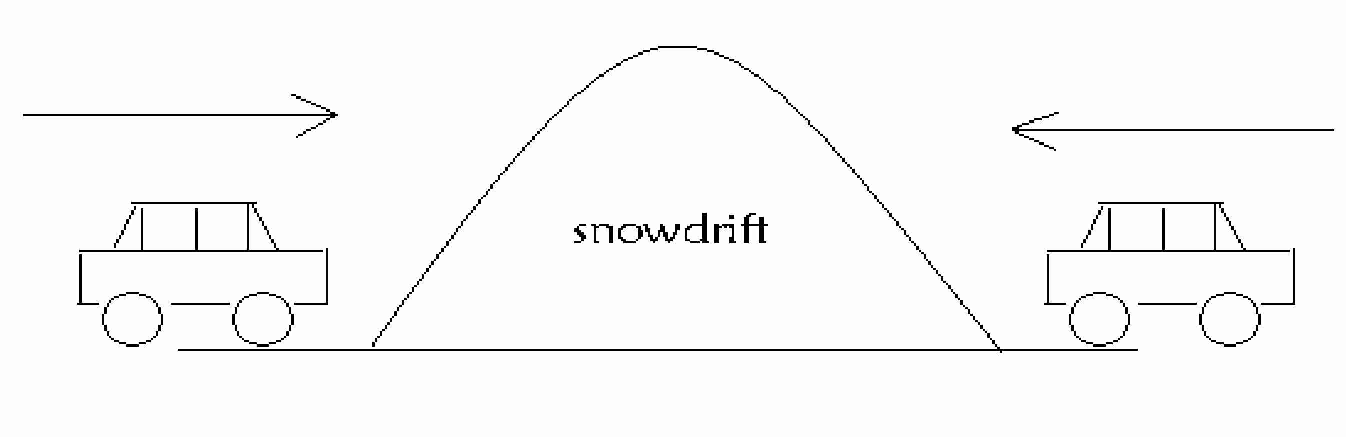
Game theory and physics

- Evolutionary game theory may capture the essentials of the characteristic interactions among individuals.

- What is the effects of population structures on the performance of behavioral strategies ?

Prisoner's Dilemma Game

Snowdrift Game



Prisoner's Dilemma Game (PDG)

Confesses leniently, resistance severely

(坦白从宽，抗拒从严)

	Confes	Resistance
Confes	both condemn 5 Year imprisonment	Sets free with a verdict of not guilty
Resistance	condemn 10 year imprisonment	both condemn 1 Year imprisonment

Snowdrift Game (SG)

	Cooperate	Defect
Cooperate	Both pass after 1 hour	Pass after 2 hours
Defect	Attains without effort	Both could not pass

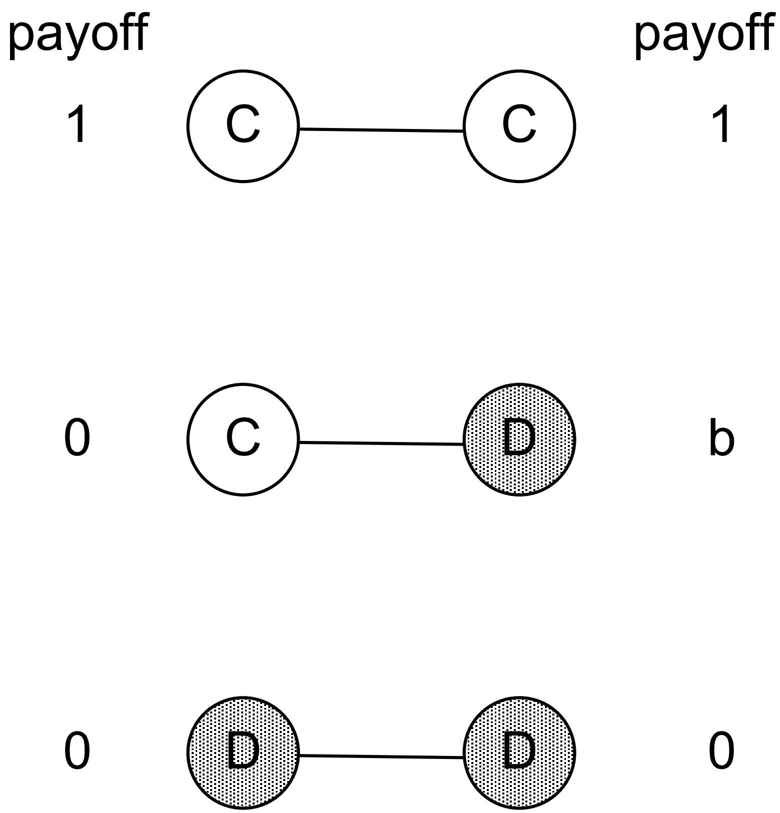
SG is more favorable to cooperation than PDG

Payoff Matrix

$$\begin{array}{ccc} & \mathbf{C} & \mathbf{D} \\ \mathbf{C} & R & S \\ \mathbf{D} & T & P \end{array}$$

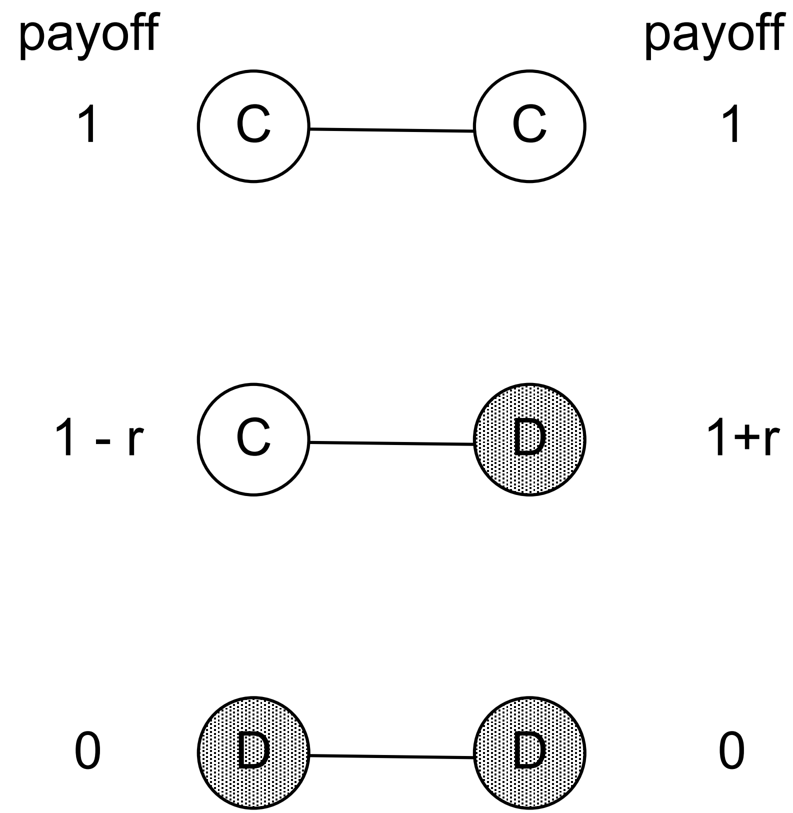
- PDG: T>R>P>S
- SG : T>R>S>P
- restriction: 2R>P+S

Prisoner’s Dilemma



1<b<2

Snowdrift Game



0<r<1

Simplified Payoff Matrix

PDG	C	D
C	1	0
D	b	0

SG	C	D
C	1	$1-r$
D	$1+r$	0

LETTERS TO NATURE

Evolutionary games and spatial chaos

Martin A. Nowak & Robert M. May

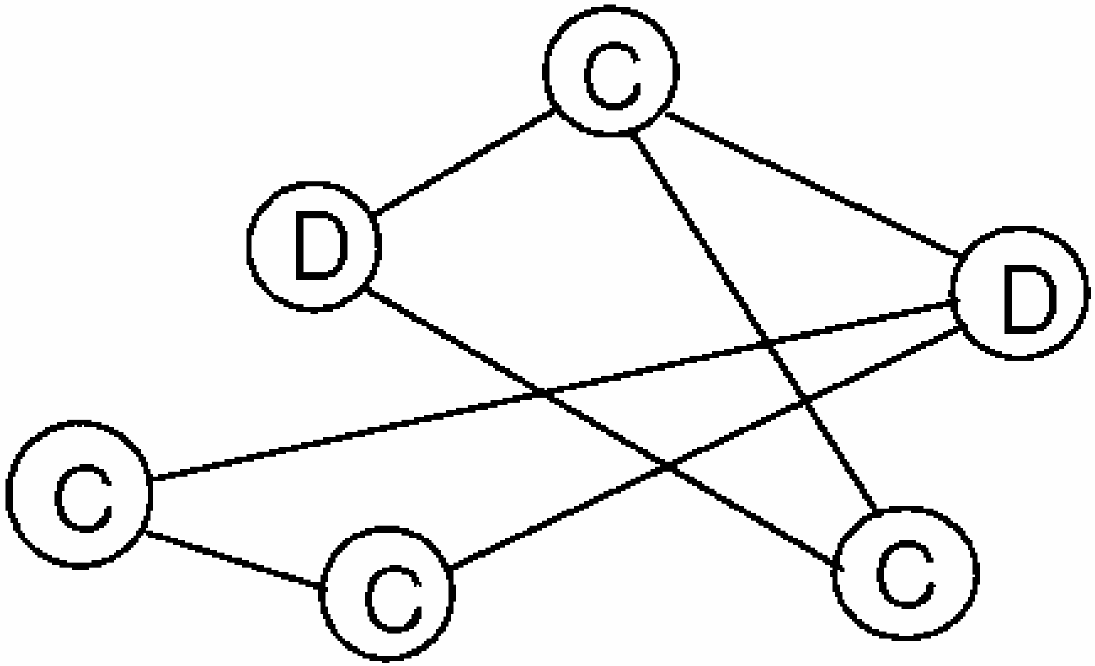
Department of Zoology, University of Oxford, South Parks Road, Oxford OX1 3PS, UK

Background

1992

In this paper, we consider only two kinds of players: those who always cooperate, C, and those who always defect, D. No explicit attention is given to past or likely future encounters, so no memory is required and no complicated strategies arise. Interesting results emerge when we place these 'players'—who may be individuals or organized groups—on a two-dimensional, $n \times n$ square lattice of 'patches': each lattice-site is thus occupied either by a C or a D. In each round of our game (or at each

The evolution Game on the complex network



The rule

In each round of our game (or at each time step, or each generation), each patch-owner plays the game with its immediate neighbours. The score for each player is the sum of the pay-offs in these encounters with neighbours. At the start of the next generation, each lattice-site is occupied by the player with the highest score among the previous owner and the immediate neighbours.

The spatial PDG can generate a large variety of qualitatively different patterns, depending on b (the advantage for defectors)

- Simulation: on 200x200 square lattice with fixed boundary condition
- Start from random initial configuration with 10% defectors (and 90% cooperators)
- Show: asymptotic pattern after 200 generations
- Color coding:

  Blue: C (following a C), **Green: C** (following a D).

  Red: D (following a D). **Yellow: D** (following a C)
- A), 1.75<b<1.8, an irregular, but static pattern
- B), 1.8<b<2, spatial chaos

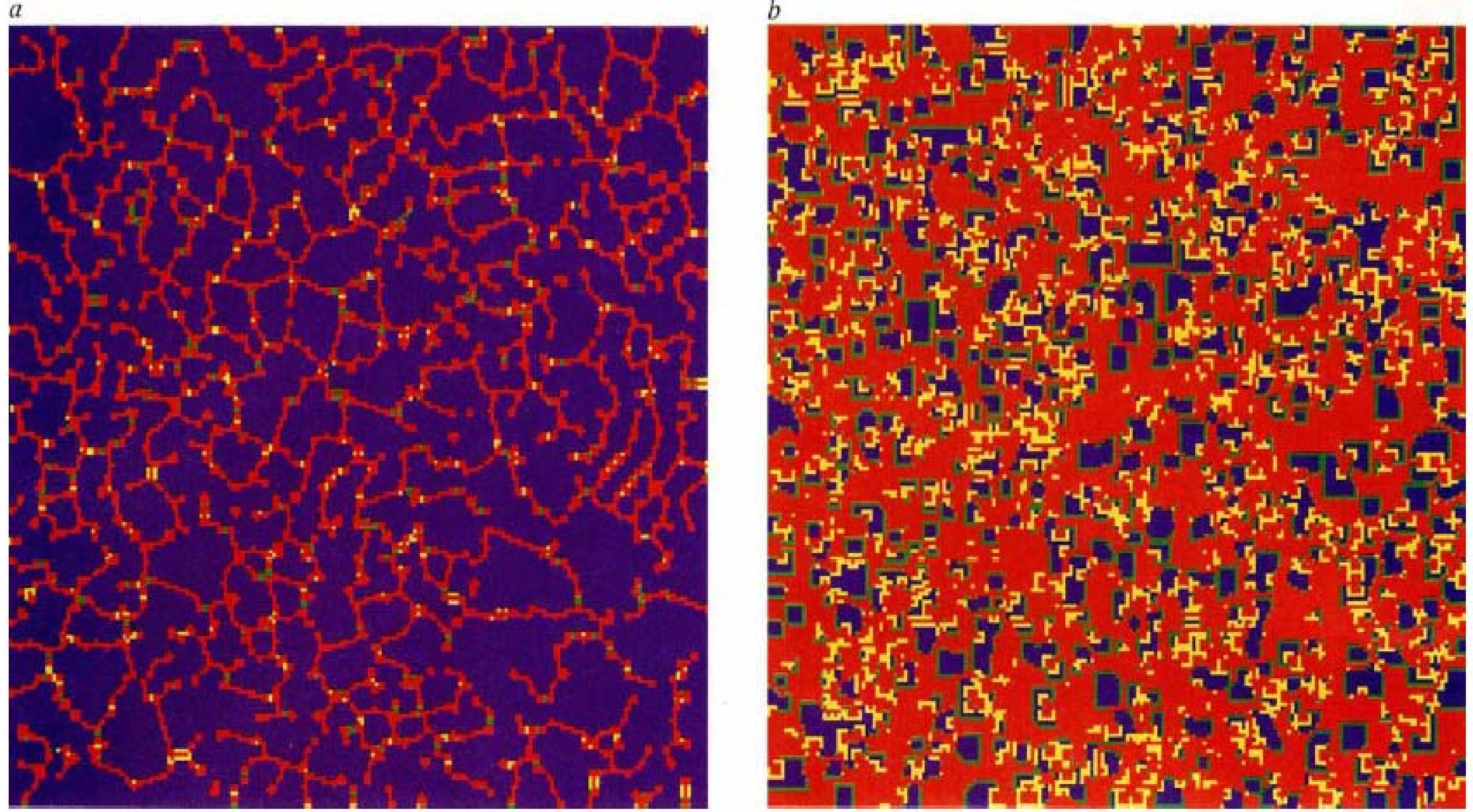
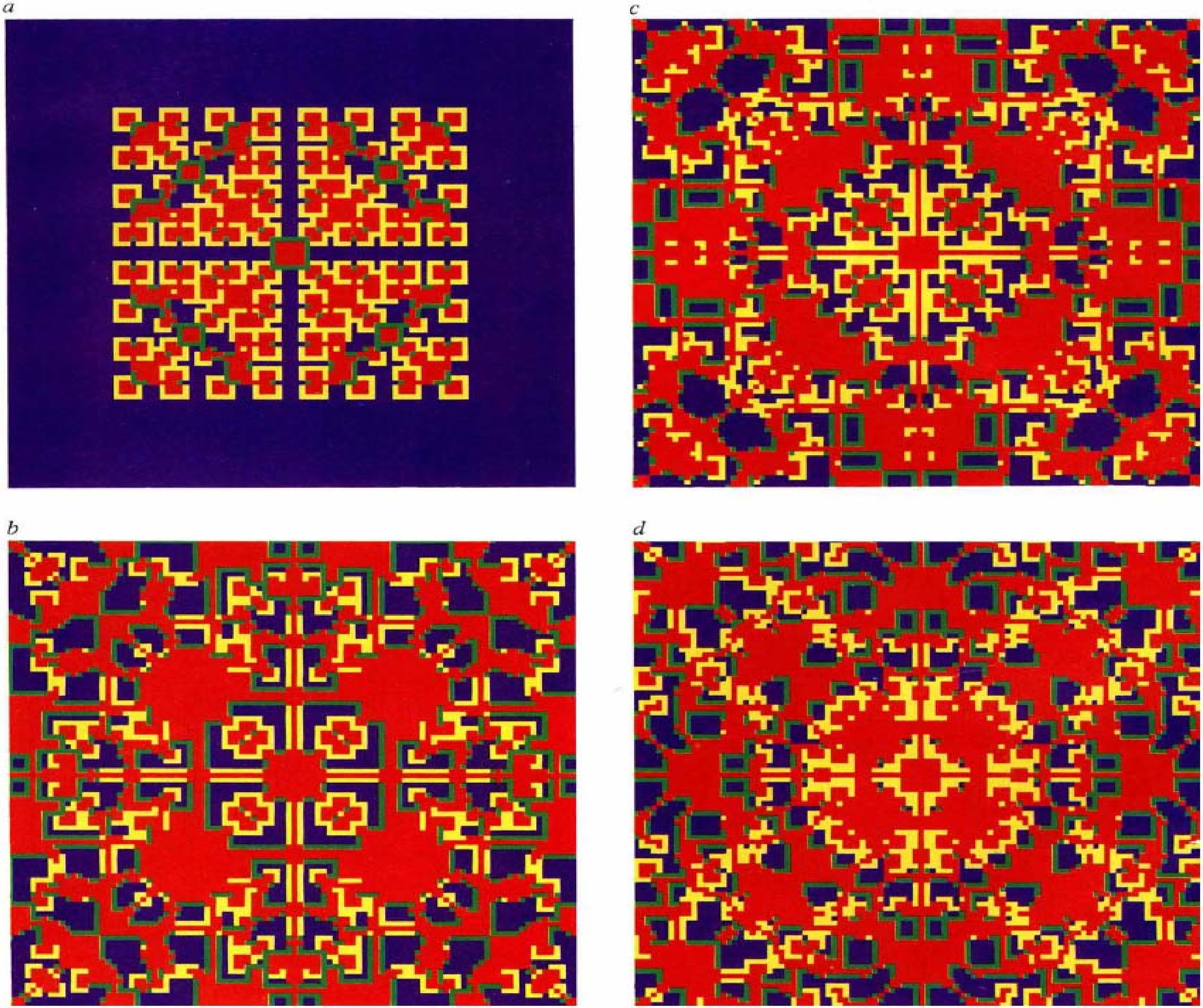


FIG. 1 The spatial Prisoners' Dilemma can generate a large variety of qualitatively different patterns, depending on the magnitude of the parameter, b, which represents the advantage for defectors. This figure shows two examples. Both simulations are performed on a 200×200 square lattice with fixed boundary conditions, and start with the same random initial configuration with 10% defectors (and 90% cooperators). The asymptotic pattern after 200 generations is shown. The colour coding is as follows: blue represents a cooperator (C) that was already a C in the preceding generation; red is a defector (D) following a D; yellow a D following a C; green a C following a D. *a*, An irregular, but static pattern (mainly of interlaced networks) emerges if $1.75 < b < 1.8$. The equilibrium frequency of C depends on the initial conditions, but is usually between 0.7 and 0.95. For lower b values (provided $b > \frac{9}{8}$), D persists as line fragments less connected than shown here, or as scattered small oscillators ('D-blinkers'). *b*, Spatial chaos characterizes the region $1.8 < b < 2$. The large proportion of yellow and green indicates many changes from one generation to the next. Here, as outlined in the text, 2×2 or bigger C clusters can invade D regions, and vice versa. C and D coexist indefinitely in a chaotically shifting balance, with the frequency of C being (almost) completely independent of the initial conditions at ~ 0.318.



***t*= 30, 217, 219, 221**

FIG. 3 Spatial games can generate an 'evolutionary kaleidoscope'. This

always maintained because the rules of the game are symmetrical. The

letters to nature

Spatial structure often inhibits the evolution of cooperation in the snowdrift game

Nature428(2004)643

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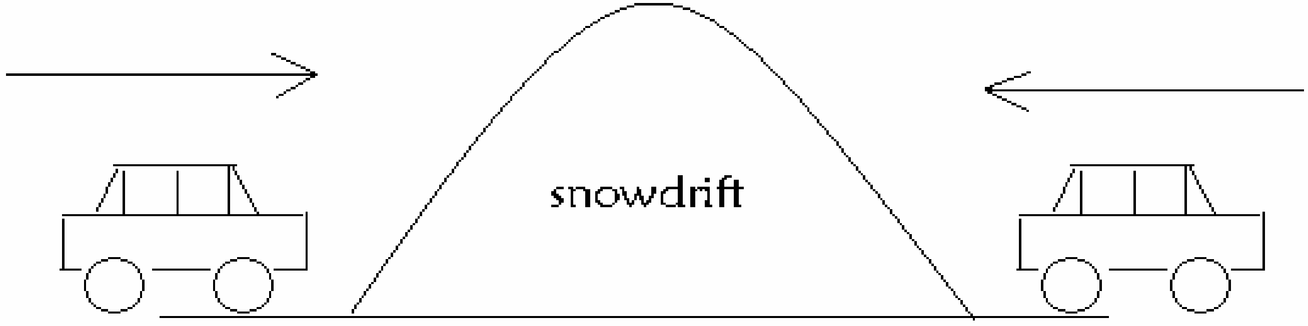


In the present evolutionary procedure the randomly chosen player (x) can adopt one of the (randomly chosen) co-player's (y) strategy with a probability depending on the difference of normalized payoff ($m_x - m_y$) as

$$W(s_x \leftarrow s_y) = \frac{1}{1 + \exp[(m_x - m_y)/T]}$$

where T indicates the noise

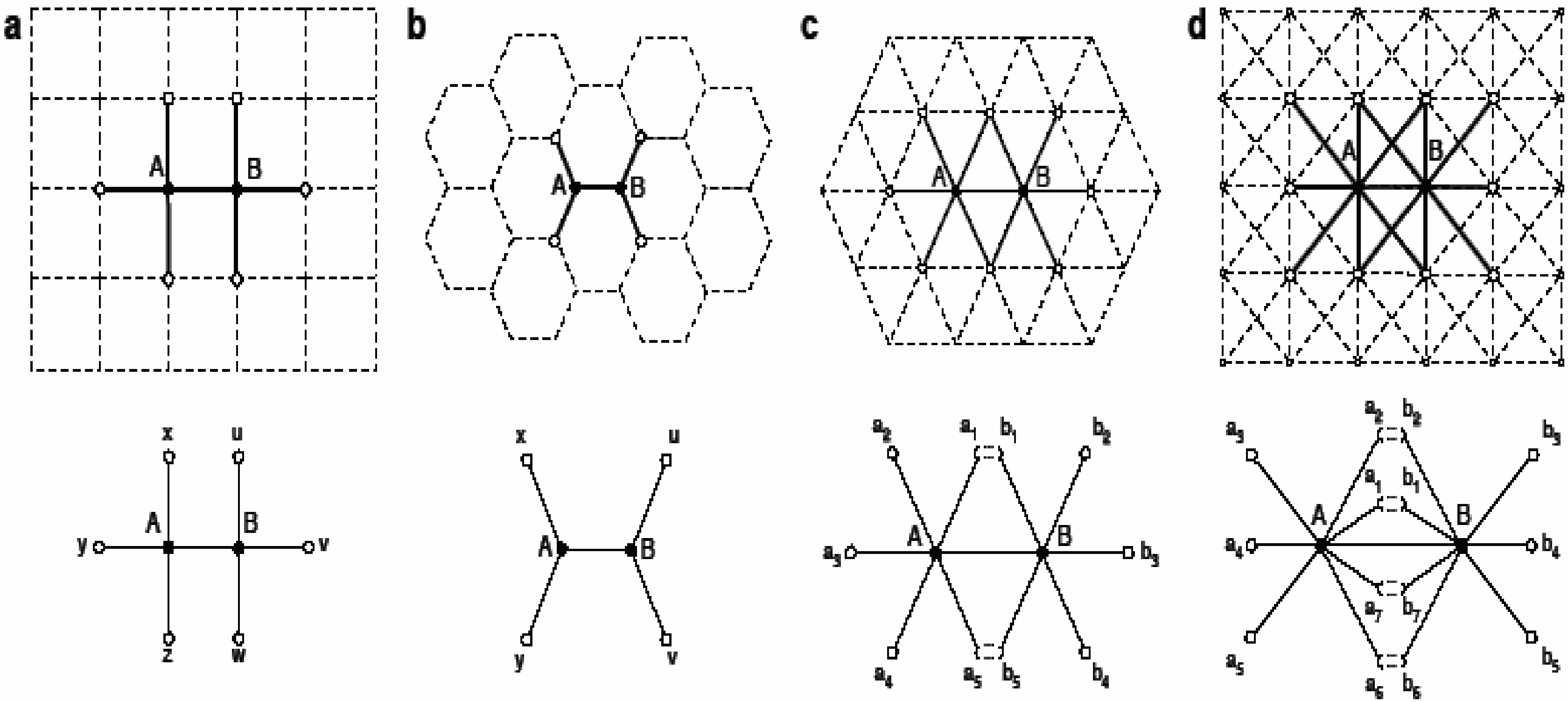
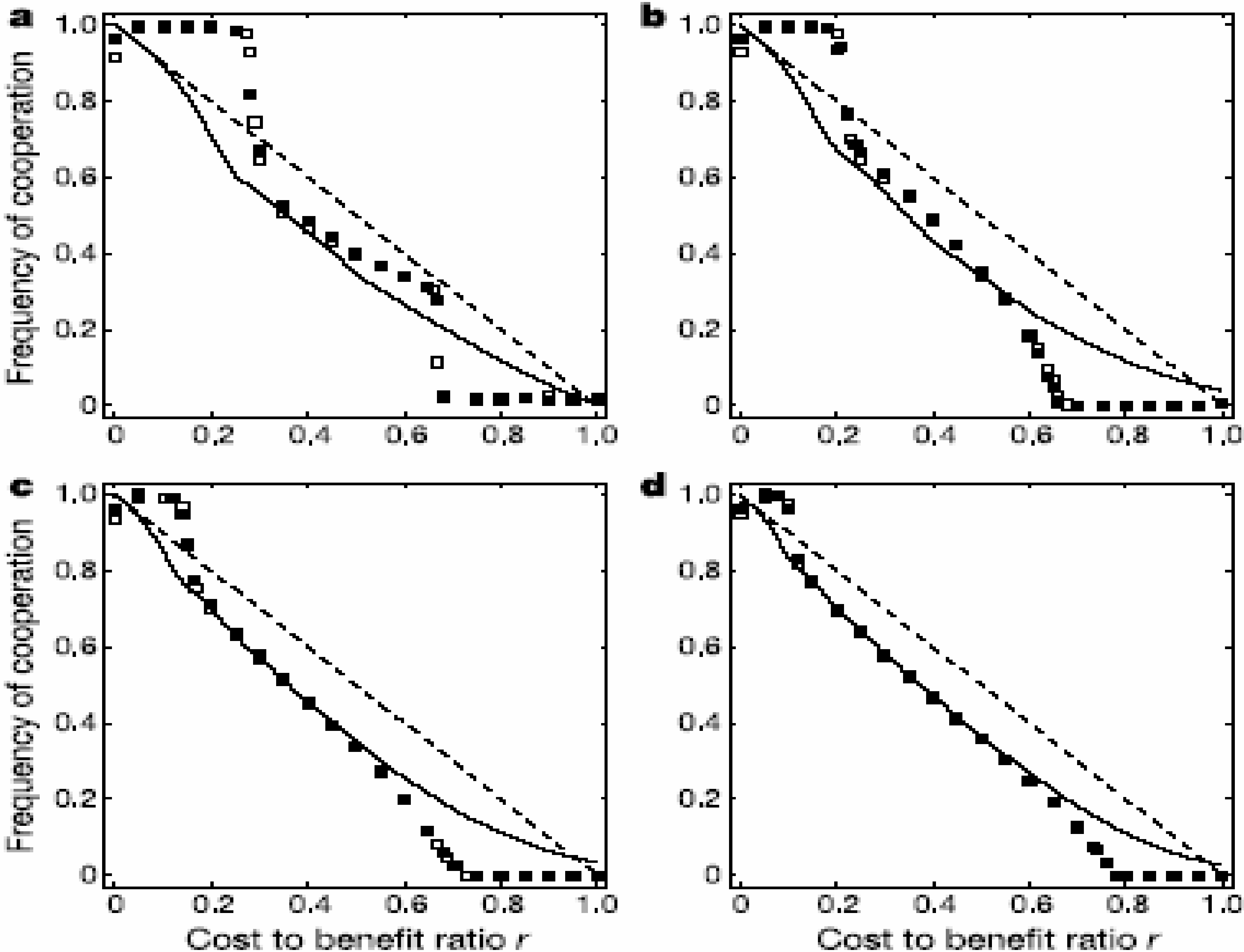


Figure 1: Four lattice configurations (top row) and the corresponding schemes used for the pair approximation with focal sites A and B (bottom row). These schemes are used to determine changes in the pair configuration probabilities $p_{A,B\to B,B}$. **a** square lattice with $N = 4$ neighbours, **b** triangular lattice ($N = 3$), **c** hexagonal lattice ($N = 6$) and **d** square lattice ($N = 8$). Note that on hexagonal and square ($N = 8$) lattices, the edges from A and B to their common neighbours are considered to be independent, i.e., all corrections arising from loops are neglected.

