Cosmological production of black hole pairs

Piero Nicolini

Frankfurt Institute for Advances Studies
Johann Wolfgang Goethe-Universität

nicolini@fias.uni-frankfurt.de

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Cosmological observations

- Fermi Gamma-ray Space Telescope (formerly GLAST)
- supported by NASA, DOE, France, Germany, Italy, Japan, and Sweden (with Austrian, Icelandic, French and Spanish research teams).
- key scientific objectives
  1. active galactic nuclei, pulsars, and supernova remnants.
  2. gamma-ray bursts
  3. gamma-ray background radiation
  4. dark matter
  5. primordial (evaporating) black holes

The Fermi lift off - 11 June 2008

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Can we observe the Hawking radiation?

- Hawking temperature

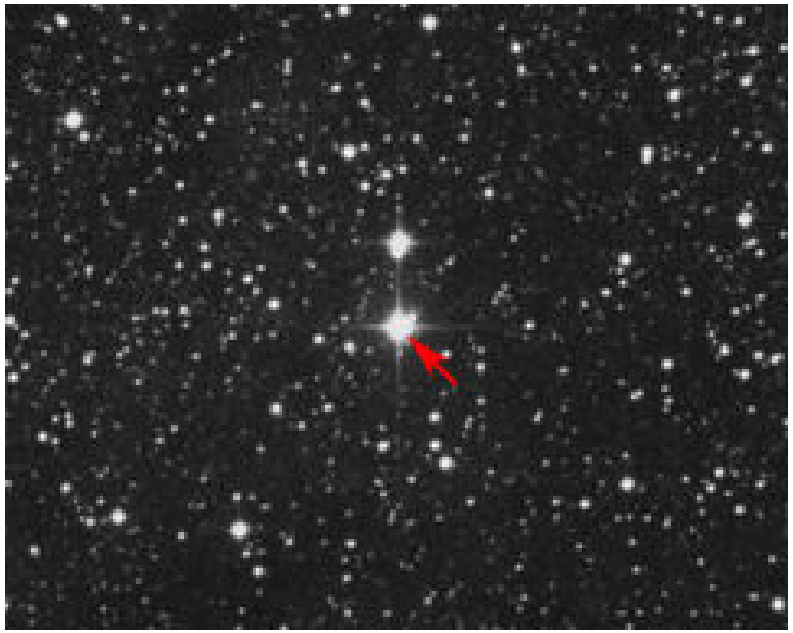
$$T_{\rm H} = \frac{\hbar c}{2\pi k_B}\,\kappa$$

- Schwarzschild

$$\begin{aligned} T_{\rm H} &= \frac{\hbar c^3}{G k_B}\frac{1}{8\pi M} \\ &\sim 10^{23}\left(\frac{1\,{\rm kg}}{M}\right){\rm K} \\ &\sim 10^{-7}\left(\frac{M_\odot}{M}\right){\rm K} \end{aligned}$$

- rules out stellar and heavier BHs

$$T_{\rm H} \ll T_{\rm CMB}$$



Cygnus X-1 at $\sim$ 6100 ly from us (1964)

$M \sim 14.8 M_\odot$

$r_{\rm h} \sim 26$ km

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Primordial black holes

- Evaporation times

$$\frac{dM}{dt} \approx -\left(k_B T_{\rm H}\right)^4 (Area)$$

$$\Rightarrow t_{\rm ev} \sim 10^{58} \left(\frac{M}{M_\odot}\right)^3 \text{Gyr}$$

- primordial BHs

$$t_{\rm ev} \stackrel{!}{=} 13.75 \text{ Gyr}$$

$$\begin{aligned} \Rightarrow \quad & M_{\rm PBH} \sim 10^{11} \text{ kg} \\ \Rightarrow \quad & T_{\rm PBH} \sim 10^{12} \text{ K} \\ \Rightarrow \quad & r_{\rm h} \sim 10^{-16} \text{ m} \\ \Rightarrow \quad & \text{EMP} \sim 1 \text{ GHz} \end{aligned}$$

[Rees, 1977][Blandford, 1977]

