The geometry and the dynamics of branes: towards emergent gravity

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part II: geometry $\leftrightarrow$ matter in the MM

- branes with extrinsic curvature $\rightarrow$ (Newtonian, at least) gravity
- compactified branes $\mathcal{M}^4 \times \mathcal{K} \subset \mathbb{R}^{10}$

  Ricci tensor $\leftrightarrow$ e-m tensor

  extrinsic curvature, moduli of $\mathcal{K} \subset \mathbb{R}^6$ mediate gravity
- might resolve problems with quantization, dispense of "landscape"

IKKT (IIB) matrix model

Ishibashi, Kawai, Kitazawa and Tsuchiya 1996

$$S[X] = -Tr\left([X^a, X^b][X^{a'}, X^{b'}]g_{aa'}g_{bb'} + \bar{\Psi}\gamma_a[X^a, \Psi]\right)$$

$$X^a = X^{a\dagger} \in Mat(N, \mathbb{C}), \qquad a = 0, ..., 9 \quad N \to \infty$$

e.o.m.: $\delta S = 0 \;\Rightarrow\; [X^a, [X^{a'}, X^{b'}]]\eta_{aa'} = 0$

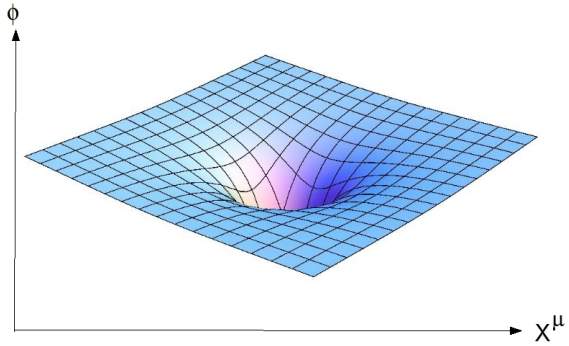
solutions:

- $[X^a, X^b] = i\theta^{ab}\,\mathbf{1}$, "quantum plane" $\mathbb{R}^4_\theta$
- $[X^a, X^b] \sim \{x^a, x^b\} = i\theta^{ab}(x)$, generic quantum space

  $\to$ brane solution

  $X^a \sim x^a : \; \mathcal{M}^4 \hookrightarrow \mathbb{R}^{10}$

- compact extra dim $\mathcal{M}^4 \times T^2$, etc.



semi-classical limit of action:

$$S_{YM} = -Tr[X^a, X^b][X_a, X_b] \sim \int d^4x \sqrt{g}\, e^{-\sigma} G^{\mu\nu} g_{\mu\nu}$$

matrix e.o.m $[X^a, [X^{a'}, X^b]]\eta_{aa'} = 0 \quad \Longleftrightarrow$

$$\boxed{\begin{aligned} \Box_G x^a &= 0, \qquad \text{“minimal surface”} \\ \nabla^\mu(e^\sigma \theta^{-1}_{\mu\nu}) &= e^{-\sigma}\, G_{\rho\nu}\theta^{\rho\mu}\partial_\mu \eta \\ & \qquad\qquad \eta \sim G^{\mu\nu} g_{\mu\nu} \end{aligned}}$$

teaser:

- perturbations of $\theta^{-1}_{\mu\nu} \to \theta^{-1}_{\mu\nu} + F_{\mu\nu}$
  lead to Ricci-flat metrix perturbations $\delta G^{\mu\nu}$ on $\mathbb{R}^4_\theta$

  Rivelles, Phys.Lett. B558 (2003)

- flat space $\mathbb{R}^4_\theta$ is always solution, no fine-tuning

towards (emergent) gravity (brane, not bulk gravity)

possible mechanisms:

- induced gravity (Sakharov) ($\rightarrow$ fine-tuning problems)
- holographic mechanism
  (bulk metric $\rightarrow$ 10D compactification, fine-tuning / anthropics)
- not quantum effect, robust: $\mathcal{M} \subset \mathbb{R}^{10}$, extrinsic curvature
  $\rightarrow$ (Newtonian, at least) gravity H.S, JHEP 0912 (2009)
- realized by compactification $M^4 \times \mathcal{K} \subset \mathbb{R}^{10}$, $\{x^\mu, y^i\} \neq 0$

  (required also for particle physics)

massless moduli of $\mathcal{K}$ $\xrightarrow{\theta^{\mu i}}$ gravitational modes (lin.)

coupling to matter $\xrightarrow{\theta^{\mu i}}$ (Newtonian) gravity

No E-H action needed! robust

H.S., JHEP 1207 (2012) 156; H.S., arXiv:1210.8364

perturbations of the geometry

consider perturbation of background brane $\mathcal{M} \subset \mathbb{R}^D$

$$\begin{aligned} x^a &\to x^a + \delta x^a \\ \delta x^a &= \varepsilon_\alpha(x)\lambda^\alpha x^a + \delta r(x)\, x^a \end{aligned}$$

where λ^α ... generators of $\mathfrak{so}(9,1)$, $\lambda^0 = \mathbf{1}$.