Frequency of cooperators as a function of the cost-to-benefit ratio r

Figure 1 Frequency of cooperators as a function of the cost-to-benefit ratio $r = c/(2b - c)$ in the snowdrift game for different lattice geometries. **a**, Triangular lattice, neighbourhood size $N = 3$; **b**, square lattice, $N = 4$; **c**, hexagonal lattice, $N = 6$; **d**, square lattice, $N = 8$. For small r, spatial structure promotes cooperation; however, for intermediate and high r, the fraction of cooperators is lower than in well-mixed populations (dotted line). This result is largely independent of whether updating is synchronous (filled squares) or asynchronous (open squares). The tendency is correctly predicted by pair approximations (unbroken line), but pair approximation underestimates the effects of local configurations at high and low r. In individual-based simulations, the range of coexistence of cooperators and defectors is delimited by two threshold values: below r_1 defectors vanish, whereas above r_2 cooperators are doomed. Both thresholds correlate with the fate of local configurations: near r_1 defector pairs tend to annihilate and vanish, whereas near r_2 single cooperators and cooperator pairs cannot survive in a sea of defectors. See Methods for simulation details.



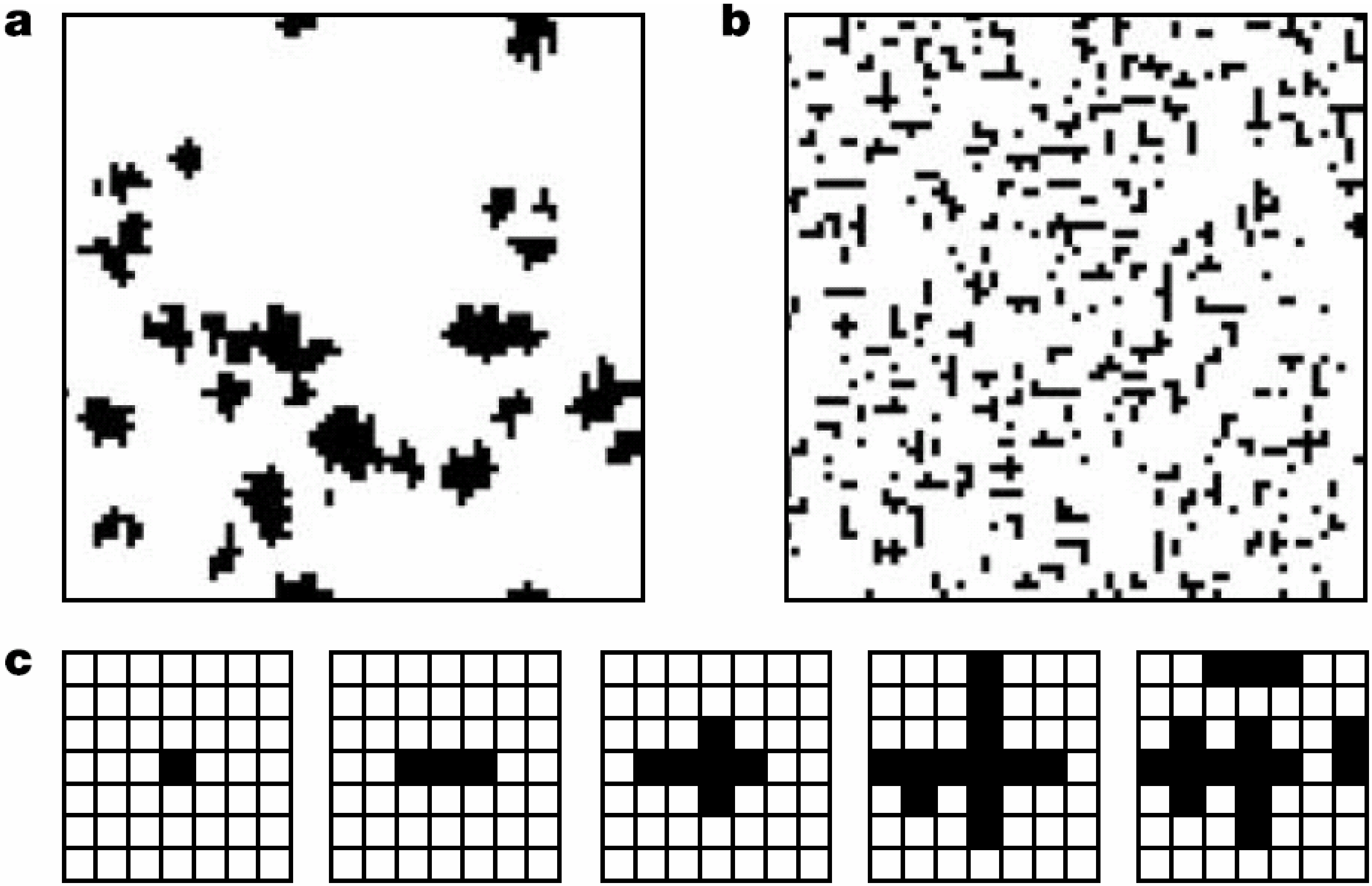
a
b
c
d
Frequency of cooperation
Cost to benefit ratio r
0
0.2
0.4
0.6
0.8
1.0

- dotted line: well-mixed populations
- filled squares: updating is synchronous
- open squares: asynchronous
- unbroken line: pair approximations

Why spatial structure often inhibits the evolution of cooperation in SG?

Figure 2 Snapshots of equilibrium configurations of cooperators (black) and defectors (white) in the spatial Prisoner's Dilemma and spatial snowdrift game on a square lattice with $N = 4$ neighbours near the extinction threshold of cooperators. **a**, In the Prisoner's Dilemma, cooperators survive by forming compact clusters ($R = 1$, $T = 1.07$, $S = -0.07$, $P = 0$). **b**, In the corresponding snowdrift game, cooperators are spread out, forming many small and isolated patches ($r = 0.62$; that is, $R = 1$, $T = 1.62$, $S = 0.38$, $P = 0$). This result also holds for other lattice structures (not shown). **c**, Microscopic pattern formation in the spatial snowdrift game. An isolated cooperator can grow into a row of cooperators and then form cross-like structures; however, cooperators cannot expand to compact clusters because the payoff structure protects the defectors in the corners. Eventually, cooperators form a dendritic skeleton. Occasionally, dendrites break off to form new seeds.

PDG SG



What is the Pattern of Evolutionary Games on Networks?

Memory-based snowdrift game (MBSG) on networks

Wang, Ren, Chen, Wang:

Memory based snowdrift game on networks,

Phys. Rev. E 74, 056113(2006).

The rules of the evolutionary MBSG

- Consider that N players are placed on the nodes of a certain network.
- In every round, all pairs of connected players play the game simultaneously.
- The total payoff of each player is the sum over all its encounters.
- After a round is over, each player will have the strategy information (C or *D)* of its neighbors.

- Subsequently, each player knows its best strategy in that round by means of self-questioning, i.e., each player adopts its antistrategy to play a virtual game with all its neighbors, and calculates the virtual total payoff.

- Comparing the virtual payoff with the actual payoff, each player can get its optimal strategy corresponding to the highest payoff and then record it into its memory.

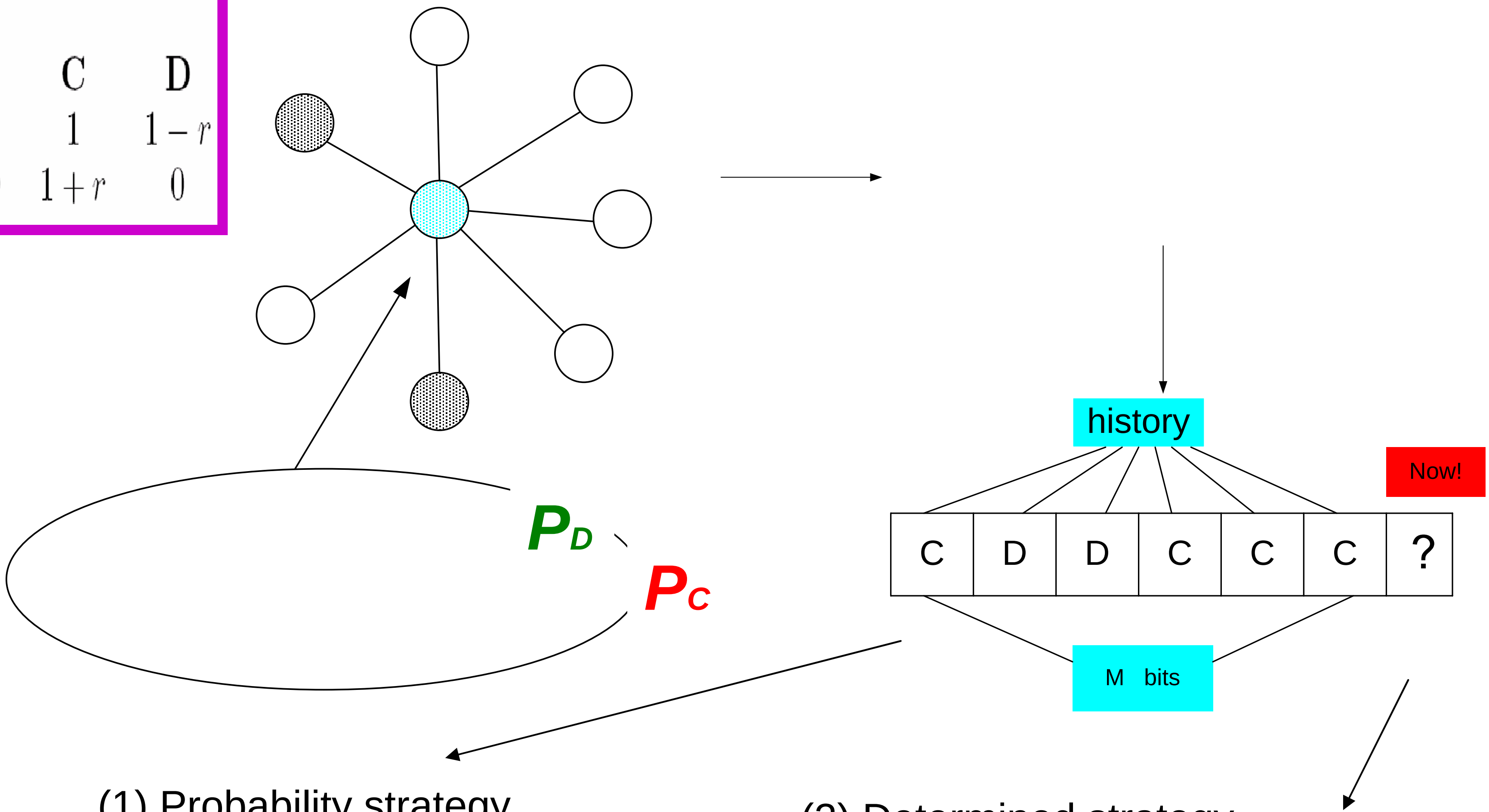
Assume: the bounded rationality of players

Players are quite limited in their analyzing power

Can only retain the last M bits of the past strategy information.

Memory-based snowdrift game on networks

SG	C	D
C	1	$1-r$
D	$1+r$	0



(1) Probability strategy

C : 4 / 6

D : 2 / 6

(2) Determined strategy

D

- At the start of the next generation, the probability of making a decision (choosing C or *D)* for each player depends on the ratio of the numbers of C and D stored in its memory:

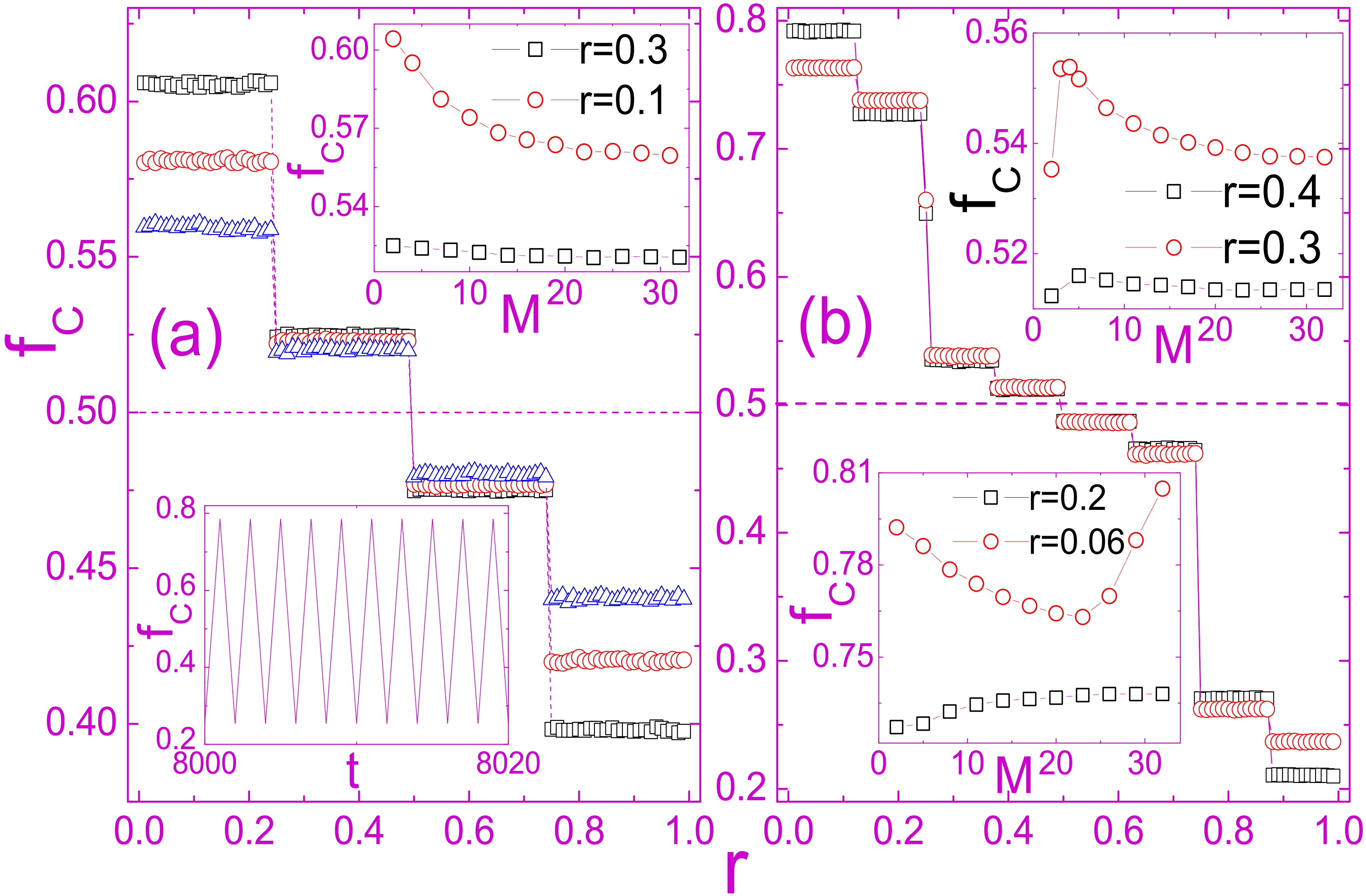
$$P_C = \frac{N_C}{N_C + N_D} = \frac{N_C}{M}, \qquad P_D = 1 - P_C$$

- All players update their memories simultaneously.
- Repeat the above process and the system evolves.

MBSG on Lattices

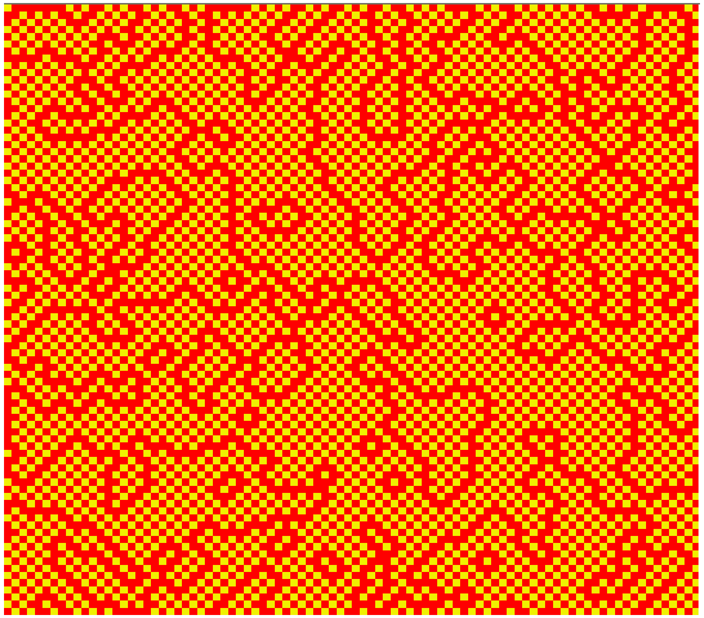
FIG. 1. (Color online). The frequency of cooperation f_C as a function of the payoff parameter r for two-dimensional (a) four-neighbor and (b) eight-neighbor lattices, respectively. (△), (○), and (□) are for $M=2$, 7, and 30, respectively. Each data point is obtained by averaging over 40 different initial states and f_C for each simulation is obtained by averaging from MC time step $t=5000$ to $t=10\,000$, where the system has reached a steady state. The top inset of (a) is f_C as a function of memory length M for two different cooperation levels. The bottom inset of (a) is a time series of f_C for $r=0.4$ in the case of $M=1$. Since for $M=1$, f_C as a function of t displays a big oscillation, we do not compute the f_C over a period of MC time steps. The inset of (b) is f_C depending on M for four cooperation levels in the range of $0<r<0.5$. The network size is $N=10\,000$.

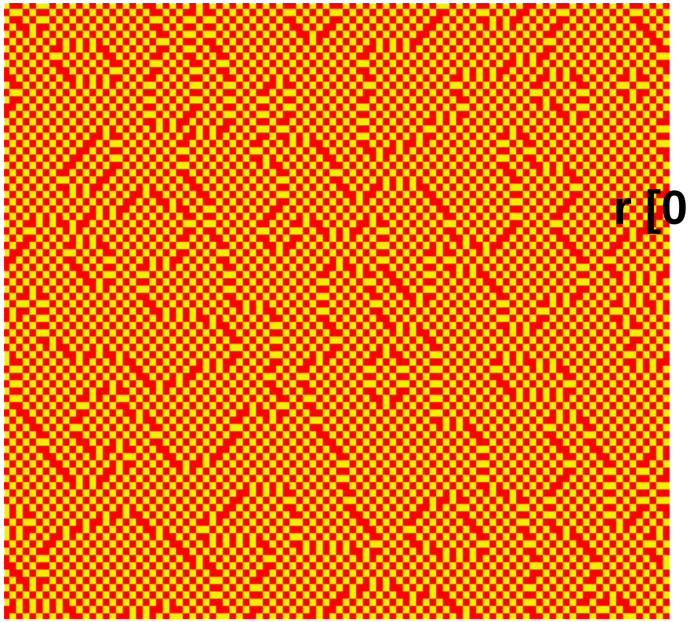
On lattices with 4 and 8 neighbors



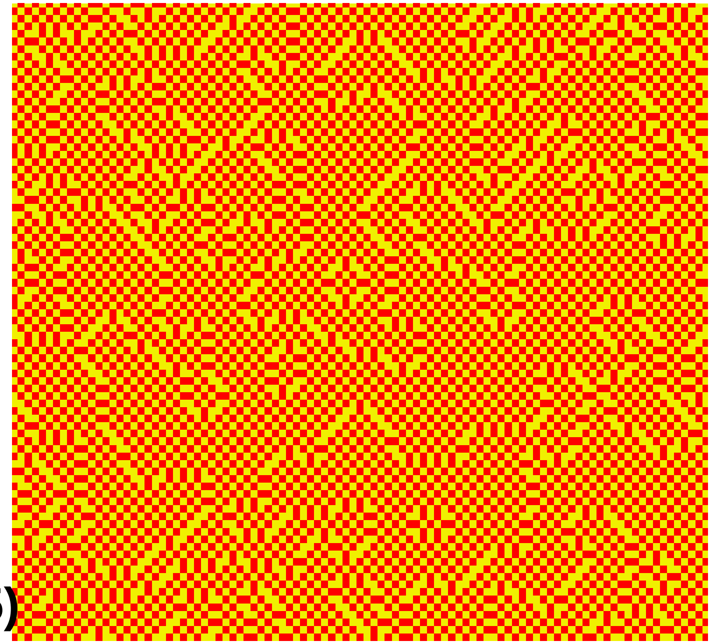
On lattices with 4 neighbors

r [0,0.25)

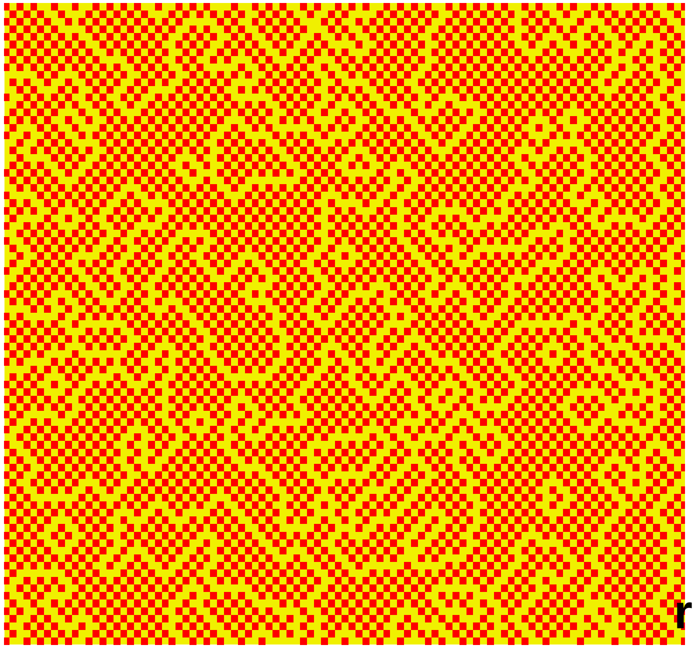




r [0. 25,0.5)



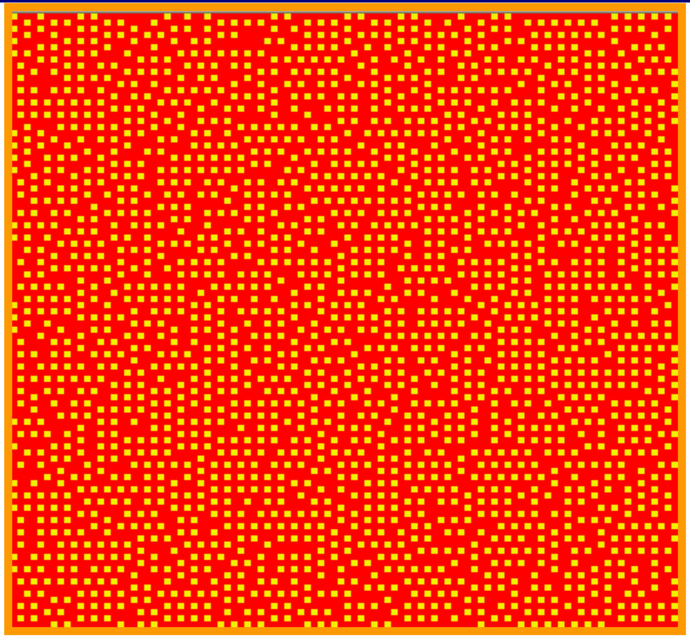
r [0.5,0.75)

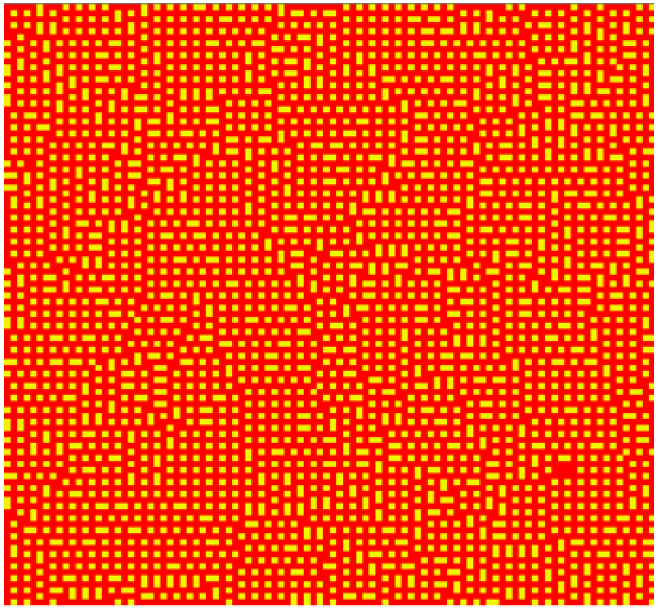


r [0.75,1)

On lattices with 8 neighbors

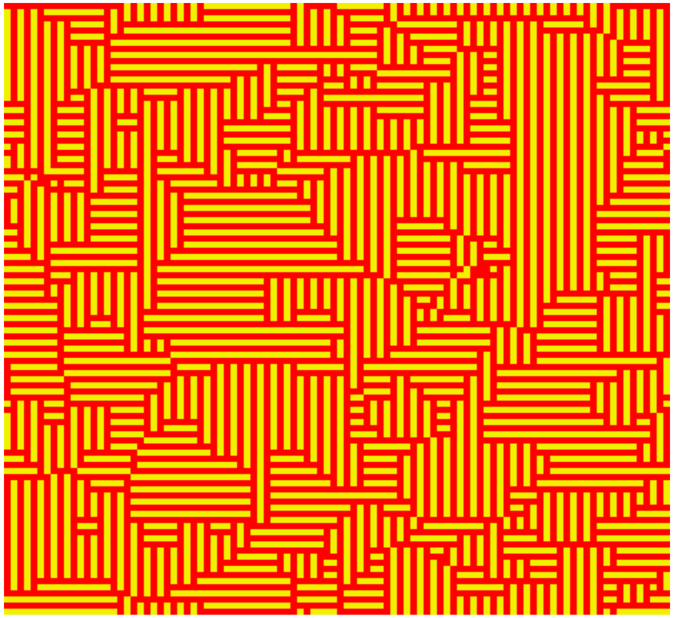
r[0, 0.125)

