Branes in Yang-Mills matrix models and their physical significance

Harold Steinacker

University of Vienna

Nordita, november 8, 2012

Motivation: $\boxed{\text{Gravity, Cosmology} \leftrightarrow \text{Quantum Mechanics}}$

- fine-tuning problems (cosm. const., “landscape”, etc.)

  “why is space-time so flat?”

- expect quantum structure of space-time at $\Lambda_{\rm Planck}$

  quantum gravity

- dark matter, dark energy ...

... maybe we’re missing something?

Gravity as emergent phenomenon? fundamental d.o.f ?

hydrodynamics,
solid state physics,
hadronic matter

} allows quantization

challenges for (emergent) gravity:

- metric is *universal*
- Lorentz invariance (Weinberg-Witten theorem)
  ($\rightarrow$ emergent space-time)
- is there a simple, more fundamental model?
  does it represent progress? can we quantize it?
- action principle (E-H action) subject to strong quantum corrections, fine-tuning $\rightarrow$ can we avoid that?
  possible alternative to Einstein-Hilbert action ?

Gravity as emergent phenomenon? fundamental d.o.f ?

hydrodynamics,
solid state physics,
hadronic matter

} allows quantization

challenges for (emergent) gravity:

- metric is *universal*
- Lorentz invariance (Weinberg-Witten theorem)
  ($\rightarrow$ emergent space-time)
- is there a simple, more fundamental model?
  does it represent progress? can we quantize it?
- action principle (E-H action) subject to strong quantum corrections, fine-tuning $\rightarrow$ can we avoid that?
  possible alternative to Einstein-Hilbert action ?

promising approach:

Yang-Mills Matrix Models: IKKT / IIB model

- stringy features $\leftrightarrow$ non-commutative (NC) gauge theory
- pre-geometric theory of gravity & fund. interactions (?)
- dynamical quantum structure of space-time, NC geometry
  needs to be *unraveled, not invented*
- relation gravity $\leftrightarrow$ NC gauge theory ,
  quantization a la Yang-Mills ($\mathcal{N} = 4$ SYM)

Outline:

- I) flat branes in matrix models
  - overview, relation with string theory
  - the role of NC,
    fluctuations and gauge theory
  - quantization
  - towards particle physics
- II) curved branes in matrix models
  - quantized Poisson manifolds, symplectic $\leftrightarrow$ Riemannian structure
  - mechanism for gravity on branes

... brane-world picture

review: H.S., CQG 27 (2010)
H.S, arXiv:1210.8364

IKKT (IIB) matrix model

Ishibashi, Kawai, Kitazawa, Tsuchiya 1996

$$S[X] = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]\eta_{aa'}\eta_{bb'} \; + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N, \mathbb{C}), \qquad a = 0, ..., 9$$

$$N \to \infty$$

gauge symmetry $X^a \to UX^aU^{-1}$, $SO(9,1)$, SUSY

$$\begin{cases} \text{1) nonpert. def. of IIB string theory (on } \mathbb{R}^{10}) \qquad (IKKT) \\ \text{2) } \mathcal{N}=4 \text{ SUSY Yang-Mills gauge thy. on "}\textit{noncommutative}\text{" } \mathbb{R}^4_\theta \end{cases}$$

$\to$ synthesis $\underbrace{\text{NC ideas} + \text{string theory}}$

not just toy model: real physics ? (brane-world scenarios)

dynamical NC branes $\mathcal{M} \subset \mathbb{R}^{10}$ ($\to$ 4D gravity H.S. 2007 ff)

Space-time from matrix models:

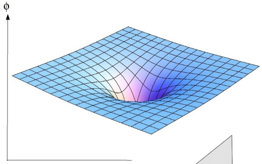
e.o.m.: $\delta S = 0 \;\Rightarrow\; [X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

solutions:

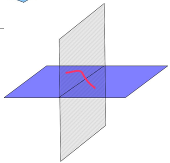
- $[X^a, X^b] = i\theta^{ab}\,\mathbf{1}$, "quantum plane" $\mathbb{R}^4_\theta$
- $[X^a, X^b] \sim i\{x^a, x^b\} = i\theta^{ab}(x)$, generic quantum space

  $\rightarrow$ space-time as 3+1-dim. brane solution

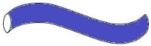
  $X^a \sim x^a : \;\; \mathcal{M}^4 \hookrightarrow \mathbb{R}^{10}$



- intersecting branes, stacks (as in string theory)

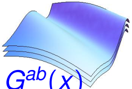


- compact extra dim $\mathcal{M}^4 \times T^2$, etc.



main result I:

- there is a universal effective metric $G^{ab}(x)$ on such branes, is dynamical
- *fluctuations* of matrices X^A around stack of branes



  $\to$ $SU(n)$ NC Yang-Mills gauge theory coupled to $G^{ab}(x)$
- fermionic matter described by Ψ also couples to $G^{ab}(x)$
- all ingredients for physics ($\to$ brane-world picture)

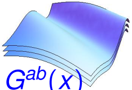
tentative result / conjecture II:

- compactified branes $\mathcal{M}^4 \times \mathcal{K} \subset \mathbb{R}^{10}$

  $\to$ mechanism for emergent gravity, Ricci tensor $\leftrightarrow$ e-m tensor
  extrinsic curvature, moduli of $\mathcal{K} \subset \mathbb{R}^6$ mediate gravity
- might resolve problems with quantization, dispense of "landscape"

main result I:

- there is a universal effective metric $G^{ab}(x)$ on such branes, is dynamical
- *fluctuations* of matrices X^A around stack of branes