Introduce $SO(9,1)$ currents

$$J^\alpha_\mu = x^a \lambda^\alpha_{ab} \partial_\mu x^b, \qquad \text{can show } \nabla^a[G] J_a = 0$$

metric perturbation

$$\begin{aligned} \delta g_{\mu\nu} &= J^\alpha_\mu \partial_\nu \varepsilon_\alpha + J^\alpha_\nu \partial_\mu \varepsilon_\alpha + 2\delta r\, g_{\mu\nu} \\ \delta G^{\mu\nu} &\sim \Pi^{\mu\nu}_{\eta\rho}\, \theta^{\eta\eta'} \theta^{\rho\rho'} \delta g_{\eta'\rho'} \\ \Pi &= \delta - \tfrac{\gamma_{..}\gamma^{..}}{2(n-1)} \end{aligned}$$

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matrix version: $[X_a, \tilde{J}^a] = 0, \quad \tilde{J}^c = \frac{1}{2}\{\lambda_{ab} X^a, [X^c, X^b]\} \sim i\theta^{\mu\nu}\partial_\mu x^c J^\alpha_\nu,$

Current conservation law with matter:

$$\begin{aligned}\delta S_{\rm YM} + \delta S_{\rm matter} &= \int d^{2n}x \left(\Lambda_0^4 \sqrt{\theta^{-1}}\, G^{\mu\nu}\delta g_{\mu\nu} + \sqrt{G}\, T_{\mu\nu}\, \delta G^{\mu\nu}\right) \\ &= \int d^{2n}x \sqrt{\theta^{-1}}\left(\Lambda_0^4 G^{\mu\nu} + T_{\rho\eta}\, \Pi^{\rho\eta}_{\mu'\nu'}\, \theta^{\mu'\mu}\theta^{\nu'\nu}\right)\delta g_{\mu\nu}\end{aligned}$$

$\rightarrow$ conservation law

$$\boxed{\nabla^\mu[G] J^\alpha_\mu \equiv x\lambda^\alpha \Box_G x = -e^{-\sigma}\Lambda_0^{-4} T_{\rho\eta}\, \Pi^{\rho\eta}_{\mu'\nu'}\, \theta^{\mu'\mu}\theta^{\nu'\nu}\, K^\alpha_{\mu\nu} + \mathcal{O}(J^\alpha),}$$

matter $\rightarrow$ deviation from harmonic embedding
in presence of extrinsic curvature

$$K^\alpha_{\mu\nu} = \frac{1}{2}(\nabla_\mu J^\alpha_\nu + \nabla_\nu J^\alpha_\mu) = x\lambda^\alpha \nabla_\mu \partial_\nu x$$

$\rightarrow$ leads to (Newtonian ...) gravity

H.S., JHEP 0912 (2009); H.S, JHEP 1207 (2012)

curvature of branes

computing $R^\mu_\nu[G]$ from metric not illuminating

<u>better</u>: overcomplete frame: H.S., arXiv:1210.8364

$$\begin{aligned} g_{\mu\nu} &= \kappa_{\alpha\beta}\,\theta^\alpha_\mu\theta^\beta_\nu, \qquad \kappa_{\alpha\beta} = tr\lambda^\alpha\lambda^\beta \\ \theta^\alpha_\mu &= \tfrac{1}{r}\,J^\alpha_\mu, \qquad r^2 = x_a x^a \end{aligned}$$

projector

$$P^{\alpha\beta} = \theta^\alpha_a\theta^\beta_b g^{ab}, \qquad P^\alpha_{\ \beta}\theta^\beta = \theta^\alpha; \qquad P^2 = P$$

view $T^*\mathcal{M} \cong P\mathcal{A}^N$ as projective module over $\mathcal{A} = \mathcal{C}(\mathcal{M})$

$\rightarrow$ Grassmann connection $\nabla = PdP$

$\rightarrow$ Levi-Civita connection $\nabla = PdP + A$ torsion free

Riemann curvature of embedding metric $g_{\mu\nu}$:

$$\begin{aligned} R_{\mu\nu} &= \theta_\mu(dPdP + dA + AA)\theta_\nu \\ R[g]_{\mu\nu} &= r^{-2}\left(\nabla_\rho J^\alpha_\mu\,\nabla_\eta J^\alpha_\nu - g_{\mu\rho}\nabla_\eta J^0_\nu - g_{\nu\eta}\nabla_\rho J^0_\mu\right)dx^\rho dx^\eta \end{aligned}$$

overcomplete frame for effective metric:

$$\begin{aligned} G_{\mu\nu} &= \kappa_{\alpha\beta}\,\Theta^\alpha_\mu \Theta^\beta_\nu = -g_{\mu\eta}\,(\mathcal{J}^2)^\eta{}_\nu, \\ \Theta^\alpha_\mu &= \tfrac{1}{r}\,J^\alpha_\nu\,\mathcal{J}^\nu{}_\mu, \\ \mathcal{J}^c{}_a &= \theta^{cb} G_{ba} = \theta^{-1}_{ab}\, g^{bc} \end{aligned}$$

projector

$$P^{\alpha\beta} = \Theta^\alpha_a \Theta^\beta_b G^{ab} = \theta^\alpha_a \theta^\beta_b g^{ab}$$

