Quantifying the rareness of clusters.

(and using rareness to constrain new physics)

Shaun Hotchkiss

arXiv:1012.2732	K. Enqvist, S.H. and O. Taanila
arXiv:1104.1145	A. Paranjape, C. Gordon and S.H.
arXiv:1105.3630	S.H.
arXiv:1108.5458	B. Hoyle, R. Jimenez, L. Verde and S.H

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If you are going to see high mass or high redshift clusters, you risk:

- Accidentally making claims of tension when no tension exists
- Missing a potentially important discovery.

Especially if you will see high mass and high redshift clusters.

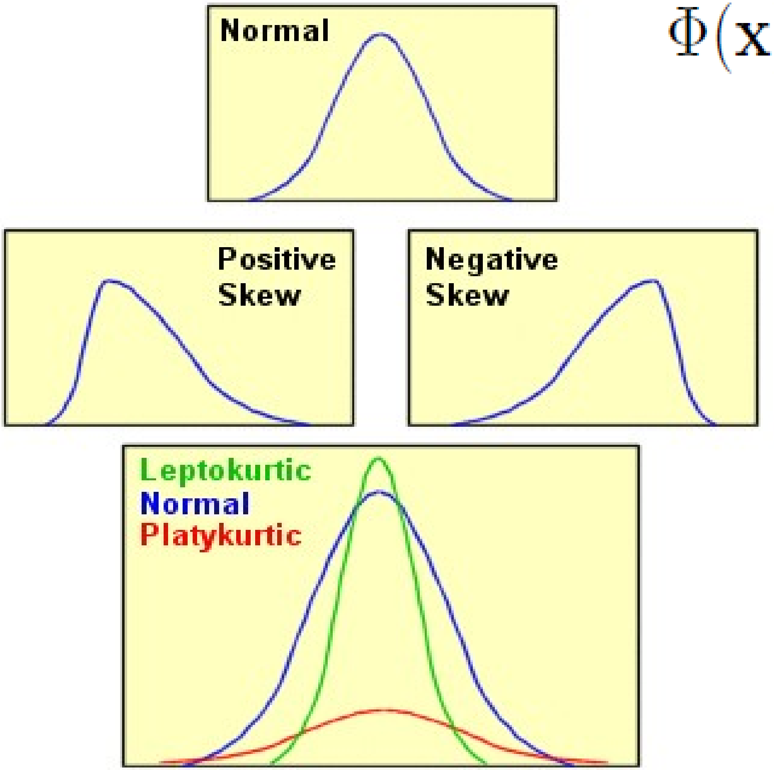
Summary and motivation

- Clusters are sensitive to non-Gaussianity and probe n-G at a unique scale
  - → most sensitive deep into the tail of probability distributions

- Care must be taken when:
  - → estimating the rareness of a cluster.
  - → estimating the rareness of an ensemble of clusters.
  - → applying mass-functions deep into their tails.

Current data requires analyses to probe masses and n-Gs untested by simulations. And, does not indicate significant tension with LCDM (subject to uncertainties on selection function).

Effect of n-G on cluster formation



$$\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\rm NL}\left[\phi^2(\mathbf{x}) - \langle\phi^2\rangle\right] + g_{\rm NL}\phi^3(\mathbf{x})$$

$+f_{NL}$ → +skewness

$+g_{NL}$ → +kurtosis

+skewness → positive tail increases

negative tail decreases

+kurtosis → positive tail increases

negative tail increases

and vice versa ($+f_{NL}$ cannot explain voids!)

Measuring non-Gaussianity

We can measure/constrain non-Gaussianity by:

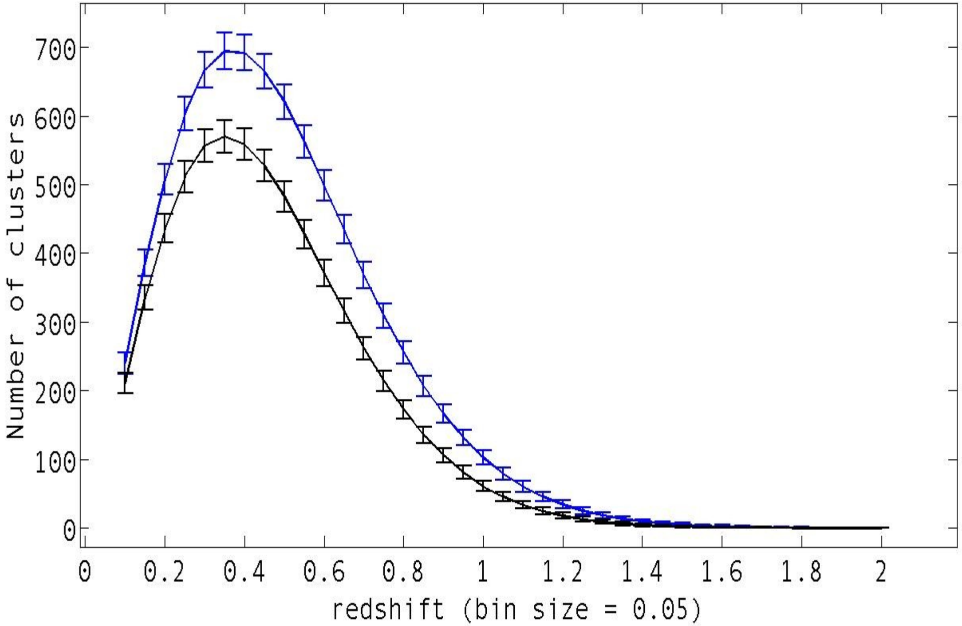
- Bispectrums - strictly zero for Gaussian, so very sensitive to deviations.
  - CMB $-10 < f_{\rm NL} < 74$
  - LSS not yet competitive

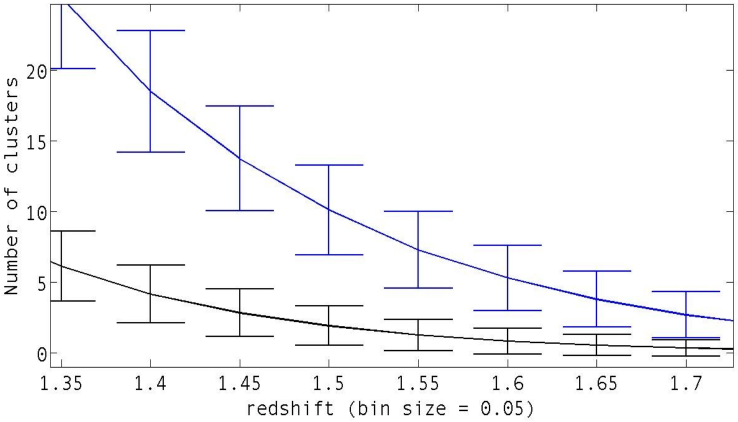
- Correction to galaxy bias. $-29 < f_{\rm NL} < 69$

- Realistic constraints from clusters in the future (assumes errors are Gaussian).