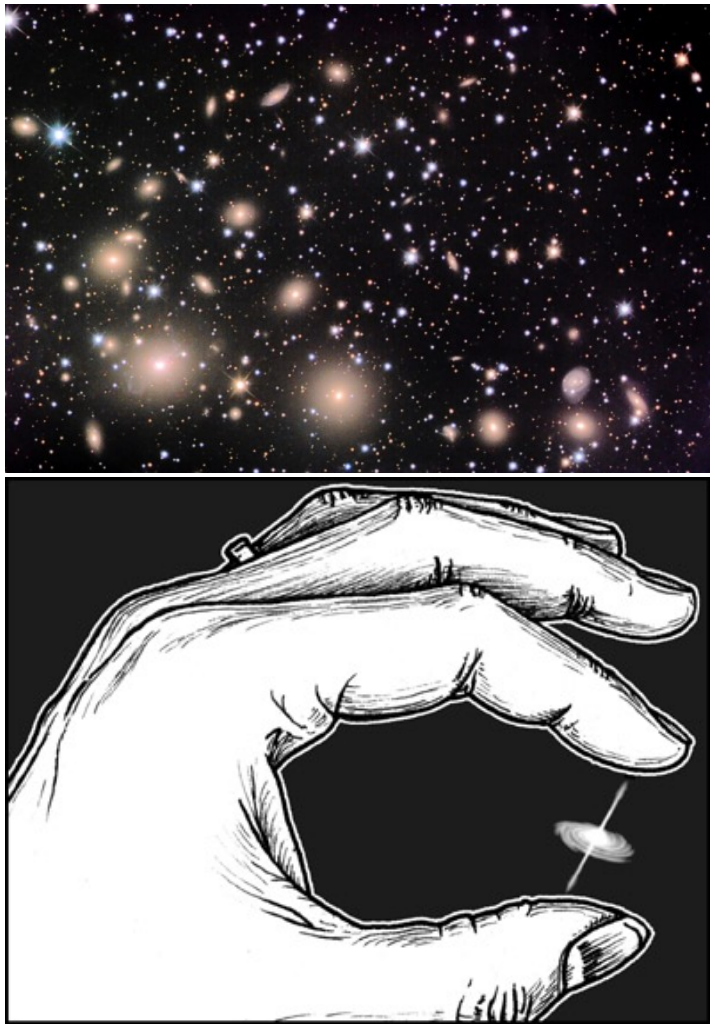


Image credit: John Cramer, as best as I can tell.

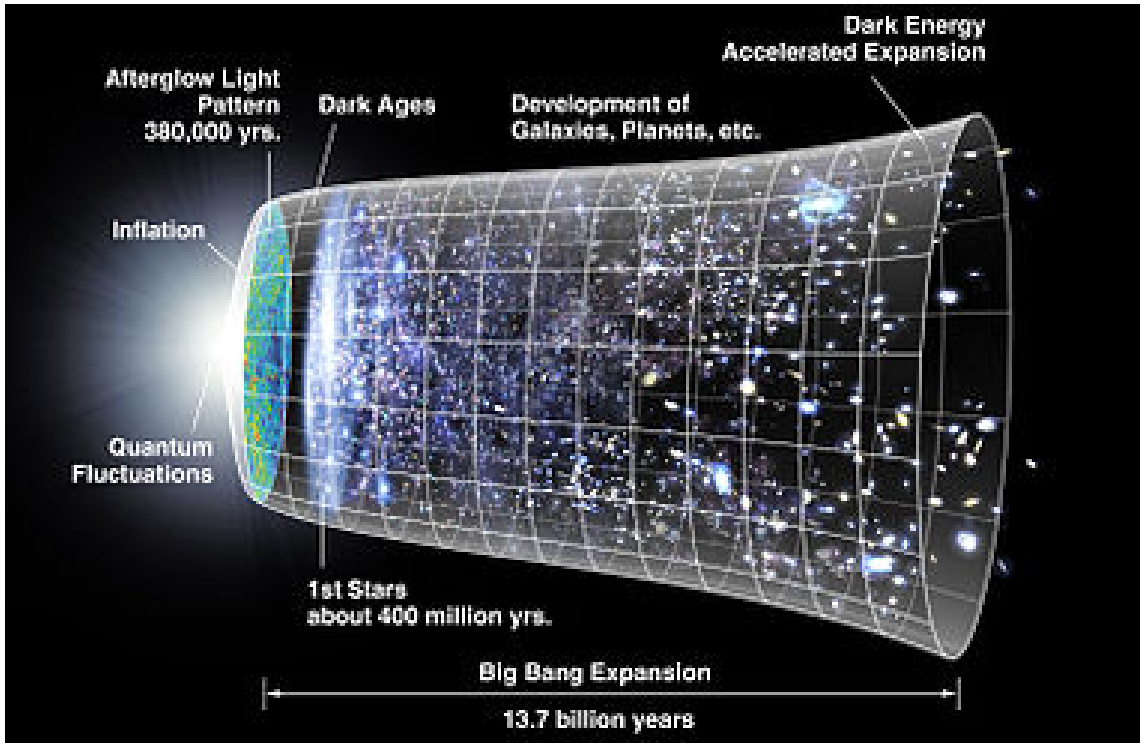
Black hole formation

Conventional mechanism

- gravitational collapse

In the inflationary era

- strong density perturbations $\Longrightarrow$ spontaneous black hole formation [Carr & Hawking, 1974]
- quantum process of pair creation