Riemann curvature

$$\mathcal{R}_{\mu\nu} = \Theta_\mu (dPdP + dA + AA)\Theta_\nu$$

can determine A for special geometries:

$$\nabla \mathcal{J}^2 = 0 \qquad \Leftrightarrow \ \nabla[g] = \nabla[G]$$

preferred dynamically, at least for vacuum (?!)

for special geometries: $Ric_{\mu\nu}[G] = Ric_{\mu\nu}[g]$

curvature $Ric_{\mu\nu}[G]$ expressed in terms of currents J^α_μ,

recall $\nabla J \sim TK_{\mu\nu}$, need extrinsic curvature !

assume compactified extra dimensions $\mathcal{M}^4 \times \mathcal{K} \subset \mathbb{R}^{10}$, $r_\mathcal{K} \ll r_\mathcal{M}$

$$\boxed{\mathrm{Ric}^{\mu\rho}[G] = -e^{-2\sigma} T_{\nu'\eta'} \Pi^{\nu'\eta'}_{\rho\eta} \mathcal{P}^{\nu\eta;\mu\rho} - e^{-2\sigma}\Lambda_0^4 g_{\nu\eta}\mathcal{P}^{\mu\nu;\rho\eta} + \mathcal{O}(\tfrac{r_\mathcal{K}}{r_\mathcal{M}}) ,}$$

(using $\mathrm{Ric}^{\mu\rho}[G] \sim \nabla^\mu J^\alpha_\mu \nabla_\eta J^\alpha_\nu - \nabla^\eta J^\alpha_\mu \nabla_\eta J^\alpha_\nu$)

coupling

$$\begin{aligned}\mathcal{P}^{\rho\eta;\mu\nu} &= r^{-2}\Lambda_0^{-4}\,\theta^{\rho\rho'}\theta^{\eta\eta'} K_{\rho'\eta'} K^\dagger_{\mu'\nu'}\,\theta^{\mu\mu'}\theta^{\nu\nu'} \\ &= \Lambda_0^{-4}\,\theta^{\rho\rho'}\theta^{\eta\eta'}\theta^{\mu\mu'}\theta^{\nu\nu'}\,(\partial_{\rho'}\partial_{\eta'}x^a\partial_{\mu'}\partial_{\nu'}x_a)\end{aligned}$$

coupling strength

$$G_N \sim \Lambda_0^{-4} r_\mathcal{K}^{-2}$$

note that Poisson structure $\theta^{\mu i}$ connects $\mathcal{M}^4$ and $\mathcal{K}$ ("split NC")

if can find compactification such that $\mathcal{P}^{\rho\eta;\mu\nu}$ Lorentz invariant in 4D

$\rightarrow$ should recover Einstein equations, up to vacuum contributions

notes:

- currents conservation law protected from quantum corrections (symmetry!)
  geometrical modes $\approx$ Goldstone bosons
- coupling to e-m tensor only in presence of extrinsic curvature
  without E-H action! (less fine-tuning ?!)
- naturally realized for compactified extra dime $\mathcal{M}^4 \times \mathcal{K} \subset \mathbb{R}^{10}$
  compactification moduli $\rightarrow$ 4D geometrical modes via $\theta^{\mu i}$
  "split NC" H.S, Prog.Theor.Phys. (2012)
- may also arise for noncompact brane: "gravity bags"
  may lead to non- Ricci-flat vacuum geom (dark matter ??)
  possibly cosmological solutions

split noncommutativity

NC spaces with geometry $\mathcal{M}^{2n} = \mathcal{M}^4 \times \mathcal{K}$ s.t.
NC structure mixes spacetime $\mathcal{M}^4$ with the compact space $\mathcal{K}$

Poisson tensor: $\{x^\mu, y^i\} \neq 0$ i.e. $[x^\mu, y^i] \neq 0$

x^μ on $\mathcal{M}^4$, y^i on $\mathcal{K}$

in particular: $\dim(\mathcal{K}) = 4 \;\Rightarrow\; \mathcal{M}$ may be isotropic! $[x^\mu, x^\nu] = 0$

always assume Π nondeg., $\mathcal{M}^{2n} = \mathcal{M}^4 \times \mathcal{K}$ symplectic space