  $\rightarrow$ $SU(n)$ NC Yang-Mills gauge theory coupled to $G^{ab}(x)$
- fermionic matter described by Ψ also couples to $G^{ab}(x)$
- all ingredients for physics ($\rightarrow$ brane-world picture)

tentative result / conjecture II:

- compactified branes $\mathcal{M}^4 \times \mathcal{K} \subset \mathbb{R}^{10}$

  $\rightarrow$ mechanism for emergent gravity, Ricci tensor $\leftrightarrow$ e-m tensor
  extrinsic curvature, moduli of $\mathcal{K} \subset \mathbb{R}^6$ mediate gravity
- might resolve problems with quantization, dispense of "landscape"

obvious solution of $[X_a,[X^a,X^b]]=0$: $X^a = \begin{pmatrix} \bar X^\mu \\ \bar X^i \equiv 0 \end{pmatrix}$

$$[\bar X^\mu, \bar X^\nu] = i\bar\theta^{\mu\nu}\,\mathbb{1}, \qquad \mu,\nu = 0,...,3$$

... Heisenberg algebra $\mathcal{A} = Mat(\infty,\mathbb{C}) =$ functions on $(\mathbb{R}^4_\theta, \theta^{\mu\nu})$

$\bar X^\mu \in Mat(\infty,\mathbb{C})$ ... coordinate functions on Moyal-Weyl plane $\mathbb{R}^4_\theta$

$$\Delta\bar X^\mu \Delta\bar X^\nu \geq |\bar\theta^{\mu\nu}|$$

$f(\bar X^\mu) \in \mathcal{A}$ ... quantized function on $\mathbb{R}^4_\theta$

quantization map (Weyl):

$$\mathcal{I}: \mathcal{C}(\mathbb{R}^4) \to Mat(\infty,\mathbb{C}) = \mathcal{A}$$

$$f(x) = \int d^4k\, \tilde f(k) e^{ik_\mu x^\mu} \mapsto \int d^4k\, \tilde f(k) e^{ik_\mu \bar X^\mu} =: F(\bar X)$$

star product:

$$f(x) \star g(x) = \mathcal{I}^{-1}(\mathcal{I}(f)\mathcal{I}(g)) \qquad (= f(x)\, e^{\frac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} g(x)\ )$$

obvious solution of $[X_a, [X^a, X^b]] = 0$: $X^a = \begin{pmatrix} \bar{X}^\mu \\ \bar{X}^i \equiv 0 \end{pmatrix}$

$$[\bar{X}^\mu, \bar{X}^\nu] = i\bar{\theta}^{\mu\nu}\, \mathbb{1}, \qquad \mu,\nu = 0,...,3$$

... Heisenberg algebra $\mathcal{A} = Mat(\infty, \mathbb{C}) =$ functions on $(\mathbb{R}^4_\theta, \theta^{\mu\nu})$

$\bar{X}^\mu \in Mat(\infty, \mathbb{C})$ ... coordinate functions on Moyal-Weyl plane $\mathbb{R}^4_\theta$

$$\Delta\bar{X}^\mu \Delta\bar{X}^\nu \geq |\bar{\theta}^{\mu\nu}|$$

$f(\bar{X}^\mu) \in \mathcal{A}$ ... quantized function on $\mathbb{R}^4_\theta$

quantization map (Weyl):

$$\mathcal{I} : \mathcal{C}(\mathbb{R}^4) \to Mat(\infty, \mathbb{C}) = \mathcal{A}$$

$$f(x) = \int d^4k\, \tilde{f}(k) e^{ik_\mu x^\mu} \mapsto \int d^4k\, \tilde{f}(k) e^{ik_\mu \bar{X}^\mu} =: F(\bar{X})$$

star product:

$$f(x) \star g(x) = \mathcal{I}^{-1}(\mathcal{I}(f)\mathcal{I}(g)) \qquad \left(= f(x)\, e^{\frac{i}{2}\overleftarrow{\partial}_\mu \theta^{\mu\nu} \overrightarrow{\partial}_\nu} g(x) \right)$$

Interpretation of matrices in M.M:

background in M.M.: 10 matrices $X^a \in Mat(\infty, \mathbb{C})$

define $\begin{cases} \text{algebra} & \mathcal{A} \cong \langle f(\bar{X}^\mu)\rangle \cong Mat(\infty, \mathbb{C}) \\ \text{quantized embedding} & X^a \sim x^a : \mathcal{M} \hookrightarrow \mathbb{R}^{10} \end{cases}$

carries info on the geometry, i.e. embedding

($\to$ Dirac-op $\Gamma_a[X^a, .]$, , Laplace op $[X_a, [X^a, .]]$)

much more info that abstract algebra $\mathcal{A}$

---

def. derivatives on $\mathbb{R}^4_\theta$

$$\partial_\mu f(X^\nu) := -i\theta^{-1}_{\mu\nu}[X^\nu, f(X)] \sim \partial_\mu f(x)$$

transversal deformations: scalar fields

$$X^a = \bar X^a + A^a = \begin{pmatrix} \bar X^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \phi^i(\bar X^\mu) \end{pmatrix}$$

behaves as <u>scalar field</u> $\phi(\bar X)$ on $\mathbb{R}^4_\theta$

plug into M.M. action, use $[\bar X^\mu, \phi] = i\theta^{\mu\nu}\,\partial_\nu \phi$