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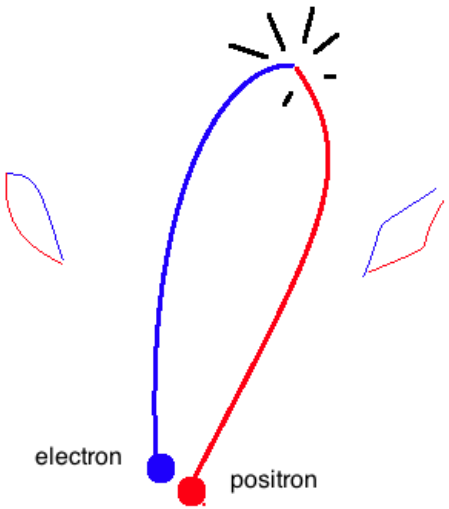
Pair creation

- dS space is stable with respect classical perturbation
- However dS space can be **quantum mechanically unstable**
- The process resembles the Schwinger electron-positron pair production

$$w = \frac{e^2\ E^2}{\pi^2 \hbar^2 c} \exp\left(-\pi\ m^2 c^3\ /e\ E\ \hbar\right)$$

e the electric charge, E the electric field.



- during the inflation, $\Lambda > 0$ supplies the energy for pair creation, i.e., different spacetime topology.

Instanton formalism

- We consider QM in $(1+1)$-dimensions
- the amplitude is given by

$$K(a,b;T) = \langle x = a | e^{iHT/\hbar} | x = b \rangle = \int \mathcal{D}[x(t)] e^{iS[x(t)]/\hbar}$$

with $S[x(t)] = \int_{t_a}^{t_b} dt \left(\frac{1}{2} m \left(dx/dt \right)^2 - V(x) \right)$ and $T = t_b - t_a$.

- by Wick rotating ($t \to \tau = it$) one gets

$$K_{\mathrm{E}}(a,b;T_{\mathrm{E}}) = \langle x = a | e^{-HT_{\mathrm{E}}/\hbar} | x = b \rangle = \int \mathcal{D}[x(\tau)] e^{-S_{\mathrm{E}}[x(\tau)]/\hbar}$$

with $S_{\mathrm{E}}[x(\tau)] = \int_{\tau_a}^{\tau_b} d\tau \left(\frac{1}{2} m \left(dx/d\tau \right)^2 + V(x) \right)$

- Saddle point(s) will contribute to the leading term of the path integral.
- An instanton is a solution to the Euclidean equations of motion with finite, non-zero action.

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Instantons in Euclidean quantum gravity

- The quantum state of the Universe is [Gibbons & Hawking, 1973]

$$\Psi = \int D\left[g_{ab}\right] \; e^{-I_{\mathrm{E}}[g]}$$

where g_{ab} are positive defined metrics and

$$\begin{aligned} I_{\mathrm{E}}[g] \quad &\sim \quad -\int d^4x\sqrt{g}\left[\frac{1}{16\pi G}\left(R-2\Lambda\right)+L_m\right] \\ &+ \quad (\text{gravitational boundary terms}) \end{aligned}$$

- Semiclassically we have $I_{\mathrm{E}}[g] \approx I$

$$P = |\Psi|^2 \sim \exp\left(-2I\right).$$

Spacetime nucleation

- Two instantons:
  1. **background** $I_{\rm bg}$
  2. **object** nucleated $I_{\rm obj}$ on the background
- the object/background pair nucleation rate is

$$\Gamma = \frac{P_{\rm obj}}{P_{\rm bg}} \sim \frac{\exp\left(-2 I_{\rm obj}\right)}{\exp\left(-2 I_{\rm bg}\right)}$$

- the background spacetime

$$ds_{\rm E}^2 = V(r)\, d\tau^2 + V(r)^{-1}\, dr^2 + r^2\, d\Omega^2$$

with $V(r) = 1 - \frac{\Lambda}{3} r^2$.