$\Delta f_{\rm NL} = 50$ Very different scales

The tail of the distribution.

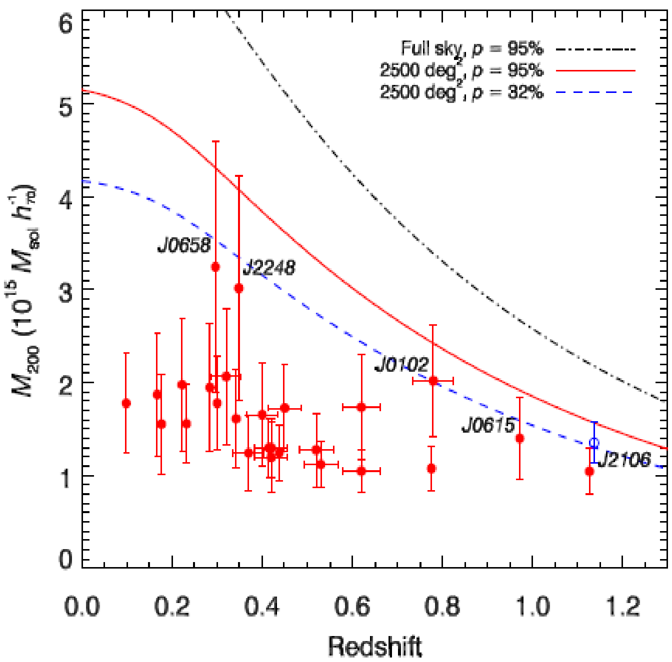




- Deep into the tail of the distribution the errors aren't Gaussian.
- If sufficiently rare, the mere existence of a pink elephant cluster could rule out LCDM. Because surveys aren't yet flux limited, it could be pink elephant clusters that give best first chances of finding tension.

pre-2011 results.

SPT



But! $f_{\rm NL} = 30 \pm 450$

which <u>includes</u> these clusters

X-rays, etc.

Jee et al.

Jimenez and verde

Hoyle et al. $\quad P(\Lambda) < 10^{-3}$

Enqvist et al.

Jee et al. (2011)

Hoyle et al. $\quad f_{\rm NL} \gtrsim 100$

Enqvist et al. the same

Cayon et al. the same

What on earth is happening?

- Either SZ selected clusters are very different to other clusters.
  - ➔ But individually, the SZ clusters seem just as improbable
- Perhaps LCDM <u>is</u> a horrendous fit, but so is <u>any</u> amount of non-Gaussianity.
- Or something fishy is going on somewhere...
  - ➔ But it can't be Eddington bias, because SPT took that into account

[1101.1286] Discovery and Cosmological Implications of SPT-CL J2106-5844, the Most Massive Known Cluster at z > 1

Authors: R. J. Foley, K. Andersson, G. Bazin, T. de Haan, J. Ruel, P. A. R. Ade, K. A. Aird, R. Armstrong, M. L. N. Ashby, M. Bautz, B. A. Benson, L. E. Bleem, M. Bonamente, M. Brodwin, J. E. Carlstrom, C. L. Chang, A. Clocchiatti, T. M. Crawford,

Abstract: Using the South Pole Telescope (SPT), we have discovered the most massive known galaxy cluster at z > 1, SPT-CL J2106-5844. In addition to producing a strong Sunyaev-Zel'dovich effect signal, this system is a luminous X-ray source and its numerous constituent galaxies display spatial and color clustering, all indicating the presence of a massive galaxy cluster. VLT and Magellan spectroscopy of 18 member galaxies shows that the cluster is at z = 1.132^+0.002_-0.003. Chandra observations obtained through a combined HRC-ACIS GTO program reveal an X-ray spectrum with an Fe K line redshifted by z = 1.18 +/- 0.03. These redshifts are consistent with galaxy colors in extensive optical, near-infrared, and mid-infrared imaging. SPT-CL J2106-5844 displays extreme X-ray properties for a cluster, having a core-excluded temperature of kT = 11.0^+2.6_-1.9 keV and a luminosity (within r_500) of L_X (0.5 - 2.0 keV) = (13.9 +/- 1.0) x 10^44 erg/s. The combined mass estimate from measurements of the Sunyaev-Zel'dovich effect and X-ray data is M_200 = (1.27 +/- 0.21) x 10^15 M_sun. The discovery of such a massive gravitationally collapsed system at high redshift provides an interesting laboratory for galaxy formation and evolution, and is a powerful probe of extreme perturbations of the primordial matter density field. We discuss the latter, determining that, under the assumption of LambdaCDM cosmology with only Gaussian perturbations, there is only a 7% chance of finding a galaxy cluster similar to SPT-CL J2106-5844 in the 2500 deg^2 SPT survey region, and that only one such galaxy cluster is expected in the entire sky.

[PDF] [PS] [BibTex] [Bookmark]

Author	Message
Fergus Simpson Joined: 25 Sep 2004 Posts: 27 Affiliation: University of Edinburgh	Posted: January 07 2011 quote

This paper looks at the detection of a large ($>10^{15}$) cluster at $z\sim1.1$ by the South Pole Telescope, and its cosmological implications. Sections 2 and 3 give a nice description of the detection and observational methods, but I'm a little concerned about section 4. One of the key claims of this work is that "there is a 7% chance of finding a cluster at least as massive and at a redshift at least as high".

Quantifying the extreme nature of a single variable, such as mass, would be a meaningful frequentist statement (although for those evangelical Bayesians out there, no such statement exists, but bear with me...). I'm just a bit concerned by this double condition which is enforced – using both mass and redshift.

For example, take a sample population of people in the UK, only one person can claim to be the shortest, and one the heaviest. But many can claim that 'no one is both shorter and heavier than me'. Perhaps each city has one such person. We don't know how many of these people there will be, or their statistical significance, until we better understand the relationship linking the two variables. What is really needed is a single measure, such as BMI in this analogy.

An appropriate statistic needs to invoke the mass-z relationship, (as depicted in Fig 5 of 1101.1290), which then allows us to compute whether one cluster is more extreme than

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Authors: R. J. Fol Carlstrom, C.
L. Chan

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Author

Fergus Simpson quote

Joined: 25 Sep 2004
Posts: 27
Affiliation: University
Edinburgh

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What were the odds of getting exactly this weekend's lottery numbers?

For example, take a sample population of people in the UK, only one person can claim to be the shortest, and one the heaviest. But many can claim that 'no one is both shorter and heavier than me'. Perhaps each city has one such person. We don't know how many of these people there will be, or their statistical significance, until we better understand the relationship linking the two variables. What is really needed is a single measure, such as BMI in this analogy.