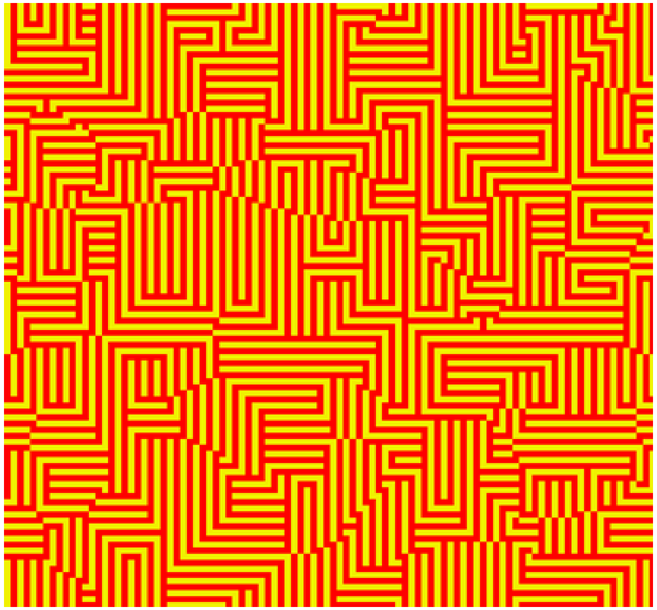




r [0.125, 0.25)

r [0.25, 0.375)



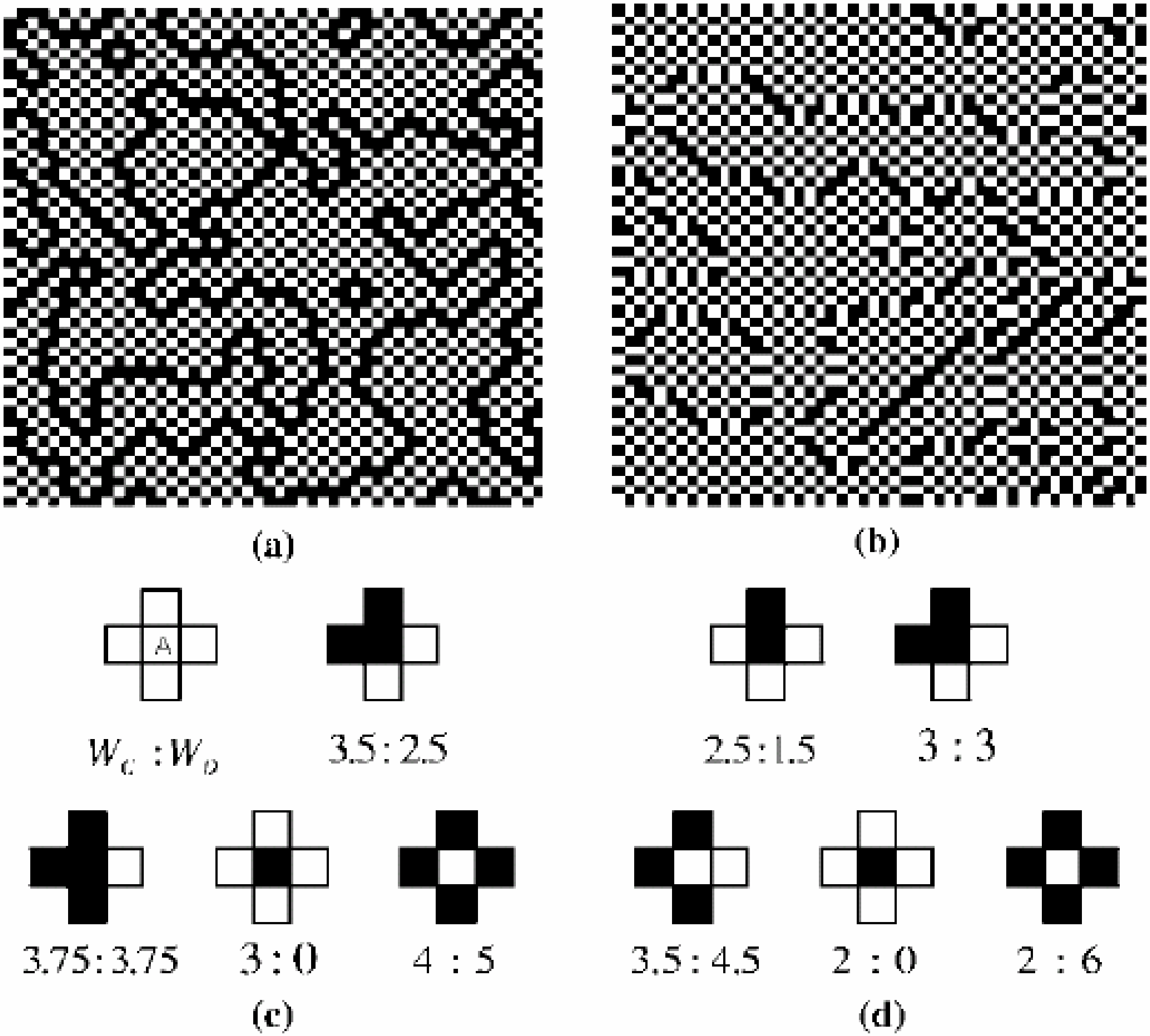


r [0.375, 0.5)

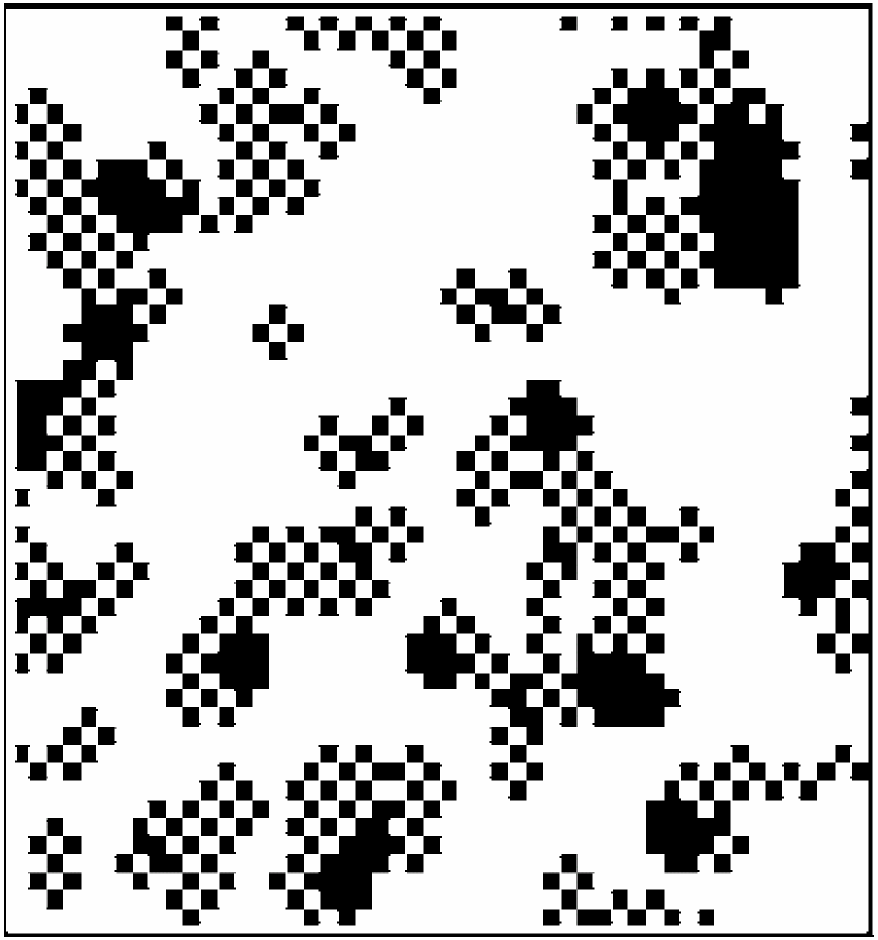
Stable local patterns (for z=4)

r [0,0.25)

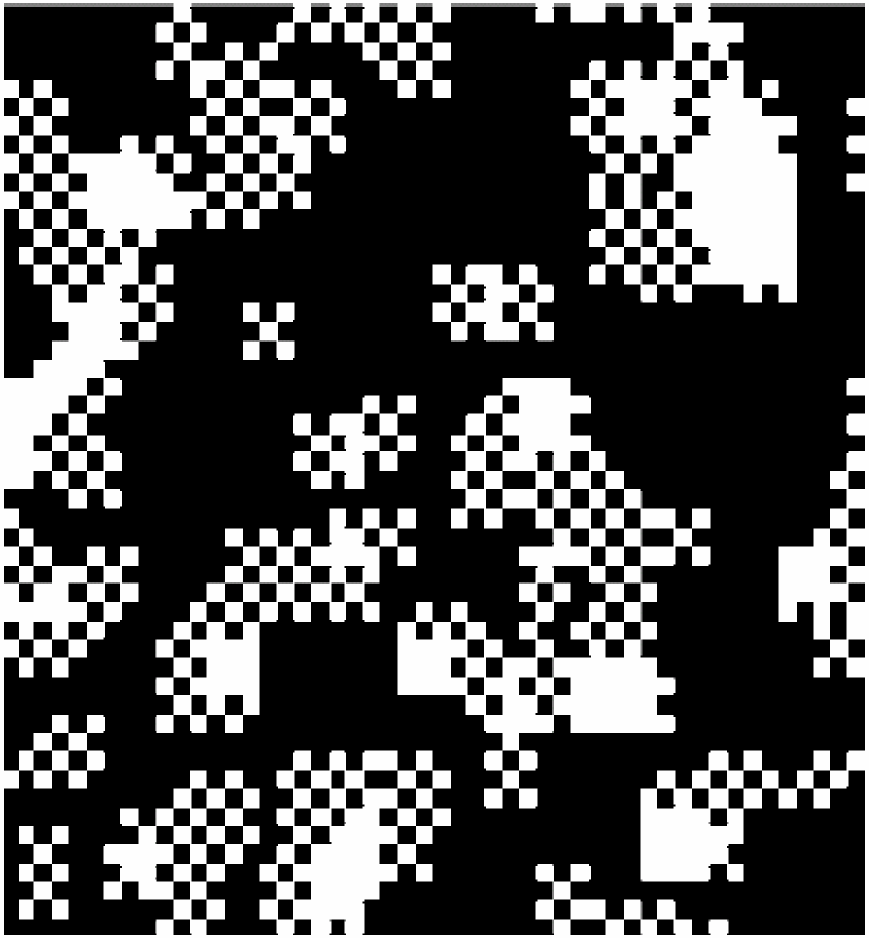
r [0. 25,0.5)



Typical patterns for one time step
(M=1, r=0.4)



(a)



(b)

Random Effect on Cooperation in Memory-Based Snowdrift Game

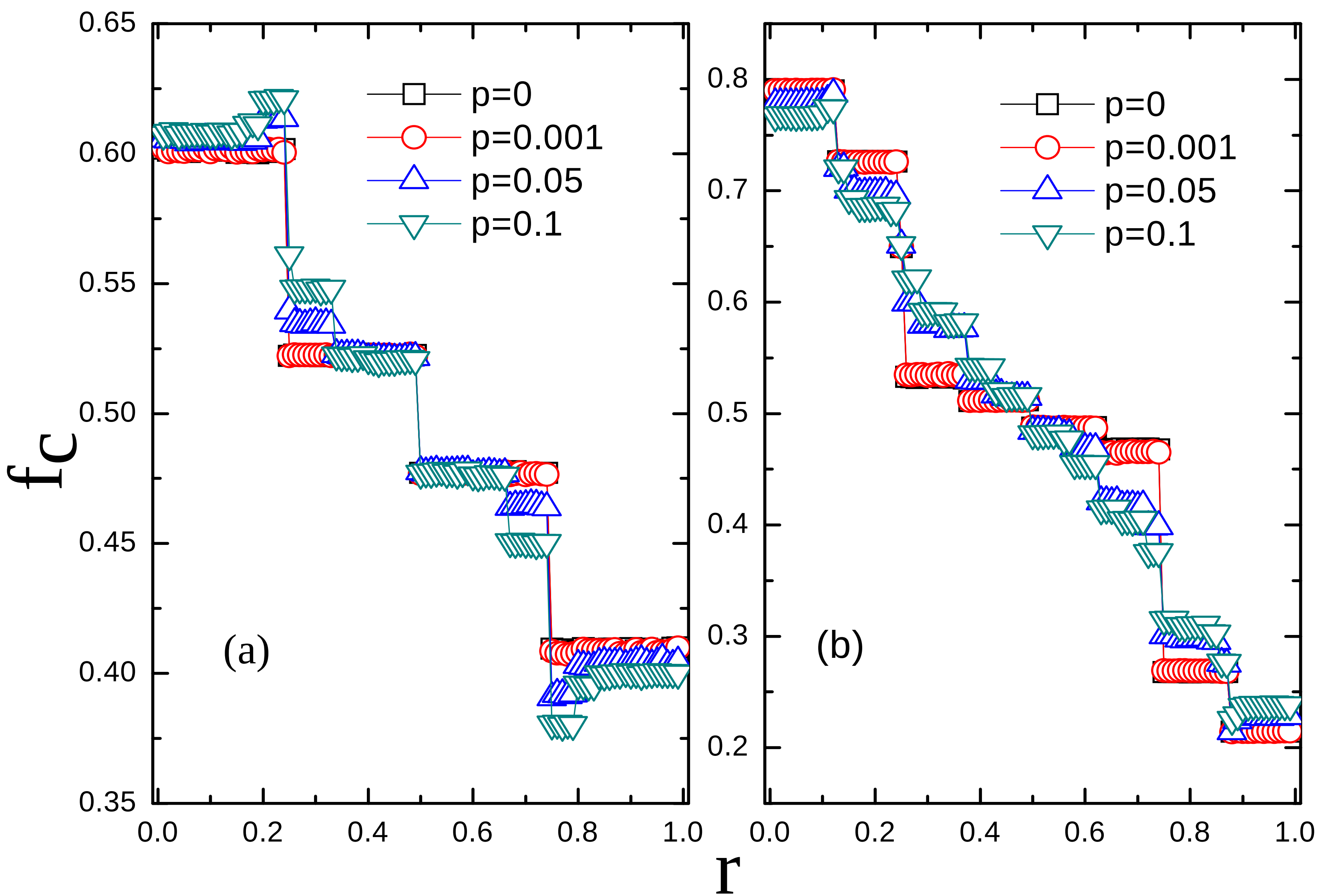
MBSG on small-world networks

Based on two-dimensional lattices, We define rewiring probability *p* with which we randomly rewire each edge of lattice. Then we get a Watts-Strogatz model.

What is the typical spatial pattern on SWNBRL4 and SWNBRL-8 ?

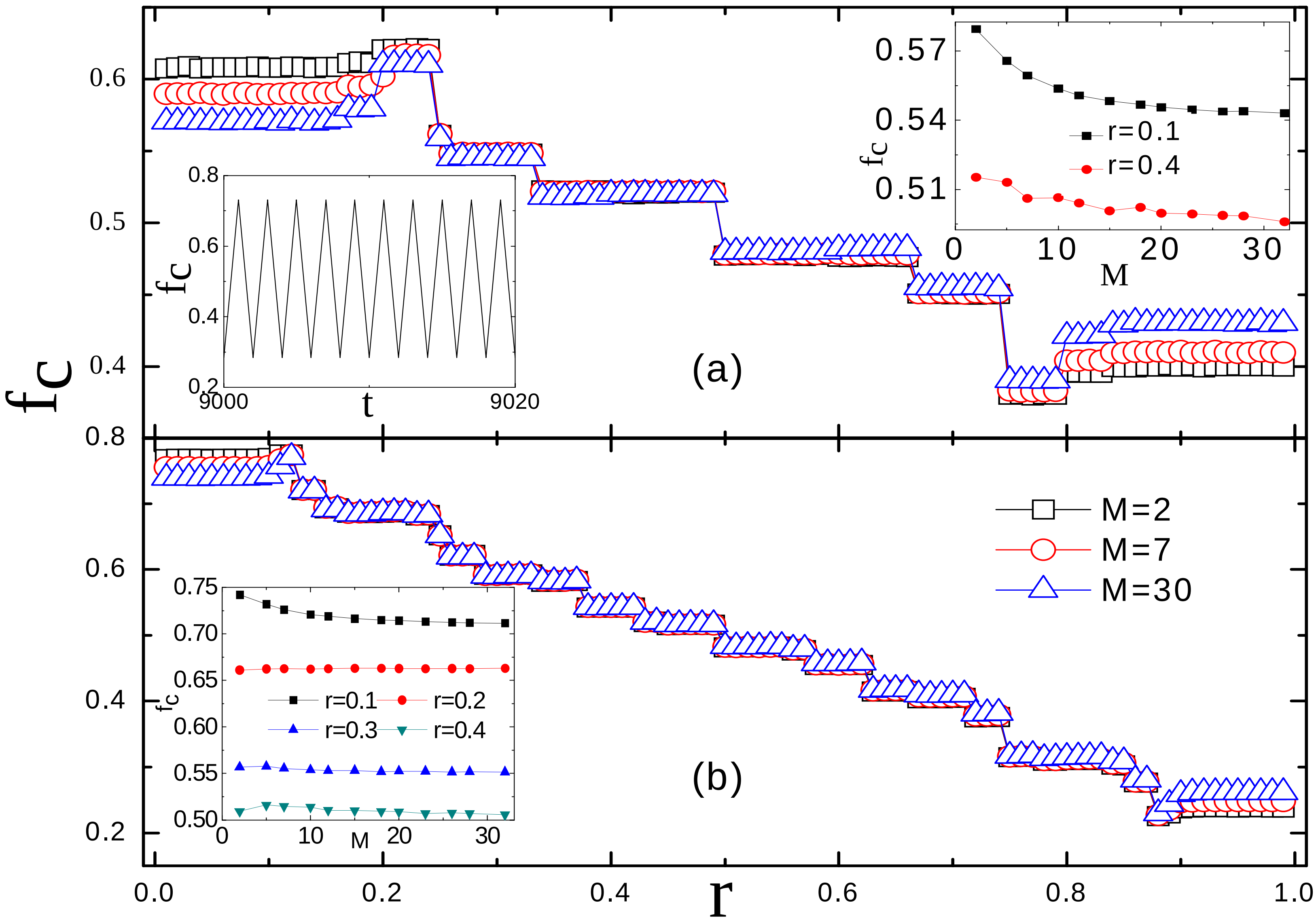
M=2, N=10000, averaging time step t=9000→10000