$\exists$ such solutions of IKKT model (Minkowski!), scales = free moduli

example: fuzzy cylinder $S^1 \times_\xi \mathbb{R}$

Chaichian Demichev Presnajder 1998

3 hermitian matrices X^1, X^2, X^3, define $U = X^1 + iX^2$,

$$\begin{aligned} UU^\dagger &= U^\dagger U = r^2 \\ [U, X^3] &= \xi U, \qquad [U^\dagger, X^3] = -\xi U^\dagger \end{aligned}$$

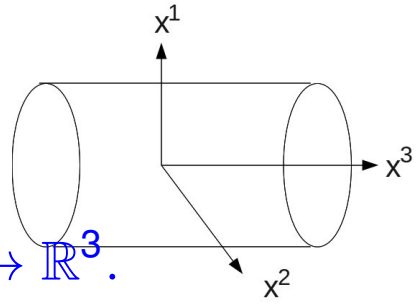
hence

$$(X^1)^2 + (X^2)^2 = r^2, \quad [X^1, X^2] = 0$$

rep. on $\mathcal{H}$:

$$\begin{aligned} U|n\rangle &= r|n+1\rangle, \qquad U^\dagger|n\rangle = r|n-1\rangle \\ X^3|n\rangle &= \xi n|n\rangle, \qquad n \in \mathbb{Z},\ \xi \in \mathbb{R} \end{aligned}$$

interpretation: quantized embedding functions



$$\begin{pmatrix} X^1 + iX^2 \\ X^3 \end{pmatrix} \sim \begin{pmatrix} Re^{iy_3} \\ x^3 \end{pmatrix} : \quad S^1 \times \mathbb{R} \hookrightarrow \mathbb{R}^3.$$

... quantization of $T^\star S^1$

compactified brane solution $\mathbb{R}^4 \times T^2$ of IKKT model

2 fuzzy cylinders (U_2, X^2) and (U_3, X^3),

$$\begin{aligned} [U_2, X^2] &= \kappa_2 U_2, \qquad [U_3, X^3] = \kappa_3 U_3, \\ U_i U_i^\dagger &= r_i^2, \qquad i = 2, 3 \end{aligned}$$

rotate cylinders along $\mathbb{R}^2_\theta$, $\quad [X^\mu, X^\nu] = i\theta^{\mu\nu}$, $\mu = 0, 1$

$$X^a = \begin{pmatrix} X^{0,1} \\ X^2 \\ X^3 \\ X^4 + iX^5 \\ X^6 + iX^7 \end{pmatrix} = \begin{pmatrix} X^\mu \\ U_2\, e^{ik_\mu^{(2)} X^\mu} \\ U_3\, e^{ik_\mu^{(2)} X^\mu} \end{pmatrix}$$

$\mathbb{R}^4 \times T^2$ solution of the matrix e.o.m.

$$\Box X^a = 0 \qquad \text{for} \qquad k_\mu^{(i)} k_\nu^{(i)} \theta^{\mu\mu'} \theta^{\nu\nu'} \eta_{\mu'\nu'} = -\kappa_i^2$$

(provided $[k_\mu^{(2)} X^\mu, k_\nu^{(3)} X^\nu] = 0$)

torus rotating along $\mathbb{R}^4$, stabilized by angular momentum

next we present some speculative explorations of non-trivial (non-Ricci-flat) vacuum solutions or excitations of the bare matrix model.

should be taken with caution since

- above mechanism for gravity in presence of compactified extra dimensions not taken into account
- ad-hoc choice of Wick-rotated Poisson structure, such that $g_{\mu\nu} \sim G_{\mu\nu}$

nevertheless, qualitative features are expected to apply

speculation 1) gravity bags

= vacuum excitations of $\mathcal{M}^4 \subset \mathbb{R}^{10}$, e.o.m. $\Box\phi = 0$

Ansatz: H.S., JHEP 0912 (2009)

$$x^a = \begin{pmatrix} x^0 \\ x^i \\ \phi^i \end{pmatrix} = \begin{pmatrix} t \\ x^i \\ g_0 \frac{\sin(\omega r)}{\omega r} \begin{pmatrix} \cos(\omega t) \\ \sin(\omega t) \end{pmatrix} \end{pmatrix}$$

$\Rightarrow$ static effective metric

$$\begin{aligned} ds^2 &= -(1-\omega^2 g^2)\, dt^2 + (\delta_{ij} + \partial_i g \partial_j g) dx^i dx^j \\ g_{00} &= -(1 + 2U_0(r)) \end{aligned}$$

gravitational potential:

$$U_0(r) = -\tfrac{1}{2}\omega^2 g_0^2 \left(\frac{\sin(\omega r)}{\omega r}\right)^2 \sim -\tfrac{1}{2}\omega^2 \tfrac{1}{r^2}, \qquad r \to \infty$$

gravity bag:

attractive, rapidly decaying gravitational field
effective vacuum energy

Modified gravity in gravity bag

place point mass M at origin

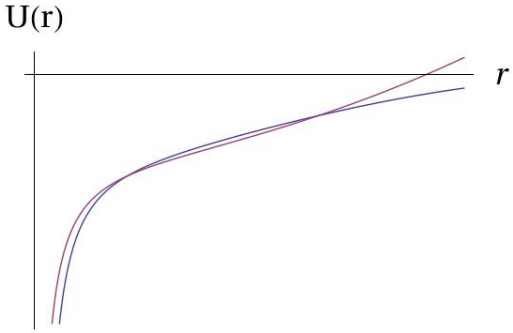
outside of ρ: $\quad g(r) = g_0 \frac{\sin(\omega r+\delta)}{\omega r} \sim g_0(\cos(\delta) + \frac{\sin(\delta)}{\omega r})$

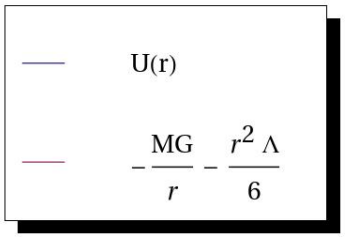
$$\Rightarrow \quad U(x) \sim \omega^2 g^2 \sim \begin{cases} const + \frac{\sin(\delta)}{r} + \ldots, & r \ll \omega^{-1} \\ \sim \frac{1}{r^2}, & r \geq \omega^{-1} \end{cases}$$

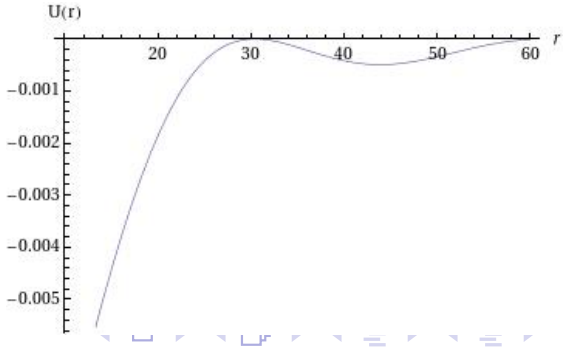
$\delta \sim M$ for small ρ $\Rightarrow$ Newtonian gravity, long-distance screening.

more precisely:

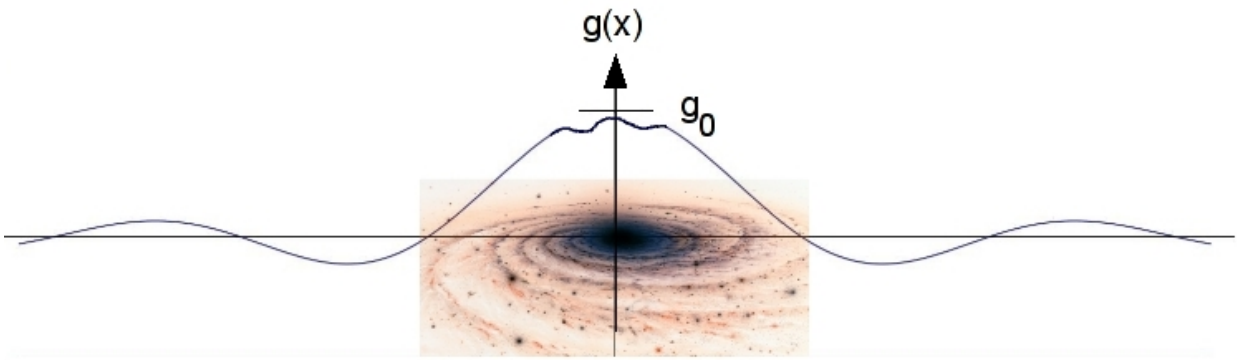
$$\begin{aligned} g_{00} &\approx -\left(1 + 2U_0 - \frac{2GM}{r} - \frac{1}{3}\Lambda_{\text{eff}} r^2\right) \\ \Delta U &= 4\pi G\left(\rho(x) + \frac{\Lambda^4}{8\pi}\right) \\ G &= \frac{2g_0^2\omega^4}{\Lambda^4} \end{aligned}$$







(modified) Newton constant $G = \frac{2g_0^2\omega^4}{\Lambda^4}$ dynamical, determined by large structures



bottom line:

gravity arises or is modified inside gravity bags

natural: initial geom. fluctuations capture matter,
become standing waves, “gravity bags” in cosm. solution

(galactic) rotation curves:

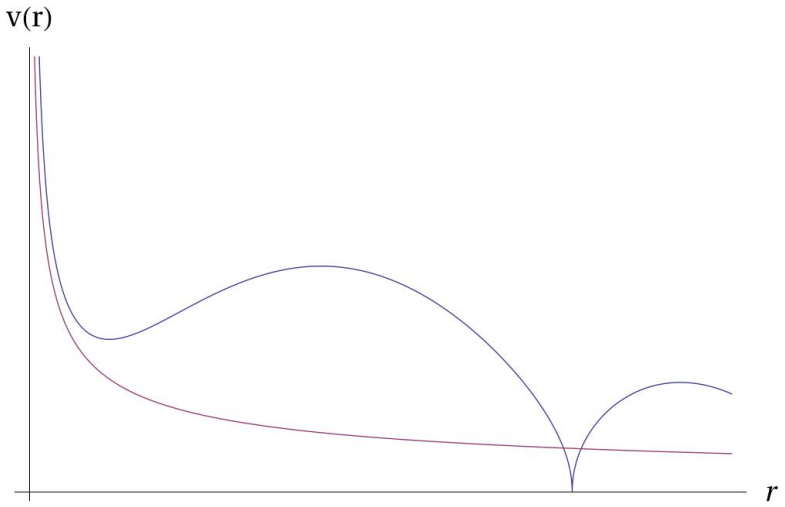
orbital velocities

$$v \approx \sqrt{2\frac{GM}{r}(1+\frac{\pi^2}{3}\frac{r^2}{L_\omega^2}) - \frac{2}{3}\Lambda_{eff} r^2}, \qquad \Lambda_{\rm eff} = -\frac{1}{2}G\Lambda^4 < 0.$$

larger for larger distances, due to vacuum energy & linear term

reminiscent of observations! ($\leftrightarrow$ "dark matter" ?!)

for point mass:



speculation 2): cosmological solution

D. Klammer, H. S., arXiv:0903.0986 (PRL 102 (2009))

<u>assume:</u> vacuum energy $\Lambda^4 \gg$ energy density ρ

$\Rightarrow$ look for harmonic embedding $\Delta x^a = 0$ of FRW metric

$$ds^2 = -dt^2 + a(t)^2(d\chi^2 + \sinh^2(\chi)d\Omega^2),$$

Ansatz

$$x^a(t,\chi,\theta,\varphi) = \begin{pmatrix} a(t)\begin{pmatrix}\cos\psi(t)\\ \sin\psi(t)\end{pmatrix} \otimes \begin{pmatrix}\sinh(\chi)\sin\theta\cos\varphi\\ \sinh(\chi)\sin\theta\sin\varphi\\ \sinh(\chi)\cos\theta\\ \cosh(\chi)\end{pmatrix} \\ 0 \\ x_c(t) \end{pmatrix} \in \mathbb{R}^{10}$$

(cf. B. Nielsen, JGP **4**, (1987))

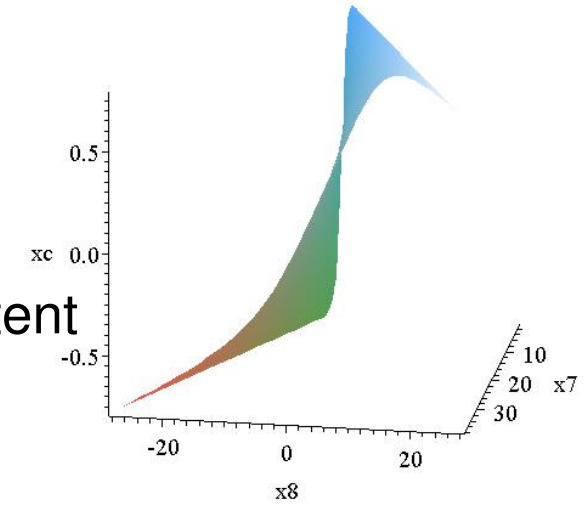
Evolution $a(t), \Psi(t), x_c(t)$ determined by $\Delta x^a = 0$

solution of M.M + leading term $\int d^4x\sqrt{G}\Lambda^4$ in Γ_{1-loop}

harmonic embedding $\Delta_g x^a = 0$ leads to

analog of Friedmann equations

$$\begin{aligned} H^2 = \frac{\dot{a}^2}{a^2} &= -b^2 a^{-10} + d^2 a^{-8} - \frac{k}{a^2}. \\ \frac{\ddot{a}}{a} &= -3d^2 a^{-8} + 4b^2 a^{-10}. \end{aligned}$$



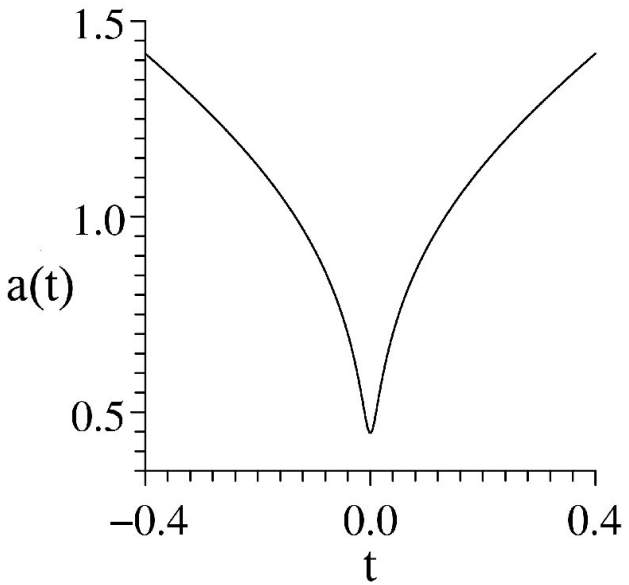
largely independent of detailed matter/energy content as long as $\Lambda^4 \gg \rho$