The background contribution

- We cast the action in the form

$$
\begin{aligned}
I_{\mathrm{E}} &= -\int_{\mathbb{M}_+} d^4x\sqrt{g}\left[\frac{\Lambda}{8\pi G} - \frac{\mathrm{T}}{2} + L_m\right] \\
&+ \quad (\text{gravitational boundary terms})
\end{aligned}
$$

  where $\mathrm{T} = T^{\mu}_{\mu}$. Here $\mathbb{M}_+$ is one of the parts the space like boundary surface Σ divides the (simply connected) spacetime $\mathbb{M}$ into.
- the background instanton reads

$$
I_{\mathrm{bg}} = -\int_{\mathbb{M}_+} d^4x\sqrt{g}\,\frac{\Lambda}{8\pi G} = -\frac{3}{2}\frac{\pi}{\Lambda G}
$$

- the background probability is

$$
P_{\mathrm{bg}} = e^{\frac{3\pi}{\Lambda G}}.
$$

The Schwarzschild-deSitter solution

- The spacetime reads

$$ds_{\mathrm{E}}^2 = V(r)\, d\tau^2 + V(r)^{-1}\, dr^2 + r^2\, d\Omega^2$$

with

$$V(r) = 1 - \frac{2MG}{r} - \frac{\Lambda}{3} r^2.$$

- the associated instanton (Nariai, i.e., $r_{\mathrm{h}} = r_{\mathrm{c}}$) reads

$$I_{\mathrm{SdS}} = -\frac{\pi}{\Lambda G},$$

the Schwarzschild-deSitter probability is

$$P_{\mathrm{SdS}} = e^{\frac{2\pi}{\Lambda G}}.$$

- Finally the rate reads [Bousso & Hawking, 1996]

$$\Gamma_{\mathrm{SdS}} = e^{-\frac{\pi}{\Lambda G}}$$

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Phenomenological repercussions

- For the Nariai solution

$$r_{\rm h} \sim 1/\sqrt{\Lambda}$$

$$\Rightarrow M \sim \frac{1}{G}(1/\sqrt{\Lambda})$$

- for $M \sim M_{\rm PBH}$

$$\Gamma_{\rm PBH} \approx 0$$

- for $\Gamma_{\rm SdS} \sim 1$

$$M \ll M_{\rm PBH} \Rightarrow t_{\rm ev} \ll \text{Age of Universe}$$

All the nucleated black holes have already evaporated off!

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Alternative topologies

- We recall from asymptotically flat space

$$ds = -V(r)\,dt^2 + V(r)^{-1}\,dr^2 + r^2\,d\Omega^2$$

with

$$V(r) = 1 - \frac{2MG}{r}\,X(r, \alpha_1, \ldots, \alpha_n).$$

- The α's label parameters like Q, J (in the non-static case) and effects like back reaction and other quantum corrections.
- The associated Hawking temperature reads

$$T_{\mathrm{H}} = \frac{1}{8\pi GMX(r_{\mathrm{h}}, \ldots)}\,(1 - 2MGX'(r_{\mathrm{h}}, \ldots))$$

- Horizon extremisation, $X'(r_{\mathrm{h}}, \ldots) = 1/2MG \Rightarrow$ evaporation switching off
  $\Rightarrow$ longer evaporation times
  $\Rightarrow$ remnant formation

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The Reissner-Nordström-deSitter solution

- Einstein-Maxwell system

$$
\begin{aligned}
I_{\mathrm{EHM}} &= \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\left(R-2\Lambda\right)-F_{\mu\nu}F^{\mu\nu}\right] \\
&+ \quad (\text{gravitational boundary terms})
\end{aligned}
$$

with

$$
F^{\mu\nu} = \left(\frac{Q}{r^2}\right)\,\delta^{0[\,\mu\,|}\delta^{r\,|\,\nu\,]}
$$