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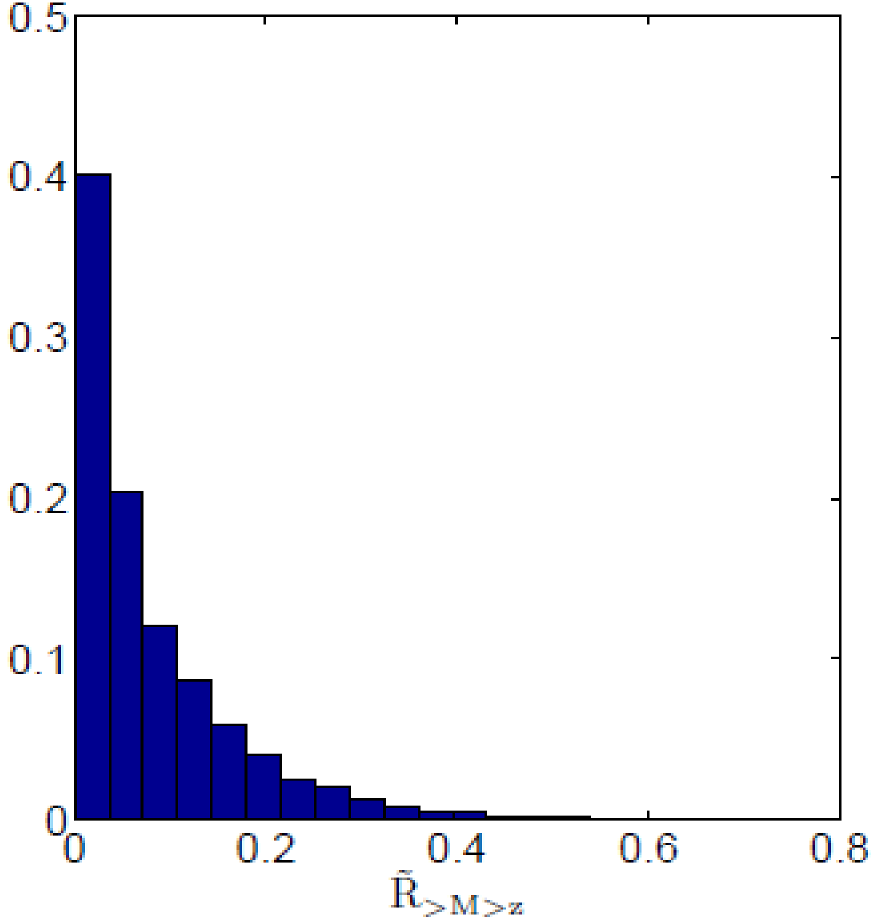
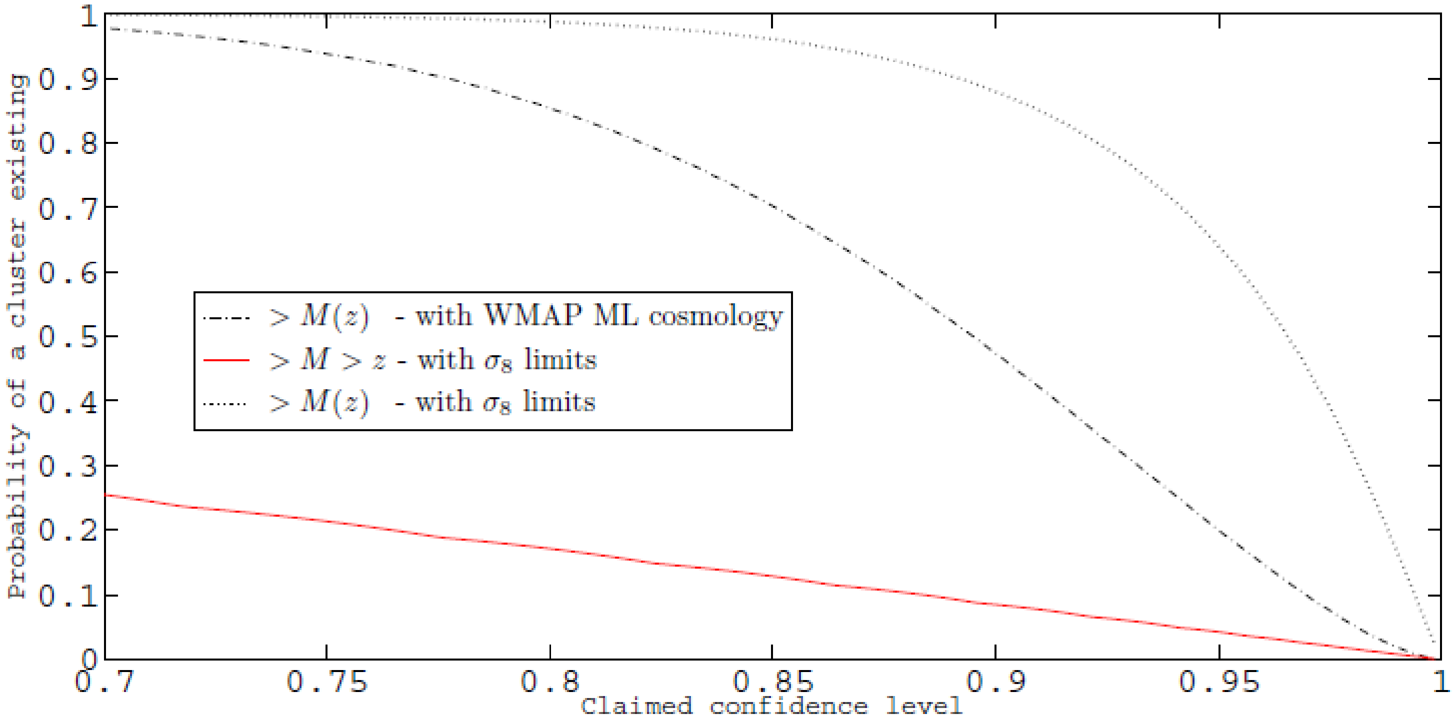
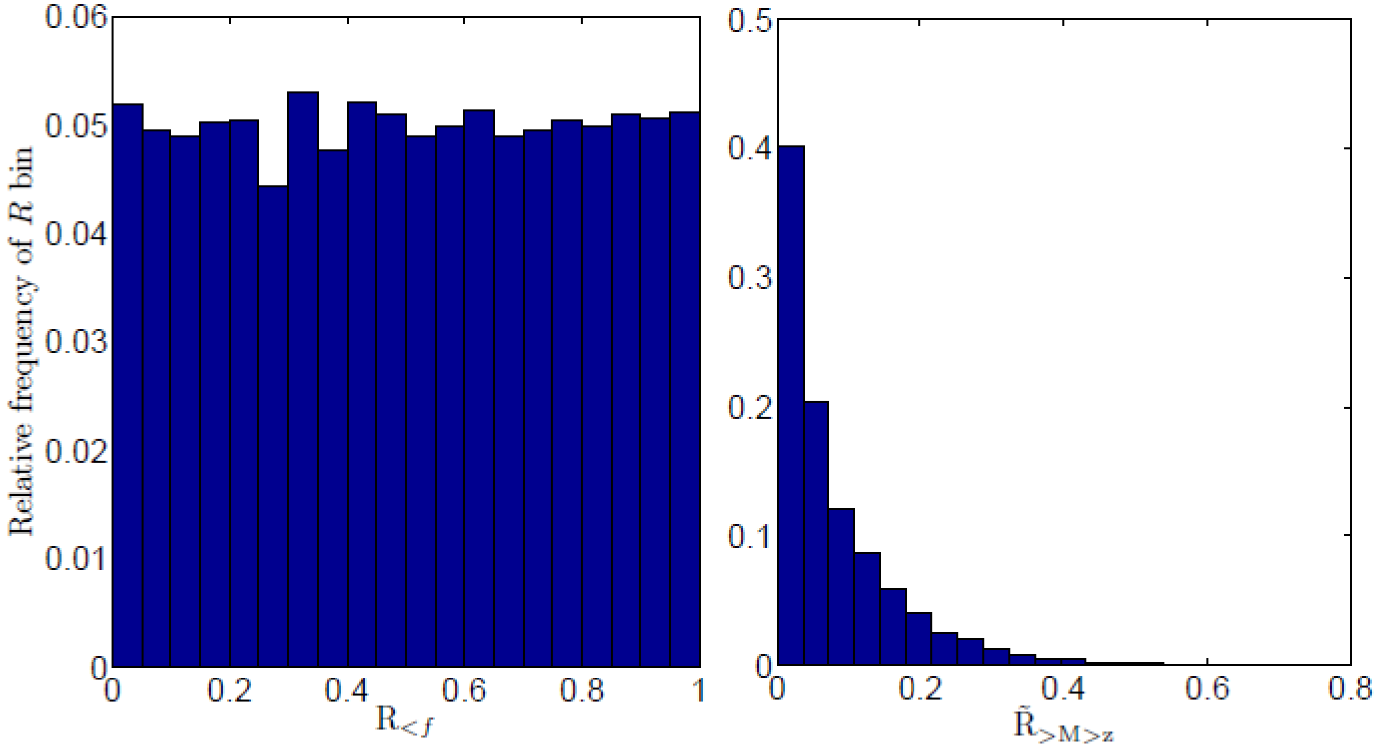
Small $\tilde{R}$ simply isn't uncommon