$$\begin{aligned}
S[X^a] &= Tr\left(\eta_{\mu\nu}\theta^{\mu\mu'}\theta^{\nu\nu'}\eta_{\mu'\nu'} + 2\eta_{\mu\nu}[\bar X^\mu, \phi^i][\bar X^\nu, \phi^j] + [\phi^i, \phi^j][\phi^i, \phi^j]\right) \\
&= \int d^4x \sqrt{|\theta^{-1}_{\mu\nu}|}\left(\mathrm{const} + 2\eta_{\mu\nu}\theta^{\mu\mu'}\theta^{\nu\nu'}\partial_{\mu'}\phi\,\partial_{\nu'}\phi + [\phi^i, \phi^j][\phi^i, \phi^j]\right) \\
&\sim \int d^4x \sqrt{|G_{\mu\nu}|}\left(\mathrm{const} + 2G^{\mu\nu}\,\partial_\mu\phi\partial_\nu\phi + e^\sigma\,[\phi^i, \phi^j][\phi^i, \phi^j]\right) \\
G^{\mu\nu} &= e^{-\sigma}\,\theta^{\mu\mu'}\theta^{\nu\nu'}\,\eta_{\mu'\nu'}, \qquad e^\sigma = \sqrt{|\theta^{\mu\nu}|}
\end{aligned}$$

transversal fluctuations $\rightarrow$ scalar fields on $\mathbb{R}^4_\theta$, eff. metric $G^{\mu\nu}$

tangential deformations: gauge fields

$$X^a = \bar{X}^a + A^a = \begin{pmatrix} \bar{X}^\mu \\ 0 \end{pmatrix} + \begin{pmatrix} A^\mu(\bar{X}^\mu) \\ 0 \end{pmatrix}$$

$S = Tr([X^a, X^b][X_a, X_b])$ is gauge-invariant: $X^a \to U^{-1} X^a U$

$\to$ fluctuations $X^\mu = \bar{X}^\mu + \theta^{\mu\nu} A_\nu$ transform as
$A_\mu \to U^{-1} A_\mu U + i U^{-1} \partial_\mu U$ gauge fields!

$$\begin{aligned} [X^\mu, X^\nu] &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} (\partial_{\mu'} A_{\nu'} - \partial_{\nu'} A_{\mu'} + [A_{\mu'}, A_{\nu'}]) \\ &= i\theta^{\mu\nu} + i\theta^{\mu\mu'}\theta^{\nu\nu'} F_{\mu'\nu'} \qquad \text{field strength} \end{aligned}$$

$\Rightarrow$ eff. action $\boxed{S = \mathrm{const} + \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'} F_{\mu\nu} F_{\mu'\nu'}}$

tangential perturbations $\to$ gauge fields on $\mathbb{R}^4_\theta$, eff. metric $G^{\mu\nu}$

fermions

Ψ ... $\mathcal{A}$ - valued M-W spinor of $SO(9,1)$
action

$$\begin{aligned} S[\Psi] &= \mathrm{Tr}\, \overline{\Psi}\Gamma_a[X^a,\Psi] \equiv \mathrm{Tr}\, \overline{\Psi}\not{D}\Psi \\ &\sim \int d^4x\, \sqrt{\theta^{-1}}\, \overline{\Psi} i\gamma^\mu(\partial_\mu + [A_\mu,.])\Psi, \end{aligned}$$

$$\gamma^\mu = \Gamma_a \theta^{\nu\mu}\partial_\nu x^a$$

note

$$\begin{aligned} \{\gamma^\mu,\gamma^\nu\} &= \{\Gamma_a,\Gamma_b\}\theta^{\mu'\mu}\partial_{\mu'}x^a\theta^{\nu'\nu}\partial_{\nu'}x^b \\ &= 2\theta^{\mu'\mu}\theta^{\nu'\nu}\eta_{\mu'\nu'} \\ &\sim 2\, G^{\mu\nu} \end{aligned}$$

Ψ decomposes into 4 Weyl fermions on $\mathbb{R}^4_\theta$,
usual dim. red of $D=10$ SYM to 4D

IKKT model with $D=10 \;\rightarrow\; \mathcal{N}=4$ SYM on $\mathbb{R}^4_\theta$

Nonabelian gauge theory

consider a stack of coincident brane solutions

$$\bar{X}^a \otimes \mathbf{1}_n = \begin{pmatrix} \bar{X}^a & 0 & \dots & 0 \\ 0 & \bar{X}^a & 0 \dots & 0 \\ & & \ddots & \\ 0 & \dots & 0 & \bar{X}^a \end{pmatrix}$$

add fluctuations

$$X^a = \bar{X}^a \otimes \mathbf{1}_n + A^a_\alpha(\bar{X})\,\lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{u}(n)$$

... $\mathfrak{u}(n)$-valued gauge fields and scalars on $\mathbb{R}^4_\theta$

IKKT model on stack of $\mathbb{R}^4_\theta \;\to\; U(n)\;\; \mathcal{N} = 4$ SYM on $\mathbb{R}^4_\theta$

relation with string theory:

- model has 10D Poincare symmetry
  $\rightarrow$ embedding $\mathbb{R}^4 \subset \mathbb{R}^{9,1}$ arbitrary
- same for all even-dimensional branes $\mathbb{R}^{2n} \subset \mathbb{R}^{9,1}$

  $\rightarrow$ recover precisely *D*-branes in IIB supergravity / string theory,
  with B - field $\leftrightarrow \theta^{-1}_{\mu\nu}$
  eff. metric $G^{\mu\nu} \leftrightarrow$ open string metric on branes
  induced metric $g_{\mu\nu} = \partial_\mu x^a \partial_\nu x^b \leftrightarrow$ closed string metric in bulk
- IKKT model has same SUSY as IIB sugra
- one-loop effective action $\Rightarrow$ interactions between D-branes consistent with IIB sugra