(a) SWNBRL4; (b) SWNBRL-8

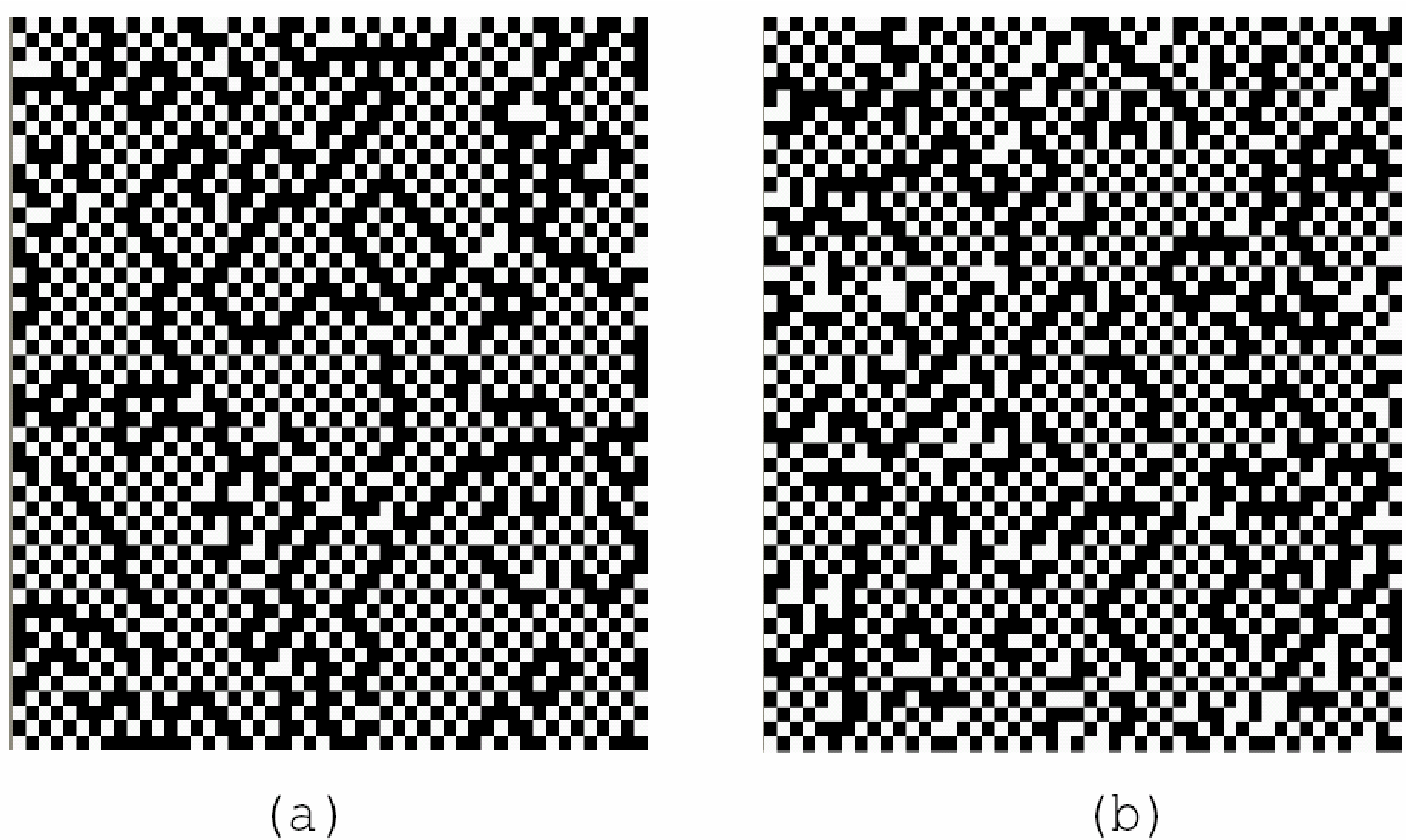


p=0.1, N=10000, averaging time step t=9000→10000

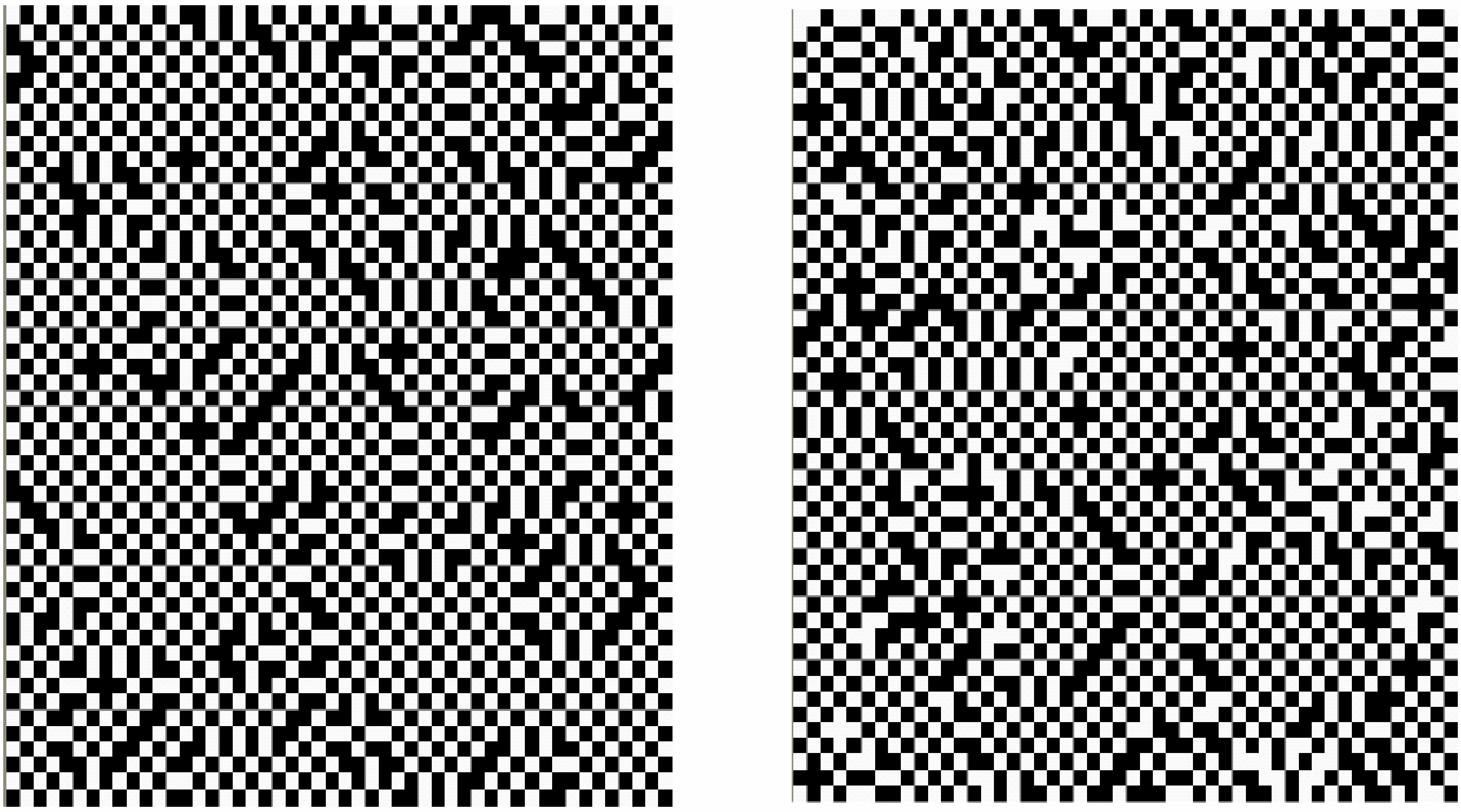
(a) SWNBRL4; (b) SWNBRL-8



Spatial pattern in SWNBRL4 (M=6)
(a) p=0, and (b) p=0.001 for r=0.1

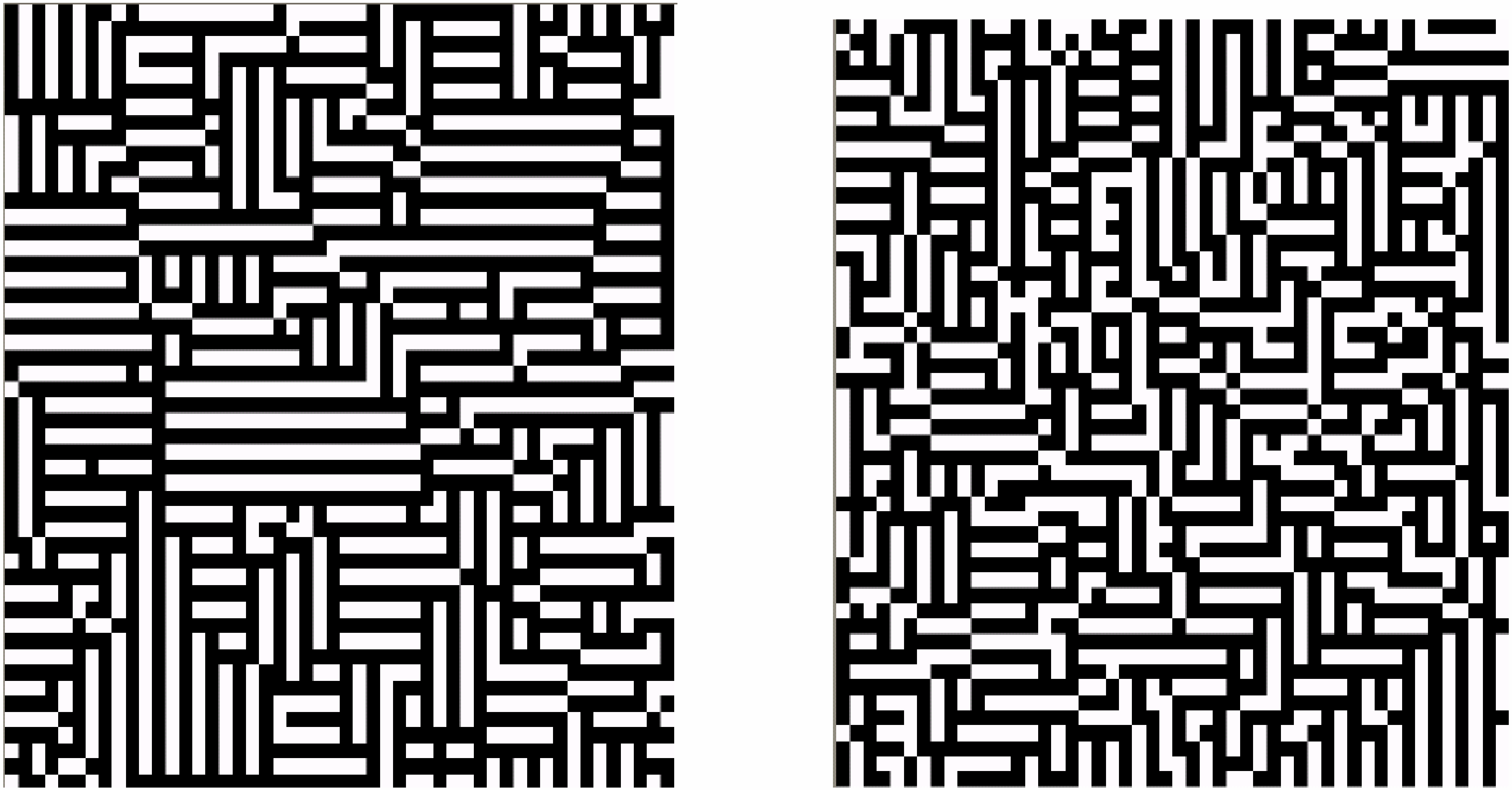


Spatial pattern in SWNBRL4 (M=6)
(c) p=0.05, and (d) p=0.1 for r=0.4



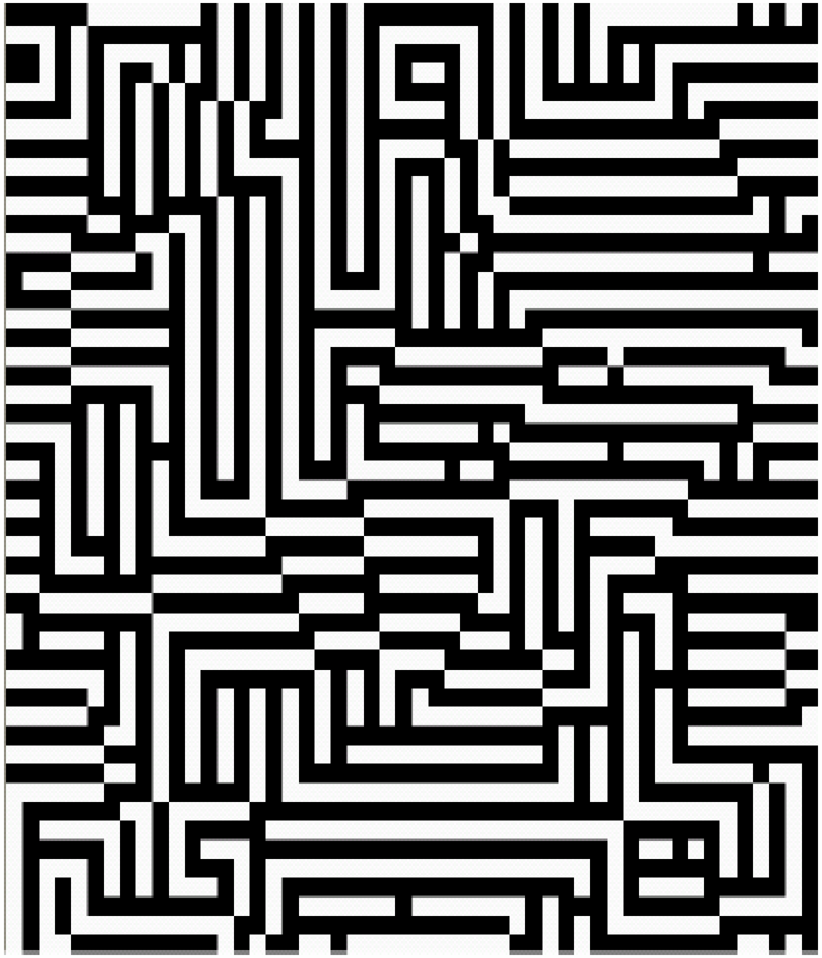
(c) (d)

Spatial pattern in SWNBRL8 (M=6)
(a) p=0.05, and (b) p=0.1 for r=0.3

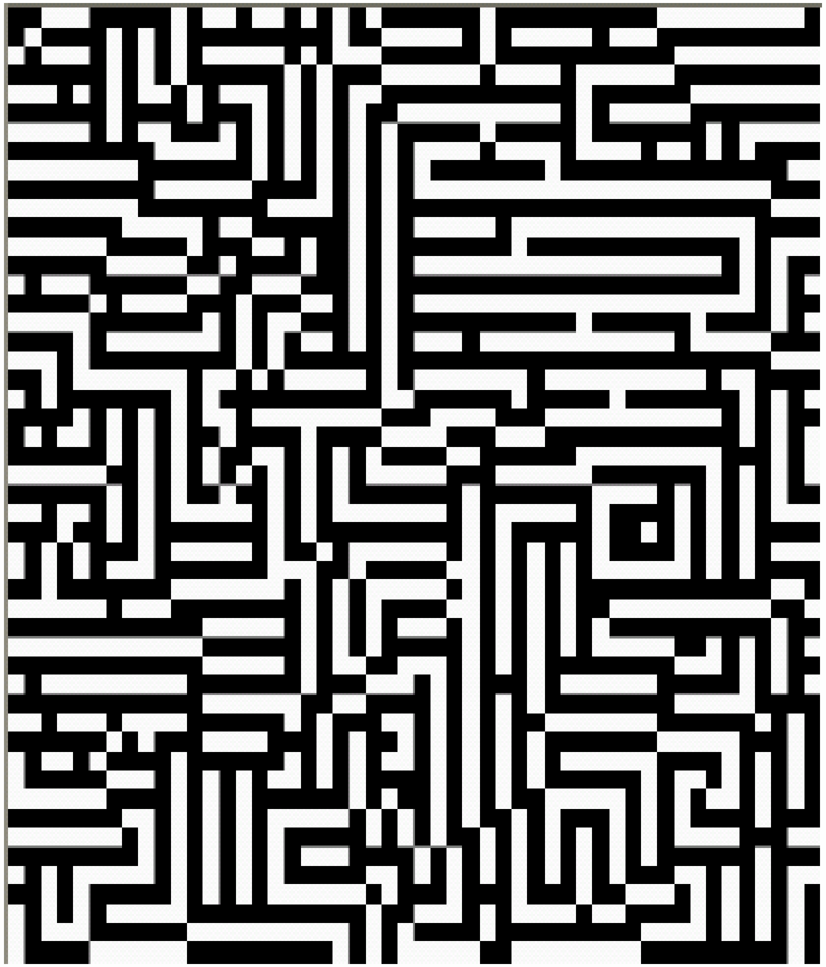


(a) (b)

Spatial pattern in SWNBRL8 (M=6)
(c) p=0.05, and (d) p=0.1 for r=0.4



(c)



(d)

Typical spatial pattern

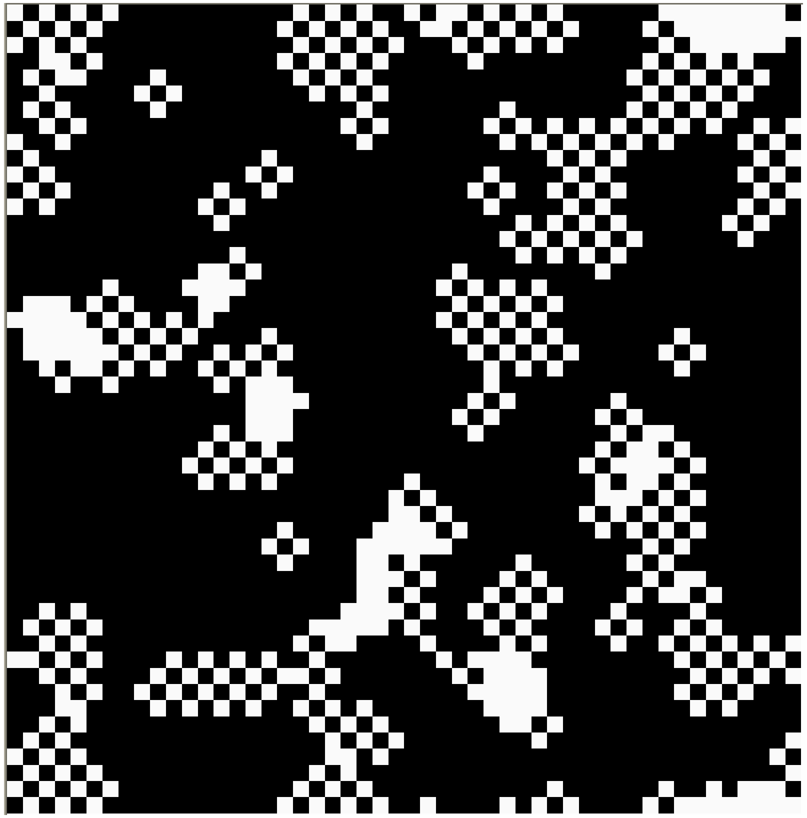
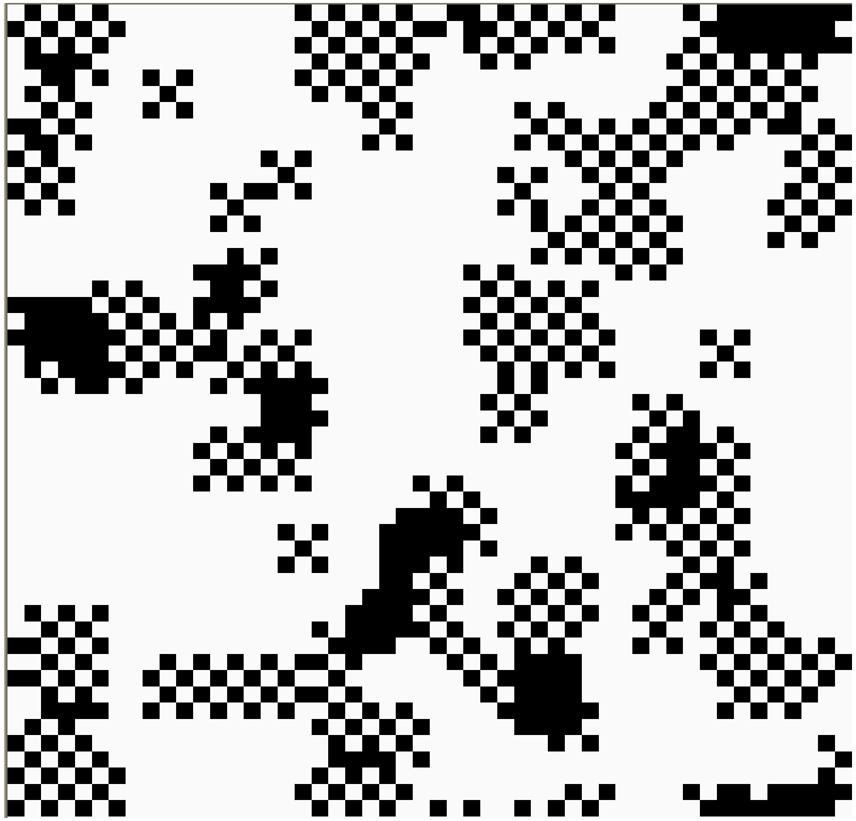
on SWNBRL4

For the time step

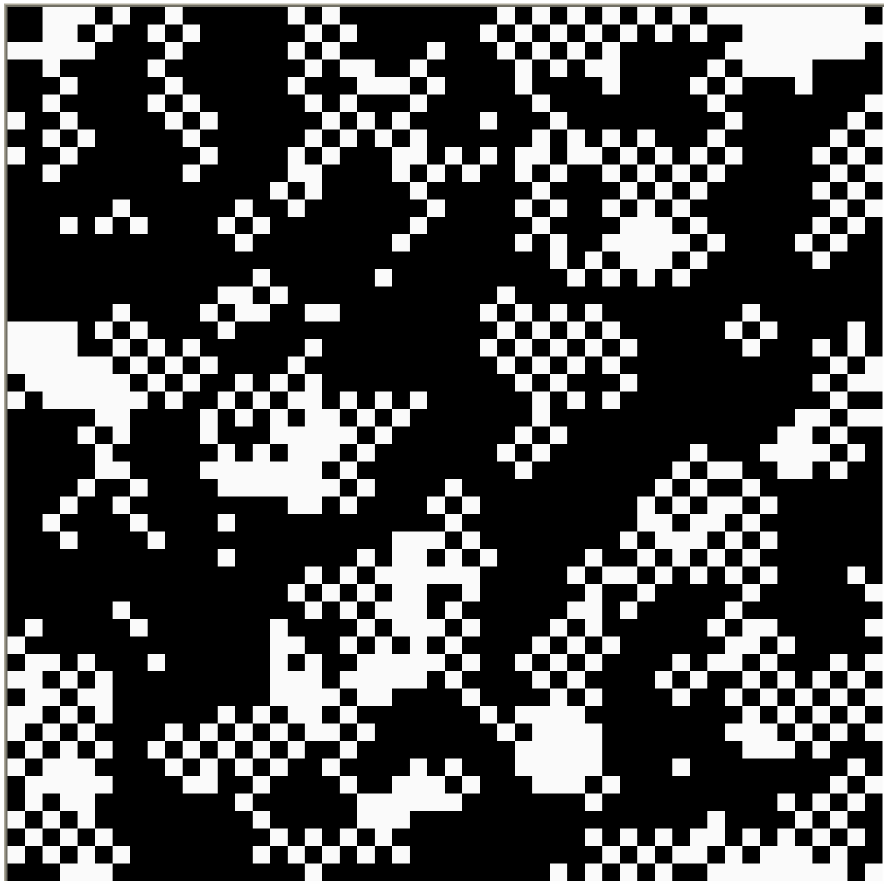
t=8001 and t=8002

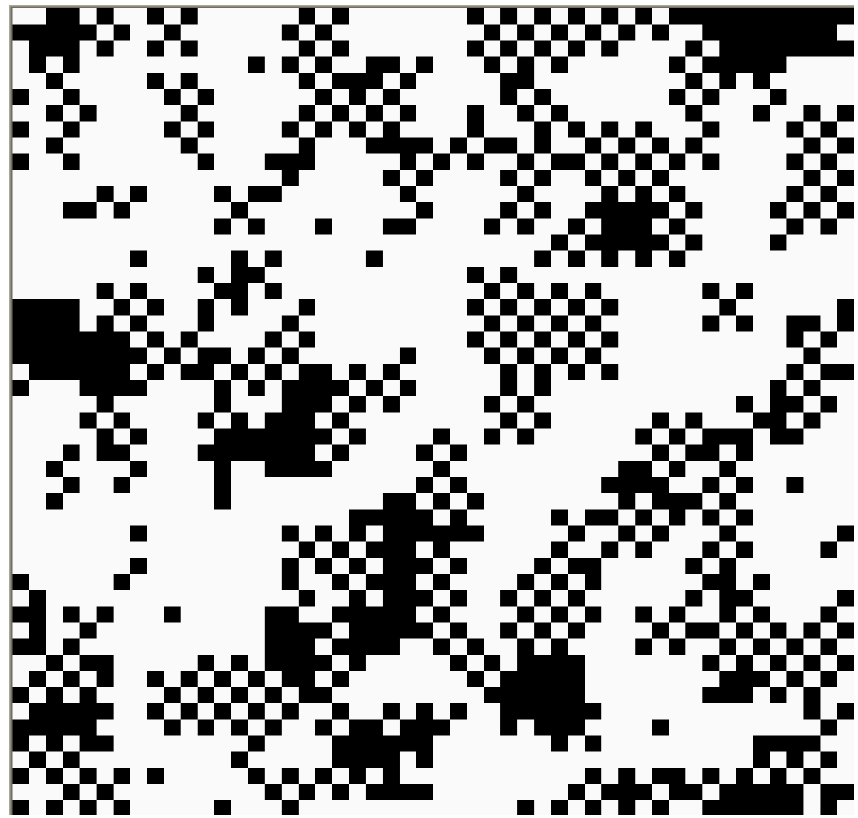
In the case of M=1, r=0.4

t=8001 and t=8002 for p=0.05

t=8001 and t=8002 for p=0.1





Summary

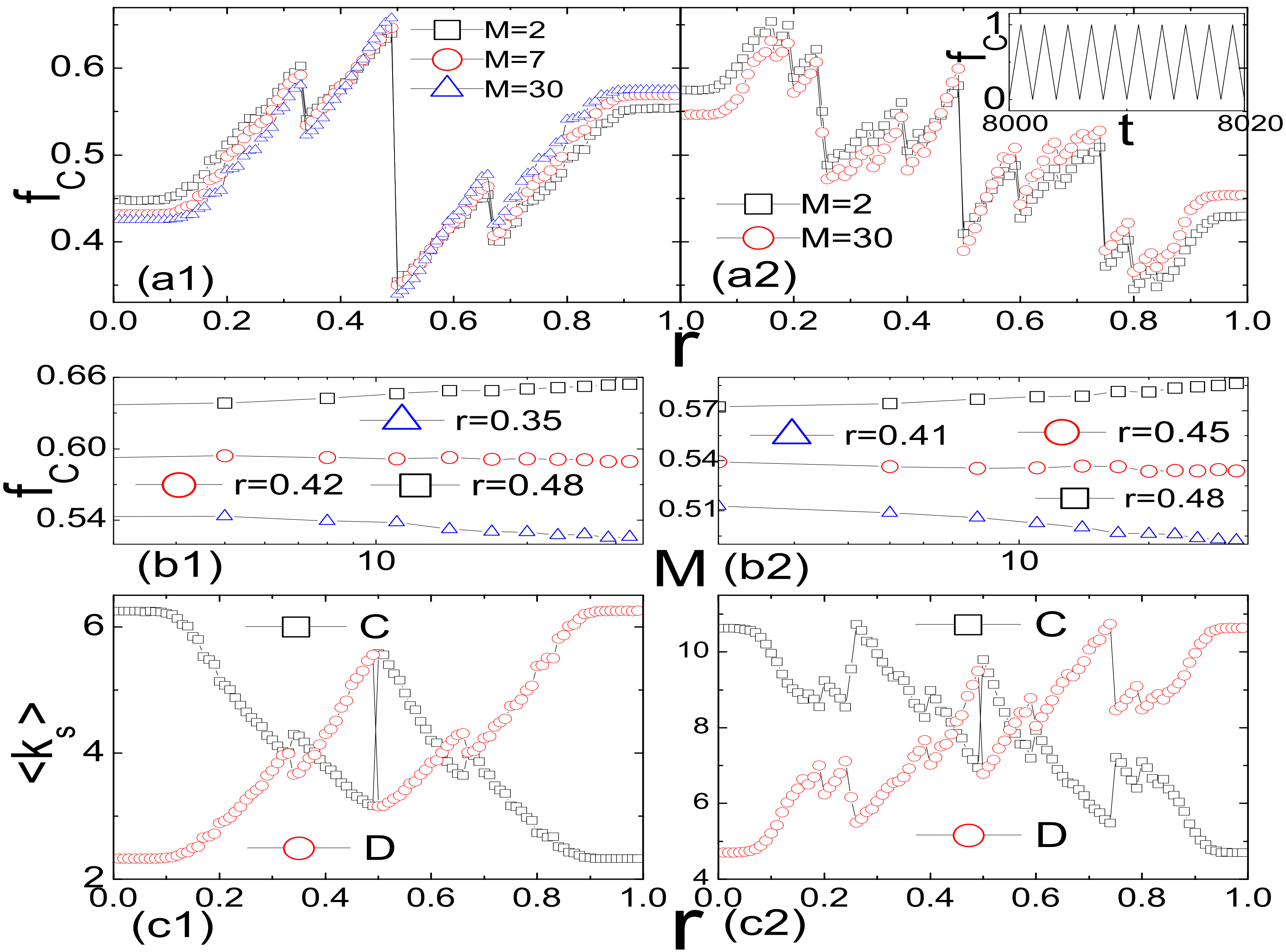
- Randomicity on small world network turns the original monotonous behavior into non-monotonous and breaks the intrinsic continuity of regular lattice, with the increasing rewiring probability

- The dependence of f_C on memory length $\boldsymbol{M}$ is different with that on regular lattice.

MBSG on Scale-Free Networks

MBSG on Scale-free networks

FIG. 5. (Color online) f_C as a function of r in BA networks with (a1) average degree $\langle k\rangle=4$ and (a2) $\langle k\rangle=8$ for different M. A time series of f_C for $M=1$ is shown in the inset of (a2). (b1) and (b2) are f_C as a function of M in the case of $\langle k\rangle=4$ and $\langle k\rangle=8$ for a special range of r. (c1) and (c2) are average degrees $\langle k_s\rangle$ of C and D players depending on r in the case of $M=7$ for $\langle k\rangle=4$ and $\langle k\rangle=8$, respectively. The network size is 10 000. Each data point is obtained by averaging over 30 different network realizations with 20 different initial state of each realization. f_C for each simulation is obtained by averaging from MC time step $t=5000$ to $t=10\,000$, where the system has reached a steady state.

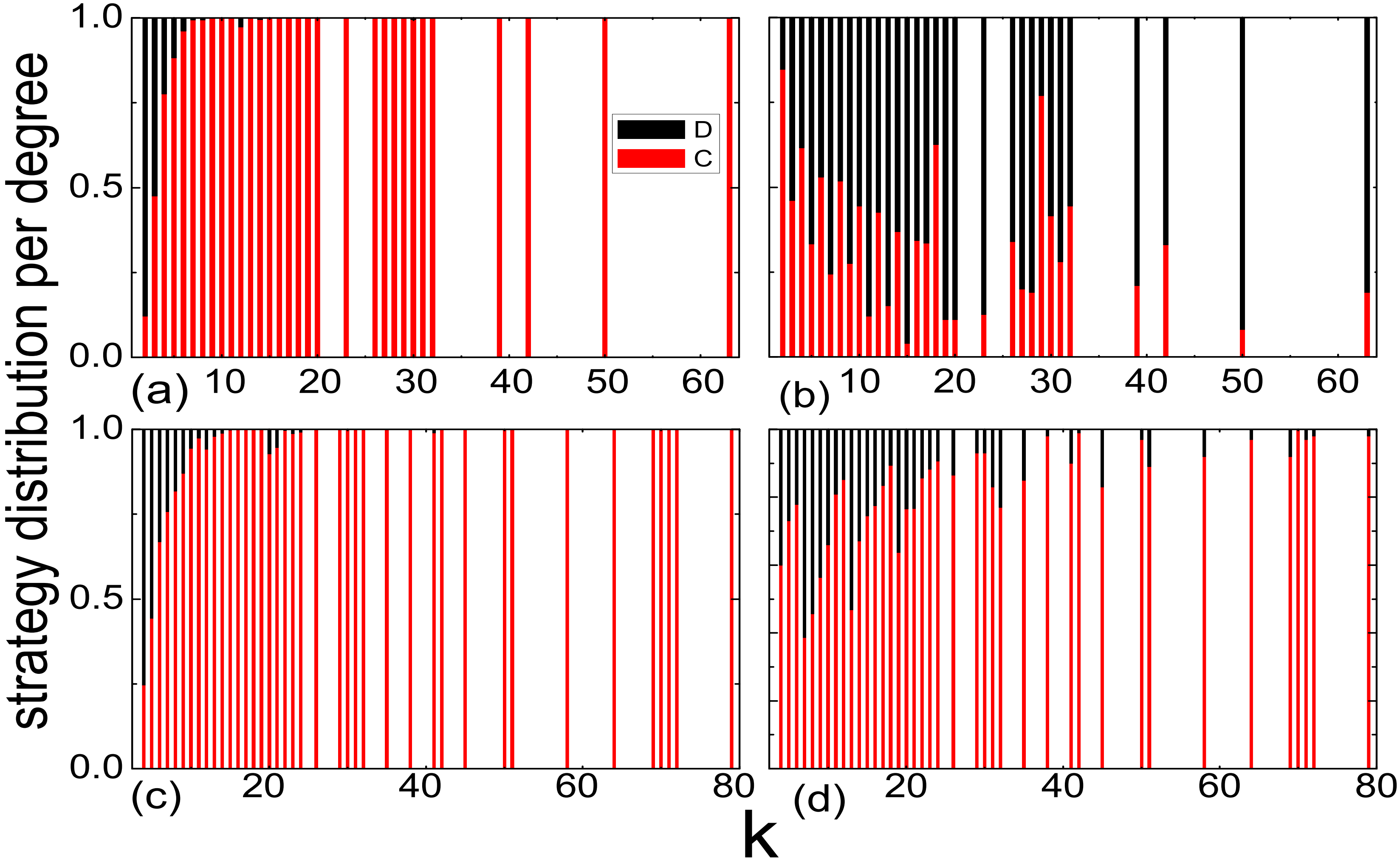


M=2
M=7
M=30
(a1)
(a2)
M=2
M=30
f_C
r
t
8000
8020
r=0.35
r=0.42
r=0.48
r=0.41
r=0.45
r=0.48
(b1)
(b2)
M
C
D
$\langle k_s \rangle$
(c1)
(c2)

Distribution of strategies in BA networks

(a) r=0.1, <k>=4, (b) r=0.49, <k>=4

(c) r=0.05 <k>=8, (d), r=0.1 <k>=8



Conclusion

- We have studied the memory-based snowdrift game on networks, including lattices and scale-free networks.

- Transitions of spatial patterns are observed on lattices, together with the step structure of the frequency of cooperation versus the payoff parameter.

- The memory length of individuals plays different roles at each cooperation level.

- In particular, nonmonotonous behavior are found on SF networks, which can be explained by the study of the occupation of nodes with given degree.

- Interestingly, in contrast to previously reported results, in the memory-based snowdrift game, the fact of high degree nodes taken over by defectors leads to a high cooperation level on SF networks.

- Furthermore, similar to the cases on lattices, the average degrees of SF networks is still a significant structural property for determining cooperative behavior.

- What role is played by memory in the evolution process of cooperative behavior?

- Something can be revealed by study of evolutionary games (MBSG).

Self-questioning games in BA network

Self-questioning games and ping-pong effect in the BA network

Kun Gao, Wen-Xu Wang and Bing-Hong Wang

Physica A 380 (2007) 528-538

Abstract

- PDG and SG with a self-questioning updating mechanism in BA network studied.
- What can this mechanism produce? Interesting non-monotonic phenomena .