Small $\tilde{R}$ simply isn't uncommon



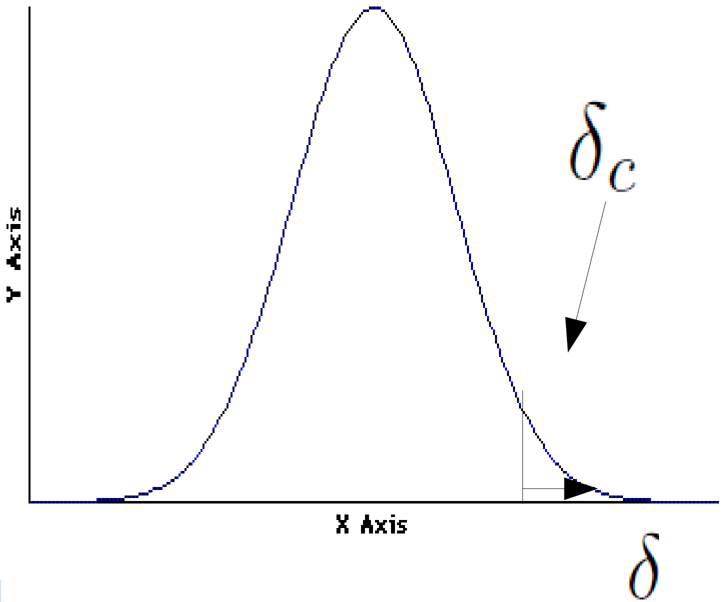
What does unbiased R look like?



$$P(R < R^*) = R^*$$

What contour should we use?

(Basically) What is the probability δ is above δ_c ?



$$n(M,z) = \frac{\bar{\rho}}{M} f\left(-\frac{\mathrm{d}\ln\sigma_M}{\mathrm{d}\ln M}\right)$$

f is the interesting dimensionless quantity.

$$\sigma_R^2 = \int_0^\infty \frac{dk}{k}\alpha_R^2(k,z)\mathcal{P}(k),$$

$$\alpha_R(k,z) = \frac{2}{3\Omega_m}D(z)\left(\frac{k}{H_0}\right)^2 T(k)W_R(k)$$

$$\mathrm{Exp}(M,z,\ldots) = \int_{z_n}^{z_f} dz \int_{M_{\min}}^{M_{\max}} dM\, f_{\mathrm{sky}}\frac{dV}{dz}n_{\mathrm{NG}}$$

What contour should we use?

$D(z)$

A different expansion history

- A different equation of state
- Cosmological back-reaction.

$\mathcal{P}(k)$

Changes to the primordial spectrum.

- Non-Gaussianities
- A different σ_8

The true, conservative, rarenesses

Cluster	$R_{<f}$	$R_{>M>z}$	$<f$ Mass at $z=0$	$>M>z$ Mass at $z=0$
J2235.3+2557 (H10)	0.58	0.49	$7.7\times10^{15}M_\odot$	$3.3\times10^{15}M_\odot$
J0546-5345 (H10)	0.76	0.61	$6.2\times10^{15}M_\odot$	$2.8\times10^{15}M_\odot$
J0910+5422 (H10)	0.86	0.79	$4.5\times10^{15}M_\odot$	$1.8\times10^{15}M_\odot$
J2215.9-1738 (H10)	0.85	0.81	$5.2\times10^{15}M_\odot$	$1.8\times10^{15}M_\odot$
J0102-4915 (W11)	0.63	0.61	$7.1\times10^{15}M_\odot$	$3.8\times10^{15}M_\odot$
J0615-5746 (W11)	0.63	0.70	$7.1\times10^{15}M_\odot$	$3.5\times10^{15}M_\odot$
J0658-5556 (W11)	0.84	0.63	$5.2\times10^{15}M_\odot$	$3.6\times10^{15}M_\odot$
J2106-5844 (W11)	0.73	0.86	$6.7\times10^{15}M_\odot$	$3.0\times10^{15}M_\odot$
J2248-4431 (W11)	0.84	0.66	$5.3\times10^{15}M_\odot$	$3.5\times10^{15}M_\odot$
J2344-4243 (W11)	0.92	0.88	$5.0\times10^{15}M_\odot$	$2.7\times10^{15}M_\odot$