IKKT model = candidate for nonperturb. description of IIB superstring

... *model for branes*

Quantization

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]}$$

expand around (Moyal-Weyl) brane background $\mathbb{R}^4_\theta$

IKKT model = NC $\mathcal{N}=4$ SYM , perturb. (UV) finite !

puzzles:

- $U(1)$ sector does not decouple from $SU(n)$ due to NC gravity !
- quantization $\rightarrow$ "strange" new IR divergences in $U(1)$ sector "UV/IR mixing"
  induced E-H terms (Grosse, H.S., Wohlgenannt JHEP 0804 (2008) 023)
- translations, symplectomorphisms are gauge transformations

$$[\Lambda(X), .] \sim i\{\Lambda(x), .\}$$

$\rightarrow$ no local observables! cf. gravity

- curved brane solutions

understood by interpreting trace-$U(1)$ sector as geometry

(H.S., JHEP 0712:049 (2007))

Quantization

$$Z = \int dX^a d\Psi\, e^{-S[X]-S[\Psi]}$$

expand around (Moyal-Weyl) brane background $\mathbb{R}^4_\theta$

IKKT model = NC $\mathcal{N}=4$ SYM , perturb. (UV) finite !
puzzles:

- $U(1)$ sector does not decouple from $SU(n)$ due to NC gravity !
- quantization $\rightarrow$ "strange" new IR divergences in $U(1)$ sector "UV/IR mixing"
  induced E-H terms (Grosse, H.S., Wohlgenannt JHEP 0804 (2008) 023)
- translations, symplectomorphisms are gauge transformations

$$[\Lambda(X), .] \sim i\{\Lambda(x), .\}$$

$\rightarrow$ no local observables! cf. gravity

- curved brane solutions

understood by interpreting trace-$U(1)$ sector as geometry
(H.S., JHEP 0712:049 (2007))

Curved backgrounds

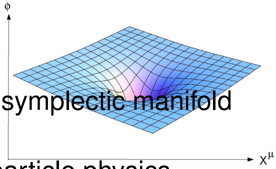
10 matrices = quantized embedding maps

$$X^a \sim x^a : \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$$

- "irreducible brane":

  deformation of Moyal-Weyl quantum plane $\mathbb{R}^{2n} \subset \mathbb{R}^{10}$

$$X^a = \begin{pmatrix} \bar{X}^\mu + A^\mu(\bar{X}) \\ \phi^i(\bar{X}) \end{pmatrix} \sim x^a(x^\mu) : \quad \mathcal{M}^{2n} \hookrightarrow \mathbb{R}^{10}$$



  quantization of algebra of functions on symplectic manifold

- multiple branes, intersections etc. $\rightarrow$ particle physics

  compactified branes $\mathcal{M}^4 \times \mathcal{K} \subset \mathbb{R}^{10}$

Noncommutative spaces and Poisson structure

$(\mathcal{M}, \theta^{\mu\nu}(x))$ ... $2n$-dimensional manifold with Poisson structure

Its quantization is NC algebra $\mathcal{A}$ such that

$$\begin{array}{rcl} \mathcal{I}: \ \mathcal{C}(\mathcal{M}) & \to & \mathcal{A} \subset \mathcal{L}(\mathcal{H}) \\ f(x) & \mapsto & \hat{f}(X) \qquad (\text{e.g. } x^\mu \mapsto X^\mu, \qquad e^{ikx} \mapsto e^{ikX}\ ) \end{array}$$

such that

$$\begin{aligned} \hat{f}\,\hat{g} &= \mathcal{I}(fg) + O(\theta) \\ [\hat{f}, \hat{g}] &= \mathcal{I}(i\{f,g\}) + O(\theta^2) \end{aligned}$$

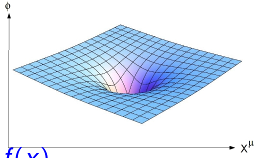
("nice") $\Phi \in Mat(\infty, \mathbb{C}) \quad \leftrightarrow \quad$ quantized function on $\mathcal{M}$

furthermore:

$$\begin{aligned} (2\pi)^n \mathrm{Tr}\,\mathcal{I}(\phi) \quad &\sim \quad \int \frac{\omega^n}{n!}\phi = \int d^{2n}x\, \rho(x)\, \phi(x) \\ \rho(x) \quad &= \quad \mathrm{Pfaff}\,(\theta^{-1}_{\mu\nu}) \ \ldots \quad \text{symplectic volume} \end{aligned}$$

in particular: $\dim \mathcal{H} \ \sim \ \mathrm{Vol}(\mathcal{M})$ large !

extracting the geometry of matrices: $X^a \sim x^a: \quad \mathcal{M} \hookrightarrow \mathbb{R}^{10}$



Lemma:

$$\Box_G f(X) := [X_a, [X^a, f(X)]] \sim -e^{\sigma}\Box_G f(x)$$

... Matrix Laplace- operator, effective metric

$$\boxed{\begin{aligned} G^{\mu\nu}(x) &= e^{-\sigma}\theta^{\mu\mu'}(x)\theta^{\nu\nu'}(x)\, g_{\mu'\nu'}(x) && \text{effective metric (cf. open string m.)} \\ g_{\mu\nu}(x) &= \partial_\mu x^a \partial_\nu x^b \eta_{ab} && \text{induced metric on } \mathcal{M}^4_\theta \text{ (cf. closed string m.)} \end{aligned}}$$