- **A shortcoming of the existing models (the learning mechanisms): Being enmeshed in a globally defective trap.**

- **This new model can avoid this globally defective trap.**

- **"Cooperative Pingpong Effect" can occur in both PDG and SG and plays an important role in the behaviors of the whole system.**

- **This new model shows nontrivial characters comparing to the existing models.**

The probability of strategy updating for Szebó and Töke model

$$W_{i \to j} = \frac{1}{1 + \exp[(M_i - M_j)/K]}$$

- ***M_i, M_j*: the total payoffs of player *i* and *j*.**
- ***K*: the parameter characterizes the noise effects to permit irrational choices.**

- $K \to 0$ deterministic updating
- $K \to +\infty$ stochastic updating

- Most of current models for the evolutionary games adopts ST learning mechanism: Players update their strategies by learning from their neighbors.

- We have proposed a memory-based SG on networks [PRE74(2006)] which abandons the learning mechanism. Instead, a self-questioning mechanism and a memory-based updating rule are presented .

- As a extension of this work, we study on the evolutionary PDG and SG with self-questioning mechanism and stochastic evolutionary rule, mainly on the scale-free network.

Our model: the self-questioning mechanism and the stochastic evolutionary rule

- In each time step, players get payoffs through the game on the basis of the payoff matrix.
- Then each player calculates a virtual payoff by self-questioning, i.e., to adopt its anti-strategy and play a virtual game with its neighbors who keep their strategies unchanged, then getting a virtual payoff.
- By comparing the real payoff and the virtual payoff, players will find out whether their current strategies are advantageous.

- In the next round, player *i* will change its current strategy to its anti-strategy with probability:

$$W_i = \frac{1}{1 + \exp[(M_i - M_i')/K]}$$

- Where M_i M_i'

the real and virtual payoff of player.

Simulation results
and statistical analysis
on BA network

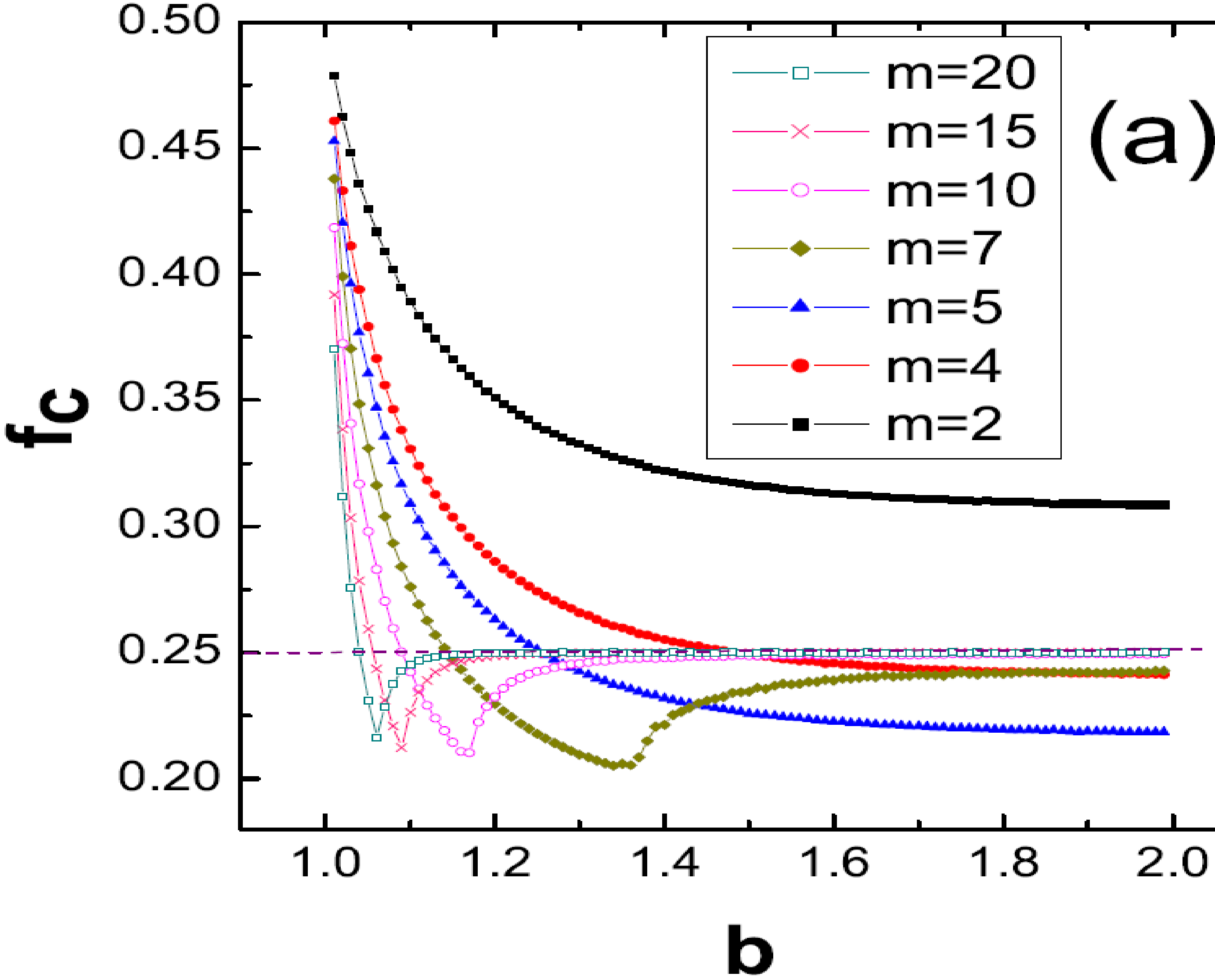
PDG

	C	D
C	1	0
D	b	0

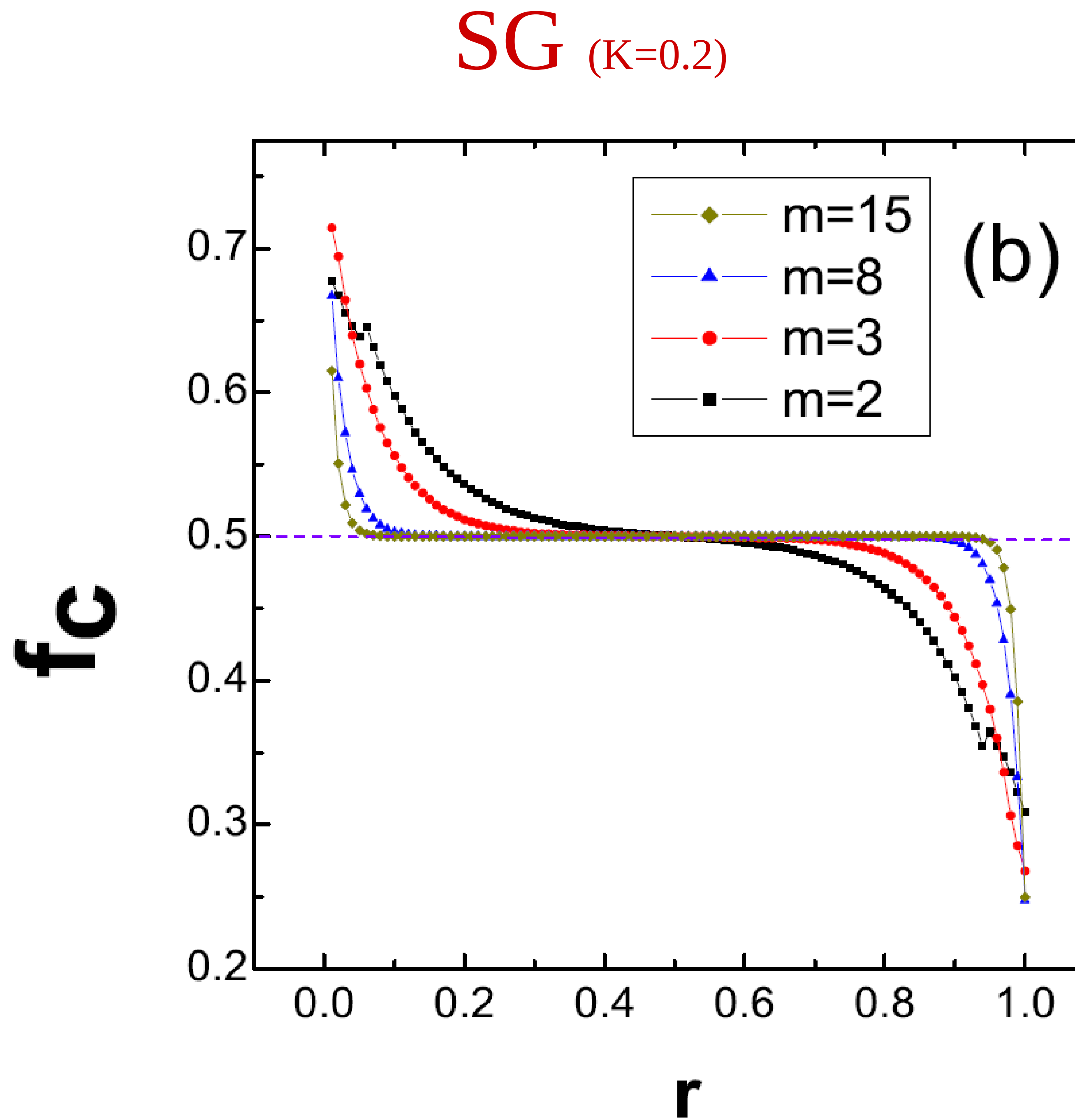
SG

	C	D
C	1	$1-r$
D	$1+r$	0

PDG (K=0.2)



0.50
0.45
0.40
0.35
0.30
0.25
0.20
fc
1.0
1.2
1.4
1.6
1.8
2.0
b
m=20
m=15
m=10
m=7
m=5
m=4
m=2
(a)


SG (K=0.2)
m=15
m=8
m=3
m=2
(b)
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0
fc
r

Statistical analysis

- If a player has **cooperative neighbors** $\boxed{n_c}$
  and **defective neighbors**, $\boxed{n_d}$

- Its total payoff is

$$\boxed{P_C = n_c R + n_d S}$$

for choosing C

$$\boxed{P_D = n_c T + n_d P}$$

for choosing D.

Statistical analysis

$$\begin{aligned} P_C - P_D &= n_c(R-T) + n_d(S-P) \\ &= \begin{cases} n_c(1-b), & \text{For the PDG} \\ n_d(1-r) - n_c r, & \text{For the SG} \end{cases} \end{aligned}$$

$$P_C - P_D = \begin{cases} kf(1-b), & \text{For the PDG} \\ k(1-f-r), & \text{For the SG} \end{cases}$$

$$\begin{aligned} \rho_c &= \rho(1 - \frac{1}{1+\exp[(P_C-P_D)/K]}) + (1-\rho)\frac{1}{1+\exp[(P_D-P_C)/K]} \\ &= \frac{1}{1+\exp[-(P_C-P_D)/K]} \end{aligned}$$

Statistical analysis

for the PDG

$$\rho_c = \frac{1}{1 + \exp[-kf(1-b)/K]}$$

for the SG

$$\rho_c = \frac{1}{1 + \exp[-k(1-f-r)/K]}$$

Cooperative Ping-pong Effect

$$\rho_c = \frac{1}{1 + \exp[-kf(1-b)/K]}$$

when $f(t) = 0$

$$\begin{aligned}\rho_c(t+1) &= \frac{1}{1 + \exp[-kf(1-b)/K]} \\ &= \frac{1}{1 + e^0} \\ &= \frac{1}{2}\end{aligned}$$

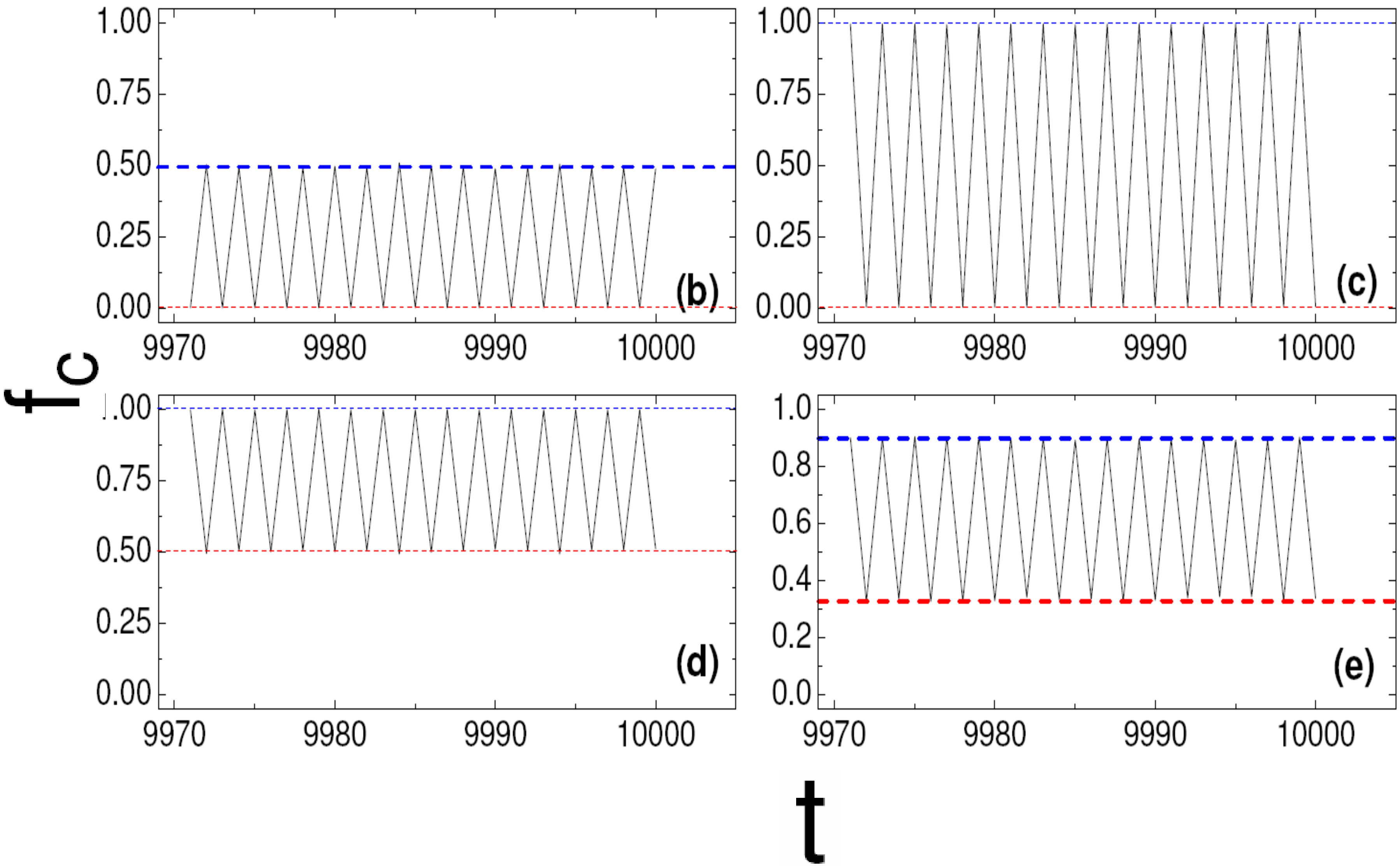
then $f(t+1) \approx 0.5$

$$\begin{aligned}\rho_c(t+2) &= \frac{1}{1 + \exp[-kf(1-b)/K]} \\ &\approx \frac{1}{1 + \exp[k(b-1)/2K]}\end{aligned}$$

with large k

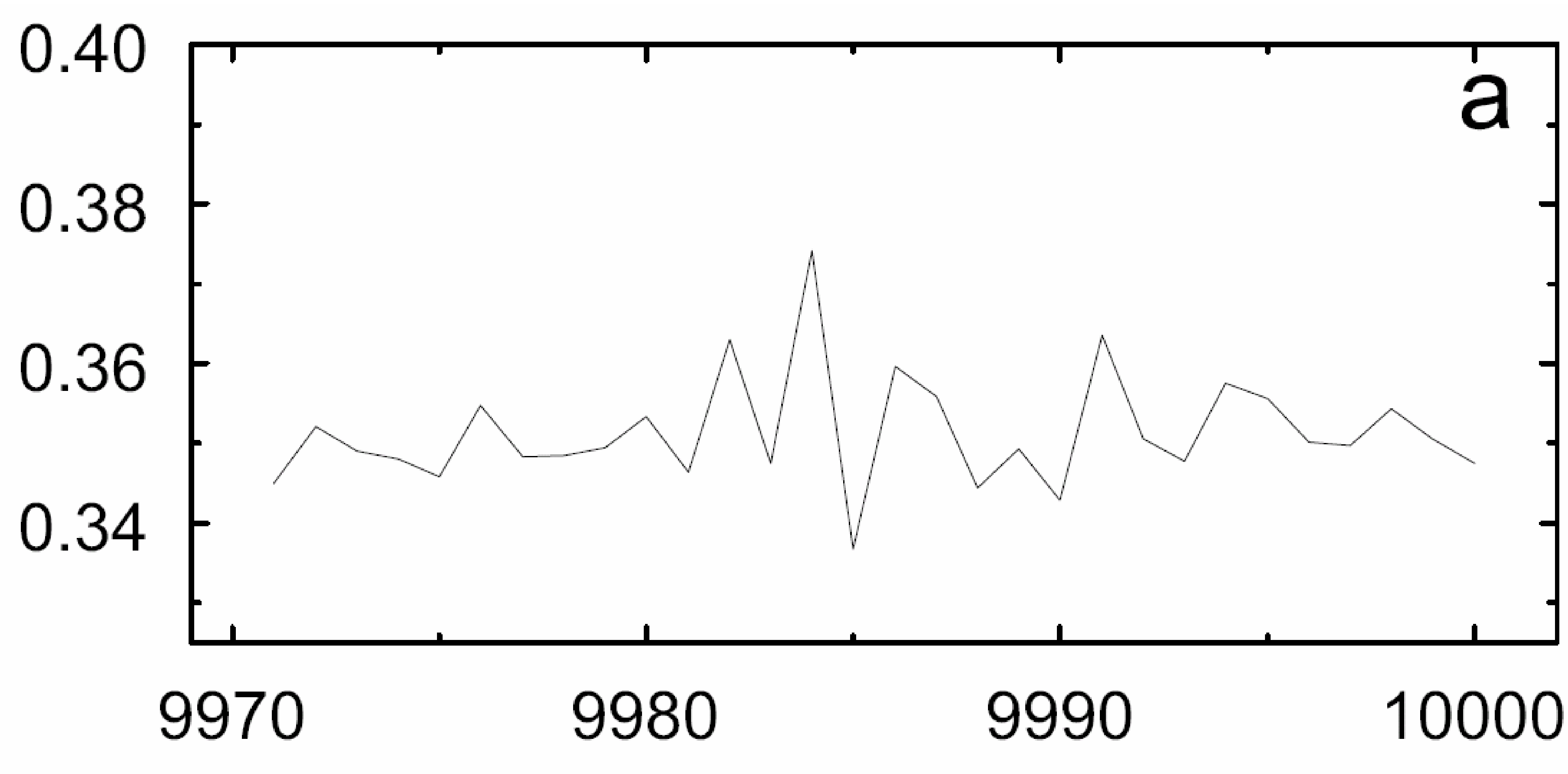
$$\rho_c(t+2) \to 0$$

Time series for the frequency of cooperation



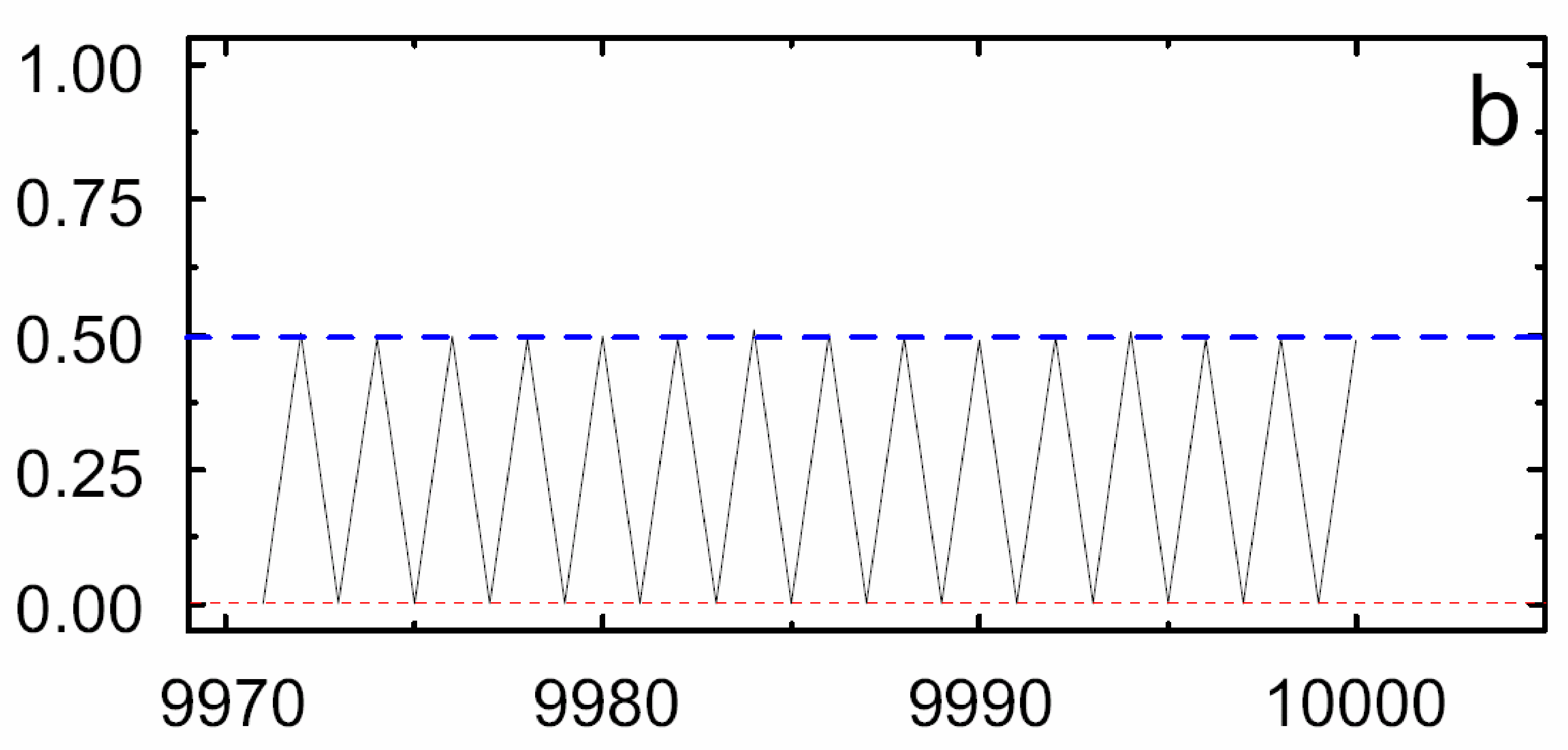
Time series for the frequency of cooperation

m=2, *K*=0.2, *b*=1.2 for PDG
(no ping-pong effect occurs)



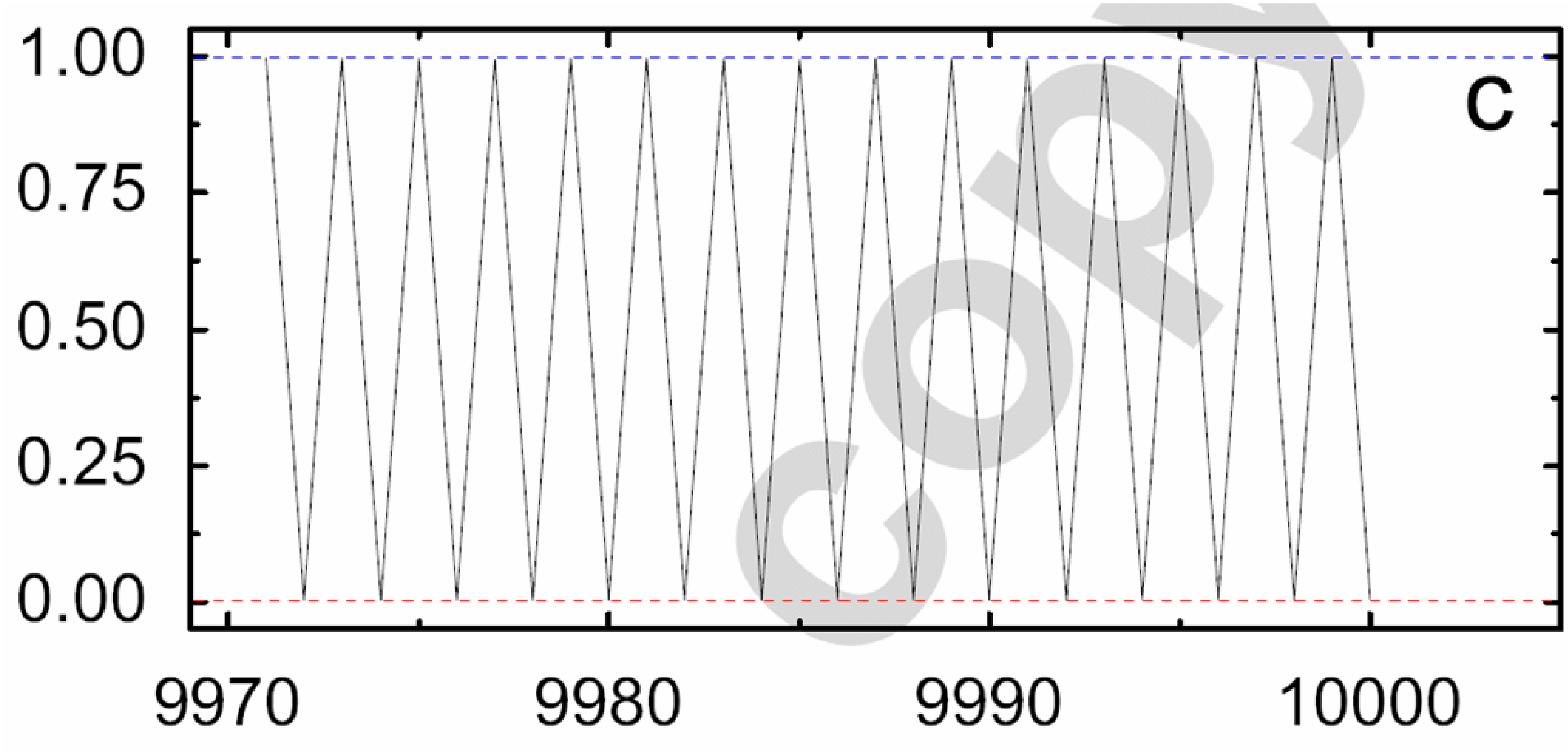
Time series for the frequency of cooperation