- the metric reads

$$
ds = -V(r)\,dt^2 + V(r)^{-1}\,dr^2 + r^2\,d\Omega^2
$$

with

$$
V(r) = 1 - \frac{2GM}{r} + \frac{GQ^2}{r^2} - \frac{1}{3}\Lambda r^2.
$$

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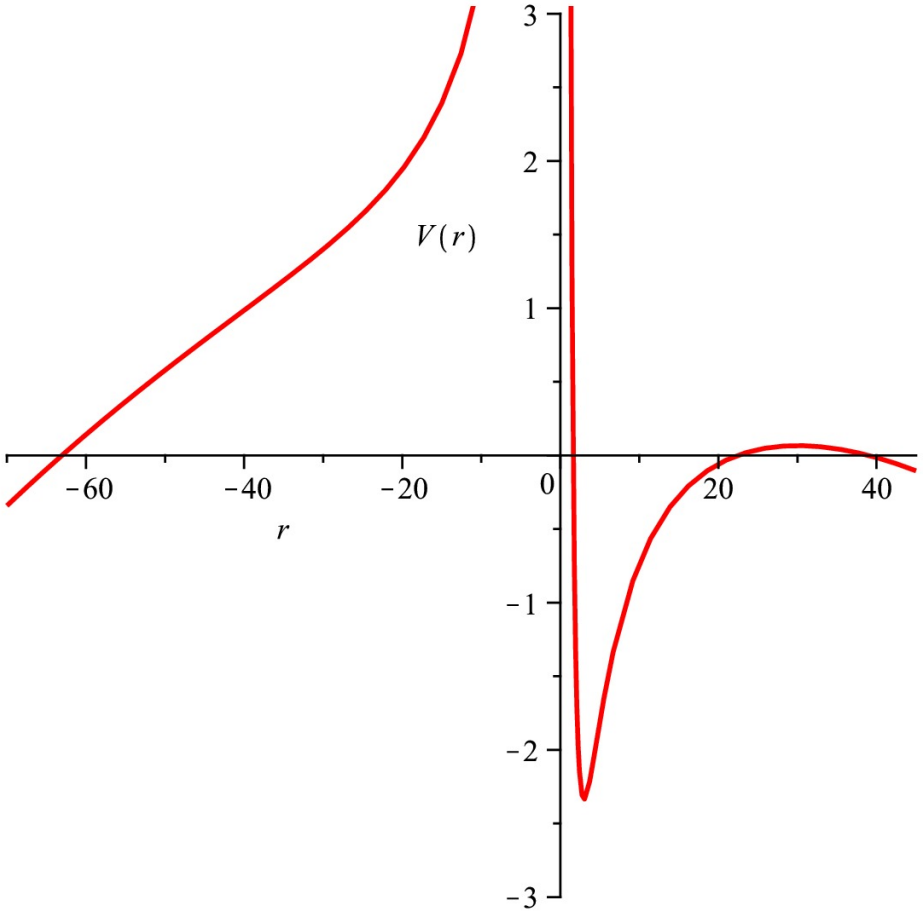
Charged istantons

- the function $V(r)$ can have four real roots.

$$r_1 < r_2 < r_3 < r_4; \quad \text{with } r_1 < 0$$

- r_2 is the inner (Cauchy) black hole horizon
- r_3 is the outer (Killing) black hole horizon
- r_4 is the cosmological horizon
- $r_2 = r_3 \neq r_4 \Rightarrow$ cold instanton I_{C}
- $r_2 \neq r_3 = r_4 \Rightarrow$ Nariai instanton I_{N}
- $r_2 \neq r_3 \neq r_4 \Rightarrow$ lukewarm inst. I_{L}
- $r_2 = r_3 = r_4 \Rightarrow$ ultracold inst.(s) I_{U}



Charged istantons

- $I_{\mathrm{L}} = -\frac{3}{2}\pi/\Lambda + \pi M\sqrt{3/\Lambda} \Rightarrow -\frac{3}{2}\pi/\Lambda \leq I_{\mathrm{L}} \leq -\frac{3}{4}\pi/\Lambda$, being $M \leq \frac{1}{4}\sqrt{3/\Lambda}$
- $-\frac{3}{2}\pi/\Lambda \leq I_{\mathrm{C}} \leq -\frac{1}{4}\pi/\Lambda$
- $-\pi/\Lambda \leq I_{\mathrm{N}} \leq -\frac{1}{2}\pi/\Lambda$
- $I_{\mathrm{U}} = -\frac{1}{4}\pi/\Lambda$

Taken from [gr-qc/9504015].