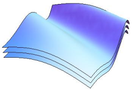
$$e^{-2\sigma} = \frac{|\theta^{-1}_{\mu\nu}|}{|g_{\mu\nu}|}$$

e.g. action for nonabelian scalar fields φ in M.M.:

$$\begin{aligned} S[\varphi] &= Tr\,[X^a,\varphi][X^b,\varphi]\,\eta_{ab} \\ &\sim \int d^4x\,\sqrt{|\theta^{-1}_{\mu\nu}|}\,\theta^{\mu'\mu}\partial_{\mu'}x^a\partial_\mu\varphi\,\theta^{\nu'\nu}\partial_{\nu'}x^b\partial_\nu\varphi\,\eta_{ab} \\ &\sim \int d^4x\,\sqrt{|G_{\mu\nu}|}\,G^{\mu\nu}(x)\,\partial_\mu\varphi\partial_\nu\varphi \end{aligned}$$

tangential fluctuation $\rightarrow$ $su(n)$ gauge fields   background

$$Y^a = \begin{pmatrix} Y^\mu \\ Y^i \end{pmatrix} = \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n \end{pmatrix}$$



include fluctuations:

$$Y^a = (1 + \mathcal{A}^\rho \partial_\rho) \begin{pmatrix} X^\mu \otimes \mathbf{1}_n \\ \phi^i \otimes \mathbf{1}_n + \Phi^i \end{pmatrix}$$

where

$$\begin{aligned} \mathcal{A}^\mu &= -\theta^{\mu\nu} A_{\nu,\alpha} \otimes \lambda^\alpha, \qquad \lambda^\alpha \in \mathfrak{su}(n) \\ \Phi^i &= \Phi^i_\alpha \otimes \lambda^\alpha \end{aligned}$$

$\Rightarrow$ effective action:

$$\boxed{S_{YM} = \int d^4x \sqrt{G}\, e^\sigma\, G^{\mu\mu'} G^{\nu\nu'}\, tr\, F_{\mu\nu}\, F_{\mu'\nu'} + 2 \int \eta(x)\, tr\, F \wedge F}$$

(H.S., JHEP 0712:049 (2007), JHEP 0902:044,(2009))

... $\mathfrak{su}(n)$ Yang-Mills coupled to metric $G^{\mu\nu}(x)$

fermions

$$S[\Psi] = \mathrm{Tr}\,\overline{\Psi} \not{D} \Psi = \mathrm{Tr}\,\overline{\Psi}\Gamma_a[X^a, \Psi]$$
$$\sim \int d^4x\, \rho(x)\, \overline{\Psi} i\gamma^\mu(x)\partial_\mu \Psi,$$

$$\gamma^\mu(x) = \Gamma_a \theta^{\nu\mu}\partial_\nu x^a, \qquad \{\gamma^\mu, \gamma^\nu\} = 2G^{\mu\nu}(x)$$

naturally SUSY (IKKT model)

couple to $G_{\mu\nu}$, but non-standard spin connection (submanifold!)

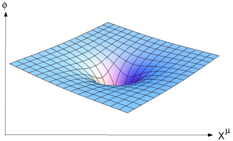
result:

- trace-$U(1)$ sector defines geometry $\mathcal{M}^{2n} \subset \mathbb{R}^{10}$
- $SU(n)$ fluctuations of matrices X^a, Ψ
  $\to$ gauge fields, scalar fields, fermions on $\mathcal{M}^{2n}$ (NOT 10 dim!)

all fields couple to metric $G^{\mu\nu}(x)$
determined by $\theta^{\mu\nu}(x)$, embedding
dynamical $\Rightarrow$ ("emergent") gravity

matrix e.o.m $[X^a,[X^{a'},X^b]]\eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\boxed{\begin{aligned} \Box_G x^a &= 0, \qquad \text{“minimal surface”} \\ \nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu\eta \\ & \qquad\qquad \eta \sim G^{\mu\nu}g_{\mu\nu} \end{aligned}}$$



covariant formulation in semi-classical limit (H.S. Nucl.Phys. B810 (2009)) dynamics of NC structure $\theta^{\mu\nu}$:

$$S_{YM} = -Tr[X^a, X^b][X^a, X^b] \sim \int d^4x \sqrt{g}\, G^{\mu\nu} g_{\mu\nu}$$

Euclidean case: can show

$$\begin{aligned} \tfrac{1}{4} G^{\mu\nu} g_{\mu\nu} &\geq 1 \\ \star\theta^{-1} &= \pm\theta^{-1} \Leftrightarrow G_{\mu\nu} = g_{\mu\nu} \Leftrightarrow S_{YM} \text{ minimal} \end{aligned}$$

minimum of S_{YM} $\Leftrightarrow$ $\theta^{\mu\nu}$ (A)SD $\Leftrightarrow$ $G_{\mu\nu} = g_{\mu\nu}$, almost-Kähler

Then

$$\begin{aligned} g_{\mu\nu} &= G_{\mu\nu}, \\ \nabla^\mu \theta^{-1}_{\mu\nu} &= 0 \end{aligned}$$

$$S_{MM} \sim Tr[X^a, X^b][X^{a'}, X^{\beta'}] = \int d^4x \sqrt{|g|}$$

... same structure as vacuum energy, “brane tension”.

NC gauge $\leftrightarrow$ gravity relation:

2 alternative, equiv. views of IKKT model:

1. as NC gauge theory on $\mathbb{R}^{2n}_\theta$

$$X^a = \begin{pmatrix} \bar{X}^\mu + \theta^{\mu\nu} A_\nu \\ \Phi^i \end{pmatrix}, \quad \mu = 1, ..., 2n$$

$\rightarrow$ $\mathcal{N} = 4\ U(N)$ SYM on $\mathbb{R}^4_\theta$

however: $U(1)$ sector does not decouple from $SU(n)$ sector !