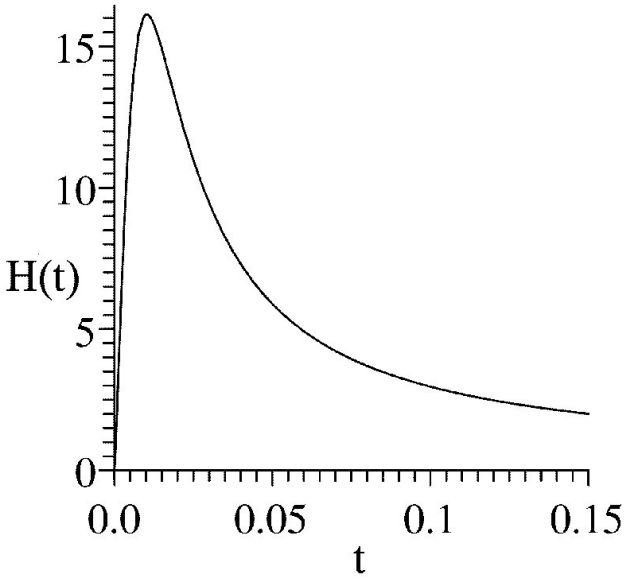
$k = -1$ (negative spatial curvature) most interesting

Implications:

1) early universe:

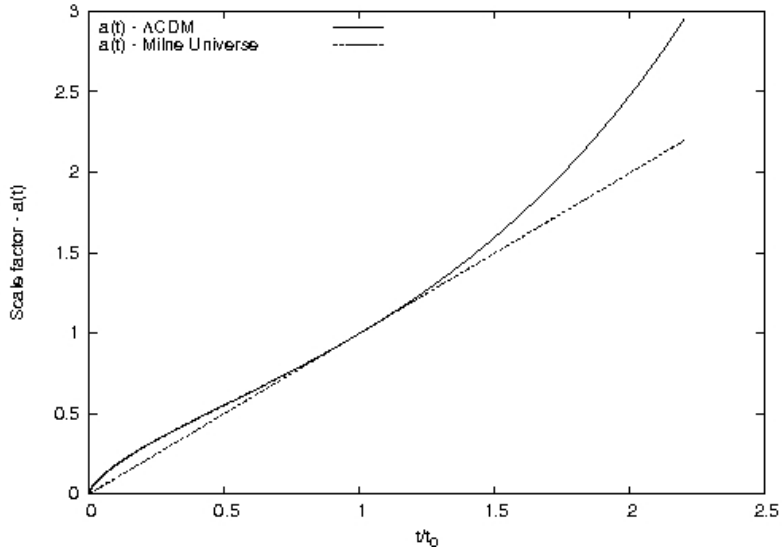
- big bounce: $\dot{a} = 0$ for $a = a_{min} \sim b^{1/4}$
  ($\exists$ bound for energy density ρ vs. vacuum energy Λ^4)
- inflation-like phase $a(t) \sim t^2$, ends at $a(t_{\text{exit}}) = \sqrt{\frac{4}{3}\frac{b}{d}}$
  geometric mechanism (no scalar field required),
  no fine-tuning





2) late evolution (now): $\dot{a} \to 1$

approaches Milne-like universe ($k = -1$, spatial curvature),



in remarkably good agreement with observation
(age $13.8 \cdot 10^9\ yr$, type Ia supernovae)
different physics for early universe (recombination etc.)

A. Benoit-Levy and G. Chardin, [arXiv:0903.2446]

CMB acoustic peak argued to be at correct scale (?)

no fine-tuning of cosm. const., no need for dark energy (?)

Summary, conclusion

- matrix-models $Tr[X^a, X^b][X^{a'}, X^{b'}]\,\eta_{aa'}\eta_{bb'}$ + fermions

  dynamical NC branes $\leftrightarrow$ emergent gravity & gauge thy
- background independent,
  fluctuations of matrices $\rightarrow$ gauge theory propagating on brane
  all ingredients for physics
  related to string theory but predictive !
- *not* same as G.R., but maybe close enough

  new mechanism (extrinsic geometry, split NC, ...)
  new light on vacuum energy $\leftrightarrow$ gravity ?!