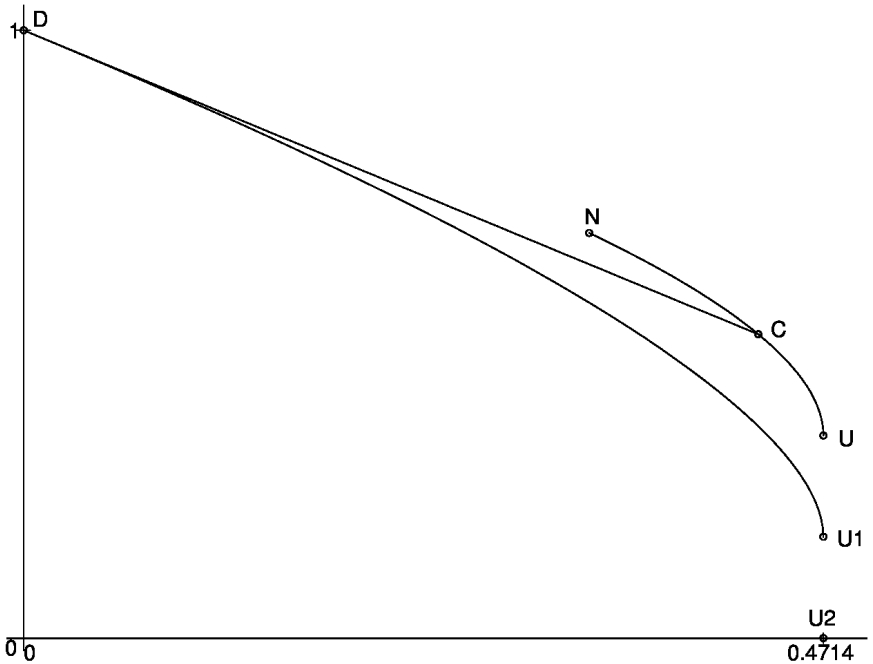


Figure 3: The action for the various instantons in the cosmological case. The action as a fraction of the action for de Sitter space, $I/I_{\text{de Sitter}}$, is plotted against the dimensionless mass $M\sqrt{\Lambda}$. The curve DU1 represents

Discussion

- Extremal black holes are less likely than non-extremal ones.
- Thermodynamics
$$Z = |\Psi|^2 = \text{density of states}$$
- From $S \equiv \ln Z$ one gets
$$S = -2I$$
and
$$\Gamma = \frac{e^{S_{BH}}}{e^{S_{dS}}}$$
- Black hole production is suppressed because de Sitter space has a higher entropy (area) than the combined area of the horizons in the instantons [Mann & Ross, 1995].

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Non-singular (neutral) black holes

- Charged remanant cannot be a reliable dark matter component.
- Quantum gravity corrected black hole spacetimes, e.g.,
  - Noncommutative geometry
  - Generalized Uncertainty Principle
  - Loop Quantum Gravity
  - Asymptotically Safe Gravity

  offer neutral extremal configuration black holes.
- Do they affect the quantum (in)stability of the deSitter spacetime?
- We consider short scale corrections to the Schwarzschild metric modelled via

$$X(r) = \frac{\gamma(3/2; r^2/4\ell^2)}{\Gamma(3/2)}$$

where

$$\gamma\left(3/2\,, r^2/4\ell^2\right) \equiv \int_0^{r^2/4\ell^2} dt\, t^{1/2} e^{-t} \xrightarrow[r \gg \ell]{} \Gamma(3/2) = \sqrt{\pi}/2$$

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The regular Schwarzschild-deSitter spacetime

- The line element

$$ds^2 = -V(r)\,dt^2 + V(r)^{-1}\,dr^2 + r^2\,d\Omega^2.$$

with

$$V(r) = 1 - \frac{4MG\gamma(3/2; r^2/4\ell^2)}{r\sqrt{\pi}} - \frac{\Lambda r^2}{3}$$

- Asymptotic behaviors

$$r \gg \ell \Rightarrow V(r) \approx 1 - \frac{2MG}{r} - \frac{\Lambda r^2}{3}.$$

$$r \sim \ell \Rightarrow V(r) \approx 1 - \frac{\Lambda_{\rm eff}}{3} r^2, \quad \Lambda_{\rm eff} = \Lambda + \frac{1}{\sqrt{\pi}} \frac{MG}{\ell^3}.$$

- no curvature singularity!