m=10, K=0.2, b=1.5 for PDG



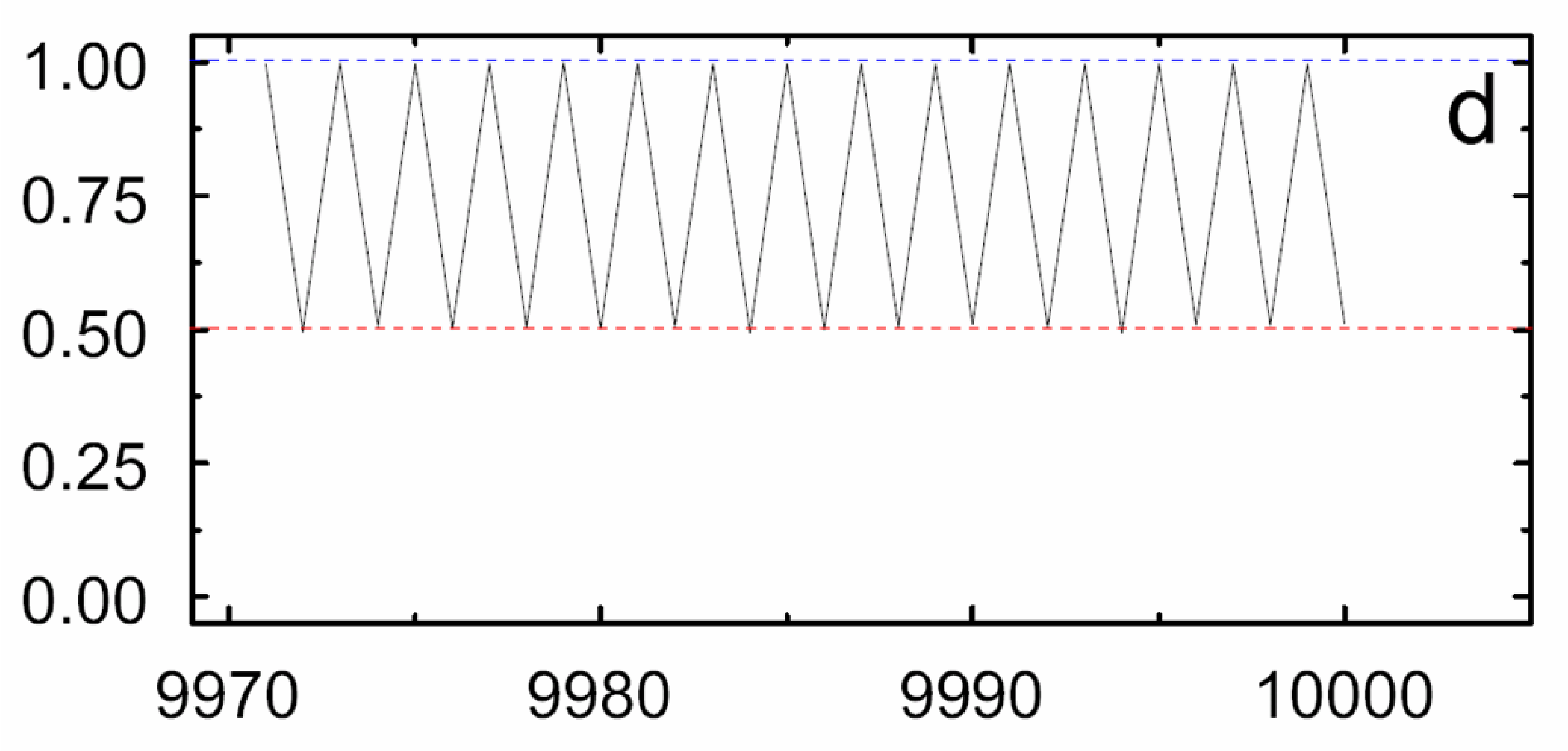
Time series for the frequency of cooperation

m=2, K=0.2, r=0.5 for SG



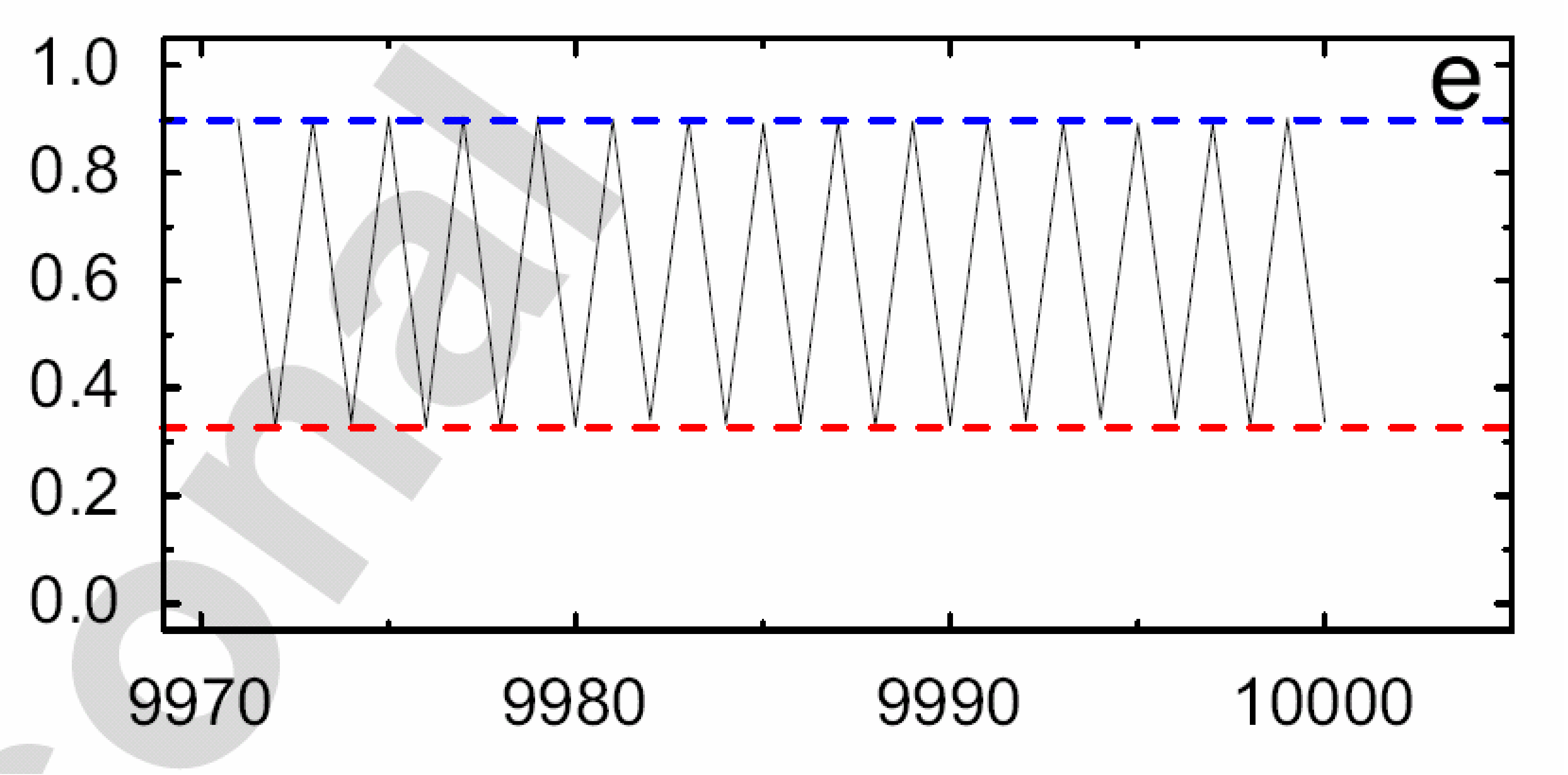
Time series for the frequency of cooperation

m=2, K=0.2, $r\to 0$ for SG

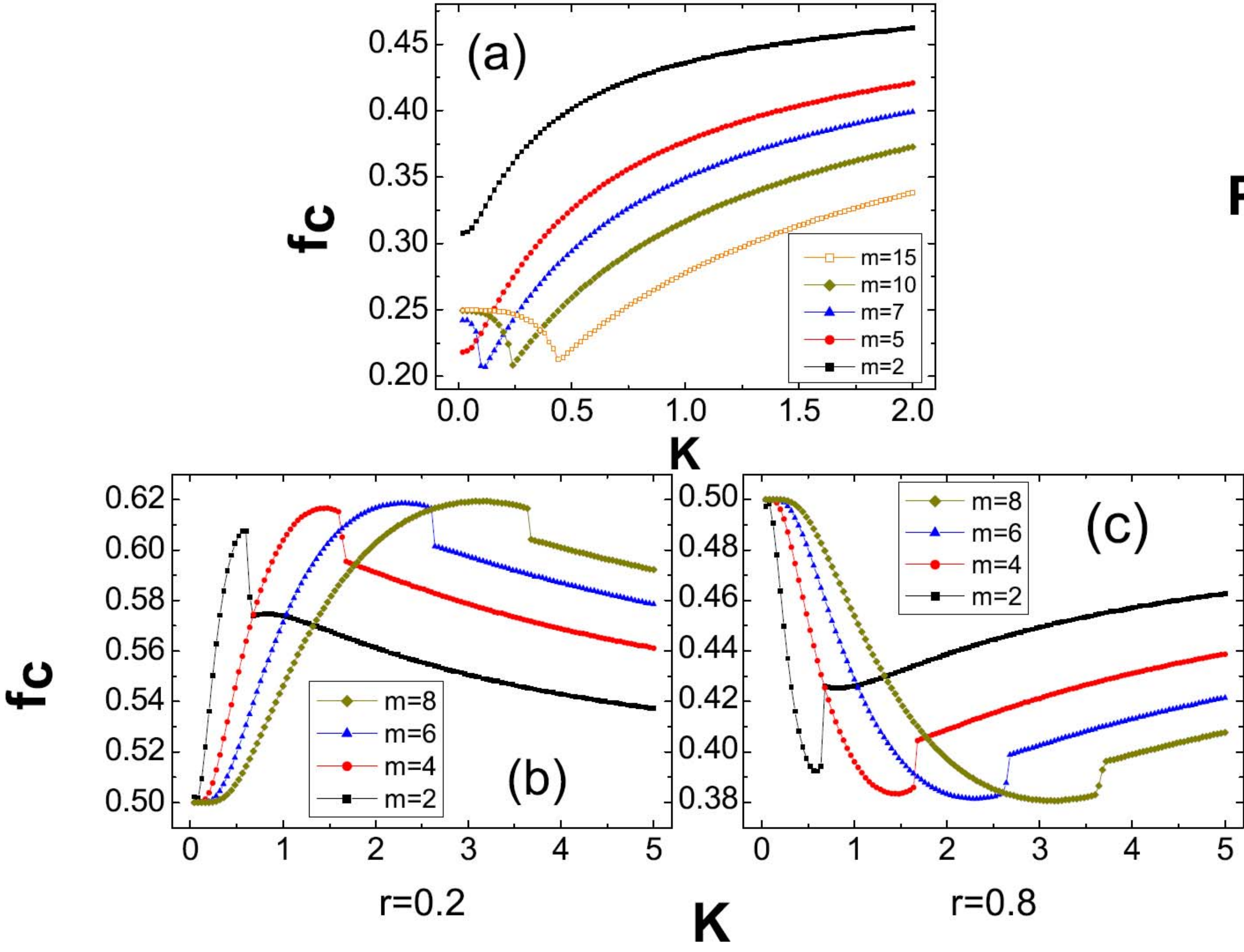


Time series for the frequency of cooperation

m=6, K=2.0, r =0.2 for SG



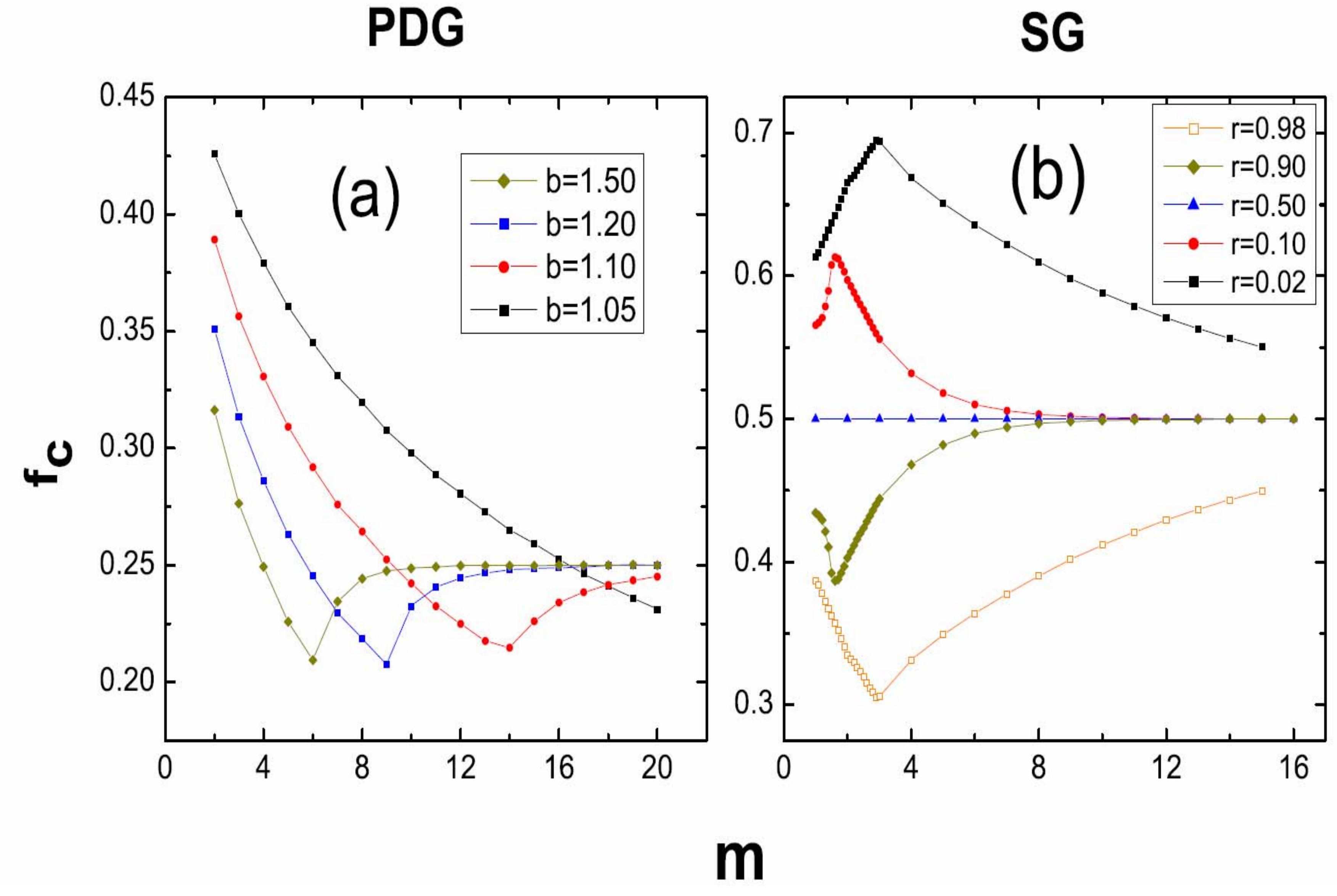
The frequency of cooperation as a function of the noise parameter in BA network



PDG

SG

The frequency of cooperation as a function of Parameter *m* (fixed noise *K*= 0.2)



Further discussion on PDG

$$\begin{array}{ccc} & \mathbf{C} & \mathbf{D} \\ \mathbf{C} & R & S \\ \mathbf{D} & T & P \end{array} \Rightarrow \boxed{\begin{array}{ccc} & \mathbf{C} & \mathbf{D} \\ \mathbf{C} & 1 & 0 \\ \mathbf{D} & b & \varepsilon \end{array}}$$

$$\boxed{0 < \varepsilon \ll 1}$$

$$\boxed{\text{PDG: } T>R>P=\varepsilon>S}$$

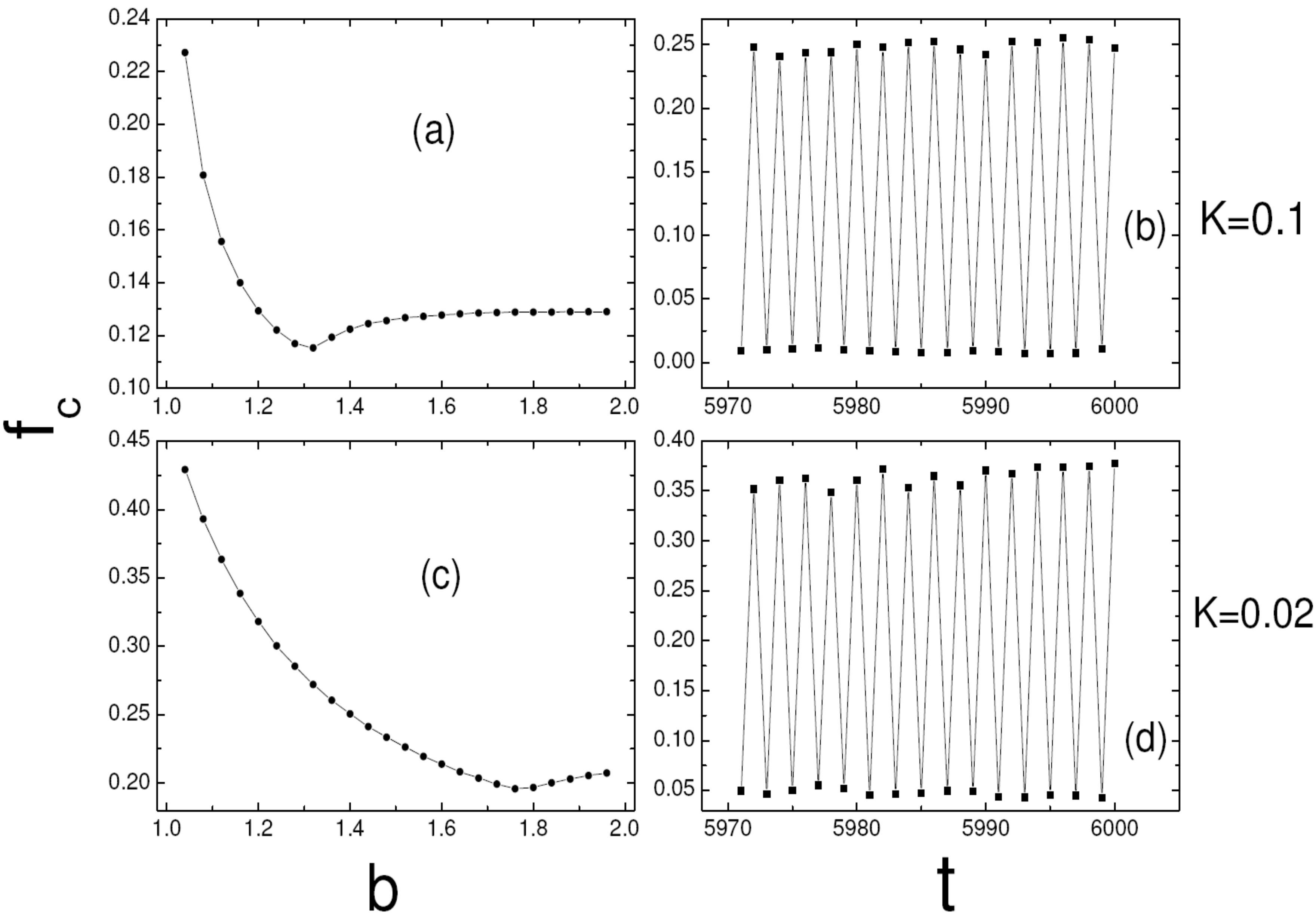
The normalized payoffs and the probability of strategy updating

$$W_{i\to j} = \frac{1}{1+\exp[(M_i/k_i - M_j/k_j)/K]}$$

$$W_i = \frac{1}{1+\exp[(M_i/k_i - M_i'/k_i)/K]}$$

$$\rho_C = \frac{1}{1+\exp\{\cancel{k}[\varepsilon - f(1+\varepsilon-b)]/K\}}$$

Cooperative frequency of PDG for *T=b*, *R=1*, *P=0.02*, *S=0*



Conclusion

- We have studied the evolutionary games on the scale-free network with a self-questioning updating mechanism.

- Compared with the previous work in this field, this model shows interesting phenomena of non-monotony and discontinuous transition etc.

Conclusion

- These phenomena are related to the so-called "Cooperative Ping-pong Effect"
- In the evolutionary games, the ping-pong effect is driven by the player's tendency of drifting with the tide.
- It happens under certain conditions in the PDG and almost everywhere in the SG.

Conclusion

- Note that in our games, although each player only pays attention to his own information of payoffs, the whole system exhibits highly self-organized characters and the players' actions are highly synchronized.

- That means the payoffs of each player have contained plenty of information about the circumstances.

- In the evolutionary games, deciding according to yourself is as effective as learning from others.

Conclusion

- Furthermore, the self-questioning mechanism has avoided the system from being enmeshed in a trap of the globally defective state, which is a shortcoming for the previous models.

- Usually the ping-pong effect emerges when the situation is not so propitious to cooperation, thus it can improve the cooperative behavior in the system.

Conclusion

- However, large vibration, as well as defection, will also waste the resources seriously.

- So these problems are worthy of further studies, in order to find out more effective mechanisms to sustain the cooperative behavior better and to make use of the resources most efficiently.

Naming Game on Network for Evolution of Language

- The naming game has been considered as an important approach for understanding and characterizing the evolution of a language and more generally of a communication system without global supervision or a prior common knowledge.

- It has been demonstrated that in these games, agents can achieve the consensus of naming an object through local pair-wise interactions in a self-organized way, which can well explain the origin and evolution of languages.

- Besides, such models were inspired by global coordination problems in artificial intelligence and peer-to-peer communication systems.

- A prototypical example is the so-called talking heads experiment, in which robots assign names to objects observed through cameras and negotiate these names with other agents.

- The naming game was also found meaningful for the new developed web tools, such as

del.icio.us

www.flickr.com

which enable web users to share classification of information in the web through tags invented by each user.

A minimal version of the naming game proposed by

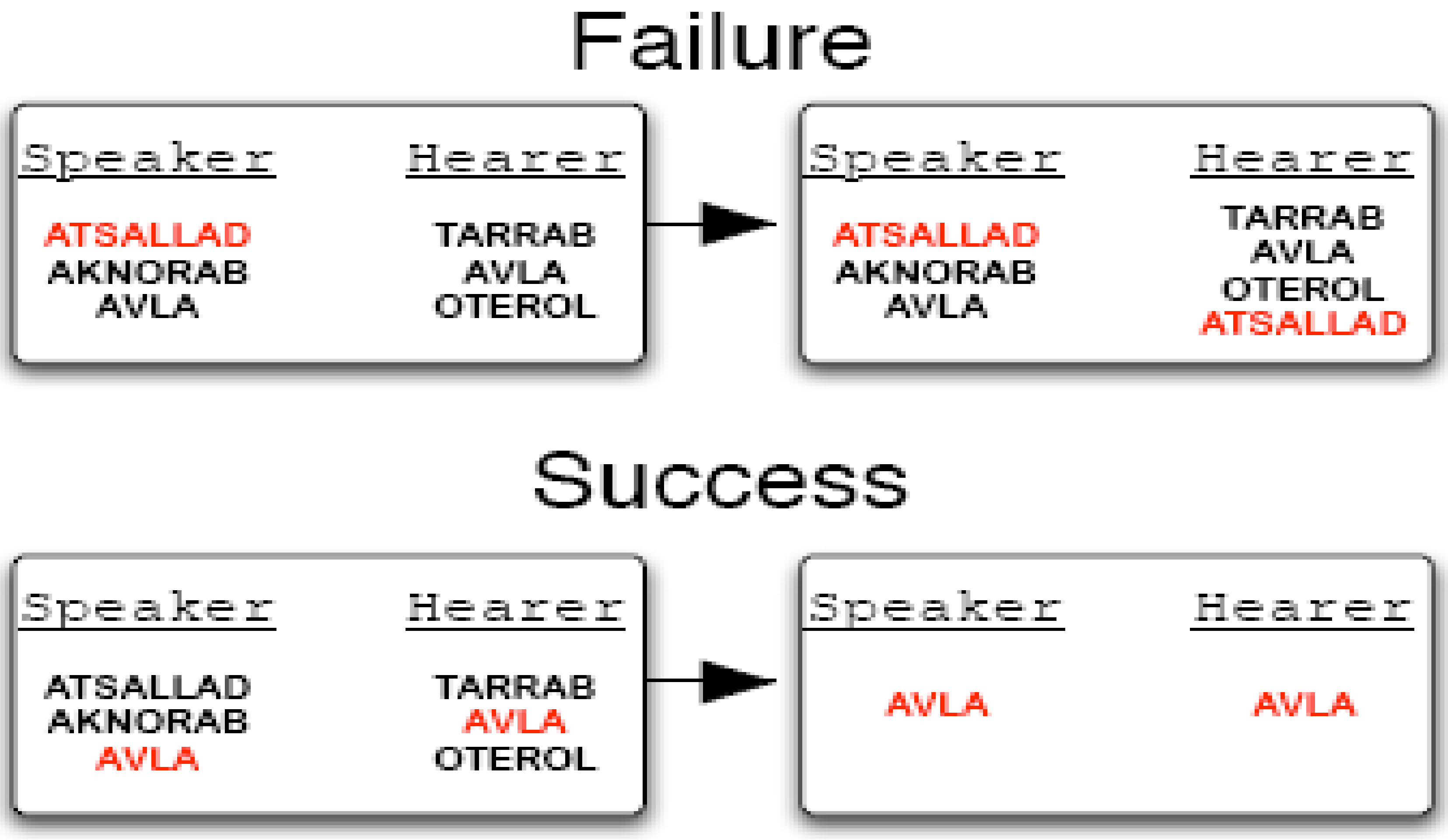
A. Baronchelli, M. Felici, et al:

J. Stat. Mech.: Theory Exp. (2006), P06014.

A. Baronchelli, L. Dall'Asta, A. Barrat, et al:

Phys. Rev. E **73**, 015102R (2006).

- Game model: one is speaker， another is hearer.



- Three rules:（Different ways to select speaker and hearer）

- ***1. direct naming game:***

  A randomly chosen speaker selects again randomly a hearer among its neighbors.

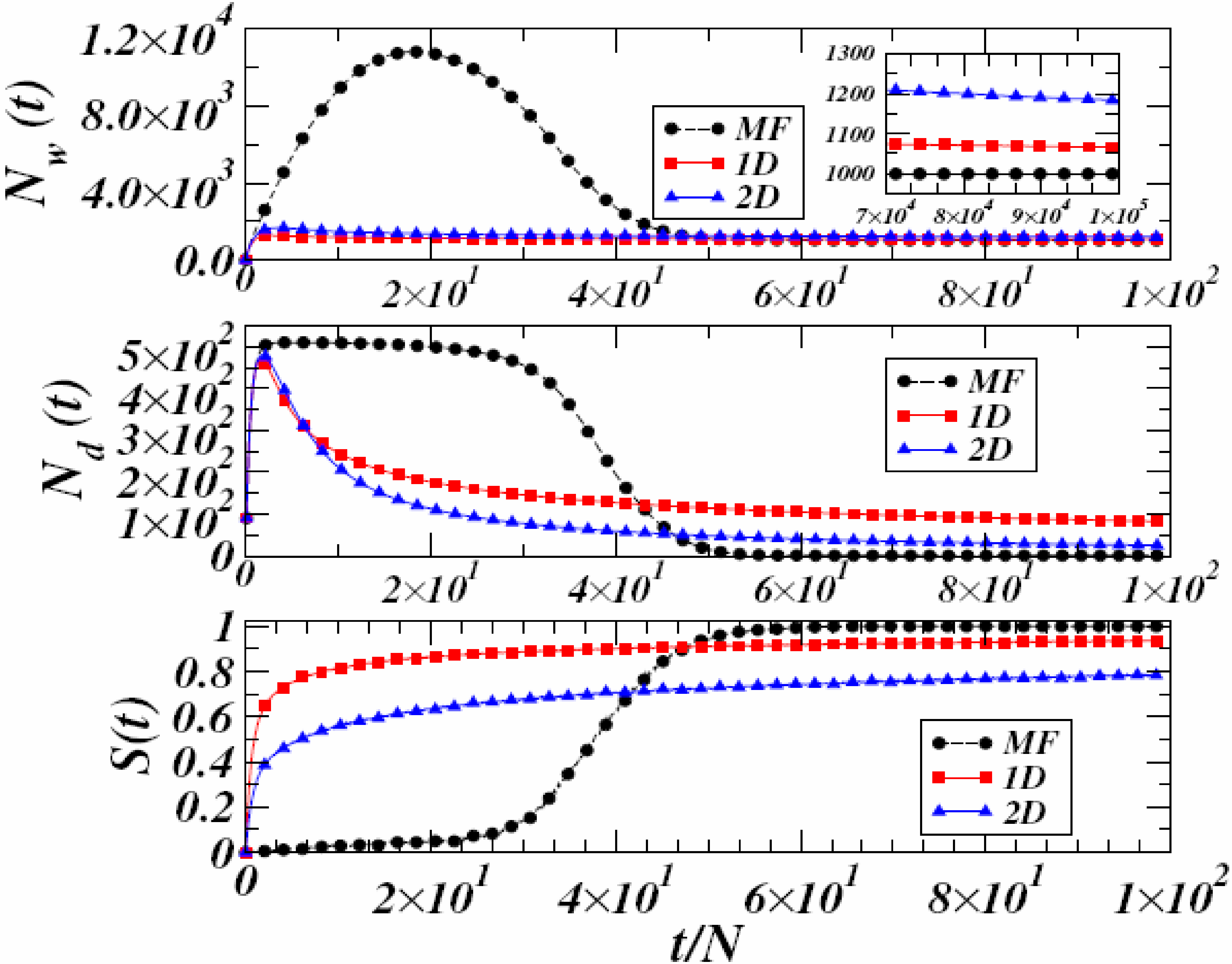
- ***2. reverse naming game:***

  Choose the hearer at random and one of its neighbors as speaker.