  (flat space is always solution!)
- suitable for quantizing gauge theory & gravity
  (IKKT model, $\mathcal{N} = 4$ SUSY in $D = 4$)
- ... more to be discovered!

references

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(almost) generic 4D geometry in M.M. (euclidean) :

1. take some nice $(\mathcal{M}^4, g_{\mu\nu})$ (e.g. asympt. flat, glob. hyperbolic, ...)
2. choose embedding $x^a : \mathcal{M} \hookrightarrow \mathbb{R}^{10}$ (Friedman etal)
3. equip $\mathcal{M}$ with (anti)selfdual symplectic form $\omega = \theta^{-1}_{\mu\nu} dx^\mu \wedge dx^\nu$,
   $\star_g(\omega) = \pm\omega$ (almost-Kähler)
   $\rightarrow$ construct quantization of $(\mathcal{M}, \omega)$:

$$\mathcal{I} : \ \mathcal{C}(\mathcal{M}) \rightarrow \mathcal{A} \cong Mat(\infty, \mathbb{C})$$

   in particular: $X^a \sim x^a$
4. $\rightarrow$ effective metric $G^{\mu\nu} \sim g^{\mu\nu}$, encoded in Δ in M.M.

relation with IIB supergravity

"probe" -brane parallel to stack of $N-$ branes

modeled via $\langle \Phi^i \rangle \sim \mathrm{diag}(1-N, 1, \ldots, 1) =: \lambda$

$\rightarrow$ 1-loop eff. action (D. Blaschke, H.S., 2011; cf. Tseytlin 1999)

$$\begin{aligned}\Gamma_{1-\mathrm{loop}} &= -\int_{\mathcal{M}} d^4x\, tr\, \frac{e^{-2\sigma}}{(\phi^i\phi_i[\lambda,[\lambda,.]])^2}\Big((\Sigma^{(Y)}_{ab}[\mathcal{F}^{ab},.])^4 - \tfrac{1}{2}(\Sigma^{(\psi)}_{ab}[\mathcal{F}^{ab},.])^4 + \ldots\Big) \\ &= (N-1)\int_{\mathcal{M}} \frac{d^4x}{(\phi^i\phi_i)^2} e^{2\sigma}\Big(-4F_{\mu\nu}F^{\nu\eta}F_{\eta\rho}F^{\rho\mu} + (F_{\mu\nu}F^{\nu\mu} - 2e^{-\sigma}D_\mu\phi_i D^\mu\phi^i)^2 \\ &\quad +16e^{-\sigma}D_\mu\phi_i D_\nu\phi^i F^{\nu\eta}G_{\eta\eta'}F^{\eta'\mu} - 8e^{-2\sigma}D_\mu\phi_i D_\nu\phi^i D^\nu\phi_j D^\mu\phi^j\Big),\end{aligned}$$

consistent with expansion of Dirac-Born-Infeld action on $AdS^5 \times S^5$

("near-horizon")

$$S_{\mathrm{DBI}} = \int_{\mathcal{M}} d^4x\, e^{-2\sigma}|\phi\cdot\phi|^2 \left(\sqrt{\left|\det\left(G_{\mu\nu} + \frac{e^\sigma}{|\phi\cdot\phi|^2}D_\mu\phi^i D_\nu\phi_i + \frac{e^\sigma}{|\phi\cdot\phi|}F_{\mu\nu}\right)\right|} - \sqrt{|\det G|}\right)$$

to $\mathcal{O}(\mathcal{F}^4)$. consistent with supergravity / string theory

higher-order terms, curvature

$$
\begin{aligned}
H^{ab} &:= \tfrac{1}{2}[[X^a, X^c], [X^b, X_c]]_+ \\
T^{ab} &:= H^{ab} - \tfrac{1}{4}\eta^{ab} H, \quad H := H^{ab}\eta_{ab} = [X^c, X^d][X_c, X_d]\,, \\
\Box X &:= [X^b, [X_b, X]]
\end{aligned}
$$

result:

for 4-dim. $\mathcal{M} \subset \mathbb{R}^D$ with $g_{\mu\nu} = G_{\mu\nu}$:

$$
Tr\left(2T^{ab}\Box X_a \Box X_b - T^{ab}\Box H_{ab}\right) \sim \frac{2}{(2\pi)^2}\int d^4x\sqrt{g}\, e^{2\sigma} R
$$

$$
Tr([[X^a, X^c], [X_c, X^b]][X_a, X_b] - 2\Box X^a \Box X^a)
$$

$$
\sim \frac{1}{(2\pi)^2}\int d^4x\sqrt{g}\, e^{\sigma}\left(\tfrac{1}{2}e^{-\sigma}\theta^{\mu\eta}\theta^{\rho\alpha}R_{\mu\eta\rho\alpha} - 2R + \partial^\mu\sigma\partial_\mu\sigma\right)
$$

(Blaschke, H.S. arXiv:1003.4132)

(cf. Arnlind, Hoppe, Huisken arXiv:1001.2223)

$\Rightarrow$ Einstein-Hilbert- type action for gravity as matrix model
pre-geometric version of (quantum) gravity, background indep.