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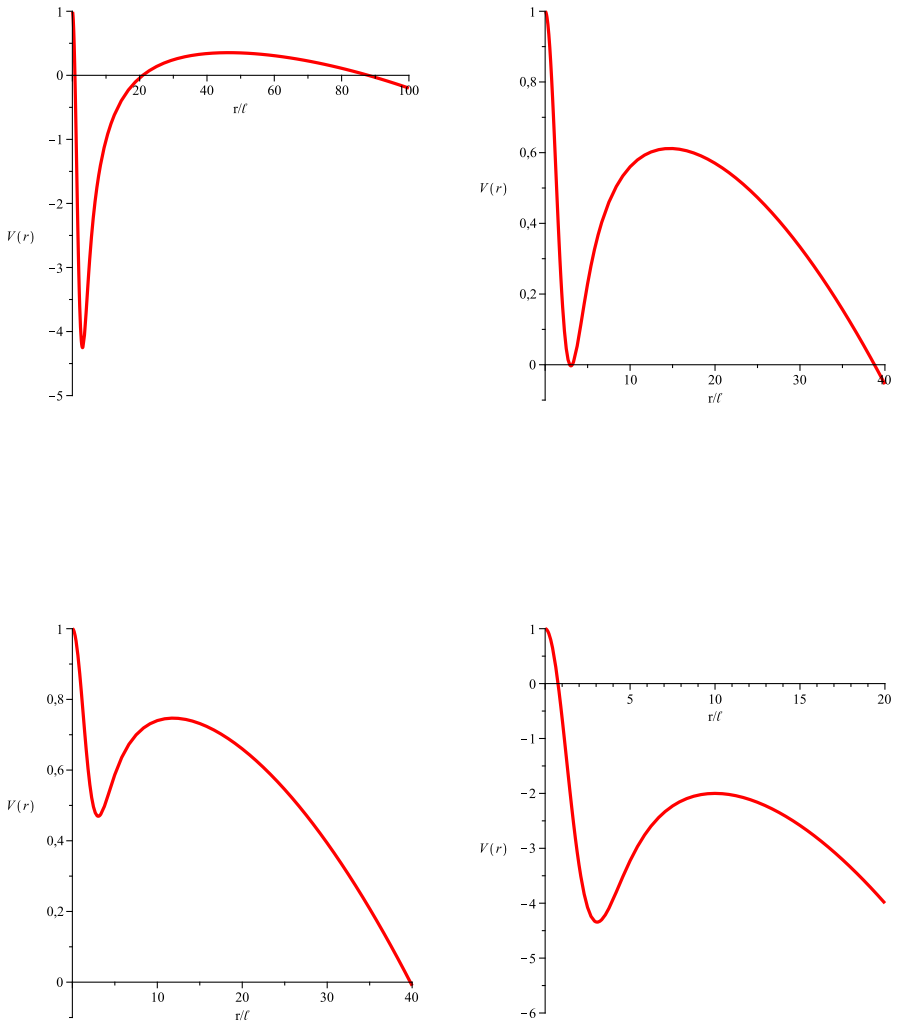
The regular Schwarzschild-deSitter spacetime

- The horizon equation

$$V(r_H) = 0.$$

- There exists a mass $M_0 = M_0(\Lambda)$ such that
  a) for $M > M_0$ there are three horizons, r_-, r_+ and r_c.
  b) for $M = M_N > M_0 \Rightarrow r_N \equiv r_+ = r_c$ (Nariai-like solution).
  c) for $M = M_0 \Rightarrow r_+ = r_-$ and r_c.
  d) $M < M_0$ there is just one (cosmological) horizon, yielding a soliton.



Gravitational Instantons

- the background gives

$$I_{\rm bg} = -\int_{\mathbb{M}_+} d^4x\sqrt{g}\,\frac{\Lambda}{8\pi G} = -\frac{3}{2}\frac{\pi}{\Lambda G} \quad \Rightarrow \quad P_{\rm bg} = e^{\frac{3\pi}{\Lambda G}}.$$

- We recall that

$$I_{\rm obj} = -\int_{\mathbb{M}_+} d^4x\sqrt{g}\left[\frac{\Lambda}{8\pi G} - \frac{\mathrm{T}}{2} + L_m\right] + (\text{grav. boundary terms})$$

- the solution is obtained by an effective stress tensor

$$T^0_0 = -\rho_\ell(r) = -\frac{M}{(4\pi\ell^2)^{3/2}}\,\exp\left(-\frac{r^2}{4\ell^2}\right).$$

$$T^{\mu\nu}\,;\,\nu = 0, \quad g_{00} = -g_{rr}^{-1}$$
$$\Rightarrow T^\mu{}_\nu = \mathrm{Diag}\left(-\rho_\ell(r),\, p_r(r),\, p_\perp(r),\, p_\perp(r)\right).$$

- One has to take into account $\mathrm{T} = T^\mu_\mu$ and calculate

$$L_m = p_r + \frac{r^2}{4\ell^2}\frac{M}{(4\pi\ell^2)^{3/2}}e^{-\frac{r^2}{4\ell^2}}$$

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Production rates

- for all istantons, rates are of the order $\Gamma \sim e^{-\frac{\pi}{\Lambda G}}$
- however for the cold Instanton

$$I_{\rm C} \approx -\frac{\pi}{\Lambda G}\left(1 + 4M_0 G\sqrt{\frac{\Lambda}{3}}\left[1 - \frac{0.67}{\sqrt{\pi}}\right]\right)$$