2. "would-be $U(1)$ sector" absorbed in $\theta^{\mu\nu}(x),\ g_{\mu\nu}(x)$

$\rightarrow$ gravity on curved branes

$$X^a \sim x^a: \quad \mathcal{M}^{2n} \hookrightarrow \mathbb{R}^{10}$$

$\rightarrow$ $\mathcal{N} = 4\ SU(N)$ (S)YM coupled to gravity on $\mathcal{M}^4$

2 interpretations for quantization:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]}$$

1. on $\mathbb{R}^4_\theta$: $X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu$, $\bar{X}^\mu$...Moyal-Weyl

   $\rightarrow$ NC gauge theory on $\mathbb{R}^4_\theta$, UV/IR mixing in $U(1)$ sector

   IKKT model: $\mathcal{N} = 4$ SYM, perturb. finite !(?)

2. on $\mathcal{M}^4 \subset \mathbb{R}^{10}$: $U(1)$ absorbed in $\theta^{\mu\nu}(x)$, $g_{\mu\nu}$

   $\rightarrow$ quantized gravity, induced E-H. action

$$S_{eff} \sim \int d^4x \sqrt{|G|} \, (\Lambda^4 + c\Lambda_4^2 \, R[G] + ...)$$

- explanation for UV/IR mixing & $U(1)$ entanglement
- good quantization for theory with gravity! (maximal SUSY)

2 interpretations for quantization:

$$Z = \int dX^a d\Psi \, e^{-S[X]-S[\Psi]}$$

1. on $\mathbb{R}^4_\theta$: $X^\mu = \bar{X}^\mu + \bar{\theta}^{\mu\nu} A_\nu,$ $\bar{X}^\mu$...Moyal-Weyl

   $\to$ NC gauge theory on $\mathbb{R}^4_\theta$, UV/IR mixing in $U(1)$ sector

   IKKT model: $\mathcal{N} = 4$ SYM, perturb. finite !(?)
2. on $\mathcal{M}^4 \subset \mathbb{R}^{10}$: $U(1)$ absorbed in $\theta^{\mu\nu}(x)$, $g_{\mu\nu}$

   $\to$ quantized gravity, induced E-H. action

$$S_{eff} \sim \int d^4x \sqrt{|G|} \, (\Lambda^4 + c\Lambda_4^2 \, R[G] + \ldots)$$

- explanation for UV/IR mixing & $U(1)$ entanglement
- good quantization for theory with gravity! (maximal SUSY)

can be put on computer (Monte Carlo; Lorentzian) !

measure effective dimensions Kim, Nishimura, Tsuchiya PRL 108 (2012) :

result:

3 out of 9 spatial directions start to expand at some 'critical time',

3+1 dims at late times

(no anthropics !!!)

(1-loop) effective action of IKKT, finiteness

background field method $X^a \to X^a + Y^a$:

$$
\begin{aligned}
\Gamma_{1-\text{loop}} &= \tfrac{1}{2}\mathrm{Tr}\Big(\log(\mathbb{1} + \Sigma^{(10)}_{ab}\Box^{-1}[\Theta^{ab}, .]) - \tfrac{1}{2}\Big(\log(\mathbb{1} + \Sigma^{(16)}_{ab}\Box^{-1}[\Theta^{ab}, .]\Big)\Big) \\
&= O(\mathrm{Tr}(\Sigma_{ab}\Box^{-1})^4), \qquad \text{due to } \mathcal{N} = 4 \\
\Box &= [X^a, [X^a, .]] \\
\Theta^{rs} &= [X^r, X^s], \qquad \Sigma_{rs} ... SO(9,1) \text{ generator}
\end{aligned}
$$

fully $SO(9,1)$ covariant (IKKT, Chepelev & Tseytlin)

background $\mathbb{R}^4_\theta$: $\equiv \mathcal{N} = 4$ SYM on $\mathbb{R}^4_\theta$, no UV div.

$SO(9,1)$ invariant formalism, broken spontaneously through $\mathbb{R}^4_\theta$

powerful tool for gauge theory (Blaschke, H.S., JHEP 1110 (2011))

towards (emergent) gravity

- need brane gravity, not bulk gravity
  very different due to $\theta^{\mu\nu}$, automatically 4D
  complicated dynamics, not well understood
- Minkowski signature:
  G, g have different causality structures
- key: (work in progress)
  consider compactification $M^4 \times \mathcal{K} \subset \mathbb{R}^{10}$
  massless moduli of $\mathcal{K} \quad \xrightarrow{\theta^{\mu\nu}} \quad$ gravitational modes
  coupling to matter $\mathcal{K} \quad \xrightarrow{\theta^{\mu\nu}} \quad$ (Newtonian) gravity, Ricci-flat (lin.)
  No E-H action needed! robust
- NC $U(1)$ gauge fields $\partial^\mu F_{\mu\nu} = 0 \quad \Rightarrow \quad R_{\mu\nu}[\bar{G} + h] = 0$

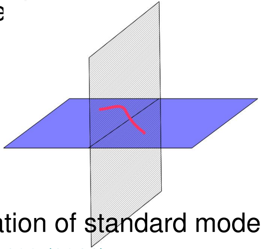
Rivelles 2002; cf. Yang 2006 ff

relation with particle physics

intersecting brane solutions

chiral fermions at intersection = 4D space
(as in string theory)

$$\begin{pmatrix} X^a_{(11)} & \psi_{(12)} \\ \psi_{(21)} & X^a_{(22)} \end{pmatrix}$$



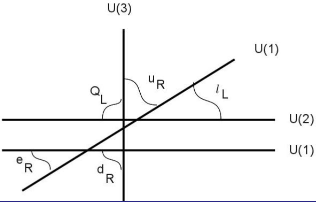
stacks of intersecting branes $\rightarrow$ realization of standard model