mass errors push R ----> 0.5

Ensemble rareness

$$\tilde{R}^H = R_1 \times R_2$$

Ensemble rareness

$$R_i^M = 1 - e^{-\lambda_i} \sum_n^i \frac{\lambda_i^n}{n!},$$

Ensemble rareness

$$R_i^M = 1 - e^{-\lambda_i} \sum_n^i \frac{\lambda_i^n}{n!},$$

$$\tilde{R}_i^C = P(R_1^M < R_{1(\text{obs})}^M \cap R_2^M < R_{2(\text{obs})}^M \cap \cdots \cap R_i^M < R_{i(\text{obs})}^M),$$

$$\begin{aligned}
R_1^C &= 1 - e^{-\lambda_1}, \\
\tilde{R}_2^C &= R_1^C - \lambda_1 e^{-\lambda_2}, \\
\tilde{R}_3^C &= \tilde{R}_2^C - \lambda_1 e^{-\lambda_3}\left(\lambda_2 - \frac{\lambda_1}{2}\right), \\
\tilde{R}_4^C &= \tilde{R}_3^C - e^{-\lambda_4}\left(\frac{\lambda_1^3}{6} + \lambda_1\lambda_2\lambda_3 - \frac{\lambda_1\lambda_2^2}{2} - \frac{\lambda_3\lambda_1^2}{2}\right).
\end{aligned}$$

Actual ensemble rarenesses

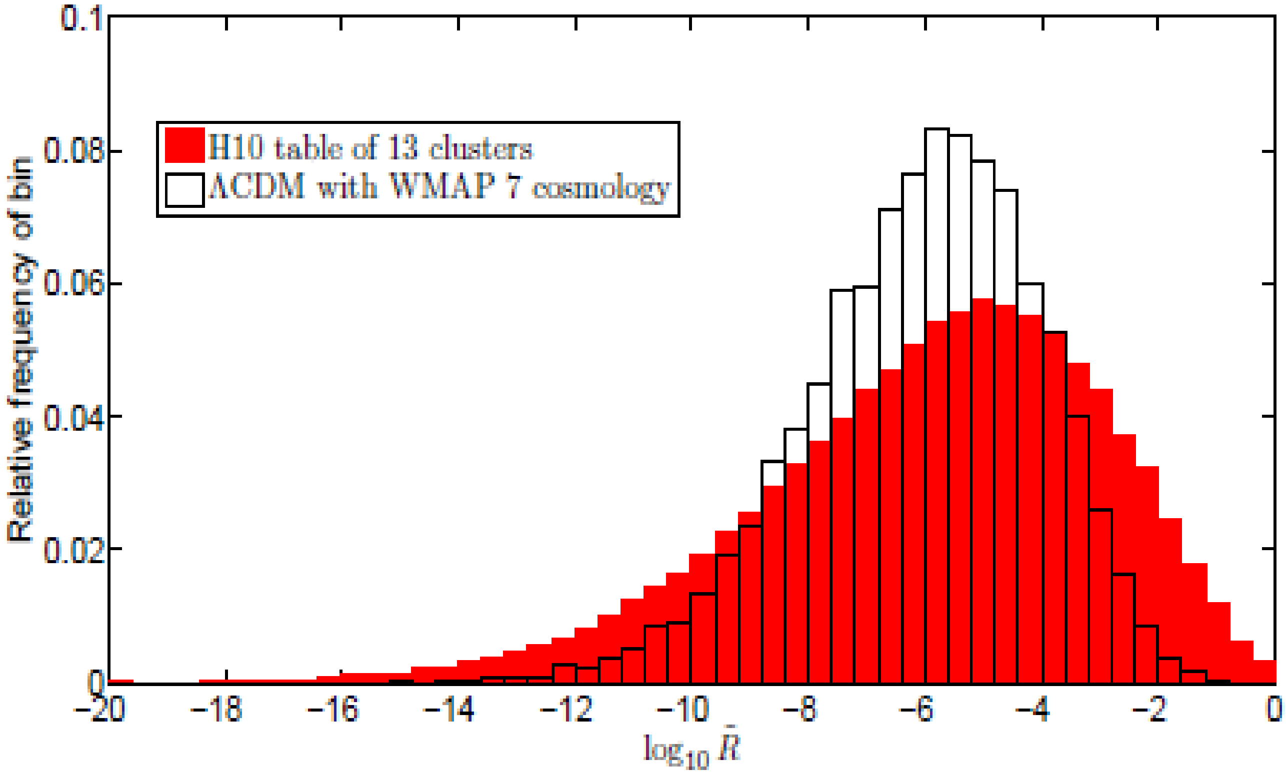
Cluster set	Mean $\tilde{R}^H_{i>M>z}$	Median $\tilde{R}^H_{i>M>z}$
H10	6.1×10^{-3}	2.6×10^{-6}
Sample (rarest)	3.0×10^{-4}	1.5×10^{-6}
Sample (random)	0.70	0.84

i	1	2	3	4	5	6	7	8	9	10
$R^M_{i<f}$	0.32	0.30	0.33	0.43	0.57	0.72	0.83	0.90	0.94	0.97
$R^M_{i>M>z}$	0.22	0.21	0.23	0.28	0.36	0.44	0.53	0.62	0.71	0.78

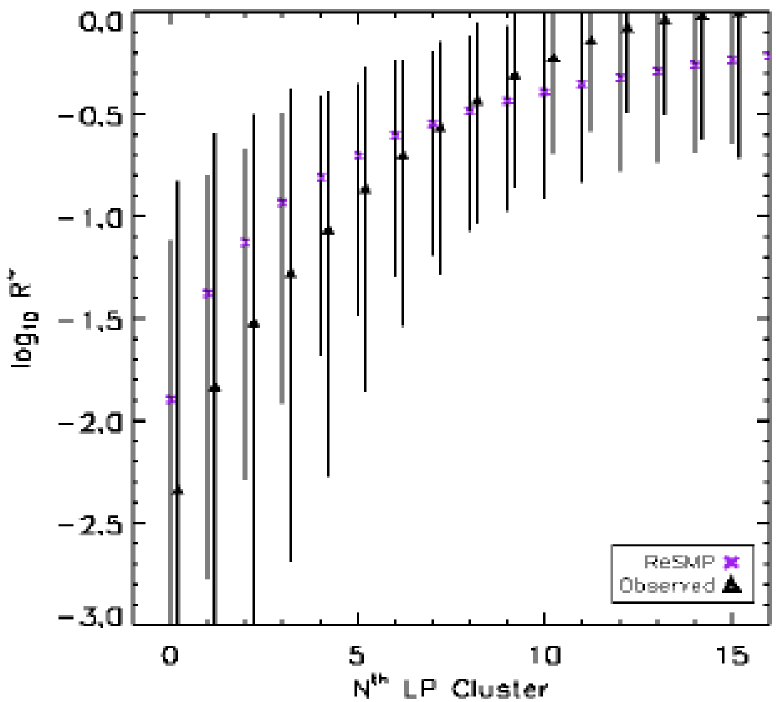
i	1	2	3	4
W11	0.32	0.30	0.30	0.31
H10	0.29	0.31	0.33	0.36