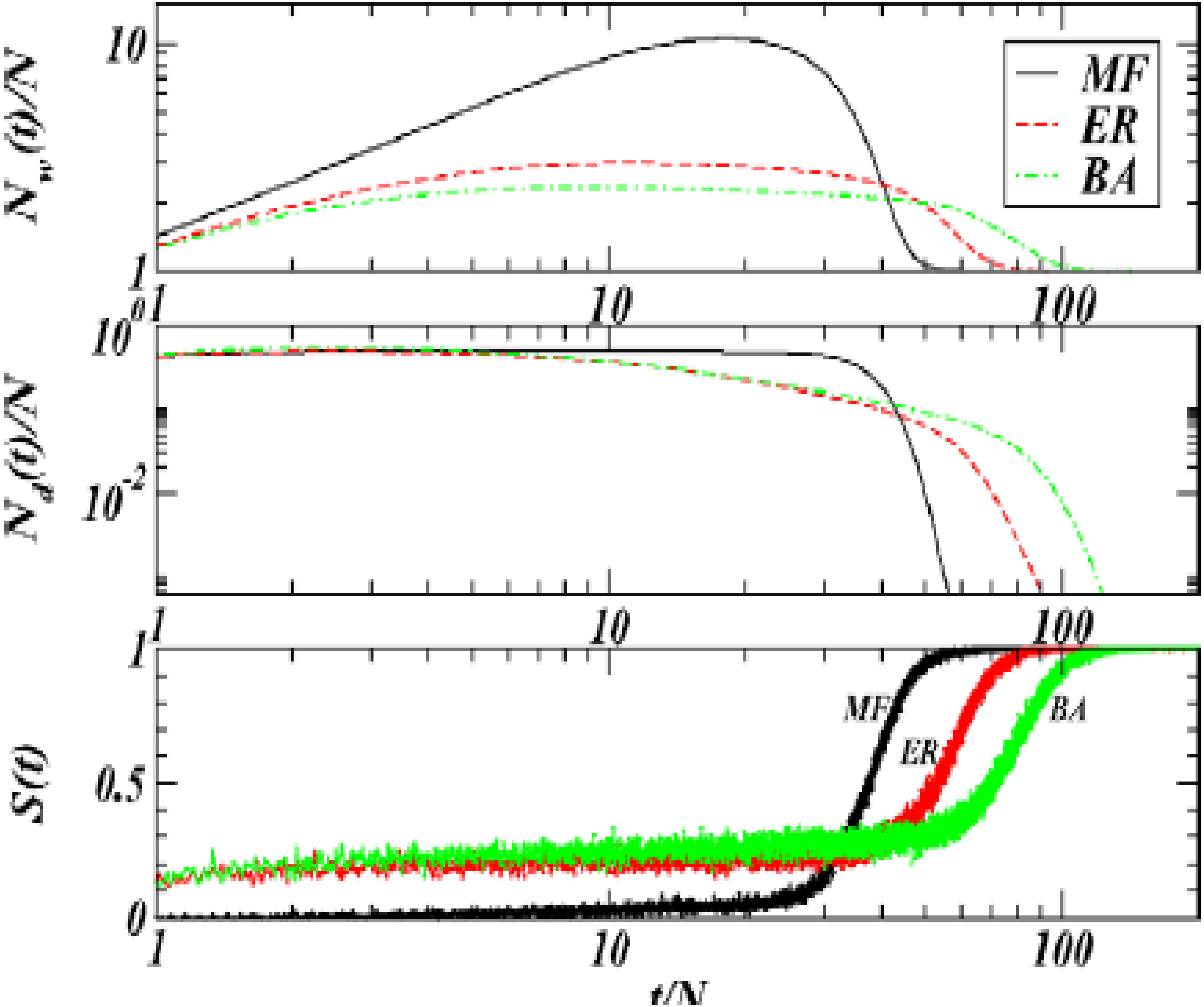
- ***3. A neutral strategy to pick up pairs of nodes*** is that of considering the extremities of an edge taken uniformly at random.

Main quantities to characterize the game

- total memory (N_w)
- number of different words (N_d)
- average success rate S
- convergence time t_c

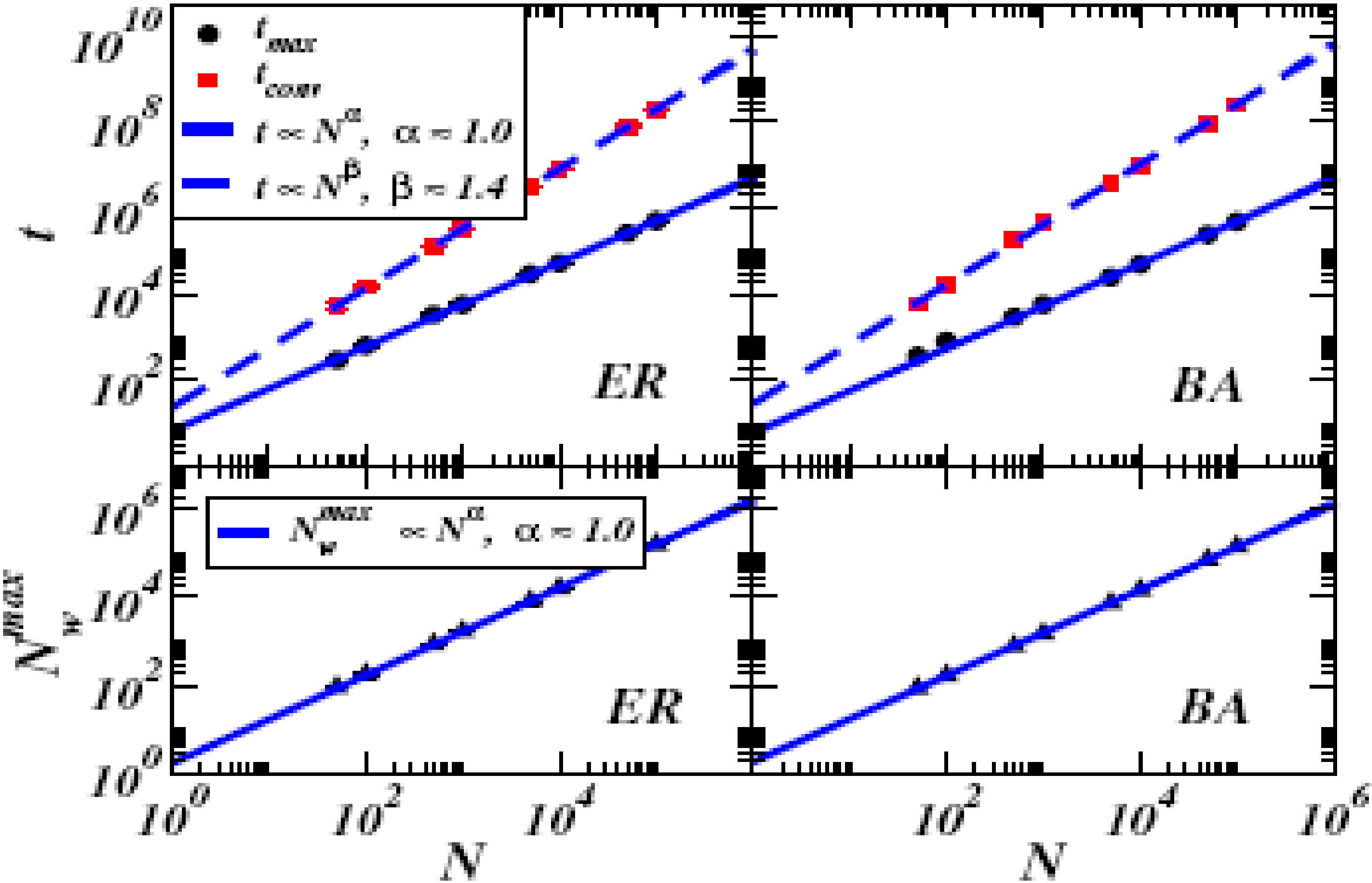


$N_w(t)$
$N_d(t)$
$S(t)$
t/N
MF
1D
2D



$N_w(t)/N$
MF
ER
BA
10
1
$N_d(t)/N$
10^0
10^{-2}
S(t)
1
0.5
0
MF
ER
BA
1
10
100
t/N

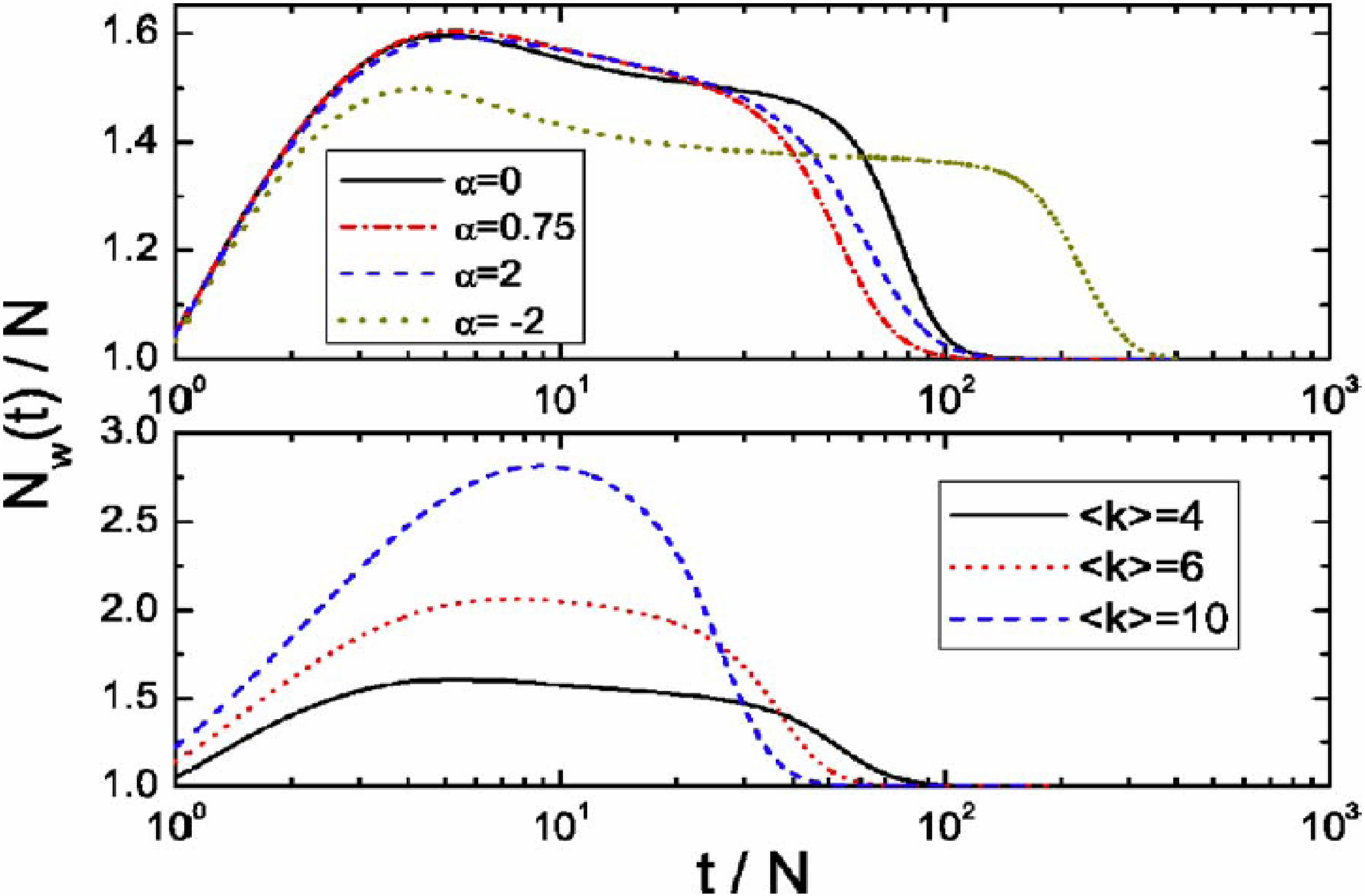
scaling behavior



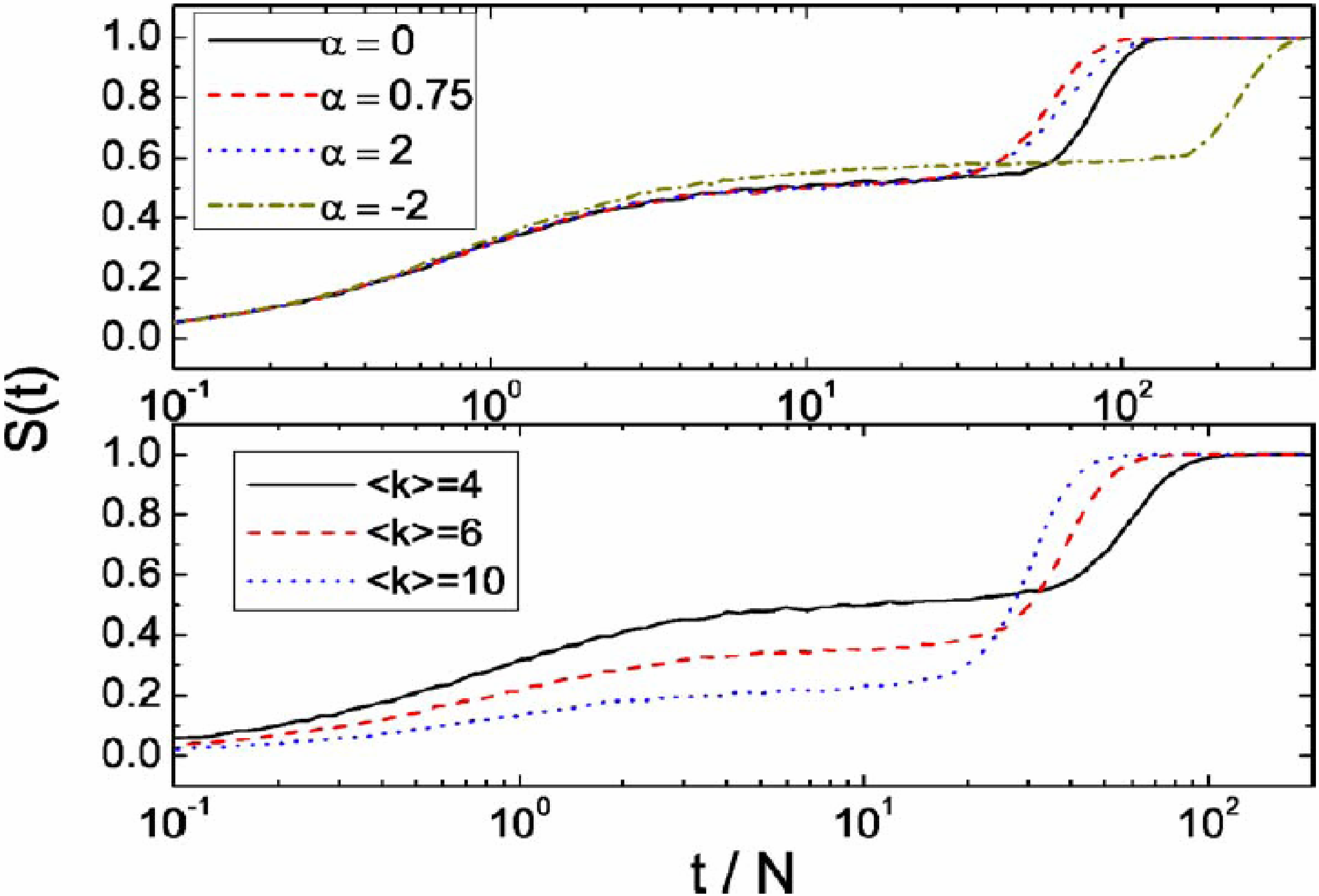
Role of connectivity-induced weighted words in language games

weight k_i^α,

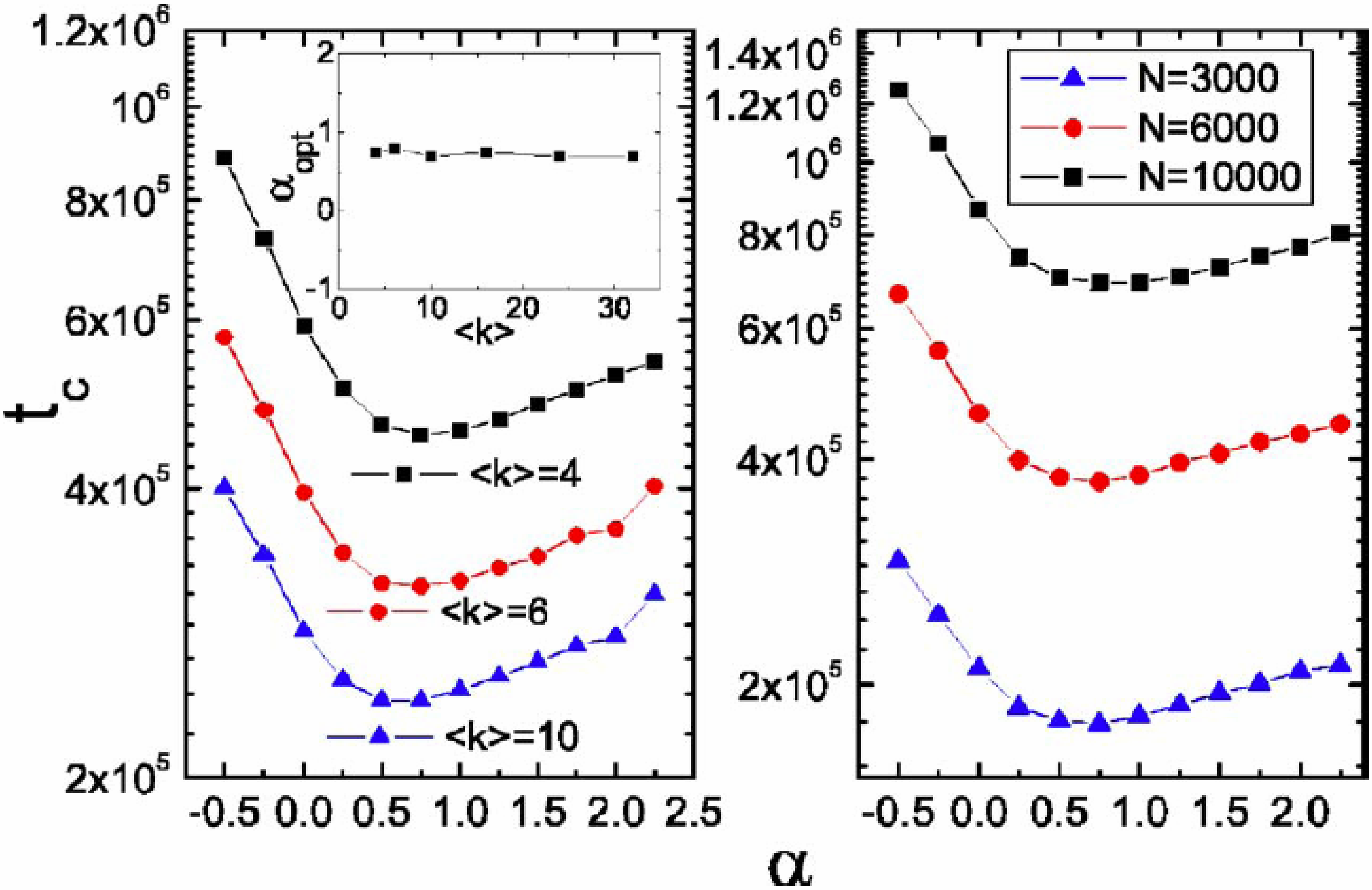
Evolution of the average memory per agent



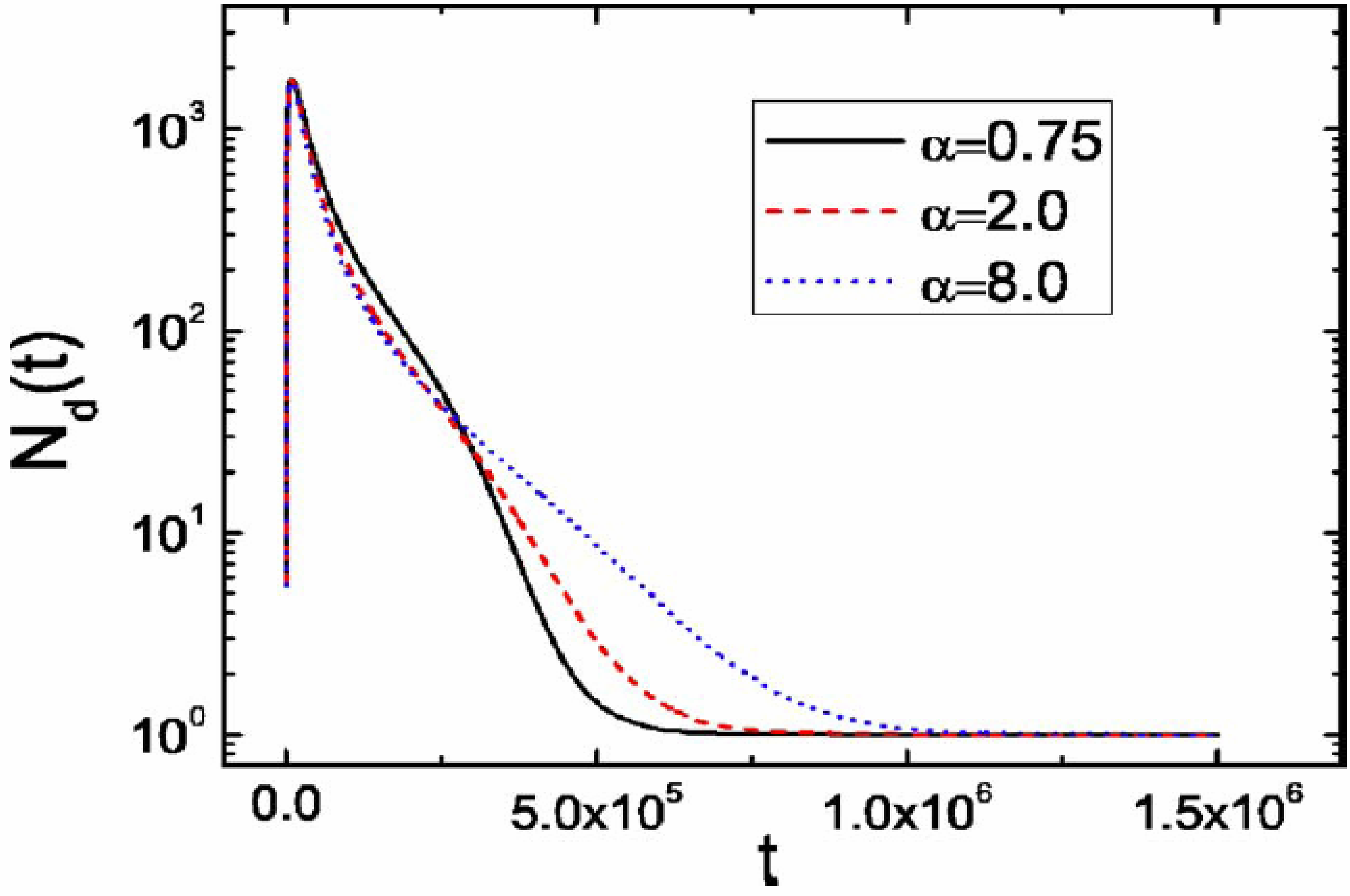
Evolution of average success rate



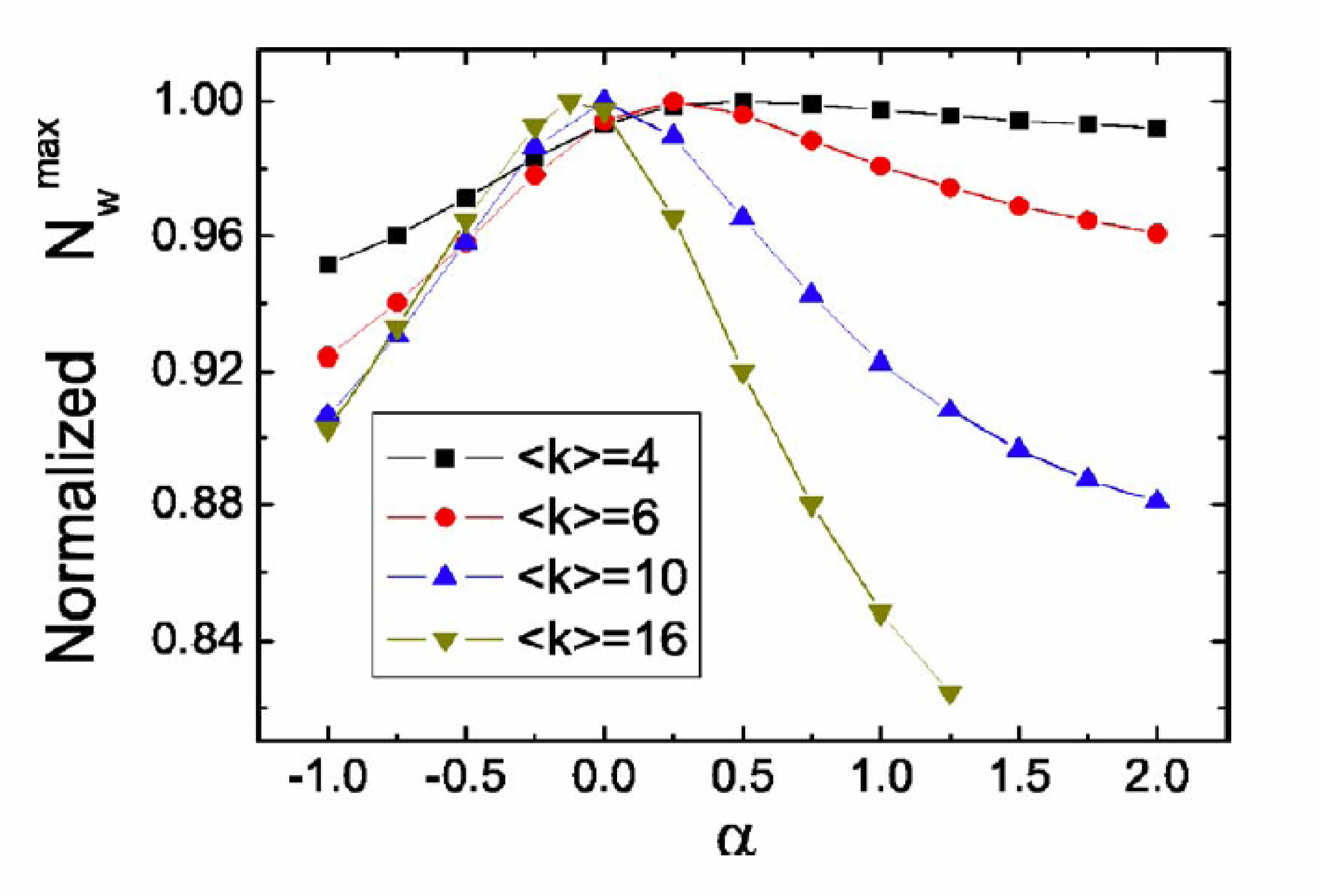
Convergence time vs α



Evolution of the number of different words in the system



Normalized maximum total memory used by agents



Conclusion 1

- We present a modified naming game by introducing weights of words in the evolution process.

$$\text{weight } k_i^{\alpha},$$

- We assign the weight of a word spoken by an agent according to its connectivity, which is a natural reflection of the agent's influence in population.