A. Chatzistavrakidis, H.S., G. Zoupanos JHEP 1109 (2011)

cf. H. Grosse, F. Lizzi, H.S. Phys.Rev. D81 (2010)

cf. string theory

clear-cut, predictive framework



1-loop $\rightarrow$ intersecting branes can form bound system!

realization of standard model

4 intersecting D7 branes $\rightarrow$ $U(3)_C \times U(2)_L \times U(1) \times U(1)$

intersections $M^4 \times K^2_{ab}$, flux on K^2_{ab}, chiral bifund. fermions

Intersection	Representation	Particle	flux
$D_a \cap D_b$	$(\bar{3}, 2)(-1, 1, 0, 0)$	Q_L	$N'_\beta - N_\beta$
$D_a \cap D_c$	$(\bar{3}, 1)(-1, 0, 1, 0)$	d_R	$N''_\beta - N_\beta$
$D_a \cap D_d$	$(\bar{3}, 1)(-1, 0, 0, 1)$	u_R	$N'_\alpha - N_\alpha$
$D_d \cap D_b$	$(1, 2)(0, 1, 0, -1)$	l_L	$N_\gamma - N''_\gamma$
$D_d \cap D_c$	$(1, 1)(0, 0, 1, -1)$	e_R	$N'_\gamma - N''_\gamma$

correct chiral particle spectrum (families from fluxes)

- compactification $\mathcal{K}$ at most 4-dimensional, “fuzzy”
- no tadpole cancellation condition
- 1-loop effective action

  $\rightarrow$ intersecting branes may form bound state (dep. on flux)

Summary, conclusion

- matrix-models $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$ $+$ fermions

  dynamical NC branes $\leftrightarrow$ emergent gravity & gauge thy

- background independent,
  fluctuations of matrices $\rightarrow$ gauge theory propagating on brane
  all ingredients for physics

- *not* same as G.R., but maybe close enough

  new mechanism (extrinsic geometry, split NC, ...)
  new light on vacuum energy $\leftrightarrow$ gravity ?!

  (flat space is always solution!)

- suitable for quantizing gauge theory & gravity
  (IKKT model, $\mathcal{N} = 4$ SUSY in $D = 4$)

references

- N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, "A Large N reduced model as superstring," Nucl. Phys. B **498** (1997) 467 [hep-th/9612115].
- H. Steinacker, "Emergent Gravity from Noncommutative Gauge Theory". *JHEP* **12** (2007) 049. `[arXiv:0708.2426v1 (hep-th)]`
- H. Steinacker, "Emergent Gravity and Noncommutative Branes from Yang-Mills Matrix Models," *Nucl. Phys.* **B 810**:1-39,2009. arXiv:0806.2032 [hep-th].
- D. N. Blaschke and H. Steinacker, "Curvature and Gravity Actions for Matrix Models," arXiv:1003.4132 [hep-th].*Class. Quant. Grav.* **27**:165010,2010.
- H. Steinacker, "Non-commutative geometry and matrix models," arXiv:1109.5521 [hep-th].
- **H. Steinacker, "Emergent Geometry and Gravity from Matrix Models: an Introduction," arXiv:1003.4134 [hep-th]. topical review, *Class. Quantum Grav.* 27 (2010) 133001.**

Fuzzy torus T^2_N

def. $U = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ & & \ddots & & \\ 0 & & \dots & 0 & 1 \\ 1 & 0 & \dots & & 0 \end{pmatrix}, \quad V = \begin{pmatrix} 1 & & & & \\ & e^{2\pi i \frac{1}{N}} & & & \\ & & e^{2\pi i \frac{2}{N}} & & \\ & & & \ddots & \\ & & & & e^{2\pi i \frac{N-1}{N}} \end{pmatrix}$

satisfy

$$\begin{aligned} UV &= qVU, \qquad U^N = V^N = 1, \qquad q = e^{2\pi i \frac{1}{N}} \\ [U, V] &= (q-1)VU \end{aligned}$$

generate $\mathcal{A} = Mat(N, \mathbb{C})$ ... quantiz. algebra of functions on T^2_N

$\mathbb{Z}_N \times \mathbb{Z}_N$ action:

$$\begin{aligned} \mathbb{Z}_N \times \mathcal{A} &\to \mathcal{A} \\ (\omega^k, \phi) &\mapsto U^k \phi U^{-k} \end{aligned}$$

similar other $\mathbb{Z}_N$

$\mathcal{A} = \oplus_{n,m=0}^{N-1} U^n V^m$ ... harmonics quantization map:

$$\begin{array}{rcl} \mathcal{I}: \quad \mathcal{C}(T^2) & \rightarrow & \mathcal{A} = Mat(N,\mathbb{C}) \\ e^{in\varphi} e^{im\psi} & \mapsto & \begin{cases} U^n V^m, & |n|,|m| < N/2 \\ 0, & \text{otherwise} \end{cases} \end{array}$$

satisfies

$$\begin{aligned} \mathcal{I}(fg) &= \mathcal{I}(f)\mathcal{I}(g) + O(\tfrac{1}{N}), \\ \mathcal{I}(i\{f,g\}) &= [\mathcal{I}(f),\mathcal{I}(g)] + O(\tfrac{1}{N^2}) \end{aligned}$$

Poisson structure $\{e^{i\varphi}, e^{i\psi}\} = \frac{2}{N} e^{i\varphi} e^{i\psi}$ on T^2 $(\Leftrightarrow \{\varphi,\psi\} = -\frac{2}{N})$

integral: $$\frac{4\pi^2}{N}\mathrm{Tr}(\mathcal{I}(f)) = \int\limits_{T^2} \omega f, \qquad \omega = d\varphi d\psi$$