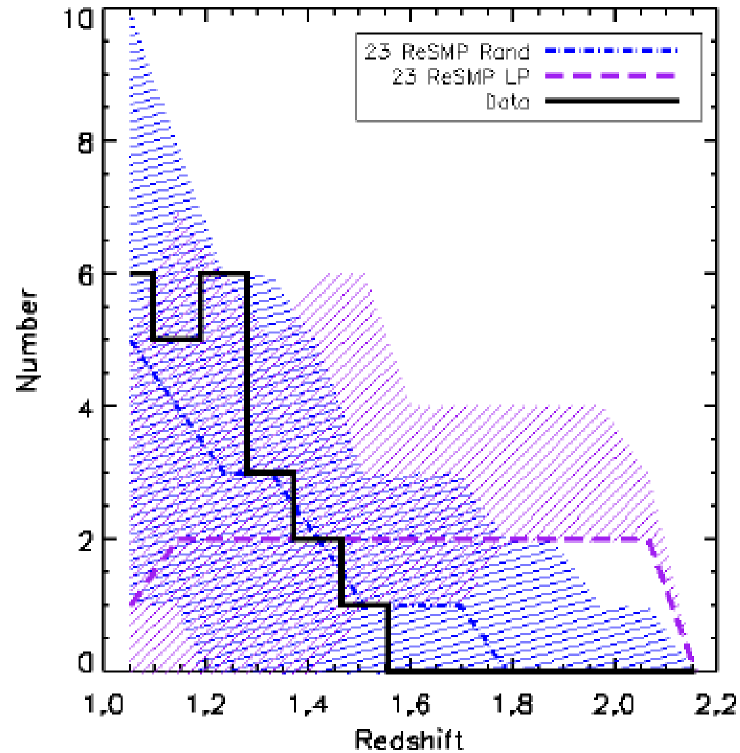
$$R^C = P(\tilde{R}^C < \tilde{R}^C_{(\mathrm{obs})}).$$

Actual ensemble rarenesses



What if we aren't conservative?





arXiv:1108.5458

Getting the n-G tail correct.

Quantitative effects of NG

- Theoretical Gaussian mass functions are getting better and better.
- Until recently weren't trusted enough, so typical method...

$$\mathcal{R}(M, z, f_{\rm NL}, g_{\rm NL}) = \frac{n_{\rm analytical}(M, z, f_{\rm NL}, g_{\rm NL})}{n_{\rm analytical}(M, z, f_{\rm NL}=0, g_{\rm NL}=0)}$$

$$n_{\rm NG} = \mathcal{R} \times n_{\rm Gauss}$$

- Sounds dodgy.... is a little dodgy... but tested against simulations.
- In the approximate limit (large mass, high redshift) it also matches the best theory.

Different non-Gaussian ratios

$$\mathcal{R}(M, z, f_{\rm NL}, g_{\rm NL}) = \frac{n_{\rm analytical}(M, z, f_{\rm NL}, g_{\rm NL})}{n_{\rm analytical}(M, z, f_{\rm NL} = 0, g_{\rm NL} = 0)}$$

- Loverde et al $$\mathcal{R}_{NG} = 1 + (\sigma S_3)\frac{\delta_c^3}{6\sigma^3}$$

- MVJ $$\mathcal{R}_{NG} = \exp\left((\sigma S_3)\frac{\delta_c^3}{6\sigma^3}\right) \times (\sigma S_3 - \text{polynomial})$$

Then use excursion sets to increase complexity of collapse model.

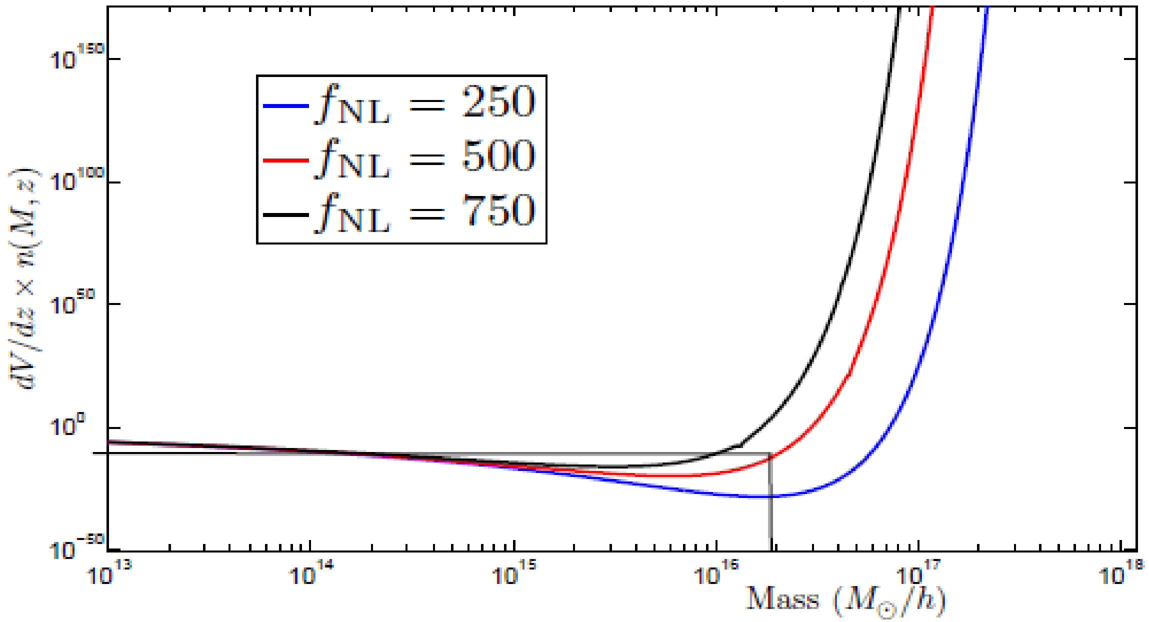
Integrate to ∞? (be careful)

Enqvist, SH, Taanilla (arXiv:1012.2732)

$$\mathrm{Exp}(M, z, \ldots) = \int_{z_n}^{z_f} dz \int_{M_{\min}}^{M_{\max}} dM\, f_{\rm sky} \frac{dV}{dz} n_{\rm NG}$$

$$\mathcal{R} \sim \exp\left(\frac{\alpha}{\sigma^3}\right)$$

$$n_{\rm Gauss} \sim ?$$



If Mmax > Mturn some of the probability is unphysical.

Once Mmax >> Mturn almost all the probability is unphysical.

But, if Mmax < Mturn results can be trusted.