Conclusion 2

- A tunable parameter is introduced, governing the word weight based on the connectivity of agents.
- We consider the scale-free topology and concentrate on the efficiency of reaching the final consensus, which is of high importance in the self-organized system.

Conclusion 3

- Interestingly, it is found that there exists an optimal parameter value, leading to the fastest convergence.
- This indicates appropriate hub's effects favor the achievement of consensus.
- The evolution of distinct words helps to give a qualitative explanation of this phenomena.

Conclusion 4

- Similar nontrivial phenomena are observed in the total memory of agents with a peak in the middle range of parameter values.

- Other relevant characters are provided as well, including the time evolution of total memory and success rate for different parameter values as well as the average degree of the network, which are helpful for understanding the dynamics of the modified naming game in detail.

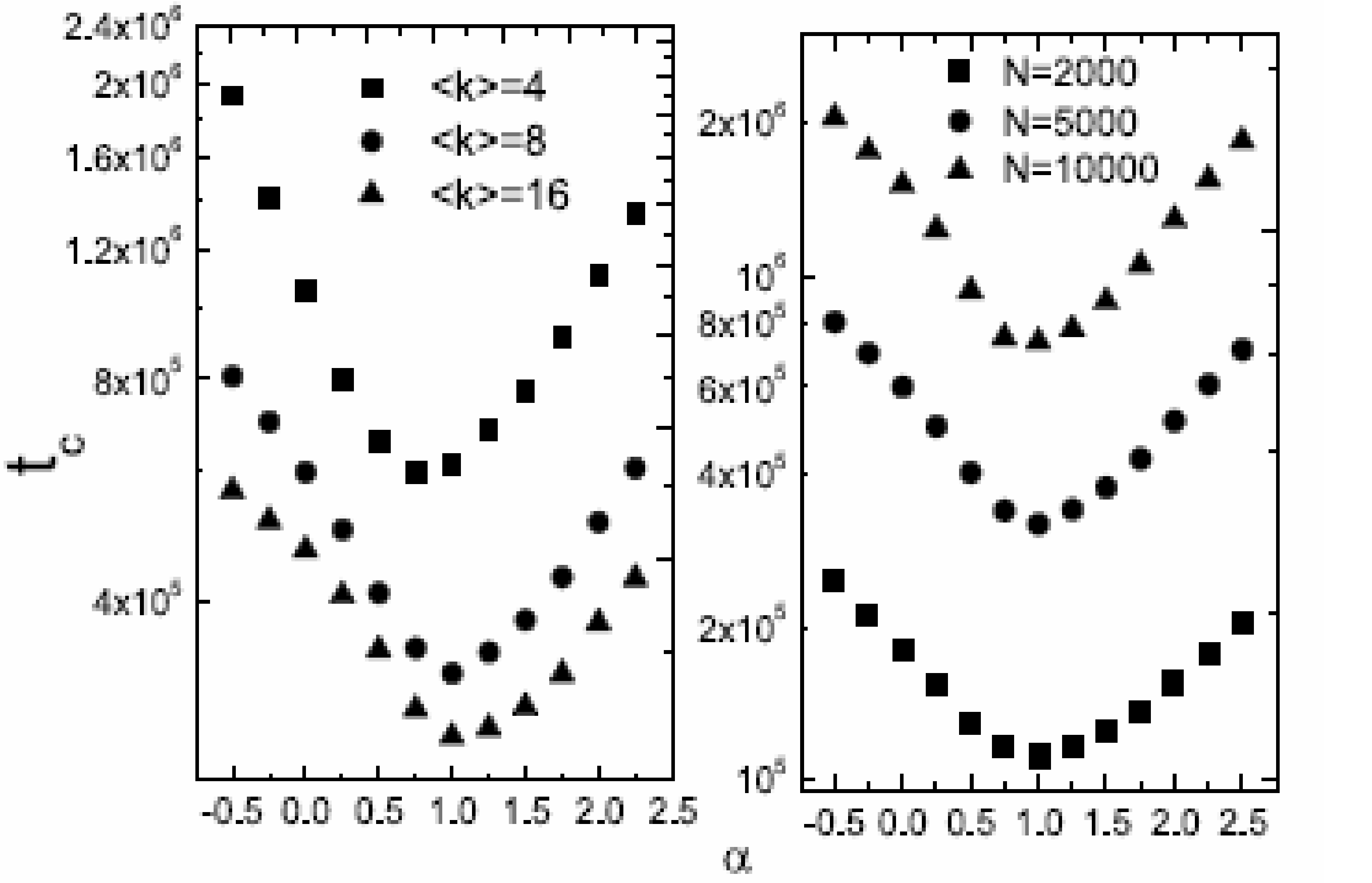
Asymmetric negotiation in structured language games

Asymmetric negotiation

- Each time step, a pair of connected nodes are randomly selected .The probability ***p_i*** of choosing one of them ***i*** as speaker is proportional to ***i*** 's weight:

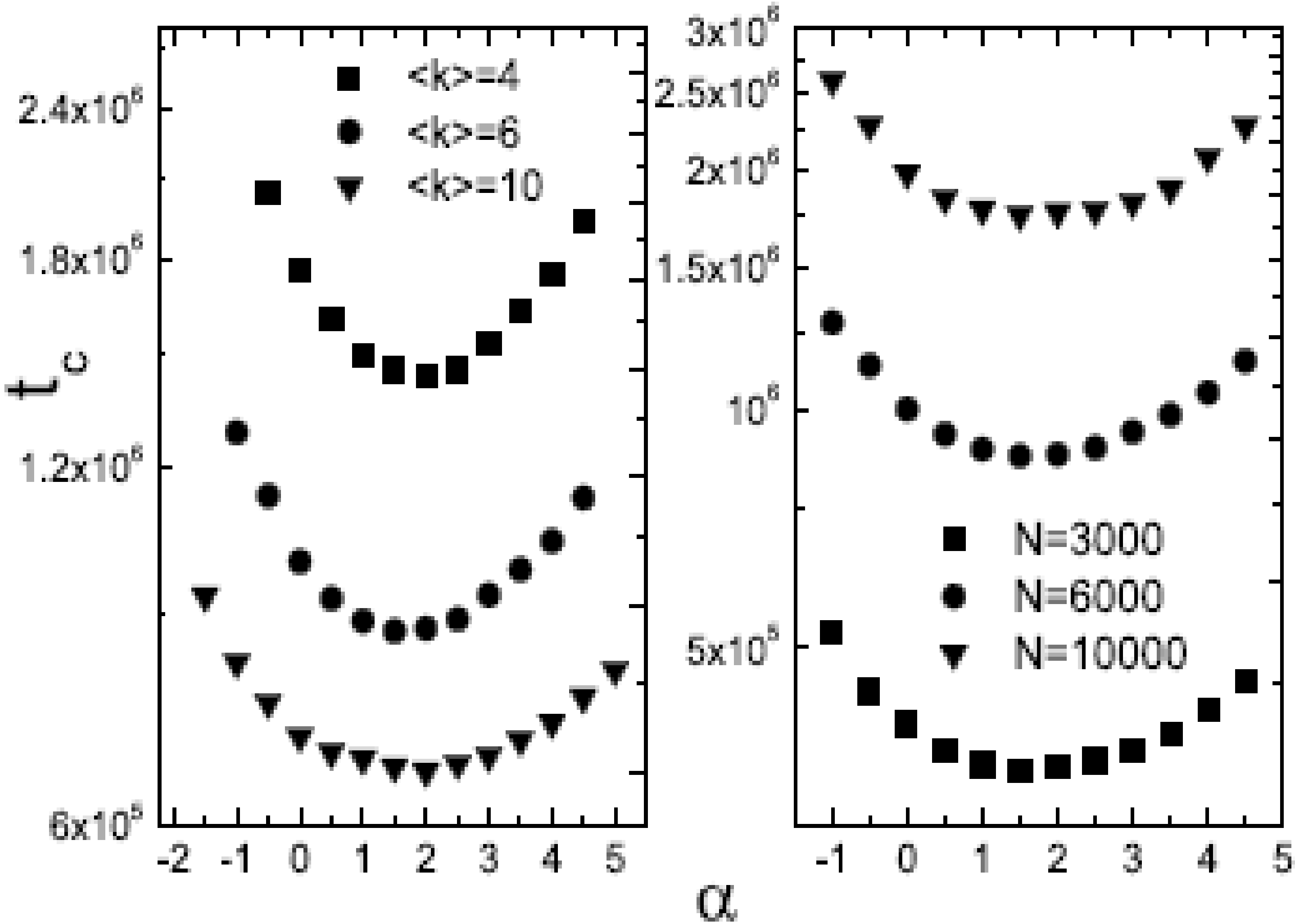
$$p_i = \frac{k_i^\alpha}{k_i^\alpha + k_j^\alpha}.$$

BA network



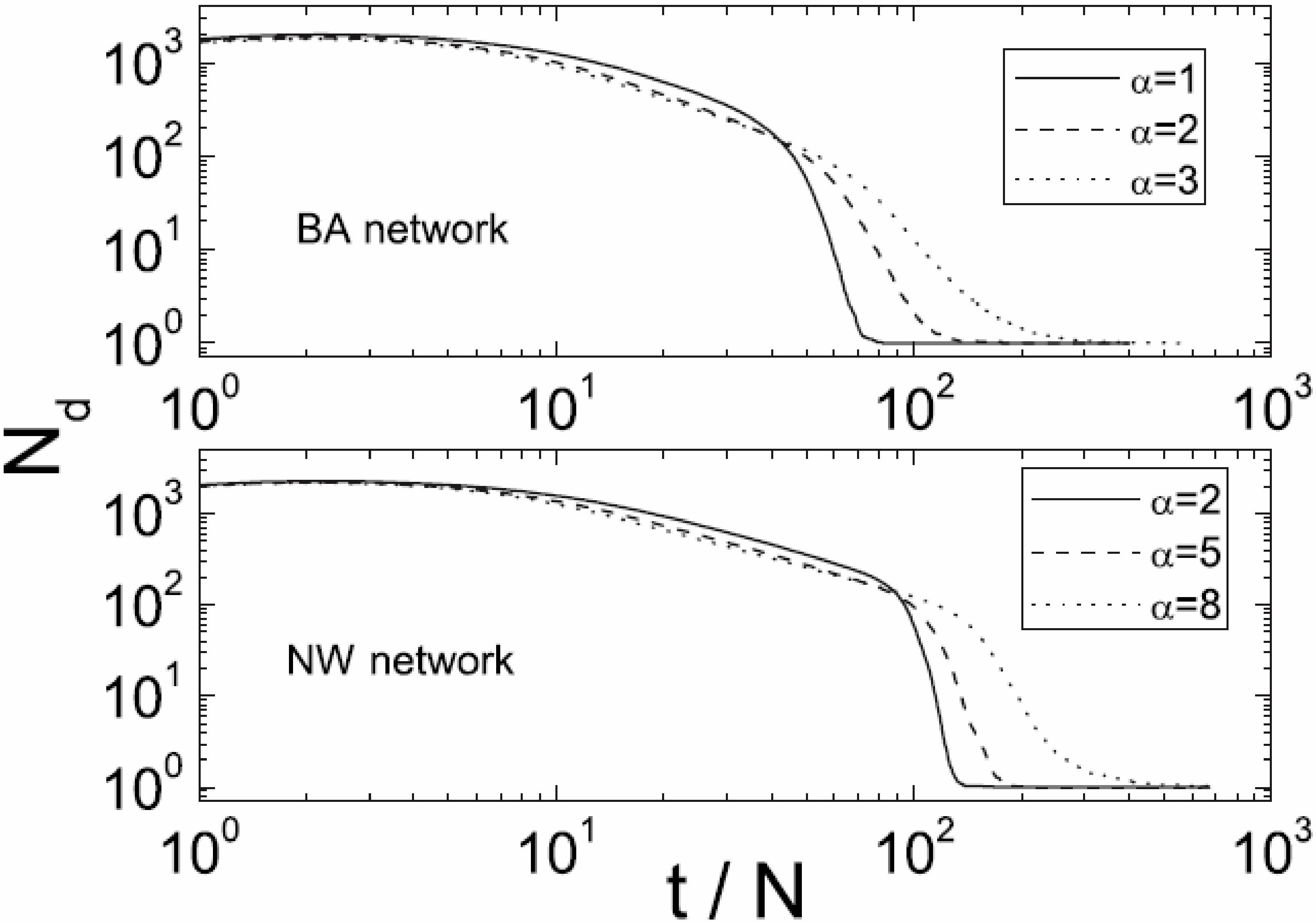
2.4x10^6
2x10^6
1.6x10^6
1.2x10^6
8x10^5
4x10^5
<k>=4
<k>=8
<k>=16
t_c
-0.5 0.0 0.5 1.0 1.5 2.0 2.5
2x10^6
10^6
8x10^5
6x10^5
4x10^5
2x10^5
10^5
N=2000
N=5000
N=10000
-0.5 0.0 0.5 1.0 1.5 2.0 2.5
α

NW network

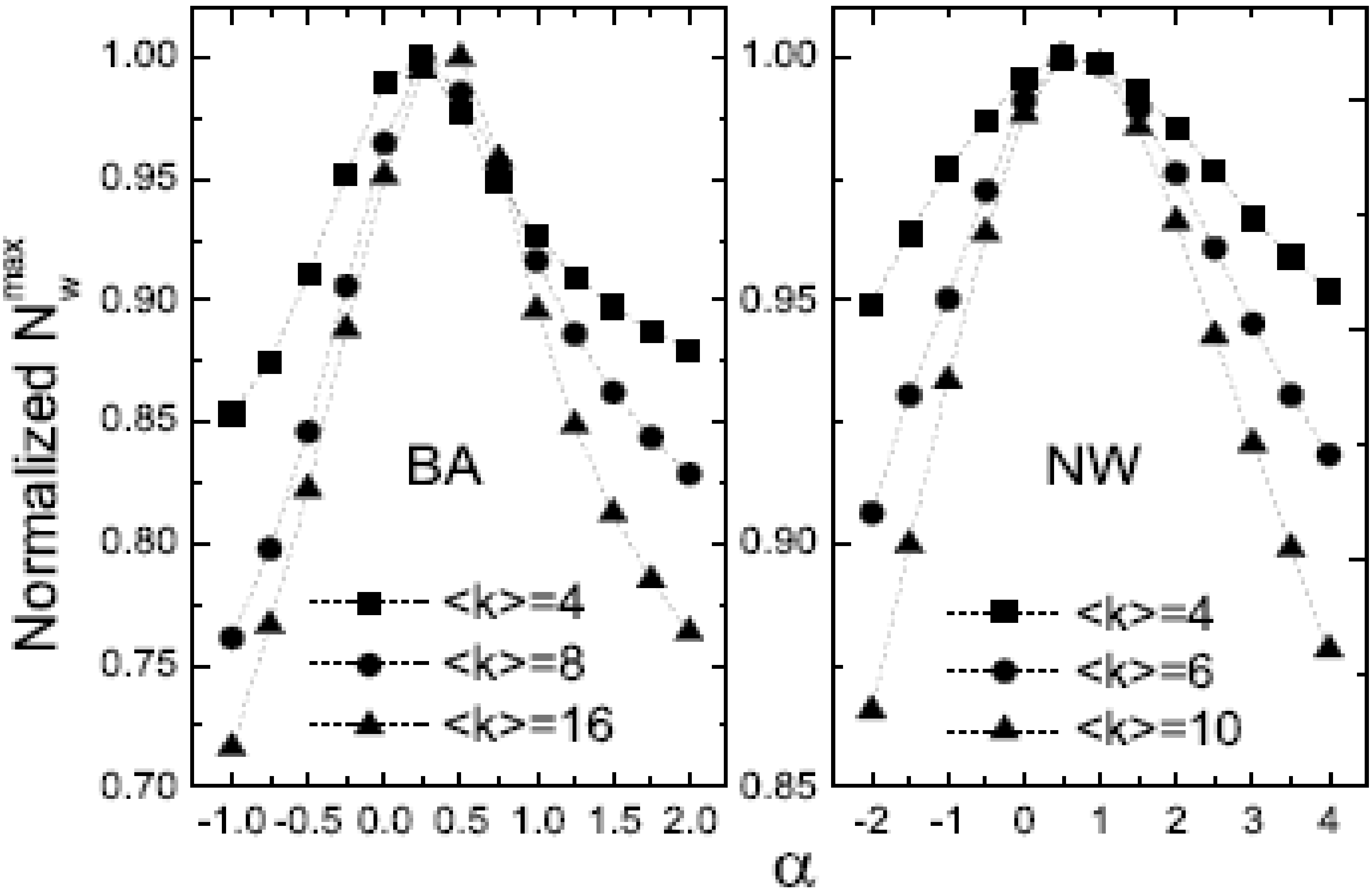


<k>=4
<k>=6
<k>=10
N=3000
N=6000
N=10000
t_c
α
2.4x10^6
1.8x10^6
1.2x10^6
6x10^5
3x10^6
2.5x10^6
2x10^6
1.5x10^6
10^6
5x10^5
-2 -1 0 1 2 3 4 5
-1 0 1 2 3 4 5

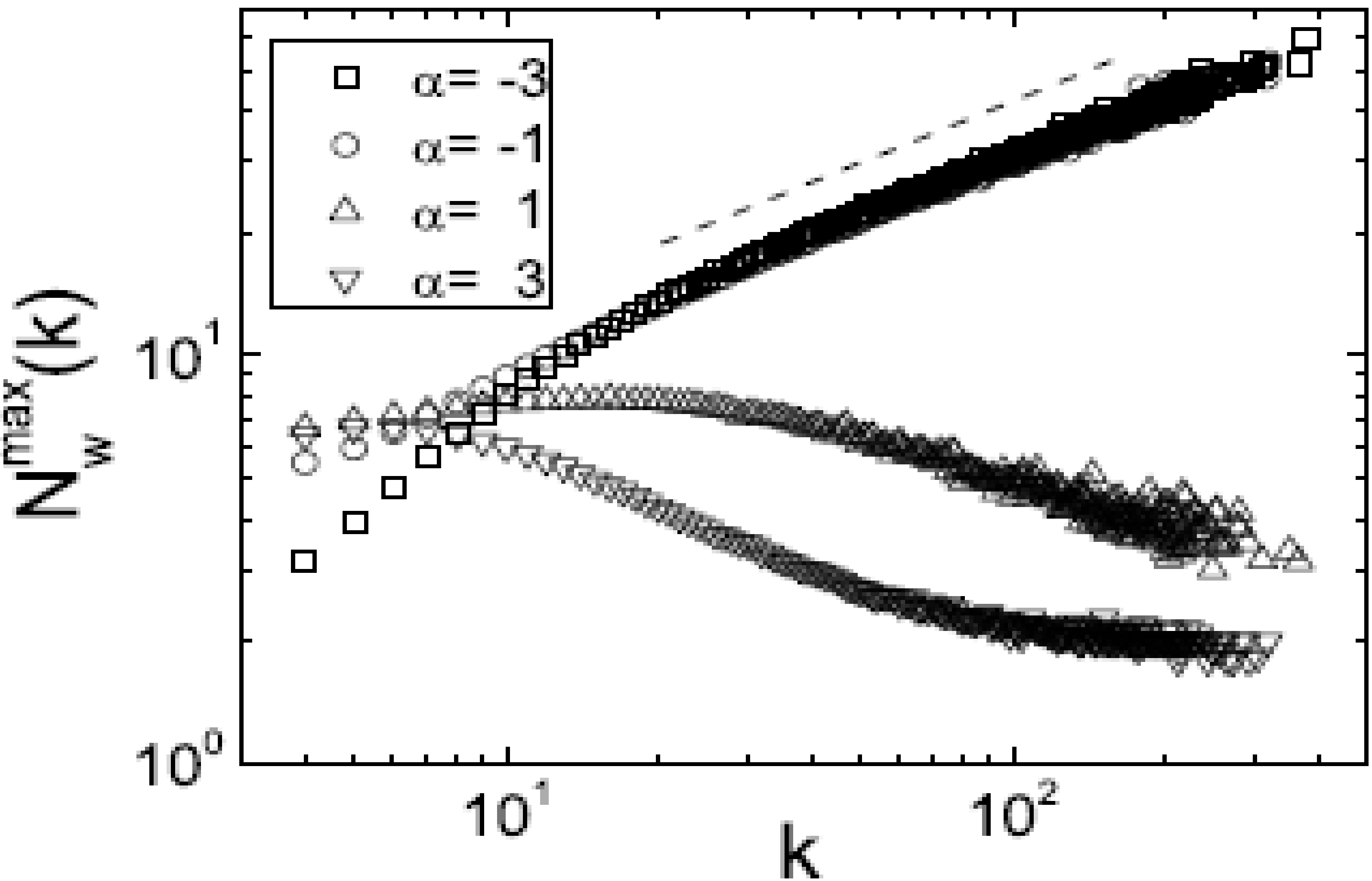
Evolution of the number of different names



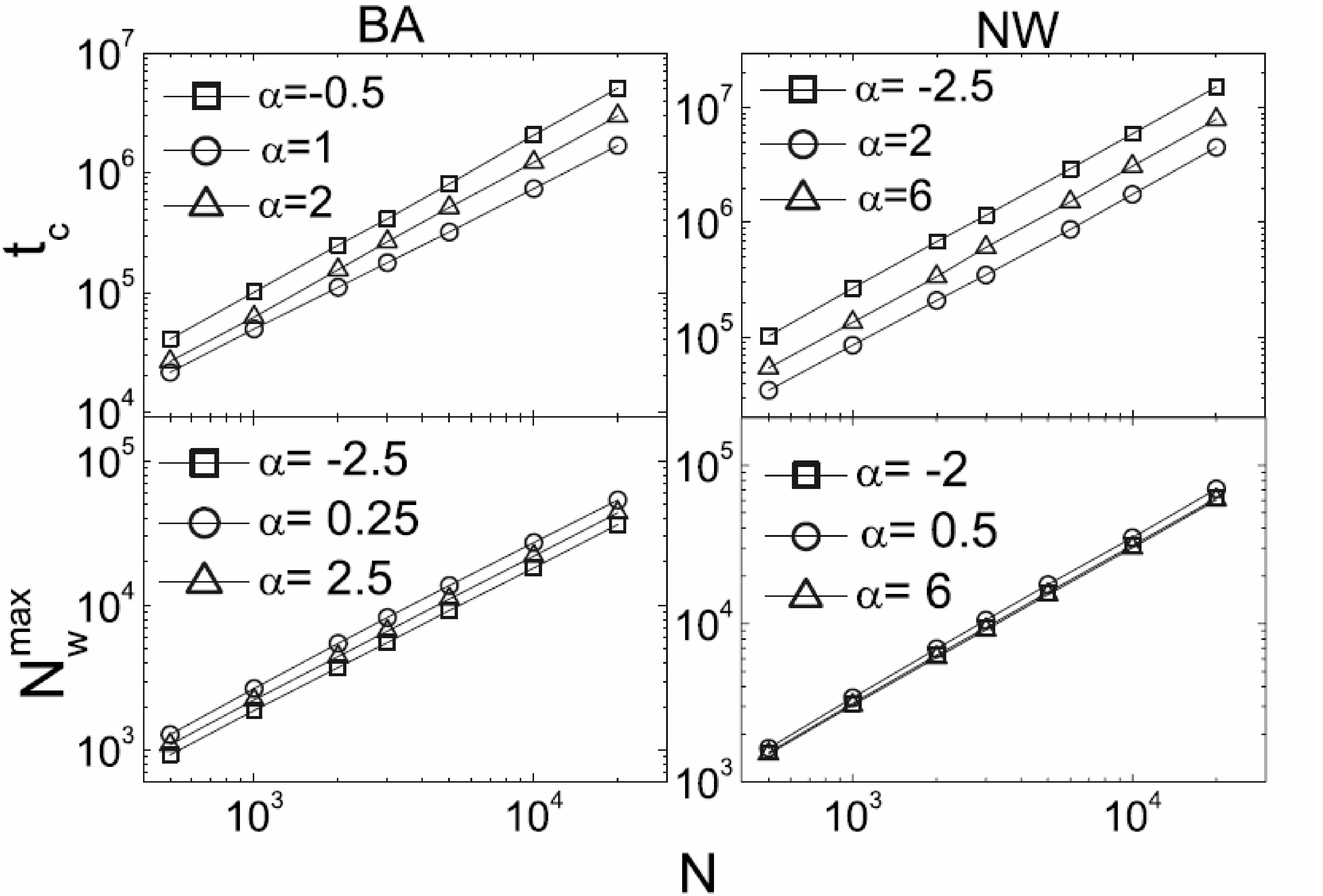
Normalized maximum total memory used by agents



Maximum memory used by agents



Convergence time and maximum total memory



Conclusion 1

- We have investigated a modified naming game with asymmetric negotiation strategy on both scale-free and small-world networks.

- The most interesting result is that there exists an optimal value of the parameter α that leads to the fastest convergence.

- This result demonstrates that a proper influence of high-degree agents in negotiation best benefits the achievement of final consensus, and high-degree agents can play both positive and negative roles in the agreement dynamics of the naming game.

Conclusion 2

- We have qualitatively explained the results for the convergence time in terms of the evolution of the total number of different names.

- We have also investigated the dependence of the total maximum memory used by agents on the parameter and found a peak in the middle range of the parameter space.

Conclusion 3

- The relationship between the maximum memory used by an agent and its degree shows different behavior compared to previously reported results in the naming game, while the convergence time and the total maximum memory show similar scaling behavior.

- It may be interesting to explore asymmetric negotiation on networks with degree correlation in future work.

Recent papers on Game Theory for understanding Evolutionary Cooperation

1 Wen-Xu Wang, Jie Ren, Guanrong Chen, and Bing-Hong Wang

Memory-based snowdrift game on networks

Physical Review E 74_(2006) 056113

2 Tang CL, Lin BY, Wang WX, Hu MB, Wang BH

Role of Connectivity-Induced Weighted Words in Language Games

Phys Rev E_75 (2007) 027101

3 C.-L. Tang, W.-X. Wang, X.Wu, and B.-H. Wang

Effects of average degree on cooperation in networked evolutionary game

European Physical Journal B 53, (2006) 411–415 B47

Recent papers on Game Theory for understanding Evolutionary Cooperation

4, Kun Gao, Wen-Xu Wang, Bing-Hong Wang,

Self-questioning games and ping-pong effect in the BA network

Physica A (2007)

5, Han-Xin Yang, Wen-Xu Wang, Bing-Hong Wang

Asymmetric negotiation in structured language games, Phys.Rev.E 77(2008)027103

Thanks

for your listenning!