T^2_N ... quantization of $(T^2, N\omega)$

metric on T^2_N ? ... "obvious", need extra structure:

embedding $\quad T^2 \hookrightarrow \mathbb{R}^4$ via $\quad x^1 + ix^2 = e^{i\varphi}, \; x^3 + ix^4 = e^{i\psi}$

quantization of embedding maps $x^a \sim X^a$: 4 hermitian matrices

$$X^1 + iX^2 := U, \qquad X^3 + iX^4 := V$$

satisfy

$$\begin{aligned} [X^1, X^2] &= 0 = [X^3, X^4] \\ (X^1)^2 + (X^2)^2 &= 1 = (X^3)^2 + (X^4)^2 \\ [U, V] &= (q-1)\,VU \end{aligned}$$

Laplace operator:

$$\begin{aligned} \Box\phi &= [X^a, [X^b, \phi]]\delta_{ab} \\ &= [U, [U^\dagger, \phi]] + [V, [V^\dagger, \phi]] = 2\phi - U\phi U^\dagger - U^\dagger\phi U - (\% V) \\ \Box(U^n V^m) &\sim ([n]_q^2 + [m]_q^2)\, U^n V^m \sim (n^2 + m^2)\, U^n V^m \end{aligned}$$

where

$$[n]_q = \frac{q^{n/2} - q^{-n/2}}{q^{1/2} - q^{-1/2}} = \frac{\sin(n\pi/N)}{\sin(\pi/N)} \sim n \qquad \text{("q-number")}$$

metric on T^2_N ? ... "obvious", need extra structure:

embedding $T^2 \hookrightarrow \mathbb{R}^4$ via $x^1 + ix^2 = e^{i\varphi}, \;\; x^3 + ix^4 = e^{i\psi}$

quantization of embedding maps $x^a \sim X^a$: 4 hermitian matrices

$$X^1 + iX^2 := U, \qquad X^3 + iX^4 := V$$

satisfy

$$\begin{aligned} [X^1, X^2] &= 0 = [X^3, X^4] \\ (X^1)^2 + (X^2)^2 &= 1 = (X^3)^2 + (X^4)^2 \\ [U, V] &= (q-1)\,VU \end{aligned}$$

Laplace operator:

$$\begin{aligned} \Box\phi &= [X^a, [X^b, \phi]]\delta_{ab} \\ &= [U, [U^\dagger, \phi]] + [V, [V^\dagger, \phi]] = 2\phi - U\phi U^\dagger - U^\dagger\phi U - (\% V) \\ \Box(U^n V^m) &\sim ([n]_q^2 + [m]_q^2)\, U^n V^m \sim (n^2 + m^2)\, U^n V^m \end{aligned}$$

where

$$[n]_q = \frac{q^{n/2} - q^{-n/2}}{q^{1/2} - q^{-1/2}} = \frac{\sin(n\pi/N)}{\sin(\pi/N)} \sim n \qquad \text{("q-number")}$$

$\boxed{\mathrm{spec}\Box \approx \mathrm{spec}\Delta_{T^2} \qquad \text{below cutoff}}$

therefore:

geometry of (embedded) fuzzy torus $T^2_N \hookrightarrow \mathbb{R}^4$ = flat

momentum space is compactified! $[n]_q$

relation with IIB supergravity

"probe" -brane parallel to stack of $N-$ branes

modeled via $\langle\Phi^i\rangle \sim \mathrm{diag}(1-N,1,\ldots,1) =: \lambda$

$\to$ 1-loop eff. action (D. Blaschke, H.S., 2011; cf. Tseytlin 1999)

$$\begin{aligned}
\Gamma_{1-\mathrm{loop}} &= -\int_{\mathcal{M}} d^4x\, tr\, \frac{e^{-2\sigma}}{(\phi^i\phi_i[\lambda,[\lambda,.]])^2}\Big((\Sigma^{(Y)}_{ab}[\mathcal{F}^{ab},.])^4 - \tfrac{1}{2}(\Sigma^{(\psi)}_{ab}[\mathcal{F}^{ab},.])^4 + \ldots\Big) \\
&= (N-1)\int_{\mathcal{M}} \frac{d^4x}{(\phi^i\phi_i)^2} e^{2\sigma}\Big(-4F_{\mu\nu}F^{\nu\eta}F_{\eta\rho}F^{\rho\mu} + (F_{\mu\nu}F^{\nu\mu} - 2e^{-\sigma}D_\mu\phi_i D^\mu\phi^i)^2 \\
&\quad +16e^{-\sigma}D_\mu\phi_i D_\nu\phi^i F^{\nu\eta}G_{\eta\eta'}F^{\eta'\mu} - 8e^{-2\sigma}D_\mu\phi_i D_\nu\phi^i D^\nu\phi_j D^\mu\phi^j\Big),
\end{aligned}$$

consistent with expansion of Dirac-Born-Infeld action on $AdS^5\times S^5$ ("near-horizon")

$$S_{\mathrm{DBI}} = \int_{\mathcal{M}} d^4x\, e^{-2\sigma}|\phi\cdot\phi|^2\left(\sqrt{\left|\det\left(G_{\mu\nu} + \frac{e^\sigma}{|\phi\cdot\phi|^2}D_\mu\phi^i D_\nu\phi_i + \frac{e^\sigma}{|\phi\cdot\phi|}F_{\mu\nu}\right)\right|} - \sqrt{|\det G|}\right)$$

to $\mathcal{O}(\mathcal{F}^4)$. consistent with supergravity / string theorie