The "resummed" mass-function

A. Paranjape, C. Gordon and S.H. (arXiv:1104.1145)

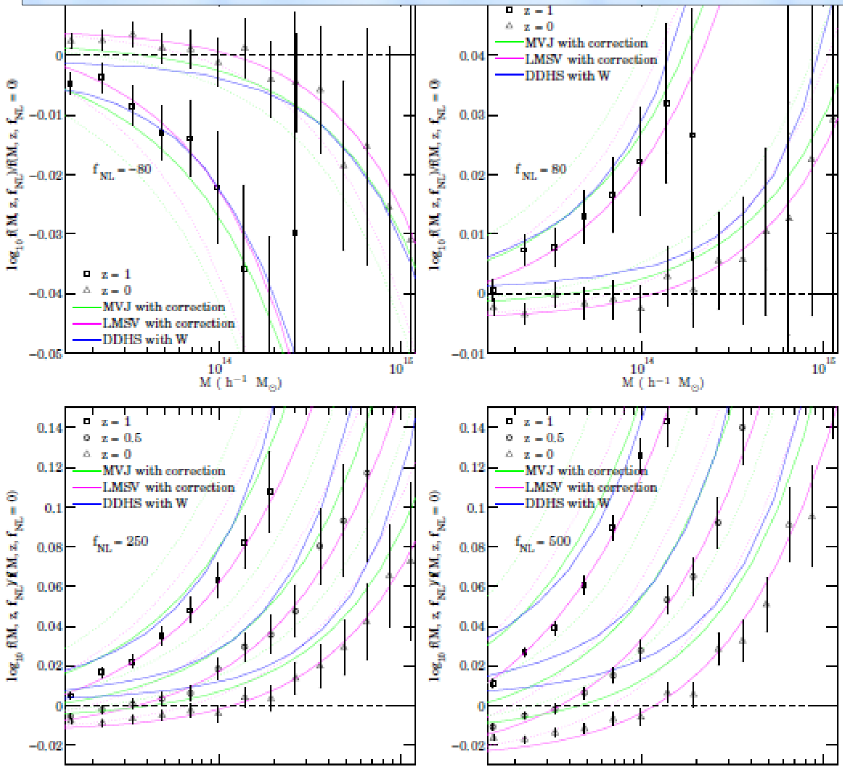
- Assume well motivated hierarchy in $\epsilon_i = \epsilon_1^i \;\; (\epsilon_1 = \sigma S_3)$
- Resum all terms of order $\epsilon\nu \;\; (\nu = \sqrt{a}\delta_c/\sigma)$

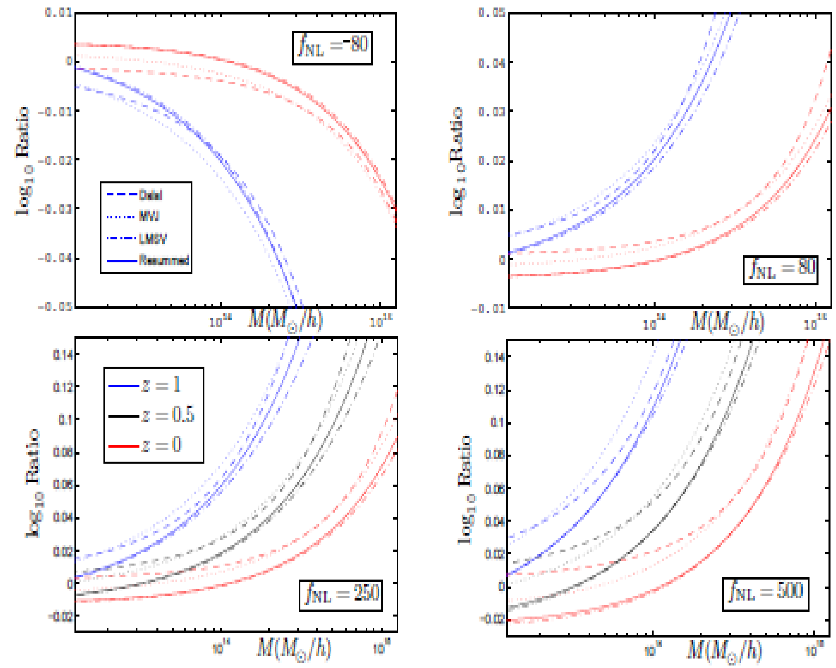
$$\mathcal{R}_{\mathrm{Resum}}(M, z, f_{\mathrm{NL}}) = (1+\varepsilon_1\nu)^{-1/2}\exp\left[\frac{1}{2}\nu^2 + \frac{1}{\varepsilon_1^2}\left(\varepsilon_1\nu - (1+\varepsilon_1\nu)\ln(1+\varepsilon_1\nu)\right)\right].$$

- Define 'a' based on choice of Gaussian mass-function to ensure stability at large ν .
- Not one free parameter to play with in ratio

Necessary so long as data does not constrain small ν and small f_{NL}

The “resummed” mass-function





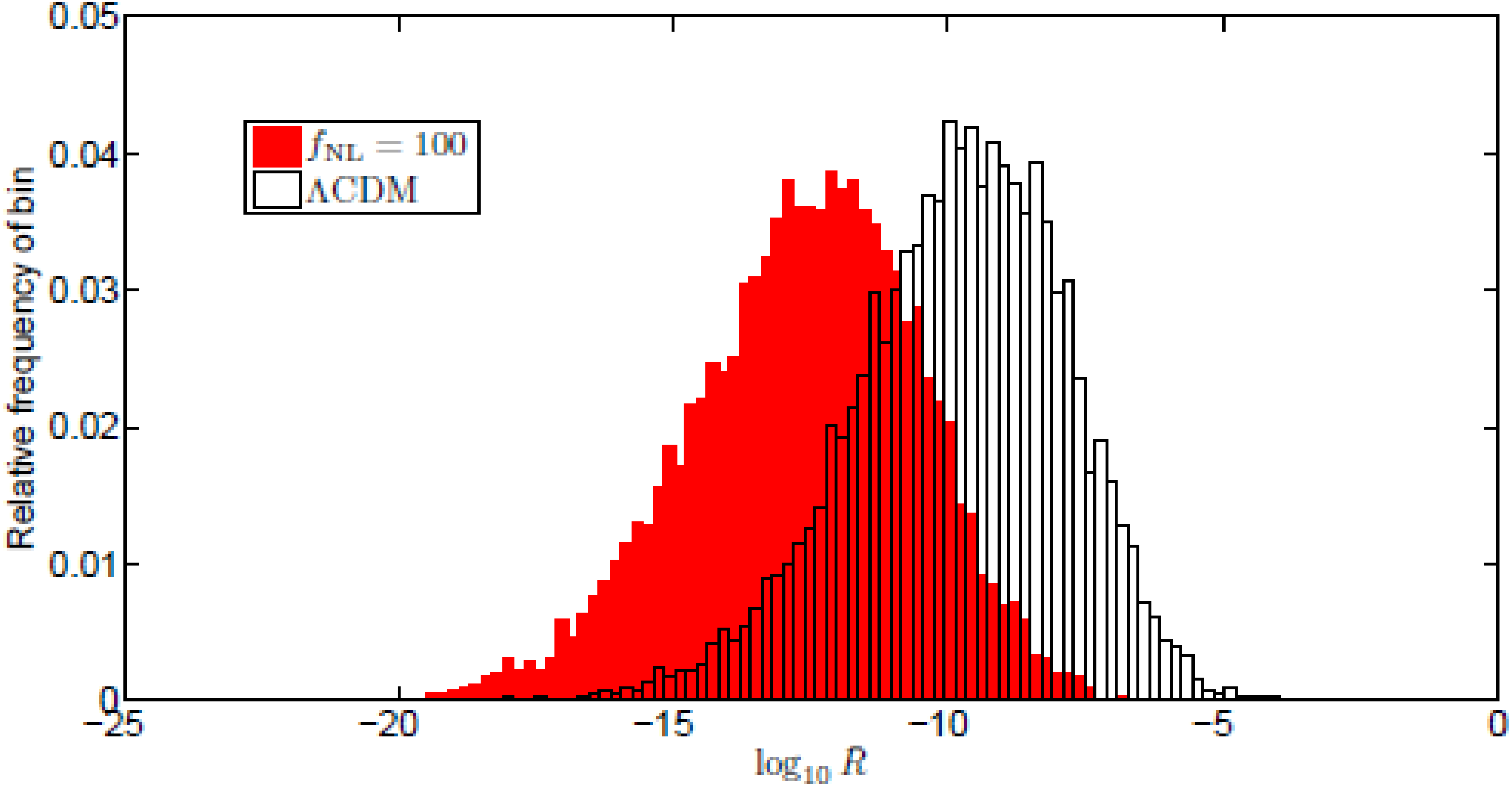
- In range well tested by simulations, it matches very well.
- As expected, it deviates outside of tested range. However this is when other formalisms are breaking down. Needs testing.