- the production rate is

$$\Gamma_{\rm C} \approx e^{-\frac{\pi}{\Lambda G}\left(1-8M_0 G\sqrt{\frac{\Lambda}{3}}\left[1-\frac{0.67}{\sqrt{\pi}}\right]\right)} > \Gamma_{\rm SdS}$$

- *Nota bene*: for $\Lambda G \sim 1$ there is no $I_{\rm C}$, but I_1.

$$I_1 = \frac{\beta_1 M}{\sqrt{\pi}}\left[2\gamma(3/2; r_1^2/4\ell^2) - \frac{1}{4}\frac{r_1^3}{\ell^3}e^{-r_1^2/4\ell^2}\right] - \frac{\beta_1 \Lambda}{12G}\, r_1^3.$$

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Single horizon topologies

- I_1 contributes only for $\Lambda G \sim 1$ (in such a case I_1 is equivalent to dS space)

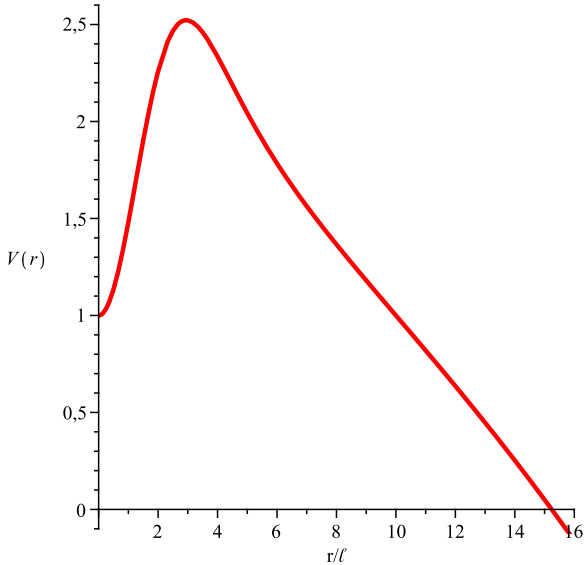
$$\Gamma_1 = e^{\frac{\beta_1 \Lambda}{6G} r_1^3} \, e^{-\frac{2\beta_1 M}{\sqrt{\pi}} \left[2\gamma(3/2; r_1^2/4\ell^2) - \frac{1}{4} \, \frac{r_1^3}{\ell^3} e^{-r_1^2/4\ell^2} \right]} \, e^{-\frac{3\pi}{\Lambda G}}$$

- regular spacetimes are geodesically complete.
  By $r \to -r$ we found another universe!
- the transformation is equivalent to a negative mass solution

$$r \to -r \quad \text{or} \quad M \to -|M|$$

$$\Rightarrow V_-(r) = 1 + \frac{4|M| G \gamma(3/2; r^2/4\ell^2)}{r\sqrt{\pi}} - \frac{\Lambda r^2}{3}.$$

- for $M < 0$ we have instability increasing with $|M|$ [Mann & PN, 2011].



Summary

- We have reviewed the occurrence of deSitter space instability due to the nucleation of
  1. Schwarzschild black holes
  2. Reissner-Nordström black holes
  3. regular Schwarzschild like black holes.
- In all cases the nucleation was relavant in inflatiorary epochs only

$$\Lambda G \sim 1$$

- Extremal (Planck size) black holes would be less likely (charged case) or even strongly disfavoured (regular case).
- Planck size black holes would not have been produced
- the case $M < 0$ leads to a potential instability and needs further investigations.
- theoretical revision + improved observations.

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tack!

References

G. W. Gibbons and S. W. Hawking (eds)
Euclidean quantum gravity.
Cambridge University Press, Cambridge, England (1973).

B. J. Carr and S. W. Hawking (1974)
Black holes in the early Universe.
Mon. Not. Roy. Astron. Soc. **168**, 399.

M. J. Rees (1977)
A better way of searching for black-hole explosions?
Nature **233**, 333.

R. D. Blandford (1977)
Spectrum of a radio pulse from an exploding black hole.
Mon. Not. Roy. Astron. Soc. **181**, 489.

R. B. Mann and S. F. Ross (1995)
Cosmological production of charged black hole pairs.
Phys. Rev. D **52**, 2254 [gr-qc/9504015].

R. Bousso and S. W. Hawking (1996)
Pair creation of black holes during inflation.
Phys. Rev. D **54**, 6312 [gr-qc/9606052].

R. B. Mann and P. Nicolini (2011)
Cosmological production of noncommutative black holes.
Phys. Rev. D **84**, 064014 [arXiv:1102.5096 [gr-qc]].

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