Rareness of f_{NL} ensembles

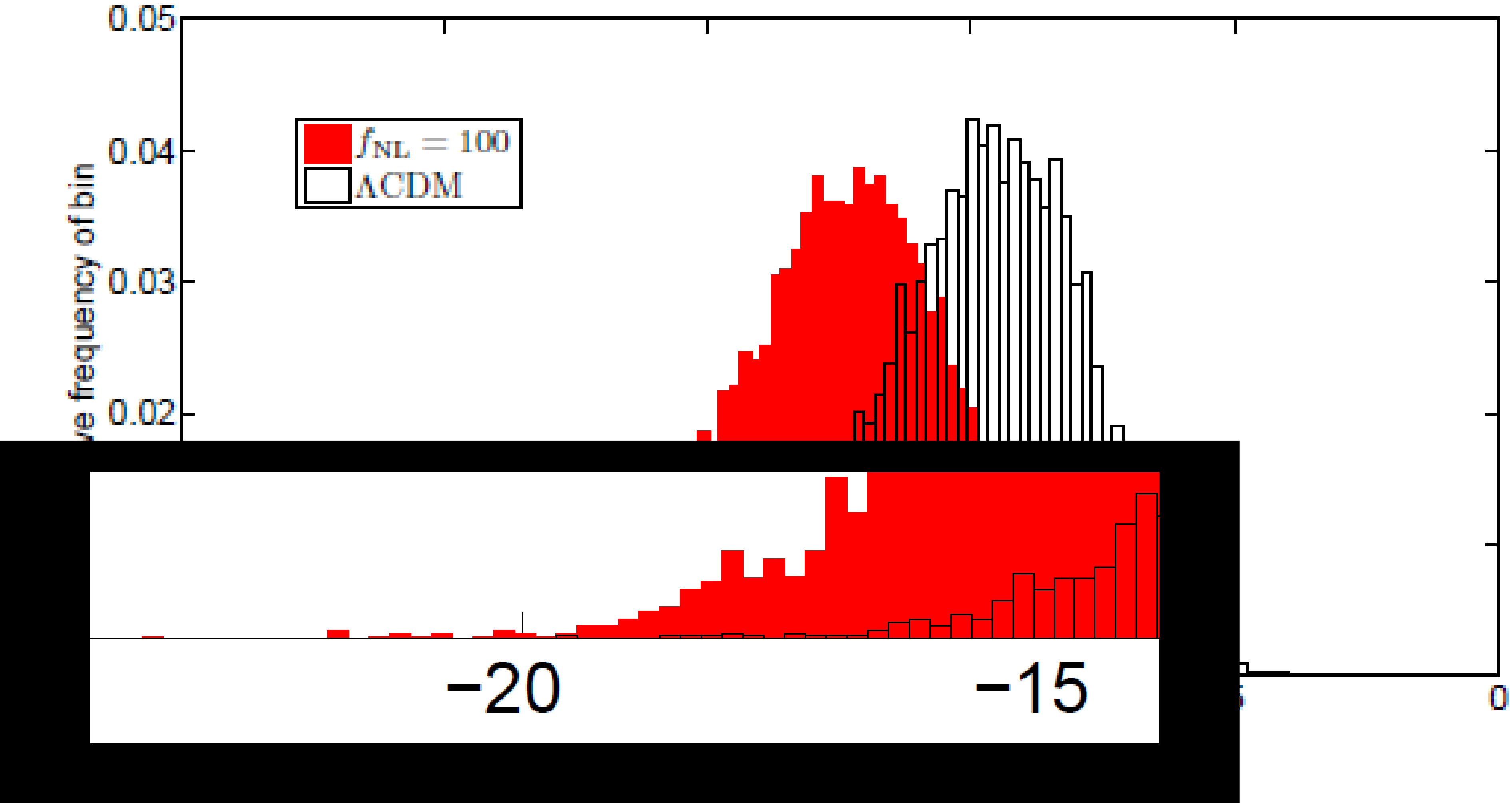
	$R^C_{i<f}$			$R^M_{i<f}$		
i	$f_{NL} = 50$	$f_{NL} = 100$	$f_{NL} = 500$	$f_{NL} = 50$	$f_{NL} = 100$	$f_{NL} = 500$
1	0.39	0.30	0.022	0.40	0.30	0.023
2	0.37	0.26	0.0050	0.38	0.24	0.0024
3	0.36	0.24	0.0012	0.34	0.20	3.0×10^{-4}
4	0.34	0.21	3.1×10^{-4}	0.32	0.17	4.5×10^{-5}

These are mean values. But there is a tail where R is very small.

Rareness of f_{NL} ensembles



Rareness of fNL ensembles



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- Care must be taken when:
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Current data requires analyses to probe masses and n-Gs untested by simulations. And, does not indicate significant tension with LCDM (subject to uncertainties on selection function).

End

Clusters in the Universe are <u>less rare</u> than they appear.

So we should be seeing many more of them soon...